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LOG950 Logistics

Helicopter routing based on hub and spoke method in off-shore oil industry

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Abstract

In oil industry, helicopters are widely used to transport people to and from offshore installations. There are several routing policies to do transportation work in order to minimize the expected number of fatalities that is an objective function. The mutual characteristic of those routing policies is each customer installation get service from heliport directly by utilizing a helicopter. It could be a limit if several installations are far away from heliport and helicopter could not support that long trip to serve them. Hence, a method that treats offshore node(s) as hub(s) and allows other non-hub nodes (spoke nodes) to receive service from chosen hubs instead of heliport has been introduced by previous work and an exact mathematical model that corresponds to the method has also been made in that research.

Hence, in this paper, we first test the mathematical model in AMPL programming. Secondly, we introduce and test five indicators that work with some heuristics developed by other researchers. The reason for doing that is we would like to find some better combinations of indicator and heuristic that could be used to choose offshore hub(s) and assign non-hub nodes to chosen hub(s) for getting the minimized expected number of fatalities. According to all solutions of two examples, variant2 of heuristic1 working with largest demand indicator, indicator α or shortest distance indicator and heuristic3 working with shortest distance indicator are relatively better combinations and shortest distance indicator works better than other indicators in both two examples in general.

Furthermore, we modify the mathematical model by adding lifeboat-related constraints. There are two different constraints introduced in the paper. One is developed under situation of utilizing random service method to serve customer nodes and another one is made under situation of using sequential service method. Based on the results shown in AMPL programming, we could conclude the sequential service method is better and smaller number of lifeboat seats takes more negative effect on the expected number of fatalities.

Key words: helicopter routing, hub and spoke method, choose-hub indicator, lifeboat constraint.

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1.0 Introduction

Helicopters are high-efficient vehicles in transposition field. They have short response time and flexible scheduling. Hence, they are used to transport passengers in many industries, such as rescue activities, tourism, oil, etc. However, the frequency of helicopter accidents is relatively high and the damage caused by them is severe and often leads to death. National Transportation Safety Board (NTSB) in U.S. recorded from 2004 to 2008, the number of accidents made by helicopters in U.S. is 850, whereas the agency counted another 3.25 years starting from 2009 to record the number of helicopter accident that is 444 (Gribkovskaia, Halskau, and Kovyalov 2012). In oil industry, helicopters are commonly used to offer two types of services. One is for transporting equipment or other supplies to offshore installations if they are needed urgently. Another main reason of employing helicopters is to serve those people who work on offshore installations for many purposes. For instance, helicopters perform search and rescue (SAR) services whole year because of its short response time. But the key role helicopters play for serving people in oil industry is performance of tasks regarding deliveries and pickups employees to and from offshore installations.

There are some advantages of utilizing helicopters as vehicles to execute this kind of task. Helicopter transportation has higher speed and is more flexibility than ships as well as it is healthier in aspects of less travel sickness (Qian, Gribkovskaia, and Halskau 2011). However, these mentioned advantages could not overwhelm the disadvantages resulting from utilizing helicopters to transport offshore employees. Most of them consider that taking the trip with a helicopter is uncomfortable because of some physical or external factors, such as experiencing zero gravity in takeoff and landing phase and enduring heavy noise, etc. Moreover, travelling by helicopter is also viewed as the most risky component of offshore-installation-related work by offshore employees (Qian, Gribkovskaia, and Halskau 2011). UK offshore Public Transport Helicopter Safety Record reports, from 1977 to 2006, the highest risk public transportation mode among all is offshore helicopter transport. It is more risky than the normal air transportation (almost 630 times higher risk) from the point of view of fatality rate per billion passenger kilometers (Qian 2012). European offshore helicopter data records that, from 1968 to 2000, there are 23 fatal and serious injury accidents in the offshore petroleum’s industry (Qian et al. 2012).

Safety improvements of helicopter transportation have recently attracted more attention from researchers. Their works could not affect the probability of an accident but it contributes to reduce the expected number of people being involved in a fatal accident (Halskau 2012). However, there are also some ways to reduce the probability that an accident takes place, such as improving the quality and maintenance of helicopters, making helidecks on the platform easier and safer to landing on as well as giving more training for pilots, etc.

The best way to minimize expected number of fatalities in routing helicopters is hub-and-spoke method (Qian, Gribkovskaia, and Halskau 2011). As helicopter routing problem can be treated as a traditional vehicle routing problem with pickups and deliveries (VRPPD) constrained by capacity of helicopters, some routing policies used to solve VRPPD could still be available for helicopter routing problem.

In Qian et al. (2012) works, he views a heliport as a hub and gives three routing policies. These three are: direct flight from heliport, Hamiltonian and general routing policy respectively, to assign other non-hub offshore installations to the hub (heliport). In general, the procedure of transporting people between a heliport and each offshore installation is that a helicopter delivers employees from the heliport to offshore installations, and then picks up employees from offshore installations as well as sending them back onshore. Helicopters can do delivery and pickup activities simultaneously on an offshore installation or doing them separately if an offshore installation could be visited twice. More exactly, if an offshore installation could only be visited exactly once, a helicopter deliver required number of employees to one node (offshore installation) and return to the heliport from the node directly with some home-bound employees picked up from the visited node. The way is named as direct routing policy. Another way is a helicopter departs from a heliport and return to the hub (heliport) after it visits several nodes (offshore installations). Each of nodes can only be visited exactly once, so pickups and deliveries are combined together. This is Hamiltonian routing policy. The way under the circumstance allowing visiting each node twice is defined as general routing policy. More specific, a helicopter does not pick up workers from nodes until it is done delivering required number of workers to all nodes within a tour.

1.1 *Offshore hub(s) solution*

Hamiltonian routing policy outperforms other two policies in terms of minimizing cost (Qian et al. 2012).

However, if the total delivery demand and total pickup demand of several nodes within a tour does not exceed the capacity of a helicopter, but the pickup demand for each node is larger than the delivery demand, which means every node has to be visited twice, then we could not use Hamiltonian cycle. Moreover, the obvious disadvantage of using Hamiltonian routing policy is that the expected number of fatalities is the highest among all three policies. Especially when the objective function is to minimize the cost under Hamiltonian cycle, the passenger risk is maximized (Qian et al. 2012). Furthermore, passengers picked up from the first visited node within a tour have longer trip and have to undergo more takeoff and landing phases, which is the most risky phase during a helicopter trip (Qian et al. 2011), than others before they come back onshore. From this point of view, it is not a good policy when one wants to minimize the expected number of fatalities.

