



Master's degree thesis

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Vehicle Routing Planning for joint venture
«Westintertrans»

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Abstract

In recent years it becomes very popular and profitable to organize transportation through the cross dock. The primary purpose of a cross dock is to enable a consolidation of differently sized shipments with the same destination to full truck loads, so that economies in transportation costs can be realized. But organizing a correct and efficient work of the cross dock and finding the cheapest way of pickup and delivery routes of full trucks with small cargos requires a lot of time, practice, energy and experience. In this study we tried to make this work faster and easier by using in practice our introduced model. The data for this study was provided by the company “Westintertrans” Ltd. The company “Westintertrans” Ltd. use its own trucks for providing all services and also it has its own cross dock. The objective of this thesis is to introduce and implement the mathematical model for finding the optimal routes of pickup and delivery and to compare the results of the model with the real routes of the company “Westintertrans” Ltd.

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1. Introduction

Transportations play a very important role in business in every country. Transportation system is the crucial element in a logistics chain, which joints the separated activities. Transportation occupies one-third of the amount in the logistics costs, and transportation systems influence the performance of logistics system very significant. Transportations are required in the whole production procedures, from manufacturing to delivery to the final customer, and returns. Only a good coordination between each link would bring the benefits to a maximum.

Without well developed transportation systems, logistics couldn't bring its advantages into full play. Besides, a good transportation system in logistics activities could provide better logistics efficiency, reduce operational cost, and promote service quality. The improvements of transport systems need the effort from both public and private sectors. A well-operated logistics system could increase the competitiveness of the government and enterprises.

The role of transportation in logistics system is more complex than just carrying goods for the owners. Its complexity can take effect only through high quality management. By means of well-handled transportation system, goods could be sent to the right place at right time in order to satisfy customers' demands. It brings efficacy, and also it builds a bridge between producers and consumers. Therefore, transport system is the base of efficiency and economy in business logistics and expands other functions of logistics system. In addition, a good transportation system in logistics activities brings benefits not only to service quality, but also to company competitiveness.

The operation of transport system determines the efficiency of moving products. The progress in techniques and management principles improves the moving loads, delivery speed, service quality, operation costs, the usage of facilities and energy savings. So, transportation takes a key part in the manipulation of logistic.

So from the point of view that transportation system is a part of logistics it is very interesting for us to try to use in practice the knowledge that we received during our education in Molde University College. A lot of researches concentrate in this field of logistics. Transport companies more and more actively looking for new ways of improving their operations. This shows that transport logistics becomes in recent years an important subject for research and study. This role of logistics in transport sphere became very important in last years, because almost all large transport companies exhaust their internal reserves of extensive growth so they forced to look for new intensive tools for receiving the competitive advantages and as a result an extra margin against their rivals.

Transportation logistics plays a very important role in Belarus economy as well. The geographical situation of the country as a transit country between west Europe (Germany, France, Holland, Poland and many others) in one hand and a huge space of Russia in the other hand gives to Belarus a great priorities and broad capabilities for extracting profit in this field. But the level of development of logistics science in Belarus is not high. More over the level of implementation of different logistics methods and models of optimization in transport companies is even the less. As the companies grow, they need more thorough planning of performed operations. The whole activities of such companies become more complex. Optimization of different side of their activities – such as truck scheduling or choosing the place for cross dock or many others – has become an

issue of a big concern for transport companies all over the world. One of such transport companies in Belarus is a joint Austrian-Belarusian venture “Westintertrans” Ltd. The complexity of problems and the scope of operations has made the logistic planning a subject of interest for research in my own opinion.

A cross docking terminals are an intermediate nodes in a distribution network, which are exclusively dedicated to the transshipment of truck loads. In contrast with traditional warehouses, a cross docks don't carry or carry at least a considerably reduced amount of stocks. Whenever the incoming trucks arrive at the yard of a cross dock, they are assigned to a dock doors, where inbound loads are unloaded and scanned to determine their intended destinations. The loads are then sorted, moved across the dock and loaded into outbound truck for an immediate delivery elsewhere in the distribution system.

The most important purpose of a cross docks is to enable a consolidation of differently sized shipments with the same destination to full truck loads, so that economy in transport costs can be realized. Success stories on cross docking, which resulted in considerable competitive advantages, are reported for several industries with high proportions of distribution costs such as retail chains (WalMart), mailing company (UPS), automobile manufacturers (Toyota) and others. In contrast with traditional point-to-point delivery an additional transshipment of cargos at the cross dock slows down the distribution processess and generates a significant amount of double handling. Consequently, efficient transshipment process is required, where inbound and outbound truckloads are synchronized, so that intermediate storage inside the cross dock terminal is kept low and on-time deliveries are ensured. A lot of problems constantly arises during the daily cross dock operations and have vital influence on a rapid transshipment processes. For example, it can be such problems as:

- Location of cross dock.
- Layout of the cross dock terminal.
- Assignment of destinations to cross dock terminal doors.
- Vehicle routing.
- Truck Scheduling.
- Resource scheduling inside the cross dock terminal.
- (Un-)Packing loads into (from) truck.

Here the decision problems to be solved during the life cycle of a cross docking terminal ordered from strategic to operational. As we can see the vehicle routing problem is in the middle of this scale. Different organizational and technical implementations lead to a large variety of possible vehicle routing problems in real world settings.

But scientific methods and solution approaches for planning of routes are not commonly applied in the company. More over vehicle routing specialized software packages are not available on Belorussian distribution market. But as we know significant cost reduction can be achieved by efficient planning of routes using known scientific approaches and commonly available software. Vehicle routing by itself is a complicated task dependent on a wide range of factors. Planning of routes belongs to the most difficult and interesting operational planning problem in logistics. So the application of modern methods is some kind of a challenge and has practical importance and interest.

For this reason the purpose of this work is to build a mathematical model for the problem of joint venture “Westintertrans” shortly described below. This model would allow to make an optimum choice.

The remaining part of the thesis is structured as follows. Section 2 discusses the literature on the topic relevant to this thesis. In section 3 the detailed problem description is given. Research objectives and the plan are presented in Section 4. Section 5 describes general assumptions for the model, input specifications and modeling considerations. Mathematical model with all her constraints and their descriptions are given in section 6. Section 7 describes the implementation of the model, its verification and validation. Conclusions are drawn in Section 8.

2. Problem description

The problem, described in this thesis, is the case of company «Westintertrans» Ltd. Therefore in this section we will give a short description of the company «Westintertrans» Ltd.

2.1 The company “Westintertrans” Ltd.

The company "Westintertrans", Ltd, Belarusian – Austrian joint venture, was founded to organize international automobile cargo transportation. It focuses on cargo transportation between Western and Eastern European countries – between Germany, Austria, Belgium, France, Italy, the Czech Republic, Switzerland, the Netherlands and Russia, Kazakhstan, Uzbekistan.

The company "Westintertrans" Ltd. exists on a market of transportation already almost 15 years and it constantly grows and extends and develops dynamically and stable.

Over 60 persons work at the enterprise’s office and there are also more than 400 drivers. All employees are highly qualified. It was confirmed by a many year cooperation with such serious partners as DHL, Hamman Group, LKW WALTER, Fixemer, Zufall, Panalpina, Rieck, Kuhne & Nagel, M&M Militzer & Munch, Revival Express, Sovtransavto Deutschland GmbH, ICT, Welz, Danzas and others.

This company has some advantages such as:

1. The enterprise has more than 220 trucks with tented trailers. The fleet of motor vehicles consists of Scania, Renault – Magnum, semi – trailers – canopy frames of European standards (90 m³, length – 13,6 m). All the trucks are equipped with mobile and satellite devices to get the latest information about the location of the truck. Also this company has its own maintenance service where trucks are repaired in the shortest terms.

2. The company owns only a new trucks, all they have a date of produce - 2007 and younger. The company has licenses for transportation of dangerous cargoes and expensive goods without convoy.

Joint venture “Westintertrans” has 4 offices – 1 in Minsk (Belarus), 1 in Dudenhofen (Germany) and 2 in Moscow (Russia). All branches represent independent structural subdivisions. The management of the whole company is carried out from the headquarters located in Minsk.

The company structure has several departments: Finance Department, Accounting, Logistics Department and 3 Sales Departments – Import, Export Departments and Department of assorted loadings. And the key interest from the point of view of optimization represents the last one - Department of assorted loadings.

The company’s German office is completely responsible for this activity. It has its own warehouse. So it takes small cargoes (incomplete truck) from everywhere in Germany, brings them to the warehouse, sorts them and forms the full complete trucks following in same destinations in Russia. So cargoes in a warehouse usually store not more than one-two days. All cargoes are different – they have different sizes, it is necessary to take away them at various times and from different places, also it is necessary to deliver them to Russia at various times and place. So it is

very difficult sometimes to plan and coordinate this work because every moment you need to decide what cargoes it is necessary to take away by your own truck, what cargoes to take away by external truck, in what order it is necessary to organize, or maybe it is necessary to refuse some cargoes at all.

In order to structure and promote scientific progress, this paper introduces a modification of the Model of vehicle routing. With the help of this model, existing literature is reviewed and future research needs are identified.

2.2 Problem description

First of all we need to formulate the problem. There are trucks which stand in different places in Germany and are unloaded and ready for new loading at various times. We can use some of them (we do not obliged to use all of them) for our pickup routes. There are also small cargoes (not for full truck) in different places in Germany and various times, there is a cross-dock in Dudenhofen in Germany, which can at take and ship small cargoes. In a cross-dock there are expenses on reception-shipment of cargoes, and there are places of delivery of cargoes in Russia and the time horizon for delivery. So we would like to build the algorithm that should consider the best strategy for a concrete situation, consider when better to collect cargo by which truck and when it is better to refuse the cargo, also this algorithm should make constantly recommendations how to operate in order to get the maximum profit, how many trucks to use. Also in algorithm it is necessary to consider carrying capacity of the truck.

On the figures below we can see the examples, which represent the processes of transporting small cargos to cross dock from the different places of loading in Germany (Figure 1) and carrying small cargos from cross dock to the different places of unloading in Russia (Figure 2). They are presented with the help of the program AutoRoute2002, which allows calculate the distances between different places (cities) in different countries with the given route.

