



# Master's degree thesis

**LOG950 Logistics**

**Maritime Distribution in NorStone**

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## **Preface**

This master thesis constitutes a part of a larger project “Ny logistikk løsning for NorStone” developed in collaboration with Møreforskning AS and Molde University College. NorStone AS, owned by the Heidelberg Cement Group, has involved Møreforskning AS in order to develop and design a new logistical solution to their seaborne distribution problem. Time span of this project is stipulated to two-three years, and the superior objectives are both comprehensive and demanding. The purpose of this project is to develop a decision support system that reduces the overall transportation cost while maintaining a high level of customer services.

The objective of this master thesis is to develop a full-scale mathematical model in order to optimize the maritime distribution in NorStone AS. This model contributes as a part of a research project involving Møreforskning AS and NorStone AS, and will be solved by a heuristic approach at a later stage, developed by our supervisor Johan Oppen.

Since the beginning of January data collection has been a continuous process. In addition to e-mail correspondence, a visit at the head office and the production site at Tau lead to better insight. We are grateful for the warm reception and the substantial support that we received from NorStone during the process. Thank you, Oddmund Oterhals and Cristina Ciobanu at Møreforskning AS, for providing us with the opportunity to write this thesis. Finally, we draw attention to Johan Oppen for his ability to follow-up and outstanding assistance throughout the process of writing this master thesis.

This paper and the following oral presentation 12/6-09 constitute the course Log 950, and are considered as a development of the proposal carried out during the fall of 2008.

## **Abstract**

Our objective has been to develop a mathematical model that describes the real world, full scale maritime distribution problem at NorStone and to generate a foundation for further development. We have searched the literature of Vehicle Routing Problems (VRP) to find an adequate basis to describe our problem and to build a model. In our effort to adopt the model to the need of the maritime distribution problem, we have connected two fields of literature in an untraditional way: VRP and Inventory Management. However, this cannot be considered to be a pure Inventory Routing Problem.

The background for the Møreforskning project is to explore if there are potential benefits by using optimization based on mathematical modeling, instead of applying manual planning. From our data selection period in 2008 the maritime distribution planning was segmented into decentralized areas, where the production sites were responsible for manually scheduling the deliveries to the customers. Furthermore, the planning horizon for deliveries was short, approximately from one to three days. In our model we want to establish a centralized distribution system where all deliveries from all production sites are planned simultaneously for a longer time horizon. The objective is to achieve better planning for application of vessels together with achieving better visibility of inventory.

We have performed a thorough data analysis of the company to be able to illustrate the real world problem and to generate a comprehensive problem description. The analysis resulted in several interesting findings that constitute an adequate basis for further discussion and the making of the mathematical model.

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# 1 Introduction

In a time when the whole world is affected by the global financial crisis and its effects, costs saving measures are highly prioritized in most industries. Consequences of the present financial crisis has reached the Norwegian stone market as well, and enforced all the actors in the market to either increase profit or reduce costs along the value chain. Within this industry, the transportation costs are enormous. The effort concerning cost saving measures should contribute to reduce transportation costs by finding alternative distribution systems, improved effectiveness of current fleet, better planning methods and reduced waiting time.

Our opinion is that this industry is characterized, to a great extent, by manual transportation planning between production sites, terminals and their customers. When an order arrives to a specific site or an “order- central”, the charterer is responsible to provide available ships/vessels. In this thesis the terms *vessel* and *ship* are used interchangeably, as well as product and fraction. The ship selection for stone transportation is based on different types of contracts, and those will be explained in chapter two. The distribution between types of contract complicates the routing problem further. In a fast moving market, shipping companies are dependent on evaluating their ability to complete contract commitments. In addition, they have to check future contract’s profitability in regard to the development in the SPOT market. Contract evaluation will be further discussed in chapter 2.3. Shipping market develops in the direction of smaller and more frequent deliveries, which makes it possible for vessels to perform in a more effective way.

This master thesis constitutes a part of the pre-project between Møreforskning AS and NorStone AS. The background for our part of the project is based on NorStone’s request for development and optimizing of a new logistical solution to their seaborne distribution, based on the current vessel fleet. The main project has three important objectives:

1. Develop a planning model for optimizing the product distribution
2. Develop a fleet model for optimizing the vessel sizes.
3. Analyze and optimize the whole value chain from production site to customer.



Our master thesis is included within first objective, in parallel with the pre-project. We considered it as difficult to solve this problem by current solution methods or mathematical programming software. As a consequence, we decided to develop a full-scale model that considers the maritime distribution in NorStone. The main focus is to optimize the distribution problem between all sites, terminals and customers in Rogaland and Hordaland. The model should be as comprehensive and realistic as possible, based on the features and information we possess at present time. In accordance with the objectives, the model is supposed to be solved by a heuristic solution method, developed by our supervisor Johan Oppen at a later stage.

During the last past winter we received some data, such as cost and distance matrices, specifications concerning the fleet, production sites and terminals, and most important: the vessels report from 2008. We considered it as natural that we used the impressions and figures from Tau production site as basis, in order to present and describe the value chain, including related factors and processes. We visited this site in the beginning of January 2009 and were guided through all the relevant steps in the value chain. Tau is the largest NorStone owned site in Rogaland and Hordaland. We have assumed that the value chain is similar at each of the other sites and differs only in size and product specifications.

All data have been collected through mail-correspondence, interviews, telephone conversations and internet. In agreement with Møreforskning AS and NorStone AS, the data-set is limited to consider all sites, terminals and customer locations in Rogaland and Hordaland during 2008. The selection is justified by two factors; the share of total tonnage and the potential of future savings within this region. NorStone distributed 4.1 million tdw in 2008, 63% were carried out in Rogaland and Hordaland.

Analyses of the annual vessels report were conducted in order to expose possibilities and challenges that can explain and improve the current maritime distribution. Some of the analyses are based only on June 2008, due to the large number of customers, vessels and product diversity. All inputs in the model are based on schedules from each site, cost and distances as revealed by the received documents.

The next chapter is dedicated to introducing the case and the factors we found as important in our analysis of the value chain and the current maritime distribution system in NorStone.

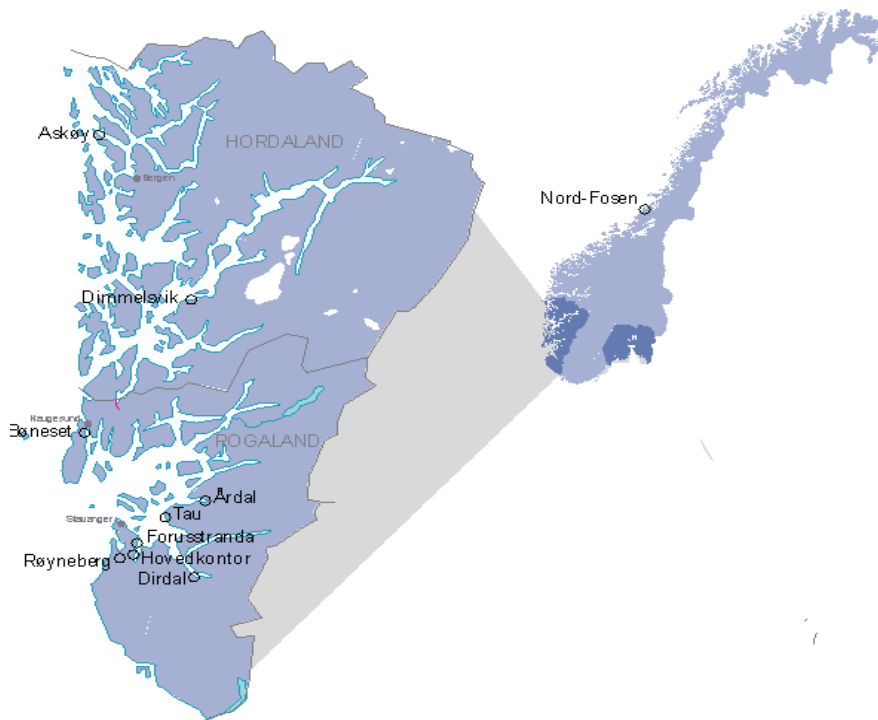
## **2 Problem description**

This chapter starts with a comprehensive presentation of the company, NorStone AS, then describes their current maritime distribution planning, their challenges and limitations, and ends with a final problem formulation. Different aspects and factors at various stages along the value chain restrict the existing distribution system. In this thesis we seek to develop a model that optimizes the maritime transportation of stone production. In order to analyze the maritime distribution we have studied NorStone's production sites, terminals and customers, in addition to assessing different types of contracts and capacity constraints.

### **2.1 *NorStone AS***

The Norwegian mountains contain some of the oldest sorts of stone in the world. The bedrock was formed billions of years ago by hardened, liquid material from the inner earth, and consists of a particular good quality concerning comprehensive strength and durability. In order to exploit the unique qualities of the Norwegian mountain, NorStone has acquired the rights to valuable deposits of Norwegian stone and sand.

NorStone is a traditional Norwegian company owned by Heidelberg Cement Group, a concern performing businesses in 50 countries around the world. NorStone is the biggest manufacturer of aggregate for asphalt and concrete in Norway, with a production capacity over 8,020,000 tons/ year (2007). NorStone has ten production sites located in different regions, six sites in the western Norway and four sites in south-eastern Norway (where NorStone operates for local markets only). In order to maintain a competitive advantage, NorStone uses four terminals (three of them in Rogaland and Hordaland) as distribution centers and to some extent used as temporary inventory holding facility. The role of the terminals is to facilitate customer pickup by truck for further transportation to inland locations along the Norwegian coastline. NorStone has 172 employees distributed on main office and production/shipping facilities. Figure 2.1 shows production sites and terminals along the Norwegian coastline.



**Figure 2.1:** Sites and terminals along the western coast of Norway Source: ([www.norstone.no](http://www.norstone.no))

The product variety comprises different stone sizes used in industries like railway, offshore, contractor and road building. However, most of the production provides for the asphalt and concrete markets. All products are delivered from the plants directly to their customers by vessels from 500 up to 40,000 tons deadweight (tdw). Deadweight, defined by Christiansen et al. [8], is the weight carrying capacity of a ship in metric tons, including the weight of cargo, weight of fuels, lube oils, supplies and everything else on the ship. NorStone delivers to customers located along the coastline of Norway and northern Europe, and they offer the guarantee that the cargo has the same quality during unloading as during loading. All vessels used for stone transportation are equipped to prevent the material from separating, mixing and suffering contamination from sea-water. The loading capacity is normally about 1000 tons/hour at each production site.

Most of the transportation to northern Europe is shipped by NorStone's joint venture partner, Stema Shipping. In 2008, approximately 1.5 million tons of NorStone's total production was picked up and distributed by Stema Shipping. These vessels have top priority at the ports and possess capacities up to 40 000 tdw. Their arrivals are relatively frequent at all sites and known weeks in advance, which makes it easier to schedule. Vessels from Stema Shipping require substantial time when loading at the production sites,

due to their size. If other vessels arrive for loading within the same time period, waiting time are generated for all other vessels until vessels from Stema Shipping have completed their pickup.

The production processes present some variety from site to site, but is mainly a stepwise process. It begins when NorStone purchases and cleans up the land for soil and wood. Then they drill and place a high amount of explosives, which after the explosion results in a pile of stone equal to approximately 110 000 tons. Further, the stone is transported from the piles to a processing plant for fine or coarse crushing. The transportation is performed by one to three big dumper trucks, a sub-process that triples the cost at this stage in the value chain. In the processing plant is the stone crushed into various grades and mix compounds of curves according to the customers' requirement. After processing and storage are the final products shipped out on a conveyor belt, through a discharger (land-side cargo handling equipment), onto predetermined vessels.

Figure 2.2 illustrates all depots/production sites, terminals and customer locations in Rogaland and Hordaland during June 2008. Due to the large amount of information regarding site production and customer locations, we choose to illustrate on our presentation the state of the facts for the best month in 2008. Each production site is marked with a vessel-icon because all transportation from these sites is done by vessels. The three terminals are marked with a truck-icon, since all transportation from the terminals is done by trucks. Each customer that was served during June 2008 are located and marked by a push-pin. The yellow trace illustrates a suggested shipment between the site at Dimmelsvik and the terminal at Bønaset.

The map confirms short distances and a significant complexity. A large amount of vessels serves customers located all over the western coast of Norway. Orders arrive continuously and they need to be executed within a period of one to three days. A large number of possibilities and constraints complicate the charterers planning problems. As an example, product variety is both a constraint and an opportunity: sand can be delivered only from Årdal, while some customers require the Dura-Split® quality which can be produced only at Tau. Capacity constraints and several maximum/minimum limitations on ports, vessels and dischargers complicate the maritime distribution problem even further. The customers are located rather evenly, with some natural density around inhabited areas as the city of Bergen and the city of Stavanger. All vessels are hired from local ship-owners, on the three

mentioned types of contracts. In order to remain competitive in the long run, an optimal, or at least close to optimal, mix of contracts is considered as necessary. Such decisions are dependent on NorStone's opinion of the present and the future markets.

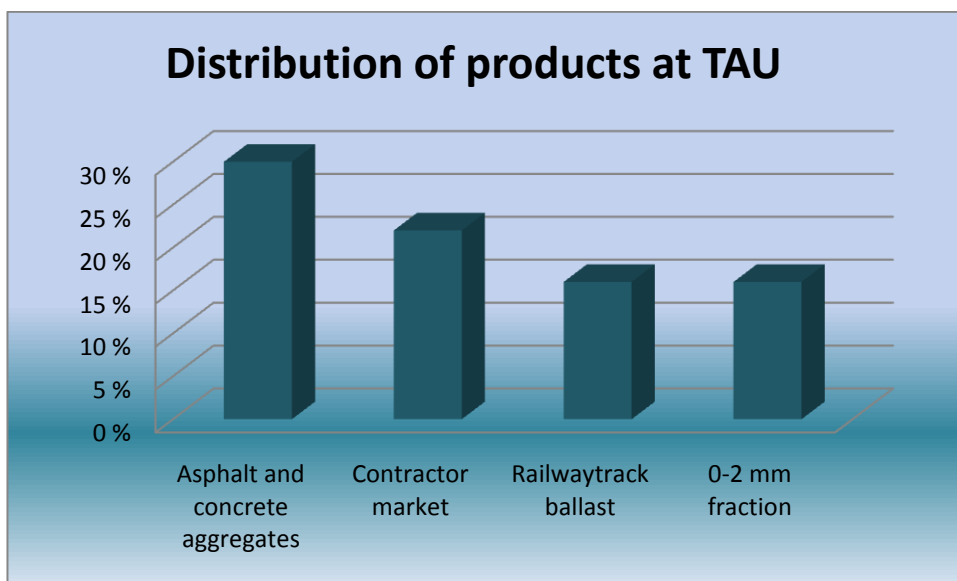


**Figure 2.2:** A Google earth-based illustration of sites, terminals and customers during June 2008, within Rogaland and Hordaland areas.

### 2.1.1 Products description

NorStone produces aggregates for asphalt and concrete, ballast for railway track, material for road building, materials for protection of oil and gas pipelines, materials for the contractor market, etc. All products are based on the same type of raw material, and the final crushing throughout the processing plant determines the given product range. The range of products is broad; Tau alone offers 11 different fractions in their assortment and those can have different quality from site to site, both in size, type and quality.

Since the products and processes are quite similar at all sites we choose to present the products delivered from Tau. The reason for this is our visit to Tau in January, and the following interviews with the site manager. Tau produces mainly aggregates for asphalt, concrete, and railway track ballast. The first fraction sorted out along the production process is the 30-60 mm or “railway track ballast”, which is a very profitable final product due to low production cost, high demand and thus a good market price. This product accounts for relatively high amount of the annual production at Tau, approximately 16 %. The most profitable fractions 2-5 mm, 5-8 mm and 8-11 mm, covered 30 % of the production. These later fractions are sold as asphalt and concrete aggregates. Approximately 22 % of the annual production in 2008 was delivered to the contractor market, while 16-17 % ended up as 0-2 mm fraction, which has the highest production cost and is least profitable. Figure 2.3 summarize the mention findings, and illustrates the percentage distribution between all products.



**Figure 2.3:** Illustrates the percentage distribution of the products delivered from the site at Tau.

Table 2.1 includes products from all sites and contains a brief description of the main products produced by NorStone.

