Market Risk Management with Stochastic Volatility Models

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1. Introduction

Risk assessment and management have become progressively more important for enterprises in the last few decades. Investors diversify and find financial distress and bankruptcy among enterprises not welcome but expected in their portfolios. Some enterprises do extremely well and keep expected profits (and realised) at a satisfactory level above risk free rates. In contrast, corporations should be run at its shareholders best interest inducing project acceptance with internal rates of return greater than the risk adjusted cost of capital. These considerations are at the heart of modern financial theories. However, often not stressed enough, for the survival of a corporation financial distress and bankruptcy costs can be disastrous for continued operations. Every corporation has an incentive to manage their risks prudently so that the probability of bankruptcy is at a minimum. Risk reduction is costly in terms of the resources required to implement an effective risk-management program. Direct cost are transactions costs buying and selling forwards, futures, options and swaps – and indirect costs in the form of managers' time and expertise. In contrast, reducing the likelihood of financial distress benefits the firm by also reducing the likelihood it will experience the costs associated with this distress. Direct costs of distress include out-of-pocket cash expenses that must be paid to third parties. Indirect costs are contracting costs involving relationship with creditors, suppliers, and employees. For all enterprises, the benefits of hedging must outweigh the cost. Moreover, due to a substantial fixed cost element associated with these risk-management programs, small firms seem less likely to assess risk than large firms[1]. In addition, closely held firms are more likely to assess risk because owners have a greater proportion of their wealth invested in the firm and are less diversified. Similarly, if managers are risk averse or share ownership increases[2], the enterprises are more likely to pursue risk management activities. Stringent actions from regulators, municipal and state ownership and scale ownership (> 10-15%), may therefore force corporations to work even harder to avoid large losses from litigations, business disruptions, employee frauds, losses of main financial institutions, etc. leading to increased probability for financial distress and bankruptcy costs.

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[1] See Booth, Smith, and Stolz (1984), DeMarzo and Duffie (1995), and Nance, Smith and Smithson (1993). The improvements in use of information technology have made it more likely that smaller companies use sophisticated risk-management techniques Moore et al. (2000).

[2] Tufano (1996) finds that risk management activities increase as share ownership by managers increases and activities decreases as option holdings increases (managerial incentives hypotheses).

Energy as all other enterprises must take on risk if they are to survive and prosper. This chapter describes parts of the portfolio of risks a European energy enterprise is currently taking and describes risks it may plan to take in the future. The three main energy market risks to be managed are financial, basis and operational risk. The financial risks are market, credit and liquidity risks. For an energy company selling its production in the European energy market, the most important risk factor is market risk, which is mainly price movement risks in Euro (€). The credit risk is the risk of financial losses due to counterparty defaults. The Enron scandal made companies to review credit policies. Finally, the liquidity risk is market illiquidity which normally is measured by the bid-ask spread in the market. In stressed market conditions the bid –ask spread can become large within a certain time period. The next main risk category for energy companies is basis risk[3] which is risk of losses due to an adverse move or breakdown of expected price differentials. Price differentials may arise due to factors as weather conditions, political developments, physical events or changes in regulations. Some markets operate with area prices that differ from the reference prices and contract for differences (CfD) are established to allow for basis risk management. The last main risk category is operational risk which is divided into legal, operational and tax risks. Legal risks are related to non-enforceable contracts. Operational risk is the risk of loss resulting from inadequate or failed internal processes, people, and systems or from external events. Tax risk can occur when there are changes to taxation regulations. Importantly, all these risks interrelate and affect one another making the use of portfolio risk assessment and management relevant. Basis and operational risk measures contribute to total relevant risk and some of the basis risk is related to market risk (CfDs). In many ways, the key benefit of a risk management program is not the numbers that are produced, but the process that energy companies go through producing the risk related numbers.

Economic capital is defined as the amount of capital an energy corporation needs to absorb losses over a certain time horizon (usually one year) with a certain confidence level. The confidence level depends on the corporation's objectives. Maintaining an AA credit rating implies a one-year probability of default of about 0.03%. The confidence level should therefore be 99.97%. For the measurement of economic capital the bottom up approach is often used. In this method the loss distributions are estimated for different types of risks (market and operational) over different business units and then aggregated. For an energy corporation the loss distributions for market risks can be divided into for example price and volume risk, basis risk into location and time risks and operational risks into business and strategic risks (related to an energy company's decision to enter new markets and develop new products/line of business). A final risk aggregation procedure should produce a probability distribution of total losses for the whole corporation. Using for example copulas, each loss distribution is mapped on a percentile-to-percentile basis to a standard well-behaved distribution. Correlation structures between the standard distributions are defined and this indirectly defines correlation structures between the original distributions. In a Gaussian copula the standard distributions are multivariate normal. An alternative is a multivariate t distribution. The use of the t distribution leads to the joint probability of extreme values of two or more variables being higher than in the Gaussian copula. When

many variables are involved, analysts often use a factor model: $U_i = a_i \cdot F + \sqrt{\left(1 - a_i^2\right)} \cdot Z_i$,

where F and Z have standard normal distributions and Z_i are uncorrelated with each other

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[3] Three components of basis risk: location basis (area supply/demand factors), time basis (grid problems) and some mixed basis issues.

and uncorrelated with *F*. Energy corporations use both risk decomposition and risk aggregation for management purposes. The first approach handles each risk separately using appropriate instruments. The second approach relies on the power of diversification of reducing risks.

The chapter is concerned with the ways market risk can be managed by European enterprises. Several disastrous losses[4] would have been avoided if good risk management practices had been enforced. The current financial crises may have been avoided if risk management had reached a higher understanding at the level of the CEO and board of directors. Normally, corporations should never undertake a trade strategy that they do not understand. If a senior manager in a corporation does not understand a trading strategy proposed by a subordinate, the trade should not be approved. Understanding means instrument valuations. If a corporation does not have the in-house capability to value an instrument, it should not trade it. The risks taken by traders, the models used, and the amount of different types of business done should all be controlled, applying appropriate internal controls. If well handled, the process can sensitize the board of directors, CEOs and others to the importance of market, basis and operational risks and perhaps lead to them thinking about them differently and aggregately.

2. Energy markets, financial market instruments and relevant hedging

The main participants in financial markets are households, enterprises and government agencies. Surplus units provide funds and deficit units obtain funds selling securities, which are certificates representing a claim on the issuer. Every financial market is established to satisfy particular preferences. Money markets facilitate flow of short-term funds, while those that facilitate flow of long-term funds are known as capital markets. Whether referring to money market or capital market securities, the majority of transactions are pertained to secondary markets (trading existing securities) and not primary markets (new issuances). The most important characteristic of secondary markets is liquidity (the degree a security can be liquidated without loss of value). If a market is illiquid, market participants may not be able to find a willing buyer and may have to sell the security at a large discount just to attract a buyer. Finally, we distinguish between organised markets (visible marketplace) and the over-the-counter market (OTC), which is mainly a telecommunication network. All market participants must decide which markets to use to achieve their goals or obtain financing.

Europe's power markets consist of more than half a dozen exchanges, most of which offer trading in both spot, futures and option contracts, giving a dauntingly complex picture of the markets. Moreover, the markets are fragmented along national lines. The commodity itself is impossible to store, at least not on the necessary scale, and is subject to extreme swings in supply and demand. And critical information about such key factors as the level of physical generation is incomplete or not available at all in certain markets. The Nordic market was one of the leaders on electricity liberalization, with Nord Pool becoming Europe's first international power exchange in 1996. Liquidity and volume have grown significantly. Nord Pool trades and clears spot and financially settled futures in Finland,

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[4] Recent examples are Orange County in 1994 (US), Barings Bank (UK) (Zang, 1995), Long-Term Capital Management (Dunbar, 2000), Enron counterparties, and several Norwegian municipals in 2007-2008.

Sweden, Denmark and Norway, listing day and week futures, three seasonal forwards, a yearly forward, contracts for difference and European-style options. Volume in its financial power market in 2009 totaled 2,162 terawatt hours, valued at 68.5 billion euros. Cleared OTC volumes in 2009 reached 942 TWh from 1,140 TWh in 2008. The European Energy Exchange (EEX) in Germany is Europe's fastest growing power futures market. EEX offers trading in physically-settled German and French power futures as well as cash-settled futures based on an index of power prices. On 1st April 2009, the Powernext SA futures activity was entrusted to EEX Power Derivatives AG. The exchange also offers trading in German, Austrian, French and Swiss spot power contracts, emission allowances and coal, and launched trading in natural gas in 2009/2010. On 1st January 2009, Powernext SA transferred its electricity spot market to EPEX Spot SE and on 1st September 2009 EPEX Spot merged with EEX Power Spot. The exchange has more than 160 members from 19 countries, including banks such as Barclays, Deutsche Bank, Lehman Brothers and Merrill Lynch. Eurex owns 23% of the exchange and supplies its trading platform. In 2009 the volume of futures traded on EEX was 1,025 TWh, and the value of futures trading was 61 billion euros. The number of transdactions at the end of 2009 was approximately 114,250. France's Powernext exchange was established in 2002 as a spot market for electricity. Futures trading were launched in 2004 and until 2009 traded physically-settled contracts with maturities from three months to three years. In 2009 the exchange entrusted the futures activity to EEX Power Derivatives AG. Moreover, 1st January 2009, Powernext SA transferred its electricity spot market to EPEX Spot SE and on 1st September 2009 EPEX Spot merged with EEX Power Spot. The transfer of activity was due to the implementation of France's TRTAM "return to tariff" law, which reinstates regulated tariffs for industrial users from EDF, France's main electricity supplier, which limits competition and is seen to distort exchange prices. Liquidity was severely dented and trading volume plunged and open interest sank from around 14 TWh in June 2006 to 11 TWh at the end of 2005. The European Energy Derivatives Exchange (Endex) is funded by financial players and Benelux energy market participants, including Fortis Bank, Endesa and RWE. It incorporates the Endex Futures Exchange, an electronic market for Dutch and Belgian power futures, and Dutch gas futures. Electrabel, Essent and NUON act as liquidity providers. Since the exchange launched in 2004 the major interest has been in Dutch power futures, though Belgian power markets have also grown. Combined, they rose 156% in year one and grew from 327 TWh in 2008 to 412 TWh in 2009. Number of transactions in Dutch power for 2009 was 45,900. In November 2009, the Endex and Nord Pool take the first steps towards a integrated cross-border intra-day electricity market. There are many other markets changing rapidly, or where futures markets may develop. The U.K., for instance, is currently building a new trading model to combat declining liquidity. A considerable amount of spot and forward trading takes place on APX Power UK, but all attempts to create a futures market for U.K. electricity have failed to attract significant volume. Most market participants have relied instead on bilateral contracts traded on the over-the-counter market. The latest initiative is Nord Pool and the N2EX market initiative started in 2009/2010. Volume is still an issue also for this initiative. European markets are moving towards greater physical integration, with more market coupling to increase the efficiency of cross-border interconnectors. Coupling between Denmark and Germany is due, with EEX and Nord Pool party to an existing agreement. Similarly, the 700MW NorNed interconnector links the Dutch APX market with Nord Pool via Norway. The future could well see consolidation among exchanges, particularly as cross-border integration becomes more widespread.

		Power Futures (TWh)		Carbon Trading (tonnes)		Spot Power (TWh)		Cleared OTC power (TWh)	
		2008	2009	2008	2009	2008	2009	2008	2009
Nord Pool	Volume (TWh)	1437	1220	121731	45765	298	286	1140	942
	Transactions	158815	136030	6685	3792	70 %	72 %	51575	40328
EEX	Volume (TWh)	1165	1025	80084	23642	154	203	n/a	n/a
	Transactions	128750	114250	4398	1959	54 %	56 %	n/a	n/a
Powernext	Volume (TWh)	79	87	n/a	n/a	203.7	196.3	n/a	n/a
	Transactions	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
APX/Endex	Volume (TWh)	327	412	n/a	n/a	n/a	n/a	n/a	n/a
	Transactions	36150	45900	n/a	n/a	n/a	n/a	n/a	n/a

* On 1st January 2009, Powernext SA transferred its electricity spot market to EPEX Spot SE and
on 1st September 2009 EEX Power Spot merged with EPEX Spot.
* On 1st April 2009, the Powernext SA futures activity was entrusted to EEX Power Derivatives AG.

Table 1. Volume (TWh) and Number of Transactions for European Power markets

A financial futures contract is a standardised agreement to deliver or receive a specified amount of a specified financial instrument at a specified price and date. The instruments are traded on organised exchanges, which establish and enforce rules of trading. Futures exchanges provide an organised market place where contracts are traded. The marketplaces clear, settle, and guarantee all transactions that occur on their exchange. All exchanges are regulated and all financial future contracts must be approved and regulations imposed before listing, to prevent unfair trading practices. The financial future contracts are traded either to speculate on prices of securities or to hedge existing exposure to security price movements. The obvious function of commodity future markets is to facilitate the reallocation of the exposure to commodity price risk among market participants. However, commodity future prices also play a major informational role for producers, distributors, and consumers of commodities who must decide how much to sell (or consume) now and how much to store for the future. By providing a means to hedge the price risk associated with the storing of a commodity, futures contracts make it possible to separate the decision of whether to physically store a commodity from the decision to have financial exposure to its price changes. For example, suppose it is Wednesday week 9 and a hydro electricity producer has to decide whether to produce his 10 MW maximum capacity of electricity from his water reservoir, which has a normal level for the time of year, next week at an uncertain spot price of S_1 or selling short a future contract to day at F_0^1. By selling the future contract, the producer has obtained complete certainty about the price he will receive for his energy production. Anyone using a future contract to reduce risk is a hedger. But much of the trading of futures contracts are carried on by speculators, who take positions in the market based on their forecasts of the future spot price. Hence, speculators typically gather information to help them forecast prices, and then buy or sell futures contracts based on those forecasts. There are at least two economic purposes served by the speculator. First, commodity speculators who consistently succeed do so by correctly forecasting spot prices and consequently their activity makes future prices better predictors of the direction of change of spot prices. Second, speculators take then opposite site of a hedger's trade when other hedgers cannot readily be found to do so. The activity makes futures markets more liquid than they otherwise would be. Finally, future prices can provide information about investor expectations of spot prices in the future. The reasoning is that the future prices reflects what inspectors expect the spot price to be at the contract delivery date and, therefore, one should be able to retrieve that expected future spot price. Options are broader class securities called contingent claims. A contingent claim is any security whose future

payoff is contingent on the outcome of some uncertain event. Commodity options are traded both on and off organised exchanges all around the world. Therefore, any contract that gives one if the contracting parties the right to buy or sell a commodity at a pre-specified exercise price is an option. European Energy Enterprises are all able to trade these securities on organised exchanges and OTC markets. Traders and portfolio managers use each of the "Greek Letters" or simply the Greeks, to measure a different aspect of the risk in a trading position. Greeks are recalculated daily and exceeded risk limits require immediate actions. Moreover, delta neutrality ($\Delta = 0$) is maintained on a daily basis rebalancing portfolios[5]. To use the delta concept, obtain delta neutrality and managing risks can be shown assuming a electricity market portfolio for company TK AS in Table 2. One way of managing the risk is to revalue the portfolio assuming a small increase in the spot electricity price from €65.27 per MW to €65.37 per MW. Let us assume that the new value of the portfolio is €65395. A €0.1 increase in price decreases the value of the portfolio by €1000.

Portfolio of Electricity Products in Tafjord Kraft book (daily):

	Number of MW (000)	Spot Prices (€)	Value € (000)
Spot position (long normal production):	1000	65.27	65270
Forward contracts			
One Year Forward Contracts	-100	52.5	-5250
One Quarter Forward Contracts	50	68.23	3411.5
Two Quarter Forward Contracts	-200	52.5	-10500
Four Quarter Forward Contracts	150	75.7	11355
One Month Forward Contracts	50	64.55	3227.5
Three Month Forward Contracts	-10	58.25	-582.5
Future Contracts			
One Week Future Contracts	100	67.25	6725
Two Weeks Future Contracts	-50	65.21	-3260.5
Options			
Call One Year Forward Options			-10000
Put One Year Forward Options			5000
Total value of Portfolio Electricity			65396

Table 2. Portfolio of Electricity Products in TK AS trading book (daily)

The sensitivity of the portfolio to the price of electricity is the delta: $\Delta = \dfrac{-1000}{+0.1} = -10000$.

