Master's degree thesis

LOG950 Logistics

Coordinated replenishments and return on investments

Maria Dryuk

Number of pages including this page: 55

Molde, May 2013

Molde University College

Mandatory statement

Each student is responsible for complying with rules and regulations that relate to examinations and to academic work in general. The purpose of the mandatory statement is to make students aware of their responsibility and the consequences of cheating. Failure to complete the statement does not excuse students from their responsibility.

Please complete the mandatory statement by placing a mark ***in each box*** *for statements 1-6 below.*		
1.	**I/we herby declare that my/our paper/assignment is my/our own work, and that I/we have not used other sources or received other help than is mentioned in the paper/assignment.**	☒
2.	**I/we herby declare that this paper** 1. Has not been used in any other exam at another department/university/university college 2. Is not referring to the work of others without acknowledgement 3. Is not referring to my/our previous work without acknowledgement 4. Has acknowledged all sources of literature in the text and in the list of references 5. Is not a copy, duplicate or transcript of other work	Mark each box: 1. ☒ 2. ☒ 3. ☒ 4. ☒ 5. ☒
3.	I am/we are aware **that any breach of the above will be considered as cheating, and may result in annulment of the examinaion and exclusion from all universities and university colleges in Norway for up to one year, according to the Act relating to Norwegian Universities and University Colleges, section 4-7 and 4-8 and Examination regulations** section 14 and 15.	☒
4.	**I am/we are aware that all papers/assignments may be checked for plagiarism by a software assisted plagiarism check**	☒
5.	**I am/we are aware that Molde University college will handle all cases of suspected cheating according to prevailing guidelines.**	☒
6.	**I/we are aware of the University College`s rules and regulation for using sources**	☒

Publication agreement

ECTS credits: 30

Supervisor: Øyvind Halskau

Agreement on electronic publication of master thesis

Author(s) have copyright to the thesis, including the exclusive right to publish the document (The Copyright Act §2).
All theses fulfilling the requirements will be registered and published in Brage HiM, with the approval of the author(s).
Theses with a confidentiality agreement will not be published.

I/we hereby give Molde University College the right to, free of charge, make the thesis available for electronic publication: ☒**yes** ☐**no**

Is there an agreement of confidentiality? ☐**yes** ☒**no**
(A supplementary confidentiality agreement must be filled in)
- If yes: **Can the thesis be online published when the period of confidentiality is expired?** ☐**yes** ☐**no**

Date: May 2013

Preface

My sincere thanks to my family who provided me with an opportunity to take part in Master of Science program in Logistics at the Molde University College, for their support and believe.

It is impossible to represent all experience and skills achieved during my study at the Molde University College in one master thesis because a set of interesting and diverse directions are presented within the Master of Science program in Logistics. This master program gives me good knowledge in the field of logistics and some other related area. It provides me with new skills and help to develop new personal qualities.

This master thesis represents my result achieved in direction chosen. With the help of my supervisor associate professor Øyvind Halskau I selected a topic for my master thesis within a field of inventory management. The writing process was interesting and challenging and I really satisfied with my results. I strongly feel that I owe Professor Øyvind Halskau for his responsive leadership, useful comments for me which I have received on my achievements and for his patience, of course.

My indebtedness and appreciation goes to all my teachers at Molde University College for their contribution to my education. Their highly qualified assistance cannot be overstated.

I am also very thankful to all my friends and co-students who support me in different ways. All these people have increased the meaning of my stay at the Molde University College by several times.

Abstract:

In order to provide the effective work of enterprise that deals with inventories, the question of determination right inventory policy has to be solved. Most inventories contain a lot of different items. In practice some additional restrictions on available capacity of several resources (for example, limited available space at warehouse, or limited budget to invest in inventory) are possible.

Coordination of the replenishments for items is often used approach. This master thesis contain an attempt to move from the common cost minimizing models towards to models which maximize the return on investments under coordination of the replenishments for items. In the first part named “Introduction” basic concepts are disclosed and different approaches to joint replenishment problem. The second part is “Some general concepts and literature overview”. The third part is problem formulation and solution and analysis of the results.

Table of context

1. Introduction

Inventory is considered as one of the most crucial questions for the companies as the total investment in inventories is huge. (Axsater 2006) There are various estimates. For example, manufacturing firm's inventory assets may represents «*from 20% to 60% of the total assets*». (Arnold, 1998) «*In general, companies invest about 30% of their fluid capital and 90% of their operational capital on the inventories*» (Stevenson, 1996) referenced by (Lee and Yao 2003). Inventory is also considered as expensive assets for a large number of companies linked with finished goods, spare parts or raw materials. There are two basic types of costs linked with having inventories: *the ordering cost* and *the holding cost*. Ordering costs are linked with adjusting the equipment at production (for example, adjusting the machines before production). For goods to be purchased from suppliers ordering costs is transaction and transportation costs. The cost of replenishment order to the supplier has also two components (Khouja & Goyal, 2008):

- *a major ordering cost* of order;
- *a minor ordering cost*.

The difference between these two types of cost is that major ordering cost is independent of the number of items in the order.

The main components of the holding cost or inventory carrying costs are the cost of capital tied up in inventory, taxes, handling and counting costs, the costs of deterioration of stock etc. More details can be found in (Silver, Pyke, & Peterson, 1998). The biggest part of the carrying cost is presented by opportunity costs of capital tired up in inventory. The cost of capital depends on the degree of risk that is contained in an investment. In practice this lead to that the opportunity cost of capital can change in between the bank's prime lending rate to 50 percent and even more. (Silver, Pyke, & Peterson, 1998)

Under probabilistic demand a firm also has to keep safety stock that is defined as «*the average level of the net stock just before replenishment arrives*». (Silver, Pyke and Peterson 1998) The safety stock is a buffer against larger than demand expected. In case of probabilistic demand the safety stock will increase the holding cost. If firm get a customer's order when at that moment item is out of stock, where two cases are possible. One of these cases is that the customer's order is backordered; in another case it is lost. Some cost linked with stockout situation may increase the total cost.

According to Muller all organization keep inventory. Why does firm need inventory?

There are some reasons (Waters, 1992) referenced by (Gribkovskaya, 2012):

- inventory can be used as a buffer between two interdependent operations to prevent breakdowns or unevenness in production rates. In this case inventory also reduce the need for output synchronization;
- to correct the mismatch between supply and demand;
- to correct forecast and delivery errors;
- to avoid delays in delivery goods to the customers;
- to capitalize the price discounts on large orders;
- to sustain stable level of operations.

Therefore, inventory management proves critical in determining the efficiency work of an enterprise and gives opportunity for improvements that can lead to getting significant competitive advantages for a company. The main questions that determine inventory system for a company are (Silver, 2008):

1. How often should inventory manager check the inventory status?
2. When the item should be ordered?
3. How large the size of order should be?

Under condition of deterministic demand which will be discussed in this master's thesis, to find an answer for the first question is not difficult. The knowledge of the inventory status at any one point gives possibility to calculate it in any achievable point of time. If replenishment of order arrives when inventory level achieve some certain value, the answer for the second question is ready under condition of deterministic demand. As usual this certain value equals 0. (Silver, Pyke and Peterson 1998) So the main question is “How much to order?”

There are different models that provide diverse solutions for various types of businesses and various types of inventories. More details can be found in (Axsater 2006) or (Silver, Pyke and Peterson 1998). Before selecting one of them it is necessary to answer for a number of related issues that help to find inventory system needed.

1.1 Types of inventory system

There are different classifications of the inventory systems. Classifications depend on criteria chosen.

1.1.1 Single versus several items inventory models

Most inventories contain more than one item. For example, several items are purchased from the same supplier. In this situation it may have sense to coordinate the control of different items.

Coordination replenishments for a family of items or joint replenishment problem (JRP) imply making replenishment decisions for family of items so that some benefits can be achieved. According to (Silver, Pyke, & Peterson, 1998) these benefits are:

- When buyer have fixed transaction costs per replenishment order, then adding one or more items to the order implies smaller size of fixed transaction costs. Thus there is *saving on unit purchasing costs*
- *Saving on unit transportation costs*. This is the case when items share the same transportation mode. The situation is similar to the one described above
- In some cases cost of placing replenishment order that are independent on size of order or fixed (setup) cost is high enough so it is more preferably to combine several item in one order to reduce setup cost for the period of time. This is *saving on ordering costs*
- Joint replenishment may facilitate receiving and inspection merchandises etc.

However, some problems may occur. Under joint replenishment some items can be reordered earlier than it is while they are treated independently. It may produce situation of *increasing average inventory level*. Coordination replenishments for a family of items is complex problem especially in case of thousands items. Inability to work with items independently makes it difficult to handle unexpected situations so there is a flexibility reduction. (Silver, Pyke, &

Peterson, 1998). My thesis considers the multi-items inventory system under coordination replenishments for a family of items.

1.1.2 Types of inventory system according to demand

There are several types of modeling demand process. As a function of calendar time it could be (Silver E. A., 2008):

- Level of demand is deterministic and constant
- Level of demand is deterministic, but changing in a known way with time
- Demand level is conformed to known stationary distribution with known parameters. For example, these distributions could be normal, Poisson or negative binomial
- Demand level is conformed to known stationary distribution with unknown parameters
- Demand level is conformed to unknown stationary distribution
- Probabilistic demand is non-stationary.

This paper mostly deals with deterministic constant level of demand.

According to another types of modeling demand process a demand could be *dependant* versus *independent*. *Independent demand systems* assume that there are no connections among demands for different items. If to summarize demands from different customers, which probably even don't know each other, i.e. independent customers, the total demand for an item can be obtained. (Waters, 1992) referenced by (Gribkovskaya, 2012)

The *dependent demand systems* assume that demand for every item linked at least with some demand for other item. Certain products tend to be demanded together. (Gribkovskaya, 2012) That means that items have complementary demand. Another type of interdependency is that items can be substitutable. If items are absent at a store, a customer may buy analogue. (E. A. Silver 2008)This master thesis deals with independent demand systems inventory model.

1.1.3 Types of inventory system according to information flow criteria

According to information flow criteria there are two groups of inventories (Silver, Pyke, & Peterson, 1998):

- Perpetual inventory[1] and continuous inventory
- Perpetual inventory and non-continuous inventory
- One-periodic inventory.

Way of placing order to supplier also defines types of inventory system. It could be single order or repetitive order (perpetual inventories). The most known example of single order is so called «*The news boy problem*». This is problem about stock quantity of an item when there is one opportunity to order before selling period starts. The demand for item is random. The trade-off is between risks of to have more than enough items and to have less. (Gallego and Moon 1993)

Perpetual inventory can be split into two groups. Information flow is a function of doing business and assumes that demand information is updated on a continuous basis in the perpetual inventory system with *continuous* inventory. The opposite situation is in the system with *non-continuous inventory*. Information is updated on a periodic basis. (Silver, Pyke, & Peterson, 1998)

This future master thesis deals with perpetual and continuous inventory.

[1] Lecture notes of inventory management course 2012

1.2 Objective functions

In perpetual inventory it is very common to minimize the total costs while a demand is met.