If minimizing the expected number of fatalities is a priority, the best policy is direct routing policy. However, it is costly to utilize this policy (Qian et al. 2012). Companies have to do tradeoff between minimized costs and minimized the expected number of fatalities. This is the first reason why we need to find a new solution in which the expected number of fatalities is smaller than in Hamiltonian cycle and the cost is cheaper than using direct routing policy.

Either Hamiltonian cycle or direct routing solution in Qian et al. (2012) work uses a heliport as a hub. However, a heliport cannot always be a hub. In real-life, some offshore installations are far away from the onshore heliport. The volume of fuel in a helicopter's tank cannot support that long trip. In other cases, helicopters could deliver employees to a certain offshore installation. However, return trip is impossible because on this offshore installation there is no equipment to refuel helicopter's tank. Some companies may prefer to purchase some helicopters with big-size tank in order to serve those nodes that are far away from heliport. Although the size of helicopter's tank is larger, the number of passenger onboard may not be increased compared to helicopters with normal size tank.

Limiting the number of passenger onboard could prevent fuel from running out during one trip as the more passengers a helicopter has onboard, the faster utilization ratio of fuel the helicopter has. Hence, it may not an optimal choice for serving those nodes that are far away from heliport. For these reasons it is not always possible to use a helicopter as a hub. In these cases, a new solution is needed.

Instead of choosing a heliport as a hub, one or more offshore nodes could be used as a hub (s). A helicopter(s) serves all nodes assigned to the offshore hub(s) by following the hub and spoke method. The cost will be less than applying direct routing policy. The expected number of fatalities is larger than the direct routing solution but much smaller than the number in Hamiltonian cycle. Helicopter departing from an offshore hub can also perform transportation tasks for those nodes that are far away from the heliport without considering tank-related capacity issue. A mathematical model discussed in the paper (Halskau 2012) could decide which offshore installation(s) could be hub(s) and which customer nodes should be assigned to which hub(s) in order to get minimized the expected number of fatalities.

1.2 Lifeboat-related constraint

In our paper, we added a new type of constraint on the mathematical model created by Halskau (2012).We called it as lifeboat-related constraint. On each offshore node, there are several lifeboats. We assumed in this paper each offshore node has the same type and same number of lifeboats. Specifically, the total number of lifeboat seats does not take influence on which node(s) could be chosen to be hub(s), but it constrained on the number of customer nodes that an offshore hub could have as well as which customer installations could be assigned to which hub. Hence, we made mathematical formulas to express lifeboat-related constraint first; afterwards we tested modified mathematical models that include this kind of constraint in AMPL computer programming.

2.0 Methodology

2.1 Quantitative research

"Quantitative research is based on the measurement of quantity or amount. It is applicable to phenomena that can be expressed in terms of quantity."(Kothari 2009)

In this paper, we use numbers, which is the value of the expected number of fatalities, to evaluate the quality of indicators and heuristics. An indicator under a heuristic that gives relatively smaller expected number of fatalities could be concluded as an optimal combination used in offshore hub(s) solution. For lifeboat-related constraint section, we also use values as measurement to do comparison and analysis about, like, which method of serving customer nodes is better and how many lifeboat seats do not affect the value of objective function, etc.

2.2 Data collection

Secondary data refers to the data collected by others. It has already been gathered, integrated, existed and documented by other scientists and researchers (Hox and Boeije 2005).

The secondary data utilized in this paper is collected from Aas et al. (2007), Qian et al. (2012) and Halskau (2012). More specifically, the distance matrix used in example1 is from Aas et al. (2007), and the distance matrix and demand of each node in example2 are from Halskau (2012). The value of π_c and π_L are from Qian et al. (2012). All analysis are based on these secondary data.

2.3 Inductive reasoning

Inductive reasoning generally means some specific measures or observations could generate broader generalized theories as well as conclusions (Decoo 1996).

In this paper, we use two examples to test all indicators under some heuristics in order to find optimal combination of an indicator and a heuristic and prove the quality and stability of the combination is high. It means the combination also could be used to find a good solution in other cases. Some heuristics in this paper offer several infeasible solutions to example1. The characteristic of this kind of infeasible solution is it has a single cluster that has only one node served by the heliport directly. The method we used to change infeasible solutions into feasible could be generalized for many cases that have the same issues as this example.

2.4 External validity

Whether the findings of our study can be generalized or not (Aas and Wallace 2009).

It refers to the extent of generalization of findings in a paper. In our paper, each indicator under all heuristics has been tested by two examples. The relatively better combination of an indicator and a heuristic could be generalized. Moreover, the method of making infeasible solutions under some heuristics into feasible had been tested by several infeasible solutions. It would also be generalized.

2.5 Heuristics

Mathematical model in some situations might take long time to get an optimal solution or even does not work. The possible situations that lead the issue mentioned above could be the large number of offshore installations or more and tighter constraints on the number of workers a helicopter could deliver or pick up to or from an offshore hub, etc. In order to solve this kind of situations, we introduce heuristics here. Hence, two problems arise. How to choose one or more offshore installation(s) among all installations to be a hub(s) and the second problem is about which non-hubs offshore installations are assigned to which hub. In Halskau (2012), the author used largest demand of a node as an indicator to choose node(s) to be offshore hub(s). Then, the author used several heuristics working with this indicator to assign non-hub nodes to chosen hub.

The work that has been done in this paper could be viewed as a continuation of the previous research done by Halskau (2012). We, in the first place, tested the mathematical model (Halskau 2012) by using AMPL computer programming. In the second place, we tried to find other possible indicators that could be used to select offshore hub(s) instead of using largest demand indicator. We tested these new indicators combining with those heuristics utilized in Halskau (2012) on the same examples we used to test mathematical model. The reason for doing it is that we would like to find an optimal combination of an indicator with a heuristic that could offer a solution that is same or closer to the optimal solution given by the exact mathematical model. Moreover, we would like to test the stability of indicators, which means we tested each indicator under all heuristics by employing two different examples and observed if an indicator under a heuristic could give relatively better value of objective function in both two examples.