Hence, the portfolio loses (gains) value at a rate of €10000 per €1 increase (decrease) in the spot price of electricity. Elimination of the risk is to buy for example an extra one year (month) forward contract for 10000/8250h (10000/740h) MW. The forward contracts gains (loses) value of €10000 per €1 increase in the electricity price. The other "Greek letter" are the Gamma $\left(\gamma = \dfrac{\partial^2 Portfolio}{\partial S^2} \right)$, Vega $\left(\upsilon = \dfrac{\partial Portfolio}{\partial \sigma} \right)$, Theta $\left(\varsigma = \dfrac{\partial^2 Portfolio}{\partial T} \right)$, and Rho $\left(\rho = \dfrac{\partial^2 Portfolio}{\partial i} \right)$. Corporations in any market must distinguish between market, basis and operational risk. The relevant risk is the market risk and the other risks are those over

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[5] Gamma and Vega neutrality on regular basis is in most cases not feasible.

which the company has control[6] (internal risk). In classical corporate finance textbooks we find the separation theorem (the separation of ownership and management), which defines all relevant risk as the market (external) risk while all other risk (internal) is diversified away building diversified portfolios. Hence, the trade-off between return versus risk (higher expected returns for higher risks) for investors must be separated from risk and return for corporations. For an investor the relevant risk is $\sigma_j \cdot \rho(R_j, R_m)$, which divided by σ_m for scaling purposes, defines the β measure (often interpreted as market sensitivity). Investors are therefore compensated only for market (systematic) risk. All other risks can be diversified away building asset portfolios[7]. For corporations the assumptions of shareholder wealth maximization are imposed. Every investment project with a positive net present value (NPV) discounted with the risk adjusted cost of capital using the Capital Asset Pricing Model ($CAPM$) approach[8], should be accepted. Operational (non-systematic) risk is irrelevant[9]. However, there are two important arguments among more (in an imperfect world) that can be extended to apply for all risks; that is, bankruptcy costs (product reputations, service products, accountants and lawyers) and managerial performance. The bankruptcy costs can be disastrous for a corporation's continued operations. It makes therefore sense for a company that is operating in the best interest of its shareholders to limit the probability of this value destruction occurring. Managerial performance evaluates company performance that can be controlled by the executives in the organisation. Idiosyncratic risks not possible to control by company executives should therefore be controlled. Hence, limiting total risk may be considered a reasonable strategy for a corporation. Many spectacular corporate failures can be traced to CEOs who made large levered acquisitions that did not work out. Corporate survival is therefore an important and legitimate objective, where both financing and investment decisions should be taken so that the possibility of financial distress (bankruptcy costs) is as low as possible. To limit the probability of possible destructive occurrences, energy corporations monitor market risks (mainly the correlated price and volume risks), basis, and operational risk. Even though a corporation manage its Greek letters (delta, gamma, theta and vega) within certain limits, the corporation is not totally risk free. At any given time, an energy corporation will have residual risk exposure to changes in hundreds or even thousands of market variables such as interest rates, exchange rates, equity markets, and other commodity market prices as oil, gas and coal prices. The volatility of one of these market variables measures uncertainty about the future value of the variable. Monitoring volatility to assess potential losses for the corporation is therefore crucial for risk management.

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[6] All internal risks are included as for example the rogue trader risk and the risk of other sorts of employee fraud.

[7] The Arbitrage Pricing Theory (APT) extends the one-factor model (CAPM) to dependence of several factors (Ross, 1976).

[8] The CAPM was simultaneously and independently discovered by Lintner(1965), Mossin (1966), and Sharpe(1964).

[9] Some companies in an investor's portfolio will go bankrupt, but others will do extremely well. The overall result for the investor is satisfactory.

3. Value at risk, expected shortfall, volatility, correlations and copulas

3.1 Value at risk and expected shortfall

Value at Risk (VaR) is an attempt to provide a single number that summarizes the total risk in a portfolio. VaR is calculated from the probability distribution of gains during time T and is equal to minus the gain at the $(100 - X)$th percentile of the distribution. Hence, if the gain from a portfolio during six months is normally distributed with a mean of €1 million and a standard deviation of €2 million, the properties from a normal distribution, the one-percentile point of the distribution is $1 - 2.33 * 2 = €3.66$ million. The VaR for this portfolio with a time horizon of six months and confidence level of 99% is therefore €3.66 million. However, the VaR measure has some incentive problems for traders. A measure with better incentives encouraging diversification (Artzner et al., 1999) is expected shortfall also called conditional VaR (CVaR). As for the VaR, the CVaR is a function of two parameters: T (the time horizon) and X (the confidence interval). That is, the expected loss during time T, conditional on the loss being less than the Xth percentile of the distribution. Hence, if the X = 1%, T is one day, the CVaR is the average amount lost over 1 day assuming that the loss is greater than the 1% percentile. The CVaR measure is a coherent risk measure while the VaR is not coherent.

The marginal VaR/CVaR is the sensitivity of VaR/CVaR to the size of the ith sub-portfolio $\left(\dfrac{\partial VaR}{\partial x_i} \ and \ \dfrac{\partial CVaR}{\partial x_i} \right)$ and is closely related to the capital asset pricing model's beta (β). If a sub-portfolio's beta is high (low), its marginal VaR/CVaR will tend to be high (low). In fact, if the marginal VaR/CVaR is negative, an increase of the weight of a particular sub-portfolio, will reduce overall portfolio risk. Moreover, incremental VaR/CVaR is the incremental effect on VaR/CVaR of the ith sub-portfolio. An approximate formula of the ith sub-portfolio is $\left(\dfrac{\partial VaR}{\partial x_i} x_i \ and \ \dfrac{\partial CVaR}{\partial x_i} x_i \right)$. Finally, using the Euler theorem: $VaR = \sum_{i=1}^{N} \dfrac{\partial VaR}{\partial x_i} x_i$ and $CVaR = \sum_{i=1}^{N} \dfrac{\partial CVaR}{\partial x_i} x_i$ where N is the number of sub-portfolios. The component VaR/CVaR of the ith portfolio is defined as $C_i^{VaR} = \dfrac{\partial VaR}{\partial x_i} x_i$ and $C_i^{CVaR} = \dfrac{\partial CVaR}{\partial x_i} x_i$.

Component VaR/CVaR is often used to allocate the total VaR/CVaR to subportfolios – or even to individual traders.

Back-testing is procedures to test how well the VaR and CVaR measures would have performed in the past and is therefore an important part of a risk management system. Var/ CVaR back-testing is therefore used for reality checks and is normally easier to perform the lower the confidence level. Test statistics for one and two-sided tests have been proposed (Kupiec, 1995). Bunch test statistics (not independently distributed exceptions) are also proposed in the literature (Christoffersen, 1998). Weaknesses in a model can be indicated by percentage of exceptions or to the extent to which exceptions are bunched.

3.2 Volatility, Co-variances/correlations and copulas

Volatility and correlation modelling of financial markets combined with appropriate forecasting techniques are important and wide-ranging topics. Volatility is defined as the

standard deviation of variable i's return $\left(y_{i,t} = \ln\left[\dfrac{P_{i,t}}{P_{i,t-1}} \right] \cdot 100 \right)$ per unit time (t-1, t), where

$P_{i,t}$ is the price of asset i at time t. Relative to time horizons, the uncertainty measured by the standard deviation increases with the square root of time $\left(\sqrt{t} \right)$. There are approximately 252 (trading) days (t) per year. Volatility estimates can normally be obtained from two alternative approaches. The first is directly from the Black & Scholes option pricing formula (1973, 1976) (implied volatility) and the second is to estimate volatility from historical data series and make conditional forecasts. Implied volatility estimates assume an actively traded market for the derivatives and therefore an up-to-date price.

Observing the price in the market, the volatility can be estimated by use of a Newton-Raphson technique. This technique's σ-measure is used extensively by market traders (the vega-measure). However, risk management is largely based on historical volatilities. The

s_i estimate of standard deviation of returns (y_i) is: $s_i = \sqrt{\left[\dfrac{1}{n-1} \cdot \sum_{t=1}^{n}\left(y_{i,t} - \bar{y}_i \right)^2 \right]}$, where $\bar{y}_i$ is

the mean for asset i of the $y_{i,t}$ and n is the number of periods. The s_i variable is therefore an

estimate of $\sigma_i \sqrt{t}$. It follows that σ_i itself can be estimated as $\hat{\sigma}_i$, where $\hat{\sigma}_i = \dfrac{s_i}{\sqrt{t}}$ and the

standard error of this estimate can be shown to be approximately $\dfrac{\hat{\sigma}_i}{\sqrt{2n}}$. A corporation that

has exposure to two different market variables will have gains and losses non-linearly related to the correlation between the changes in the variables. The correlation coefficient (ρ)

between two variables R_1 and R_2, is defined as $\rho_{1,2} = \dfrac{E(R_1 R_2) - E(R_1)E(R_2)}{SD(R_1)SD(R_2)}$, where $E()$

denotes expected value and $SD()$ denotes standard deviation. As the covariance between R_1 and R_2 can be defined as $E(R_1 R_2) - E(R_1)E(R_2)$ the correlation between R_1 and R_2 can be

written as $\rho_{1,2} = \dfrac{\text{cov}(R_1, R_2)}{SD(R_1)SD(R_2)}$. Two variables are defined as statistically independent if

knowledge about one of them does not affect the probability distribution for the other. That is, if $f(R_2 \mid R_1 = y) = f(R_2)$ for all y, where $f()$ is the probability density function. However, a correlation coefficient of zero between two variables does not imply independence. The correlation coefficient measures only linear dependence. There are many other ways in which two variables can be related. For example, for the values of R_1 normally encountered, there is very little relation between R_1 and R_2. However extreme values of R_1 tend to lead to extreme values[10] of R_2. The marginal distribution of R_1 (sometimes also referred to as the unconditional distribution) is its distribution assuming we know nothing about R_2 and vice versa. To define the joint distribution between R_1 and R_2, how can we make an assumption about the correlation structure? If the marginal distributions are normal then the joint

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[10] The quote is: "During a crisis the correlations seem all to go to one"!

distribution of the variables are bivariate normal[11]. In the bivariate normal case a correlation structure can be defined. However, often there is no natural way to define a correlation structure between two variables. It is here copulas come to our rescue. Regardless of probability distribution shapes, copulas are tools providing a way of defining default correlation structures between two or more variables. Copulas therefore have a number of applications in risk assessment and management. Formally, a Gaussian copula can be defined for the cumulative distributions of R_1 and R_2, named F_1 and F_2, by mapping $R_1 = r_1$ to $U_1 = u_1$ and $R_2 = r_2$ to $U_2 = u_2$, where $F_1(r_1) = N(u_1)$ and $F_2(r_2) = N(u_2)$ and N is the cumulative normal distribution function (Cherubini et al., 2004).. This means $u_1 = N^{-1}[F_1(r_1)], u_2 = N^{-1}[F_2(r_2)]$ and $r_1 = F_1^{-1}[N(u_1)], r_2 = F_2^{-1}[N(u_2)]$. The variables U_1 and U_2 are then assumed to be bivariate normally distributed. The key property of a general copula is that it preserves the marginal distribution of R_1 and R_2 while defining a correlation structure between them. In addition to the Gaussian copula we also have the *Student-t copula* (the tail correlation is higher in a bivariate *Student-t*-distribution than that in a bivariate normal distribution). For more than two variables a multivariate Gaussian copula can be used. Alternatively, a factor model for the correlation structure between the U_i can be used:

$U_i = a_i \cdot F + \sqrt{(1 - a_i^2)} \cdot Z_i$ where F and the Z_i have standard normal distributions and the Z_i

are uncorrelated with each other and uncorrelated with F. Other distributions can be used to obtain for example a *Student-t* distribution for U_i (Demarta and McNeil, 2004). Copulas will is this paper be used to apply a simple model for estimating the value at risk on a portfolio of electricity accounts (households/firms) and to value credit derivatives and for the calculation of economic capital.

To illustrate and implement these market risk management concepts for the European energy markets, the Nord Pool and EEX energy markets are quite evolved and liquid markets for energy in Scandinavia and central Europe, respectively. In both markets, prices for energy are established seven days a week for the spot market and from Monday to Friday (not holidays) for the front week/month futures/forwards contracts. Hence, to establish the necessary concepts and define volatilities, co-variances and copulas fir these markets we use the financial EEX and Nord Pool base and peak load prices from Monday to Friday. We use all available prices from Monday to Friday for front week and front month contracts in the two energy markets. The price series are shown in Figure 1 (note the change in currency from NOK to Euro (€) for contracts with physical delivery after December 31st 2005). Prices seem to move randomly over time for both markets and contracts and is clearly non-stationary. The prices seem to show movements similar to other commodity markets and Solibakke (2006) have shown that energy markets seem to exhibit similar features to other markets. The EEX markets show a much higher frequency of price spikes and after adjusting for NOK and Euro differences the EEX market seem to have higher peak prices than the Nord Pool market. Due to the obvious non-stationary prices we calculate the returns in percent (logs) and these return series will be the main objects of our investigations.

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[11] There are many other ways in which two normally distributed variables can be dependent on each other. There are similar assumptions for other marginal distributions.

When distributions from energy market time series are compared with the normal distribution, fatter tails are observed (excess kurtosis). The standardized fourth moment is much higher than the normal distribution postulates[12]. Hence, distributions with heavier tails, such as Paretian and Levy are proposed in the international literature for modelling price changes. Moreover, the time series from energy markets show sometimes too many observations around their mean value and the tails show different characteristics at the negative (left) side relative to the positive (right) side of the distribution. In particular, the spikes at the EEX market may give some positive skewness to the EEX markets price changes.
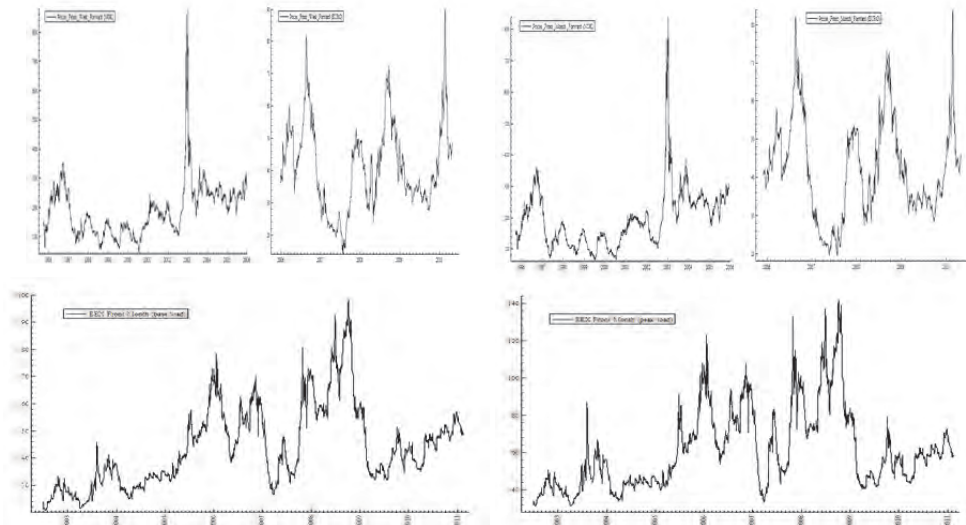
Fig. 1. Price series for Nord Pool and EEX. Nord Pool Front Week and Front Month (base). EEX front Month (base load) and Front Month (peak load).