According to (Khouja & Goyal, 2008) there are many types of multi-product inventory models. As it has already mentioned the objective of these models is usually minimization of total cost and at the same time demand should be satisfied. This approach has many advantages (see, for example, (Waters, 1992)), but there are some weaknesses. Main weakness is that many models which minimize costs do not have so called company-wide approach. That means they do not contain links between the inventory and other parts of the company. (Gribkovskaya, Halskau, & Olstad, 2012)

If inventory is considered as an investment, then it is rationale consider *profit*, *return on investment*, or *residual income* as optimizing criteria. *Profit* is a well known criterion. The example of profit maximization model is the classical newsboy problem. The objective function of such model is maximization the expected profit in a single period model with probabilistic demand. (Hadley & Whitin, 1963) referenced by (Khouja, 1995) The major disadvantage of profit as performance measure is that it doesn't answer about size of the investment in order to get the needed profits (Arcelus & Srinivasan, 1987)

Another performance measure is *residual income*. Residual income equals to the difference between the profit and capital charge that is the unit's cost of capital multiplied by the investment base (for detail see, (Arcelus & Srinivasan, 1987)). This measure include risk premium in the calculations. Risk premium represents different opportunity costs that mean different capital charges to different investments. At the same time residual income has disadvantage that is similar to profit as performance measure. This disadvantage is that large investment units bring more residual income compare to small one. But it can occur that small investments are more efficient. (Arcelus & Srinivasan, 1987)

Return on investment (ROI) is good in providing comparisons between *ROI'*s of different items and overall ROI's. According to (Li, Mina, Otake, & Voorhis, 2008) return on investment «*is a widely utilized performance measure in business investment analysis*». *ROI* approach is not the same as cost minimization or profit maximization in that the *ROI* represents the measure of the ratio between profit and investment; the two others deal with only the absolute values. According to (Trietsch, 1995) «*ROI is ratio between the profit (before tax) and the owners' (or shareholders') equity (i.e. the investment on which they measure their return, minus debts)* ». Under profit it is assumed difference between gross income and different types of costs as purchasing, transportation, manufacturing, packaging, storage, shrinkage and selling costs. Using the definition in relation to the inventory it is necessary to make some adjustments:

- Owners' have the right to determine its equity.
- ROI after tax is strongly correlate ROI before tax

«*ROI is especially preferable when there is a working capital scarcity or high opportunity cost of investment*» (Rosenberg, 1991) referenced by (Li, Mina, Otake, & Voorhis, 2008) The main argument in favor of ROI is that when there is capital budget limitation and this limitation is below the capital requirements needed to maximize profit, «*then the opportunity cost of the funds tied up in inventory is no longer fixed, as inventory theory suggests*». (Arcelus & Srinivasan, 1987) In this case the opportunity cost is the return on the last investment that was rejected. Investment that was rejected is identified then decision is done so a new optimizing criterion is needed. According to (Arcelus & Srinivasan, 1987) in this situation the Profitability Index (*PI*) is widely used index in capital budgeting. Profitability Index is the present value of the benefits earned per dollar invested. Under predetermined discount rate and for a single-period model maximization of the *PI* equals *ROI* maximization. That is why *ROI* is considered as a short-term performance objective.

However, there are some drawbacks. Under *ROI* maximization managers have causes to not behave in efficient way. For example, there is an incentive to increase the *ROI* level through at least not increasing the capital base. Some investment projects which have the return greater than the firm's cost of capital can, but decrease the overall ROI per unit can be rejected. (Arcelus & Srinivasan, 1987)). That is reason that «*ROI and cost minimization or profit maximization are two important and complementary criteria in investment decision-making*». (Li, Mina, Otake, & Voorhis, 2008) Other possible objective functions are listed at (Silver E. A., 2008).

1.3 Joint replenishment problem (JRP)

Under multi-product problem the main questions are optimal order quantities for items from the same supplier and the length of cycle. This implies that the production process can be completed in the chosen cycle. ((Khouja & Goyal, 2008), (Silver, Pyke, & Peterson, 1998)) Optimal order quantities and the length of cycle are affected degree of coordination. There are several possible degrees of coordination: no coordination, joint replenishment for group of items and complete coordination, when all items are ordered together. Lack of coordination and complete coordination are two extreme cases. This subchapter is devoted to joint replenishment problem of items.

1.3.1 Grouping strategies for *JRP*

Under constant demand joint replenishment of items can be done in two ways, which is named «*indirect grouping strategies*» and «*direct grouping strategies*». A general assumption for the two strategies is that the replenishment cycle is constant. A group or family assumes the set of items with the identical replenishment cycle. Under replenishment cycle it is understood the time that are between two consecutive placement of orders to the supplier of an individual item. (Eijs, Heuts, & Klei, 1992)

An *indirect grouping strategy* assumes that intervals for placement of order for a family of items to supplier is constant intervals and the replenishment cycle (time between two successful placement of orders to the supplier) of each item (or group) equals to basic cycle time or multiplication of basic cycle time by integer number. In this case a group is represented by items with the identical basic cycle time and the frequency of replenishments. (Eijs, Heuts, & Klei, 1992). (Silver, Pyke, & Peterson, 1998) got an optimal solution that minimized total relevant cost per unit time T and the number of intervals m_i of length T at which to place an order of item i. Results showed that the best choice of m_i doesn't linked with holding costs. As it is assumed in practice the number of intervals m_i is integer number and it is not mandatory that optimal cost is integer. The task is to find the best integer m_i.

Direct grouping strategies is opposite to previous approach in a way of integer multiplication of basic cycle, as a consequence the family replenishments are not equally spaced. (Eijs, Heuts, & Klei, 1992) Under direct grouping strategies the number of groups is predetermined. Every item will be in one out of *m* groups. The items belonging to the same group have the same cycle time T_j. The aim of this strategy is providing best grouping strategy and minimization the total cost through optimal cycle time for each group. (Wang, He, Wu, & Zeng, 2012)

1.3.2 *JRP* and Just-in-Time (*JIT*).

In 1985 materials that were bought by manufactures at USA were around 60% of total sales revenue. At the same time Japanese producers' index was much lower. The success and resulting performance of Japanese *JIT* system has been described in many articles. JIT system is described by cooperation between the supplier and buyer. As usual this cooperation is between the central factory and her satellite factories/suppliers which are close to the main enterprise. These satellite factories deliver materials and components to the central factory quite often. These relationships are long-term relationships that are building on high share of bilateral trust (for example, share information) and openness. (Hsu, 2009)

At the same time transportation costs are significant part of costs of the bought goods. The consolidation of replenishments of several items has been a trend in JIT as it provides possibility to reduce transportation costs through combination of materials and components to be purchased. Every time when orders are placed to the supplier, a major ordering cost will be incurred. They are transportation costs and order processing costs. In case of JRP of a number of items major ordering costs are shared among all items at the order. (Hsu, 2009)

1.3.3 *JRP* at a single stocking point versus multi-echelon inventories

To create and sustain a competitive advantage a firm can deal with other firms in interorganizational relationships. These interorganizational relationships can include collaboration and coordination among channel partners through sharing information and streamline cross company operations. (Chen & Chen, 2005)

The first step of streamline cross company operations is in grouping products or customers under their logistics needs and characteristics was offered by (Fuller, O'Conor, & Rawlinson, 1993) referenced by (Chen & Chen, 2005). Further (Hammer, 2001) described supply problems linked with situations under which a customer acquires different products from one supplier or one supplier offers one product to different customers. Chen & Chen illustrated «*supply networks where the product-based or customer-based cooperatives in the channel fall into the general framework of a joint replenishment program*».

		Joint replenishment	
		No	Yes
Channel cooperative	No	Policy I	Policy II
	Yes	Policy III	Policy IV

Figure 1. Policies describes joint effects of multi-echelon cooperation and multi-product replenishment ((Chen & Chen, 2005))

According to (Chen & Chen, 2005) the multi-echelon literature concentrated on channel coordination problem for inventory replenishments from vendor to retailer through supply chain based on minimization of the channel-wide costs objective function.

The authors also propose four policies to describe effects of joint replenishment into multi-echelon cooperation. These policies are presented at Figure 1. According to this Figure 1 there are two policies (Policy I and Policy III) that assume no channel cooperation. In this case each cell of the supply chain minimizes or maximizes its own performance without taking into account the activities of their partners and costs what can be linked with such type behavior. Under this type channel cooperation two cases occur. They are individual replenishment and *JRP* for a multi-item problem. Under individual replenishment policy the relevant costs for retail dealer have a major ordering cost, minor ordering cost and inventory holding cost. The total relevant cost is sum relevant costs of each item with respect to that every item has its own optimal order quantity as well as its own replenishment cycle. (Chen and Chen 2005)

The task of retail dealer under Policy III is to identify a common cycle of placement order to supplier for the all items. The total relevant cost under this policy is similar to previous case, except for major ordering costs. These costs occur once over the cycle. (Chen and Chen 2005)

Under Policy II place of order occurs separate for each item according to decision of every participant of supply chain. The total cost of one particular item is represented by summation of all outlays for this item for the whole supply chain under consideration. In the article it is three-leveled inventory system. Under Policy IV the cooperation between members of a supply chain as well as joint replenishment of items occur. For more detail see (Chen and Chen 2005).

The vital factor that helps to understand the behavior of the organization and at the end leads to competitive results is *performance measurement*. Performance measurement helps to provide decisions about distribution human and others types of resources among different areas in a business. (Waggoner, Neely, & Kennerley, 1999) According to (Neely, Gregory, & Platts, 1995) «*performance measurement can be defined as the process of quantifying the efficiency and effectiveness of action*». Under effectiveness it is understood the amount customers' needs that have been met. The rate of using resources of organization subject to predetermined level of customers' satisfaction is *efficiency*. (Anvari, Nayeri, & Razavi, 2011)

There are different types of methods that provide performance measurement of organizations. At the same time there is no universal approach to performance measurement that is accepted all authors together as well as categorization of these methods. (Shepherd & Günter, 2006) An example of these methods could be (Chan, Qi, Chan, Lau, & Ip, 2003) that split performance measures into two groups. They are qualitative and quantitative measures. Customer's satisfaction, information/material flow integration are examples of qualitative measures. Quantitative measures are built on three parts. First measures are based on cost (for example, minimization of cost of investments in inventories) as well as profit maximization or return on investments maximization. The second are measures based on customer «*as maximizing the percentage of meeting orders (fill rate), minimizing product delivery delays*» etc. (Chan, Qi, Chan, Lau, & Ip, 2003) The third are measures based on productivity. As example, it could be maximization of the usage of capacity and resources. (Chan, Qi, Chan, Lau, & Ip, 2003) Some of them will be considered below.

This master thesis will deal with two related topics. The first one is common cost minimizing approach under coordination of replenishments for a family of items with a restriction on capital that can be invested to the inventory. The second one is an attempt to provide replenishment politics for an inventory when the objective is to maximize the return on investment. This will in many situation lead to closed formulas that can be analyzed and used as guidelines for replenishment politics for an inventory.

2. Some general concepts and literature overview

This chapter has an aim to introduce the reader to the basic concepts and notation used in the thesis. The first part is devoted to achievements in return on inventory investment area. Second and third part studies joint replenishment problem literature and different methods to solve it.

2.1 Return on inventory investment literature overview

«If you do not know where to go, no one wind will be favorable»

Seneca

Classical economic order quantity (*EOQ*) probably is the most well-known in the inventory management. *EOQ* is using in many practical applications. *EOQ* model was built by F. W. Harris in 1913. However, an extensive application of this model is associated with the name of R. H. Wilson. There are some assumptions to this model. They are constant and continuous demand d_i for item *i*, ordering A (or in production it is the cost of setting up production) and holding v_i costs for item *i* and *r* is an interest rate. Purchasing cost (unit cost) is independent of the order size. No stock-out situation is allowed. The items are always available in the market. The total number of items is *n.* In this case the *EOQ* model provides the order quantities equal to the $(Q_i)_{HW}$ (Axsater, 2006), (Gribkovskaya, 2012):

$$(Q_i)_{HW} = \sqrt{\frac{2A_i d_i}{v_i r}} \qquad i = 1, \dots n \quad (2.1.1)$$

Based on the minimization of the total relevant costs function:

$$TRC(Q_i) = \sum_{i=1}^{n} \frac{d_i}{Q_i} A_i + \frac{1}{2} \sum_{i=1}^{n} Q_i v_i r \qquad i = 1, \dots n \quad (2.1.2)$$

Different answer can be obtained if the objective to minimize *return on investment (ROI)* is used. See, for example (Trietsch, 1995), (Gribkovskaya, Halskau, & Olstad, 2012), (Gribkovskaya, 2012). This approach consider inventory as investments. More fully the difference in the two approaches described at (Gribkovskaya, Halskau, & Olstad, 2012) as citation from Chamberlain:

«*Two companies may show the same return on sales, but if one requires twice as much investment to achieve the result, it would be stretching a point to claim that their performances were equally good*».

If (Trietsch, 1995) is right, then the first attempts to adopt the economic order quantity model *(EOQ)* to the objective of maximizing return on investment *(ROI)* were done at early 1930s by (Raymond, 1931).

Next big stage in exploration of maximization of *ROI* in order to get optimal batch-size was done by (Eilon S. , 1960), (Eilon S. , 1964). To get optimal batch-size in batch production the objective should be define. Four different objectives were introduced. They were the minimization of total cost, the maximization of profit for the batch, maximum return and maximum rate of return. The mathematician model that was built is similar to the cost function from inventory management. Minimization of the total cost leads to batch size that is quite similar to the economic order quantity of Wilson. According to Eilon maximization of profit provides the same result as the minimization of cost per unit. This leads to identical optimal batch-size. When *ROI* was using as criterion, it provided $ROQ \leq EOQ$, where *ROQ* is solution of *ROI*-maximizing objective function.