3.0 Model test in AMPL and heuristics test

3.1 Mathematic model test

We put the mathematic mode (Halskau 2012) in AMPL programming and test it by using two examples. For example1, cost distance matrix (Aas et al. 2007) is shown in table 1. Some delivery demand and pickup demand of each offshore node are cited from Halskau (2012) (Table 2). We named the example utilized in Halskau (2012) as example2 in this paper (Table 3 and Table 4). The article (Qian et al. 2012) gives the value of π_L and π_c (Table 5) that are needed to calculate the value of objective function in the model.

Table 1 Distance Matrix of Example1

	0	1	2	3	4	5	6	7	8	9	10
0	0	360	360	385	590	590	605	620	620	590	670
1	360	0	0	80	235	240	245	255	260	235	310
2	360	0	0	80	235	240	245	255	260	235	310
3	385	80	80	0	255	250	260	265	270	230	310
4	590	235	235	255	0	5	45	60	60	65	155
5	590	240	240	250	5	0	50	65	75	70	155
6	605	245	245	260	45	50	0	15	15	30	110
7	620	255	255	265	60	65	15	0	10	30	95
8	620	260	260	270	60	75	15	10	0	30	95
9	590	235	235	230	65	70	30	30	30	0	100
10	670	310	310	310	155	155	110	95	95	100	0

Table 2 Delivery Demand and Pickup Demand of Each Node of Example1

	Delivery Demand	Pickup Demand	Total Demand
0	0	0	0
1	2	5	7
2	9	8	17
3	8	7	15
4	8	6	14
5	8	5	13
6	2	7	9
7	5	6	11
8	7	4	11
9	3	7	10
10	5	3	8
Sum	57	58	115

Table 3 Distance Matrix of Example2

	0	1	2	3	4	5	6
0	0	28	40	36	45	67	71
1	28	0	28	41	60	64	91
2	40	28	0	22	45	36	76
3	36	41	22	0	22	32	54
4	45	60	45	22	0	41	32
5	67	64	36	32	41	0	64
6	71	91	76	54	32	64	0

Table 4 Delivery Demand and Pickup Demand of Each Node of Example2

	Delivery Demand	Pickup Demand	Total Demand
0	0	0	
1	2	5	7
2	9	8	17
3	8	7	15
4	8	6	14
5	8	5	13
6	2	7	9
Sum	37	38	75

Table 5 The Value of π_L and π_c

π_L ($\times 10^{-6}$)	π_c($\times 10^{-6}$)
0.65	0.86

The optimal solution of example1 and example2 given by AMPL programming are shown in table 6 and 7, respectively.

Table 6 The Optimal Solution of Example1 in AMPL

Example1	Number of Passenger Landing (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub1 (Customer node 4, 7 and 10)	**191**	**61470**	**52988.3**
Hub 2 (Customer node 5 and 6)			
Hub 3 (Customer node 8 and 9)			

Table 7 The Optimal Solution of Example2 in AMPL

Example2	Number of Passenger Landing (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 2 (Customer node 1and 5)	**118**	**4306**	**3779.86**
Hub 3 (Customer node 4 and 6)			

The reason why the optimal solution of objective function has to time 10^{-6} is that the probability of fatal accidents happened during takeoff and landing period (π_L) is 0.65 per million pairs of takeoff and landings and the probability of cruise accidents (π_c) is 0.86 per million flight hours (Qian et al. 2012).

3.2 Heuristics test with largest demand indicator

The indicator used to choose offshore hub(s) in this section is based on total demand of each offshore node. Offshore node(s) that has largest demand could be selected as offshore hub(s). As example2 had been tested under heuristics with largest demand indicator (Halskau 2012) , we only used example1 to test this indicator under all heuristics done by Halskau (2012) in our paper.

3.2.1 Heuristic 1

Step 1: setting the offshore installation that has the biggest demand as a hub.

Step 2: assigning those adjacent nodes to the hub. The helicopter capacity can constrain the number of customer nodes assigned to the hub.

Step3: choosing another hub by using the same way and then going back to the step 2.

The solution is shown below (Table 8).

Table 8 The Solution of Heuristic1

Example1	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 2 (Customer node 1 and 3)	61	15240	**54233.4**
Hub 4 (Customer node 5 and 6)	58	21710	
Hub 7 (Customer node 8,9 and10)	69	25970	
Sum	**188**	**62920**	

3.2.2 Variety of heuristic 1

First choosing *m* hubs (the number of hubs is the same as the number of helicopter *m*) that are those nodes with the biggest demands. Then non-hub nodes are assigned to these hubs. There are two approaches to assign them.

3.2.2.1 Variant 1

First, we sequence total demand of each offshore hub in decreasing order, then assigning non-hub installations to the first offshore hub on the list, which has the largest demand among all offshore hubs. It does not stop to assign nodes until adding one more node in this tour makes the total pickup demand or total delivery demand excess the capacity of a helicopter. Then it starts to assign spoke nodes to the hub that has the second largest demand on the list. Assignment task is done when all non-hub nodes are assigned. The solution is indicated in table 9

Table 9 The Solution of Variant1 of Heuristic1

Example1	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities $(\times 10^{-6})$
Hub 2 (Customer node 1 and 9)	51	14590	**53731.1**
Hub 3 (Customer node 5 and 6)	59	19835	
Hub 4 (Customer node 7 and 8)	58	22560	
HP 0 (Customer node 10)	8	5360	
Sum	**176**	**62345**	

3.2.2.2 Variant 2

There is no priority to offshore hubs. The first assigned node and the offshore hub that has a customer node first are pair nodes that have shortest distance between them. In short words, the priority is given in assignment process to the customer node that has the shortest distance to any chosen hubs. Capacity of a helicopter also works on limiting the number of customer nodes each hub could have. The solution is indicated in table 10.

Table 10 The Solution of Variant 2 of Heuristic1

Example1	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities $(\times 10^{-6})$
Hub 2 (Customer node 1 and 7)	53	15405	**53094.7**
Hub 3 (Customer node 8 and 9)	57	19130	
Hub 4 (Customer node 5 and 6)	58	21710	
HP 0 (Customer node 10)	8	5360	
Sum	**176**	**61605**	

3.2.3 Heuristic 2

It is a sweep heuristic (Gillett and Miller 1974). Collecting installations by using a line starting from heliport and sweeping clockwise or counter clockwise to collect nodes. It does not stop until adding the next node will violate the capacity constraint. Repeating this collection step until several clusters can cover all nodes. Then we choose the node that has the largest demand as the offshore hub in each cluster. Table 11 shows the solution by using this heuristic.