Uni-variate and bi-variate return characteristics, densities (frequency distribution, normal distribution and the Epanechnikov kernel), volatilities and correlations for the Nord Pool front Week and Month contracts and the EEX Front Month base and peak load contracts are reported in Figure 2. For all the density plots (panel A-D) we distinguish three main arguments: the middle, the tails, and the intermediate parts (between the middle and the tails). When moving from a normal distribution to the heavy-tailed distribution, probability mass shifts from the intermediate parts of the distribution to the tails and the middle. As a consequence, small and large changes in a variable are more likely than they would be if a normal distribution were assumed. Intermediate changes are less likely. The QQ-plots confirm this non-normal story for all return distributions. The contract volatilities (panel E-F) show clearly different shapes between Nord Pool and EEX. However, the products within the same market show similar volatility patterns. The asymmetry (panel G-H) is much clearer at EEX than at Nord Pool. In particular, the EEX market seems to exhibit much more

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[12] See the first studies of this feature: Mandelbrot (1963) and Fama (1963, 1965).
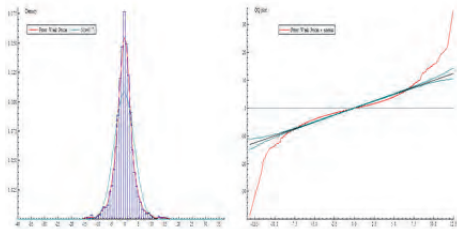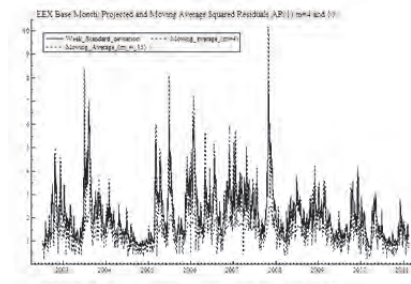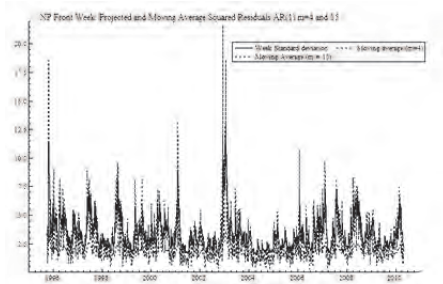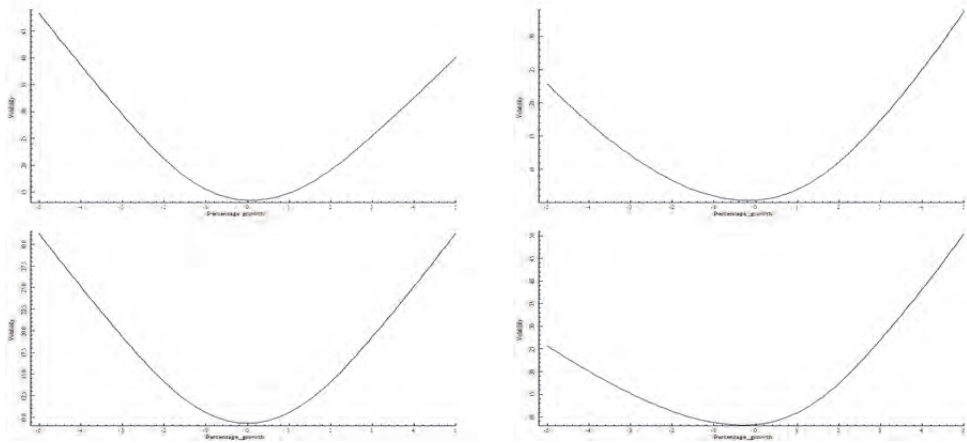
A: Nord Pool Front Week
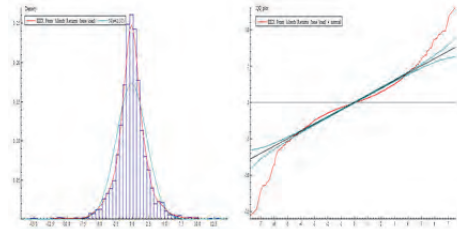
B: EEX Front Month (base load)
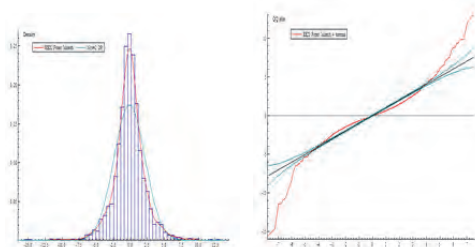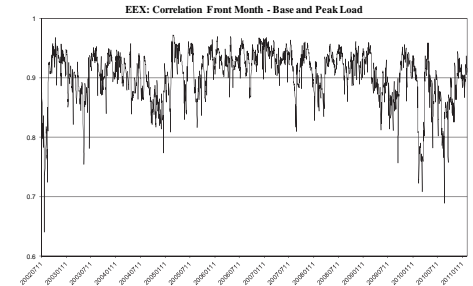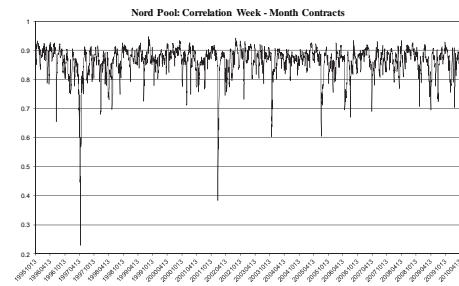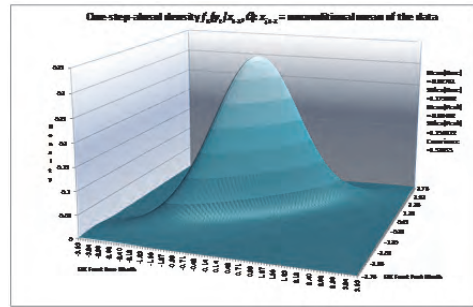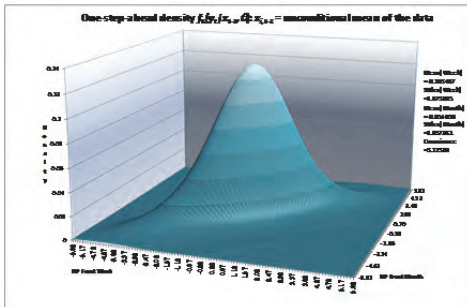

C: Nord Pool Front Month (base load)
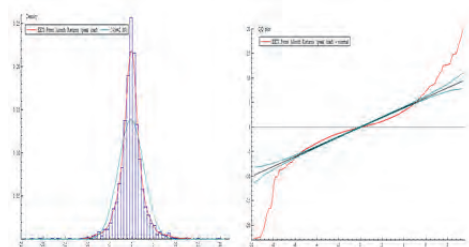
D: EEX Front Month Peak Load
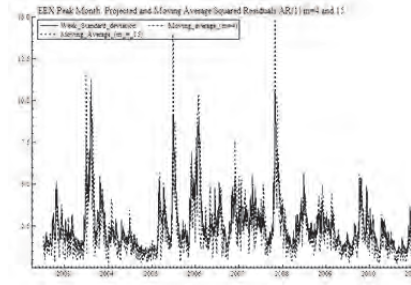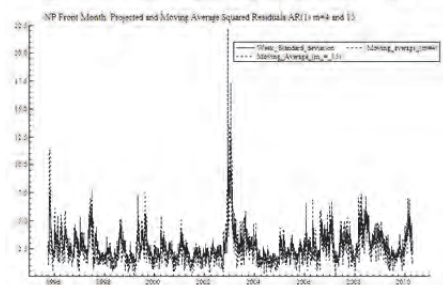
E: Nord Pool volatility clustering (conditional volatility)

F: EEX volatility clustering (conditional volatility)

G: Nord Pool Asymmetry Measures (conditional volatility)   H: EEX Asymmetry Measures (conditional volatility)

I: Nord Pool: Bivaraite Week-Month Density and Correlation   J: EEX: Bivariate Months Density and Correlation

Fig. 2. Characteristics of Nord Pool and EEX Front Week/Month Forward/Future Contracts
Nord Pool (www.nordpool.no) and *EEX* (www.eex.de)

positive asymmetry, that is – higher volatility from positive than negative price changes. In contrast, the Nord Pool week future contract report a low but significant negative asymmetry, in line with equity markets where the asymmetry is well known under "the leverage effect". Finally, panels I-J in Figure 2 report the bi-variate relationships in the Nord Pool and EEX markets. The distributions for the two markets show similar densities but

clearly different mean and standard deviations. The correlations seem at a higher level in the Nord Pool bi-variate front week and month contracts relative to the EEX bi-variate front month base and peak load contracts. However, in some time periods the correlations are as low as 0.23/4 for the Nord Pool market. Generally, the correlation seems high between financial instruments within the two energy markets.

The densities for the energy markets returns suggest heavy tail distributions that have relative to the normal distribution, more probability mass in the tails and in the middle, and less mass in the intermediate parts of the distribution. That is, small and large price changes are more likely and intermediate changes are less likely, relative to a normal distribution. An alternative to the normal distribution is the power law. The power law asserts that it is approximately true that the value υ of a variable has the property that, when x is large $\mathsf{Prob}(\upsilon > x) = K \cdot x^{-\alpha}$ where K and α are constants. The extreme Power Law has been found to be approximately true for variables at many and diverse applications. The equation is useful when we use extreme value theory for risk management purposes and is valuable for VaR and CVaR calculations. Extreme value theory can be used to improve *VaR* estimates and to deal with situations where the *VaR* confidence level is very high. The theory provides a way of smoothing and extrapolating the tails of an empirical distribution.

Gnedenko (1943) stated that, for a wide range of cumulative distributions *F(x)*, the distribution of $F_u(y) = \dfrac{F(u+y) - F(u)}{1 - F(u)}$ converges to a generalised Pareto distribution as the threshold u is increased. The generalised Pareto distribution is defined with the formula

$$G_{\xi,\beta}(y) = 1 - \left(1 + \xi \frac{y}{\beta}\right)^{-\frac{1}{\xi}-1}$$. The distribution has two parameters that have to be estimated from the data set (ξ, β). The ξ parameter is the shape parameter and determines the heaviness of the tail of the distribution (a normal distribution has $\xi = 0$). The parameter β is the scale parameter. Estimating ξ and β can be done with maximum likelihood methods. We first differentiate the cumulative distribution function with respect to y and obtain the

probability density function $g_{\xi,\beta}(y) = \dfrac{1}{\beta}\left(1 + \xi \frac{y}{\beta}\right)^{-\frac{1}{\xi}-1}$. We choose first u close to the 95% percentile point of the empirical distribution. The focus is for observations x > u. We now assume that there are n_u such observations and they are υ_i $(1 \le i \le n_u)$. The likelihood

function becomes: $\displaystyle\prod_{t=1}^{n_u} \dfrac{1}{\beta}\left(1 + \dfrac{\xi(\upsilon_i - u)}{\beta}\right)^{-\frac{1}{\xi}-1}$. Finally maximize its logarithm:

$\displaystyle\sum_{t=1}^{n_u} \ln\left[\dfrac{1}{\beta}\left(1 + \dfrac{\xi(\upsilon_i - u)}{\beta}\right)^{-\frac{1}{\xi}-1}\right]$. The probability that $\upsilon > u + y$ conditional on $\upsilon > u$ is

$1 - G_{\xi,\beta}(y)$. The probability that $\upsilon > u$ is 1 – *F(u)*. The unconditional probability that $\upsilon > x \,(x > u)$ is now $\left[1 - F(u)\right]\left[1 - G_{\xi,\beta}(x - u)\right]$. If n is the total number of observations, an

estimate of $[1-F(u)]$ calculated from the empirical data is $\dfrac{n_u}{n}$. The unconditional

probability that $\upsilon > x$ is therefore $Prob(\upsilon > x) = \dfrac{n_u}{n}\left[1 - G_{\xi,\beta}(x-u)\right] = \dfrac{n_u}{n}\left[1 + \xi\dfrac{(x-u)}{\beta}\right]^{-\frac{1}{\xi}}$.

For the equivalence to the power law, set $u = \dfrac{\beta}{\xi}$ and the equation reduces to

$Prob(\upsilon > x) = \dfrac{n_u}{n}\left[\xi\dfrac{x}{\beta}\right]^{-\frac{1}{\xi}}$ so that the probability of the variable being greater than x is $Kx^{-\alpha}$

(the power law) where $K = \dfrac{n_u}{n}\left[\dfrac{\xi}{\beta}\right]^{-\frac{1}{\xi}}$ and $\alpha = \dfrac{1}{\xi}$, implying that the $Prob(\upsilon > x)$ is consistent

with the power law. To calculate the VaR with a confidence level of q it is necessary to solve

the equation: $F(VaR) = q$. We now use $q = 1 - \dfrac{n_u}{n}\left[1 + \xi\dfrac{VaR - u}{\beta}\right]^{-\frac{1}{\xi}}$ so that

$VaR = u + \dfrac{\beta}{\xi}\left[\dfrac{n}{n_u}\cdot(1-q)^{-\xi} - 1\right]$. Finally, the expected shortfall[13] (CVaR) becomes

$CVaR = \left[\dfrac{VaR + \beta - \xi u}{1 - \xi}\right]$.
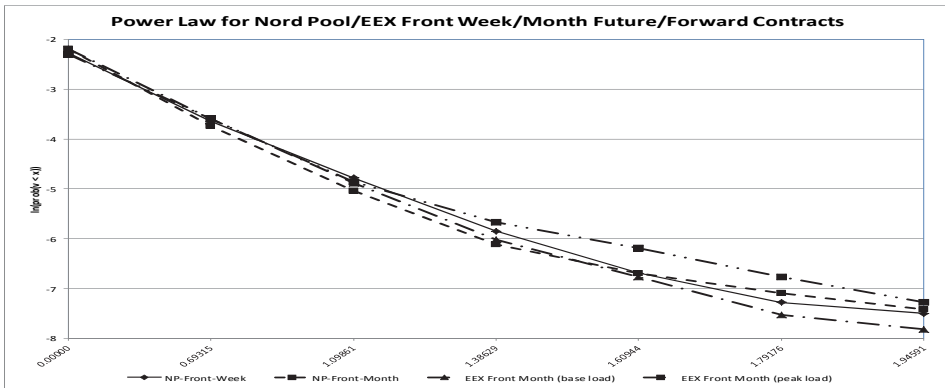
Fig. 3. The Power Law: Log plot for Electricity price increases: x is the number of standard deviations; υ is the electricity price increase/decrease4. Stochastic volatility and risk assessment/management

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[13] The choice of u does not influence the estimate of $Prob(\upsilon > x)$ much. u should be approximately equal to the 95th percentile of the empirical distribution.

A test of whether the power law[14] holds for the energy markets is to plot $\ln\big[\text{Prob}(\upsilon > x)\big]$ against $\ln x$. For the time series from the energy markets Nord Pool and EEX, define x as the number of standard deviations by which electricity prices decreases in one day. Figure 3 shows that the logarithm of the probability of the electricity price decreasing by more than x standard deviations is approximately linearly dependent on $\ln x$ for $x > 3$. The power law therefore seems to hold for energy market applications and we can therefore apply the extreme value theory for VaR and CVaR calculations.

4. Stochastic volatility and risk assessment/management

4.1 The stochastic volatility model

The model building approach implies a need for a scientific model for the mean and volatility using the MCMC (Markov Chained Monte Carlo) methodology to generate distributions for $y=\delta P$. A stochastic volatility (SV) model provide alternative models and methodologies to EWMA and (G)ARCH models. SV models specify a process for volatility and in the form used by Gallant et al. (1997) is formulated as:

$$y_t = a_0 + a_1\left(y_{t-1} - a_0\right) + \exp(v_{1t} + v_{2t}) \cdot u_{1t}$$
$$v_{1t} = b_0 + b_1\left(v_{1,t-1} - b_0\right) + u_{2t}$$
$$v_{2t} = c_0 + c_1\left(v_{2,t-1} - c_0\right) + u_{3t}$$
$$u_{1t} = z_{1t}$$
$$u_{2t} = s_1\left(r_1 \cdot z_{1t} + \sqrt{1 - r_1^2} \cdot z_{2t}\right)$$
$$u_{3t} = s_2\left(\frac{r_2 \cdot z_{1t} + \left((r_3 - (r_2 \cdot r_1))/\sqrt{1-r_1^2}\right) \cdot z_{2t} +}{\sqrt{1 - r_2^2 - \left((r_3 - (r_2 \cdot r_1))/\sqrt{1-r_1^2}\right)^2} \cdot z_{3t}}\right)$$

where z_{it}, $i = 1, 2 \ and \ 3$ are standard Gaussian random variables. The parameter vector is $\rho = (a_0, a_1, b_0, b_1, s_1, c_0, c_1, s_2, r_1, r_2, r_3)$. The r_i's are correlation coefficients from a Cholesky decomposition; enforcing an internally consistent variance/covariance matrix. Early references are Rosenberg (1972), Clark (1973) and Taylor (1982) and Tauchen and Pitts (1983). More recent references are Gallant, Hsieh, and Tauchen (1991, 1997), Andersen (1994), and Durham (2003), see Shephard (2004) and Taylor (2005) for more background and references. The model has three stochastic factor and extensions to four and more factors can be easily implemented through the model setup. The inclusion of a Poisson distribution to model jumps with the use of intensities, are applicable. Long memory can be formulated. The long-memory stochastic volatility model can be described as $u_{1t} = \left(1 - L\right)^{-d} \cdot z_{\omega t}$ and

$z_{\omega t} = \sum_{j=1}^{L_\omega} a_j \cdot z_{\omega t-j} + z_{1t}$, valid for $|d| < 1/2$, as described by Sowell (1990). Other extensions

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[14] The power law can be rewritten as: $\ln\big[\text{Prob}(\upsilon > x)\big] = \ln K - \alpha \ln x$ very useful for regressions and the observing the possibility of empirically estimating $\ln K$ and α when the measure $\ln[\text{Prob}(\upsilon > x)]$ can be calculated.

of stochastic volatility models for better data fit are possible. Splines and t-errors have for example been applied (Gallant and Tauchen, 1997). Liquid financial market normally reports a much better model fit introducing three (or more) stochastic factors. The applicable extensions will be called upon when needed.

Note that writing the variance rate (volatility) as: $\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} y_{i-1}^2$ where y_i^2 is observation $i's$

squared return, is a particularly simple model for updating volatility estimates over time. The Exponentially Weighted Moving Average (EWMA) model, where weights α_i decrease exponentially as we move back through time ($\alpha_{i+1} = \lambda \cdot \alpha_i, 0 \le \lambda \le 1$) is such a simple model. The formula becomes[15]: $\sigma_i^2 = \lambda \cdot \sigma_{i-1}^2 + (1-\lambda) \cdot y_{i-1}^2$ and can relatively easy be implemented by using for example the Excel spreadsheet and the Solver routine. Adding a constant term to this equation establish the (G)ARCH (generalised autoregressive conditional hetero-scedastic) model. However, the number of EWMA/GARCH model reports/papers and the simple fact that both methodologies have limited theoretical justifications, the chapter will focus exclusively on the scientific SV model implementation for the Nord Pool and EEX energy markets. In fact, it is only the SV-model estimation and simulation that makes a bi-variate Nord Pool – EEX market density estimation possible. The SV-model implementation use the computational methodology proposed by Gallant and McCulloch (2010) for statistical analysis of a stochastic volatility model derived from a scientific process. The scientific stochastic volatility model cannot generate likelihoods (latent variables) but it can be easily simulated. The VaR can now be calculated as the appropriate percentile of the distribution. The one-day 99.9% VaR for a 100 k simulation ΔP series is the value for the 100th-worst outcome. The 99.9% CVaR measure is the average of observations below the 99.9% percentile; that is, the average of the 100 observations.