**Table 2.1:** Overall description of the main products produced by NorStone

<b><u>PRODUCTS</u></b>	<b><u>SPESEIFICATIONS</u></b>
<b>ASPHALT AGGREGATES</b>	Aggregates from NorStone can be found in the motorways in several European countries. Dura-Splitt® is their trademark for asphalt aggregates produced at NorStone's plant at Tau in Rogaland and known for its durability and solidity. The aggregate determines the quality of the asphalt regarding resistance, friction and brightness.
<b>CONCRETE AGGREGATES</b>	High quality is both expected and required within the production of concrete aggregates. Concrete consist of cement, water, rocks and sand. Rocks and sand are named aggregates. Concrete aggregates are of most importance in regard to the quality of the concrete. Concrete aggregate from the plant in NorStone's plant in Årdal is used as a Norwegian reference within aggregate product. Characteristics of quality are measured by these criteria: sizing, roundness and non-reactivity.
<b>OFFSHORE PRODUCTS</b>	Offshore products are basically applied to cover underwater pipes and preparing the sea-bed for installations. NorStone has delivered to all oil-field in the Norwegian part of the North Sea since 1980. These offshore rocks are delivered from the plant in Dirdal and Nord-Fosen Pukkverk, and can be delivered in large quantities. The plants can load vessels with a capacity equal to 25,000 tdw. Special vessels are used to unload the products.
<b>RAILWAY TRACK BALLAST</b>	Track ballast, as well as offshore rocks, have high requirement concerning quality. NorStone produce and ship large quantities of railroad rock within these quality requirements.
<b>CONTRACTOR MARKET</b>	The stone is used in the contractor market, were it is applied as filling compound and grading in projects like: development of roads and lots.

### 2.1.2 Production sites

NorStone operates six different production sites along the western coast of Norway. The illustration in figure 2.2 expose that most customers and production sites are located in the area close to the city of Bergen and the city of Stavanger. Volumes and capacities vary from site to site. Dimmelsvik is the smallest site, with an annual production of 200 000 tons. In contrast is Jelsa the largest, with an annual production that is 30 times bigger than Dimmelsvik, approximately 5 million tons. Table 2.2 is a concise description of applications, capacities and relevant information listed for each of the sites. These six production sites complicate the charterer's problem even more, while it simultaneously makes it easier to meet the customers' requests. Vessels can be loaded at different production sites when the discharger is occupied, and that reduces the waiting time considerable. All ports at the production sites are open day and night for shipment of products; still a considerable amount of waiting should be taken into account, especially during peaks.

**Table 2.2:** Description of the six production sites operated by NorStone AS

<u>Production site</u>	<u>Specifications</u>
<b>TAU</b>	Tau produces approximately 2.3 mil tons of different stone products per year, mainly aggregates of asphalt and concrete, railway track ballast and offshore products. The port at Tau is capable of loading vessels up to 25 000 tdw, with an average loading capacity equal to 1500 ton per hour. The best trademark for asphalt aggregates, Dura-Splitt® is only produced at Tau. This department had 893 port calls during 2008, and the customers were spread all over the northern part of Europe. Approximately 200 of these calls were performed by local ship-owners that execute contract-based transportation. The rest of the calls were performed by Stema Shipping.
<b>ÅRDAL</b>	The production site in Årdal produces close to 1 mil ton of natural sand each year, mainly different fraction up to 22mm. These products/fractions are applied in the concrete industry. Årdal is often restricted by low production capacity, related to the rate of turnover. The storages are filled up during Tuesday, Wednesday, Thursday, but often sold out during the weekend, due to restricted production capacities. The port can load vessels up to 8000 tdw.
<b>DIMMELSVIK</b>	Dimmelsvik is the smallest of NorStone's production sites with an annual production of 200 000 tons. This site produces only to the local markets and has the smallest shipping capacity, 1500 tons.



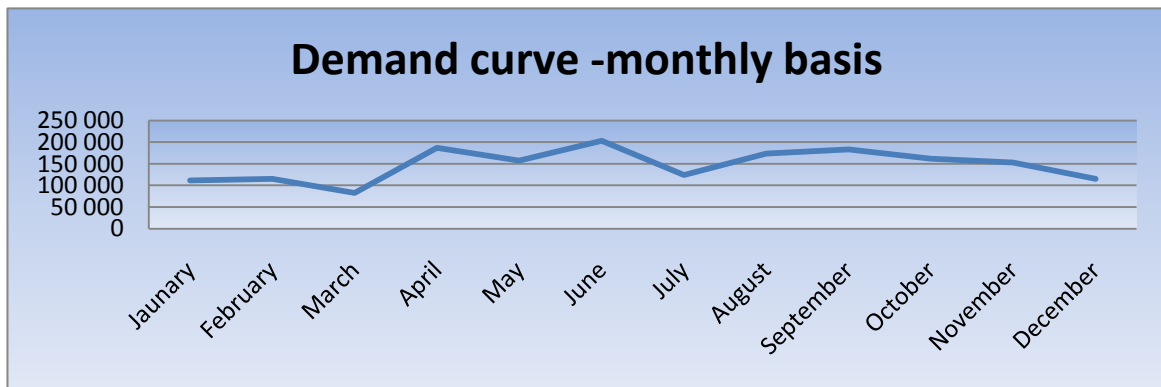
<b>ASKØY</b>	Askøy produced between 700 000-800 000 tons in 2008. The products are mainly the same as produced at Tau, however the quality deviate from the Dura-Splitt®. The deviation in quality is so significant that some areas of application cannot use products from Askøy. The shipping capacities are considered as good, since this port loads vessels up to 30 000 tons.
<b>JELSA</b>	This site had an annual production equal to 5.5 mil ton in 2008, and budgets with a total production amount at 10 mil tons in 2009. Jelsa operates with more or less the same range of products as Tau. This site is 50% owned by Heidelberg Cement Group and 50% owned by Stema Shipping. Most of the goods are transported abroad.
<b>DIRDAL</b>	Approximately 800 000 tons of stones to the offshore industry was delivered from Dirdal in 2008. Mainly sand for insulation, offshore products up to 200mm, ballast and masses for covering of subsea pipes and some deliveries to the local market. A huge shipping capacity, approximately 60 000 tons.

### **2.1.3 Terminals description**

In addition to the six production sites in Rogaland and Hordaland, NorStone has three terminals located alongside the Norwegian western coast at Forus, Laksevåg and Bønaset. These terminals are used in order to serve local customers within a specific area and product distribution is based on trucks, but the terminals are supplied by vessels. In the dataset the transportation between production sites and terminals are considered as a customer order. The role of the terminals is partly to create a buffer in product delivery for the local customers, who do not need large quantities at once, but rather smaller quantities and more often. Therefore, terminals are a potential extra inventory holding spot for the production sites.

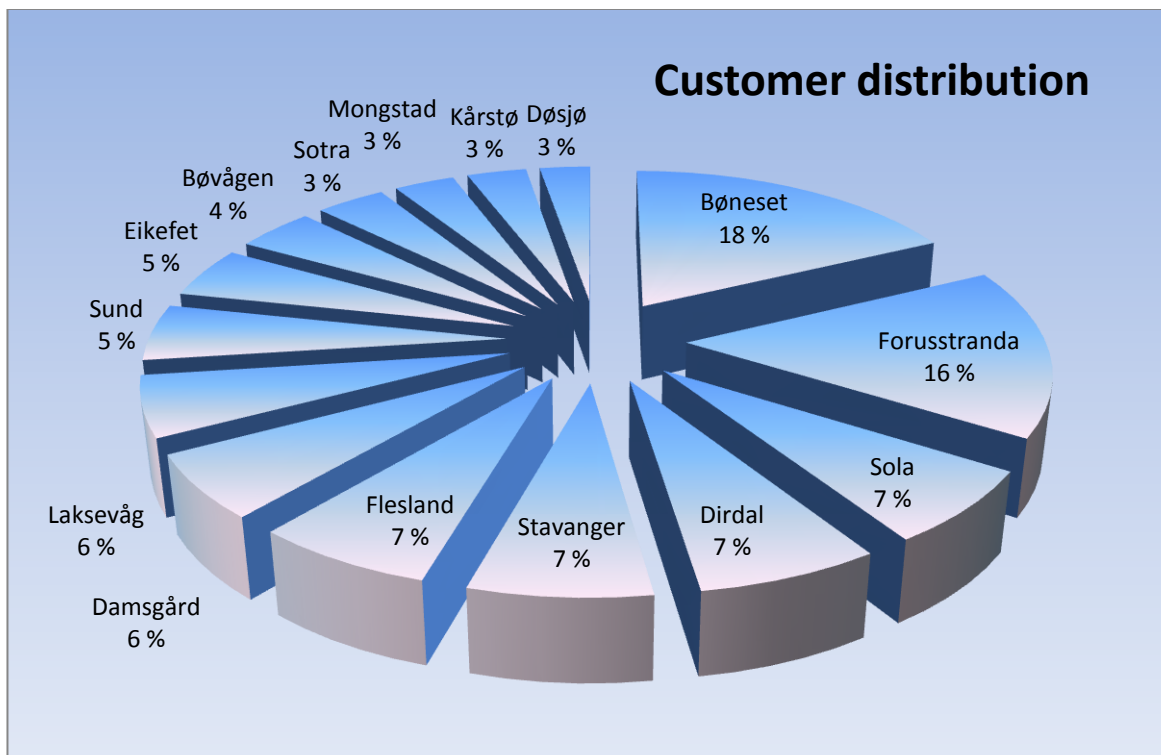
### **2.1.4 Shipment overview**

During our visit to Tau in January, we were told that the site at Tau had 80% fixed customers, although fixed do not always represent fixed customer locations. Data collected from 2008 illustrates a relatively smooth demand curve, with some peaks during April and June. Figure 2.4 illustrates the annual demand in Rogaland and Hordaland during 2008. Based on the peaks between March and July, we decided to use the data from June as basis for a further analysis and studies.



**Figure 2.4:** *The annual demand in Rogaland and Hordaland.*

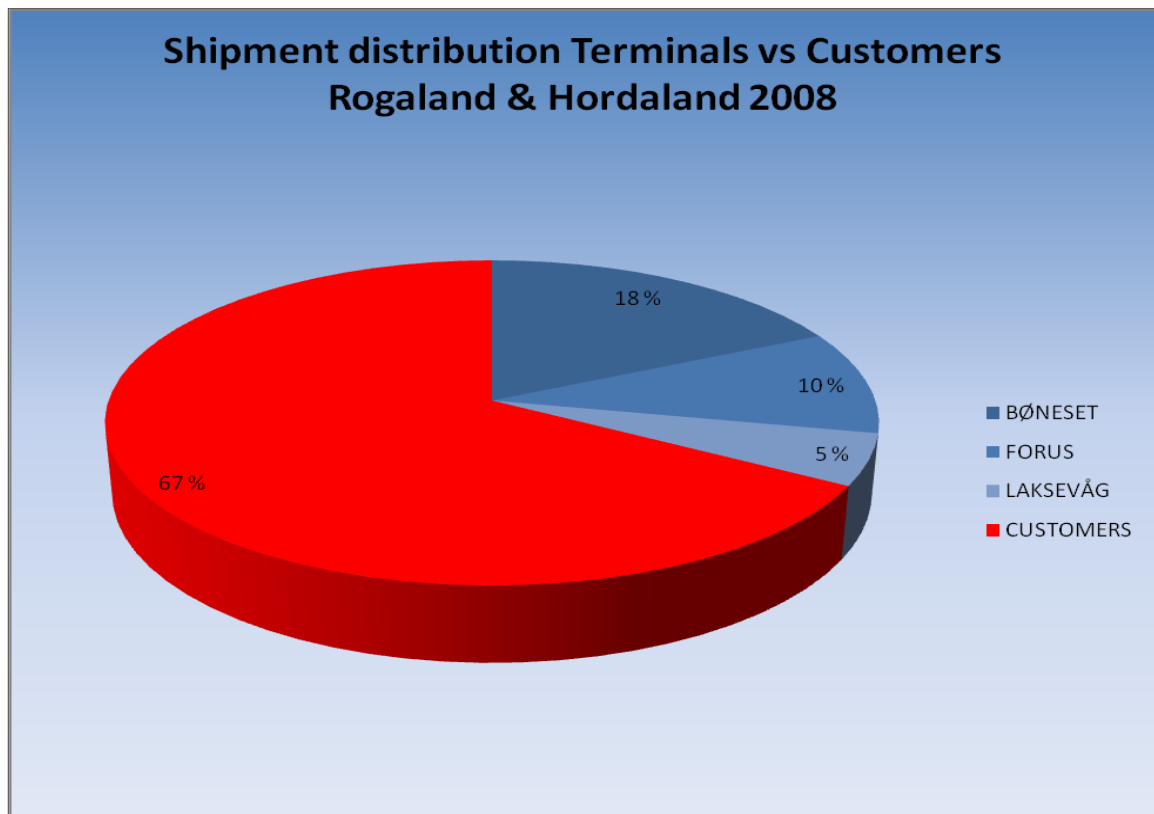
The developed cost and distance matrix presents the exact distances between all locations, as well as illustrating the costs of all possible shipments. Such data matrices are necessary in order to generate an adequate vehicle routing model that optimizes the maritime distribution of NorStone. During June, 38 customer locations (including three terminals) in Hordaland and Rogaland were visited at least once. The reason for including the terminals in our analysis is caused by the order process which treats those shipments as customer orders. Figure 2.5 illustrates a percentage distribution of transported goods in June 2008. We discovered that approximately 40 % of all shipments are executed from the sites to the terminals at Laksevåg, Forus and Bøneset.



**Figure 2.5:** *Percentage distribution of the customer locations served in June 2008*

Figure 2.6 is an annual distribution chart of shipments to terminals and customers. Of all shipments, 33 % are carried out between production sites and terminals, which is equivalent to 650 000 tons. The total amount of tonnage distributed in Rogaland and Hordaland is calculated to approximately 1 900 000 tons. During 2008 were about 80 customer locations served at least once by one of the 44 heterogeneous vessels.

NorStone operates in a market that moves in the direction of demanding smaller and more frequent deliveries. The average freight per shipment was approximately 1600 tons in 2008.



**Figure 2.6:** Percentage distribution between terminals and customers in Rogaland and Hordaland for the whole year.

### 2.1.5 Time windows

The model we are trying to develop must be capable of generating solutions that takes time windows into consideration. It is important that the vessels arrive within the opening hours at each customer locations. The sites and terminals are open day and night, however at almost all the customer locations the vessels can only be served at the ports from 07:00 to 16:00 hours. In contrast to soft time windows, opening hours are not interchangeable and therefore considered as hard time windows. Waiting time is a significant problem during the peaks, and needs to be taken care of by improving the routing scheduling. Vessels

operated by Stema Shipping have first priority at all sites when they arrive at sites and they occupy the ports for about 15-20 hours during each loading. Therefore, in situations where the site is occupied, smaller vessels have to either wait for loading or call on other sites that can provide the customer needs. Moreover, the operators in NorStone is restricted to complete the customer orders within a predetermine time span of earliest and latest time of delivery.

## **2.2 Value Chain**

This section describes the value chain as a stepwise process, including raw material, production, inventory, shipment and order- processing. In the end we present a figure that illustrates the cost allocation in the supply chain.

### **2.2.1 Raw material processing**

Exposure, explosion and transportation of the raw material are very time-consuming processes. It is an important factor that all of these steps take place in parallel, and hence minimizes the waiting time at each of the processing activities. Figure 9.1 and 9.2 in the appendix is a picture from the quarry at Tau, and illustrates how the land is cleaned up for wood and soil, before they place explosives and transport the stone over to the processing plant.

### **2.2.2 Production processing**

Big blocks of stone are transported down to the processing plant continuously then tipped into a primary-crusher and crushed down to fractions between 0-250 mm. These fractions are sorted and then transported by a conveyor belt for intermediate storage before it is transported to the secondary crusher. The fractions resulting from the secondary crushing are either final products like gravel or fractions that need to be stored for a further processing activity. Final products resulting from crushing and straining are stored in dedicated places around the production machine. All fractions are sold either in a mix or individually, according to the customers preferences. For illustration photos of the processing plant at Tau, see figure 9.3, 9.4 and 9.5 in the appendix.

### **2.2.3 Inventory processing**

Storage of the final product is the last step before loading onto vessels. Every fraction is transported and stored separately into large inventory bins/compartments. At Tau, those bins have a capacity from 25 000 to 50 000 tons, and photographed in the figure 9.6 in the appendix. Some of the low demanded fractions are stored remotely in specific areas at the

sites and generate an increased inventory holding cost. A conveyor belt that crosses under each of the inventory bins is developed in order to select, mix and transport a given amount of each fraction. Some of the sites remain short on inventory during the peaks. As an example, Årdal produces from Monday to Thursday, but often get short at the end of the weekend. At Tau is the inventory levels controlled by the production manager, and adjusted to the market situation by following the fluctuations on the inventory.