Table 11 The Solution of Heuristic2

Example1	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 2 (Customer node 1 and 10)	47	14000	**53204.6**
Hub 4 (Customer node 5 and 6)	58	21710	
Hub 7 (Customer node 8 and 9)	53	20250	
HP 0 (Customer node 3)	15	5775	
Sum	**173**	**61735**	

3.2.4 Heuristic 3

It is Fisher-Jaikumars (FJ) heuristics (Fisher and Jaikumar 1981). In helicopter routing problem, offshore hubs are viewed as seed nodes. The same objective, which is to minimize the expected number of fatalities in an accident, is used to select offshore hubs. Hence, m offshore installations with biggest demand are selected as hubs. Then three possible approaches could be used in assignment process.

3.2.4.1 Clark and Wright heuristic (1964)

The objective function is to maximize the saving cost.

(1) Saving cost = $\Delta_{is} = c_{0s} + c_{i0} - c_{si} \; \forall i, s, i = 1,2 \dots n, s = 1,2 \dots m, i \neq s$

Mathematical model:

$$(2) \; max \sum_{i=1}^{n} \sum_{s=1}^{m} \Delta_{is} W_{is}$$

Subject to

$$(3) \sum_{\substack{s=1, \\ s \neq i}}^{m} W_{is} = 1 \quad \forall i, i = 1,2, \dots, n$$

$$(4) \sum_{\substack{i=1, \\ i \neq s}}^{n} d_i W_{is} \leq Q \quad \forall s, s = 1,2, \dots, m$$

$$(5) \sum_{\substack{i=1, \\ i \neq s}}^{n} p_i W_{is} \leq Q \quad \forall s, s = 1,2, \dots, m$$

Parameter:

Δ_{is} = saving cost.

d_i = delivery demand of customer node *i*.

p_i = pickup demand of customer node *i*.

c_{0s}= the distance between heliport 0 and offshore hub *s* or the cost of a helicopter departing from heliport 0 to the offshore hub *s*. ∀*s* =1,2,…,m

c_{i0} = the distance between heliport 0 and customer node *i* or the cost of a helicopter departing from heliport 0 to the customer node *i*. ∀i =1,2,…,n

c_{si} =the distance between offshore hub *s* and the customer node *i* served by offshore hub *s* or the cost a helicopter departs from offshore hub *s* to its customer node *i*. $\forall i, s, i = 1,2 \dots n, s = 1,2 \dots m, i \neq s$

Variable:

$W_{is} = 1$ if offshore hub *s* servers customer node *i*, 0 otherwise, $i \neq s$.

The objective function (2) is to maximize the saving cost. Constraint (3) ensures one customer node only could be assigned to exact one hub. Constraint (4) and (5) means the sum of delivery demand and pickup demand for a hub and its customer nodes could not exceed the capacity of a helicopter, respectively. The solution is in table 12.

Table 12 The Solution of Heuristic 3 with Largest Demand Indicator (Clark and Wright heuristic assigning approach)

Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 2 (Customer node 5 and 6)	61	19365	**56570**
Hub 3 (Customer node 8 and 9)	57	19130	
Hub 4 (Customer node 1,7 and 10)	66	27145	
Sum	**184**	**65640**	

3.2.4.2 Transportation-work-related cost

The objective function is to minimize the total transportation work.

$$\text{Transportation-work-related cost} \doteqdot \Theta_{is} = c_{is}(d_i + p_i) \tag{6}$$

Mathematical model:

$$min \sum_{i=1}^{n} \sum_{s=1}^{m} \Theta_{is} W_{is} \tag{7}$$

Constraints in this situation are same as (3)-(5) shown in the previous situation. Table 13 shows the solution.

Table 13 The Solution of Heuristic 3 with Largest Demand Indicator (Transportation-work-related cost assigning approach)

Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 2 (Customer node 5 and 6)	61	19365	**56570**
Hub 3 (Customer node 8 and 9)	57	19130	
Hub 4 (Customer node 1,7 and 10)	66	27145	
Sum	**184**	**65640**	

3.2.4.3 Cost or Distance

The objective function is to minimize the total distance or cost.

(8) Cost or distance = c_{is}

Mathematical model:

(9)

$$min\sum_{i=1}^{n}\sum_{s=1}^{m}\mathrm{c}_{is}W_{is}$$

In this case, constraints used in previous situations are kept. The result is shown below (Table 14).

Table 14 The Solution of Heuristic 3 with Largest Demand Indicator (Cost or Distance assigning approach)

Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 2 (Customer node 6 and 8)	57	18385	**56630.2**
Hub 3 (Customer node 5 and 9)	61	20180	
Hub 4 (Customer node 1,7 and 10)	66	27145	
Sum	**184**	**65710**	

4.0 Direct routing policy

Direct routing policy means each customer node gets service from heliport directly by a helicopter. It offers the smallest value of objective function, which is to minimize the expected number of fatalities, among all policies. However, the weakness of this policy is that expense is costly. The solution given by the policy is shown below (Table 15).

Table 15 The Solution of Direct Routing Policy

Routing	Number of Passenger Landings (PL)	Transportation Work (PC)
0-1-0	7	2520
0-2-0	17	6120
0-3-0	15	5775
0-4-0	14	8260
0-5-0	13	7670
0-6-0	9	5445
0-7-0	11	6820
0-8-0	11	6820
0-9-0	10	5900
0-10-0	8	5360
Sum	**115**	**60690**
The Expected Number of Fatalities ($* 10^{-6}$)	**52268.15**	

5.0 New indicators of filtering offshore hubs

According to Halskau (2012), the method to choose offshore hubs from all offshore nodes is based on the demand. Those nodes that have the largest demand could be offshore hubs and the number of offshore hubs is the same as the number of helicopters.

In this section, we tried to find several new indicators working for selecting offshore hubs and tested them under all heuristics mentioned above in order to evaluate quality and reliability of combination of a heuristic and an indicator. We still used example1 and example2 to test each new indicator that works with heuristics.