4.2 The Nord Pool and EEX front week/month stochastic volatility models

The $y_{i,t}^{3644}$ ($i = NP\ Front\ Week\ /\ Month$) and the $y_{i,t}^{2189}$ ($i = EEX\ Front\ Month\ Base\ /\ Peak\ Load$) is the percentage change (logarithmic) over a short time interval (day) of the price of a financial asset traded on an active speculative market. The SV model implementation established a mapping between a statistical model and a scientific model and the adjustment for actual number of observations and number of simulation must be carefully logged for final model assessment. For the SV model implementation reasonable starting values are important. The implementation of the scientific model is a lengthy sequential process which is finalized with a 25 CPU parallel computing run applying the Open-message passing interface[16] (Open-MPI).

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[15] To understand why this equation corresponds to weights that decrease exponentially, substitute σ_{i-1}^2

with $\lambda \cdot \sigma_{i-2}^2 + (1-\lambda) \cdot u_{i-2}^2$. The substitution produce: $\sigma_i^2 = (1-\lambda) \cdot \sum_{j=1}^{m} \left(\lambda^{j-1} \cdot u_{i-j}^2\right) + \lambda^m \cdot \sigma_{i-m}^2$. For large

m the last term $\lambda^m \cdot \sigma_{i-m}^2$ is small enough to be ignored.

[16] Open-MPI is a high-performance, freely available, open source implementation of the MPI standard that is researched, developed, and maintained at the Open System Lab at Indiana University (www.open-mpi.org).

SV model extensions are condition specific. The extensions are analysed from both the score model ($f_k()$) and from characteristics of the EMM implementation. The $f_k()$ indicates the starting values and active SV model parameters for the EMM estimation. The normalised scores quasi t-statistics indicate score failures and need for SV model extensions. Finally, the Bayesian log posterior χ^2 test statistic and the Epanechnikov kernel density plots of parameters and functional statistics (stats) assesses SV model optimality or fit. These optimization routines together with an associated 25 iterative run for a comprehensive model assessments, establish the empirical foundation of the Bayesian MCMC estimation reports. The implementation of the 3x8-/2x12-core CPUs generates 240,000 simulated paths for the stochastic volatility model. The Bayesian MCMC M-H algorithm α^* optimal model from the 24-core CPU parallel run model is reported in Table 3. The mode, mean and standard errors are reported for the four series. For all models the optimal Bayesian log posterior value is reported together with the χ^2 test statistic. Moreover, all the score diagnostics (not reported) are all well below 2.0 in value[17]. The first important observation from Table 3 is the four χ^2(df) rejection statistics for the multifactor SV models. None of the SV models are rejected at the 5% significance level. Moreover, the model diagnostics do not identify score moments that are rejected (> 2). The SV models are therefore found accepted for extended commodity market analyses. Table 3 suggests some important differences between Nord Pool and EEX. The Nord Pool week contracts show the largest negative drift, inducing a positive risk premium that is traded the last week before contract maturity. The three other monthly forward products show all lower but negative drift. The volatility seems highest for the Nord Pool week contracts (which also have the shortest time to maturity)[18]. Finally, the analysis shows interesting mean – volatility correlation structures for the EEX market. The asymmetry is found for both volatility factors. The first factor report a positive asymmetry (largest) and the second volatility factor reports a negative factor. From the initial plots in Figure 2, the positive factor seems to dominate asymmetry for EEX. For the Nord Pool the correlation structure seems close to zero and insignificant. That is, asymmetry and non-linearity seems higher for the EEX market than for Nord Pool, which is close to negligible.

The multi-equation SV model reported in Table 3 can now be easily simulated at any length. First, Figure 4 reports plot of standard deviation versus returns for the original series with 3644 observations for Nord Pool (left: panel A and B) and 2189 observations for EEX (right: panel C and D) in the upper part of the figures and a simulated series with 100 k observations right below. From these plots we can find signs of positive volatility asymmetry for the EEX market, while Nord Pool shows little or no volatility asymmetry. However, the standard deviations over time (t) seem quite symmetric around negative and positive returns for all contracts. The asymmetry coefficients in Table 3, where we find that Nord Pool shows close to zero and insignificant asymmetry while the EEX market reports significant and positive asymmetry.

In particular, note that relative to the negative asymmetry found for equity markets the asymmetry for the EEX energy market is positive. The positive asymmetry can be explained by production/grid capacity constraints. Figure 5 shows volatility scatter plots which are

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[17] The standard errors are biased upwards (Newey, 1985 and Tauchen, 1985) so the quasi t-ratios are downward biased relative to 2.0. Hence, a quasi-*t*-statistic above 2.0 indicates failure to fit the corresponding score.

[18] See Samuelson (1965) for the volatility hypothesis.

plots of $y_t - y_{t-1}$ versus y_{t-1}. The raw data (Nord Pool: 3644 and EEX: 2189 points) are plotted in the upper part and a simulated data set (100 k points) is plotted in the lower part of each plot. Interestingly, the SV specification seems to mimic the general characteristic of the raw time series.

Front Week Contract Scientific Model. Parallell Run

Parameter values Scientific Model.

θ	Mode	Mean	Standard error
a_0	-0.3445300	-0.3453300	0.0363680
a_1	0.1609800	0.1612400	0.0115440
b_0	0.9583000	0.9454000	0.0465370
b_1			
c_1	0.9672900	0.9648300	0.0052904
s_1	0.3292400	0.3242200	0.0180660
s_2	0.1114500	0.1140200	0.0085650
r_1	0.0339180	0.0364510	0.0219700
r_2			

log sci_mod_prior	3.5624832	$\chi^2(6)$
log stat_mod_prior	0	-3.32910
log stat_mod_likelihood	-4397.58339	{0.13111}
log sci_mod_posterior	-4394.02091	

Front Month Contract Scientific Model. Parallell Run

Parameter values Scientific Model.

θ	Mode	Mean	Standard error
a_0	-0.0988820	-0.1009600	0.0222770
a_1	0.1534000	0.1518500	0.0154420
b_0	0.2070900	0.2071800	0.0344310
b_1	0.9567500	0.9570600	0.0061345
c_1			
s_1	0.1167100	0.1169700	0.0084579
s_2	0.1366500	0.1366500	0.0329160
r_1	0.4152200	0.4163500	0.0920760
r_2	-0.2458700	-0.2458700	0.0961530

log sci_mod_prior	4.5115377	$\chi^2(7)$
log stat_mod_prior	0	-10.26600
log stat_mod_likelihood	-1907.22335	{0.05298}
log sci_mod_posterior	-1902.71181	

Front Month Contract Scientific Model. Parallell Run

Parameter values Scientific Model.

θ	Mode	Mean	Standard error
a_0	-0.1179100	-0.1085800	0.0299480
a_1	0.1038300	0.1127900	0.0150280
b_0	0.8209700	0.8358000	0.0226330
b_1	0.7949800	0.7997200	0.0068112
c_1			
s_1	0.2316000	0.2303400	0.0024430
s_2			
r_1			
r_2			

log sci_mod_prior	4.7847347	$\chi^2(6)$
log stat_mod_prior	0	-3.51990
log stat_mod_likelihood	-4488.39850	{0.13323}
log sci_mod_posterior	-4483.61377	

Front Month Contract Scientific Model. Parallell Run

Parameter values Scientific Model.

θ	Mode	Mean	Standard deviation
a_0	-0.1490200	-0.1461100	0.0296170
a_1	0.1505900	0.1488200	0.0153380
b_0	0.4335400	0.4269000	0.0310010
b_1	0.9604900	0.9570500	0.0062079
c_1			
s_1	0.1273000	0.1322500	0.0086580
s_2	0.2673400	0.2560800	0.0245790
r_1	0.5503200	0.5346100	0.0772270
r_2	-0.2647600	-0.2786900	0.0522500

log sci_mod_prior	5.1621327	$\chi^2(7)$
log stat_mod_prior	0	-5.67350
log stat_mod_likelihood	-1673.34850	{0.11953}
log sci_mod_posterior	-1668.18637	

Table 3. Scientific Stochastic Volatility Characteristics for Nord Pool/EEX: the θ-parameters

The mean and variance results for the Nord Pool and EEX energy market contracts are summarised below. The Nord Pool week future contracts show a negative daily mean of -0.323 inducing a yearly negative drift of -81.4% (-0.323 * 252 days). That is, a strategy of selling futures Friday the week before maturity and buying back/closing out the last day of trading/ at maturity seem to be a very profitable strategy. The high negative drift (risk premium) suggests a high yearly return. However, the volatility measured by the daily standard deviation is 3.49% indicating a yearly volatility of 55.44%. The Nord Pool one-month forward contracts have a mean daily drift of -0.134% (-33.85% per year). The volatility measured by the daily standard deviation is 2.61% indicating a yearly volatility of 41.5%. Generally, both the mean and standard deviation numbers from these Nord Pool contracts are high for financial markets. The drift numbers for the EEX contracts are for the front month base (peak) -0.089 (-0.168) inducing a yearly negative drift of -22.36% (-42.22%). The EEX base (peak) month volatility measured by the daily standard deviation is 1.48% (2.04%) indicating a yearly volatility of 23.52% (32.41%).

A: Nord Pool Std Deviation vrs Returns Week

C: EEX Std Deviation vrs Returns Month (base)

B: Nord Pool Std Deviation vrs Returns Month

D: EEX Std Deviation vrs Returns Month (peak)
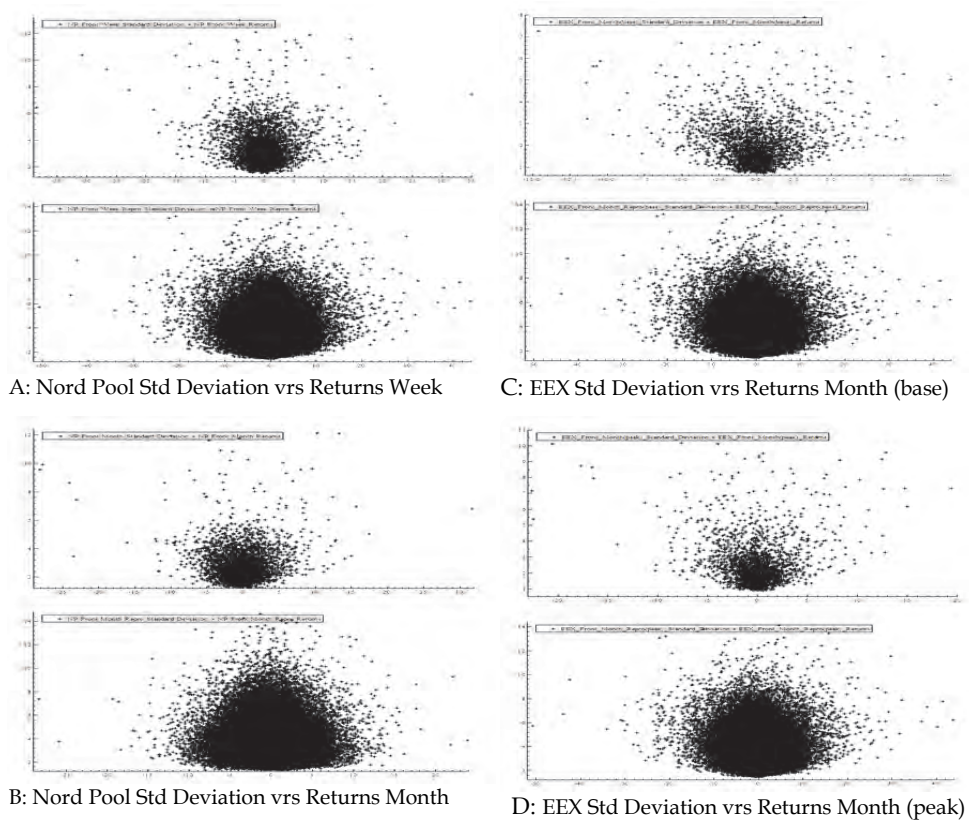
Fig. 4. Nord Pool and EEX Standard deviations versus Returns.
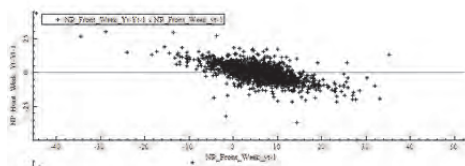
A: Nord Pool Front Week Yt-1-Yt vrs Yt-1   C: EEX Front Month (base load) Yt-1-Yt vrs Yt-1
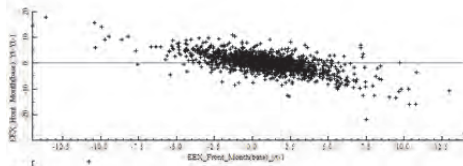
B: Nord Pool Front Month Yt-1-Yt vrs Yt-1   D: EEX Front Month (peak load) Yt-1-Yt vrs Yt-1
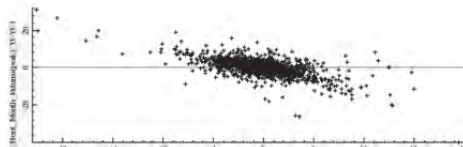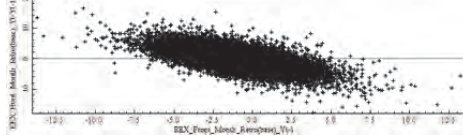
Fig. 5. Nord Pool and EEX Return differences $y_t - y_{t-1}$ versus Returns y_{t-1}.

A: Mean Simulations (100 k)


B: Exponential Volatility Simulations (100 k)


C: Volatility Factor Simulations (100 k)


D: Subsamples Volatility Factor Simulations (100 k)

E: Distributional Density Characteristics (100 k)

F: QQ-plot Characteristics (100 k)

G:
Nord Pool Covariance Week – Month Contracts

H: Nord Pool Correlation Week – Month Contracts
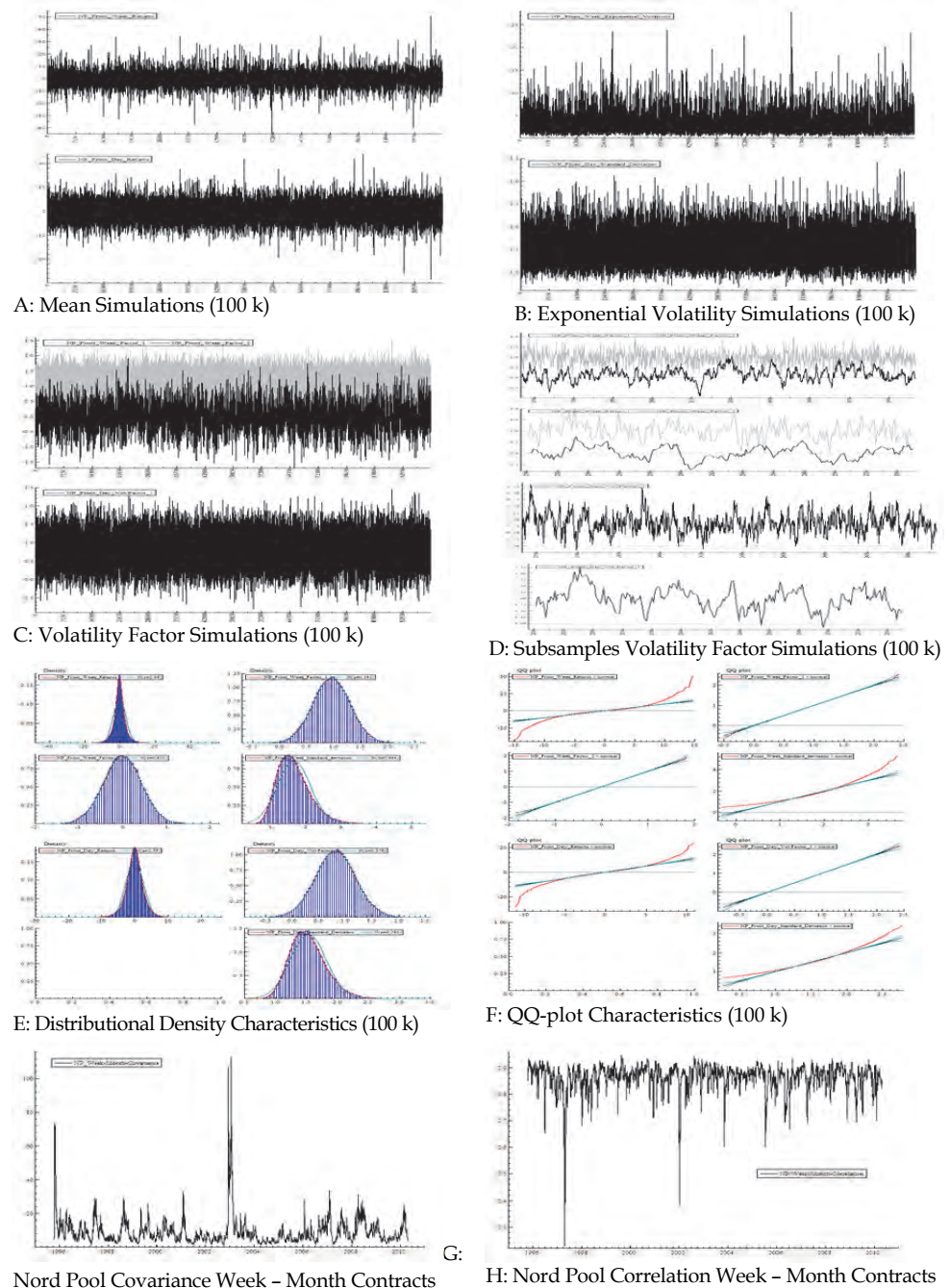
Fig. 6. Nord Pool SV model Characteristics for Future Week and Forward Month Contracts
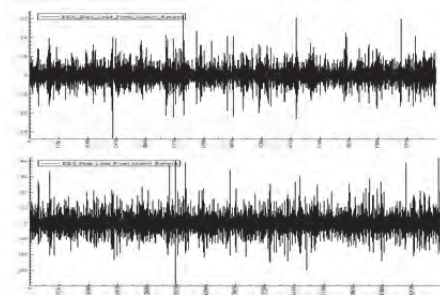
A: Mean Simulations (100 k)
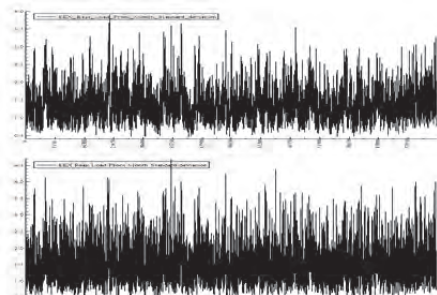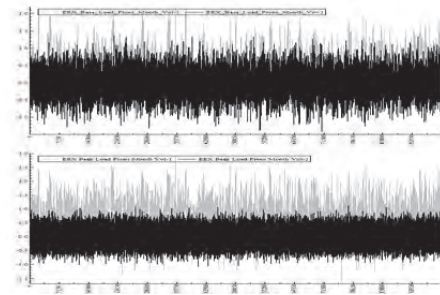
B: Exponential Volatility Simulations (100 k)