#### **2.2.4 Order processing**

Product quality, order amount and waiting time for order, vessel and products, are vital parameters in order to optimize the order scheduling, whether it is manually or by solving a mathematical model. NorStone plans to restructure their order processing during 2009, and works on establishing a customer service office that is capable of managing product orders for the entire company at Sandnes. When a customer order is received the charterer is responsible for generating feasible solutions and to optimizing the fleet utilization. The size of the orders varies from approximately 200 to 5000 (tdw) and they deliveries are usually made to a fixed set of regular customers. In addition, NorStone deliver considerable amounts of products to large oil and gas projects, as well as Norwegian road and railway projects. A heterogeneous fleet makes it easier to maximize the fill rate per shipment, but the tough delivery time complicates their scheduling remarkably. Delivery time of one-three days after receiving an order is a result of offering and maintaining a superior customer service level.

#### **2.2.5 Shipment processing**

Vessels used by NorStone between ports all over the world can be used both for cargo loading and unloading, as well as for loading fuel, fresh water, supplies, and discharging waste. Most important characteristics are the physical limitations imposed by the vessels draft, length and width. However, such limitations are not a problem at NorStone's production sites and terminals, but some of the customer's ports need to be served by specific vessels.

About 12-15 vessels constitute NorStone's present chartered fleet. These vessels are heterogeneous, hired from external ship-owners, which means that all vessels have different capacities and specifications. Current fleet ranges from 500 to 5000 tdw, all vessels hired through three different types of contracts: TC, COA and SPOT. We describe their main features in the following sections.

### 1. Time charter (TC)

The vessels are hired for a specific amount of time, usually from one-three years, where the charterer controls the operations of the vessel and decides the routing. This contract is defended through the “economy of scale”- principle, which in practice means that it is profitable to apply TC contracts as much as possible, as long as the amount and size of orders are adequate. At present time, NorStone operates with 2 vessels on TC-contract; Kongsvaag and Dynabulk. The cost of a time charter contract differs from contract to contract, depending on size, objective, fabrication year and operating costs. The costs of current TC-vessels are a fixed amount per day, in addition to an accumulated cost of bunker per shipment.

### 2. Contract of Affreightment (COA)

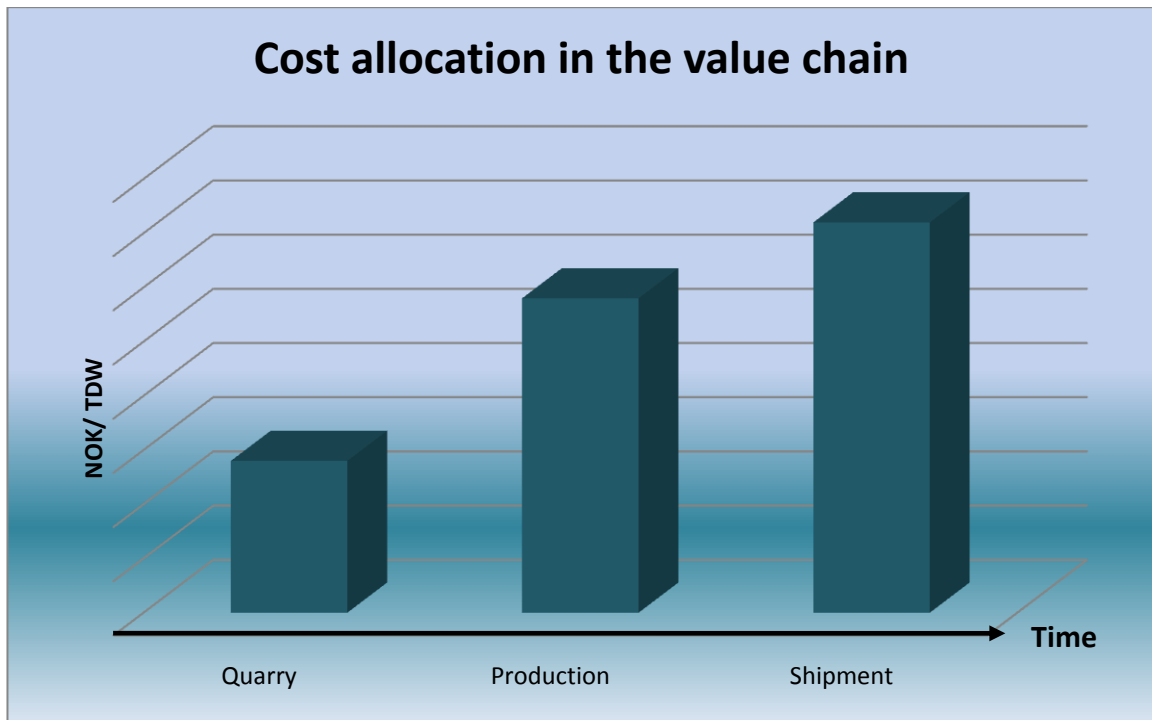
Most applied in NorStone’s case is the “Contract of Affreightment” (COA), which is a contract to carry specified quantities of cargo between specified ports within a specific time frame for an agreed payment per ton. This contract consists of two parties; the ship-owner and the charterer, where the ship-owner agrees to transport the charterer’s goods with his vessel/vehicle. NorStone hired several vessels on these conditions during 2008. Normally, COA is twice as expensive as the time charter contract. The price is determined by distance and quantity, independent of which products are freighted.

### 3. SPOT

SPOT contracts are often applied during peak seasons when the existing capacities are no longer sufficient. This contract type consists of the same elements as the COA, but operates through shorter notice-time. The vessels are no longer managed by the charterer, but by the ship-owners, who determines whether they accept to complete the order or reject it. The SPOT prices are dependent on the market situation (supply and demand). However, it is considered as very expensive in the long run (in general 4 times as high as TC-contracts, and 2 times the cost of COA) and therefore used only in specific situations when other vessel capacity is not available.

## **2.2.6 Cost allocation in the value chain**

Figure 2.7 illustrates the relationship between the accumulated costs in the value chain, and witness that the most expensive part of the value chain is given by the shipment activities. The raw material processing corresponds to the half of the cost of production processing, and constitute the first and the least expensive process in the value chain.



**Figure 2.7:** Shows the cost allocation in the value chain.

### **2.3 Contract evaluation**

Contract evaluation considers three important aspects; the fleet size, composition issues and whether to accept long-term contracts. Based on those aspects:

The shipping company has to evaluate whether it has sufficient fleet tonnage to fulfill the contract commitments together with its existing commitments, and if so, whether the contract is profitable (Christiansen et al. [8]).

Thus, in order to check the contract's profitability, assumption regarding the development of the future spot market (in a given time period) needs to be considered. A common strategy among the shipping companies is to prefer as large contract coverage as possible when low spot rates are anticipated. The distribution of contracts depends on the trade-off between customer relations and costs. [8]

NorStone's current shipping policy is based on a long-term evaluation of the ship-owners they cooperate with. Considering the present market, SPOT-contracts would be very profitable in the short run, due to low demand and favorable bunker rates. Quality in transportation, the ability to deliver within given terms, and benefits from long-term relationship is important factors that determine the distribution of contracts. NorStone strive to maintain their "best in class" philosophy and, in order to do so, they offer customer service at highest level by providing deliveries within one-three days.

Maintaining this request is possible due to a heterogeneous fleet and a broad cooperation with various ship-owners. Altogether, 44 heterogeneous vessels distributed approximately 1.9 million tdw of stone in Hordaland and Rogaland during 2008. Only 12-15 of these vessels are committed to NorStone through TC or COA contracts, the rest are hired in through the SPOT market. Beside the SPOT market, NorStone collaborates with three local ship-owners and manage approximately 100%, 50% and 30% of their fleet.

Dynabulk and Kongsvaag represent the fleet of TC-vessels. In respect to our data selection, Kongsvaag is the vessel with most shipments in the region. Kongsvaag carried out 105 shipments in Rogaland and Hordaland, out of a total number of 164 shipments during 2008. Dynabulk is mostly used for shipments with a longer duration and a decisive reason can be the size of the vessel which is beneficial for accommodating to long duration shipments. In 2008, only 6 out of 112 shipments were executed by Dynabulk in the selected region.

When it was decided that Rogaland and Hordaland would be the selected region, NorStone made an inquiry to make an in-depth analysis of the utilization of one COA and one TC-vessel, namely Falksund and Kongsvaag. These two vessels are representative for approximately 30% of the total tonnage distributed in Rogaland and Hordaland.

Falksund is hired on COA-conditions, however 100 % administrated by NorStone. In accordance with the inquiry made by NorStone, we decided to conduct the analysis in respect to their interests.



## ***2.4 Status Quo 2008- analyses of empirical data.***

The current maritime distribution at NorStone, the way we see it, seems to be characterized by manual planning, a lot of reorganization and a great desire of maintaining superior customer service, despite the cost of sub optimizing their routing problems. Our impression is that the mix of reaching for a high service level (through minimum delivery-time) and manual planning, leads to a stressful and inflexible situation, which most likely generates uneconomical solutions.

The annual vessel report for 2008 describes the current situation and exposes a high average fill-rate, some multiple pickups and none or an insignificant amount of backhauls. We have calculated the fill rate and developed an overview that describes both average and individual fill-rate for each vessel. These findings are based on annual figures and don't include backhauls. Backhauls are under evaluation, the Møreforskning/NorStone project are working on estimating the potential savings by better utilization of backhauls. However, at present time this work is not completed, therefore we do not have a basis for implementing it in our model. Table 2.3 is an annual overview that illustrates the rates of effectiveness and the applications of a specific selection of vessels, both individual and compared towards the total. This selection constitutes 24 of 44 vessels and is based on the share of freighted tonnage or the number of shipments and during 2008. The mix of contract types is determined by the charterer at NorStone. Table 2.3 is based on data analyses from 2008 illustrating, among others, the fill-rate of each vessel. This figure corresponds with the "economy of scale" principal, meaning that most shipments are executed by vessels on TC or COA- contracts. If the SPOT market is applied, these vessels are only executed for one or two shipments during the whole month.

In total, approximately 1.9 million tdw were distributed along the western coast of Norway during the whole year. Altogether, 1577 shipments between production sites and customers/terminals were executed, with an average freight equal to 1493 tons. In order to fulfill customers' needs, NorStone hired 44 different vessels from local ship-owners. Most vessels were hired through the SPOT market; still, these vessels distributed less than 10 % of the total tonnage. Some vessels had a fill rate below 90 %, which probably is unprofitable and can be viewed as a potential for improvement. In total, the average fill-rate was close to 94 % concerning the whole fleet.

**Table 2.3:** Analysis of the effectiveness of the current fleet and ship scheduling (2008)

Vessel	Tot. Amount of tonnage	No. of shipments	Average freight	Capacity**	Fill-rate ***	Share of tot. Tonnage	Share of tot. no. of shipments
FALKSUND	347070	208	1669	1500	100 %	18,5 %	13,2 %
FALKVIK	293742	374	785	750	100 %	15,7 %	23,7 %
FALKVÅG	200337	231	867	1000	87 %	10,7 %	14,6 %
KONGSVAAG	182847	90	2032	2500	81 %	9,7 %	5,7 %
NYFJELL	159262	105	1517	1300	100 %	8,5 %	6,7 %
FALKNES	131598	66	1994	2100	95 %	7,0 %	4,2 %
CABAL	50859	32	1589	1735	92 %	2,7 %	2,0 %
SANDSUND	48810	57	856	900	95 %	2,6 %	3,6 %
NYSAND	48109	61	789	800	99 %	2,6 %	3,9 %
MERCATOR	41367	22	1880	2100	90 %	2,2 %	1,4 %
NORHOLM	36031	7	5147	4700	100 %	1,9 %	0,4 %
SULE VIKING	33680	17	1981	2200	90 %	1,8 %	1,1 %
NORSUND	31133	10	3113	3700	84 %	1,7 %	0,6 %
NYVÅG	30519	47	649	800	81 %	1,6 %	3,0 %
NORNE	28988	7	4141	4500	92 %	1,5 %	0,4 %
FINNØYGLIMT	27642	41	674	714	94 %	1,5 %	2,6 %
BASEN	21066	38	554	581	95 %	1,1 %	2,4 %
FAKTOR	20309	18	1128	1200	94 %	1,1 %	1,1 %
FINNØYFJORD	20059	34	590	571	100 %	1,1 %	2,2 %
ALRITA	19336	22	879	880	100 %	1,0 %	1,4 %
DURABULK	17999	6	3000	3500	86 %	1,0 %	0,4 %
DYNABULK	17959	5	3592	3600	100 %	1,0 %	0,3 %
ANDERS BAS	13138	12	1095	1200	91 %	0,7 %	0,8 %
NYBORG	11097	22	504	500	100 %	0,6 %	1,4 %
<b>Sum of selection</b>	<b>1832955</b>	<b>1532</b>	<b>1709</b>	<b>1805</b>	<b>94 %</b>	<b>97,7 %</b>	<b>97,1 %</b>
<b>The total fleet*</b>	<b>1876438</b>	<b>1577</b>	<b>1493</b>	<b>---</b>	<b>94 %</b>	<b>100 %</b>	<b>100 %</b>
<b>Deviation</b>	<b>43483</b>	<b>45</b>	<b>-216</b>	<b>---</b>	<b>0,4 %</b>	<b>2,3 %</b>	<b>2,9 %</b>

\*During 2008 the total fleet counted 44 heterogeneous vessels. This table presents only 24 of them. The selection is based on either the share of total tonnage or number of shipments.

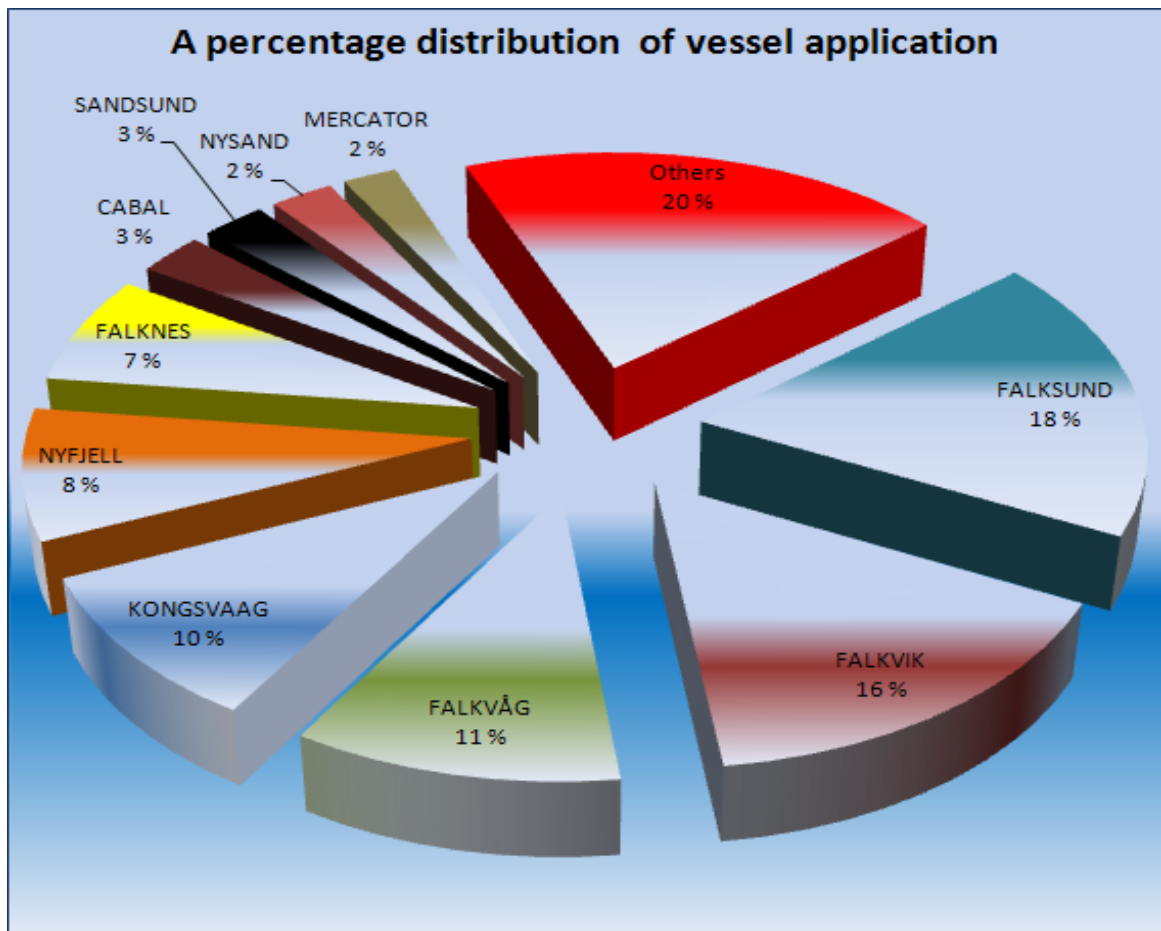
\*\*Due to lack of data and in order to fulfill the analysis of 2008 we assumed (calculated) the capacities for some vessels to be equal to the highest amount of tonnage pr shipment.