5.1 Heuristics test with shortest distance indicator

The new indicator is based on distance (cost) between the heliport and each offshore node. A node that has shortest distance (lowest cost) to heliport could be chosen as offshore huband the number of offshore hubs is still the same as the number of helicopters. This way may make the expected number of fatalities during the trip from heliport to offshore hubs lower. By using shortest distance indicator, some heuristics mentioned above with largest demand indicator need to be modified.

5.1.1 Heuristic 1

Step 1: setting the offshore installation that has the shortest distance to the heliport as a hub.

Step 2: assigning those adjacent nodes to the hub. The helicopter capacity can constrain the number of customer nodes assigned to the hub.

Step 3: choosing another hub by using the same way and then going back to the step 2.
The solution is shown in table 16.

Table 16 The Solution of Heuristic1 with Shortest Distance Indicator

Example1	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 1 (Customer node 2 and 3)	71	15240	**53457.95**
Hub 4 (Customer node 5 and 6)	58	21710	
Hub 9 (Customer node 7,8 and10)	70	25060	
Sum	**199**	**62010**	
Example2	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 1 (Customer node 2 and 3)	71	2183	**4060.49**
Hub 4 (Customer node 5 and 6)	58	2441	
Sum	**129**	**4624**	

5.1.2 Variety of Heuristic 1

First sequencing distance from the heliport to each offshore node in decreasing order, choosing the first *m* node, which is the same as the number of helicopter, as offshore hubs. Then we assign non-hub nodes to these hubs. There are two approaches to assign them.

5.1.2.1 Variant 1

Assigning adjacent non-hub installations to the first offshore hub on the list, which has the shortest distance to the heliport among all offshore hubs. It does not stop to assign nodes until adding one more node in this tour makes the total pickup demand or total delivery demand excess the capacity of a helicopter. Then it starts to assign spoke nodes to the offshore hub that has the second shortest distance to the heliport. Assignment task is done when all non-hub nodes are assigned (Table 17)

Table 17 The Solution of Variant 1 of Heuristic1 with Shortest Distance Indicator

Example1	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 2 (Customer node 5 and 6)	61	19365	**53142.25**
Hub 3 (Customer node 8 and 7)	59	20130	
Hub 1 (Customer node 4 and 9)	55	16800	
HP 0 (Customer node 10)	8	5360	
Sum	**183**	**61655**	
Example2	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 1 (Customer node 2 and 4)	69	2380	**4051.24**
Hub 3 (Customer node 5 and 6)	59	2234	
Sum	**128**	**4614**	

5.1.2.2 Variant 2

There is no priority to offshore hubs. We give the priority in assignment process to the customer node that has the shortest distance to any chosen hubs. Capacity of a helicopter also works on limiting the number of customer nodes each hub could have (Table 18).

Table 18 The Solution of Variant 2 of Heuristic1 with Shortest Distance Indicator

Example1	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities $(\times 10^{-6})$
Hub 2 (Customer node 4)	45	14450	**52981.2**
Hub 3 (Customer node 9 and 7)	57	19075	
Hub 1 (Customer node 5, 6 and 10)	67	21125	
HP 0 (Customer node 8)	11	6820	
Sum	**180**	**61470**	
Example2	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities $(\times 10^{-6})$
Hub 1 (Customer node 2 and 5)	67	2344	**3958.36**
Hub 3 (Customer node 4 and 6)	61	2162	
Sum	**128**	**4506**	

5.1.3 Heuristic 2

It is a sweep heuristic. Collecting installations by using a line starting from heliport and sweeping clockwise or count clockwise to collect nodes. It does not stop until adding the next node will violate the capacity constraint. Repeating this collection step until several clusters can cover all nodes. Then choosing the node that has the shortest distance to heliport as the offshore hub in each cluster (Table 19)

Table 19 The Solution of Heuristic 2 with Shortest Distance Indicator

Example1	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 2 (Customer node 1 and 10)	47	14000	**52594.6**
Hub 4 (Customer node 5 and 6)	58	21710	
Hub 9 (Customer node 7 and 8)	54	19540	
HP 0 (Customer node 3)	15	5775	
Sum	**174**	**61025**	
Example2	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 1 (Customer node 2 and 5)	67	2344	**3958.36**
Hub 3 (Customer node 4 and 6)	61	2162	
Sum	**128**	**4506**	

5.1.4 Heuristic 3

It is Fisher-Jaikumars (FJ) heuristics. In helicopter routing problem, offshore hubs are viewed as seed nodes. The same objective, which is to minimize the expected number of fatalities in an accident, is used to select offshore hubs. Hence, m offshore installations with shortest distance to heliport are selected as hubs. Then three possible approaches could be used in assignment process.

5.1.4.1 Clark and Wright heuristic

The objective function is to maximize the saving cost.

(1) Saving cost = $\Delta_{is} = c_{0s} + c_{i0} - c_{si}\ \forall i, s, i = 1,2 \dots n, s = 1,2 \dots m, i \neq s$

Mathematical model:

$$(2)\ max \sum_{i=1}^{n} \sum_{s=1}^{m} \Delta_{is} W_{is}$$

Subject to

$$(3) \sum_{\substack{s=1, \\ s \neq i}}^{m} W_{is} = 1 \quad \forall i, i = 1,2, \dots, n$$

$$(4) \sum_{\substack{i=1, \\ i \neq s}}^{n} d_i W_{is} \leq Q \quad \forall s, s = 1,2, \dots, m$$

$$(5) \sum_{\substack{i=1, \\ i \neq s}}^{n} p_i W_{is} \leq Q \quad \forall s, s = 1,2, \dots, m$$

Parameter:

Δ_{is} = saving cost.

d_i = delivery demand of customer node *i*.

p_i = pickup demand of customer node *i*.

c_{0s}= the distance between heliport 0 and offshore hub s or the cost of a helicopter departing from heliport 0 to the offshore hub s. $\forall s$ =1,2,…,m

c_{i0} = the distance between heliport 0 and customer node i or the cost of a helicopter departing from heliport 0 to the customer node i. $\forall i$ =1,2,…,n

c_{si} =the distance between offshore hub s and the customer node i served by offshore hub s or the cost a helicopter departs from offshore hub s to its customer node i. $\forall i, s, i = 1,2 \dots n, s = 1,2 \dots m, i \neq s$

Variable:

$W_{is} = 1$ if offshore hub s servers customer node i, 0 otherwise, $i \neq s$.