In 1964 one more attempt was done by (Tate, Burbidge, & Duckworth, 1964). (Tate, Burbidge, & Duckworth, 1964) states that $ROQ = EOQ$. Nevertheless, further research provides support to Eilon's approach. For, example (Trietsch, 1995) showed that $ROQ \leq EOQ$. The author also developed the single item model under *ROI* maximization as well as generalized the solution for several items (the combined order case) and for several items with independent orders.

In inventory management there are other criteria (apart from *ROI*) that have already mentioned which are considered as an appropriate. According to (Gribkovskaya, Halskau, & Olstad, 2012) cost minimization models do not answer about volume of investment that should be done in order to get certain profit. *EOQ* – models based on profits don't take into account that investment in firms can be different. Taking *ROI* as the objective, the model and calculations become more difficult than *EOQ* and are not used so common.

Other attempts to develop an *ROQ* model were done by (Schroeder & Krishnan, 1976) and (Arcelus & Srinivasan, 1987). One of them developed the single-item *ROQ* model with the objective of maximizing the *ROI* in inventory. The article of (Schroeder & Krishnan, 1976) also devoted to a discussion of *ROI* as a criterion. For example, *ROI* is not appropriate criteria for nonprofit organizations (schools, hospitals etc.). From point of view of authors *ROQ* fits to firms that operates with finished goods (retailers, wholesalers) as their assets may coincide with inventories. If inventories are represented by raw material or in-process inventories, they may not be considered as investment by themselves. In this case cost minimization model may be better. (Arcelus & Srinivasan, 1987) mainly focused in optimizing the selling price based on a monopolistic approach. The article extends the deterministic *EOQ* model to reflect various optimizing criteria. The goal of this paper is in developing decision rules for operations linked with control of finished goods inventories mostly for retailers. According to (Arcelus & Srinivasan, 1987) in retailing inventories are evaluated in the same way as any other investment, based on their ability to bring profits, rather than on the traditional least-cost approach. In their models demand is a function of price, with price defined as a markup of unit cost. The decision variables are the order quantity and the markup rate.

(Rosenberg, 1991) explored behavior of monopoly firm linked with price-inventory decisions under various criteria. He made an attempt to decide between profit and *ROI* as criterion for the inventory. The author showed that «*decentralized price-inventory decision-making is optimal when the return criterion is used*». (Rosenberg, 1991) An interesting example which explains the difference between *ROI* and profit as an inventory objective was also provided by author. In his example the author highlighted that the same input data for different models brings different results. For example, model based on *ROI* maximization approach provide only half profit of profit maximizing model. From return on investment point of view, the ROI model has value 3.5 times bigger than profit maximizing model.

Further researches are provided by (Halskau & Thorstenson, 1998). This article discusses how *ROI* maximization order quantities are determined for inventory control aims. The results obtained from using *ROI* are related to results obtained from using cost minimization criterion. Also in this article difference which depends on the cost structure of the planning situation between two resulting order policies is shown.

Some of the studies on this topic are (Otake, Min, & Chen, 1999), (Li, Min, Otake, & Van Voorhis, 2008), (Gribkovskaya, Halskau, & Olstad, 2012). (Otake, Min, & Chen, 1999) have done an attempt to develop and analyze behavior of inventory and investment in setup operations policies when *ROI* maximization as objective function is used. In this paper *ROI* model was established. Also in this article the unique global optimal solution is described when there is a possibility to invest in setup operations exists. The authors prove that there is a reduction in inventory level under a situation which is characterized by need of additional investments of money in setup operations. This unique optimal solution in closed-form was obtained «*when the setup cost is a rational or linear function of the level of investment*». (Otake, Min, & Chen, 1999)

(Li, Mina, Otake, & Voorhis, 2008) also developed model for inventory and capital investment in setup and quality operations when *ROI* maximization is objective function. In addition there is an investment budget constraint. The authors focused on a way of building of such an *ROI* maximization model and providing analysis of this model. One of aims of this article is the determination of the unique global optimal solution. Authors also explored conditions under which the inventory is reduced. The achievements of the authors also include the study of the question how increasing (or decreasing) of the investment budget affects investment strategies. May be fundamental shift of investment strategies is needed to get maximum *ROI*.

To the best of our knowledge there are not so many studies which represent a combination joint replenishment problems and return on investment maximization. It is (Wee, Lo, & Hsu, 2009). (Wee, Lo, & Hsu, 2009) developed a multi-objective joint replenishment inventory models with deteriorated items. In the multi-objective inventory model, the decision maker makes an attempt to optimize two or more objectives under various constraints at the same time. In this article authors use profit and return on inventory investment maximizations as objectives. With help of inverse weight fuzzy non-linear programming authors derive solution that «*satisfies the decision maker's desirable achievement level of the profit objective, ROII objective and shortage cost constraint goal under the desirable possible level of fuzzy demand*». (Wee, Lo, & Hsu, 2009)

This master thesis is partly based on the research made by (Gribkovskaya, Halskau, & Olstad, 2012). The main conclusion achieved in this article is that if all logistical activities of firm are outsourced and there is no investment in equipment of any kind as well as reduces the capital tied up in the inventory, then «*the return on investment can be improved substantially compared to the case where one uses the classical Harris-Wilson order size*». (Gribkovskaya, Halskau, & Olstad, 2012) From the other side if there is changing the strategy from profit maximization to maximization of return on investment, the net profit will be reduced. Also in this article the upper bound on amount of the capital to invest in the inventory that could be reduced before the return on investment starts to decrease is derived. The main content of this paper are presented also in (Gribkovskaya, 2012).

2.2 Joint replenishment grouping strategy

If (Khouja & Goyal, 2008) are right, *joint replenishment problem (JRP)* has been intensively research since the 1960s. As it has already been mentioned because of the major ordering cost, replace order for a group of items may lead to significant cost savings. These cost savings are more substantial the higher the major ordering cost.

According to (Eijs, Heuts, & Klei, 1992) in case of constant demand, joint replenishment of orders can be split into two strategies, named as *indirect grouping strategies* and *direct grouping strategies*.

Indirect grouping strategy. The basic cycle time *T* is decision variable in the indirect grouping model. Two fulfilled replenishments of item *i* for all *i*=1,…,*N*, where N is the number of items in the family have the number of basic cycles equals k_i. The task is to find combination of (*T*, k_i) for ∀ *i*=1,…,*N* that will give the lowest *total relevant cost* (*TRC*) of the family. If it is assumed that *A* is the major ordering cost and a_i is the minor ordering cost of item *i.* Demand per period for item *i* is D_i and h_i is the inventory carrying cost per unit of item *i* per period. In this case *TRC* can be represent as

$$TRC = \frac{1}{T}\left(A + \sum_{i=1}^{N} \frac{a_i}{k_i}\right) + \frac{1}{2} T \sum_{i=1}^{n} k_i D_i h_i,$$

where $k_i \in \{1, 2, \dots\}$. (Eijs, Heuts, & Klei, 1992)

Through derivative of TRC regards to basic cycle time *T* and the number of basic cycles k_i between two fulfilled replenishments of item *i*, the optimal T^* and ${k_i}^*$ can be obtain. The problem is that the value of ${k_i}^*$ is needed to get T^*. The opposite is true. It should be notice that the number of basic cycles k_i is considered as a continuous variable. (Eijs, Heuts, & Klei, 1992)

According to (Olsen, 2005) some methods how to find an inventory replenishment policy were developed by Goyal (1973) and Shu (1971). These methods produced sub-optimal solutions. (Goyal, 1974) has proposed a search procedure for findings the best set of k_i's. (Silver E. , 1976) has provided a heuristic in order to define the optimal or near optimal set of k_i's. Then with help of these k_i the optimal or near optimal cycle time *T* and a minimal total cost are got. After that according to (Olsen, 2005) there were a number of a modification of Silver's and Goyal's (1974) methods which gave closer to optimal results in many cases. An extensive survey of early work can be found in (Goyal & Satir, 1989).

RAND algorithm was proposed by (Kaspi & Rosenblatt, 1991). This algorithm improves previous achievement of these authors by determining minimum T_{min} and maximum T_{max} values for basic cycle time *T*. A set of initial values of basic cycle time *T* are taken from this range. For every basic cycle time *T* from the set of initial values, the algorithm proposed is used in order to find values for k_i. Also extensive experiments in order to compare their results with other methods were done. It was concluded that RAND was better than all other strategies and was «*almost as good as the optimal solutions*». (Kaspi & Rosenblatt, 1991) The authors didn't compare results achieved with full enumeration because that is quite expensive to use for large number of *n*.

(Fung & Ma, 2001) developed some new bounds on the basic cycle time. They also provide two new algorithms. (Lee & Yao, 2003) explored the optimality structure of the JRP and derived a global optimum search algorithm for the JRP under power-of-two policy. They also proved that the optimality structure of the JRP is piece-wise convex. (Nilsson, Segerstedt, & Sluis, 2007) presented a novel heuristics based on a spreadsheet technique. (Wang, He, Wu, & Zeng, 2012) proposed a new differential evolution (DE) algorithm for JRP using both direct and indirect grouping strategy.

The way to determine the replenishment cycles of the group provides difference between two strategies. For *direct grouping strategies* the number of groups *m* is predetermined. Every item in the family belongs to one out of *m* groups. Within each particular group all items have the same cycle time T_j. The goal of direct grouping strategies provides the best separation into groups and the optimal cycle time for each group to get minimum of total cost. If it is assumed that

M is given exogenously and S_j and T_j are the set of items in group *j* and the time between two successive replenishments of all items in group *j* correspondingly, then according to (Eijs, Heuts, & Klei, 1992)

$$TRC = \sum_{j=1}^{M} \left(\frac{A + \sum_{i \in S_j} a_i}{T_j} + \frac{1}{2} T_j \sum_{i \in S_j} D_i h_i \right)$$

The task to divide *n* items into *m* groups is not simple because of big number of possible combinations. According to (Eijs, Heuts, & Klei, 1992) Chakravarty (1981) and Bastian (1986) proved so called «*consecutiveness property*» theorem. The main idea is that «*when the items are arranged in increasing order with respect to the ratio* $\frac{D_i h_i}{a_i}$*, then the optimal groups can be created from this sequential list*». (Eijs, Heuts, & Klei, 1992) Let consider 4 items, which is arranged in increasing order of the ratio $\frac{D_i h_i}{a_i}$. That means that item 1 has the item with the smallest ratio. In this case, the groups $S_1 = \{1,2\}$ and $S_2 = \{3,4\}$ can be optimal, at the same time $S_1 = \{1,3\}$ and $S_2 = \{2,4\}$ cannot. (Eijs, Heuts, & Klei, 1992) Based on this ranking scheme, there are several algorithms were proposed for direct grouping. (Eijs, Heuts, & Klei, 1992) mentioned that (Page & Paul, 1976) was one of these authors who apply this scheme. It was proved that a method of adjusting the order intervals (Equal Order Interval Method) of products very often gives better cost solution than Lagrangian Multiplier Method. There are some examples which demonstrate effective and efficient application of Equal Order Interval Method with help of heuristics. (Chakravarty, 1985) developed fast converging heuristics to create *m* groups where $m = 2,3, \dots M$. (Bastian, 1986) provided heuristics «*for forming the groups which turns out to be an optimal algorithm for the case that there are no major set-up costs*». It was founded by (Eijs, Heuts, & Klei, 1992) that the last heuristics was the best algorithm with respect to as cost as well as complexity. (Strijbosch, Heuts, & Luijten, 2002) used combination of two types of strategies to develop a cyclical packaging plan for a pharmaceutical company. (Wang, He, Wu, & Zeng, 2012) (Olsen 2005) applied an evolutionary algorithm based on direct grouping strategy to *JRP.*

(Eijs, Heuts, & Klei, 1992) have analysis and comparison of two strategies mentioned above for multi-item inventory systems. The authors generated problems in a random way, which then were used to test indirect and direct grouping strategy. The main conclusion was that the indirect grouping strategy exceeded the direct grouping strategy in case high major ordering cost. The main reason for that is many products can be ordered together under indirect grouping strategy.

3 The Order Quantity and other parameters for *ROI* maximization

The problem in this case is to provide replenishment politics for a family of items inventory when the objective is to maximize *ROI*.

There is a list of assumptions. The family has *n* items with following characteristics: demand rate is constant and deterministic. The unit variable cost is independent from replenishment quantity; the cost factors do not vary with time, no shortages. The entire order quantity is delivered simultaneous.