\*\*\*Fill-rate is defined as one shipment, without backhauls.

Table 2.3 presents several interesting aspects:

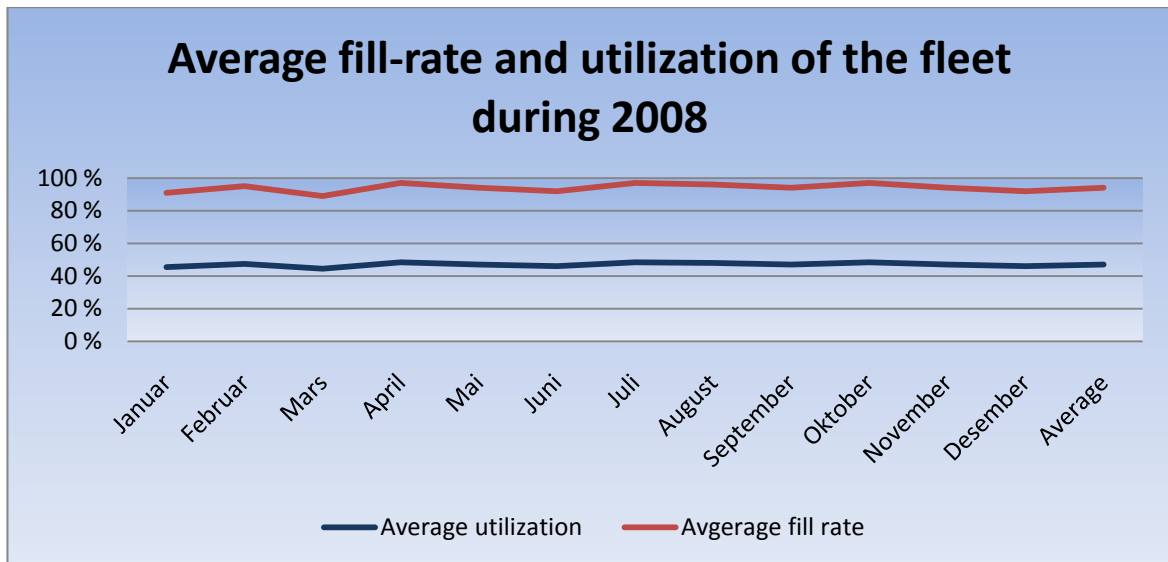
- ✓ 80 % of all shipments are executed by 20 % of the fleet. In accordance with the Pareto-principle, table exposed that 10 out of 44 vessels shipped 80% of the total tonnage during 2008.
- ✓ 5 vessels delivered close to 60 % of the total amount of tonnage, even though only two of these vessels operate through TC-contracts. Assuming that it is profitable for NorStone to have Kongsvaag and Dynabulk on TC-contracts, we assume that it would be even more profitable to have Falkvik, Falksund, Falkvåg and Nyfjell operating on these conditions as well.
- ✓ In January we were told that the capacities per shipment were utilized 100%, our analysis exposed that the average fill-rate is 94%. The rate of utilization is equal to the half of the fill-rate, since bringing backhauls are not widely exploited. And that means 94% of 50%.
- ✓ We discovered that almost half of the fleet (20 out of 44 vessels) delivers only 2.3 % of the total tonnage. These 20 vessels are hired on SPOT contracts and responsible for only 45 shipments during the whole year.
- ✓ We were also told that the average freight is approximately 1200 tons/shipment, during 2008 the real number was 1493, - a deviation of 24.5 %.

The data set we analyzed confirms the Pareto-principle which states that a minor selection (20 %) is responsible for a major part (80 %) of the result. Figure 2.8 is a percentage distribution of vessel application and confirms the 80-20 rule. We calculated that 10 out of 44 vessels shipped 80 % of the total tonnage during 2008. Falksund shipped 18 % of all tonnage, which corresponds to 252 shipments with an average freight calculated to 1377 tons. Falkvik possess half the capacity of Falksund, but executed 403 shipments and shipped 16 % of the total tonnage.



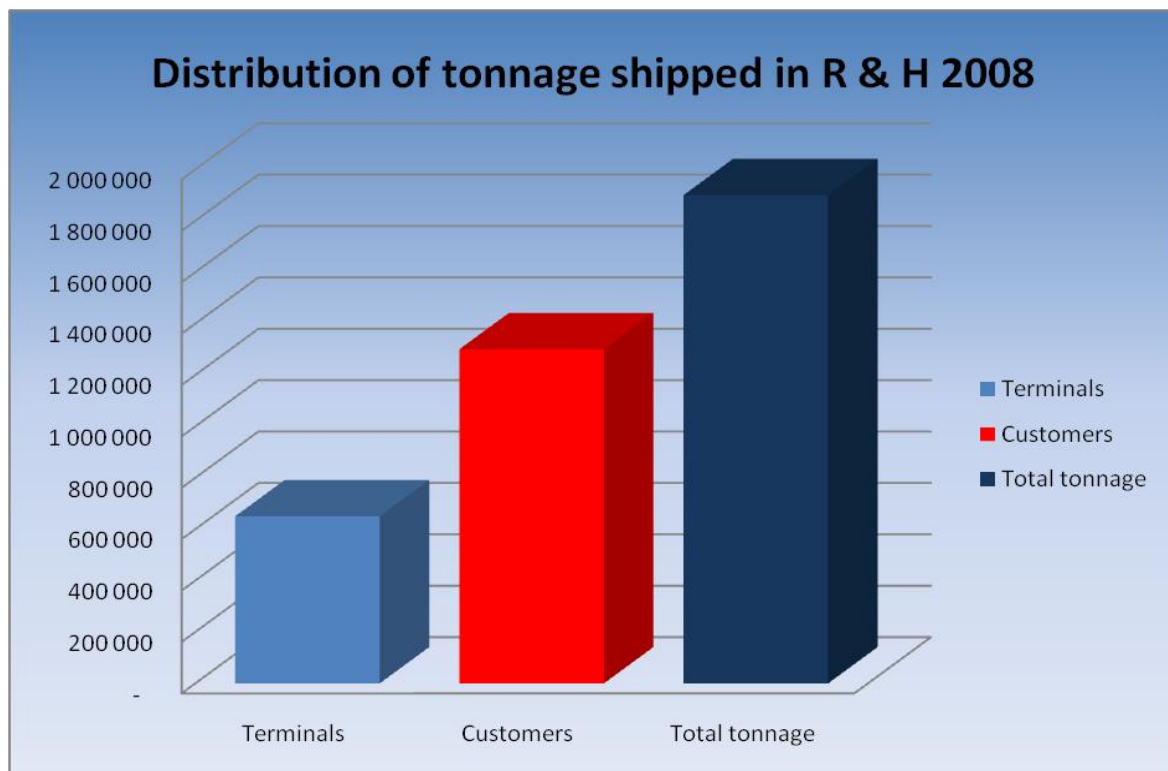
**Figure 2.8:** A percentage distribution of vessel application in Hordaland and Rogaland, 2008.

The figure 2.9 shows the average fill-rate and the rate of utilization of the whole fleet during 2008. The rate of utilization is equal to the half of the fill-rate since the vessels did not bring backhauls. The average fill-rate was 94 %, however it is not 100 %, which exposes a potential of better utilization. A rate of utilization equal to 47 % is a result of the missing backhauls, caused by the short distances, rapid delivery times and the complexities of the market. The earnings potential of including backhauls by cooperating with external actors can be significant.



**Figure 2.9:** shows the average fill-rate and the parallel rate of utilization of the whole fleet, 2008.

The next figure, figure 2.10, is a supplement to the analysis in figure 2.6. However, it illustrates the total amount of tonnage transported to terminals, in comparison to the customers and the total tonnage.



**Figure 2.10:** Illustrates the distribution of tonnage shipped to terminals and customers in 2008.

## **2.5 Problem formulation**

Today's market situation and the continuous need on improving distribution processes, cutting unnecessary costs while adapting to customers' demand is a challenge for every company.

For NorStone, the main challenge seems to be the transportation link along the value chain. Therefore, finding a system which can improve the rate of utilization for the main vessels became an important issue. Then, the main objective of this master thesis is to *describe, analyze and develop a full scale model for the seaborne shipping problem at NorStone.*

Our approach is to describe the current situation and to develop a mathematical model that can help to minimize the shipping costs between production sites and customers in Rogaland and Hordaland. Basic vehicle routing problem (VRP)-models are used as a foundation to our mathematical model, and furthermore, we aim to capture the major complexity of this problem by adding special extensions to the model. In addition we have made some combinations of different models that make our model unique from former models within vehicle routing problems (VRPs). VRP is introduced in section 3.2.

In the following we introduce to the reader the theories we found as meaningful for our attempt to answer to this problem formulation.

### 3 Literature review

This chapter is organized into two sections: a brief introduction about maritime transportation as a worldwide industry and then the vehicle routing problem with several variants of both classical and special extensions. As a main area of research we try to focus on materials and literature referring to scheduling and optimization within maritime transportation.

#### 3.1 Maritime Transportation

The statistics presented in Christiansen et al. [8] points out the total international seaborne trade which has increased by 67 % since 1987, the cargo carrying capacity of the world fleet that has increased by 25% since 1980, and the productivity has increased from 5.4 tons carried per deadweight ton to 7.2 within the same time span, These statistics states the dependence of the world economy on the seaborne trade. However, a vessel requires major capital investment and need to be operated in a profitable manner. In order to remain competitive, the industry actors needs exploit the potential of improving their financial performances and reduce shipping cost through fleet and operations planning. [8]

**Table 3.1:** Comparison of operational characteristics of freight transportation modes [9]

Operational characteristic	Mode				
	Ship	Aircraft	Truck	Train	Pipeline
Barriers to entry	small	medium	small	large	large
Industry concentration	low	medium	low	high	high
Fleet variety (physical & economic)	large	small	small	small	NA
Power unit is an integral part of the transportation unit	yes	yes	often	no	NA
Transportation unit size	fixed	fixed	usually fixed	variable	NA
Operating around the clock	usually	seldom	seldom	usually	usually
Trip (or voyage) length	days–weeks	hours–days	hours–days	hours–days	days–weeks
Operational uncertainty	larger	larger	smaller	smaller	smaller
Right of way	shared	shared	shared	dedicated	dedicated
Pays port fees	yes	yes	no	no	no
Route tolls	possible	none	possible	possible	possible
Destination change while underway	possible	no	no	no	possible
Port period spans multiple operational time windows	yes	no	no	yes	NA
Vessel-port compatibility depends on load weight	yes	seldom	no	no	NA
Multiple products shipped together	yes	no	yes	yes	NA
Returns to origin	no	no	yes	no	NA

NA – not applicable.

In comparison with other transportation modes (rail, truck and air), ship routing and scheduling problems are different since vessels operate under other conditions. Table 3.1 confirms that ships are of the most flexible mode of transportation, even though it still

involves a considerable amount of operational, political, regulatory and economic aspects. According to the European Commission, a central policy of the EU is to: improve the quality and efficiency of the European transportation system by shifting traffic to maritime and inland waterways, revitalizing the railways and linking up different modes of transport [19].

In accordance with the Commission's objective Sigurd et al. [50] discussed whether there is a realistic possibility to switch some of the demand from road transportation to ships. These authors concluded that the underlying planning problem is how to find recurring liner routes, hence these routes needed to adapt with both quantity and the frequency demanded by the attending companies.

Christiansen et al. [8] discussed literature regarding the transportation planning, and discovered that transportation planning as a special field has been widely studied. However, most of the attention has been devoted on trucks and aircraft transportation. The low attention drawn in this particular literature to maritime transportation problems, appeared unintelligible especially considering the major capital investments and operating costs within this mode. Four possible explanations were listed by the authors:

- ✓ Low visibility
- ✓ Maritime transportation planning problems are less structured.
- ✓ In maritime operations there is more uncertainty
- ✓ The ocean shipping industry has a long tradition and is fragmented.

The ocean shipping industry is known to be very conservative and there has been little willingness to develop new optimization methods. Despite the prospects of saving money, they choose to continue applying less efficient methods. Several reasons are discussed. Ronen [45] claims that ship scheduling generates a much larger variety in problem structures and operating environments than standard vehicle scheduling. Another reason is related to the fact that ocean shipping industry has a long tradition for applying simple methods of routing their fleet. In regard to the given history of ship routing, there is potential of significant savings if the industry is interested and willing to develop new decision support system software, based on optimization methods. [9]

The interest of research in maritime transportation has increased rapidly during the last decades. However, the pioneer was Ronen [45], who traced papers back to 1950s and published the first review of operational research (OR) work in ship routing and



scheduling, after which he followed up with second review a decade later, Ronen [47]. A similar review that considered the further development from 1994 to 2004 was presented in Christiansen et al. [9]. Significant researches on maritime transportation optimization issues were carried out by, among others, in Ronen [46], [48], Psaraftis' [42] and Mehrez et al. [37].

Crainic and Laporte [13] discussed the main issues in freight transportation and operations, and presented OR models and methods, organized according to the three classical decision making levels: strategic, tactical and operational. Usually *strategic decisions* covers ship design, fleet size and mix, market selection, and port/terminal locations. These decisions require some tactical and operational information and reveal the significant overlap between strategic decision on one hand, and the tactical and operational decisions on the other hand. Models used to determine fleet size and ship network design are strategic decisions that are based on evolution of ship routing strategies, which are considered as *tactical decisions*. In addition to ship routing, are fleet deployment, crane scheduling and ship management are considered as tactical decisions, while the *operational decisions* covers ship loading, environmental routing and cruising speed selections. [8]

Related problems in the maritime transportation industry are widely studied by Fagerholt [22], who considered the experience and future research of a computer-based decision support system for vessel fleet scheduling.

The need for proper fleet routing and scheduling together with rapid development of computing power indicates that there is a potential for improvement by using optimization-based decision support systems (Fagerholt [22]).

His paper is summarized by stating that research focus should be directed towards optimization algorithms, and emphasize on user interaction and the environment that the algorithm function together with.

### **3.2 The vehicle routing problem (VRP)**

Vehicle routing problems are applied as a collective term that includes important problems within the fields of transportation, distribution and logistics. Even though conditions vary from one setting to the next, these problems are faced daily by companies and organizations worldwide. The VRP was first introduced by Dantzig and Ramser [14] and has since been one of the most widely studied problems in combinatorial optimization. Figure 3.1 is inspired by Toth and Vigo [52] and illustrates the huge number of variants of

the basic problem. Extensions of the basic problems of VRP are discussed in the section 3.3.2.

Much progress has been made since the earliest publications and several strong formulations, polyhedral studies and exact algorithms have been proposed. Smaller prototype problems are often basis for development of algorithm research and software. However, by implementing sufficient flexibility, most algorithms can be adapted and applied into other practical contexts. Moreover, a large number of heuristics have also been developed. [12]

The VRP consists of designing the optimal set of routes, with consideration to three important aspects: every route starts and ends at the depot, all customers are visited exactly once, and the demand of a vehicle route cannot exceed the capacity of the vehicle. [30]

Finding exact solutions for the classical VRP and extensional variants are hard to obtain. This makes it desirable to develop fast and accurate heuristics that both coincide with the objectives and computes the problems within sufficient terms.

The literature emphasizes three types of solution methods that manage VRP:

### **Manual planning**

The planners use their own knowledge as a planning tool to compose and improve new routes. These routes are often as good as routes based on modeling, but restricted by the size of the problem. Manual planning is based on a mix of gained experience and former optimization process.

### **Exact solutions based on mathematical models**

Modeling is a mathematical description of the reality, based on re-structured deterministic data. Predetermined simplifications are made in order to adapt into a real world situation.

Exact algorithms can find the optimal solution. Those techniques, Branch & Bound [53] and Branch & Cut [40] etc., are very restricted and can only be used to solve small problems, approximately up to 100 nodes.

### **Heuristics**

Heuristics, defined by Pearl [41], are “*strategies using readily accessible, though loosely applicable, information to control problem solving in human beings and machines*”. The literature distinguishes between heuristics and metaheuristics. Heuristics produces high quality solutions that are based on limited exploration of the set of possible solutions and

solved within an approvable computing time. In comparison to heuristic, metaheuristics are known for generating solution with even higher quality, due to the fact that these techniques emphasis on a deeper exploration of the most promising areas within the solution set. [35].

Different heuristic techniques are developed in order to solve variants of the VRPs. We have listed the 2 most accounting techniques below;

- ✓ **Classical heuristics** are mostly 2 phase algorithms or constructive methods. “Clark and Wright savings”- algorithms [10] is often applied as a constructive method, while Fisher and Jaikumar [23] have developed alternative 2-phase algorithms that are capable of solving the same VRPs. All 2-phase algorithms are decomposed into 2 components, first dividing the vertices into feasible clusters, and then construct the actual routes. [35]
- ✓ **Metaheuristics** as a solution technique are applied to solve VRPs through a Tabu search described by Brandão and Mercer [3], simulated annealing presented by Osman [39] or for instance a genetic algorithm initially proposed by Holland [32]. These strategies are used to guide other heuristics.