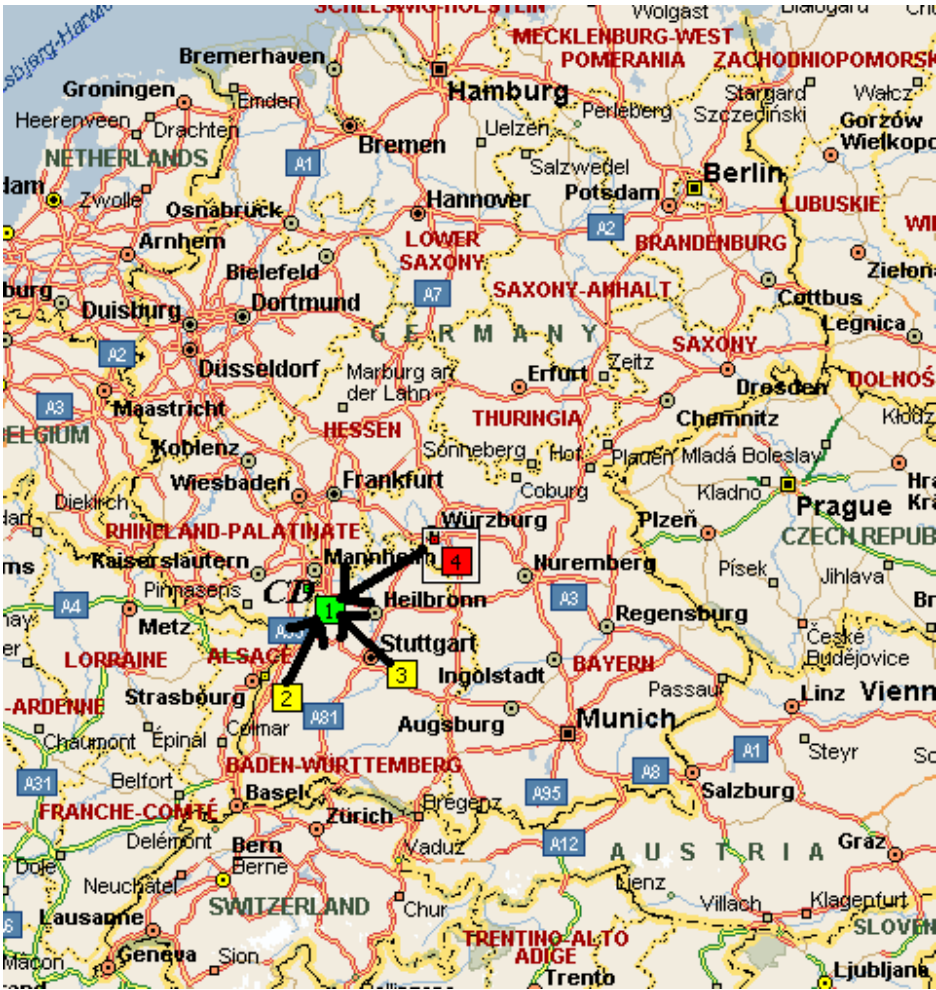


Figure 1: Cross dock and operations for carrying cargos to cross dock from the places of loading in Germany

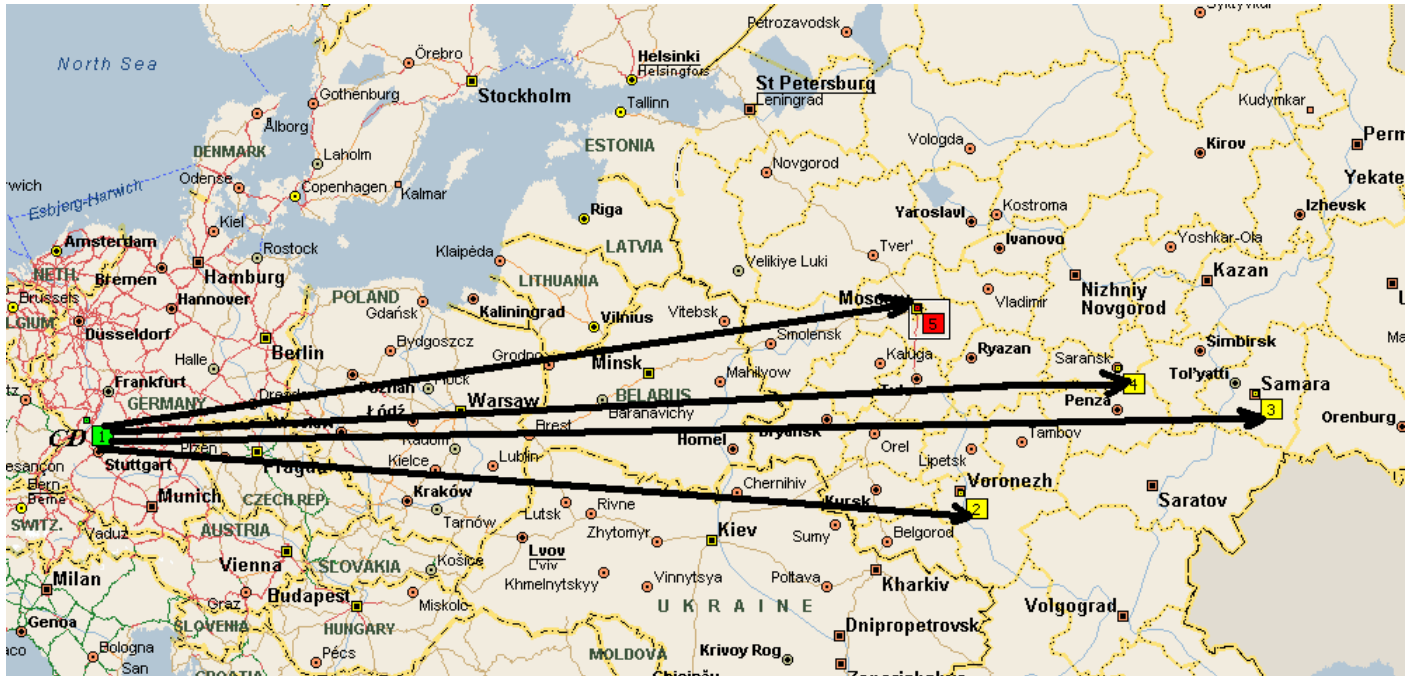


Figure 2: Cross dock and operations for carrying cargos from cross dock to the places of unloading in Russia

2.3 Cross dock

The problem treated in this thesis is a case of cross dock operations performed by trucks owned by the company “Westintertrans” Ltd. The cross dock was established in 2009 as a base for collecting small cargoes for combining them to full trucks. The cross dock is situated in Germany in D67373 Dudenhofen. Later in 2010 its became an independent company “Advice Logistics”, that provides services for its clients independently from the company “Westintertrans” Ltd. So

from the beginning the cross dock served only needs of the company “Westintertrans” Ltd. And its used only park of trucks of the “Westintertrans” Ltd. But now the company “Advice Logistics” can attract and use also the transport of others companies to serve its own needs and it can do it in terms of her own profitability and economic efficiency. But for our problem and model we will do assumption that company use only its own (internal) trucks without attracting external trucks in case to avoid an additional costs. Because, as we know, there are a lot of disadvantages arises while using the external transport, especially that the usage of hired transport is rather expensive.

2.4 Park of trucks

The company “Westintertrans” Ltd. has her own large park of trucks – 220 trucks. Nearby half of them every week unloaded in Germany and loaded there again with the end destination in Russia and on the same week other half of trucks unloaded in Russia and loaded there again with the end destination in Germany . So on the next week they delivery their cargos to the end destinations, so they became vise versa. But despite of such huge amount of trucks the company “Westintertrans” Ltd. is not always can served needs of company “Advice Logistics”. There are some reason for that:

- Sometimes it is more profitable for the company “Westintertrans” Ltd. to take the full-truck loading, it is possible if the price for suck cargo is high.

- The truck is unloaded far away from the cross dock. (The cross dock is situated in the South of Germany, so if the truck is unloaded in the North or Northeast of the Germany, it is not efficient to use this truck for cross dock operations.)
- And also a lot of others causes can be the reason for that.

But because of the huge amount of its own trucks such situations have place not very often in the company “Westintertrans” Ltd.

The fleet of vehicles consists of Scania, Renault – Magnum, semi – trailers – canopy frames of European standards (90 m³, length – 13,6 m, capacity - 20 tons). All the trucks are equipped with mobile and satellite devices to get the latest information about the location of the truck.

For our model we will assume that all trucks have the only measure of the capacity – the weight of the cargoes, which can not exceed the maximum of 20 tons.

2.5 Places of unloading the trucks and places of loading/unloading the cargoes

As we already mentioned, every week the trucks of the company “Westintertrans” Ltd. unloaded in different places in Germany. These places has following characteristics – time of unloading the truck, concrete name and postcode of the city and the exact address, the distances from the place of unloading the truck to the every places of loading the cargo.

Also every week the company “Advice ” and the company “Westintertrans” Ltd. receive the propositions of different incomplete cargos, that need to be taken in different places of loading in Germany and to be delivered to different places in Russia. So these places of loading and unloading the cargos have similar characteristics – possible time for loading/unloading, concrete name and postcode of the city and the exact address, the distances to/from the cross dock, and the demand for each place in tons. Also for our model we will need the data about distances between the places of loading, and also between the places of unloading.

My work experience in the company “Westintertrans” Ltd. last already for 4 years and from my practice I would like to mention, that assorted ladings is very popular from Germany (or other European countries) to Russia, but not vise versa. Noone has a cross dock in Russia and does not collect and sort incomplete cargos on the cross dock there. This kind of transportation is usual and widely-spread only in direction from Europe to Russia. And our case is not exception from this situation – ours cross dock is situated in Germany.

3. Literature review

At cross docks incoming deliveries of inbound trucks are unloaded, sorted, moved across the cross dock terminal and finally loaded into outbound trucks, which immediately leave the cross dock towards their next destination in the distribution chain. Accordingly, a cross dock terminal is a consolidation point in the distribution networks, where multiple smaller shipments can be merged to full truck loads in order to realize economy in transport system. In this context, the vehicle routing problem associated with pickup and delivery of small cargoes to and from cross dock terminal is especially important to ensure a rapid turnover and on-time delivery. As using of cross docks terminals is a comparatively new logistics strategy, there is not yet a massive body of academic literature on this subject. But due to its high real life significance several vehicle routing procedures have been introduced during last years, which all treat specific cross dock settings.

However, most of papers have investigated the physical design of cross dock terminals (Ratliff, Vate and Zhang, 1999; Bartholdi III and Gue, 2004) and its locations (Gumus and Bookbinder, 2004). There are very few papers that deal with the transport problems, associated with cross docking. They have studied two types of network models.

The first type of models is characterized by an open network in which distribution flow begins from the single supplier and finishes at the single customer via a cross dock terminal without forming any loop. Papers in this area include Sung and Song (2003), Jayaraman and Ross (2003) and Chen et al.(2006). Sung and Song (2003) have discussed problem of deciding whether to open a cross dock terminal or not, and problem of assigning trucks for transportation from a supplier

to a single destination via one of open cross dock terminals. They have proposed a tabu search algorithm for transport problem. Jayaraman and Ross (2003) have investigated the same problem. Given a cost for opening each supplier, they have discussed, how to decide whether supplier should be opened or closed. Simulated annealing methods were used in this paper. In Chen et al. (2006), time windows constraints for suppliers and customers are given, and the inventory costs at cross dock terminal is also taken into account. These authors have proposed a hybrid metaheuristics combining simulate annealing and tabu search.