3.1 ROQ vs EOQ without budget constraint

In this part of the thesis it is assumed that the average capital tied up in the inventory is the only capital invested. It is also assumed that our organization is third party logistic firm. It responsibilities are replacement of orders and storage at warehouse. A transportation cost is certain amount of money to the supplier per order, so it can be define as the fixed cost per order.

3.1.1 Cost minimization model: the combined order case

In the classical and the simplest approach of the inventory theory for a family of items model can be formulated as

$$Minimize \quad TRC(Q_i) = \frac{d_1}{Q_1}A + \frac{1}{2}\sum_{i=1}^{n} Q_i v_i r\,, \qquad (3.1.1)$$

$$st \quad \frac{d_1}{Q_1} = \frac{d_2}{Q_2} = \cdots = \frac{d_n}{Q_n} \qquad (3.1.2)$$

$TRC(Q_i)$ = total relevant cost

Q_i= order quantity for item i

d_i = demand for item *i*

A = the fixed cost per order

v_i = unit cost for item *i*

r = interest rate

n = number of items

By placing only one order for all items, fixed cost could be saved and individual $Q^*{}_i$ for all *i* might not be optimal.

The order quantity obtained by such method equals to

$$Q^*{}_i = d_i \sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}} \quad (3.1.3)$$

Some important properties of the order size are considered. These properties come from equation (3.1.3):

- It is obvious that if the ordering cost *A* will increase in case of any item, the order quantities for all of the items will increase.
- If the unit cost v_i for any item *i* increase, all order quantities for all items will decrease.
- If demand d_i for any item *i* increase, order quantity Q_i will increase. Different situation happens for order quantity Q_j, it decreases. If, for reasons of simplicity, the expression (3.1.3) will be re-written as $x\sqrt{x}$, then we have linear function x and a convex upward function $\sqrt{x}$. For all the values which start with the unit, the value of first function exceeds the value of the second function.
- If interest rate *r* increases, order quantity Q_i will decrease.

Now it is easy to get the number of orders

$$\frac{d_i}{Q_i^*} = \sqrt{\frac{\sum_{i=1}^{n} d_i v_i r}{2A}}. \quad (3.1.4)$$

The optimal cost can be calculate and we get the formula (3.1.5) if we put (3.1.3) into (3.1.1):

$$TRC(Q_i^*) = \sqrt{2A \sum_{i=1}^{n} d_i v_i r} \quad (3.1.5)$$

There are some outcomes from this formula. If the ordering cost *A* or demand d_i or the unit cost v_i increase, the total relevant costs will also increase. The same result can be obtained if interest rate *r* increases.

Now we are going to have the ratio of total relevant cost to optimal relevant cost that equals to

$$\frac{TRC(Q_i)}{TRC(Q_i^*)} = \frac{\frac{d_1}{Q_1} A + \frac{1}{2} \frac{Q_1}{d_1} \sum_{i=1}^{n} d_i v_i r}{\sqrt{2A \sum_{i=1}^{n} d_i v_i r}} = \frac{d_1 \sqrt{A}}{Q_1 \sqrt{2 \sum_{i=1}^{n} d_i v_i r}} + \frac{Q_1 \sum_{i=1}^{n} d_i v_i r}{2d_1 \sqrt{2A \sum_{i=1}^{n} d_i v_i r}}$$

If we multiply the first term $\frac{\sqrt{2}}{\sqrt{2}}$, we will have that

$$\frac{TRC(Q_i)}{TRC(Q_i^*)} = \frac{Q^*{}_1}{2Q_1} + \frac{Q_1}{2Q^*{}_1} = \frac{1}{2}\left(\frac{Q_1^*}{Q_1} + \frac{Q_1}{Q_1^*}\right).$$

The result obtained means that relative cost increase if the batch quantity Q_1 deviate from the optimal batch size Q_1^*. If $\frac{Q_1^*}{Q_1} = \frac{3}{2}$, the ratio $\frac{TRC(Q_i)}{TRC(Q_i^*)}$ equals to 1,08. That means even there is the increase of batch quantity by 50%, the cost increases only by 8%.

One more outcome that comes from (3.1.5) is that according to this model cost of order equals to inventory holding cost. If we place (3.1.3) to the part of (3.1.1) which hold inventory holding cost, we will have following expression:

$$\frac{1}{2}\sum_{i=1}^{n} Q_i^* v_i r = \frac{1}{2}\sqrt{2A\sum_{i=1}^{n} d_i v_i r} \quad (3.1.6)$$

As it is follow from (3.1.6) inventory holding cost equals to half of total relevant cost. It is easy to interpret if the objective function (3.1.1) is re-written with respect to equation $Q_i = \frac{d_i}{d_1} Q_1$ that follows from (3.1.2):

$$TRC(Q_i) = \frac{d_1}{Q_1} A + \frac{Q_1}{2d_1}\sum_{i=1}^{n} d_i v_i r$$

If we increase order quantity Q_1, cost of order will decrease, but at the same time inventory holding cost will increase. So we can iterate order quantity Q_1 until we find a state under which there is no potential to improve. This occurs when cost of order equals to inventory holding cost.

3.1.2 Cost minimization model with independent orders

The disadvantage of the combined order case is that all items are ordered together. Variation of their parameters is not counted, but different order cycles may fit better to it. Now it is allow ordering item independently. We will modify our previous model in a way that fit to indirect joint replenishment strategy

$$TRC = \frac{1}{T}\left(A + \sum_{i=1}^{N} \frac{a_i}{k_i}\right) + \frac{1}{2} T \sum_{i=1}^{N} k_i d_i v_i r, \quad (3.1.7)$$

where $k_i \in \{1,2,\ldots\}$ and a_i is a minor ordering cost of item *i*.

For simplicity it is assumed that basic cycle time *T* equals to one from previous example:

$$T = \sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}} \quad (3.1.8)$$

Through derivative of TRC with regards to the number of basic cycles k_i between two fulfilled replenishments of item *i*, the value of k_i can be obtained. For details see Apendix ll.

$$k_i = \frac{1}{T}\sqrt{\frac{2a_i}{d_i v_i r}}$$

$$k_i = \sqrt{\frac{a_i \sum_{i=1}^{n} d_i v_i}{A d_i v_i}} \quad (3.1.9)$$

As it can be seen from equation (3.1.9) that demand d_i for item *i,* the fixed cost A per order, unit cost v_i for item *i* also determine the number of basic cycles equals k_i for item *i.* It should be mentioned that the interest rate r do not affect k_i's. The increase of a_i lead to increase of value k_i. The increase of A decreases value k_i. To answer how increase of d_i or v_i affect k_i we need to modify the (3.1.9) in a way:

$$k_i = \sqrt{\frac{a_i \frac{1}{d_i} \sum_{i=1}^{n} d_i v_i}{A d_i v_i \frac{1}{d_i}}} = \sqrt{\frac{a_i \sum_{j=1, j\neq i}^{n} \frac{d_j}{d_i} v_j + v_i}{A v_i}}$$

So increase of d_i decreases the value k_i and increases k_j. Now we re-write (3.1.9) in a similar way

$$k_i = \sqrt{\frac{a_i \frac{1}{v_i} \sum_{i=1}^{n} d_i v_i}{A d_i v_i \frac{1}{v_i}}} = \sqrt{\frac{a_i \sum_{j=1, j\neq i}^{n} \frac{v_j}{v_i} d_j + d_i}{A d_i}}$$

So increase of v_i decreases the value k_i and increases k_j. One interesting outcome from (3.1.9) is that there is a very small probability to have integer k_i. In case when k_i is non-integer, k_i should be rounded up to the nearest integer number as it will be shown in example further.

Now order quantity Q_i could be determined by formula:

$$Q_i = k_i d_i T = \frac{1}{T}\sqrt{\frac{2a_i}{d_i v_i r}} d_i T = \sqrt{\frac{2a_i d_i}{v_i r}} \quad (3.1.10)$$

It is interesting that order quantity Q_i could be determined without knowledge of the number of basic cycles k_i and basic cycle time *T.* Also Q_i could be determined without knowledge of parameters for other items. Order quantity Q_i increases with increase of a minor ordering cost a_i and demand d_i of item *i.* At the same time unit cost v_i for item *i* and interest rate r decrease order quantity Q_i.

We can find a lower bound for the total relevant cost for our number of basic cycles k_i obtained by substituting (3.1.8) and (3.1.9) into (3.1.7).

$$TRC = \frac{1}{\sqrt{2}}\left(\sqrt{Ar\sum_{i=1}^{n} d_i v_i} + 2\sqrt{r}\sum_{i=1}^{N}\sqrt{a_i d_i v_i}\right) \quad (3.1.11)$$

The lower bound (3.1.11) link with non-integer number of basic cycles k_i. Linked with integer number of k_i.total relevant cost will be bigger, but in case where order cost per replenishment cycle that equals to $A + \sum_{i=1}^{n} a_i$ is identical to the cost of order A from case the combined order case, it provides better solution.

3.1.3 ROI maximization model

Now assuming a wholesale outsourcing let p_i be the unit selling price for item *i*. If it is assume that fixed cost that associated with the firm's overhead equals zero, then a general concept the ROI can be described as the net profit divided by the average capital employed in the investment project.

$$Maximize \quad ROI(Q_i) = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \frac{d_1}{Q_1}A - \frac{1}{2}\sum_{i=1}^{n} Q_i v_i r}{\frac{1}{2}\sum_{i=1}^{n} Q_i v_i} \quad (3.1.12)$$

$$st \qquad \frac{d_1}{Q_1} = \frac{d_2}{Q_2} = \cdots = \frac{d_n}{Q_n} \quad (3.1.13\ )$$

The objective of second model is to maximize return on investment. Further the author is going to get optimal order size. Then the results will be compared. Finally conclusions and comments will be done.

In order to find formulas for the order quantity Q_i $(3.1.12) - (3.1.13)$ are simplified by using the following equality: $Q_i = \frac{d_i}{d_1} Q_1$. With a standard method of finding the extreme value of the function, orders quantity Q_i for every item i in order to get the maximum ROI can be expressed as:

$$(Q_i)_{ROI} = \frac{2d_i A}{\sum_{i=1}^{n} (p_i - v_i) d_i} \quad (3.1.14)$$

Furthermore, some important properties of the order size are considered. These properties come from view of the expression $(3.1.14)$:

- It is obvious that if the ordering cost *A* will increase in case of any item, the order quantities for all of the items will increase.
- If the selling price p_i for any of the items increases, the denominator will increase. All order quantities will decrease in this way. The opposite is also true if selling price p_i for any of the items decrease.
- If the unit cost v_i for any item *i* increases, all order quantities for all items except item *i* will increase.
- If demand d_i increase that is more difficult decide how the order size will change since it is included both in numerator and denominator. To answer this question equation $(3.1.14)$ are modified to:

$$(Q_i)_{ROI} = \frac{2d_i A \frac{1}{d_i}}{\frac{1}{d_i} \sum_{j=1}^{n} (p_j - v_j) d_j} = \frac{2A}{\sum_{j=1, j \neq i}^{n} (p_j - v_j) \frac{d_j}{d_i} + (p_i - v_i)}$$

If demand d_i increases, order quantity $(Q_i)_{ROI}$ will increase. At the same time $(Q_j)_{ROI}$ will decrease.

- If ratio $p_i - v_i$ that means gross profit per one unit of item *i,* order quantity $(Q_i)_{ROI}$ will decrease.

One of the interesting outcomes of equation $(3.1.14)$ is that the value of $(Q_i)_{ROI}$ does not depend on the interest rate r. To understand this outcomes let's have a look at objective function. It can be represented as:

$$ROI(Q_i) = \frac{\text{profit}}{\text{average level of capital tied up at inventory}} - \frac{\text{costs of orders}}{\text{average level of capital tied up at inventory}} - \text{r} \cdot \frac{\text{average level of capital tied up at inventory}}{\text{average level of capital tied up at inventory}} \qquad (3.1.15)$$

From equation (3.1.15) it can be seen that the value of Q_i can be defined from relationships between the first and the second members of the sum that don't contain any interest rate r. It can be concluded that obtained results depend on modeled situation.

Now we are interested to get value of the objective function ROI linked with order quantity $(Q_i)_{ROI}$.*fo*r every item i. By putting order quantity $(Q_i)_{ROI}$ for every item i obtained with equation (3.1.14), the objective function ROI looks like:

$$ROI(Q_i)_{ROI} = \frac{(\sum_{i=1}^{n}(p_i - v_i)d_i)^2}{2A \ \sum_{i=1}^{n} d_i v_i} - r \quad (3.1.15)$$

As it can be seen from (3.1.15) $ROI(Q_i)_{ROI}$ decreases when there are increases in order cost A or interest rate r or unit cost v_i for item *i.* In case of unit cost v_i for item *i*, it is not obvious as v_i is in both numerator and denominator. Unit cost v_i decreases numerator and increases denominator, so $ROI(Q_i)_{ROI}$ decreases. If demand d_i for any item *i* increases, $ROI(Q_i)_{ROI}$ increases as numerator grow faster than denominator.