C: Volatility Factor Simulations (100 k)
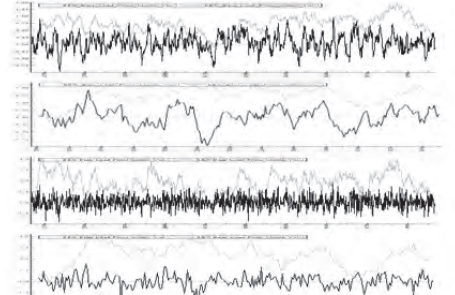
D: Subsamples Volatility Factor Simulations (100 k)
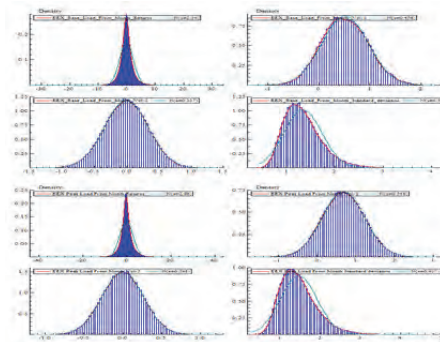
E: Distributional Density Characteristics (100 k)
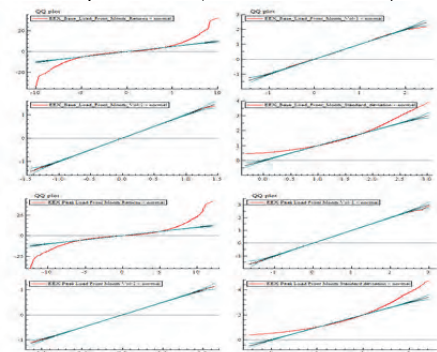
F: QQ-plot Characteristics (100 k)
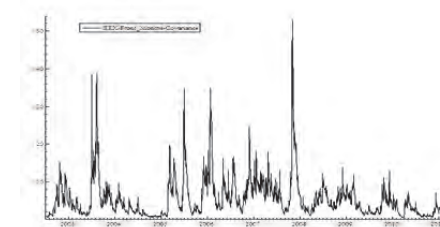
G: EEX Covariance Month Base-Peak Contracts
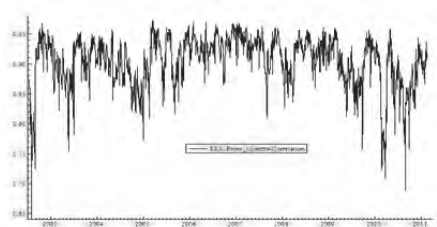
H: EEX Correlation – Month Base-Peak Contracts

Fig. 7. EEX SV model Characteristics for Future Month Contracts (base and peak load)

Distributional features of the mean and volatility equations from a functional simulation (100 *k*) of the Nord Pool and EEX commodity markets are reported in Figure 6 (Nord Pool) and Figure 7 (EEX). The top plots report a full-simulation of the mean (left) and the exponential volatility (right); the middle report the full-sample paths of the two volatility factors together with sub-samples for the two volatility factors (right). From the plots to the right we see that the first factor reports a quite choppy behaviour with lower persistence (solid-line) while the second factor is smoother with higher persistence (dotted-line). The result confirms the interpretation of Table 3. The two factors seem to represent quite different processes inducing volatility processes that originate from informational flow from several sources. In the middle bottom plots (panel E and F) we have reported the densities (left) and the *QQ*-plots (right) for the mean, the two volatility factors and the exponential volatility (standard deviation). The one/two volatility factors seem normally distributed while the mean have inherited the non-normal features from the original plots in Figure 2 and the exponential volatility seem log-normal distributed as would be expected using the exponential functions for normally distributed variables. Finally in the bottom plots (panel G and H) the co-variance is reported in the left plot and the correlation to the right. For both markets the correlation seems high with only minor exceptions towards a correlation of 0.25 for the Nord Pool market and toward 0.5 for the EEX market.

Irrespective of markets and contracts, Monte Carlo Simulations should lead us to a deeper insight of the nature of the price processes that can be described by stochastic volatility models. The results are close to the moment based (non-linear optimizers) techniques adjusting for a more robust model specification (but at a higher dimension). The Bayesian M-H α^* technique also helps to keep the model parameters in the region where the predicted shares are positive.

4.3 Market risk management measures and the conditional moments forecasts

For the mean and volatility forecasting we can simply use the fitted SV model in each iteration to generate samples for the forecasting period. Point forecasts of the return (y_{t+1}) and volatility $\left(e^{v_{1,t+1} + v_{2,t+1}} \right)$ are simply the sample means of the two random samples. Similarly, the sample standard deviations can be used as the standard deviations of forecast errors. The MCMC method produces a predictive distribution of the mean and volatility. The predictive distributions are more informative than simple point forecasts. Quartiles are readily available for VaR and CVaR calculations for example. Figure 8 reports densities for the mean and the exponential volatility for a 100 *k* simulation of the optimally estimated SV models. The percentiles of the densities can be extracted and associated VaR and CVaR values are therefore also reported in Figure 8 using percentage notation. From Figure 8 and for the Nord Pool week contracts (long positions) the 99.9% VaR (CVaR) is -0,1729 (-0,2165), giving an average daily loss of €172,919 (€216,509) for a 1 million Euro portfolio. The 99.9% VaR and CVaR for an EEX peak front month contract portfolio of 1 million Euro is €103,044 and €124,408, respectively. The SV-model results give us also immediate access to the *Greek* Letters (a contract with an exercise price must be quoted). Hence, as VaR and *Greek* letters are accessible for every stochastic run both methods will be available for reporting in distributional forms. The VaR and CVaR is calculated using extreme value theory (EVT[19])

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[19] For applications of the EVT, it is important to check for log-linearity of the Power Law (Prob($\upsilon > x$) = Kx-α). See section 3.2 above.

for smoothing out the tail results. Applying the estimated SV-model for 10 k simulations and 1 million Euro invested in the front contracts, a maximum likelihood optimization of 97.5%, 99.0%, 99.5% and 99.9% VaR and expected shortfall (CVaR) calculations are reported in Figure 9. The VaR and CVaR densities using EVT are credible, are clearly related to the VaR and CVaR values reported using the optimal SV-model percentiles in Figure 8, and the density means seem higher. In fact, optimal forecast percentiles are only in the left part of the EVT-tails. The EVT-tails of the VaR and CVaR densities must be of considerable interest to risk managers engaged in commodity markets. The mean and standard deviation for the EVT calculated VaR (CVaR) can be extracted from the underlying distributions. For example, from Figure 9, the Nord Pool week future contracts Var (CVaR) numbers with associated standard errors becomes 0.1809;0.0217 (0.2239;0.0332), 0.1243;0.0115 (0.1604;0.0183), 0.1026;0.0084 (0.1363;0.0139), and 0.0763;0.0052 (0.1069;0.0093) for 99.9%, 99.5%, 99.0% and 97.5% percentiles, respectively. SV model simulations and the EVT calculated VaR and CVaR numbers seem to indicate higher values for both markets and all contracts relative to SV optimal forecast model. High volatilities induce risky instruments and rather high VaR/CVaR values for the European energy market.
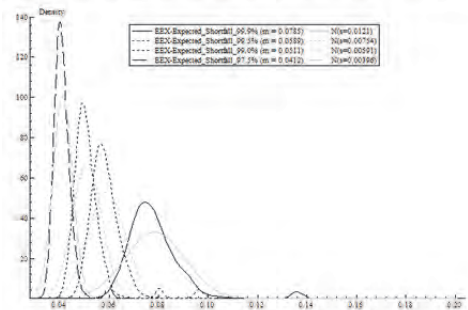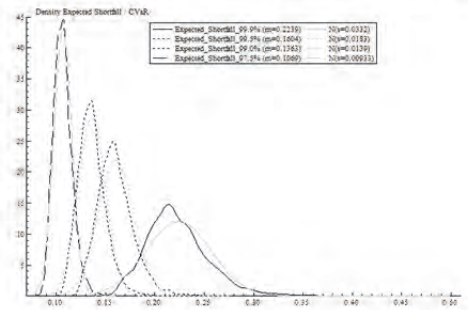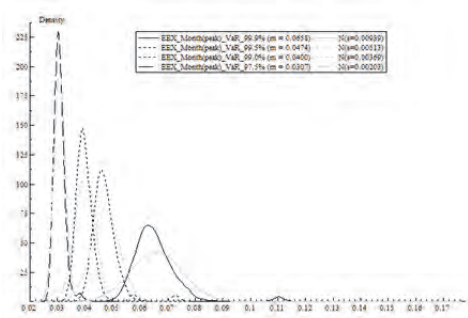

A: Nord Pool Forecasted Mean Densities   B: EEX Forecasted Mean Densities
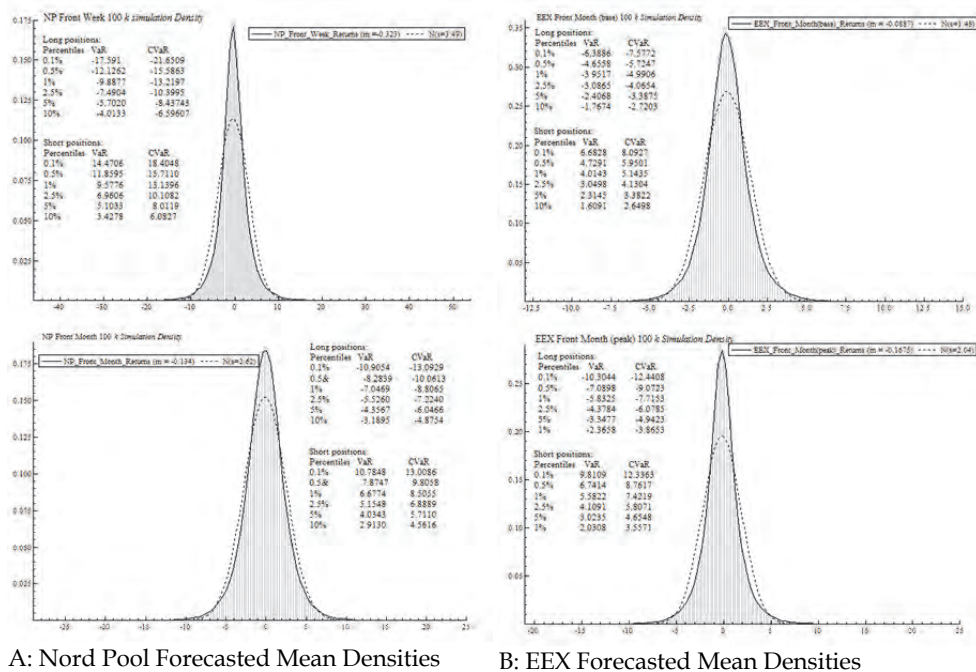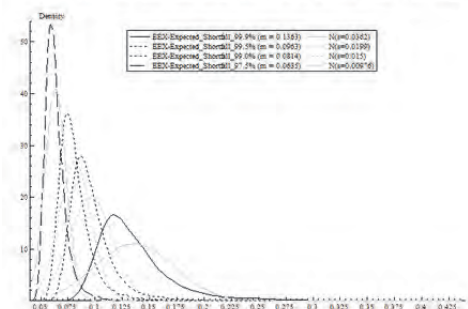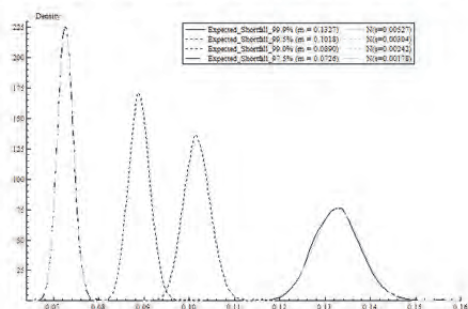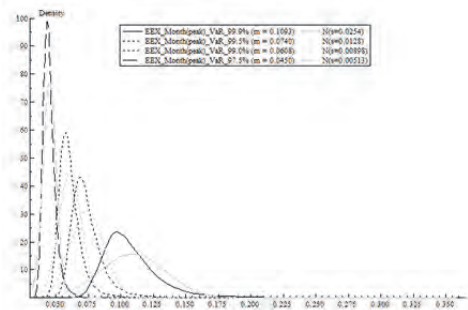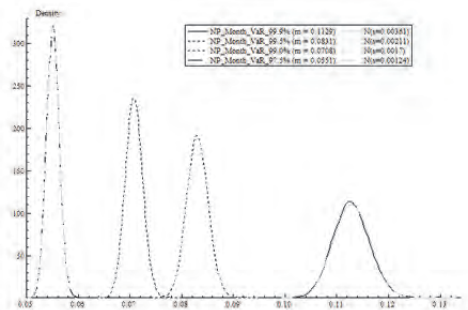
Fig. 8. Forecasted Densities with associated VaR and CVaR values for Nord Pool and EEX
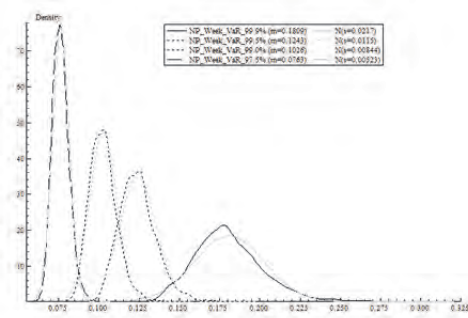
A: NP Front Week VaR and CVaR Densities

C: EEX FM (base load) VaR and CVaR Densities

B: NP Front Month VaR and CVaR Densities

D: EEX FM (peak load) VaR and CVaR Densities

Fig. 9. VaR and CVaR (expected shortfall) Densities Nord Pool and EEX using EVT

The *Greek* letters can be calculated for all stipulated contract prices using the Broadie and Glasserman formulas (1996). The Gamma (γ) letter is not stochastic but deterministic and can be derived using the classical deterministic formula. Applying the estimated SV-model for 10 k simulations, the Greek letter densities (delta, (gamma), rho and theta) are reported in Figure 10 for ATM call and put options (only the delta density is reported). The Nord Pool front week call-option delta density for example has a mean of 0.4484 (below 0.5 due to negative drift) with associated standard error of 0.0078. Gamma is deterministic and becomes 0.3742. The values for rho and theta are 6.5592 and 1.2582 with associated standard errors of 0.1110 and 0.1653, respectively. Considering the relatively high values for VaR and CVaR in these commodity markets there may be some value in a procedure helping the risk management activities. Fortunately, a procedure for post estimation analysis and forecasting is accessible. The post estimation analysis we will apply is the final and third step described by Gallant and Tauchen (1998), the re-projection step (see appendix I). The step brings the real strengths to the methodology in building scientific valid models for commodity markets.