### 3.2.1 Basic models for the VRP

This section is based on Toth and Vigo [52], and appears as a very concise extract of the basic models for the VRP.

The literature mentions three different basic modeling approaches. *Vehicle flow models* are mostly used for the basic version of VRP, and uses integer variables which are associated with each arc or edge of the graph. It is common to describe the vehicle routing problems by creating graphs, in which the graphs contains a set of points denoted as nodes and as set of arc or edges. Every arc represents a one-way street between al nodes, while an edge are considered as a two-way street. Those variables count the number of times the arc or edge is traversed by a vehicle.

The vehicle flow models are particularly suited for cases in which the cost of the solution can be expressed as the sum of the costs associated with the arcs, and when the most relevant constraints concern the direct transition between the customers within the route, so they can be effectively modeled through an appropriate definition of the arc set and of the arc cost (Toth and Vigo [52]).

The second approach is based on the *commodity flow formulation*, where the additional integer variables represent the flow of commodities along the path traveled by the vehicle.

Balinski and Quandt [1], proposed a third approach which is the *set-partitioning* formulation of the VRP. This model is based on an exponential number of binary variables, each associated with a different feasible circuit, in which the main issue is to determine a collection of circuits to a minimum cost. Every solution must serve all customers once and satisfy all additional constraints. Usually, such approaches require a large number of variables to deal with. The fact that the Set-Partitioning model allows for extremely general route costs appears to be its main advantage.

These 3 approaches represent the main mathematical programming formulations that are applied in order to model basic VRP. In accordance with the goals of this thesis, the maritime distribution problem in NorStone should be solved by developing a mathematical model that minimizes the total cost of serving all customers and fulfils all additional constraints. The best approach for our further work is the vehicle flow models, because our problem requires use of multiple products. Commodity flow models are limited in the sense of multiple products. Moreover, the set-partitioning model is limited to the usage of column generation as solution method. Therefore we concluded that the best solution was to apply the vehicle flow model, as a basic modeling approach for further extensions. The following formulation below is the vehicle flow model, which is extensively presented in [52].

### *Objective function*

$$(3.01) \quad \text{Minimize} \sum_{i \in V} \sum_{j \in V} c_{ij} X_{ij}$$

### *Subject to*

$$(3.02) \quad \sum_{i \in V} X_{ij} = 1 \quad \forall j \in V \setminus \{0\},$$

$$(3.03) \quad \sum_{j \in V} X_{ij} = 1 \quad \forall i \in V \setminus \{0\},$$

$$(3.04) \quad \sum_{i \in V} X_{i0} = K,$$

$$(3.05) \quad \sum_{j \in V} X_{0j} = K,$$

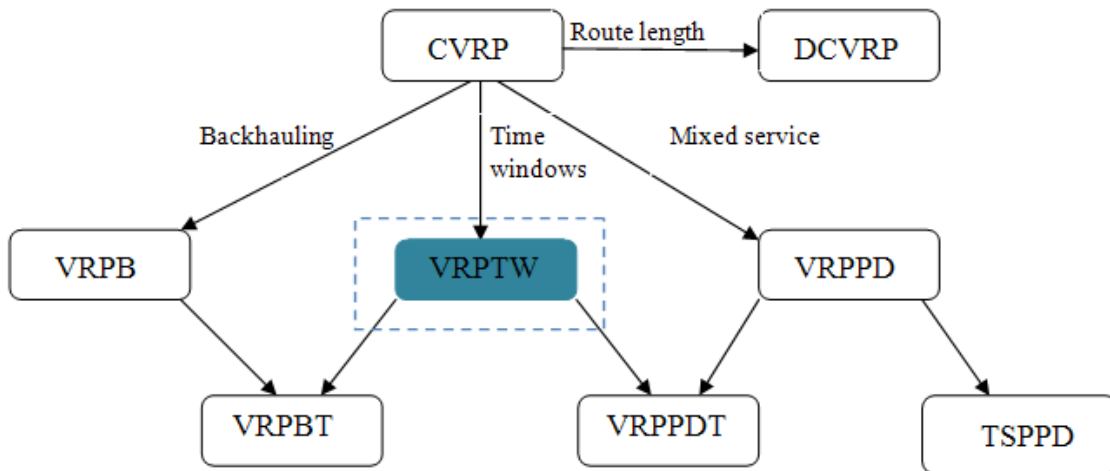
$$(3.06) \quad \sum_{i \notin S} \sum_{j \in S} X_{ij} \geq r(S) \forall S \subseteq V \setminus \{0\}, S \neq \emptyset,$$

$$(3.07) \quad X_{ij} \in \{0,1\} \forall i, j \in V.$$

This is a two index formulation that uses binary variables to indicate if a vehicle traverses an arc in the optimal solution. Constraints (3.02) and (3.03) imposes that exactly one arc enters and leaves each vertex (customer); (3.04) and (3.05) imposes the degree requirements for the depot vertex. The capacity-cut constraints (3.06) impose each connectivity of the solution, and the vehicle capacity requirements; (3.07) impose that the variable  $x_{ij}$  takes the value 1 if arc  $(i,j)$  is included in the optimal solution, 0 otherwise.

### 3.2.2 Basic extensions of the VRP

A short presentation of the basic extensions of the VRP will be presented in the following sections. Figure 3.1 illustrates how the capacitated VRP (CVRP) is the origin for other extensions. However, the dotted line indicates that VRP with time windows (VRPTW) is the basis for our further work.



**Figure 3.1:** The basic problems of the VRP class and their interconnections, inspired by [52]

#### 3.2.2.1 Capacitated Vehicle Routing Problem (CVRP)

*Capacitated VRP (CVRP)* is considered to be the basic version of VRP and generalizes the well known Travelling Salesman Problem (TSP). TSP consists of travelling a round trip, called a Hamiltonian cycle, to a minimum cost. The salesman starts in any given node and

visits all other nodes, before he returns to the start node. The CVRP is considered as the simplest and most studied member of the basic VRPs. The problem is characterized by a single central depot, deterministic demand (known in advance), and identical vehicles with maximum loading capacities. This problem appears to be hard to solve by exact algorithms, but can effectively be solved by heuristics. [52]

Ralphs et al. [44], considered the VRP and imposed capacity constraints on the integer programming formulation of their routing model, providing the link between the underlying routing and packing structures. This problem contained both the Bin Packing Problem (BPP) and the TSP. The BPP consist of either to minimize the number of used bins, or to maximize the number of items packed into the number of bins, without exceeding the capacity of the bins and without splitting items. [34]

### **3.2.2.2 Distance-Constrained VRP (DVRP)**

A variant of the basic version of CVRP is called the Distance-Constrained VRP (DVRP). The problem is separated from regular CVRP since a constraint on maximum length replaces the capacity constraint for each route. The optimal solution is the solution that generates the minimum length of all routes. If the problem is restricted by both the vehicle capacity and the maximum distance, the literature defines the problem as *Distance-constrained CVRP* (DCVRP). [52]

### **3.2.2.3 VRP with backhauls (VRPB)**

*VRP with backhauls* is different from basic VRP since the customer set is partitioned into two subsets; (i) *linehaul customers*, which require that a given quantity of product(s) is delivered, and (ii) *backhaul customers*, which require that a given quantity of inbound products are picked up. [52]

The literature mentions the grocery industry as the most common way of applying this theory in practice. Golden et al [27], argues that a mixed distribution-collection context leads to a significant saving in transportation costs by visiting backhaul costumers in distribution routes.

### **3.2.2.4 VRP with pickups and deliveries (VRPPD)**

*VRP with pickups and deliveries* consist of a set of transportation requests that need to be satisfied by a heterogeneous fleet based at multiple terminals. These orders contain a pick up point, a delivery point and demanded quantity of goods/persons. The VRPPD problem

involves time, capacity and depot constraints; time windows must be satisfied at each stop, visiting each pickup and delivery location exactly once, not exceeding the capacity constraints and in addition to the depot constraints that ensure that the vehicles return to the appropriate terminals. [16]

Hoff and Løkketangen [31], discussed practical problems of a TSP with pickups and deliveries (TSPPD-which is an extension of the general TSP), and proposed to relax the constraints of only checking Hamiltonian tours, and also try solutions that can visit customers in a way giving rise to a “lasso” model. The specific literature mentions several solution methods to this problem, Bräysy et al. [4], [5], indicates that the methods can be formulated both for the TSPPD and for the corresponding and extended problem vehicle routing problem with pickups and deliveries (VRPPD). Gendreau, Hertz and Laporte [26], showed that one solution method is to solve the problem as TSP with backhauling.

### **3.2.2.5 VRP with time windows (VRPTW)**

*The VRP with time windows* is an important generalization of the classical VRP. Time constraints force the service at every customer  $i$  must start within a given time interval  $[a_i, b_i]$ , called a time window. Each vehicle must stop at the customer location for  $s_i$  (additional service time) time instants. In case of early arrival at the location of customer  $i$ , the vehicle generally is allowed to wait until time instant  $a_i$ , until the service may start, but arrivals after  $b_i$  are prohibited. [52]

Since the 80s, time windows have been heavily studied in the literature. However, it was first presented as case studies during the late 60s and the early 70s by Pullen and Webb [43], and Madsen [36]. The VRPTW, as well as the basic VRP, is difficult to solve by exact methods and categorized to be NP-hard, meaning that the time required to solve these problems increase rapidly in accordance with the problem sizes. Thus, most research is concentrated on heuristics. In contrast to exact algorithms, which often only minimize the distances, Cordeau et al. [12], emphasize that most heuristics consider a hierarchical objective which first minimizes the number of vehicles and then distances. An approach that corresponds with the heuristics developed in Dauter-Pères [15].

The motivation for introducing soft instead of hard time windows is that by allowing controlled time window violations for some customers, it may be possible to obtain better schedules and significant reductions in the transport costs (Fagerholt [21]).

Moreover, he imposed inconvenience costs for serving customers outside their time windows.

The VRPTW is normally modeled as an asymmetric problem because the time window requirements induce an implicit orientation of each route even if the original matrices are symmetric. In the majority of all cases are time windows defined by assumptions that impose the vehicle to leave the depot at time instance 0. Moreover, it is also normally to assume that the cost matrix coinciding with the travelling time matrix. [51]

The formulation below is presented in Cordeau et al. [11], and is an extension of the vehicle flow models, and describes a multi- commodity network flow model with time window and the capacity constraints.

### Objective function

$$(3.08) \quad \text{Minimize} \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijk}$$

### Subject to

$$(3.09) \quad \sum_{k \in K} \sum_{j \in \Delta^+(i)} x_{ijk} = 1 \quad \forall i \in N,$$

$$(3.10) \quad \sum_{j \in \Delta^+(0)} x_{0jk} = 1 \quad \forall k \in K,$$

$$(3.11) \quad \sum_{i \in \Delta^-(j)} x_{ijk} - \sum_{i \in \Delta^*(j)} x_{jik} = 0 \quad \forall k \in K, j \in N,$$

$$(3.12) \quad \sum_{i \in \Delta^-(n+1)} x_{i,n+1,k} = 1 \quad \forall k \in K,$$

$$(3.13) \quad x_{ijk} (w_{ik} + s_i + t_{ij} + w_{jk}) \leq 0 \quad \forall k \in K, (i,j) \in A,$$

$$(3.14) \quad a_i \sum_{j \in \Delta^+(i)} x_{ijk} \leq w_{ik} \leq b_i \sum_{j \in \Delta^+(i)} x_{jik} \quad \forall k \in K, j \in N,$$

$$(3.15) \quad E \leq w_{ik} \leq L \quad \forall k \in K, i \in \{0, n+1\},$$

$$(3.16) \quad \sum_{i \in N} d_i \sum_{j \in \Delta^+(i)} x_{ijk} \leq C \quad \forall k \in K,$$

$$(3.17) \quad x_{ijk} \in \{0,1\} \quad \forall k \in K, (i,j) \in A.$$

This is a nonlinear formulation were the objective function (3.08) is to minimize the total



cost. (3.09) restrict the assignment of each customer to exactly one vehicle route, given that  $N = V \setminus \{0, n+1\}$  represents the set of customers. Constraints (3.10)-(3.12) imposes that vehicle  $k$  must follow the flow on the path. While (3.13)-(3.15) guarantee schedule feasibility with respect to time and constraints (3.16) guarantee feasibility with respect to capacity aspects. The constraint (3.14) ensure that time windows are respected and force  $w_{ik} = 0$  whenever vehicle  $k$  does not visit customer  $i$ . (3.17) requires binary conditions on the flow variables.

### 3.2.3 Special extensions of VRP

So far we have concluded that our model is based on the vehicle flow model, and mainly developed in regard to time windows. In order to apply the theoretical framework into our concept, we considered it as necessary to present four special extensions of the VRP. The extensions below constitute important aspects of the maritime distribution in NorStone.

#### 3.2.3.1 Fleet size and mix

Designing an optimal fleet is one of the major issues for the shipping companies, and concerns the type of ships, their sizes and the number of ships of each size. Fagerholt [20] studied fleet size problems, considering the issues of deciding the minimum cost fleet in a real liner shipping problem. The ship type was given and he assumed that the fleet is hired on TC contracts. This problem can be considered as a VRP with multiple use of vehicles, where the main costs components is the TC-costs and the operational costs. He proposed a solution that worked well for the studied problem and was easy to adapt and implement.

The fleet can either be homogeneous or heterogeneous. A homogeneous fleet considers same type, size and cost of vessels and is described as a fleet planning problem in the literature, while a heterogeneous fleet includes decisions concerning mix of different vessels sizes.[8]

Mixed Fleet VRP is often applied in practice, and modeled by adding an extra index on the capacity constraints. Golden et al [28], studied this topic in-depth, and Christiansen et al. [8], presented a model for the strategic fleet size and mix problem with predefined routes:

#### *Objective function*

$$(3.18) \quad \text{Minimize} \sum_{v \in V} \sum_{r \in R_v} c_{Vur} U_{vr} + \sum_{v \in V} c_{Fv} S_v$$

*Subject to*

$$(3.19) \quad \sum_{v \in V} U_{vr} - U_v S_v \leq 0 \quad \forall v \in V,$$

$$(3.20) \quad \sum_{v \in V} \sum_{r \in R_v} a_{ir} U_{vr} \geq d_i \quad \forall i \in N,$$

$$(3.21) \quad \sum_{r \in R_v} t_{vur} U_{vr} \leq t \quad \forall v \in V,$$

$$(3.22) \quad U_{vr} \geq 0 \text{ and integer } \forall v \in V, r \in R_v$$

$$(3.23) \quad S_v \in \{0,1\} \quad \forall v \in V.$$

The objective function (3.18) minimizes the cost of sailing the used routes in addition to the fixed cost of the ships in operation. Constraints (3.19) ensure that the fixed costs for the ships in operation are taken into account. While (3.20) say that each port pair is serviced at least the required number of times, then constraints (3.21) ensure that each ship finishes all its routes within the planning horizon. Finally, the formulation involves integer and binary requirements on the variables.

### **3.2.3.2 VRP with Multiple Use of Vehicles (VRPM)**

Most models assume that vehicles are used only once within a given period. However, multiple uses of vehicles are often applied in practice and NorStone is no exception. Vessels performed multiple trips during the same period, in which the aim is to reduce the number of used vessels. This variant is poorly studied in the literature, a statement that is clearly defended in Gendreau [51]. However, a model that included VRPM was published by Hajri-Gabouj and Darmoul [29].

### **3.2.3.3 Multiple products**

Christiansen et al. [8], argues that some shipments require multiple nonmixable products to be carried onboard simultaneously, in which the charterer is responsible of splitting the tonnage into defensible piles or tanks (dependent on the vessel). Usually, this problem is solved by separating the vessels carrying space into fixed tanks. However, several vessels operate by dynamic holds or load the cargo into piles. The sizes of the piles are dependent on the requested orders, and therefore not considered as fixed, a mathematical description of this problem is presented in Christiansen et al. [8]. In order to allow nonmixable products onboard simultaneously, the authors added even more constraints. This formulation assumes that the arrival node is both a loading and an unloading port. The

added constraints describe the number of occupied tanks, in addition to the intervals of the number of occupied tanks after servicing the loading and unloading nodes. Further, all vessels are restricted by a constraint that imposes the initial tank occupancy condition for each of them. Moreover, in respect to the planning horizon, the numbers of occupied tanks are assumed to be zero when the vessels are first available for scheduling.