Now we are going to calculate the objective function ROI linked with order quantity $(Q_i)_{cost\ min}$ .for every item i

$$ROI(Q_i)_{costmin} = \frac{\sqrt{2r}\sum_{i=1}^{n}(p_i - v_i)d_i}{\sqrt{2A\sum_{i=1}^{n} d_i v_i}} - 2r \qquad (3.1.16)$$

There is a similar effect that is increase of order cost A or unit cost v_i for item *i* decrease $ROI(Q_i)_{costmin}$. If demand d_i for any item *i* increase, $ROI(Q_i)_{costmin}$ increases. If it assumed that we have linear function x at numerator and a convex upward function $\sqrt{x}$ at denominator. For all values which start with the unit, the value of first function exceeds the value of the second function. If there is an increase of interest rate r, it is not easy to answer. Let assume that the first term of (3.1.16) equals to $b\sqrt{x}$ and the second one is x. The first term is equal to second term when $x = 0$ and $x = b^2$. The answer depends on parameters.

On the figure 1 the graphic illustration (based on example that will be considered further) of the process of change the *ROI* with changing order quantity Q_1 can be seen. With increase of order quantity Q_1 the *ROI* is also changed. The function of *ROI* gets its maximum value, and then it starts to decrease.

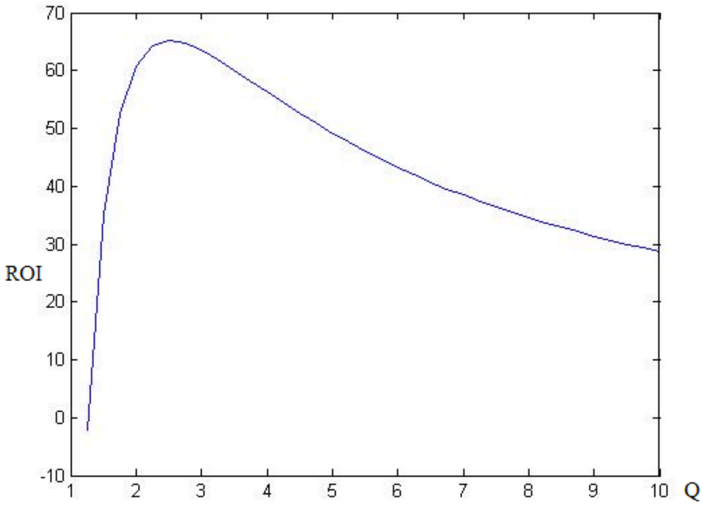

Figure 1: The graphic illustration of the dependence between *ROI* and the order quantity Q1.

As it will be proved further and if the graph is considered in more detailed it can be seen that the value of *ROI* decreases while moving from *ROI* maximization model to cost minimization.

3.2 Numerical example

Let's consider a family of six different. The table below has the input parameters such as price $\{p_1,\ p_2, \dots, p_6\}$, unit cost per item i $\{v_1,\ v_2, \dots, v_6\}$ and demand $\{d_1,\ d_2, \dots, d_6\}$. We will assume the *A*=200 $ and interest rate *r*=10%. (Gribkovskaya, 2012)

Table 1: Inputs for the numerical example: data for price, demand and, unit cost per item

d1	500	v1	25$	p1	35$
d2	350	v2	150$	p2	200$
d3	400	v3	130$	p3	170$
d4	800	v4	50$	p4	70$
d5	470	v5	80$	p5	100$
d6	620	v6	75$	p6	100$

It is interesting to get Harris-Wilson order quantities $(Q_i)_{HW}$. The results are presented at Table 2.

Table 2: Order quantities $(Q_i)_{HW}$

$(Q1)_{HW}$	$(Q2)_{HW}$	$(Q3)_{HW}$	$(Q4)_{HW}$	$(Q5)_{HW}$	$(Q6)_{HW}$
283	97	111	253	153	182

For our example with combined order case introduced above order quantities are calculated and presented at Table 3

Table 3: Order quantities for combined order case

$(Q1)_{comb}$	$(Q2)_{comb}$	$(Q3)_{comb}$	$(Q4)_{comb}$	$(Q5)_{comb}$	$(Q6)_{comb}$
64	45	52	103	61	80

Now we will modify our input parameters in such way that minor ordering cost a_i will be introduced wherein total ordering cost equals to sum of major ordering cost A_{new} and all minor ordering cost a_i as $A_{new} + \sum_{i=1}^{n} a_i$. This total ordering cost equals to the value from previous example *A*=200. In this case all a_i equals to each other and to 20 $, so A_{new} =60 $.

Table 4: Order quantities for combined order case

$(Q1)_{indep}$	$(Q2)_{indep}$	$(Q3)_{indep}$	$(Q4)_{indep}$	$(Q5)_{indep}$	$(Q6)_{indep}$
89	35	35	80	48	58

It can be seen easily that values of order quantities are different from one case to others. There is significant difference between Harris-Wilson order quantities $(Q_i)_{HW}$ and two others. Now we are interested in having values of total relevant cost for each case which are presented in Table 5.

Table 5: TRC compared between Harris-Wilson model, the combined order case model and independent order case model.

$TRC(Q_i)_{HW}$	$TRC(Q_i)_{comb}$	$TRC(Q_i)_{indep}$	$TRC(Q_i)_{comb}/TRC(Q_i)_{HW}$	$TRC(Q_i)_{indep}/TRC(Q_i)_{HW}$	$TRC(Q_i)_{indep}/TRC(Q_i)_{comb}$
7453,57	3105,48	2953,07	0,41	0,39	0,95

From Table 5 it is obvious that joint replenishments provide significant savings.

For our example with combined order case introduced above value $(Q_i)_{ROI}$ are calculated and presented in Table 6 below. It can be seen that $(Q_i)_{ROI}$ are extremely small.

Table 6: Order quantities in the case if ROQ model is used

$(Q1)_{ROI}$	$(Q2)_{ROI}$	$(Q3)_{ROI}$	$(Q4)_{ROI}$	$(Q5)_{ROI}$	$(Q6)_{ROI}$
3	2	2	4	2	3

Results for profit and *ROI* in case of both models are presented in the table below.

Table 7: Profit and ROI for the ROI maximization model compared to same values for cost minimization approach under combined order case.

$Profit(Q_i)_{ROI}$	$Profit(Q_i)_{comb}$	$ROI(Q_i)_{ROI}$	$ROI(Q_i)_{comb}$	$Profit(Q_i)_{ROI}$/ $Profit(Q_i)_{comb}$	$ROI(Q_i)_{ROI}$ / $ROI(Q_i)_{comb}$
39639	76295	64	5	0,5	13

Profit in case of *cost minimization model* under combined order case is two times bigger than in case *ROI maximization model*. At the same time *ROI*'s in our example is unrealistic, as for $ROI(Q_i)_{comb}$ that equals 5 or 500% so for $ROI(Q_i)_{ROI}$ that is 6400%, see section 3.3.1 for another approach.

3.3 ROQ vs EOQ. Case of budget constraint

In practice managers may face with system wide goals on service level or costs at a company. For example, there can be a goal to achieve a fill rate service level of 97% for particular year. As practice shows managers deal with situation that can be characterized as limited available capacity of several resources. It could be limitation on available space at warehouse, budget limitation to invest in inventory, limitation on available workforce capacity. (De Schrijver, Aghezzaf, & Vanmaele, 2011) To bring our model to reality budget constraint will be introduced in this subchapter.

Something similar was done in the (Gribkovskaya, 2012) and (Gribkovskaya, Halskau, & Olstad, 2012). According (Gribkovskaya, 2012) in determining the corresponding order quantity, first the criterion of minimization of total relevant costs is used.

The objective function keep the same form as it is at (3.3.1):

$$Minimize \quad TRC(Q_i) = \sum_{i=1}^{n} \frac{d_i}{Q_i} A_i + \frac{1}{2} \sum_{i=1}^{n} Q_i v_i r \quad (3.3.1)$$

Subject to

$$\frac{1}{2} \sum_{i=1}^{n} Q_i v_i \leq C \quad (3.3.2)$$

where C -available budget.

Through the application of the Lagrange method for the optimization problem to the model (3.3.1) - (3.3.2), the explicit formulas for Q_i were obtained:

$$(Q_i)_C = \frac{2C}{\sum_{i=1}^{n} \sqrt{A_i d_i v_i}} \sqrt{\frac{A_i d_i}{v_i}} \quad (3.3.4)$$

If the ordering costs A_i are equal for all types of items, then order quantities could be obtained without knowledge of ordering costs A_i. (Gribkovskaya, 2012)

The formula to calculate the order quantity in the case of *ROI* maximization were obtained as well

$$(Q_i)_{C_{max}} = 2\sqrt{\frac{A_i d_i}{v_i r}} \frac{\sum_{i=1}^{n} \sqrt{A_i d_i v_i} C}{\sum_{i=1}^{n} (p_i - v_i) d_i - \Phi}, \quad (3.3.5)$$

where Φ represents fixed cost per time unit and these cost is independent from the demand. (Gribkovskaya, 2012)

The difference between these two theses is that in this thesis the major ordering costs A are equal for every item *i* and occur once over the replenishment cycle for group of items.

3.3.1 Cost minimization model: combined order case

To the classical and the simplest approach of the inventory theory for a family of items that are presented in (3.1.1), the budget constraint is added. The model under consideration can be formulated as:

$$Minimize \quad TRC(Q_i) = \frac{d_1}{Q_1} A + \frac{1}{2} \sum_{i=1}^{n} Q_i v_i r \quad (3.3.6)$$

$$st \quad \frac{d_1}{Q_1} = \frac{d_2}{Q_2} = \cdots = \frac{d_n}{Q_n} \quad (3.3.7)$$

$$\frac{1}{2} \sum_{i=1}^{n} Q_i v_i \leq C \quad (3.3.8)$$

We can rewrite the budget constraint with using equality $Q_i = \frac{d_i}{d_1} Q_1$:

$$\frac{Q_1}{2d_1} \sum_{i=1}^{n} d_i v_i \leq C \quad (3.3.9)$$

We can get value of Q_1 from $(3.3.9)$. We cannot spend more money than C, so inequality can be replaced with equality sign.

$$\frac{Q_1}{2d_1}\sum_{i=1}^{n} d_i v_i = C \quad (3.3.10)$$

From equation above we can define the order quantity Q_1

$$Q_i = \frac{2d_i C}{\sum_{i=1}^{n} d_i v_i} \quad (3.3.11)$$

One of the interesting outcomes in this case is that the value of Q_i does not depend on the ordering cost *A*. Furthermore, some important properties of the order size are considered:

- It is obvious that if the budget available *C* will increase in case of any item, the order quantities Q_i for all of the items will increase.
- If the unit cost v_i for any item *i* increase, all order quantities Q_i for all items will decrease.
- If demand $\mathrm{d_i}$ increase that is more difficult decide how the order size will change since it is included both in numerator and denominator. To answer this question equation are modified to:

$$\mathrm{Q_i} = \frac{2\mathrm{d_i C}\,\frac{1}{\mathrm{d_i}}}{\frac{1}{\mathrm{d_i}}\sum_{\mathrm{i=1}}^{\mathrm{n}}\left(d_j v_j\right)} = \frac{2\mathrm{C}}{\sum_{\mathrm{j=1,j\neq i}}^{\mathrm{n}} v_j \frac{\mathrm{d_j}}{\mathrm{d_i}} + \mathrm{v_i}}$$

If demand $\mathrm{d_i}$ increases, order quantity $\mathrm{Q_i}$ will increase.

Now it is easy to get the number of orders

$$\frac{d_i}{Q_i} = \frac{\sum_{i=1}^{n} d_i v_i}{2C}$$

The optimal cost can be calculate by

$$TRC(Q_i) = \frac{\sum_{i=1}^{n} d_i v_i}{2C} A + Cr \quad (3.3.12)$$

There are some outcomes from this formula:

- If the ordering cost *A* or demand d_i or the unit cost v_i increase or even interest rate r, the total relevant costs $TRC(Q_i)$ will also increase.
- If there is an increase of budget *C*, $TRC(Q_i)$ increases. If we consider the first term of equation it could be represented as $\frac{k}{x}$ that is hyperbole. This

function decreases from zero to plus infinity. The second term is kx that is linear function. Under large value of x the second term will exceed the first one.