In the second type of network models, each truck leaves the cross dock terminal to pick up or deliver products and returns to cross dock terminal after completing its route. Only two publications, that of Lee, Jung and Lee (2006) and Wen, Larsen, Clausen, Cordeau, Laporte (2009), have studied a transport problem of this type. This problem consists of a single cross dock terminal, multiple suppliers and multiple customers. The task is to assign routes to a set of trucks at the cross dock terminal, so that suppliers and customers are visited within their time windows. Lee, Jung and Lee (2006) assume that all trucks should arrive simultaneously at the cross dock terminal from their pickup routes, but Wen, Larsen, Clausen, Cordeau, Laporte (2009) overcome this assumption. Instead of this, the dependency among the trucks is determined by the consolidation decisions in their paper.

For the problem in this Master Thesis we will use the open type of network model, like in type one, but the distribution flow will not start strictly from a single supplier and will not end strictly at a single customer. In the model will be possibility to visit many suppliers and customers via a cross-dock without forming any loop if the loading capacity of truck will allow. So we consider a kind of mix of two types of existing models – a kind of a new problem. The model in this paper

built on the base of the model from the Wen, Larsen, Clausen, Cordeau, Laporte (2009). This model will be describes in the Section 6 “Mathematical formulation of the model”. Also in this section we will describes the changes in the model, that we will do for transform the model for our type of the network and for our problem in general.

In order to get more information about existing mathematical models the article of Mula, Peidro, Diaz-Madronero, Vicens (2009) was very useful. This paper presents a review of the mathematical programming models for supply chain production and transportation planning. They identify the current and future research in this field and propose a framework, based on following elements: supply chain structures, decision levels, modeling approaches, purposes, shared information, limitations, novelty and application. They briefly describe each paper, but they do not describe or formulate models that have been considered in the details. Their work provides the readers with a starting point to studing the literature about best management methods for the different supply chain production and transportation planning problems.

My paper addresses the vehicle routing problem with cross dock terminal (VRPCD), where a set of homogeneous trucks are used to transport goods from the suppliers to the corresponding customers via a cross-dock. The objective of the VRPCD is to minimize the total traveled distance while respecting time window constraints at the nodes and a time horizon for the whole transportation operation.

4. Research Objectives and Plan

The pickup and delivery less-than-full truck cargos system has to be estimated under some projected conditions. This leads to the conclusion, that the appropriate way for the analysis is to describe and analyze the system with the help of mathematical model that can be evaluated analytically.

The one from the main objectives of this thesis is to design and develop a mathematical model for evaluation of alternative possibilities. The model has to represent pickup and delivery operations performed by trucks of the company “Westintertrans” Ltd. as efficient as possible, including the possibility to use the different amount of trucks and also including the time-windows constraints for places of loading and unloading. The mathematical model will be than write on the AMPL-programming language for receiving the final results of the model and have the possibility to estimate and analyze the solution, to analyze the possibility implementation of the model in real life and to have the possibility to make verification and validation of the model.

In order to make a model adequate and useful, following steps must be performed:

- As a base for the AMPL-model, a basic mathematical model will be created.

The basic mathematical model will describe all deterministic elements of the model. These elements are: the distances between places of loading of cargos and cross dock, between cross dock and places of delivery of the cargo, between the

places of unloading of trucks and the places of loading, opening hours of the places of loading/unloading of cargos and trucks, demand for the places of loading/unloading of cargos, the fixed time for unloading and reloading at the cross-dock and the time for unloading and reloading one ton of the cargo;

- The data with of needed information for the model from the company “Westintertrans” Ltd. should be collected;
- When all the deterministic elements are modeled by the mathematical model, the AMPL-model will be designed. Its require some additional knowledge on the AMPL-programming. For details – look “AMPL: A Modeling Language for Mathematical Programming” by Robert Fourer, David M. Gay, and Brian W. Kernighan;
- The next step is an AMPL-model running and receiving the quantative results;
- On the last stage the analysis of created model will be carried out. Different number of internal trucks in use, number of the places of loading and unloading will be evaluated based on number of total sum of costs, which is the main measure of efficiency.

The important part of the AMPL-model is the continuous verification of the model. Some changes in the model may lead to the behaviors that are not possible for the real-life system, as in real life there is also a human factor and some decisions are made by company’s director or dispatcher on the cross dock considering the circumstances. Therefore, the model has to be adequate for the real world and verified on every change.

5. Input specification

This section contains basic model assumptions and general data considerations. We also describe the modeling of major inputs: places of pickup and delivery cargos and unloading of trucks, distances between these places and time-windows constraints. Distances between places will be described by the time that is necessary and sufficient to overcome these distances.

5.1 Data collection and considerations

As a primary source of information were used the data from the company “Westintertrans” Ltd. and her programs “Spedition” and “AutoRoute2002”. Another sources of the information are the offers from company’s customers for transportation of less-than-full truck cargos from Germany to Russia. The data from “Spedition” contains the information about date, place and time of unloading of trucks. The fragment of the program “Spedition” are shown in Figure 3. There we can see the information about current transportations. On the window in the center of the program we can see the information about the time of unloading the concrete truck (it is the sms from the driver with the relevant information). But the program also has an archive, in which we can find the same information about the transportations, that was made months or years ago.

The screenshot displays the 'Spedition' software interface, which is a spreadsheet-style application for managing shipping orders. The main window contains a table with the following columns: 'Номер' (Order Number), 'Дата разгрузки' (Unloading Date), 'Разгрузка' (Unloading), 'Заказ' (Order), 'Тип машины' (Vehicle Type), 'Разрешения' (Permits), 'Комментарий' (Comments), a grid of days from 06 to 19, 'Заказчик' (Client), 'Экспедитор' (Dispatcher), 'Экспедитор в Минске' (Dispatcher in Minsk), 'Экспедитор' (Dispatcher), 'Блоки' (Blocks), 'Простои' (Delays), 'Гараж' (Garage), 'Неделя' (Week), and 'Приписка' (Attachment). The table lists various orders with details such as origin and destination cities (e.g., Moscow, Minsk, Brest, Novosibirsk), vehicle types (e.g., AM, AT, CD, CI), and specific dates and times for loading and unloading. A pop-up window is visible over the table, displaying contact information for 'Полна' (Polina) and 'Игорь' (Igor), including phone numbers and addresses in Novosibirsk.

Figure 3: The fragment of the program “Spedition”

By combining the information, that we collected, we get the all data that is necessary for our model. So on the table in Appendix A we can find this information (in this table we can find the information, that is necessary for running our model 1 time, only for one example).

By considering the first place of loading of the cargo we will describe how we get all information in the table. So the information for first column “Place of unloading of truck” and second column “Time of unloading” we can get from the company’s program “Spedition”. For our model we bring the information not about of all the trucks, but only about of few of them that are situated not very far from cross dock. The information in the first column contains the postcode and name of the city of unloading the truck, for example D77815 Buhl. And in the second column we can find the time of unloading the truck in this place in minutes today, for example 600 minutes that means 10.00 in the morning (600minutes/60 minutes in hour=10 hours).

The information for the next six columns and for the last one we received from the customer’s orders and offers. They required to transport their goods (demand in tons) from the place of pickup in Germany to the concrete place of delivery in Russia and they have the special time requirements for that. The places of loading marks in the table in the same way as the places of unloading the trucks – postcode and the name of the city. Columns “a11” and “b11” represent the time in minutes in which the cargo should be picked up, for example from 480 till 2640 minutes, that means from today 8.00 in the morning till tomorrow 20.00. The place of unloading in Russia marks by the name of the city – column “Place of unloading”. And correspondingly columns “a12” and “b12” represent the time in minutes in which the cargo should be delivered, for example from 480 till 12720 minutes, that means that delivery is possible from today 8.00 in the morning during eight days till 20.00 of the ninth day. This is a standard time horizon for such transportation.

All distances in the table in kilometers (Distance from the place of unloading of truck to the place of loading in km, Distance from the place of loading to the CD in

km, Distance from the CD to the place of unloading in km) define with the help of the program “AutoRoute2002”. For this we need just to enter in the program the names of the cities or their codes and the program gives us the information about distance between them. But for our model these distances in kilometers are not suitable, because the other information we have in time parameters – in minutes: time-windows constraints, the fixed time for unloading and reloading at the cross-dock and the time for unloading and reloading one ton of the cargo. So we need to convert distances in kilometers into distances in time. For that we will make assumption that truck in general goes with the speed 60 km/h, so the distance of 60 km = the distance of 60 minutes. But that is true only for short distances (less or about 600 km). Because by the international law, driver should have eight hours of rest every day. Accordingly with the company’s statistics, common truck can run in general 600 km per day. So we will assume that after every 600 kilometers (600 minutes) truck will need eight hours (480 minutes) of rest. So if our distances are considerably more than 600 km, for converting them in time we will use the next formula:

$$\text{Distance in km} + (\text{integer from dividing (Distance in km/600)}) * \\ *60(\text{minutes in hour}) * 8 (\text{hours of rest})$$

For example, distance in kilometers is 2562 km, so in time it will be $2562 + \text{integer}(2562/600) * 60 * 8 = 2562 + 4 * 60 * 8 = 2562 + 1920 = 4482$ minutes.

In the same way we fill the information in the table for every place of loading of the cargo. And then based on this data we will solve the AMPL-model.

5.2 General assumptions

As we already saw, there are some assumption was made during the work at the model. The main assumption for modeling the pickup and delivery processes through cross dock is that truck in common goes with the constant speed 60 km/h without any delays and traffic jams. So to run 60 km we need 60 minutes accordingly to this our assumption.

Most of the places of pickup and delivery and also of unloading the truck are closed from 19.00 until 7.00. These places cannot be visited at this time, and if so, the truck has to wait until morning to perform the corresponding operation. But for our model we use only the time horizon constraints (pickup during 2 days, deliver during 8 days and so on independently if it will be day or night).

As our model includes demands specifications for the places of pickup and delivery, the following assumption was made: the only measure of the demand is tons of the cargo – no differences in length, width, high. And the capacity of the truck is 20 tons. So to check if we still not exceed the capacity of the truck we need only summarize all cargos in truck by tons.

It is also assumed, that we always have enough internal trucks available to start the planned route, so the external trucks will never hired to perform such operation.

5.3 Reloading/Unloading operation durations at the cross dock

The duration of reloading and unloading operations usually depends on the amount of trucks that have to be reloaded/unloaded. Even though in real life the duration of reloading/unloading operation will depend on the every truck and the amount of trucks in the cross dock, in this model we assume that it standard and do not depends from any factors. Therefore, cross dock will have standard loading/unloading times, which will not change from truck to truck. Accordingly to the company's statistics the fixed time for unloading and reloading at the cross-dock is equal to 61 minutes in average, the time for unloading and reloading one ton of the cargo is equal to 15 minutes in average.

6. Mathematical formulation of the model

In this section we will discuss the mathematical formulation of the model.