The objective function (2) is to maximize the saving cost. Constraint (3) ensures one customer node only could be assigned to exact one hub. Constraint (4) and (5) means the sum of delivery demand and pickup demand for a hub and its customer nodes could not exceed the capacity of a helicopter, respectively. Table 20 shows the result by using this way to do assignment of non-hub nodes.

Table 20 The Solution of Heuristic3 with Shortest Distance Indicator (Clark and Wright heuristic assigning approach)

Example1	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 1 (Customer node 4,7 and 10)	73	22975	**53048.55**
Hub 2 (Customer node 6 and 8)	57	18385	
Hub 3 (Customer node 5 and 9)	61	20180	
Sum	**191**	**61540**	
Example2	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 1 (Customer node 2 and 5)	67	2344	**3958.36**
Hub 3 (Customer node 4 and 6)	61	2162	
Sum	**128**	**4506**	

5.1.4.2 Transportation-work-related cost

The objective function is to minimize the total transportation work.

(6) Transportation-work-related cost ÷ $\Theta_{is} = c_{is}(d_i + p_i)$

Mathematical model:

$$(7)\ min \sum_{i=1}^{n} \sum_{s=1}^{m} \Theta_{is} W_{is}$$

Constraints in this situation are same as (3)-(5) shown in the previous situation (Table 21).

Table 21 The Solution of Heuristic3 with Shortest Distance Indicator (Transportation-work-related cost assigning approach)

Example1	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 2 (Customer node 5 and 6)	61	19365	**52988.35**
Hub 3 (Customer node 8 and 9)	57	19130	
Hub1 (Customer node 4,7 and 10)	73	22975	
Sum	**191**	**61470**	
Example2	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 1 (Customer node 2 and 5)	67	2344	**3958.36**
Hub 3 (Customer node 4 and 6)	61	2162	
Sum	**128**	**4506**	

5.1.4.3 Cost or Distance

The objective function is to minimize the total distance or cost.

(8) Cost or distance = c_{is}

Mathematical model:

(9)

$$min \sum_{i=1}^{n} \sum_{s=1}^{m} \mathrm{c}_{is} W_{is}$$

In this case, constraints used in previous situations are kept (Table 22).

Table 22 The Solution of Heuristic3 with Shortest Distance Indicator (Cost or Distance assigning approach)

Example1	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 1 (Customer node 4,7 and 10)	73	22975	**53048.55**
Hub 2 (Customer node 6 and 8)	57	18385	
Hub 3 (Customer node 5 and 9)	61	20180	
Sum	**191**	**61540**	
Example2	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 1 (Customer node 2 and 5)	67	2344	**3958.36**
Hub 3 (Customer node 4 and 6)	61	2162	
Sum	**128**	**4506**	

5.2 Improvement of solutions given by some heuristics

After testing largest demand indicator and shortest distance indicator under all heuristics, we found some solutions of example1 are infeasible. Each of infeasible solutions has four clusters. The characteristic of this kind of solution is that only one of four clusters has a single node that has to be served by heliport directly. We named this type of cluster as single cluster and the node in single cluster is called single node. This kind of solution requires four helicopters generally. However, in example1, the number of helicopter is assumed as three. Hence, the single node has to be integrated into one or more other clusters in the same solution, thereby forming three clusters.

Three heuristics generating this kind of solution are variant1 of heuristic1 (H1v1), variant2 of heuristic2 (H1v2) and heuristic2 (H2). The table 23 below shows the original solutions, which has four clusters, given by each of three heuristics working with two different indicators.

Table 23 Solutions of Three Heuristics in Example1 with Two Sorts of Indicator

Example1	Largest Demand Indicator	Shortest Distance Indicator
H1v1	1. HP 0 (single customer node 10) 2. Hub2 (customer node1 and 9) 3. Hub3 (customer 5 and 6) 4. Hub4 (customer 7 and 8)	1. HP 0 (single customer node 10) 2. Hub2 (customer node 5 and 6) 3. Hub3 (customer 8 and 7) 4. Hub1 (customer 4 and 9)
H1v2	1. HP 0 (single customer node 10) 2. Hub2 (customer node 1 and 7) 3. Hub3 (customer 8 and 9) 4. Hub4 (customer 5 and 6)	1. HP 0 (single customer node 8) 2. Hub2 (customer node 4) 3. Hub3 (customer 9 and 7) 4. Hub1 (customer 5,6 and 10)

5.2.1 The method of integrating a single node into other clusters

The prerequisite of integrating a single node into other clusters is each offshore node could be visited more than once.

The first step is calculation of total delivery and pickup demand of each cluster. The second step is to sequence other three clusters based on decreasing order of total demand of each hub. It decides which cluster would get some demand from the single node first. The capacity of a helicopter limits the type of demand (pickup or delivery) as well as the number of each type of demand of the single node that can be assigned to cluster(s).

The solution given by H2 with largest demand indicator is used to show the process of integrating a single node into other clusters. The table below indicates the total delivery and pickup demand of each cluster (Table 24).

Table 24 Heuristic 2 with Largest Demand Indicator in Example1: Total Demand of Each Cluster

H2 with largest demand indicator	(Delivery Demand, Pickup Demand)
Cluster 1. HP 0 (single customer node 3)	(8,7)
Cluster 2. Hub2 (Customer node 1 and 10)	(16,16)
Cluster 3. Hub4 (Customer 5 and 6)	(18,18)
Cluster 4. Hub7 (Customer 8 and 9)	(15,17)

Sequencing total demand of each hub into decreasing order (Table 25).

Table 25 Decreasing Sequence of Total Demand of Each Hub

	Demand
Hub 2	17
Hub 4	14
Hub 7	11

Hence, the priority of assigning node3 to clusters follows the order: cluster (2,1,10), cluster (4,5,6) and cluster (7,8,9).

The capacity of a helicopter is 20. Hence, the integrated solution is shown in the table 26.

Table 26 The Integration Solution

	(Delivery Demand, Pickup Demand)
1. HP 0 (Single customer node 3)	(8,7)-(4,4)-(2,2)-(2,1)
2. Hub2 (Customer node 1 and 10) + node3 (4,4)	(16,16) + (4,4) = (20,20)
3. Hub4 (Customer 5 and 6) + node3 (2,2)	(18,18) +(2,2) = (20,20)
4. Hub7 (Customer 8 and 9) + node3 (2,1)	(15,17) +(2,1) = (17,18)

5.2.2 Method of serving single node in integration solution

After assignment of the single node, we need to decide how to serve the single node within a cluster that shares it. There are several situations.