If we take the derivative of (3.3.12) and set equal to zero, we will get optimal available budget

$$C = \sqrt{\frac{A\sum_{i=1}^{n} d_i v_i}{2r}} \quad (3.3.13)$$

Available budget *C* increases if there is increase of ordering cost *A* or demand d_i or the unit cost v_i. With respect to interest rate r the available budget *C* decreases.

Now we need to get the $Q_{i_{comb}}$ and $TRC(Q_i)_{comb}$ with respect to (3.3.13) that is available budget for cost minimization approach under combined order case.

$$Q_{i_{comb}} = d_i \sqrt{\frac{2A}{r\sum_{i=1}^{n} d_i v_i}} \quad (3.3.14)$$

The formula (3.3.14) is identical to (3.1.3) with the same properties.

3.3.2 ROI maximization model

Now let's consider how *ROI* approach can be implemented, if a firm decided to use it for the inventory policy. It is assumed that our firm uses subcontracting for their set up operations and don't have other investments apart the inventory. That is the simplest case, which is convenient to use as example to apply the method. In this case the problem can be formulated as follows:

$$Maximize \quad ROI(Q_i) = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \frac{d_1}{Q_1}A - \frac{1}{2}\sum_{i=1}^{n} Q_i v_i r}{\frac{1}{2}\sum_{i=1}^{n} Q_i v_i} \quad (3.3.15)$$

$$st \quad \frac{d_1}{Q_1} = \frac{d_2}{Q_2} = \cdots = \frac{d_n}{Q_n} \quad (3.3.16)$$

$$\frac{1}{2}\sum_{i=1}^{n} Q_i v_i \le C$$

Following the same logic that was applied in case of cost minimization model. We get that orders quantity Q_i equal to

$$Q_i = \frac{2d_i C}{\sum_{i=1}^{n} d_i v_i} \quad (3.3.17)$$

Now we are interested to get value of the objective function ROI linked with order quantity Q_i.*fo*r every item i. By putting order quantity Q_i. for every item i obtained with equation (3.3.17), the objective function ROI looks like

$$ROI(C)_{comb} = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i}{C} - \frac{A\sum_{i=1}^{n} d_i v_i}{2C^2} - r \quad (3.3.18)$$

As it can be seen from (3.3.18) $ROI(Q_i)_{comb}$ decreases when there are increases in order cost A or interest rate r or unit cost v_i for item *i*. In this way a behavior of $ROI(Q_i)_{comb}$ is similar to (3.1.15). If there is an increase of d_i, the answer depends on parameters. To describe the impact of increase in available budget *C*, a graph (Figure 2) for the data from our example were built.

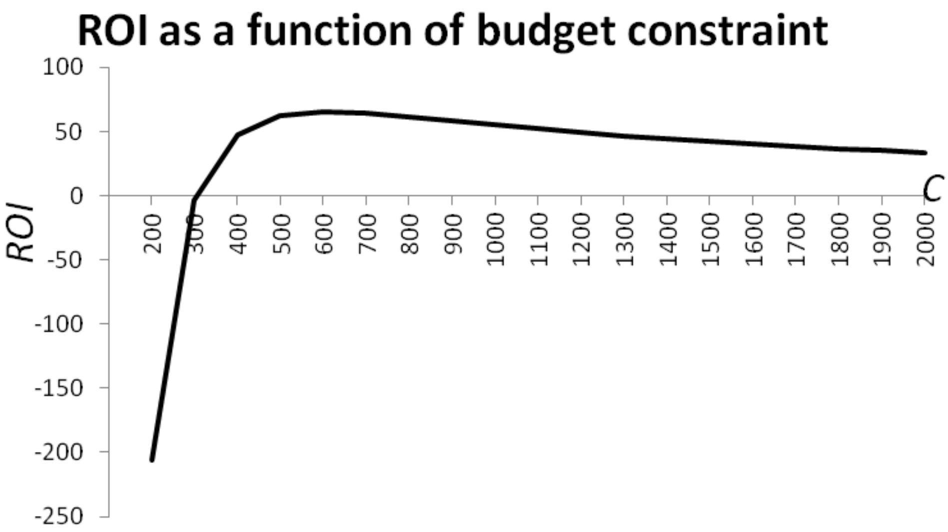

Figure 2: The graphic illustration of the dependence between *ROI* and the available budget.

With increase of available budget *C* the function of *ROI* gets its maximum value, and then it starts to decrease. It is interesting to determine the amount of money to invest in order to get maximum *ROI*. If we derives (3.3.18), we will have

$$C_{max} = \frac{A\sum_{i=1}^{n} d_i v_i}{\sum_{i=1}^{n}(p_i - v_i)d_i} \quad (3.3.19)$$

In order to get maximum *ROI* available budget should be equals to (3.3.19). Furthermore, some important properties come from view of the expression (3.3.19):

- It is obvious that if the ordering cost *A* will increase, the available budget *C* increases.
- If the selling price p_i for any of the items increases, the denominator will increase and as a consequence *C* decreases.
- If the unit cost v_i for any item *i* increases, the available budget *C* increases.
- If demand d_i increase and as it is included both in numerator and denominator, the answer depends on input parameters.
- If ratio $p_i - v_i$ per one unit of item *i* increases, *C* decreases.

To bring our model to reality some new variables will be introduced. Extended *ROI* model as a function of order size can be formulated as follows:

$$ROI(Q_i) = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi - \frac{d_1}{Q_1}A - \frac{1}{2}\sum_{i=1}^{n} Q_i v_i r}{\frac{1}{2}\sum_{i=1}^{n} Q_i v_i + L} \quad (3.3.20)$$

where Φ is additional cost, for example, it could be fixed cost per unit time. It should be mentioned here that there are no connections between demand and this cost. L is capital, which is used by firm apart inventory.

From (3.3.20) we get

$$C_{max} = \frac{2A\rho + \sqrt{4(A\rho)^2 + 8\varphi L\rho A}}{4\varphi}, \qquad (3.3.21)$$

where $\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi - Lr = \varphi$ and $\sum_{i=1}^{n} d_i v_i = \rho$

If we put L equals to zero, then

$$C_{max} = \frac{A\sum_{i=1}^{n} d_i v_i}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi} \quad (3.3.22)$$

The formula (3.3.22) obtained is close to (3.3.19), apart fixed cost Φ per unit time increase of which decreases available budget.

Now we can find the formula for the order quantity in case of ROI maximization

$$(Q_i)_{Cmax} = \frac{2d_i A}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi} \quad (3.3.23)$$

The formula $(3.3.23)$ obtained is similar to the case without budget constraint. The formula $(3.3.23)$ doesn't have any parameters which need additional calculations.

Let go back to our numerical example and recalculated order quantities with respect to $(3.3.21)$, where Φ=27 000$ and L=1 000$.

Table 8: Order quantities in the case if ROQ model is used

$(Q1)_{ROI}$	$(Q2)_{ROI}$	$(Q3)_{ROI}$	$(Q4)_{ROI}$	$(Q5)_{ROI}$	$(Q6)_{ROI}$
29	20	23	46	27	36

With value obtained $ROI(Q_i)_{ROI}$ equals to 6. Profit$(Q_i)_{ROI}$ equals to 48249 $ that is 0,6 of Profit$(Q_i)_{comb.}$ The result is still unrealistic, but it seems more reasonable.

3.3.3 Cost minimization model: independent order case

In previous case the optimal solution was found without any derivation and problems. However, if we have the more general situation, where we have partial coordination using k_i, it becomes more complicated.

It is assume that we know all $k_i \in \{1,2,\dots,m\}$ and there is a capital restriction *C*. The basic cycle time equals to

$$T = \frac{Q_i^*}{d_i} = \sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}}$$

To make the model more simple, the condition that

$$Q_i = k_i Q_i^* \quad (3.3.24)$$

is used, where

$$Q_i^* = d_i \sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}}$$

As it is known $Q_i^* = \frac{d_i}{d_1} Q_1^*$.

Hence we have the following situation

$$Minimize \quad TRC = \frac{1}{T}\left(A + \sum_{i=1}^{n} \frac{a_i}{k_i}\right) + \frac{1}{2} T \sum_{i=1}^{N} k_i d_i v_i r,$$

$$st \quad \frac{1}{2}\sum_{i=1}^{n} Q_i v_i \leq C$$

If we re-write the problem with respect to $(3.3.24)$ and replace inequality as equality

$$Q_1^* \sum_{i=1}^{n} k_i d_i v_i = 2d_1 C$$

then order quantities can be obtained

$$Q_i = k_i \frac{2d_i C}{\sum_{i=1}^{n} k_i d_i v_i} \quad (3.3.25)$$

If we assume that we have optimal available budget *C*, then we can use k_i's from 3.1.2:

$$Q_i = \sqrt{\frac{d_i a_i}{v_i}} \frac{2C}{\sum_{i=1}^{n} \sqrt{a_i d_i v_i}} \quad (3.3.26)$$

Some interesting properties of obtained results is that

- the ordering cost *A* do not affect Q_i.
- If the ordering cost a_i for any item *i* increase, the Q_i decreases.
- If the unit cost v_i for any item *i* increases, the Q_i decreases.
- If demand d_i increases for any item *i* increase, the Q_i decreases.

If it assumed that available budget equals to 12108 $, as it follows from 3.1.2 then we have order quantities that are presented into table

Table 9: Order quantities in the case if ROQ model is used

$(Q1)_{indep}$	$(Q2)_{indep}$	$(Q3)_{indep}$	$(Q4)_{indep}$	$(Q5)_{indep}$	$(Q6)_{indep}$
89	35	35	80	48	58

There is no big difference in values of order quantities between case without budget constraint and this one in values as we use the value of available budget equals to from case 3.1.2. As we use the same formulas for k_i and T the lower bound for the total relevant cost can be defined the same way as it is at 3.1.2. The value of $TRC(Q_i)_{indep}$ equals to same value as from case without budget constraints. The main difference between these two cases is that order quantities are not considered independently any more.

4 Conclusion and further research

This master thesis is devoted to intensive research of joint replenishment problem under condition when all items are ordered together. In some situation this approach brings significant savings and is more preferable than classical approach presented here by Harris-Wilson *EOQ* model. Nevertheless, despite all benefits there are some possibilities to improve it. This master thesis has an attempt to adopt results obtained from case when all items are ordered together to situation under which whole set of items are divided into several groups. This strategy has yielded positive results from the point of view total relevant cost in a situation when there is no budget constraint. In the case of budget constraint, the result is confirmed. The only difference that was found in case of budget constraint is that order quantities are not considered independently any more.

Abstracting from this master thesis contain an attempt to move from classical cost minimization approach to *ROI* maximization approach. Despite the fact that the input parameters for cost minimization model and *ROI* maximization model are the same, applications of these two models in some cases leads to different results. This linked with substantial difference in approaches. First is that cost represents absolute measure of firm's efficiency and *ROI* provide a relative measure. Second is that according to *ROI* model goods should not be managed independently. In this way the result of previous work of (Gribkovskaya, 2012) are supported.

There are many possibilities for future researches. One of possible variants is to combine different strategies for joint replenishment problem, for example, when all items are independent and *ROI* maximization approach. Some special situations as discounts or backorders can be considered as well as some additional constraints can be added.