In the specific literature, these problems are mostly attached to the gas and oil industry. Among others, Scott [49] and Baush et al. [2], considered multiple products in a maritime transport setting. Scott [49], considered a problem that involved shipping of multiple products from a refinery to several depots. All shipments were carried out by heterogeneous vessels with fixed tankers. Baush et al. [2], presented a DSS that optimizes the fleet of coastal tankers and barges transported liquid bulk products among sites, distribution centers and industrial customers.

The figure 3.2 is a picture of “M/V Faktor” during unloading, and illustrates how the vessel is capable of carrying multiple products onboard simultaneously. The different products are separated into piles, which provide the opportunities of executing more than one order per shipment. Nonmixable cargos will, because of the shapes of the piles, take care of the limitations of carrying multiple products on a shipment.



**Figure 3.2:** A photo of “M/V FAKTOR” during unloading. (Source: [www.sandfrakt.no](http://www.sandfrakt.no))

### **3.2.3.4 The Inventory Routing Problem (IRP)**

Routing and inventory decisions are also required in the IRP, as well as in VRP. However, IRP is distinguished from VRP since there are no customer orders. Instead of customer orders, the delivery company decides which customers/terminals to visit and the size of the

shipment per day/period. Another distinction is the planning horizon, IRP deals with a longer horizon than VRPs. The inventory levels need to be controlled by restrictions that impose the delivery company to operate in a way that their customers not are allowed to run out of products. [6]

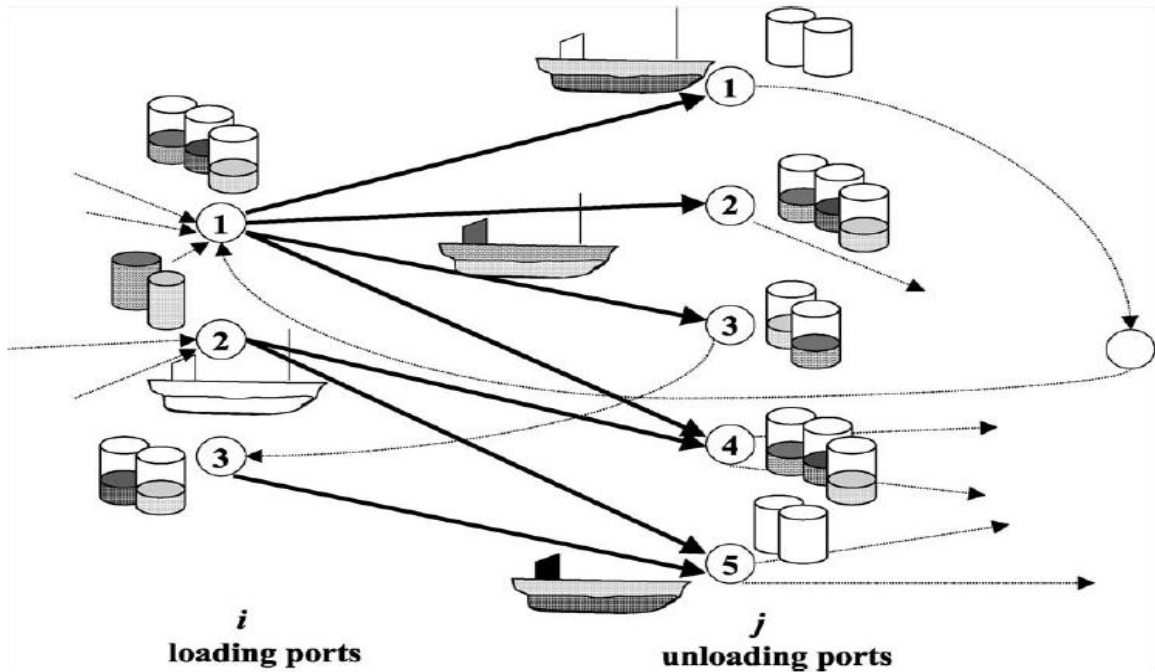
Campbell et al. [6], indicates that IRP is considered as a long-term problem, however almost all proposed solution approaches simplifies the problem by solving only short term versions. Dror and Ball [17], were the first to develop an approach that concerned a 5-days perspective. A weekly schedule that applied node and arc exchanges to reduce costs in the planning period was proposed by Dror and Levy [18], and used as a basis for the rolling-horizon approach discussed in Jaillet et al.[33] This approach contained a 2 weeks of scheduling, in which only the first week were implemented.

Different studies of the inventory routing problem for a single product (s-IRP) can be found in Christiansen [7], Flatberg [24], and Fox and Herden [25], but in regard to our thesis, the inventory routing for multiple products is more interesting, a problem that is scarcely considered in the literature. However, a project between Møreforskning AS and the Norwegian company Omya Hustadmarmor led to a development of a decision-support system (DSS) for maritime inventory routing. The project led to an annual saving of 5 % of the company's total cost, and increased the predictability and flexibility throughout the whole supply chain. [15]

Oppen and Løkketangen [38], proposed a Tabu Search based heuristic for Livestock Collection Problem (LCP). This problem deals with transportation of live animals to slaughterhouses and is based on the basic VRP, extended with production and inventory constraints. In contrast to the IRP, they considered the orders as data and therefore knew where to go and what to collect. A slaughter plan is required to regulate the inventory level sufficiently, and is based on the inventory level of the previous day plus all animals received during the present day, and restricted by a minimum safety level and a lairage capacity.

Christiansen [8], considered a multiple inventory routing problem where the focus is to optimally determine the quantity and timing of shipments to be shipped. The ship routing part of the problem is disregarded, and the transportation between loading and unloading ports is executed to a known cost (spot charter). Moreover, it is assumed that the shipper does not control and operate the fleet of ships, which is in contrast with s-IRP. Production and consumption rates vary over time, but balanced in the long run. They also assumed

that there are always a sufficient number of heterogeneous ships. The authors assume that ship voyages contain both a single loading and unloading port. The costs between the ports consist of a fixed set-up cost, and a variable cost per unit shipped. In figure 3.3 is the situation illustrated, bold arcs are in the model, while stippled are not considered.



**Figure 3.3:** A multiple product inventory problem illustrated in Christiansen [8]

In the next chapter we connect the above described theories with the problem formulated earlier in the thesis and possible approaches.

## **4 Application of theory into our concept**

This section is reviewing the theories from chapter three and we take a look at the potential pitfalls when combining different models and extensions to the models. In this chapter we describe some models and extensions to the model that may not be possible to combine. First, we have to decide which models and extensions that should be omitted from our conceptual model. Then, we are going to further discuss which combinations can be applied. The mathematical model takes basis in the CVRP model because we see that the deterministic demand from customers make a good basis for further extensions.

### ***4.1 Backhauls / pickup and deliveries***

Backhauls, as presented earlier in section 3.2.2.3, are a part of the local transportation problem for NorStone that is not widely exploited. The discussion about backhauls considers whether the potential lost savings or increase in profits are significant enough to justify the usage of the company's resources to find good solutions for backhauls. The application of backhauls is to be compared with VRP with pickup and deliveries, and in our problem can be an opportunity to perform multiple pickups at one route. But we must note that the pickups are made at the productions sites and there are rarely any deliveries at those locations. In regard to the small percentage of implementation of this method in the real world problem, we are going to make a simplification on this aspect. For now we are not going to consider backhauls, and manage the tour from the customer locations to a production site for pick up of new orders as empty and let the distance of deadheading be minimized by the model.

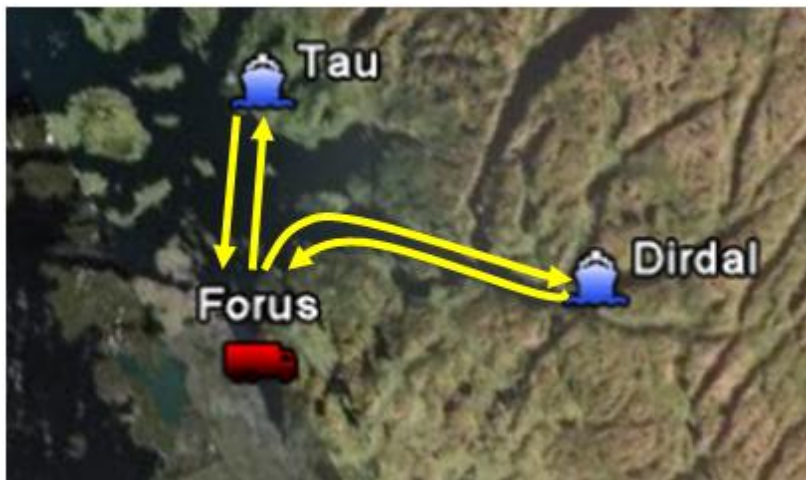
### ***4.2 Time windows***

Time windows are an important aspect of our real-world problem and in order to solve the maritime distribution problem, it is important that the mathematical model include time window constraints. A process of constraint relaxation from hard into soft time windows could be an option to help obtain good solutions to the problem, in a shorter amount of time. However, relaxations of these time windows cannot be performed without significantly changing the solutions. An optimization model to the maritime distribution problem without considering time windows will not generate satisfactory results, since this is a scheduling problem where customer nodes are visited and there are working hours to consider at all nodes (production sites, terminals and customers). In our model, opening hours at shipping facilities at the production sites are assumed to be open 24 hours, seven days a week. There are also other time limitations like the potential to build waiting time

for vessels, at the unloading locations, into the model and this is an area we have chosen to disregard in the current model. However, this can be an area of imminent potential for further research.

### ***4.3 Multiple tours per vessel***

Optimization of current fleet of vessels is the objective of our mathematical model to find the optimal maritime distribution schedule. Optimization can display that the present fleet of vessels are too comprehensive and a better utilization of the fleet is fewer vessels and more frequent tours per vessel. In our case study the geographical area is limited, at the same time it also consists of shipments with long duration due to for instance long fjords. Still, a majority of the shipments completed in a month, are of relatively short duration; thereby a solution consisting of one vessel carrying out multiple shipments per day is to be expected. Due to the fact that the model aims to optimize the distribution problem, it will hopefully minimize deadheading of each vessel in the fleet mix. The figure 4.1 is an illustration of multiple tours per vessel from the production sites at Tau and Dirdal, to the terminals at Forus.



**Figure 4.1:** *Illustrates multiple use of vessels between sites and terminals*

### ***4.4 Heterogeneous fleet / Fleet mix***

In tramp shipping, which includes NorStone, fleet size and contract evaluation are closely related. It is important that NorStone finds the best split between fixed long-term contracts (COA and TC) and SPOT contracts; these decisions are based on estimation of future prices and demand. In regard to the already high complexity of the model, for simplification of the model, decided to disregard heterogeneous contracts, and only consider one costs aspect and that is costs of travelling between the nodes. We consider

heterogeneous sizes on the fleet because it generates increased flexibility to the model compared to homogeneous. It is necessary that the model have these options in order to optimize the maritime distribution problem. The reason for this is the deterministic customer orders, if the size of the fleet were to be homogeneous; the mathematical model would not be operational. When considering the aspect of different contract types of the vessels, this is an issue that is excluded from the model in order to simplify it. In addition to eliminating the option of using SPOT contracts, this will reduce the complexity of the model.

#### ***4.5 Inventory constraints at production sites***

In the NorStone case, the transportation between production sites and terminals can be considered as an inventory routing problem (IRP). In this case all quantities are treated as known and the model is simplified in such a way that the terminals are considered to be customers. In order to accommodate to this assumption the IRP theory is not a suitable extension to our problem. However, it is possible to picture an extension of the model at a later stage. Where the terminals are considered to be remote depots and require to be refilled within specific inventory levels (upper and lower safety stock). Instead of using deterministic quantities to refill the inventory at the terminals, another application is to consider them as variable quantities. When changing from deterministic to variable quantities at the terminals this part is in accordance to the IRP model, it adds to the number of variables in the problem, but it generates an increased flexibility.

#### ***4.6 Multiple products on shipment***

Vessels completing stone shipments are able to carry multiple products in piles onboard. In the empirical data study we found several shipments with transportation of multiple products. Transportation of aggregates is capacitated by tdw, because the weight of the aggregates always exceeds the capacity of tdw carried prior to any other capacity violations, such as volume. A solution is to place a limitation on number of orders on a shipment to one, in order to simplify the problem, but an order has the opportunity to carry multiple products. An implementation of this option to the model is a decision to allow for multiple products on a shipment, in accordance to the multiple products theory. In addition, the aggregates deployed from a production site cannot be mixed together, so they have to be sorted in piles. Usually there will incur some overlapping of the aggregates, but this is a potential problem that are left out of the model.



In the next chapter we are going to present the model, by first giving a short introduction, followed by an explanation of which assumptions and simplifications we have made to be able to build it. Then we explain the mathematical model step by step, starting with objective function and continuing with the constraints. At the end of the chapter we give a short reflection on what we have achieved in the model.

## **5 Model**

### ***5.1 Introduction of the model***

In this chapter we present our model for the real world full scale problem to the reader. Since we are not presenting any solutions to the problem, we have disregarded that the model is nonlinear. In order to simplify the understanding of the model, we have chosen to give a brief introduction to the concepts presented in the model.

In the literature review we presented the vehicle flow model as the model with the most resemblance to our concept, and we have chosen to generate a model with basis in this model. In addition, we added some special extensions.

The illustration of the real world problem requires a complex computation of time in our concept, we have to consider discrete time periods for days and hours within the day, in addition to a continuous time variable for the time horizon. The introduction of discrete time periods into the model is necessary to calculate the daily inventory level at the production sites. In addition the model has to calculate time periods within the day to capture the opening hours at the customer locations. The continuous variables are measured in hours from the start of the time horizon until the end. The reason for combining discrete time periods and continuous time variables is in order to generate an optimizing model for the maritime distribution problem, it is important to consider the aspects of inventory management at the production sites and the time windows at the customer locations.

### ***5.2 Simplifications and assumptions of the model***

At this stage of building the model we have decided to make some simplifications to the model in order to handle some of the complexity of the problem. There has been a question of what aspects we firstly wish to research and what is of the best interest for the company. We have assumed that an efficient employment of the fleet of vessels is one of the major problems to handle. In regards to this we have considered it to be important to have a longer planning horizon than it is present, in order to better schedule the fleet of vessels. When considering the complexity of the model described in the introduction of the model, we decided to simplify some aspects of the full scale problem.

We have left out the possibility of handling different contract types in the model, there are no fixed costs incurring by choosing vessel. Furthermore, the costs of travelling with a vessel are only dependent on the duration from the production sites to the customers.

Costs of inventory are not considered in the model, because we have regarded the costs of inventory to be irrelevant to the main objective of the maritime distribution problem. The number of orders on a shipment is limited to one, and since an order is attached to a customer location we have excluded the possibility of delivering to multiple customers at one shipment. However, the model is open for an order with multiple products, but this is simplified to only be capacitated by the total tonnage carried on the shipment for the entire order.

### **5.3 Model description**

The concept is to model time windows at customers, inventory constraints at production sites and capacity constraints of vessels in the same model to capture the complexity of the maritime distribution problem. The production sites produce several product types and send shipments by vessels to customers, the number and locations of customers can change from any time horizon to the next. The terminals are used as depots and to transmit products to customer locations by trucks.

The model contains a node set  $N$  that contains all locations. This node set is split into subsets of loading and unloading nodes. Let  $S \subset N$  be the set of production sites,  $R \subset N$  the set of terminals and  $C \subset N$  the set the customers. The set  $S \subset N$  are defined as loading nodes and the set  $R \cup C \subset N$  are defined as unloading (customer) nodes. The quantity given in the orders are deterministic values, the concept is to accommodate the model to assign the appropriate vessels, in order to bring the demanded order sizes to the customers. In the model we only consider the possibility of carrying one order per shipment, but it is possibly to carry multiple products on an order. The fleet of heterogeneous vessels is represented by the set  $V$ , and we let the set  $P$  be the products.