The re-projection methodology gets a representation of the observed process in terms of observables that incorporate the dynamics implied by the non-linear system under consideration. The post estimation analysis of simulations entails prediction, filtering and general SV model assessment. Having the GSM estimate of system parameters for our models, we can simulate a long realization of the state vector. Working within this simulation, univariate as well as multivariate, we can calibrate the functional form of the conditional distributions. To approximate the SV-model result using the score generator $\left(\hat{f}_K\right)$ values, it is natural to reuse the values of the previous projection step. For multivariate applications, the optimal BIC/AIC criterion (Schwarz, 78) would be a sufficient criterion. The dynamics of the first two one-step-ahead conditional moments (including co-variances) may contain important information for all market participants. Starting with the univariate case, Figure 11 shows the first moment $\left(E[y_0 \mid x_{-1}]\right)$ densities to the left and the second moment $\left(Var[y_0 \mid x_{-1}]\right)$ densities to the right. The first moment information conditional on all historical available data shows the one-day-ahead density. This is informative for daily risk assessment and management[20]. To calculate the one-step-ahead VaR and CVaR we again use the extreme value theory to smooth out the tails. VaR (CVaR) numbers for the contracts are reported in Table 4. For the Nord Pool front week for example the VaR (CVaR) for 99.9%, and 97.5% are 3.33 (4.10) and 1.55 (2.06), respectively. The one-day-ahead forecasts conditional on all history of price changes and volatilities reduces in this case, the

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[20] We use a transformation for lags of xt to avoid the optimisation algorithm using an extreme value in xt-1 to fit an element of yt nearly exactly and thereby reducing the corresponding conditional variance to near zero and inflating the likelihood (endemic to all procedures adjusting variance on the basis of observed explanatory variables). The trigonometric spline transformation is:

$$\hat{x}_i = \begin{cases} 1/2\left\{x_i + 4/\pi\left(\arctan\left[\pi/4\left(x_i + \sigma_{tr}\right)\right]\right) - \sigma_{tr}\right\} & -\infty < x_i < -\sigma_{tr} \\ x_i & -\sigma_{tr} < x_i < \sigma_{tr} \\ 1/2\left\{x_i + 4/\pi\left(\arctan\left[\pi/4\left(x_i - \sigma_{tr}\right)\right]\right) + \sigma_{tr}\right\} & \sigma_{tr} < x_i < \infty \end{cases}$$. The transform has negligible effect

on values of xi between -σtr and +σtr but progressively compress values that exceed ±σtr so they can be bounded by ±2σtr.

Univariate (long positions)

| | Nord Pool | | | | EEX | | | |
| | Front Week | | Front Month | | Base Month | | Peak Month | |
Confidence levels:	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR
99.90 %	0.0333	0.0410	0.0240	0.0287	0.0195	0.0245	0.0246	0.0302
99.50 %	0.0237	0.0298	0.0176	0.0216	0.0129	0.0171	0.0171	0.0218
99.00 %	0.0198	0.0256	0.0152	0.0189	0.0107	0.0144	0.0140	0.0186
97.50 %	0.0155	0.0206	0.0122	0.0156	0.0079	0.0111	0.0104	0.0145
95.00 %	0.0124	0.0172	0.0102	0.0134	0.0060	0.0090	0.0080	0.0118
90.00 %	0.0096	0.0140	0.0082	0.0112	0.0043	0.0070	0.0059	0.0093

Bivariate (long positions)

| | Nord Pool | | | | EEX | | | | Nord-Pool & EEX | | | |
| | Front Week | | Front Month | | Base Month | | Peak Month | | Front Month | | Base Month | |
Confidence levels:	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR
99.90 %	0.0378	0.0464	0.0343	0.0416	0.0228	0.0285	0.0307	0.0379	0.0150	0.0178	0.0220	0.0275
99.50 %	0.0266	0.0338	0.0240	0.0303	0.0148	0.0197	0.0210	0.0272	0.0114	0.0138	0.0144	0.0191
99.00 %	0.0220	0.0289	0.0201	0.0261	0.0123	0.0166	0.0171	0.0230	0.0099	0.0121	0.0119	0.0160
97.50 %	0.0170	0.0230	0.0155	0.0209	0.0090	0.0128	0.0125	0.0178	0.0079	0.0101	0.0087	0.0124
95.00 %	0.0133	0.0190	0.0122	0.0173	0.0068	0.0103	0.0094	0.0143	0.0064	0.0086	0.0066	0.0099
90.00 %	0.0098	0.0152	0.0092	0.0139	0.0048	0.0080	0.0067	0.0111	0.0048	0.0070	0.0047	0.0077

Table 4. Univariate and Bivariate VaR and CVaR measures for Conditional First Moments[21]

A: NP Front Week Delta Call/Put_ATM Densities

C: EEX FM (base) Delta Call/Put_ATM Densities

B: NP Front Month Delta Call/Put_ATM Densities

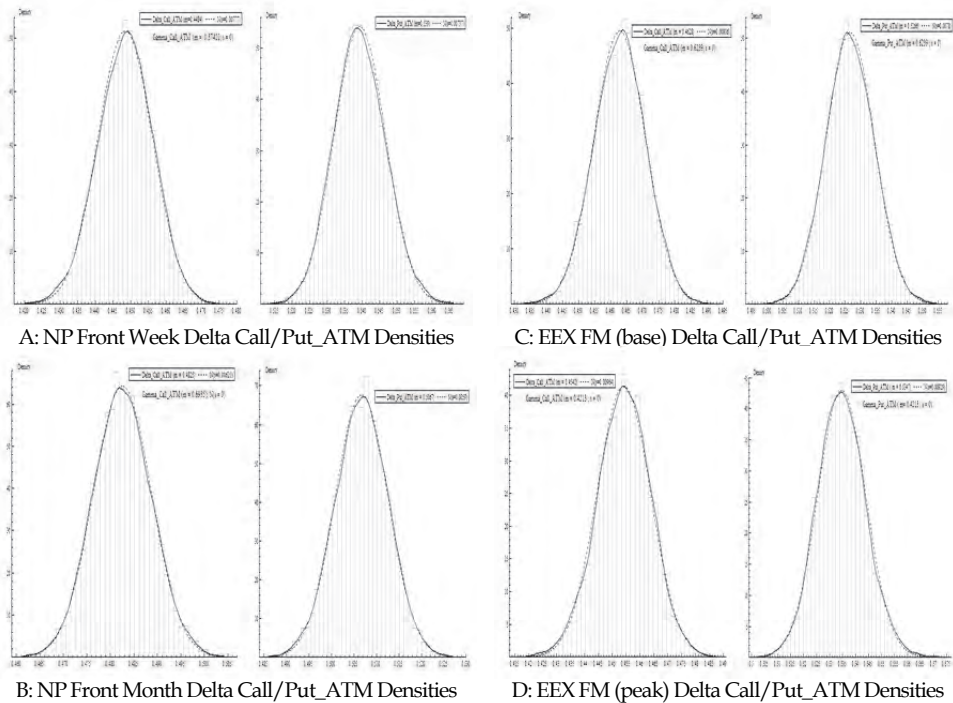D: EEX FM (peak) Delta Call/Put_ATM Densities

Fig. 10. Greek letter densities (delta, (gamma), rho theta) for Nord Pool and EEX

[21] Greek letters (delta, gamma, rho and theta) are also available from univariate and bivariate conditional first moments. For the front week series the delta for a call (put) ATM option contract is 0.1999 (0.7868).

VaR and CVaR numbers to approximately 20% of the original unconditional forecasts. Moreover, the three bivariate return distributions may add some information to the market participants. The bivariate distributions for the Nord Pool and the EEX are plotted to the right in Figure 11 and Figure 12 reports the bivariate density for the front month (base load) contracts at Nord Pool and EEX. The general conclusions from the bivariate densities of the Nord Pool and EEX markets in Table 4 are increased VaR and CVaR numbers. The exception is the front month contracts (base load) between the Nord Pool and EEX markets where we find that the Nord Pool market shows a relative strong decrease for the VaR and CVaR numbers while the EEX market show a small increase from the univariate analysis. Hence, comparing with classical forecasting in Figures 8 and 9, the use of the whole history of observed data series implies a significant reduction in the relevant risk indicating relevant information from the history of the time series. The use of forecasted conditional first moment reduces the VaR and CVaR values with a factor of 0.2. The other side of the picture is the daily calculations with often very computer intensive algorithms.
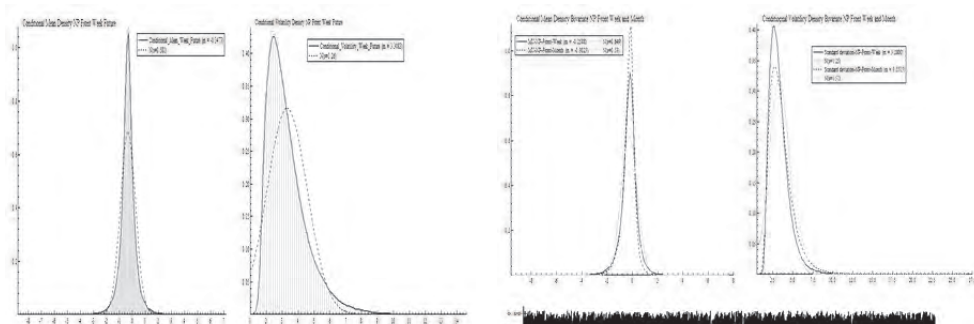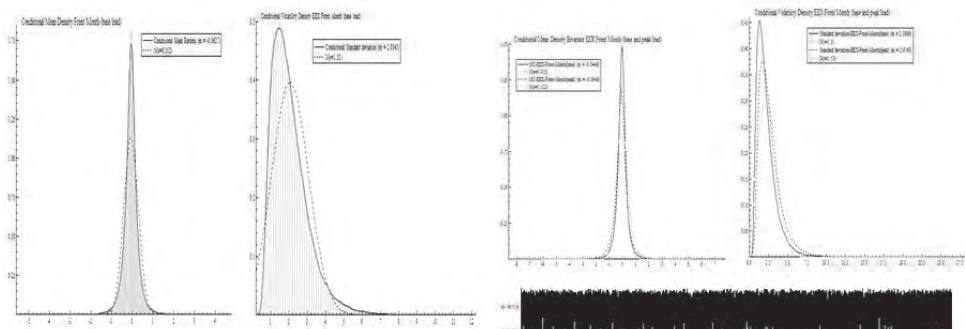

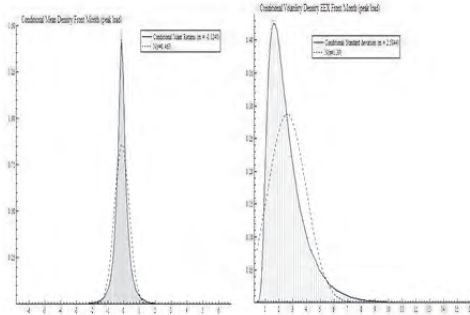A: Re-projected Mean and Volatility NP Week
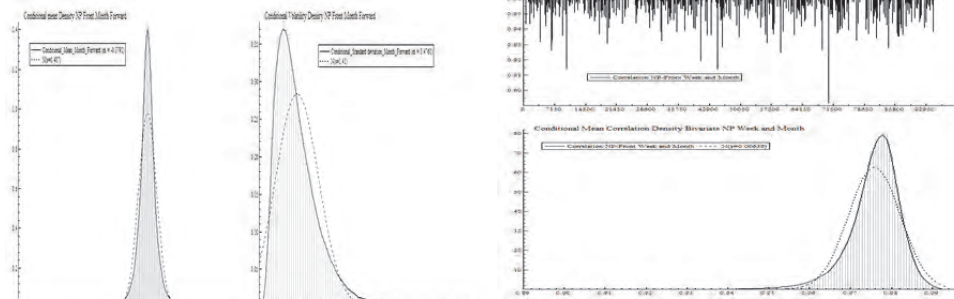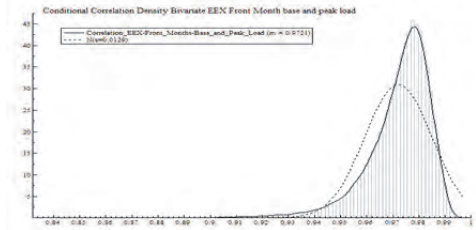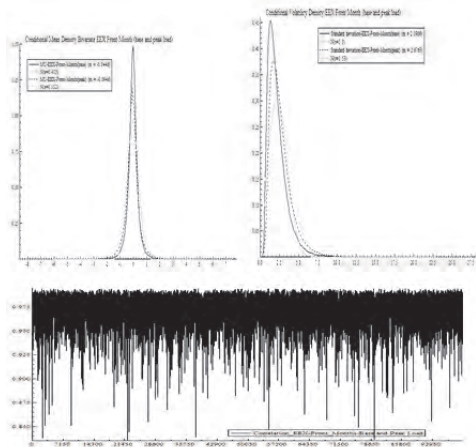
B: Re-projected Mean and Volatility NP Month

E: Bivariate Re-projected NP Week-Month

C: Re-projected EEX Month (base load)

D: Re-projected EEX Month (peak load)

F: Bivariate Re-projected EEX F-M (base and peak ld)

Fig. 11. Univariate and Bivariate Characteristics for Nord Pool and EEX contracts
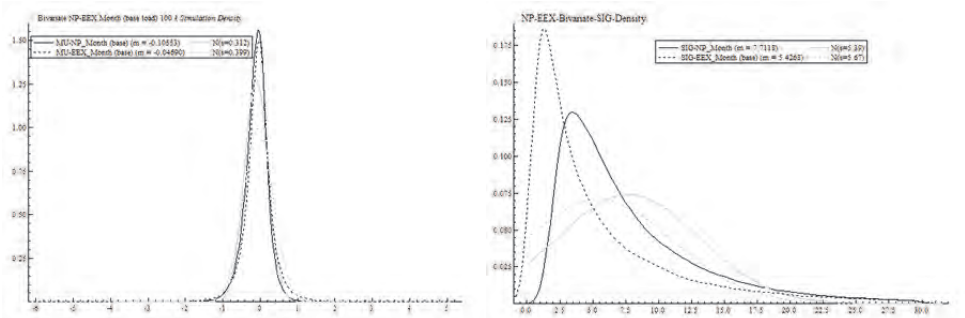

Fig. 12. Bivariate Characteristics between Nord Pool and EEX. Month (base load) contracts

For the second moment we find a log-normal distribution. The explicit variance and standard deviation distributions are interesting for several applications with a special emphasis on derivative computations. However, as we could expect the volatility does not change much from the original simulated SV model. The volatility is assumed latent and stochastic. However, the filtered volatility, the one-step-ahead conditional standard deviation evaluated at data values (x_{t-1}), may give us some extra information. The filtered volatility is a result of the score generator (f_K) and therefore volatility with a purely ARCH-

type meaning. Figure 13 shows a representation of the filtered volatility at the unconditional mean of the data series. The density displays the typical shape for data from a financial market: peaked with fatter tails than the normal with some asymmetry. Figure 13 also plots the distributions for several data values (x_{t-1}) from -5%/-5% and +5%/+5%. Interestingly, the largest values in absolute terms of x_{t-1} have the widest densities. That is, conditional mean densities are dependent on the x_{t-1} observations making one-day-ahead VaR and CVaR dependent on historical information. Alternatively, a Gauss-Hermite quadrature rule[22] can be used and is also reported in Figure 13. Hence, the one-step-ahead filtered volatility seems therefore to contain more information than the general SV-model. Based on the observation day t it is therefore of interest to use the one-step-ahead standard deviation for several applications. The filtered volatility and the Gauss-Hermite quadrature can be used for one-step-ahead price of any derivative.