We built our mathematical model on the base of the model Wen, Larsen, Clausen, Cordeau, Laporte (2009). We did some changes in it to provide the better compliance with the requirements of our problem – to create the open network and to provide the possibility to visit many suppliers on our pickup route and many customers on our delivery route.

So we now present a mixed integer linear programming formulation for the VRPCD. Denote set of pickup nodes by $P = \{1, \dots, n\}$, set of delivery nodes by $D = \{n + 1, \dots, 2n\}$. To involve in the model our set of nodes where trucks unloaded and ready to go to the pickup node we introduce in the model one more set of nodes - $T = \{2n+1, \dots, m\}$. And we should expand some next sets and include in them this new set T . Each request i is identified by his node pair $(i, i + n)$, where i is the pickup node and $i + n$ is the associated delivery node. The cross dock terminal is represented by two nodes and denoted by the set $O = \{o_1, o_2\}$, where the first node represent the ending point for pickup routes, and the second one for the starting point for delivery routes. In comparison with the mathematical model of Wen, Larsen, Clausen, Cordeau, Laporte (2009) we modified this set by excluding from the set two nodes – starting point for pickup route and ending point for delivery route as we would like to get an open network. Further, define $N = P \cup O \cup D \cup T$ (as we can see we include here our new set T). The set E denotes all the feasible arcs in the network. It consists of the arcs $\{(i, j) : i, j \in P \cup o_1, i \neq j\}$ and the arcs $\{(i, j) : i, j \in D \cup o_2, i \neq j\}$ and the arcs $\{(i, j) : i, j \in D \cup T, i \neq j\}$. The set E was

also a little modified in comparison with Wen, Larsen, Clausen, Cordeau, Laporte (2009) because of our changes in set O and introduction the set T . Let K be the set of vehicles.

The parameters are denoted as follows:

c_{ij} = the travel time between node i and node j ($(i,j) \in E$);

d_i = the amount of demand of request i ($i \in P$);

$[a_i, b_i]$ = the time window for node i ($i \in N$);

A = the fixed time for unloading and reloading at the cross-dock;

B = the time for unloading and reloading one ton of the cargo;

Q = the vehicle capacity.

The variables are:

$$x_{ij}^k = \begin{cases} 1 & \text{if vehicle } k \text{ travels from node } i \text{ to node } j ((i, j) \in E; k \in K), \\ 0 & \text{otherwise} \end{cases},$$

$$u_i^k = \begin{cases} 1 & \text{if vehicle } k \text{ unloads request } i \text{ at the cross - dock } (i \in P; k \in K), \\ 0 & \text{otherwise} \end{cases},$$

$$g^k = \begin{cases} 1 & \text{if vehicle } k \text{ has to unload at the cross - dock } (k \in K), \\ 0 & \text{otherwise} \end{cases},$$

$$r_i^k = \begin{cases} 1 & \text{if vehicle } k \text{ reloads request } i \text{ at the cross - dock } (i \in P; k \in K), \\ 0 & \text{otherwise} \end{cases},$$

$$h_k = \begin{cases} 1 & \text{if vehicle } k \text{ has to reload at the cross-dock } (k \in K), \\ 0 & \text{otherwise} \end{cases};$$

s_i^k = the time at which vehicle k leaves node i ($i \in N; k \in K$);

w_k = the time at which vehicle k starts reloading at the cross-dock ($k \in K$);

t_k = the time at which vehicle k finishes unloading at the cross-dock ($k \in K$);

v_i = the time at which request i is unloaded by its pickup vehicle at the cross-dock ($i \in P$).

In addition, M is an arbitrarily large constant.

So our model can be formulated as follows:

Mathematical model
Formulation:
$\min \sum_{(i,j) \in E} \sum_{k \in K} c_{ij} x_{ij}^k$
Subject to
(1) $\sum_{j:(i,j) \in E} \sum_{k \in K} x_{ij}^k = 1, \forall i \in P \cup D$
(2) $\sum_{j:(i,j) \in E} \sum_{k \in K} x_{ij}^k \leq 1, \forall i \in T$

(3) $\sum_{i \in P} \sum_{j: (i,j) \in E} d_i x_{ij}^k \leq Q, \forall k \in K$
(4) $\sum_{i \in D} \sum_{j: (i,j) \in E} d_i x_{ij}^k \leq Q, \forall k \in K$
(5) $\sum_{j: (h,j) \in E} x_{hj}^k = 1, \forall h \in T \cup o_2, k \in K$
(6) $\sum_{i: (i,h) \in E} x_{ih}^k - \sum_{j: (h,j) \in E} x_{hj}^k = 0, \quad h \in P \cup D, k \in K$
(7) $\sum_{j: (j,o_1) \in E} x_{jo_1}^k = 1, k \in K$
(8) $s_j^k \geq s_i^k + c_{ij} - M(1 - x_{ij}^k), \forall (i,j) \in E, k \in K$
(9) $a_i \leq s_i^k \leq b_i \forall i \in N, k \in K$
(10) $\sum_{(i \in T, j \in P)} c_{ij} x_{ij}^k \leq 300, k \in K$
(11) $u_i^k - r_i^k = \sum_{j \in P \cup O_1} x_{ij}^k - \sum_{j \in D} x_{i+n,j}^k, \quad i \in P, k \in K$
(12) $u_i^k + r_i^k \leq 1, \quad i \in P, k \in K$
(13) $\frac{1}{M} \sum_{i \in P} u_i^k \leq g_k \leq \sum_{i \in P} u_i^k, k \in K$
(14) $t_k = s_{o_1}^k + A g_k + B \sum_{i \in P} d_i u_i^k, k \in K$

(15)	$w_k \geq t_k, k \in K$
(16)	$w_k \geq v_i - M(1 - r_i^k), \forall i \in P, k \in K$
(17)	$v_i \geq t_k - M(1 - u_i^k), \forall i \in P, k \in K$
(18)	$\frac{1}{M} \sum_{i \in P} r_i^k \leq h_k \leq \sum_{i \in P} r_i^k, k \in K$
(19)	$s_{o2}^k = w_k + Ah_k + B \sum_{i \in P} d_i r_i^k, k \in K$
(20)	$x_{ij}^k, u_i^k, r_i^k, g_k, h_k \in \{0,1\}, \forall i \in P, \forall (i,j) \in E, k \in K$
(21)	$s_i^k, t_k, w_k \geq 0, \forall i \in N, k \in K$
(22)	$v_i \geq 0, \forall i \in P$

The objective is to minimize total distance traveled, which is expressed in time. We can divide all constraints on two parts: vehicle routing constraints (constraints (1) to (10)) and the consolidation decisions at cross dock terminal (constraints (11) to (19)).

We can formulate both the pickup part and the delivery part as VRPTWs. Constraints (1) mean that each node is visited once by one truck. Constraints (2) for set T mean that each unloaded truck can be used only once or can be not used at all.

Constraints (3) and (4) help us to ensure that for each truck, the cargo on the pickup route and on the delivery route does not exceed the vehicle capacity. Constraints (5) ensure that each truck's pickup route must start from one of the places where trucks unloaded and delivery route must begin from O_2 . Constraints (6) are the flow conservation constraints. Constraints (7) force each truck to end the pickup route at O_1 . Constraints (8) compute traveling time between two nodes, if they are visited consecutively by the same truck. Constraints (9) state that each node is visited within its time window and the whole operation is completed within the time horizon. Constraints (10) help to decide if we should bring the cargo, or we should refuse it.

For the consolidation decisions at cross dock terminal, whether a truck k should unload or reload product i depends on its pickup and delivery routes. This dependence which shows the relationship between the pickup part and the delivery part is expressed by constraints (11) and (12). In these constraints the following three cases are considered:

- ✓ if truck k picks up i but does not deliver $i+n$, then it unloads his product at the cross dock terminal,
- ✓ if truck k doesn't pick up i but delivers $i+n$, then it have to to reload the product at the cross dock terminal,
- ✓ if the truck neither picks up i nor delivers $i + n$, then it neither unloads nor reloads the product at cross dock.

Constraints (13) to (19) define the internal working flows and deadlines for all the trucks at the cross dock terminal. Constraints (13) force g_k to be 1 if the truck

needs to unload. Constraints (14) indicate that the unloading duration for the truck k consists of a fixed time (A) for the preparation of unloading, and the time for unloading the cargo, equal to the unit time for unloading a one ton of the cargo (B) multiplied by the number of tons ($\sum_{i \in P} d_i u_i^k$) to be unloaded from the truck.

Constraints (15) and (16) ensure that the truck cannot begin reloading until it finishes unloading, and all the cargos to be reloaded on it are ready. The ready time of product i is represented by constraint (17). This ready time depends on the time at which the pickup truck of product i finishes unloading. Constraints (18) and (19) for reloading operations are similar to (13) and (14).

Constraints (20) state that variables for vehicle and route selected, for vehicle and request selected for unloading at the cross dock, for vehicle and request selected for reloading at the cross dock, for vehicle and request selected to be forced for unloading at the cross dock, for vehicle and request selected to be forced for reloading at the cross dock are binary.

There are also non-negativity requirements on all variables (21) and (22).

In comparison with the model of Wen, Larsen, Clausen, Cordeau, Laporte (2009) we did some changes:

- ✓ We introduce the constraints (2) which are similar to (1) but they are only for set T and mean that each unloaded truck can be used only once or can be not used at all.
- ✓ We modify constraints (5) to state that each vehicle's pickup route must depart from one of the places where trucks unloaded (but not from the

cross dock as in the model of Wen, Larsen, Clausen, Cordeau, Laporte (2009)) and delivery route must leave from O_2 .

- ✓ Constraints (7) are also modified in order that it is not necessary to force each vehicle to finish the delivery route at the cross dock.
- ✓ We introduce constraints (10), which help to decide if we should bring the cargo, or we should refuse it. We can take the cargo if the distance between the place of unloading of the truck and the place of pickup the cargo less than 300 km and we should refuse the cargo if this distance is more than 300 km – it will be cost too much for the company and it is not efficient.
- ✓ Also we modify constraints (11) in accordance with our changes in set O .

7. Model implementation, verification and validation

In this section we will discuss the implementation, verification and validation of the mathematical model in AMPL 90 (business version).

7.1 Implementation software

For implementation of the mathematical model we used AMPL, which was chosen for three reasons:

- AMPL offers an interactive command environment for setting up and solving mathematical programming problems. A flexible interface enables several solvers to be available at once so a user can switch among solvers and select options that may improve solver performance.
- Once optimal solutions have been found, they are automatically translated back to the modeler's form so that people can view and analyze them.
- All of the general set and arithmetic expressions of the AMPL modeling language can also be used for displaying data and results; a variety of options are available to format data for browsing, printing reports, or preparing input to other programs.

To build and solve models with AMPL we need to have some knowledge about AMPL modeling language.