## Notation

### Sets:

$N$	Nodes
$S \subset N$	Production sites
$R \subset N$	Terminals
$C \subset N$	Customers
$S \cup R \cup C = N$	
$S \cap R = \emptyset, S \cap C = \emptyset, R \cap C = \emptyset$	
$P_s$	Product line at site $s, s \in S, p_s \subset P$
$V$	Vessels
$O$	Orders

### Parameters:

$D$	Number of days in the time horizon
$T$	Maximal number of tours per vessel carrying an order in the time horizon
$c_{ij}$	Shipment cost between node $i$ and node $j$ , assume same cost structure for all vessels, $(i, j) \in N$
$a_{ij}$	Travel time from node $i$ to node $j$ , assume same speed for all vessels, $(i, j) \in N$
$q_{op}$	Quantity in order $o$ of product $p, o \in O, p \in P$
$r_{ps}$	Daily production rate of product $p$ at production site $s, p \in P, s \in S$
$b_{ps}$	Capacity of inventory bin for product $p$ at production site $s, p \in P, s \in S$
$d_v$	Draft vessel $v, v \in V$
$(de)_i$	Depth node $i, i \in N$
$f_v$	Capacity vessel $v$ in tons, $v \in V$
$S_o$	Possible exit nodes for order $o, S_o \subset S$
$j_o$	Destination node for order $o, j_o \in R \cup C$
$g_i^v$	Loading/unloading time per ton for vessel $v$ at node $i, v \in V, i \in N$
$(i)_p$	Initial inventory of product $p$ production site $s, p \in P, s \in S$
$w_v$	Arrival time to initial position for vessel $v, v \in V$
$i_v$	Initial position of vessel $v, i_v \in S$
$e_j$	Earliest arrival time at customer $j, j_o \in R \cup C$

$l_j$	Latest arrival time at customer $j$ , $j_o \in R \cup C$
$(ea)_o$	Earliest arrival time of order $o$ , $o \in O$
$(la)_o$	Latest arrival time of order $o$ , $o \in O$
$M$	Large number

### Variables:

$X_{ij}^{vot}$	1 if vessel $v$ travels from $i$ to $j$ carrying order $o$ on tour $t$ , 0 otherwise
$I_{ps}^d$	Inventory level of product $p$ at site $s$ at the end of day $d$
$W_i^{vot}$	Time when vessel $v$ arrives at node $i$ carrying order $o$ on tour $t$
$Z_i^{od}$	1 if order $o$ is delivered from site $i$ on day $d$ , 0 otherwise
$Y_s^{pd}$	Amount of product $p$ delivered from site $s$ during day $d$
$\delta_{oi}$	Day that order $o$ departs from node $i$
$\lambda_i^{uqvo}$	1 if vessel $u$ arrives at node $i$ with order $q$ before vessel $v$ arrives with order $o$ , 0 otherwise
$\beta_i^{vo}$	1 if vessel $v$ arrives at node $i$ carrying order $o$ , 0 otherwise

## 5.4 Model

### 5.4.1 Objective function

$$(1) \quad \text{Minimize} \quad \sum_{o \in O} \sum_{v \in V} \sum_{t=1}^T \sum_{i \in S_o} X_{ij}^{vot} c_{ij} \sum_{p \in P} q_{op} \\ + \sum_{o \in O} \sum_{v \in V} \sum_{t=1}^T \sum_{i \in S} X_{jo^i}^{vot} a_{ij}$$

The variable  $X_{ij}^{vot}$  is a binary variable that indicates if a vessel  $v$  travels from node  $i$  to customer  $j$  carrying order  $o$  in time  $t$ . The parameter  $c_{ij}$  is the cost per tons dead weight carried from site  $i$  to customer  $j$ , multiplied with the parameter  $q_{op}$  which is the order size of order  $o$  product  $p$ . The parameter  $a_{ij}$  is a distance matrix from node  $(i, j) \in N$ . The

objective of (1) is to minimize the total shipment cost of transport of aggregates to the customers. In addition the objective function minimizes deadheading on the way to the next production site  $s$ .

## 5.4.2 Subject to

### 5.4.2.1 Constraints on inventory at production sites

We assume that if the inventory levels are feasible at midnight every day, they are feasible at all other times.

$$(2) \quad I_{ps}^{d-1} + r_{ps} - Y_s^{pd} = I_{ps}^d \quad \forall p \in P, s \in S, d = 1, \dots, D$$

The set of constraint (2) is describing the inventory balance of product  $p$  at site  $s$  at the end of day  $d$ . In the variable all tours  $t$  taken from site  $s$  during the entire day  $d$  are summarized, this is illustrated in constraints (3).

$$(3) \quad Y_s^{pd} = \sum_{o \in O} q_{op} Z_s^{od} \quad \forall p \in P, s \in S, d = 1, \dots, D$$

The set of constraint (3) is calculating the consumption of product  $p$  at the end of day  $d$  at site  $s$ .

$$(4) \quad I_{ps}^d \leq b_{ps} \quad \forall p \in P, s \in S, d = 1, \dots, D$$

Constraints (4) are restrictions on inventory bins of product  $p$  at site  $s$ , they decide the capacity of product  $p$  at site  $s$ . The inventory of product  $p$  at site  $s$  at day  $d$  cannot exceed the capacity of each bin for product  $p$  at site  $s$ .

$$(5) \quad I_{ps}^0 = (ii)_{ps}, \quad \forall p \in P, s \in S$$

Constraints (5) state the initial inventory of product  $p$  at site  $s$  at the start of the time horizon  $D$ .

$$(6) \quad (int) \frac{\sum_{v \in V} \sum_{t=1}^T \left[ X_{ij_o}^{vot} \left( X_{jq^i}^{vqt-1} W_i^{vqt-1} + 24 \right) \right]}{24} = \delta_{oi},$$

$$\forall (o, q) \in O, i \in S$$

Constraints (6) determine the day  $\delta_{oi}$  when order  $o$  departs from production site  $s$ . The day has to be an integer number in order to relate it to the index of day  $d$ , the dynamic inventory is the reason for generating a variable that is able to identify the day in the time horizon.

$$(7) \quad MZ_i^{o\delta_{oi}} \geq \delta_{oi}, \forall \delta_{oi} \neq 0, i \in S, o \in O$$

$$\left[ \begin{array}{c} \delta_{oi} > 0 \\ \Downarrow \\ Z_i^{o\delta_{oi}} = 1 \end{array} \right], Z_i^{od} = 0 \forall d \neq \delta_{oi}$$

$$(8) \quad Z_i^{od} \leq \delta_{oi}, \forall o \in O, d = 1, \dots, D, i \in S$$

$$\left[ \begin{array}{c} \delta_{oi} = 0 \\ \Downarrow \\ Z_i^{od} = 0 \end{array} \right], \forall d \neq 0$$

The set of constraints (8) are determining if no order  $o$  is shipped from node  $i$  on day  $d$  and in (7) that order  $o$  is shipped from node  $i$  on day  $\delta_{oi}$ . The constraint sets (7) and (8) ensures that  $Z_i^{o\delta_{oi}} = 1$  if  $\delta_{oi} > 0$  and that  $Z_i^{od} = 0$  for all  $d$  if  $\delta_{oi} = 0$ .

#### 5.4.2.2 Constraints on vessels

There are capacity constraints on the vessels; they are weight constraints in tdw of each vessel. By using the binary variable  $X_{ij}^{vot}$  we identify the distance travelled and the vessel applied to transport the products.

$$(9) \quad X_{ij}^{vot} \sum_{p \in P} q_{op} \leq f_v \quad \forall (i, j) \in N, v \in V, o \in O, t = 1, \dots, T$$

Constraints (9) state that the sum of product  $p$  carried on vessel  $v$  has to be smaller or equal the capacity of the carried weight on vessel  $v$ .

$$(10) \quad X_{ij}^{vot} d_v \leq (de)_i, \quad \forall (i, j) \in N, v \in V, o \in O, t = 1, \dots, T$$

Constraints (10) secures that the vessels  $v$  arriving at node  $(i, j) \in N$ , meaning all nodes, sites, terminals & customers, are within the port capacity at the nodes.

### 5.4.2.3 Time windows

$$(11) \quad X_{ij}^{vot} \left( W_i^{vqt-1} + g_i^v \sum_p q_{op} + a_{ij_o} - W_{j_o}^{vot} \right) \leq 0, \\ \forall i \in S, v \in V, (o, q) \in O, t = 1, \dots, T$$

$$(12) \quad X_{ji}^{vot} \left( W_{j_o}^{vot} + g_{j_o}^v \sum_p q_{op} + a_{ij_o} - W_i^{vot} \right) \leq 0, \\ \forall i \in S, v \in V, o \in O, t = 1, \dots, T$$

Constraints (11) and (12) are time constraints that secure the continuous flow of vessels  $v$  in the node network  $(i, j) \in N$ . Constraints (11) are illustrating the shipment from the production site  $i \in S$ , which include time of loading of vessel  $v$  at node  $i$  and the travel time from node  $i$  to node  $j$ . Constraints (12) are picturing the trip from the customer site  $j \in R \cup C$ , including unloading time at node  $j$  at arrival and travel time to production site  $i$ .

$$(13) \quad X_{i_v j_o}^{vot} \left( w_v + g_{i_v}^v \sum_p q_{op} + a_{i_v j_o} - W_{j_o}^{vot} \right) \leq 0, \\ \forall i \in S, v \in V, o \in O, t = 2, \dots, T$$



In the set of constraint (13) the point of origin in the time horizon is stated.

$$(14) \quad \sum_{i \in S_o} X_{ij_o}^{vot} W_{j_o}^{vot} \geq (ea)_o, \forall v \in V, t = 1, \dots, T, o \in O$$

$$(15) \quad \sum_{i \in S_o} X_{ij_o}^{vot} W_{j_o}^{vot} \leq (la)_o, \forall v \in V, t = 1, \dots, T, o \in O$$

Constraints (14) and (15) illustrates the order delivery time, an order  $o$  has to arrive within these time windows. Constraints (14) state the earliest possible arrival time for order  $o$ , while constraints (15) state the latest arrival time for order  $o$ .

$$(16) \quad \text{int} \left( \sum_{i \in S_o} X_{ij_o}^{vot} W_{j_o}^{vot} \right) \text{mod } 24 \geq e_j, \forall v \in V, t = 1, \dots, T, o \in O$$

$$(17) \quad \text{int} \left( \sum_{i \in S_o} X_{ij_o}^{vot} W_{j_o}^{vot} \right) \text{mod } 24 \leq l_j, \forall v \in V, t = 1, \dots, T, o \in O$$

In constraints (16) and (17) the time windows for the arrival of vessel  $v$  at the customer location  $j \in R \cup C$  is determined. In constraints (16) the predetermined opening time for customer  $j$  is established and in (17) the closing time for customer  $j$  is stated. The opening hours usually are given as integers, therefore the variable  $W_{j_o}^{vot}$  have to be made integer in these constraints. In addition, variable  $W_{j_o}^{vot}$  run continuous from the start of the time horizon until the end and then the model is identifying at what hour it is on day  $d$ .

$$(18) \quad \sum_{i \in S} X_{ij_o}^{vot} W_{j_o}^{vot} = W_{j_o}^{vot}, \forall v \in V, o \in O, t = 1, \dots, T$$

Constraints (18) set the correct time if vessel  $v$  arrives at node  $j_o$  on tour  $t$  carrying order  $o$ , otherwise the time is set to 0. These constraints help eliminating time variables that are not needed in the model.

#### 5.4.2.4 Logical Constraints

Logical constraints will for instance ensure that a vessel do not make multiple shipments at the same time to different places.

$$(19) \quad \sum_{v \in V} \sum_{t=1}^T \sum_{i \in S_o} X_{ij_o}^{vot} = 1, \forall o \in O$$

Constraints (19) ensure that all orders  $o$  have to be covered on one tour  $t$  by one vessel  $v$  from one production site  $s$ .

$$(20) \quad \sum_{i \in S_o} X_{ij_o}^{vot} = \sum_{i \in S} X_{j_o i}^{vot}, \forall v \in V, o \in O, t = 1, \dots, T,$$

Constraints (20) forces vessel  $v$  to leave the node  $i$  after having delivered the last order  $o$  during the time horizon  $D$ , it will then always go to the closest production site  $s$ . In addition it ensures that a vessel  $v$  that arrives at a node  $j \in R \cup C$  carrying order  $o$  on tour  $t$  has to leave the same node on the same tour  $t$  with the same order  $o$ .

$$(21) \quad \sum_{o \in O} X_{j_o i}^{vot} \geq \sum_{o \in O} X_{i j_o}^{vot+1}, \forall v \in V, t = 1, \dots, T, i \in S$$

For all production sites  $i$ , constraints (21) secure that vessel  $v$  returning after having delivered an order  $o$  on tour  $t$  has to leave the same node  $i$  for tour  $t + 1$ .

$$(22) \quad \sum_{o \in O} \sum_{i \in S} X_{ij_o}^{vot} \leq 1, \forall v \in V, t = 1, \dots, T$$

Constraints (22) make sure that the sum of orders on vessel  $v$  on shipment  $t$  is no more than one order  $o$ .

$$(23) \quad \sum_{o \in O} \sum_{i \in S} X_{ij_o}^{vot-1} - \sum_{o \in O} \sum_{i \in S} X_{ij_o}^{vot} \geq 0, \forall v \in V, t = 1, \dots, T$$

Constraints (23) secure that no later tours can be completed unless tour  $t$  is used.

$$(24) \quad \sum_{v \in V} \sum_{t=1}^T \sum_{i \in S \setminus S_o} \sum_{j \in RUC \setminus j_o} X_{ij}^{vot} = 0, \forall o \in O$$

The set of constraint (24) are excluding all shipments of order  $o$  from site  $s$  that cannot deliver product  $p$ , and all shipments to other nodes than  $j_o$ .

#### 5.4.2.5 Opposite time windows

In this section we introduce  $\beta_{j_o}^{vo}$ , an indicator variable, which is needed in the model to enforce a constraint on combinations where two vessels actually visit the same port.

$$(25) \quad \sum_{t=1}^T \sum_{i \in N} X_{ij_o}^{vot} = \beta_{j_o}^{vo}, \forall v \in V, o \in O, t = 1, \dots, T$$

The set of constraint (25) enforces  $\beta_{j_o}^{vo} = 0$  if vessel  $v$  does not travel out or in of node  $(i, j) \in N$  and enforces  $\beta_{j_o}^{vo} = 1$  if vessel  $v$  travels out or in of node  $(i, j) \in N$ .

$$(26) \quad \sum_{t=1}^T \sum_{j \in N} X_{ji}^{vot} W_i^{vot} \geq \lambda_i^{uqvo} \left( \sum_{t=1}^T \sum_{j \in N} X_{ji}^{uqt} W_i^{uqt} + g_i^u \sum_{p \in P} q_{qp} \right),$$

$$\forall i \in N, (u, v) \in V, (o, q) \in O$$

Constraints (26) ensure that arrivals at all nodes  $(i, j) \in N$  are not possible at the same time for vessel  $u$  carrying order  $q$  as vessel  $v$  carrying order  $o$ .

$$(27) \quad \beta_{j_o}^{vo} \beta_{j_q}^{uq} \left[ \left( \sum_{t=1}^T \sum_{i \in S_o} X_{ij_o}^{vot} W_{j_o}^{vot} \right) - \left( \sum_{t=1}^T \sum_{i \in S_q} X_{ij_q}^{uqt} W_{j_q}^{uqt} \right) \right] \leq M \lambda_i^{uqvo},$$

$$\forall i \in N, (u, v) \in V, (o, q) \in O$$

If left side of constraints (27) is positive then  $\lambda_i^{uqvo} = 1$ . It forces  $u$  to arrive earlier than  $v$  and enforces  $\lambda_i^{uqvo} = 0$  if  $W_{j_o}^{vot} \neq W_{j_q}^{uqt}$ .

$$(28) \quad \beta_i^{vo} \beta_i^{uq} \geq \lambda_i^{uqvo},$$

$$\forall i \in N, (u, v) \in V, (o, q) \in O$$

Constraints (28) secure if  $\beta_i^{vo} \neq \beta_i^{uq}$  then  $\lambda_i^{uqvo} = 0$  for node  $i$ , vessel  $(u, v)$  and order  $(o, q)$ . It is basically saying that if there is no problems with simultaneously arrivals at the nodes the application of opposite time windows are not necessary.

$$(29) \quad \lambda_i^{uqvo} + \lambda_i^{vouq} \leq 1,$$

$$\forall i \in N, (u, v) \in V, (o, q) \in O$$

Constraints (29) ensure that if vessel  $v$  with order  $o$  is equal to 1 then vessel  $u$  with order  $q$  is equal to 0 and vice versa. These constraints are telling that only one vessel can arrive first.

#### 5.4.2.6 Integer values and non negativity on the variables

$$(30) \quad X_{ij}^{vot} \in \{0,1\} \forall i \in N, v \in V, o \in O, t = 1, \dots, T$$

$$(31) \quad I_{pst} \geq 0 \forall p \in P, s \in S, t = 1, \dots, T$$

$$(32) \quad W_i^{vot} \geq 0 \forall i \in N, v \in V, o \in O, t = 1, \dots, T$$

$$(33) \quad Z_i^{od} \in \{0,1\} \forall i \in N, o \in O, d = 1, \dots, D$$

$$(34) \quad Y_s^{pd} \geq 0 \quad \forall s \in S, p \in P, o \in O, d = 1, \dots, D$$

$$(35) \quad \delta_{oi} \geq 0 \quad \forall i \in N, o \in O$$

$$(36) \quad \lambda_i^{uqv} \in \{0,1\} \quad \forall i \in N, (v,u) \in V, (o,q) \in O$$

$$(37) \quad \beta_i^{vo} \in \{0,1\} \quad \forall i \in N, v \in V, o \in O$$

## 5.5 Reflections on the Model

In the process of building the mathematical model we wanted to implement as much as possible of the real world into it, when considering the vast complexity we was forced to enforce some simplifications to the model, such as just address one cost element and consider the overall weight onboard a vessel to be the only constraint. These simplifications have made it possible to prioritize to model aspects of the real world problem that we anticipated to be of most importance, such as finding solutions to the combination of discrete time periods and continuous time variables and to calculate time windows. By making use of opposite time windows we have managed to model capacity constraints at the ports.