Figure 13 also reports the conditional variance functions. The conditional variance functions are reported for both univariate and bivariate simulated data series. We can interpret the conditional variance graphs as representing the consequences of a shock to the system that comes as a surprise to the economic agents involved. From the plots we see that the EEX responses from positive shocks are higher than from negative shocks. The SV model positive ρ signals positive mean and volatility correlation inducing positive asymmetry (higher volatility from positive price changes). As noted earlier in this chapter, the asymmetry seems close to zero for the Nord Pool market but the EEX market reports clearly positive asymmetry.
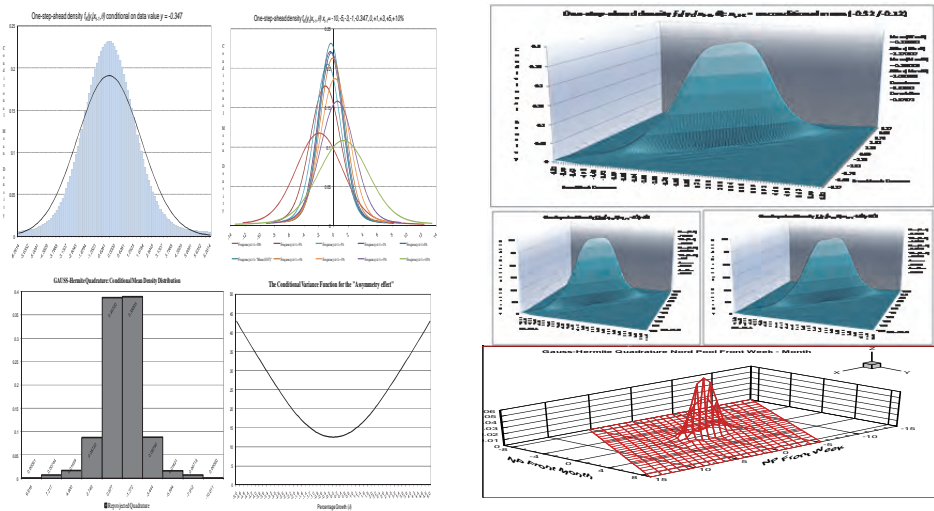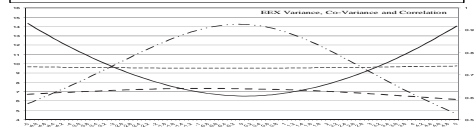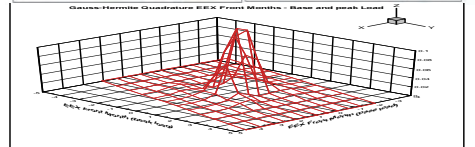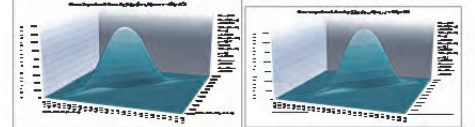

A: Re-projected Filtered Volatility NP Week

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[22] A Gaussian quadrature over the interval $(-\infty, +\infty)$ with weighting function $W(x) = e^{-x^2}$ (Abramowitz and Stegun 1972, p. 890). The abscissas for quadrature order n are given by the roots x_i of the Hermite polynomials $H_n(x)$, which occur symmetrically about 0. An expectation with respect to the density can be approximated as: $E(g(y)) = \sum_{j=1}^{npts} g(abcissa[j]) \cdot weight[j]$.
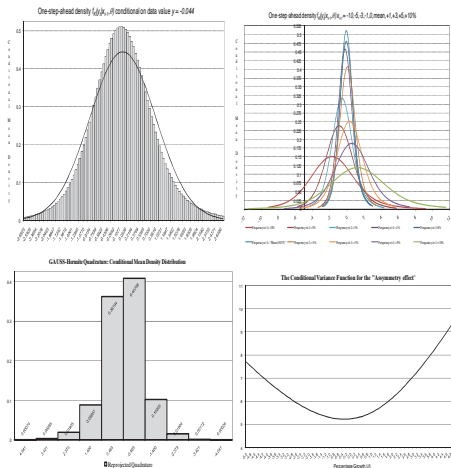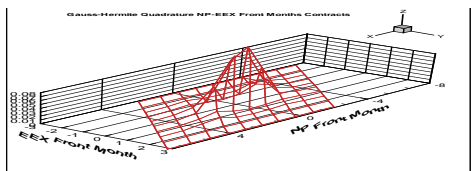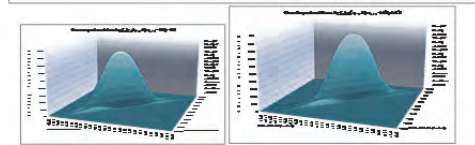
B: Re-projected Mean and Volatility NP Month
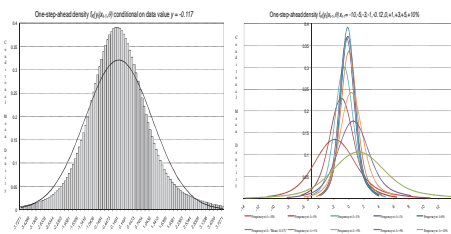

C: Re-projected EEX Month (base load)
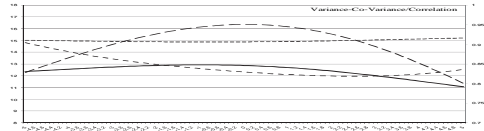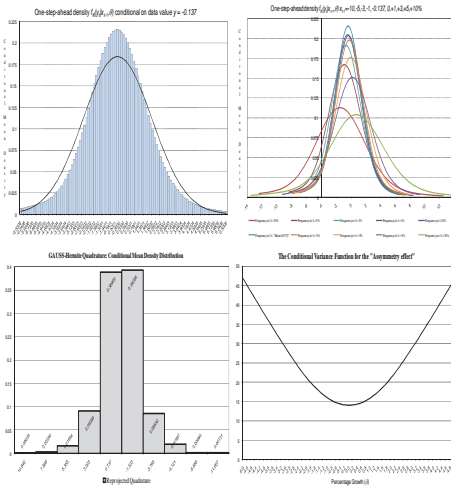
E:Bivar Filtered Volatility NP Week-Month
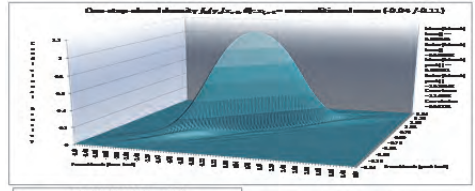
F:Bivar Filtered Volatility EEX Month(base-peak)

D: Re-projected EEX Month (peak load)   G:Bivar Filtered Volatility NP - EEX Month (base)
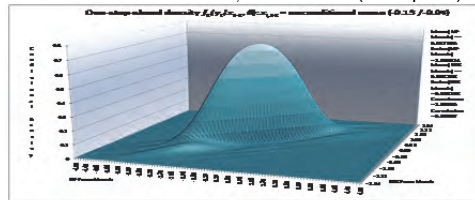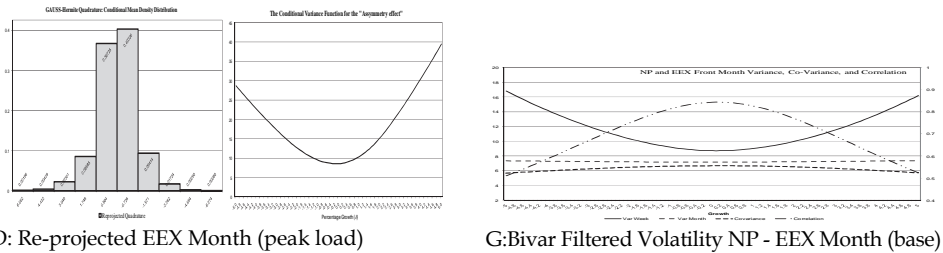
Fig. 13. Bivariate Contract Characteristics between Nord Pool and EEX

The multivariate post estimation analysis gives access to covariances/correlations for any simulated series combinations. The number of multivariate series is dependent on problem at hand. In this paper we analyse three bivariate 100 k simulated series: (1) the Nord Pool one-week future and one-month forward contracts; (2) the EEX base and peak one month future contracts, and (3) the Nord Pool and EEX front month (base load) contracts. Bivariate forecasts, one-step-ahead conditional mean, volatility and correlations are all interesting measures. Figure 13, middle part to the right (panel E, F and G), reports the bivariate conditional mean forecasts, dependent on changing historical information (x_{t-1}). The Gauss-Hermite quadrature adds to the mean density information and finally for all bivariate investigations, the conditional variance functions, co-variance functions and correlations are reported. The asymmetry story holds also for the bivariate analysis and the co-variances and correlations seem to decrease during high volatility periods. The correlation seems symmetric and is at its minimum when volatility and price changes (growth) are high either negative or positive. Hence, the quadrature and variance/covariance information from the post estimation analysis seems to add extra insight to scientifically valid models, the VaR/CVaR measures for risk management and Greek letters for portfolio managment. Implicitly, Figure 13 panel G reports diversification effects between Nord Pool and EEX. The bivariate Nord Pool and EEX analysis report lower VaR and CVaR measures for all percentiles of the bivariate distributions relative to the two Nord Pool and EEX univariate analyses (front month (base load)).

Finally, for illustrative purposes and the use of EVT for conditional moments and the VaR, CVaR and Greek letters density measures, we perform 5 k SV-model simulations for the Nord Pool Front week SV model, extract the conditional density using the $f_k(\theta)$ score model and calculate VaR, CVaR and Greek letters density measures. It takes considerable time and computer resources to do this exercise. However, the VaR/CVaR measures are interesting. Figure 14 upper right plot shows 30 subsamples of the 5 k front week SV-model unconditional return density simulations. In the upper right plot, the front week conditional density returns are plotted. In the middle plots of Figure 14 the VaR and CVaR measures are reported for the conditional densities. Interestingly, the VaR/CVaR density measures are at a considerably lower level than the same unconditional VaR/CVaR measures in Figure 9 above. The lower plots in Figure 14 report the Greek letter delta densities for call and put ATM options for front week contracts. Interestingly, the Greek letter density measures have also changed from the same unconditional measures in Figure 10 above. There seems to be some extra information in the conditional densities from the SV models.
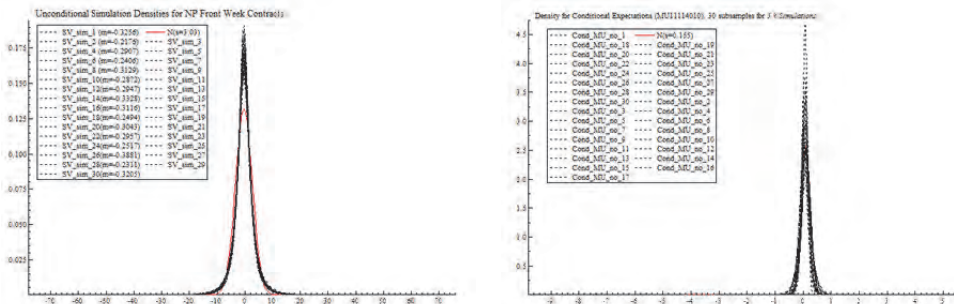
Panel A: Unconditional density plots (30 sub-samples) Panel B: Conditional density plots (30 sub-samples)
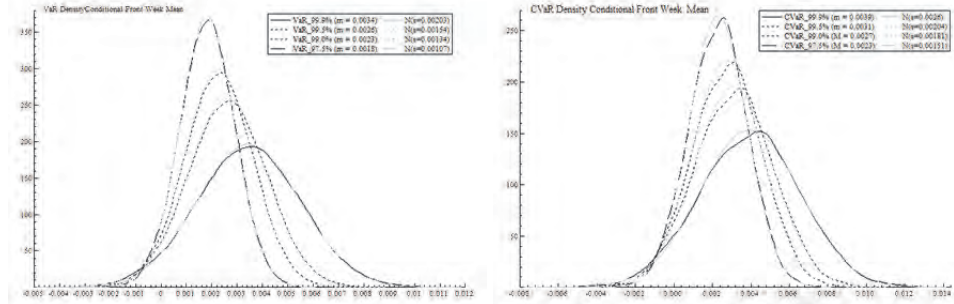
Panel C: VaR from 5 k conditional expectations   Panel D: CVaR from 5 k conditional expectations
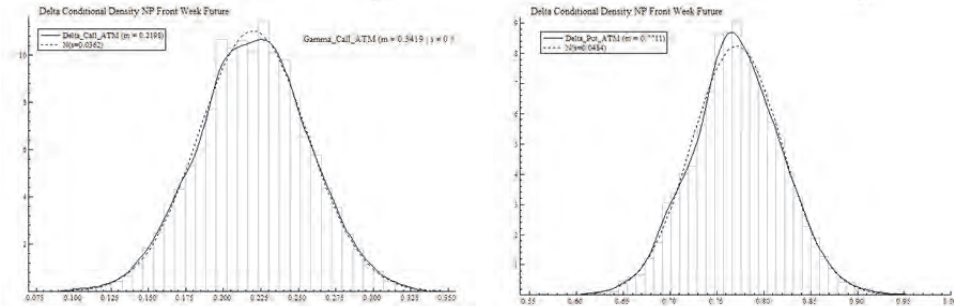
Panel E: ATM_Δ_Call 5 k conditional expectations   Panel F: ATM_Δ_Put 5 k conditional expectations

Fig. 14. (Un-)Conditional expectations, VaR/CVaR measures and Greek letters

5. The credit and liquidity risks

The chapter has focused mainly on stochastic volatility models. Other risks often found in energy will be briefly discussed and incorporated in the Economic Capital concept. For energy enterprises with a large number of customers we will use the one-factor Gaussian copula. A energy wholesale and retail company will have a portfolio of account payables for short electricity positions from households and industry. The risk for the energy company is default of these account payables. We define T_i as the time customer i defaults (we assume that all customers will default eventually, but the default time may be many years into the future.) We denote the cumulative probability distribution of T_i by Q_i. In order to define a

correlation structure between the T_i using the one-factor Gaussian copula model, we map, for each i, the default time T_i to a variable U_i that has a standard normal distribution on a percentile-to-percentile basis. For the correlation structure between the U_i, we assume the factor model $U_i = a_i \cdot F + \sqrt{\left(1 - a_i^2\right)} \cdot Z_i$ where F and the Z_i have standard normal distributions and the Z_i are uncorrelated with each other. The mapping between the the U_i and the T_i, imply that $Prob\ (U_i < U) = Prob\ (T_i < T)$, when $U = N^{-1}[Q_i(T)]$. Now using the one factor model which we can write as $Z_i = \dfrac{U_i - a_i \cdot F}{\sqrt{\left(1 - a_i^2\right)}}$, the probability that $U_i < U$ conditional on the factor value F is $Prob\left(U_i < U \mid F\right) = Prob\left(Z_i < \dfrac{U_i - a_i \cdot F}{\sqrt{\left(1 - a_i^2\right)}} \right) = N\left(\dfrac{U_i - a_i \cdot F}{\sqrt{\left(1 - a_i^2\right)}} \right)$.

Finally, $Prob\left(T_i < T \mid F\right) = Prob\left(Z_i < \dfrac{N^{-1}\left[Q_i(T)\right] - a_i \cdot F}{\sqrt{\left(1 - a_i^2\right)}} \right)$. Assuming that the time to default Q, is equal for all i and equal Q and that the copula correlation between the default times of any two customers is the same and equal ρ, inducing that $a_i = \sqrt{\rho}$ for all i. Hence, we have

$Prob\left(T_i < T \mid F\right) = N\left(\dfrac{U_i - \sqrt{\rho} \cdot F}{\sqrt{(1 - \rho)}} \right)$. When the customer portfolio is larger the expression provides a good estimate of the percentage of customers defaulting by time T conditional on F. Therefore, we have defined the probability Y that the default rate will be greater than $N\left(\dfrac{N^{-1}\left[Q_i(T)\right] - \sqrt{\rho} \cdot N^{-1}(Y)}{\sqrt{(1 - \rho)}} \right)$ and therefore

$VaR\left(T, X\right) = AP \cdot (1 - R) \cdot N\left(\dfrac{N^{-1}\left[Q_i(T)\right] + \sqrt{\rho} \cdot N^{-1}(X)}{\sqrt{(1 - \rho)}} \right)$, where X is the confidence level and $Y=1-X$, and AP is accounts payable. The probability for an energy enterprise with €250 of retail exposures, probability of default is 4%, the recovery rate averages 75% and the copula correlation parameter is ρ = 0.25, is $N\left(\dfrac{N^{-1}\left[0.04\right] + \sqrt{0.25} \cdot N^{-1}(0.999)}{\sqrt{(1 - 0.25)}} \right) = 0.40618$. Losses with one-year time horizon and a 99.9% confidence level when the worst case loss rate occurs are therefore: $VaR\left(1, 99.9\%\right) = 250 \cdot (1 - 0.75) \cdot 0.40618 = €25.387$. For a confidence level of 97.5% the VaR will become €11.672 $\left(250 \cdot (1 - 0.75) \cdot 0.18675\right)$. Several other methodologies for credit risk and default rates are available. For the default rate the Merton (1974) model, where we use equity prices and option theory to estimate default probabilities, is useful. The Credit Risk Plus software from Credit Suisse Financial[23] Products and CreditMetrics from J.P. Morgan[24] are commercial tools for the risk calculations.

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[23] See www.credit-suisse.com/investment_banking/holt/
[24] See www.jpmorgan.com/pages/jpmorgan/

Finally, liquidity risk is the cost of liquidation in stressed market conditions within a certain time period. Bid-ask spread is normally a good measure for unwinding positions. If we define ai and si as the mean and standard deviation of the proportional bid-ask spread, we can write the cost of liquidation as $\sum_{i=1}^{n}\frac{1}{2}(\alpha_i + \lambda\sigma_i)\gamma_i$, where λ_i is the required confidence level (1% = 2.33) and γ_i is the size of the instrument/commodity. A liquidity adjusted VaR can therefore be calculated as $VaR + \sum_{i=1}^{n}\frac{1}{2}(\alpha_i + \lambda\sigma_i)\gamma_i$.