We have to write three files – one with the model (***.mod), another with the data (***.dat) and the last one is ***.run-file. The results of the AMPL run can be viewed through automatically generated report in ***.sol-file. By default the report contains only the following information - the meaning of the targeted function. Other information can be requested to be present in the report (about meanings of the sets, parameters, variables, constraints and the cost function).

A short summary on AMPL software can be found in the book “AMPL: A Modeling Language for Mathematical Programming” by Robert Fourer, David M. Gay, and Brian W. Kernighan.

7.2. AMPL solution:

The files primerWEST.mod, primerWEST.dat and primerWEST.run you can find in Appendix B.

In this section we will only introduce the data from the file primerWEST.sol, which consist the information about result of running the model.

Next picture introduce the running of this file by AMPL in DOS.


```

C:\WINDOWS\system32\cmd.exe
Microsoft Windows [Version 5.2.3790]
(C) Copyright 1985-2003 Microsoft Corp.

M:\>c:

C:\>cd ampl90

C:\AMPL90>ampl primerwesttrue.run
AMPL Version 20021031 (Win32)
CPLEX 9.0.0: optimal integer solution; objective 25498
345 MIP simplex iterations
0 branch-and-bound nodes

C:\AMPL90>_

```

Figure 4: The running of the file primerWEST.run by AMPL in DOS

File primerWEST.sol:

For the case of 2 trucks

Time = 9853

Route :=

```

1 3 T2 1
2 1 T2 1
3 O2 T2 1
4 O2 T1 1
5 4 T1 1
6 8 T1 1
7 17 T1 1
8 9 T1 1
9 7 T1 1
10 20 T2 1
11 5 T1 1

```

For the case of 3 trucks

Time = 14683

Route :=

```

1 3 T1 1
2 1 T1 1
3 O2 T1 1
4 O2 T3 1
5 O2 T2 1
6 16 T1 1
7 17 T3 1
8 9 T3 1
9 7 T3 1
10 20 T2 1
11 5 T2 1

```

For the case of 4 trucks

Time = 19951

Route :=

```

1 O2 T1 1
2 1 T1 1
3 O2 T2 1
4 O2 T3 1
5 O2 T4 1
6 16 T4 1
7 17 T1 1
8 9 T2 1
9 19 T2 1
10 20 T3 1
11 5 T4 1

```

13 2 T2 1
O3 6 T1 1
O3 10 T2 1;

13 2 T1 1
15 4 T3 1
O3 6 T1 1
O3 8 T3 1
O3 10 T2 1;

13 2 T1 1
14 3 T2 1
15 4 T3 1
O3 6 T4 1
O3 7 T1 1
O3 8 T2 1
O3 10 T3 1;

7.3 Output analysis

So, as we can see, the best result with the minimum time is in the case of 2 trucks – 9853 minutes (it means that the last cargo will be delivered to the end customer after 6 days and 20 hours from today). So the company should select two trucks for performing the transportation of these cargos – from points 11 (D77815 Buhl) and 13 (D73466 Lauchheim) and the routes of trucks will be following:

Truck1: 11-5-4-O2-O3-6-8-9-7-17 or D77815Buhl - D69123Heidelberg - D67354Romerberg - cross dock – Vladimir - Nizhnij Novgorod – Kazan - Samara

Truck2: 13-2-1-3-O2-O3-10-20 or D73466Lauchheim - D86720Noerdlingen - D89269Vohringen- D71701Schwieberdingen - cross dock – Moscow.

The illustrations of these two route we can see on the following figures:

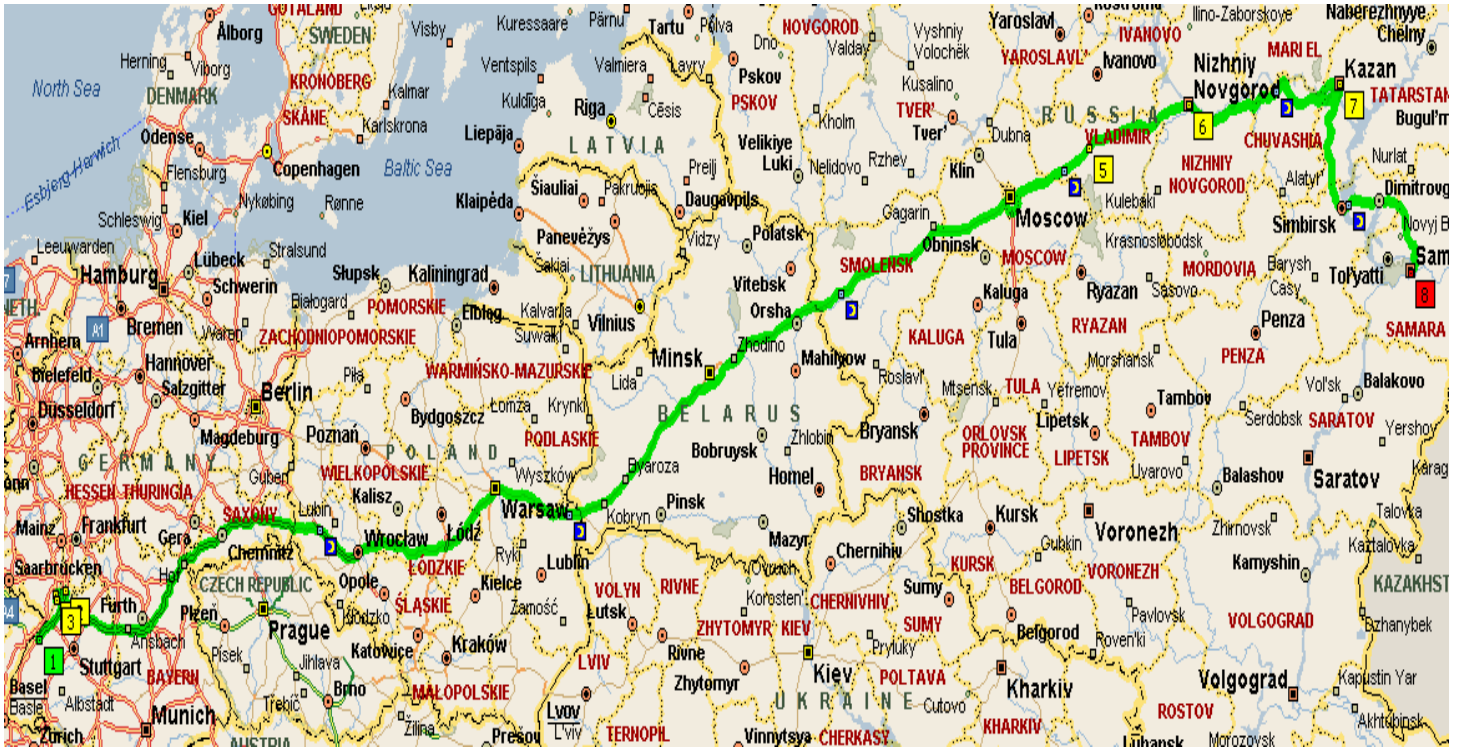


Figure 5. The representation of the route of truck 1

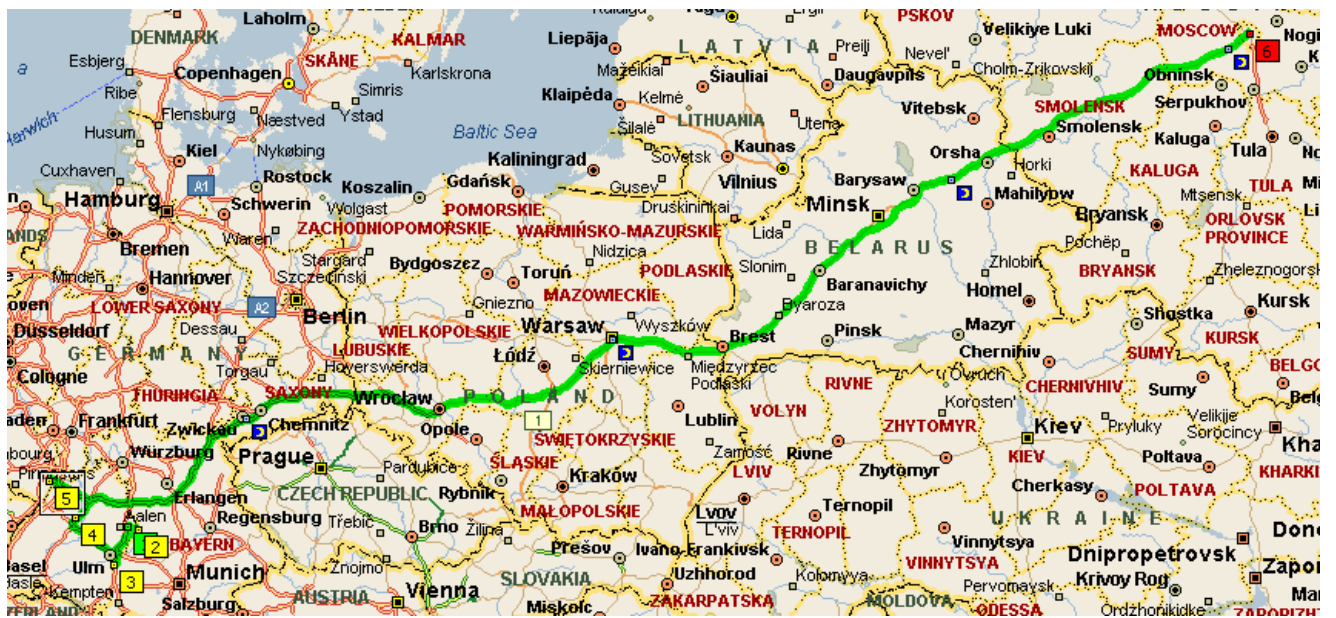


Figure 6. The representation of the route of truck 2

Also in this case we can see that Truck1 pickup cargos from the places of pickup 4 and 5 and then unload at the cross dock the cargo from the place 5 and reload the cargos from places 1, 2 and 3. Truck2 pickup cargos from the places of pickup 1, 2 and 3 and then unload all of them at the cross dock and reload the cargo from the place 5.

The same procedures were running during two weeks (10 working days – from 4th till 15th of April). So we collect the results from the model during these week in the table below and then compare them with the real routes, which were done by the company “Westintertrans” Ltd.