5.2.2.1 A cluster shares both delivery and pickup demand of a single node

The single node would be served by offshore hub of the cluster directly.

5.2.2.2 A cluster shares some or all delivery demand of a single node only. Or some or all pickup demand of a single node only is assigned to a cluster.

There are two ways to serve the shared single node:

The first approach is the shared single node could be served by offshore hub of the cluster directly. The second approach is a customer node that is closest to the shared single node within this cluster offer service.

We need to compare the number of passenger landings (PL) and transportation work (PC) value given by the two approaches in order to find better approach. In the second approach, there is a problem related to service sequence in a cluster. Hence, we compare PL and PC value generated by the first approach and each possible visiting sequence in the second approach. We set:

A=offshore hub;
B= the customer node in the cluster that is closest to the single node;
C= the single node;
d= delivery demand;
p= pickup demand.

Only some or all of delivery demand of the single node is assigned to a cluster. The routing from the first approach is:

A-C (d)-A**;**

The possible visiting sequence in the second approach could be:

A-B(d)-C(d)-B(p)-A;
A-C(d)-B(d)-B(p)-A;
A-B(d)-B(p)-C(d)-A

Only some or all of pickup demand of the single node is assigned to a cluster. The routing from the first approach is:

A-C(p)-A**;**

The possible routings given by the second approach are:

A-B(d)-C(p)-B(p)-A;
A-C(p)-B(d)-B(p)-A;
A-B(d)-B(p)-C(p)-A

Conclusion 1: The offshore hub of a cluster would serve the shared single node in the cluster directly.

Based on the calculations in example1, the first approach gives the optimal PL and PC value in both two discussed situations above.

Here we only show two examples that could support the conclusion1. In the solution given by H1v2 with shortest distance indicator, the single node is node 8. We decided to assign four of delivery demand of node 8 to cluster (3, 9, 7) where node 3 is offshore hub (Table 27).

Table 27 Integration Solution of Cluster (3, 9, 7)

	Delivery Demand of A Cluster	Pickup Demand of A Cluster
Single node 8	7	4
Hub3 (Customer node 9 and 7)	16	20
Capacity of a helicopter	20	
Integrated solution for this cluster: Hub3 (Customer node 9,7 and 8 (4,0))		

In this cluster, the customer node 7 is closest to node 8. Hence, there are four ways to serve single node 8 (Table 28).

Table 28 Four Possible Ways of Serving Single Node 8

	PL	PC	
3-9/3-7/3-8(4,0)	65	21695	Best
3-9/3-7(d)-8(d)-7(p)-3	69	21715	
3-9/3-8(d)-7(d)-7(p)-3	70	21770	
3-9/3-7(d)-7(p)-8(d)-3	75	21805	

In the solution given by H1v1 with largest demand indicator, the single node is node 10. We decided to assign two pickup demand of node 10 to cluster (4, 8, 7) where node 4 is the offshore hub (Table 29).

Table 29 Integration Solution of Cluster (4, 8, 7)

	Delivery Demand of A Cluster	Pickup Demand of A Cluster
Single node 10	5	3
Hub4 (Customer node 8 and 7)	20	16
Capacity of a helicopter	20	
Integrated solution for the cluster: Hub4: Customer node 8,7 and 10 (0,2)		

In this cluster, the customer node8 is closest to node10. Hence, there are four ways to serve single node10 (Table 30).

Table 30 Four Possible Ways of Serving Single Node 10

	PL	PC	
4-7/4-8/4-10(0,2)	62	24050	Best
4-7/4-8(d)-10(p)-8(p)-4	64	24050	
4-7/4-10(p)-8(d)-8(p)-4	71	25380	
4-7/4-8(d)-8(p)-10(p)-4	66	24810	

Returning to the solution given by H2 with largest demand indicator, according to the conclusion1, node3 in each cluster is served by the offshore hub directly. The final integrated solution of H2 is shown below (Table 31).

Table 31 The Integration Solution of Heuristic2 with Largest Demand Indicator in Example1

	PL	PC
Hub 2: node 1, 10 and 3(4,4)	63	17520
Hub 4: node 5, 6 and 3(2,2)	66	25090
Hub 7: node 8, 9 and 3(2,1)	59	22905
Sum	188	65515
The Expected Number of Fatalities($\times 10^{-6}$)	56465.1	

If we assigned node3 to cluster (2,1,10) first and let the total delivery and pickup demand meet the capacity of a helicopter respectively. Then we assigned all left delivery demand and pickup demand of node3 to cluster (7, 8, 9) because it does not make total delivery and pickup demand of the cluster violate the capacity of a helicopter. The solution is shown below (Table 32).

Table 32 Integration Solution with an assignment approach that is based on largest left capacity of each cluster

	PL	PC
Hub 2: node 1, 10 and 3(4,4)	63	17520
Hub 7: node 8, 9 and 3(4,3)	67	26445
Hub 4: node 5 and 6	58	21710
Sum	188	65675
The Expected Number of Fatalities($\times 10^{-6}$)	56602.7	

The table below shows the comparison of PL, PC and the expected number of fatalities given by two solutions of assigning single node3 (Table 33).

Table 33 The Comparison of Two Integration Solutions

Solution 1	PL	PC	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 2: node 1, 10 and 3(4,4)	188	65515	56465.1
Hub 4: node 5, 6 and 3(2,2)			
Hub 7: node 8, 9 and 3(2,1)			
Solution 2	PL	PC	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub 2: node 1, 10 and 3(4,4)	188	65675	56602.7
Hub 7: node 8, 9 and 3(4,3)			
Hub 4: node 5 and 6			

Conclusion 2: Splitting single node's delivery demand and pickup demand into many fractions may generate less transportation work and takes no influence on the PL value. In other words, the increase of visiting time to a single node is possible to decrease the transportation work and does not increase the number of passenger landing if the sequence of assigning a single node is based on the decreasing order of total demand of hubs in a solution.

The two tables show both the original solution and integrated solution given by largest demand indicator and shortest distance under all heuristics respectively (Table 34 and Table 35).