Appendix I

In order to find formulas for the number of orders constraint, Model 2 are simplified by using the following equality: $Q_i = \frac{d_i}{d_1} Q_1$. The objective function now is:

$$ROI(Q_1) = \frac{\sum_{i=1}^{n} (p_i - v_i) d_i - \frac{d_1}{Q_1} A - \frac{1}{2} \sum_{i=1}^{n} \frac{d_i}{d_1} Q_1 v_i r}{\frac{1}{2} \sum_{i=1}^{n} \frac{d_i}{d_1} Q_1 v_i}$$

$$ROI(Q_1) = \frac{\sum_{i=1}^{n} (p_i - v_i) d_i - \frac{d_1}{Q_1} A - \frac{1}{2} \frac{Q_1 r}{d_1} \sum_{i=1}^{n} d_i v_i}{\frac{1}{2} \frac{Q_1}{d_1} \sum_{i=1}^{n} d_i v_i}$$

$$ROI'(Q_1) =$$

$$\frac{\left(\frac{d_1}{Q_1^2} A - \frac{1}{2} \frac{r}{d_1} \sum_{i=1}^{n} d_i v_i\right)\left(\frac{1}{2} \frac{Q_1}{d_1} \sum_{i=1}^{n} d_i v_i\right) - \left(\sum_{i=1}^{n} (p_i - v_i) d_i - \frac{d_1}{Q_1} A - \frac{1}{2} \frac{Q_1 r}{d_1} \sum_{i=1}^{n} d_i v_i\right)\left(\frac{1}{2 d_1} \sum_{i=1}^{n} d_i v_i\right)}{\left(\frac{1}{2} \frac{Q_1}{d_1} \sum_{i=1}^{n} d_i v_i\right)^2} =$$

$$= \frac{\left(\frac{A}{2Q_1} \sum_{i=1}^{n} d_i v_i - \frac{1}{4} \frac{Q_1 r}{d_1^2} \left(\sum_{i=1}^{n} d_i v_i\right)^2\right) - \left(\left(\sum_{i=1}^{n} (p_i - v_i) d_i\right) * \left(\frac{1}{2d_1} \sum_{i=1}^{n} d_i v_i\right) - \frac{1}{2Q_1} A \sum_{i=1}^{n} d_i v_i - \frac{1}{4} \frac{Q_1 r}{d_1^2} \left(\sum_{i=1}^{n} d_i v_i\right)^2\right)}{\left(\frac{1}{2} \frac{Q_1}{d_1} \sum_{i=1}^{n} d_i v_i\right)^2}$$

$$= \frac{\frac{A}{2Q_1} \sum_{i=1}^{n} d_i v_i - \frac{1}{4} \frac{Q_1 r}{d_1^2} \left(\sum_{i=1}^{n} d_i v_i\right)^2 - \left(\sum_{i=1}^{n} (p_i - v_i) d_i\right) * \left(\frac{1}{2d_1} \sum_{i=1}^{n} d_i v_i\right) + \frac{A}{2Q_1} \sum_{i=1}^{n} d_i v_i + \frac{1}{4} \frac{Q_1 r}{d_1^2} \left(\sum_{i=1}^{n} d_i v_i\right)^2}{\left(\frac{1}{2} \frac{Q_1}{d_1} \sum_{i=1}^{n} d_i v_i\right)^2}$$

$$= \frac{\frac{A}{Q_1} \sum_{i=1}^{n} d_i v_i - \left(\sum_{i=1}^{n} (p_i - v_i) d_i\right) * \left(\frac{1}{2d_1} \sum_{i=1}^{n} d_i v_i\right)}{\left(\frac{1}{2} \frac{Q_1}{d_1} \sum_{i=1}^{n} d_i v_i\right)^2}$$

$$\frac{\frac{A}{Q_1} \sum_{i=1}^{n} d_i v_i - \left(\sum_{i=1}^{n} (p_i - v_i) d_i\right) * \left(\frac{1}{2d_1} \sum_{i=1}^{n} d_i v_i\right)}{\left(\frac{1}{2} \frac{Q_1}{d_1} \sum_{i=1}^{n} d_i v_i\right)^2} = 0$$

$$\frac{\frac{A}{Q_1} - \left(\sum_{i=1}^{n} (p_i - v_i) d_i\right) * \left(\frac{1}{2d_1}\right)}{\frac{1}{4} \left(\frac{Q_1}{d_1}\right)^2 \sum_{i=1}^{n} d_i v_i} = 0$$

$$\frac{A}{Q_1} - \left(\sum_{i=1}^{n} (p_i - v_i) d_i\right) * \left(\frac{1}{2d_1}\right) = 0$$

$$\frac{A}{Q_1} = \frac{\left(\sum_{i=1}^{n} (p_i - v_i) d_i\right)}{2d_1}$$

$$Q_1 = \frac{2 d_1 A}{\sum_{i=1}^{n} (p_i - v_i) d_i}$$

$$(Q_i)_{ROI} = \frac{2 d_i A}{\sum_{i=1}^{n} (p_i - v_i) d_i}$$

Now we are interested to get value of our objective function linked with $(Q_i)_{ROI}$

$$\mathrm{ROI}(\mathrm{Q_i})_{\mathrm{ROI}} = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \frac{d_1}{\frac{2d_1 A}{\sum_{i=1}^{n}(p_i - v_i)d_i}} A - \frac{1}{2}\sum_{i=1}^{n}\frac{2d_i A}{\sum_{i=1}^{n}(p_i - v_i)d_i} v_i r}{\frac{1}{2}\sum_{i=1}^{n}\frac{2d_i A}{\sum_{i=1}^{n}(p_i - v_i)d_i} v_i}$$

$$= \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \frac{1}{2}\sum_{i=1}^{n}(p_i - v_i)d_i - \frac{1}{2}\sum_{i=1}^{n}\frac{2d_i A}{\sum_{i=1}^{n}(p_i - v_i)d_i} v_i r}{\frac{1}{2}\sum_{i=1}^{n}\frac{2d_i A}{\sum_{i=1}^{n}(p_i - v_i)d_i} v_i}$$

$$= \frac{\frac{1}{2}\sum_{i=1}^{n}(p_i - v_i)d_i - \frac{Ar}{\sum_{i=1}^{n}(p_i - v_i)d_i}\sum_{i=1}^{n} d_i v_i}{\frac{A}{\sum_{i=1}^{n}(p_i - v_i)d_i}\sum_{i=1}^{n} d_i v_i} = \frac{\frac{(\sum_{i=1}^{n}(p_i - v_i)d_i)^2 - 2Ar\sum_{i=1}^{n} d_i v_i}{2\sum_{i=1}^{n}(p_i - v_i)d_i}}{\frac{A\ \sum_{i=1}^{n} d_i v_i}{\sum_{i=1}^{n}(p_i - v_i)d_i}}$$

$$= \frac{(\sum_{i=1}^{n}(p_i - v_i)d_i)^2 - 2Ar\sum_{i=1}^{n} d_i v_i}{2A\ \sum_{i=1}^{n} d_i v_i} = \frac{(\sum_{i=1}^{n}(p_i - v_i)d_i)^2}{2A\ \sum_{i=1}^{n} d_i v_i} - r$$

Now we are going to calculate

$$\mathrm{ROI}(\mathrm{Q_i})_{\mathrm{costmin}} = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \frac{d_1}{d_1\sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}}} A - \frac{1}{2}\sum_{i=1}^{n} d_i\sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}}\, v_i r}{\frac{1}{2}\sum_{i=1}^{n} d_i\sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}}\, v_i} =$$

$$= \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \frac{\sqrt{\sum_{i=1}^{n} d_i v_i r}}{\sqrt{2A}} A - \frac{r}{2}\sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}}\sum_{i=1}^{n} d_i v_i}{\frac{1}{2}\sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}}\sum_{i=1}^{n} d_i v_i}$$

$$= \frac{2\sum_{i=1}^{n}(p_i - v_i)d_i\ \sqrt{\sum_{i=1}^{n} d_i v_i r}}{\sqrt{2A}\sum_{i=1}^{n} d_i v_i} -$$

$$- \frac{\frac{\sqrt{\sum_{i=1}^{n} d_i v_i r}}{\sqrt{2A}} 2A\ \sqrt{\sum_{i=1}^{n} d_i v_i r}}{\sqrt{2A}\sum_{i=1}^{n} d_i v_i} - \frac{\frac{r}{2}\sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}}\sum_{i=1}^{n} d_i v_i}{\frac{1}{2}\sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}}\sum_{i=1}^{n} d_i v_i}$$

$$= \frac{\sqrt{2r}\sum_{i=1}^{n}(p_i - v_i)d_i}{\sqrt{2A\sum_{i=1}^{n} d_i v_i}} - r - r = \frac{\sqrt{2r}\sum_{i=1}^{n}(p_i - v_i)d_i}{\sqrt{2A\sum_{i=1}^{n} d_i v_i}} - 2r$$

Appendix II

$$TRC = \frac{1}{T}\left(A + \sum_{i=1}^{N} \frac{a_i}{k_i}\right) + \frac{1}{2}T\sum_{i=1}^{N} k_i d_i v_i r, \quad (3.1.7)$$

$$TRC' = \left(\frac{1}{T}\left(A + \sum_{i=1}^{n} \frac{a_i}{k_i}\right) + \frac{1}{2}T\sum_{i=1}^{n} k_i d_i v_i r,\right)'$$

$$-\frac{a_i}{Tk_i^2} + \frac{1}{2}Td_i v_i r = 0$$

$$\frac{a_i}{k_i^2} = \frac{Td_i v_i r}{2}$$

$$k_i = \frac{1}{T}\sqrt{\frac{2a_i}{d_i v_i r}}$$

$$k_i = \sqrt{\frac{a_i \sum_{i=1}^{n} d_i v_i}{Ad_i v_i}}$$

$$TRC = \frac{1}{\sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}}}\left(A + \sum_{i=1}^{n} \frac{a_i}{\sqrt{\frac{a_i \sum_{i=1}^{n} d_i v_i}{Ad_i v_i}}}\right) + \frac{1}{2}\sqrt{\frac{2A}{\sum_{i=1}^{n} d_i v_i r}}\sum_{i=1}^{n}\sqrt{\frac{a_i \sum_{i=1}^{n} d_i v_i}{Ad_i v_i}}\, d_i v_i r,$$

$$TRC = \sqrt{\frac{Ar\sum_{i=1}^{n} d_i v_i}{2}} + \sqrt{\frac{\sum_{i=1}^{n} d_i v_i r}{2A}}\sum_{i=1}^{n}\sqrt{\frac{a_i A d_i v_i}{\sum_{i=1}^{n} d_i v_i}}$$

$$+ \frac{1}{2}\sqrt{\frac{2Ar}{\sum_{i=1}^{n} d_i v_i}}\sum_{i=1}^{n}\sqrt{\frac{a_i d_i v_i \sum_{i=1}^{n} d_i v_i}{A}},$$

$$TRC = \sqrt{\frac{Ar\sum_{i=1}^{n} d_i v_i}{2}} + \sqrt{\frac{r}{2}}\sum_{i=1}^{n}\sqrt{a_i d_i v_i} + \sqrt{\frac{r}{2}}\sum_{i=1}^{n}\sqrt{a_i d_i v_i},$$

$$TRC = \frac{1}{\sqrt{2}}\left(\sqrt{Ar\sum_{i=1}^{n} d_i v_i} + 2\sqrt{r}\sum_{i=1}^{n}\sqrt{a_i d_i v_i}\right)$$

Appendix III

$$ROI(Q_i) = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi - \frac{d_1}{Q_1}A - \frac{1}{2}\sum_{i=1}^{n} Q_i v_i r}{\frac{1}{2}\sum_{i=1}^{n} Q_i v_i + L} \quad (3.3.20)$$

We will insert order quantities Q_i obtained:

$$Q_i = \frac{2d_i C}{\sum_{i=1}^{n} d_i v_i}$$

and derivate it with respect to available budget:

$$ROI'(C) = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \frac{A\sum_{i=1}^{n} d_i v_i}{2C} - Cr}{C} =$$

$$= \frac{\left(\sum_{i=1}^{n}(p_i - v_i)d_i - \frac{A\sum_{i=1}^{n} d_i v_i}{2C} - Cr\right)' C - \left(\sum_{i=1}^{n}(p_i - v_i)d_i - \frac{A\sum_{i=1}^{n} d_i v_i}{2C} - Cr\right)}{C^2} =$$

$$= \frac{\frac{A\sum_{i=1}^{n} d_i v_i}{C} - \sum_{i=1}^{n}(p_i - v_i)d_i}{C^2}$$

$$\frac{\frac{A\sum_{i=1}^{n} d_i v_i}{C} - \sum_{i=1}^{n}(p_i - v_i)d_i}{C^2} = 0$$

$$\frac{\frac{A\sum_{i=1}^{n} d_i v_i}{C} - \sum_{i=1}^{n}(p_i - v_i)d_i}{C^2} = 0$$

$$\frac{A\sum_{i=1}^{n} d_i v_i}{C} - \sum_{i=1}^{n}(p_i - v_i)d_i = 0$$

$$\frac{A\sum_{i=1}^{n} d_i v_i}{C} = \sum_{i=1}^{n}(p_i - v_i)d_i$$

$$C = \frac{A\sum_{i=1}^{n} d_i v_i}{\sum_{i=1}^{n}(p_i - v_i)d_i}$$

$$ROI(Q_i) = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi - \frac{d_1}{Q_1}A - \frac{1}{2}\sum_{i=1}^{n} Q_i v_i r}{\frac{1}{2}\sum_{i=1}^{n} Q_i v_i + L}$$

where Φ is additional cost. L is capital, which is used by firm apart inventory.