№	Number of Truck	Date of loading	Places of pickup	Places of delivery	Date of unloading
1	1	04.04.2011	D77815Buhl + D69123Heidelberg + D67354Romerberg	Vladimir - Nizhnij Novgorod – Kazan - Samara	10.04.2011
	2	04.04.2011	D73466Lauchheim + D86720Noerdlingen + D89269Vohringen+D71701Schwieberdingen	Moscow	10.04.2011
2	1	06.04.2011	D65462Gustavsburg +D69123Heidelberg+D68219Mannheim	Smolensk+Domodedovo +Kaluga	11.04.2011
	2	06.04.2011	D74912Kirchartd + D79227Schallstadt+ D78727Oberndorf	Nizhnij Novgorod	11.04.2011
	3	06.04.2011	D51545Waldbroel +D57223Kreuztal-Buschhuetten	Lipetsk	11.04.2011
3	1	08.04.2011	D78112Sankt Georgen im Schwarzwald +D78224Singen+D79780Stuehlingen +D70736Fellbach	Moskow+Dzerzhinsk+Penza+ Saratov	14.04.2011
	2	08.04.2011	D56070Koblenz+D68542Heddesheim	Kurgan	14.04.2011
4	1	12.04.2011	D66882Hutschenhausen+ D66497Contwig+D69469Weinheim +D63762Grossostheim	Novomoskovsk + Moscow+ Yaroslavl	19.04.2011
	2	12.04.2011	D59067Hamm+D59457Werl	Magnitogorsk	19.04.2011
	3	12.04.2011	D58135Hagen+D57223Kreuztal-Buschhuetten	Chelyabinsk	19.04.2011
5	1	15.04.2011	D89231Neu-Ulm+ D88161Lindenberg +D86405Meitingen+D77694kehl am rhein	Ul'yanovsk + Ufa + Magnitogorsk + Chelyabinsk	20.04.2011
	2	15.04.2011	D77815Buhl+D71701Schwieberdingen +D71732Tamm	Ekaterinburg	20.04.2011

Figure 7. The representation of the routes receiving by running our model

№	Number of Truck	Date of loading	Places of pickup	Places of delivery	Date of unloading
1	C933TC199	04.04.2011	D91301Forschheim + D69123Heidelberg + D67354Romerberg	Vladimir + Nizhnij Novgorod +Kazan	12.04.2011
	M464KK 197	04.04.2011	D73466Lauchheim+D71701Schwieberdingen + D86720Noerdlingen + D89269Vohringen	Moscow+Samara	9.04.2011
2	AK 4755-7	06.04.2011	D65462Gustavsborg +D69123Heidelberg+D68219Mannheim	Smolensk +Kaluga	13.04.2011
	T433TY 199	06.04.2011	D74912Kirchartd + D79227Schallstadt+ D78727Oberndorf	Nizhnij Novgorod+Domodedovo	11.04.2011
3	Y084MA 197	08.04.2011	D78112Sankt Georgen im Schwarzwald +D78224Singen+D79780Stuehlingen +D70736Fellbach	Moskow+Dzerzhinsk+Pen za+ Saratov	15.04.2011
	AE1975-7	08.04.2011	D56070Koblenz+D68542Heddesheim	Kurgan	18.04.2011
4	AI3572-7	12.04.2011	D66882Hutschenhausen+ D66497Contwig+D63762Grossostheim +D69469Weinheim	Novomoskovsk + Moscow+ Yaroslavl	20.04.2011
	C931TC199	12.04.2011	D59067Hamm+D59457Werl	Magnitogorsk+ Moscow	21.04.2011
	AB9847-7	13.04.2011	D53639Konigswinter+D57223Kreuztal-Buschhuetten+D67373Dudenhofen	Chelyabinsk+ Moscow	22.04.2011
5	K709TT199	15.04.2011	D89231Neu-Ulm+ D88161Lindenberg +D86405Meitingen+D77694kehl am rhein	Ul'yanovsk + Ufa + Magnitogorsk + Chelyabinsk	26.04.2011
	AB1735-7	16.04.2011	D77815Buhl+D71701Schwieberdingen +D71732Tamm	Ekaterinburg	26.04.2011

Figure 8. The representation of the real routes of the company “Westintertrans” Ltd.

So, as we can see from the tables above, real routes was not always the same with the results from our model. So we have next differences:

1. In first case we have 2 trucks in results from our model and in real life. But in real life first pickup route starts from another place of unloading of truck. Its not efficient but it have place because of the problem of driver with his visa in real life – the reason, which we cannot take into account in our model. Also in second pickup route in real life truck pick up cargoes in non-optimal sequence – our model shows us the optimal way. So in this case our model gives us the more optimal route. The company

“Westintertrans” Ltd. has such internal structure that all trucks divided between 5 managers and every manager responsible for his own part of trucks. The goal of each manager is to receive the maximum profit on his truck. So in real life it not gainful for manager to send the not-full-truck to one place of unloading in Russia like our model proposes. So delivery routes in real life a little bit different from the model routes because of this reason. And one more difference between model and real life results is different date of unloading. The model gives us an ideal date and time, but in real life everything is not so easy. It is very common in Russia that trucks unload at the places of delivery with the idle time above permitted standard – we can see it almost in every our case.

2. In second case in real life the company decided that it is not efficient to take cargo from the D57223Kreuztal-Buschhuetten to Lipetsk and refuse it. So we have only 2 trucks and 2 route instead of 3 in the model. And two more points as in the first case - different dates of unloading and different delivery routes. The reasons for these are the same as in the first case.
3. In third and fifth cases we’ve got almost the same results, but only the differences in the dates of loading and unloading are have place.
4. In forth case the model also gives us more optimal pickup route, than in real life (as in the case 1). Also here we have the same differences - different dates of unloading and different delivery routes for the same reasons.

But usually in most points the results coincide. And it is logical. Managers of the company “Westintertrans” Ltd. are very practical and sensible. They chose the cargos by using their logic, knowledge and experience.

The results from the model show the less costly results than real routes. But it is not always because of the errors and oversights in manager's work. The differences between result of our model and real routes can be explained by the assumptions, which were made for formulation our mathematical model (Section 5.2). Also in our model we do not take into account that the driver is a living person with his own need and desires. And this factor can have a considerable effect and influence in real life, but it's very difficult to include this factor in model.

Also we do not care about such indicator as profit per car per day. So when manager of the transport company discuss the proposition of the cargo, it is usual that the price for transportation calculate the transport company and the customer can agree with the price and so the transportation will be complete. Or customer can be disagreeing with the price of the transport company and so he will look for another cheaper transport company. But sometimes customer defines the price for transportation for himself and this price can be even more than transport company calculate. It can be reasonable excess in some cases (express delivery, high importance of the cargo and so on) or just an extra profit for transport company. So sometimes it is more beneficial for the transport company to take such cargos even the route will have bigger expenses than others routes. The extra profit will cover these bigger expenses. But in our model we do not take into consideration the price for transportation, we just calculate the expenses. By that reason the real routes of the company “Westintertrans” Ltd. not always the same as in our model.

But in general our model can give a valuable recommendation which cargo to take and by which truck by other things being equal. And program gives its results in a very short time – that is also very important and useful. So using the model can save a lot of time and energy, which managers can use for further optimization of the transportation.

7.3. Verification and validation the model

The model verification means ensuring that the computer program of the computerized model and its implementation are correct. We will rely on the logical verification of our model.

Analysis of the figures above and the results of the model helps us to understand that the model behaves the way it was meant.

Through the pictures it is easy to verify the model after any changes. Also the checking of the data about the cargos weight ensure us that the capacity constraints are observed. All mentioned above allows us to conclude, that the implementation of the model is correct. One more proof of the correct work of the model is the coincidence of the results of our model and real routes of the company “Westintertrans” Ltd.

It is often difficult to separate verification and validation, as these two processes are closely related, and often the same techniques are used for both. Various validation techniques are known. Those used for validating our model are listed below.

- **Event Validity:** The results of the model are compared to those of the real system to determine if they are similar. This technique was used to validate the fulfillment of the model's requirements. It was determined that such events as efficient truck chosen and truck movements are consistent with provided data;
- **Face Validity:** The face validity is asking people knowledgeable and with experience about the system whether the model's behavior is reasonable. This tool can be used in determining if logic in the conceptual model is correct and if the model's input-output relationship is reasonable. Using this technique we were discovere that the behavior of our model can be considered as reasonable, but the small corrections were made to avoid some possible negative situations;
- **Operational Graphics:** This technique means values of various performance measures, e.g., efficient sequences in pickup or delivery routes, are shown graphically. We used such graphics to do the output analysis.

Despite of the fact that validation and verification usually doing on the bis massive of the instances, these weeks are very representative for the company "Westintertrans" Ltd. Not every week it has so many cargos and trucks loaded through the cross dock. So this massive of data is considerably big and representative for it. So we can conclude that model is very good describes an environment, it is valid and can be used in practice.

8. Conclusions

In this paper, a mixed integer programming formulation for the VRPCD is proposed. The proposed algorithm is implemented and tested on data sets provided by the company “Westintertrans” Ltd. Experimental results show that this algorithm can produce high quality solutions within very short computational time.

So, the AMPL model for the collecting assorted loadings through the cross dock was created. The model can be used as a decision-support tool for tactical route planning, as well as an evaluation tool for operational strategies. Using the AMPL model different quantities of trucks have been tested. It was noted that as the utilization of the trucks goes down, the contribution of every next truck becomes less visible and only bring additional costs. Output analysis shows that this model can be successfully used in practice.

On the future the model can be extended with using the external trucks for pickup and delivery cargos. Also some others modification and improvements can be made.

The AMPL model is seen as a tool for analysis of the behavior of the transport company with many trucks in order to perform her activity efficiently. But the disadvantage is that the model is not so transparent and requires some special knowledge in AMPL modeling language and mathematical programming to understand the outcome on any kind of changes that can be applied to the system. But, the model can be easily extended with additional features to become even closer to the real life.

References

Wen, M., Larsen, J., Clausen, J., Cordeau, J.-F., and Laporte, G. (2009c). Vehicle routing with cross-docking. *Journal of the Operational Research Society*, 60(12):1708–1718.

N. Boysen, M. Flidner: Cross Dock Scheduling: Classification, Literature Review and Research Agenda, Technical Report, 2009.

Shaw, P. Maher: Using Constraint Programming and Local Search Methods to Solve Vehicle Routing Problems, *Principle and Practice of Constraint Programming|CP98*, Springer-Verlag, 1998.

Wooyeon Yu, W., and Egbelu, P.J., 2008. Scheduling of inbound and outbound trucks in cross docking systems with temporary storage. *European Journal of Operational Research*. Vol. 184, pp. 377-396.

Vahdani, b., and Zandieh, m., 2010. Scheduling truck in cross-docking systems: Robust meta-Heuristics. *Computers & Industrial Engineering*. Vol. 58, pp.12-24.

Larbi, R., Alpan, G., and Penz, B., 2009. Scheduling Transshipment Operations in A Multiple Inbound and Outbound Door Crossdock, *International Conference on Computers and Industrial Engineering.*, pp. 227-232.

Miao, Z., Lim, A., and Ma, H., 2009. Truck Dock Assignment Problem with Operational Time Constraint within Crossdocks. *European Journal of Operation Research*. Vol.192, pp.105-115.

Mula, J., Peidro, D., Madronero, M.D., and Vicens, E., 2009. Mathematical programming models for supply chain production and transport planning. *European Journal of Operation Research*. Doi:10.1016/j.ejor.2009.09.008.