### 5.5.1 Size of the problem

It can be important to estimate the size of the problem before processing starts, because a potential solution to the problem can be dependent of the problem size. Moreover, standard optimization tools have not the capacity of generating solutions if the problem instances are too big. In such cases a simplification of the problem size has to be made, or a heuristic that generate a near optimal solution has to be generated.

The problem size can be defined by calculating all empirical data of the real world problem. In our case we have several nodes, site, terminal and customer nodes, in which generates a vast number of variables exponentially increasing by the number of instances. The number of variables and constraints are defining the problem size, as well as parameters. A large number of integer variables makes to problem harder to solve than a problem size constituted of larger number of continuous variables.

In our selection of the real life problem, the numbers of site nodes are 6, furthermore the problem size contains of approximately 40 customer nodes, 40 vessels, 30 different product types and a time horizon of 28 days. This will generate a problem size too large to

be handled by a standard solver. Potential methods to solve such problems can be to simplify the problem instances by considering homogenous products, a decrease in the number of vessels or customer nodes. This can be done by excluding vessel that is less used and insignificant customer nodes, by consider clustering of customer nodes. Another option instead of using homogenous products is to also cluster products with similar applications/specifications.

## 6 Conclusion

In our master thesis we have considered a maritime distribution problem, in connection to the distribution of stone products from NorStone to its customers. A description of the company's value chain was presented in order to describe the current situations as thorough as possible. We performed time consuming analysis' to obtain a greater insight of ongoing processes, in order to discover fields with potential of improvement, but underestimated the time spent on processing data. In this process we discovered several important findings, such as: 33 % of all transportation is performed between productions sites and terminals, close to 50 % of the fleet of vessels only shipped 2.3 % of the total tonnage and we calculated that the Pareto-principal is applicable to the utilization of the current fleet of vessels. Further, we presented a literature review that referred to scheduling and optimization theories within maritime transportation, which formed the basis of our application of the theory into the concept.

We have developed a Mixed Integer Problem (MIP) model that minimizes the costs of transportation and deadheading. Our model is considering the possibility of vessels making multiple tours a day with multiple products, furthermore, it generate schedules of the maritime distribution for multiple time periods. In order to give an adequate description of the problem, we considered shipment duration time to run over multiple days/time periods. This approach is prolonging the planning horizon, instead of performing day to day scheduling. An approach we assume will provide an improved maritime scheduling for the fleet.

There are several factors that impose uniqueness in our model, in regard to standard VRP models. We wanted to illustrate the complexity of the maritime distribution problem by building an adequate mathematical model. One special feature that distinguish this model from others and makes it unique, is the combination of continuous time variables and discrete time periods, in order to manage VRP with time windows and inventory extensions in the same model. A necessary process in order to model some of the aspects we want to illustrate. Such as, taking simultaneous time of arrival at the same customer locations into consideration, then we had to model opposite time windows at the nodes, in order to deny entry for vessels if there already is one present. This is a process that is much harder to model than just modelling regular time windows.

The possibility of implementing our model into AMPL, for a limited selection, was considered in the beginning. Subsequent to completing the model and confirming that, as

expected, the complexity was too extensive. We understood that a process of making the model linear was comprehensive and would increase the number of variables, adding to the complexity. Furthermore, we had to find a suitable data set for a problem of small instances, solvable by applying the standard solver in AMPL. In order to find a suitable data set for solving small instances, numerous and serious simplifications have to be made to the model. These simplifications would presumably lead to a sample size so small that we disregarded the possibility of gaining any other benefits than confirmation of an operational model. Another reason was due to underestimation of time spent on processing data, we found ourselves short on time. Despite the problem of getting the model completed for testing, we are confident that the model contributes to the main project and for further development. In the process of writing our master thesis, we have succeeded in sorting and structuring the necessary information for the development of an adequate mathematical model. In addition we discovered that several of our findings are in accordance to the initial assumptions, and is useful as a confirmation in the further work.

In accordance with the objectives, this master thesis only constitutes a part of a larger project between Møreforskning AS and NorStone AS. Our mathematical model will hopefully contribute in developing a new logistical solution to the maritime distribution in NorStone and the completion of decision support system.



## 7 Further work

In the real world problem of ship routing it seems to be an infinite number of various applications to implement in a mathematical model. In a descriptive model such as the one we have built, there are in practice few limitations concerning what to implement. However, it is a difficult task to accomplish and therefore it is usual to make assumptions and simplifications, in order to capture the aspects of the problem considered to be of most imminent importance. In the following sections we will mention some interesting extensions to the maritime distribution problem, which we have kept out of our model.

An interesting objective for our mathematical model is to include the possibility of choosing between various contract types; TC, COA or SPOT. In such cases all cost elements have to be accounted for, in regard to the amount of information that has to be handled, this was left out of our model since our main objective was to generate a mathematical model for distribution optimization of the current fleet. If the objective was to account for contract type, presumably a fixed cost in relation to specific contract types has to be implemented, and added to the objective function.

An extension to the section above could be to do further analysis on fleet design and mix, as proposed by Dautère-Pérès et al. [15] in the Hustadmarmor case, such as performing analysis on finding right size on the fleet and profitability analysis if they were to acquire larger vessels. In our model we have limited the number of orders on a shipment to one, this automatically exclude the possibility of delivering to more than one customer at the same shipment, since the order is attached to a specific customer. In the real world problem some shipments actually are completed to multiple customers at one shipment. If there are going to be completed further research on vessel size in the fleet of vessels, a combination of this extension and multiple orders to multiple customers on one shipment is a natural expansion.

Terminals generate potential possibilities of partly eliminating or reduce problems with fluctuations of product demand at the production sites, if used as a secondary inventory holding location. In a situation like this it is plausible to handle this part as an Inventory Routing Problem, with demand from the terminals considered to be variable.

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## 9 Appendix



## 9.1 Model

### 9.1.1 Objective function

$$(1) \quad \begin{aligned} \text{Minimize} \quad & \sum_{o \in O} \sum_{v \in V} \sum_{t=1}^T \sum_{i \in S_o} X_{ij}^{vot} c_{ij} \sum_{p \in P} q_{op} \\ & + \sum_{o \in O} \sum_{v \in V} \sum_{t=1}^T \sum_{i \in S} X_{jo^i}^{vot} a_{ij} \end{aligned}$$

### 9.1.2 Subject to

#### 9.1.2.1 Constraints on inventory at production sites

$$(2) \quad I_{ps}^{d-1} + r_{ps} - Y_s^{pd} = I_{ps}^d \quad \forall p \in P, s \in S, d = 1, \dots, D$$

$$(3) \quad Y_s^{pd} = \sum_{o \in O} q_{op} Z_s^{od} \quad \forall p \in P, s \in S, d = 1, \dots, D$$

$$(4) \quad I_{ps}^d \leq b_{ps} \quad \forall p \in P, s \in S, d = 1, \dots, D$$

$$(538) \quad I_{ps}^o = (ii)_{ps}, \quad \forall p \in P, s \in S$$

$$(639) \quad (int) \frac{\sum_{v \in V} \sum_{t=1}^T \left[ X_{jo}^{vot} \left( X_{jq^i}^{vqt-1} W_i^{vqt} + 24 \right) \right]}{24} = \delta_{oi}, \quad \forall (o, q) \in O, i \in S$$

$$(7) \quad MZ_i^{o\delta_{oi}} \geq \delta_{oi}, \quad i \in S, o \in O$$

$$(840) \quad Z_i^{od} \leq \delta_{oi}, \quad \forall o \in O, d = 1, \dots, D, i \in S$$

#### 9.1.2.2 Constraints on vessels

$$(9) \quad X_{ij}^{vot} \sum_{p \in P} q_{op} \leq f_v \quad \forall (i, j) \in N, v \in V, o \in O, t = 1, \dots, T$$

$$(10) \quad X_{ij}^{vot} d_v \leq (de)_i, \quad \forall (i, j) \in N, v \in V, o \in O, t = 1, \dots, T$$

### 9.1.2.3 Time windows

$$(11) \quad X_{ij}^{vot} \left( W_i^{vot-1} + g_i^v \sum_p q_{op} + a_{ij_o} - W_{j_o}^{vot} \right) \leq 0, \\ \forall i \in S, v \in V, o \in O, t = 1, \dots, T$$

$$(12) \quad X_{ji}^{vot} \left( W_{j_o}^{vot} + g_{j_o}^v \sum_p q_{op} + a_{ij_o} - W_i^{vot} \right) \leq 0, \\ \forall i \in S, v \in V, o \in O, t = 1, \dots, T$$

$$(13) \quad X_{ivj_o}^{vot} \left( w_v + g_{iv}^v \sum_p q_{op} + a_{ivj_o} - W_{j_o}^{vot} \right) \leq 0, \\ \forall i \in S, v \in V, o \in O, t = 2, \dots, T$$

$$(14) \quad \sum_{i \in S_o} X_{ij_o}^{vot} W_{j_o}^{vot} \geq (ea)_o, \forall v \in V, t = 1, \dots, T, o \in O$$

$$(15) \quad \sum_{i \in S_o} X_{ij_o}^{vot} W_{j_o}^{vot} \leq (la)_o, \forall v \in V, t = 1, \dots, T, o \in O$$

$$(16) \quad \text{int} \left( \sum_{i \in S_o} X_{ij_o}^{vot} W_{j_o}^{vot} \right) \bmod 24 \geq e_{j_o}, \forall v \in V, t = 1, \dots, T, o \in O$$

$$(17) \quad \text{int} \left( \sum_{i \in S_o} X_{ij_o}^{vot} W_{j_o}^{vot} \right) \bmod 24 \leq l_{j_o}, \forall v \in V, t = 1, \dots, T, o \in O$$

$$(18) \quad \sum_{i \in S} X_{ij_o}^{vot} W_{j_o}^{vot} = W_{j_o}^{vot}, \forall v \in V, o \in O, t = 1, \dots, T$$

### 9.1.2.4 Logical Constraints

$$(19) \quad \sum_{v \in V} \sum_{t=1}^T \sum_{i \in S_o} X_{ij_o}^{vot} = 1, \forall o \in O$$

$$(20) \quad \sum_{i \in S_o} X_{ij_o}^{vot} = \sum_{i \in S} X_{j_o i}^{vot}, \forall v \in V, o \in O, t = 1, \dots, T,$$

$$(21) \quad \sum_{o \in O} X_{joi}^{vot} \geq \sum_{o \in O} X_{ij_o}^{vot+1}, \forall v \in V, t = 1, \dots, T, i \in S$$

$$(22) \quad \sum_{o \in O} \sum_{i \in S} X_{ij_o}^{vot} \leq 1, \forall v \in V, t = 1, \dots, T$$

$$(23) \quad \sum_{o \in O} \sum_{i \in S} X_{ij_o}^{vot-1} - \sum_{o \in O} \sum_{i \in S} X_{ij_o}^{vot} \geq 0, \forall v \in V, t = 1, \dots, T$$

$$(24) \quad \sum_{v \in V} \sum_{t=1}^T \sum_{i \in S \setminus S_o} \sum_{j \in RUC \setminus j_o} X_{ij}^{vot} = 0, \forall o \in O$$

### 9.1.2.5 Constraints to enforce correct values for $\lambda_i^{uqvo}$

$$(25) \quad \sum_{t=1}^T \sum_{i \in N} X_{ij_o}^{vot} = \beta_j^{vo}, \forall v \in V, o \in O, t = 1, \dots, T$$

$$(26) \quad \sum_{t=1}^T \sum_{j \in N} X_{ji}^{vot} W_i^{vot} \geq \lambda_i^{uqvo} \left( \sum_{t=1}^T \sum_{j \in N} X_{ji}^{uqt} W_i^{uqt} + g_i^u \sum_{p \in P} q_{qp} \right),$$

$$\forall i \in N, (u, v) \in V, (o, q) \in O$$

$$(27) \quad \beta_{j_o}^{vo} \beta_{j_q}^{uq} \left[ \left( \sum_{t=1}^T \sum_{i \in S_o} X_{ij_o}^{vot} W_{j_o}^{vot} \right) - \left( \sum_{t=1}^T \sum_{i \in S_q} X_{ij_q}^{uqt} W_{j_q}^{uqt} \right) \right] \leq M \lambda_i^{uqvo},$$

$$\forall i \in N, (u, v) \in V, (o, q) \in O$$

$$(28) \quad \beta_i^{vo} \beta_i^{uq} \geq \lambda_i^{uqvo},$$

$$\forall i \in N, (u, v) \in V, (o, q) \in O$$

$$(29) \quad \lambda_i^{uqvo} + \lambda_i^{vouq} \leq 1,$$

$$\forall i \in N, (u, v) \in V, (o, q) \in O$$

### 9.1.2.6 Non negativity on the variables

$$(30) \quad X_{ij}^{vot} \in \{0,1\} \forall i \in N, v \in V, o \in O, t = 1, \dots, T$$

$$(31) \quad I_{pst} \geq 0 \forall p \in P, s \in S, t = 1, \dots, T$$

$$(32) \quad W_i^{vot} \geq 0 \forall i \in N, v \in V, o \in O, t = 1, \dots, T$$

$$(33) \quad Z_i^{od} \in \{0,1\} \forall i \in N, o \in O, d = 1, \dots, D$$

$$(34) \quad Y_s^{pd} \geq 0 \forall s \in S, p \in P, o \in O, d = 1, \dots, D$$

$$(35) \quad \delta_{oi} \geq 0 \forall i \in N, o \in O$$

$$(36) \quad \lambda_i^{uqv} \in \{0,1\} \forall i \in N, (v, u) \in V, (o, q) \in O$$

$$(37) \quad \beta_i^{vo} \in \{0,1\} \forall i \in N, v \in V, o \in O$$

## 9.2 Pictures of the value chain at Tau



**Figure 9.1:** *Picture of quarry*



**Figure 9.2:** *Picture of dumper in quarry*



**Figure 9.3:** *Picture of primary-crusher*



**Figure 9.4:** *Picture of conveyor belt transporting stone*



**Figure 9.5:** *Picture of production facility*



**Figure 9.6:** *Picture of inventory bins*



**Figure 9.7:** *Picture of discharger, land-side cargo handling equipment*



### **9.3 Terms used in maritime transportation planning :**

<i>Shipping</i>	refers to moving cargoes by ships.
<i>The shipper</i>	is the owner of the transported cargo.
<i>A shipment</i>	is a specified amount of cargo that must be shipped together from a single origin to a single destination.
<i>Routing</i>	is the assignment of a sequence of ports to a vessel
<i>Scheduling</i>	is assigning times (or time windows) to the various events on a ship's route.
<i>Deployment</i>	refers to the assignment of the vessels in the fleet to trade routes.
<i>A voyage</i>	consists of a sequence of port calls, starting with the port where the ship loads its first cargo and ending where the ship unloads its last cargo and becomes empty again.
<i>A cargo</i>	is a set of goods shipped together from a single origin to a single destination.
<i>A load</i>	is the set of cargoes that is on the ship at any given point in time.
<i>A full shipload</i>	consist of a single cargo that for practical reasons cannot be carried with other cargoes.
<i>A product</i>	is a set of goods that can be stowed together in the same compartment. In this is fraction and product used interchangeably
<i>A loading port</i>	is a pickup location (corresponds to a pickup node)
<i>A unloading port</i>	is a delivery location (corresponds to a delivery node)
<i>A discharger</i>	is a land-side handling equipment applied to load vessels

Source: Christiansen et al. [8]

#### **9.4 Multiple use of vessels, extract from vessels report 2008:**

Date	Vessel	Production site	Customer	Customer location	Product	Tonnage
2008.06.09	NORNE	ASKØY	NORSTONE	DIRDAL	0-2	1 100,00
2008.06.09	NORNE	ASKØY	NORSTONE	FORUS	5-8	1 009,00
2008.06.12	NORNE	DIRDAL	NORSTONE	BØNESET	0-32	2 500,00
2008.06.12	NORNE	DIRDAL	VASSBAKK & STOL	KÅRSTØ	0-32,16-32	2 002,00
2008.06.14	NORNE	ASKØY	SANDGREVSTUR	SUND	0-2	2 500,00
2008.06.14	NORNE	TAU	SANDGREVSTUR	SUND	2-5, 5-8	2 005,00

The table capture multiple use of vessel from site to multiple terminals at the same shipment, executed by the vessel Norne.