$$ROI(Q_i) = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi - \frac{d_1}{\frac{2d_1 C}{\sum_{i=1}^{n} d_i v_i}}A - \frac{1}{2}\sum_{i=1}^{n}\frac{2d_i C}{\sum_{i=1}^{n} d_i v_i} v_i r}{\frac{1}{2}\sum_{i=1}^{n}\frac{2d_i C}{\sum_{i=1}^{n} d_i v_i} v_i + L}$$

$$\frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi - \frac{\sum_{i=1}^{n} d_i v_i}{2C}A - Cr}{C + L}$$

$$= \frac{\sum_{i=1}^{n}(p_i - v_i)d_i}{C + L} - \frac{\Phi}{C + L} - \frac{\sum_{i=1}^{n} d_i v_i}{2C(C + L)}A - \frac{Cr}{C + L}$$

$$ROI'(C) = \left(\frac{\sum_{i=1}^{n}(p_i - v_i)d_i}{C + L} - \frac{\Phi}{C + L} - \frac{\sum_{i=1}^{n} d_i v_i}{2C(C + L)}A - \frac{Cr}{C + L}\right)'$$

$$\frac{-\sum_{i=1}^{n}(p_i - v_i)d_i}{(C + L)^2} + \frac{\Phi}{(C + L)^2} + \frac{(2C + L)\sum_{i=1}^{n} d_i v_i}{2C^2(C + L)^2}A + \frac{r(C + L) - Cr}{(C + L)^2} = 0$$

$$\frac{-C^2\sum_{i=1}^{n}(p_i - v_i)d_i}{(C + L)^2} + \frac{C^2\Phi}{(C + L)^2} + \frac{(2C + L)\sum_{i=1}^{n} d_i v_i}{2C^2(C + L)^2}A + \frac{C^2 r(C + L) - Cr}{(C + L)^2} = 0$$

$$-2C^2\sum_{i=1}^{n}(p_i - v_i)d_i + 2C^2\Phi + (2C + L)A\sum_{i=1}^{n} d_i v_i + 2C^2 r(C + L) - 2C^3 r = 0$$

$$-2C^2\sum_{i=1}^{n}(p_i - v_i)d_i + 2C^2\Phi + 2CA\sum_{i=1}^{n} d_i v_i + L\sum_{i=1}^{n} d_i v_i A + 2LC^2 r = 0$$

$$2C^2\sum_{i=1}^{n}(p_i - v_i)d_i - 2C^2\Phi - 2CA\sum_{i=1}^{n} d_i v_i - L\sum_{i=1}^{n} d_i v_i A - 2LC^2 r = 0$$

$$2\left(\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi - Lr\right)C^2 - 2CA\sum_{i=1}^{n} d_i v_i - L\sum_{i=1}^{n} d_i v_i A = 0$$

If we assume that

$$\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi - Lr = \varphi$$

and

$$\sum_{i=1}^{n} d_i v_i = \rho$$

$$2\varphi C^2 - 2CA\rho - L\rho A = 0$$

$$C_{1,2} = \frac{2A\rho \pm \sqrt{4(A\rho)^2 + 8\varphi L\rho A}}{4\varphi}$$

The discriminant is positive and more than $2A\rho$. We have only one root

$$C_1 = \frac{2A\rho + \sqrt{4(A\rho)^2 + 8\varphi L\rho A}}{4\varphi}$$

List of references:

Anvari, Mohammad Reza Akhavan, Mahmoud Dehghan Nayeri, and Seyed Mostafa Razavi. "How to Measure Supply Chain Performance (case study)." *International Review of Business Research Papers*, 2011: 230-244.

Arcelus, F. J., and G. Srinivasan. "Inventory policies under various optimizing criteria and variable markup rates." *Management Science*, 1987: 756-762.

Arnold, J.R.T. *Introduction to Materials Management .* USA: Prentice Hall, 1998.

Axsater, Sven. *Inventory control.* Boston: Springer, 2006.

Bastian, M. "Joint Replenishment in Multi-Item Inventory Systems." *The Journal of the Operational Research Society*, 1986: 1113-1120.

Chakravarty, A. K. "An Optimal Heuristic for Coordinated Multi-Item Inventory Replenishments." *The Journal of the Operational Research Society*, 1985: 1027-1039.

Chan, Felix T S, H J Qi, H K Chan, Henry C Lau, and Ralph W L Ip. "A conceptual model of performance measurement for supply chains." *Management Decision*, 2003: 635-642.

Chen, Tsung-Hui, and Jen-Ming Chen. "Optimizing supply chain collaboration based on joint replenishment and channel coordination." *Transportation Research Part E: Logistics and Transportation Review*, 2005: 261–285.

De Schrijver, Steven K., El-Houssaine Aghezzaf, and Hendrik Vanmaele. "Aggregate constrained inventory systems with independent multi-product demand: Controlpracticesandtheoreticallimitations." *Science direct.* 2011. http://www.sciencedirect.com/science/article/pii/S0925527311003732 (accessed 11 24, 2012).

DeSchrijver, Steven K., El-Houssaine Aghezzaf, and Hendrik Vanmaele. "Aggregate constrained inventory systems with independent multi-product demand: Control practices and theoretical limitations." *Int. J.Production Economics*, 2011.

Eijs, M.J.G., R.M.J. Heuts, and J.P.C. Klei. "Analysis and comparison of two strategies for multi-item inventory systems with joint replenishment costs." *European Journal of Operational Research*, 1992: 405-412.

Eilon, S. "Dragons in pursuit of the EBQ." *Opl Res. Q.*, 1964: 347-354.

Eilon, S. "Economic Batch-Size Determination for Multi-Product Scheduling." *Operational Research Quarterly*, 1960: 77-79.

Eilon, Samuel. *Management practice and mispractice.* London: Routledge, 1992.

Fikhtengol'ts, G. M. *The Fundamentals of Mathematical Analysis.* London: Pergamon Press, 1965.

Fuller, J.B., J. O’Conor, and R. Rawlinson. "Tailored logistics: the next advantage." *Harvard Business Review*, 1993: 87–98.

Fung, R.Y.K., and X. Ma. "A new method for joint replenishment problems." *Journal of the Operational Research Society*, 2001: 358–362.

Gallego, Guillermo, and Ilkyeong Moon. "The Distribution Free Newsboy Problem: Review and Extensions." *The Journal of the Operational Research Society*, 1993: 825-834.

Goyal, S.K. "Determination of optimum packaging frequency of items jointly replenished." *Management Science*, 1974: 436–443.

Goyal, S.K., and A.T. Satir. "Joint replenishment inventory control: Deterministic and stochastic models." *European Journal of Operations Research*, 1989: 2–13.

Gribkovskaya, Anna. "Inventory under return on investment maximization." Molde University College, 2012.

Gribkovskaya, Anna, Øyvind Halskau, and Asmund Olstad. "Return on investment and inventory theory." *Annual nordic logistic research network conference.* Naantali, Finland, 2012. 187-201.

Guder, Faruk, and James L. Zydiak. "Ordering policies for multi-item inventory systems subject to multiple resource constraints." *Computers & Operations Research*, 1999: 583-597.

Hadley, G, and TM Whitin. *Analysis of inventory Systems.* Englewood Cliffs: Prentice-Hall, 1963.

Halskau, Ø, and A Thorstenson. "The EOQ and the ROQ and Their Relations to the ROI." *Working Paper.* Norwegian School of Logistics, 1998.

Hammer, M. "The superefficient company." *Harvard Business Review*, 2001: 82–91.

Hoque, M.A. "An optimal solution technique for the joint replenishment problem with storage and transport capacities and budget constraints." *European Journal of Operational Research*, 2006: 1033–1042.

Hsu, Shu-Lu. "Optimal joint replenishment decisions for a central factory with multiple satellite factories." *Expert Systems with Applications*, 2009: 2494–2502.

Jackson, Peter, William Maxwell, and John Muckstadt. "The Joint Replenishment Problem with a Powers-of-Two Restriction." *IIE Transactions*, 1985: 25-32.

Kaspi, M., and M.J. Rosenblatt. "On the economic ordering quantity for jointly replenished items." *International Journal of Production Research*, 1991: 107–114.

Khouja, Moutaz. "The newsboy problem under progressive multiple discounts." *European Journal of Operational Research* , 1995 : 458-466.

Khouja, Moutaz, and Suresh Goyal. "A review of the joint replenishment problem literature: 1989–2005." *European Journal of Operational Research*, 2008: 1–16.

Lee, F.C., and M.J. Yao. "A global optimum search algorithm for the joint replenishment problem under power-of-two policy." *Computers & Operations Research*, 2003: 1319–1333.

Li, J., K. J. Mina, T. Otake, and T. V. Voorhis. "Inventory and investment in setup and quality operations under Return On Investment maximization." *European Journal of Operational Research*, 2008: 593–605.

Li, Jie, K. Jo Min, Toshitsugu Otake, and Timothy Van Voorhis. "Inventory and investment in setup and quality operations under Return On Investment maximization." *European Journal of Operational Research*, 2008: 593–605.

Neely, Andy, Mike Gregory, and Ken Platts. "Performance measurement system design: A literature review and research agenda." *International Journal of Operations & Production Management*, 1995: 80 - 116.

Nilsson, A., A. Segerstedt, and E. Sluis. "A new iterative heuristic to solve the joint replenishment problem using a spreadsheet technique." *International Journal of Production Economics*, 2007: 399–405.

Olsen, Anne L. "An evolutionary algorithm to solve the joint replenishment problem using direct grouping." *Computers & Industrial Engineering*, 2005: 223–235.

Otake, Toshitsugu, K. Jo Min, and Cheng-Kang Chen. "Inventory and investment in setup operations under return on investment maximization." *Computers & Operations Research*, 1999: 883-899.

Page, E., and R. J. Paul. "Multi-Product Inventory Situations with One Restriction." *Operational Research Quarterly*, 1976: 815-834.

Pirkul, H, and O A Aras. "Capacitated Multiple Item Ordering Problem with Quantity Discounts." *IIE Transactions*, 1985: 206-11.

Raymond, F.E. *Quantity and Economy in Manufacture.* New-York: McGraw-Hill, 1931.

Rosenberg, David. "Optimal Price-Inventory Decisions: Profit vs. ROII." *IIE Transactions*, 1991: 17-22.

Schroeder, Roger G., and Ramakrishnan Krishnan. "RETURN ON INVESTMENT AS A CRITERION FOR INVENTORY MODELS." *Decision Sciences*, 1976: 697–704.

Shepherd, Craig, and Hannes Günter. "Measuring supply chain performance: current research and future directions." *International Journal of Productivity and Performance Management* , 2006: 242-258.

Silver, E. A. "Inventory Management An Overview Canadian Publications Practical Applicatiosn and Suggestions for Future Research." *INFOR: Information Systems and Operational Research (Formerly: CORS Journal)*, 2008: 15-28.

Silver, E.A. "A simple method of determining order quantities in joint replenishments under deterministic demand." *Management Science*, 1976: 1351–1361.

Silver, Edward A., David F. Pyke, and Rein Peterson. *Inventory Management and Production Planning and Scheduling.* USA: John Wiley & Sons, 1998.

Stevenson, WJ. *Production/operations management.* Chicago: IL: Irwin, 1996.

Tate, T.B., J.L. Burbidge, and W.E. Duckworth. "In defence of the economic batch quantity." *Operational Research Society*, 1964: 329-345.

Trietsch, Dan. "Revisiting ROQ: EOQ for company-wide ROI maximization." *The Journal of the Operational Research Society*, 1995: 507-515.

Waggoner, Daniel B., Andy D Neely, and Mike P. Kennerley. "The forces that shape organisational performance measurement systems:: An interdisciplinary review." *International Journal of Production Economics*, 1999: 53–60.

Wang, Lin, Jing He, Desheng Wu, and Yu-Rong Zeng. "A novel differential evolution algorithm for joint replenishment problem under interdependence and its application." *International Journal of Production Economics*, 2012: 190–198.

Waters, C. D. J. *Inventory Control and Management.* England: John Wiley & Sons, 1992.

Wee, Hui-Ming, Chien-Chung Lo, and Ping-Hui Hsu. "A multi-objective joint replenishment inventory model of deteriorated items in a fuzzy environment." *European Journal of Operational Research*, 2009: 620–631.