Chen, R., Fan, B., and Tang, G., 2009. Scheduling Problems in Cross Docking. *Lecture Notes in Computer Science*. Vol. 5573 LNCS, pp.421-429.

Jan P.H.M. Willemas, Jan-Eric Slot, Andrzej K.Hajdasinski (2009) Cross Docking Information. A new perspective in managing information. *Internal Journal of Systems Sciences*. 1(24).

www.wikipedia.org

Apte, U.M., Viswanathan, S., 2000. Effective Cross-Docking for Distribution Efficiencies, *International Journal of Logistics: Research and Applications*, 3, 291-302.

Bartholdi III, J.J., Gue, K.R., 2004. The best shape for a cross-dock, *Transportation Science*, 38, 235-244.

Chen, P., Guo, Y., Lim, A., Rodrigues, B., 2006. Multiple crossdocks with inventory and time windows, *Computers & Operations Research*, 33, 43-63.

Gumus, M., Bookbinder, J.H., 2004. Cross-docking and its implications in location/distribution system, *Journal of Business Logistics*, 25, 199-229.

Lee, Y.H., Jung, J.W., Lee, K.M., 2006. Vehicle routing scheduling for cross-docking in the supply chain, *Computers & Industrial Engineering*, 51, 247-256.

Sung, C.S., Song, S.H., 2003. Integrated service network design for a cross-docking supply chain network, *Journal of the Operational Research Society*, 54, 1283-1295.

Babics Tamas., 2004. Cross-docking in the dales supply chain: integration of information and communication relationships, *Periodica Polytechnica*, vol.33, No 1-2, pp 69-76 (2005).

Baldacci, R., Toth, P., and Vigo, D. (2007). Recent advances in vehicle routing exact algorithms. *4OR: A Quarterly Journal of Operations Research*, 5(4):269–298.

Branke, J., Middendorf, M., Noeth, G., and Dessouky, M. (2005). Waiting strategies for dynamic vehicle routing. *Transportation Science*, 39(3):298–312.

Braysy, I. and Gendreau, M. (2005). Vehicle routing problem with time windows, part II: Metaheuristics. *Transportation Science*, 39(1):119–139.

Clarke, G. and Wright, J. W. (1964). Scheduling of vehicles from central depot to number of delivery points. *Operations Research*, 12(4):568–581.

Cortes, C. E., Matamala, M., and Contardo, C. (2010). The pickup and delivery problem with transfers: Formulation and a branch-and-cut solution method. *European Journal of Operational Research*, 200(3):711–724.

Du, T., Wang, F. K., and Lu, P.-Y. (2007). A real-time vehicle-dispatching system for consolidating milk runs. *Transportation Research Part E: Logistics and Transportation Review*, 43(5):565–577.

Gaskell, T. J. (1967). Bases for vehicle fleet scheduling. *Operational Research Quarterly*, 18(3):281–295.

Suman, B. and Kumar, P. (2006). A survey of simulated annealing as a tool for single and multiobjective optimization. *Journal of the Operational Research Society*, 57(10):1143–1160.

Warehouse, cross-dock or direct ship? Making the right choices. Consultants in Material Handling Logistics, Operations Design for Warehousing, Manufacturing and Distribution. www.GrossAssociates.com, 167 Main Street, Woodbridge, New Jersey

Bartholdi, J.J. & K.R. Gue. (2000). Reducing labor costs in an LTL crossdocking terminal. *Operations Research*, 48(6), 823-832.

Klincewicz, J.G. & M.B. Rosenwein. (1997). Planning and consolidating shipments from a warehouse. *Journal of the Operational Research Society*, 48(3), 241-246.

Best Practices for Cross Docking and Load Only Services (2009). www.producetraceability.org

Dwi Agustina, C.K.M.Lee and Rajesh Piplani. (2010). A Review: Mathematical Modles for Cross Docking Planning. *International Journal of Engineering Business Management*, Vol. 2, No. 2 (2010), pp. 47-54

Hanne L. Petersen, Stefan Ropke (2010). *Vehicle Routing with Cross-Docking*. Trafikdage på Aalborg Universitet 2010 ISSN 1603-9696

Appendix A

Place of unloading of our truck	Time of unloading	Place of loading1	a11	b11	Place of unloading1	a12	b12	Distance from the place of unloading of truck to the place of loading1 in km	Distance in time in minutes	Distance from the place of loading1 to the CD1 in km	Distance in time in minutes	Distance from the CD to the place of unloading1 in km	Distance in time in minutes	Dem and 1 in tons
D77815Buhl	600	D89269Vohringen	480	2640	Vladimir	480	12720	224	224	243	243	2562	4482	7
D96106Heubacher	480	D89269Vohringen	480	2640	Vladimir	480	12720	280	280	243	243	2562	4482	7
D73466Lauchheim	660	D89269Vohringen	480	2640	Vladimir	480	12720	87	87	243	243	2562	4482	7
D67550Worms	1920	D89269Vohringen	480	2640	Vladimir	480	12720	285	285	243	243	2562	4482	7
D56727Mayen	600	D89269Vohringen	480	2640	Vladimir	480	12720	416	416	243	243	2562	4482	7
Place of unloading of our truck	Time of unloading	Place of loading2	a21	b21	Place of unloading2	a22	b22	Distance from the place of unloading of truck to the place of loading2	Distance in time	Distance from the place of loading2 to the CD	Distance in time	Distance from the CD to the place of unloading2	Distance in time	Dem and 2
D77815Buhl	600	D86720Noerdlingen	360	1260	Samara	480	12720	265	265	221	221	3425	5825	4
D96106Heubacher	480	D86720Noerdlingen	360	1260	Samara	480	12720	181	181	221	221	3425	5825	4
D73466Lauchheim	660	D86720Noerdlingen	360	1260	Samara	480	12720	23	23	221	221	3425	5825	4
D67550Worms	1920	D86720Noerdlingen	360	1260	Samara	480	12720	92	92	221	221	3425	5825	4
D56727Mayen	600	D86720Noerdlingen	360	1260	Samara	480	12720	394	394	221	221	3425	5825	4
Place of unloading of our truck	Time of unloading	Place of loading3	a31	b31	Place of unloading3	a32	b32	Distance from the place of unloading of truck to the place of loading3	Distance in time	Distance from the place of loading3 to the CD	Distance in time	Distance from the CD to the place of unloading3	Distance in time	Dem and 3
D77815Buhl	600	D71701Schwieberdingen	1800	4080	Nizhnij Novgorod	1800	14160	112	112	119	119	2793	4713	3
D96106Heubacher	480	D71701Schwieberdingen	1800	4080	Nizhnij Novgorod	1800	14160	233	233	119	119	2793	4713	3

D73466Lauchheim	660	D71701Schwieberdingen	1800	4080	Nizhnij Novgorod	1800	14160	161	161	119	119	2793	4713	3
D67550Worms	1920	D71701Schwieberdingen	1800	4080	Nizhnij Novgorod	1800	14160	144	144	119	119	2793	4713	3
D56727Mayen	600	D71701Schwieberdingen	1800	4080	Nizhnij Novgorod	1800	14160	292	292	119	119	2793	4713	3
Place of unloading of our truck	Time of unloading	Place of loading4	a41	b41	Place of unloading4	a42	b42	Distance from the place of unloading of truck to the place of loading4	Distance in time	Distance from the place of loading4 to the CD	Distance in time	Distance from the CD to the place of unloading4	Distance in time	Dem and 4
D77815Buhl	600	D67354Romerberg	480	2640	Kazan	480	22800	104	104	12	12	3180	5580	5
D96106Heubacher	480	D67354Romerberg	480	2640	Kazan	480	22800	270	270	12	12	3180	5580	5
D73466Lauchheim	660	D67354Romerberg	480	2640	Kazan	480	22800	196	196	12	12	3180	5580	5
D67550Worms	1920	D67354Romerberg	480	2640	Kazan	480	22800	240	240	12	12	3180	5580	5
D56727Mayen	600	D67354Romerberg	480	2640	Kazan	480	22800	188	188	12	12	3180	5580	5
Place of unloading of our truck	Time of unloading	Place of loading5	a51	b51	Place of unloading5	a52	b52	Distance from the place of unloading of truck to the place of loading5	Distance in time	Distance from the place of loading5 to the CD	Distance in time	Distance from the CD to the place of unloading5	Distance in time	Dem and 5
D77815Buhl	600	D69123Heidelberg	480	2640	Moscow	480	12720	100	100	31	31	2377	3817	10
D96106Heubacher	480	D69123Heidelberg	480	2640	Moscow	480	12720	268	268	31	31	2377	3817	10
D73466Lauchheim	660	D69123Heidelberg	480	2640	Moscow	480	12720	192	192	31	31	2377	3817	10
D67550Worms	1920	D69123Heidelberg	480	2640	Moscow	480	12720	238	238	31	31	2377	3817	10
D56727Mayen	600	D69123Heidelberg	480	2640	Moscow	480	12720	195	195	31	31	2377	3817	10

Figure A: The data from the company “Westintertrans” Ltd. for solving our example

Appendix B

AMPL model:

File primerWEST.mod:

```
set PICKUP;
set DELIVERY;
set CD;
set PID;
set PIDICD;
set O3;
set O2;
set PIO2;
set DIO3;
set NODESTRUCK;
set LOADNODES;
set PICKUPNODESTRUCK;
set DELIVERYLOADNODES;
set PIDICDINTILN;
set ROAD within {PIDICDINTILN} cross {PIDICDINTILN};
set TRUCK;

param cost {PIDICDINTILN,PIDICDINTILN} >=0;
param demand {PID} >=0;
param capacity >=0;
param a {PIDICDINTILN} >=0;
param b {PIDICDINTILN} >=0;
param M >=0;
param n >0;
param W = 1/M;
param A >=0;
param B >=0;

var time_leave {i in PIDICDINTILN, k in TRUCK} >= a[i], <= b[i];
var time_unloaded {TRUCK} >= 0;
var time_reloading {k in TRUCK} >= 0;
var time_free {PICKUP} >= 0;
```

```

var Route {ROAD, TRUCK} binary;
var Unl_cd {PICKUP, TRUCK} binary;
var Rel_cd {PICKUP, TRUCK} binary;
var Must_unl {TRUCK} binary;
var Must_rel {TRUCK} binary;

```

```

minimize Time: sum {(i,j) in ROAD, k in TRUCK} Route[i,j,k]*cost [i,j];