6. Economic capital and RAROC for European energy enterprises

Economic or Risk Capital is defined as the amount of capital an energy company needs to absorb over a certain time horizon (usually one year) with a certain confidence level (Rosenberg and Schuermann, 2004). Confidence levels should be chosen based on credit ratings. An energy corporation usually wants to establish and maintain an AA-rating., which normally have a one-year probability of default of 0.03%. That is, a confidence level of 99.7%. Note that when we calculate the risk capital, this means that we want to have enough economic resources inside and outside the company to cover unexpected losses. Unexpected loss is the difference between expected and actual loss, so that expected losses are already priced in the corporation's capital structure. Hence, the risk capital for a corporation that want to maintain an AA rating is the difference between expected loss and the 99.7% point on the probability distribution of losses.

Due to the fact that energy companies are not publicly traded companies and equity prices therefore is rarely available for estimation of default probabilities, the approach most often used estimates different types of risk in different business units and then aggregates to measure total or overall risk. This mainly means that we calculate probability distributions for total losses per type or total losses per business unit. At the end a final aggregation gives a probability distribution of total losses for the whole corporation. For an energy corporation market risk (price and volume), basis risk (locational/time risk), and operational risk (operational and legal) is the three main risk classes.

We use two approaches. The first is the simple *Hybrid* approach the second is the use of copulas to facilitate correlation structure between market variables (the copula approach). For the remaining example we apply fictive capital estimates for the different risks/business units. We assume three business areas for example hydro-power productions with market (price and volume) and operational risk, a network division and telecommunication division with market and operational risk. Typical shapes of loss distributions for market risk is close to the normal distribution while the operational risk may have quite extreme shape. Most of the time losses are modest, but occasionally they are large. A distribution can be characterized by the second, third and fourth moments. The following table summarizes the properties of typical loss distributions:

The business mix is clearly the most important factor for the relative importance. For an energy company also trading derivatives market risk, basis risk and operational risk are all important. Moreover, we find interactions between market, basis and operational risk. When a derivative is traded for example, and the counterparty defaults, operational risk exists only if market variables have moved so that the value of the derivative to the financial

institution is positive. A corporation in the energy sector has the following *economic capital* (*E*) estimates (Panel A) and correlation (Panel B) between market, basis and operational risk for three business units in Table 5:

	Second Moment	Third Moment	Fourth Moment
Market risk	High	Zero	Low
Basis risk	Low	High	Zero
Operational risk	Low	High	High

Panel A	Business Units (billion €)		
Economic Capital	Hydro power generation (B1)	Network operation (B2)	Telephone communication (B3)
Market risk (M)	150	45	82
Basis Risk (B)	95	38	50
Operational Risk (O)	55	25	34

Panel B Correlation

Structure	MB1	BB1	OB1	MB2	BB2	OB2	MB3	BB3	OB3
MB1	1	0.35	0.2	0.4	0	0.1	0.3	0	0.05
BB1	0.35	1	0.15	0.15	0.25	0.25	0.05	0.1	0
OB1	0.2	0.15	1	0.15	0	0.2	0.1	0.1	0
MB2	0.4	0.15	0.15	1	0.2	0.1	0	0	0.1
BB2	0	0.25	0	0.2	1	-0.1	0.1	0.2	0.05
OB2	0.1	0	0.2	0.1	-0.1	1	0	0.1	0
MB3	0.3	0.05	0.1	0	0.1	0	1	0.1	0
BB3	0	0.1	0.1	0	0.2	0.1	0.1	1	0.05
OB3	0.05	0	0	0.1	0.05	0	0	0.05	1

Table 5. Economic Capital and Relevant Risk for a European Energy Enterprise

The correlation is checked for consistency using Cholesky decomposition. The hybrid approach involves calculating the economic capital for the individual risks using $E_{total} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} E_i \cdot E_j \cdot \rho_{ij}}$ which is exactly correct if the distributions are normal. When they are non-normal, the hybrid approach gives an approximate answer – but one that reflects any heaviness in the tails of the individual loss distributions. Economic capital can be calculated in several ways. The market risk economic capital for the hydropower, network and telecommunication units: $\sqrt{150^2 + 45^2 + 82^2 + 2 \cdot \left(150 \cdot 45 \cdot 0.4 + 150 \cdot 82 \cdot 0.3 + 45 \cdot 82 \cdot 0\right)}$ and equals 233.41. The basis risk economic capital for the three business units becomes 159.37 and the operational risk becomes 98.32. The risk capital for the hydropower

generation unit is 245.34. The total risk capital for the network unit is 108.19, the telecommunication unit is 133.94, and the total enterprise wide risk capital becomes: 299.73. We find significant diversification benefits. The sum of the economic capital estimates for market, network and telecommunication risk is 233.41+159.37+98.38 = 491.09 and the sum of the economic capital estimates for three business units are 245.34+108.19+133.94 = 487.48. Both of these are greater than the total economic capital estimate of 299.73. These economic capital estimates are exactly correct.

The second approach is the use of copulas for the different risk measures. We will apply both normal copulas and *Student-t* copulas for the calculation of Economic capital. In this example we assume the same correlation structures as for the hybrid approach and we consider nine factors (market, basis and operational risk for 3 business unit) represented with a mean and standard deviation. We perform Monte Carlo simulations assuming normal and for the illustration of heavy tails, *student-t* distributions with 4 and 2 degrees of freedom for illustrational purposes. MC can also easily incorporate asymmetry (not reported). The procedure is as follows. From any original distribution each loss distribution is mapped on a percentile-to-percentile basis to a standard well-behaved distribution. A correlation structure between the standard distributions is defined and this indirectly defines a correlation structure between the original distributions. The copula therefore gives us well-behaved distributions classified as multivariate Gaussian or multivariate student-t. We simulate 100 k iterations for each E_{total}. For the normal distributions we find a E_{total} of 305.06 with an associated standard deviation of 47.48. The student-t distribution with 4 degrees of freedom shows a mean of 304.21 with associated standard deviation of 51.82. The student-t distribution with 2 degrees of freedom reports a mean of 318.58 with associated standard deviation of 222.41. Finally, we calculate the VaR and CVaR densities from 10 k MCMC iterations. The VaR (upper) and CVaR (lower) for 99.9% 99.5%, 99.0% and 97.5% confidence levels are reported in Figure 15 for the normal, student-t with 4, and student-t with 2 degrees of freedom, respectively. For the normal distribution and VaR (CVaR) 99.9% confidence level the mean is 453.08 (467.08) with associated standard deviation of 6.4 (8.4). For the student-t with 4 df (2df) the VaR 99.9% mean is 668.57 (2423.9) with associated standard deviation of 48.6 (512.9). The student-t distribution with 2 degrees of freedom shows quite a large VaR/CVaR expected loss. Alternatively, the standard deviation of the total loss from n sources of risk can be calculated directly from the relation

$$\sigma_{total} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_i\sigma_j\rho_{ij}}$$ where σ_i is the standard deviation of the loss from the ith source of

risk and ρ_{ij} is the correlation between risk i and j. For our example with three risks and 3 business units the σ_{total} becomes 99.35. From the relationships we can calculate the capital requirements. For example, the excess of the 99.9% worst case loss over the expected loss is 3.09 (normal distribution) times the number calculated for σ_{total}. The same worst-case loss numbers for a one-sided student-t-distribution is 7.17 and 22.33 for 4 and 2 degrees of freedom, respectively. For the normal distribution for example we get a worst case loss of 647.43, which is 342.4 (99.35* 3.44) over the expected loss of 305.03.

As for the Hybrid approach, the MC mean/mode economic capital (risk) shows considerable diversification effects also by using the copula approach. From an assumption of 574 separately for the total economic capital the correlation structure report

diversification effects lowering the total economic capital to approximately 305 as the new risk measure for the corporation as a whole. Capital requirements at 99.9%, 99.5% and 99% worst-case loss scenarios for the corporation become 450.69, 434.64 and 414.88, respectively, for the normal distributions case. For the student-t distribution with two (four) degrees of freedom illustrating a medium (an extreme) heavy tail case, the excess 99.9% and 99.0% worst case losses grows to 1131.8 (931.4) and 464.1 (424.6), respectively.
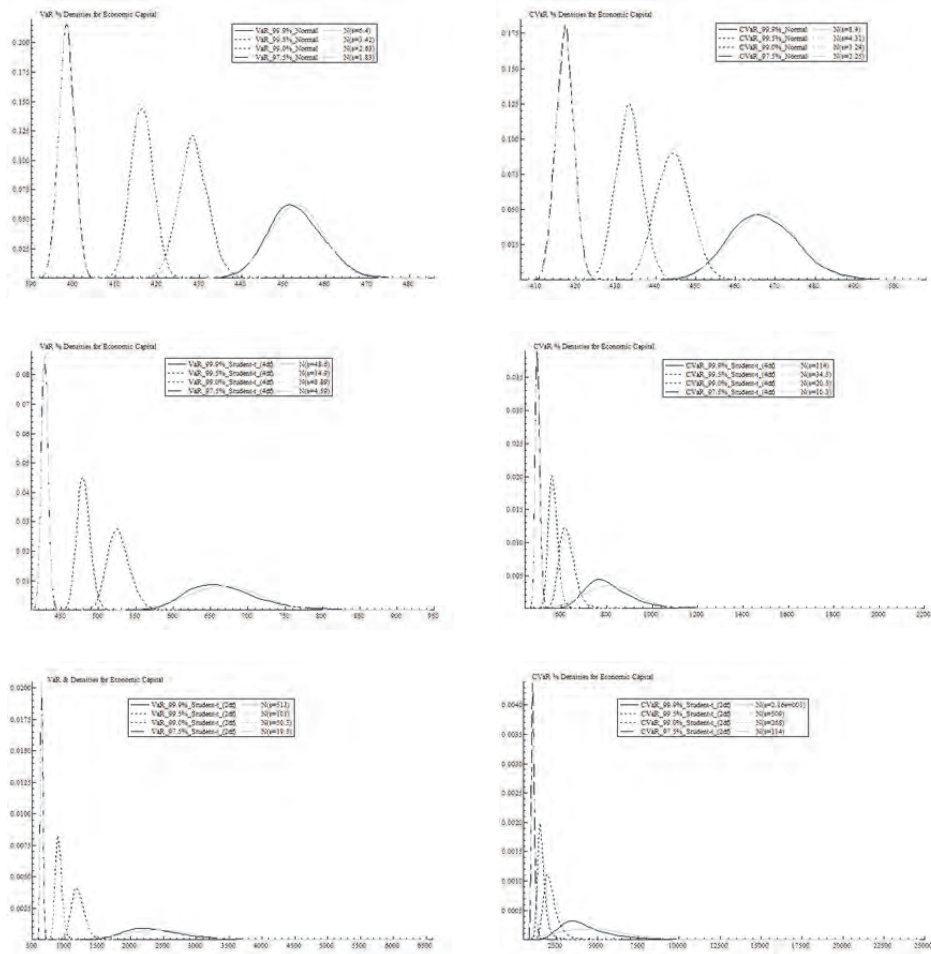

Fig. 15. Distributions of VaR and CVaR for Normal and Student-t distributions

The diversification benefits are to be allocated by an amount $x_i \cdot \dfrac{\partial E}{\partial x_i}$ to the ith business unit, where E is the total risk capital and x_i is the investment in the ith business unit. By using the Euler's theorem we ensure that the total of the allocated capital is E. Euler's theorem says:

$VaR = \sum_{I=1}^{N} \frac{\partial(VaR)}{\partial x_i} x_i$ where N is the number of components. We can therefore set

$C_i = \frac{\partial(VaR)}{\partial x_i} x_i$ where C_i is the component VaR for the ith component. We define ΔE_i as the

increase in the total risk capital when we increase x_i by Δx_i. A discrete approximation for the

amount allocated to business unit i becomes: $\frac{\Delta E_i}{\Delta y_i}$ where $Prob(\upsilon > x)$. When we increase

the size of the hydropower generation by 1% its economic capital amounts for market, basis and operational risk increases to 151.5, 95.95, and 55.55, respectively. New economic capital (hybrid approach) becomes 301.75, so that ΔE_{HP} = 301.75 – 299.73 = 2.02. Increasing the size of the network division by 1%, implies an increase in the economic capital for market, basis and operational risk to 45.45, 38.38 and 25.25, respectively. The total economic capital becomes 300.11, so that ΔE_{NT} = 300.11 – 299.73 = 0.38. The numbers for telecommunication is ΔE_{TC} = 300.33 – 299.73 = 0.60. The economic capital allocation gains are therefore divided between hydropower generation, network, and telecommunication by 2.02/0.01 = 202, 0.38/0.01 = 38[25], and 0.30/0.01=60, respectively.

7. Summaries and conclusions

The paper set out to measure volatility/correlation and market/operational risks for a general corporation in European energy markets. Starting with a relevant risk discussion the corporation may perform risk analysis based on either the argument of asymmetric information relative to owner or based on costs related to financial distress/bankruptcy costs.

For the Nordpool and the EEX energy markets the paper shows estimates of product and market volatility/correlations and makes one-step-ahead forecasts. The paper performs a model-building approach applying Monte Carlo simulation. Stochastic volatility models are estimated and simulated for risk management purposes. From the power law, the extreme value theory are used for VaR and CVaR calculations (smoothing out tails). The normal distribution assumptions make these analyses a relatively easy exercise for VaR and CVaR – distributions. Non-normality can be easily implemented applying Copulas. Finally, risk aggregation is shown for market and operational risk for normal as well as student-t distributions.

8. Appendix I : The theory of reprojection and the conditional mean densities

Having the SV model coefficients estimate $\hat{\theta}_n$ at our disposal, we can elicit the dynamics of the implied conditional density of the observables $\hat{p}(y_0 \mid y_{-L}, ..., y_{-1}) = p(y_0 \mid y_{-L}, ..., y_{-1}, \hat{\theta}_n)$. Analytical expressions are not available, but an unconditional expectation $E_{\hat{\theta}_n}(g) = \int ... \int g(y_{-L}, ..., y_0) p(y_{-L}, ..., y_0, \hat{\theta}_n) d_{y_{-L}} ... d_{y_0}$ can be computed by generating an simulation $\{\hat{y}_t\}_{t=-L}^{N}$ from the system with parameters set to $\hat{\theta}_n$ and using

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[25] Does not equal the total economic capital of 299.73, because we approximated the partial derivatives.

$E_{\hat{\theta}_n}(g) = 1/N\sum g(\hat{y}_{t-L},...,\hat{y}_t)$. With respect to unconditional expectation so computed,

define $\hat{\theta}_K = \dfrac{\arg\max}{\theta \in \Re^{\theta_K}} E_{\hat{\theta}_n} \log f_K(y_0 \mid y_{-L},...,y_{-1},\theta)$, where $f_K(y_0 \mid y_{-L},...,y_{-1},\theta)$ is the SNP

score density. Now let $\hat{f}_K(y_0 \mid y_{-L},...,y_{-1}) = f_K(y_0 \mid y_{-L},...,y_{-1},\hat{\theta}_K)$. Theorem 1 of Gallant and

Long (1997) states that $\lim\limits_{K\to\infty} \hat{f}_K(y_0 \mid y_{-L},...,y_{-1}) = \hat{p}(y_0 \mid y_{-L},...,y_{-1})$. Convergence is with

respect to a weighted Sobolev norm that they describe. Of relevance here is that convergence in their norm implies that $\hat{f}_K$ as well as its partial derivatives in

$(y_{-L},...,y_{-1},y_0)$ converges uniformly over $\Re^\ell$, $\ell = M(L+1)$, to those of $\hat{p}$. They propose to

study the dynamics of $\hat{p}$ by using $\hat{f}_K$ as an approximation. The result justifies the approach.

Hence, the conditional mean density is from 5 k iterated use of the re-projection procedure. For every simulation from the normally distributed coefficients, re-projected scores $\hat{f}_K(y_0 \mid y_{-L},...,y_{-1})$ are estimated and the conditional moments (mean and variance) and the filtered volatility are reported. The power law is also evaluated for the conditional mean series. Figure 16 report the power law test results for simulated and conditional mean 100 k data series. The power law seems to work well for both markets and the four series.
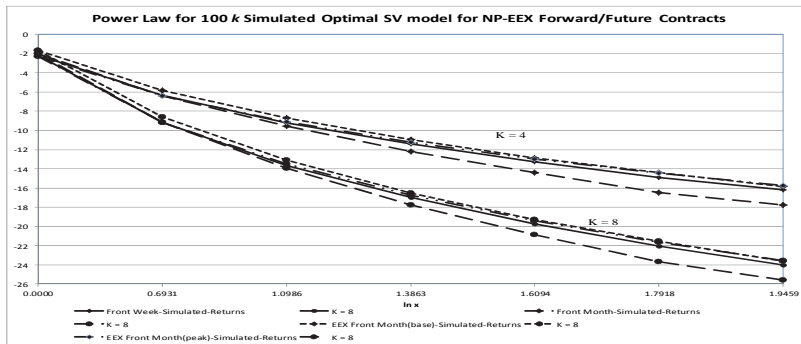
$$Prob(\upsilon > x)$$

Fig. 16. The Power Law for SV-simulated and Conditional mean series: Log plots for return increases: x is the number of standard deviations ; υ is the NP / EEX price increases/decreases.

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