```

```

subject to Node_Once1 {i in PICKUP}: sum {k in TRUCK, j in PIO2 : i<>j} Route [i,j,k]=1;
subject to Node_Once2 {j in DELIVERY}: sum {k in TRUCK, i in DIO3 : i<>j} Route [i,j,k]=1;
subject to Node_Once3 {i in NODESTRUCK}: sum {k in TRUCK, j in PICKUP} Route [i,j,k]<=1;
subject to Node_Once4 {j in LOADNODES}: sum {k in TRUCK, i in DELIVERY} Route [i,j,k]<=1;
subject to V_Capacity1 {k in TRUCK}: sum {i in PICKUP, j in PIO2: i<>j} Route [i,j,k]*demand [i] <=capacity;
subject to V_Capacity2 {k in TRUCK}: sum {i in DELIVERY, j in DELIVERY: i<>j} Route [i,j,k]*demand [i] <=
    <=capacity;
subject to Start1 {k in TRUCK}: sum {h in NODESTRUCK,j in PICKUP} Route [h,j,k]=1;
subject to Start2 {k in TRUCK, h in O3}: sum {j in DELIVERY} Route [h,j,k]=1;
subject to Conservation1 {k in TRUCK, h in PICKUP}: sum {i in PICKUPINODESTRUCK: h<>i} Route [i,h,k]-
    -sum {j in PIO2: h<>j} Route [h,j,k]=0;
subject to Conservation2 {k in TRUCK, h in DELIVERY}: sum {i in DIO3: h<>i} Route [i,h,k]-
    -sum {j in DELIVERYLOADNODES: h<>j} Route [h,j,k]=0;
subject to Finish1 {k in TRUCK, h in O2}: sum {j in PICKUP} Route [j,h,k]=1;
subject to Finish2 {k in TRUCK}: sum {j in DELIVERY, h in LOADNODES} Route [j,h,k]=1;
subject to Travel_Time {(i,j) in ROAD, k in TRUCK}: time_leave[j,k] >= time_leave [i,k] + cost [i,j] -
    -M * (1-Route [i,j,k]);
subject to Link1 {i in PICKUP, k in TRUCK}: Unl_cd [i,k] - Rel_cd [i,k] = sum {j in PIO2: i<>j} Route [i,j,k] -
    - sum {j in DELIVERY: i+n<>j} Route [i+n,j,k];
subject to Link2 {i in PICKUP, k in TRUCK}: Unl_cd [i,k] + Rel_cd [i,k] <= 1;
subject to Force1 {k in TRUCK}: W* sum {i in PICKUP} Unl_cd [i,k] <= Must_unl [k];
subject to Force2 {k in TRUCK}: Must_unl [k] <= sum {i in PICKUP} Unl_cd [i,k];
subject to Duration1 {k in TRUCK, j in O2}: time_unloaded[k] = time_leave [j,k] + A* Must_unl [k] +
    + B * sum{i in PICKUP}demand [i] * Unl_cd [i,k];
subject to Logic1 {i in PICKUP, k in TRUCK}: time_reloading [k] >= time_free [i] - M * (1-Rel_cd [i,k]);
subject to Logic2 {i in PICKUP, k in TRUCK}: time_free [i] >= time_unloaded [k] - M * (1-Unl_cd [i,k]);
subject to Force3 {k in TRUCK}: W* sum {i in PICKUP} Rel_cd [i,k] <= Must_rel [k];
subject to Force4 {k in TRUCK}: Must_rel [k] <= sum {i in PICKUP} Rel_cd [i,k];
subject to Duration2 {k in TRUCK, j in O3}: time_leave[j,k] = time_reloading [k] + A* Must_rel [k] +
    +B * sum{i in PICKUP}demand [i] * Rel_cd [i,k];
subject to Reloading {k in TRUCK}: time_reloading [k] >= time_unloaded [k];
subject to Take_Or_Refuse {k in TRUCK}: sum {i in NODESTRUCK, j in PICKUP} Route[i,j,k]*cost [i,j] <=
    <= 300;

```

File primerWEST.dat:

set PICKUP := 1 2 3 4 5;
set DELIVERY := 6 7 8 9 10;
set CD := O2 O3;
set PID := 1 2 3 4 5 6 7 8 9 10;
set PIDICD := 1 2 3 4 5 6 7 8 9 10 O2 O3;
set O3 := O3;
set O2 := O2;
set PIO2 := 1 2 3 4 5 O2;
set DIO3 := 6 7 8 9 10 O3;
set TRUCK := T1 T2;
set NODESTRUCK := 11 12 13 14 15;
set LOADNODES:= 16 17 18 19 20;
set PICKUPINODESTRUCK := 1 2 3 4 5 11 12 13 14 15;
set DELIVERYILOADNODES:= 6 7 8 9 10 16 17 18 19 20;
set PIDICDINTILN := 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 O2 O3;

param cost : 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
O2 O3 :=
1 0 93 144 240 238 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000
10000 10000 10000 10000 10000 243 10000
2 93 0 184 218 216 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000
10000 10000 10000 10000 10000 221 10000
3 144 184 0 115 114 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000
10000 10000 10000 10000 10000 119 10000
4 240 218 115 0 26 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000
10000 10000 10000 10000 10000 12 10000
5 238 216 114 26 0 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000
10000 10000 10000 10000 10000 31 10000
6 10000 10000 10000 10000 10000 0 882 81 617 185 10000 10000 10000 10000 10000 10000 10000 10000
5 882 81 617 185 10000 10000
7 10000 10000 10000 10000 10000 882 0 687 560 1048 10000 10000 10000 10000 10000 10000 10000 10000
10000 882 5 687 560 1048 10000 10000
8 10000 10000 10000 10000 10000 80 687 0 386 416 10000 10000 10000 10000 10000 10000 10000 10000
80 687 5 386 416 10000 10000
9 10000 10000 10000 10000 10000 617 560 386 0 1283 10000 10000 10000 10000 10000 10000 10000 10000
10000 617 560 386 5 1283 10000 10000
10 10000 10000 10000 10000 10000 185 1048 416 1283 0 10000 10000 10000 10000 10000 10000 10000 10000
10000 185 1048 416 1283 5 10000 10000
11 224 265 112 104 100 10000 10000 10000 10000 10000 0 10000 10000 10000 10000 10000 10000 10000
10000 10000 10000 10000 10000 10000 10000
12 280 181 233 270 268 10000 10000 10000 10000 10000 10000 0 10000 10000 10000 10000 10000 10000
10000 10000 10000 10000 10000 10000 10000

```

13      87  23 161 196 192 10000 10000 10000 10000 10000 10000 10000 0 10000 10000
10000 10000 10000 10000 10000 10000 10000
14      285  92 144 240 238 10000 10000 10000 10000 10000 10000 10000 10000 0 10000
10000 10000 10000 10000 10000 10000 10000
15      416  394 292 188 195 10000 10000 10000 10000 10000 10000 10000 10000 10000 0
10000 10000 10000 10000 10000 10000 10000
16      10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000
10000 10000 0 10000 10000 10000 10000 10000 10000
17      10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000
10000 10000 10000 0 10000 10000 10000 10000 10000
18      10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000
10000 10000 10000 10000 0 10000 10000 10000 10000
19      10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000
10000 10000 10000 10000 10000 0 10000 10000 10000
20      10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000
10000 10000 10000 10000 10000 10000 0 10000 10000
O2      10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000 10000
10000 10000 10000 10000 10000 10000 10000 0 5
O3      10000 10000 10000 10000 10000 10000 4482 5580 4713 5580 3817 10000 10000 10000 10000
10000 10000 10000 10000 10000 10000 10000 0 ;

```

```

param demand := 1 7 2 4 3 3 4 5 5 10 6 7 7 4 8 8 9 5 10 10 ;
param capacity := 20;
param a := 1 480 2 360 3 1800 4 480 5 480 6 480 7 480 8 1800 9 480 10 480 11 600 12 480 13 660 14
1920 15 600 16 0 17 0 18 0 19 0 20 0 O2 0 O3 0;
param b := 1 2640 2 1260 3 4080 4 2640 5 2640 6 12720 7 12720 8 14160 9 22800 10 12720 11 2100
12 2100 13 2100 14 3600 15 2100 16 22800 17 22800 18 22800 19 22800 20 22800 O2 22800 O3 22800
;

```

```

param M := 10000000000000000000;
param n := 5;
param A := 61;
param B := 15;

```

```

set ROAD:=(1,2),(1,3),(1,4), (1,5), (1,O2),
(2,1),(2,3),(2,4), (2,5), (2,O2),
(3,1),(3,2),(3,4), (3,5), (3,O2),
(4,1),(4,2),(4,3), (4,5), (4,O2),
(5,1),(5,2),(5,3), (5,4), (5,O2),
(6,7),(6,8),(6,9), (6,10),(6,16),(6,17),(6,18), (6,19),(6,20),
(7,6),(7,8),(7,9), (7,10),(7,16),(7,17),(7,18), (7,19),(7,20),
(8,6),(8,7),(8,9), (8,10),(8,16),(8,17),(8,18), (8,19),(8,20),
(9,6),(9,7),(9,8), (9,10),(9,16),(9,17),(9,18), (9,19),(9,20),
(10,6),(10,7),(10,8),(10,9),(10,16),(10,17),(10,18), (10,19),(10,20),
(11,1),(11,2),(11,3), (11,4), (11,5),
(12,1),(12,2),(12,3), (12,4), (12,5),
(13,1),(13,2),(13,3), (13,4), (13,5),

```

(14,1),(14,2),(14,3), (14,4), (14,5),
(15,1),(15,2),(15,3), (15,4), (15,5),
(O2,O3),
(O3,6),(O3,7),(O3,8), (O3,9), (O3,10);

File primerWEST.run:

```
model primerWEST.mod;  
data primerwest.dat;  
option omit_zero_rows 1;  
solve;  
display Time> primerWEST.sol;  
display Route > primerWEST.sol;  
exit;
```

File primerWEST.sol:

For the case of 2 trucks

Time = 9853

```
Route :=  
1 3 T2 1  
2 1 T2 1  
3 O2 T2 1  
4 O2 T1 1  
5 4 T1 1  
6 8 T1 1  
7 17 T1 1  
8 9 T1 1  
9 7 T1 1  
10 20 T2 1  
11 5 T1 1  
13 2 T2 1  
O3 6 T1 1  
O3 10 T2 1;
```

For the case of 3 trucks

Time = 14683

```
Route :=  
1 3 T1 1  
2 1 T1 1  
3 O2 T1 1  
4 O2 T3 1  
5 O2 T2 1  
6 16 T1 1  
7 17 T3 1  
8 9 T3 1  
9 7 T3 1  
10 20 T2 1  
11 5 T2 1  
13 2 T1 1  
15 4 T3 1  
O3 6 T1 1  
O3 8 T3 1  
O3 10 T2 1;
```

For the case of 4 trucks

Time = 19951

```
Route :=  
1 O2 T1 1  
2 1 T1 1  
3 O2 T2 1  
4 O2 T3 1  
5 O2 T4 1  
6 16 T4 1  
7 17 T1 1  
8 9 T2 1  
9 19 T2 1  
10 20 T3 1  
11 5 T4 1  
13 2 T1 1  
14 3 T2 1  
15 4 T3 1  
O3 6 T4 1  
O3 7 T1 1  
O3 8 T2 1  
O3 10 T3 1;
```