Table 34 Original and Integrated Solution under All Heuristics with Largest Demand Indicator in Example1

Example1	**Original Solution (Largest Demand)**	**Objective Function**	**Integrated Solution (Largest Demand)**	**Objective Function ($\times 10^{-6}$)**
H1	Hub2 (1,3) Hub4 (5,6) Hub7 (8,9,10)	54233.4	X	X
H1v1	HP 0 (10) Hub2 (1,9) Hub3 (5,6) Hub4 (7,8)	53731.1	Hub2 (1,9 10 (5,0)) Hub3 (5,6,10 (0,1)) Hub4 (7,8,10(0,2))	53886.8
H1v2	HP 0 (10) Hub2 (1,7) Hub3 (8,9) Hub4 (5,6)	53094.7	Hub2 (1,7,10 (4,1)) Hub3 (8,9,10 (1,2)) Hub 4 (5,6)	53164.4
H2	HP 0 (3) Hub2 (1,10) Hub4 (5,6) Hub7 (8,9)	53204.5	Hub2 (1,10,3(4,4)) Hub4 (5,6,3 (2,2)) Hub7 (8,9,3(2,1))	56465.1
H3v1	Hub2 (5,6) Hub3 (8,9) Hub4 (1,7,10)	56570	X	X
H3v2	Hub2 (5,6) Hub3 (8,9) Hub4 (1,7,10)	56570	X	X
H3v3	Hub2 (6,8) Hub3 (5,9) Hub4 (1,7,10)	56630.2	X	X

Table 35 Original and Integrated Solution under All Heuristics with Shortest Distance Indicator in Example1

Example1	**Original Solution (Shortest Distance)**	**Objective Function**	**Integrated Solution (Shortest Distance)**	**Objective Function ($\times \mathbf{10^{-6}}$)**
H1	Hub1 (2,3) Hub4 (5,6) Hub9 (7,8,10)	53457.95	X	X
H1v1	HP 0 (10) Hub2 (5,6) Hub3 (8,7) Hub1 (4,9)	53142.25	Hub2 (5,6,10(1,0)) Hub3 (8,7,10(0,3)) Hub1 (4,9,10(4,0))	53211.95
H1v2	HP 0 (8) Hub2 (4) Hub3 (9,7) Hub1 (5,6,10)	52981.2	Hub2 (4,8(3,4)) Hub3 (9,7,8(4,0)) Hub1 (5,6,10)	53108.75
H2	HP 0 (3) Hub2 (1,10) Hub4 (5,6) Hub9 (7,8)	52594.6	Hub2 (1,10,3(4,4)) Hub4 (5,6,3(2,2)) Hub9 (7,8,3(2,1))	55687.45
H3v1	Hub1 (4,7,10) Hub2 (6,8) Hub3 (5,9)	53048.55	X	X
H3v2	Hub2 (5,6) Hub3 (8,9) Hub1 (4,7,10)	52988.35	X	X
H3v3	Hub1 (4,7,10) Hub2 (6,8) Hub3 (5,9)	53048.55	X	X

Although the integrated solutions increase the value of objective function a little bit, three helicopters now could perform those routings.

In order to find which heuristic works better in example, we need to use integrated solutions instead of three original solutions given by H1v1, H1v2 and H2 in final solutions (Table 36).

Table 36 Final Feasible Solutions in Example1 under All Heuristics with Largest Demand and Shortest Distance Indicators respectively

Example1	**Final Feasible Solution (Largest Demand)**	**Objective Function**	**Final Feasible Solution (Shortest Distance)**	**Objective Function $(* 10^{-6})$**
H1	Hub2 (1,3) Hub4 (5,6) Hub7 (8,9,10)	54233.4	Hub1 (2,3) Hub4 (5,6) Hub9 (7,8,10)	53457.95
H1v1	Hub2 (1,9 10 (5,0)) Hub3 (5,6,10 (0,1)) Hub4 (7,8,10(0,2))	53886.8	Hub2 (5,6,10(1,0)) Hub3 (8,7,10(0,3)) Hub1 (4,9,10(4,0))	53211.95
H1v2	Hub2 (1,7,10 (4,1)) Hub3 (8,9,10 (1,2)) Hub 4 (5,6)	53164.4	Hub2 (4,8(3,4)) Hub3 (9,7,8(4,0)) Hub1 (5,6,10)	53108.75
H2	Hub2 (1,10,3(4,4)) Hub4 (5,6,3 (2,2)) Hub7 (8,9,3(2,1))	56465.1	Hub2 (1,10,3(4,4)) Hub4 (5,6,3(2,2)) Hub9 (7,8,3(2,1))	55687.45
H3v1	Hub2 (5,6) Hub3 (8,9) Hub4 (1,7,10)	56570	Hub1 (4,7,10) Hub2 (6,8) Hub3 (5,9)	53048.55
H3v2	Hub2 (5,6) Hub3 (8,9) Hub4 (1,7,10)	56570	Hub2 (5,6) Hub3 (8,9) Hub1 (4,7,10)	**52988.35**
H3v3	Hub2 (6,8) Hub3 (5,9) Hub4 (1,7,10)	56630.2	Hub1 (4,7,10) Hub2 (6,8) Hub3 (5,9)	53048.55

Currently, H3v2 with shortest distance indicator shows the solution that is the same as the one given by AMPL programming. On the other hand, it gives bad value of objective function when it works with largest demand indicator.

5.3 Center node Indicator

In this section, we introduce center node indicator to choose offshore hubs from all offshore nodes. How it works is to choose one or more nodes that have the smallest sum of distance from it to all other nodes, including the heliport. The number of chosen offshore hub is the same as the number of helicopter. In general, the procedure of using this indicator to choose offshore hubs is:

Step 1: To calculate a row sum of each node that means it sums up the distance from one node to all other nodes, which include the heliport.

Step 2: Pick out one or more nodes that have smallest row sum as offshore hubs. If there are two or more offshore nodes having the same row sum, the one that has the largest demand would be the offshore hub in this paper.

Table 37 and Table 38 show the row sum of each node in example1 and example2, respectively.

Table 37 Row Sum of Each Node in Example1

Example1:

	0	1	2	3	4	5	6	7	8	9	10	Row Sum
0	0	360	360	385	590	590	605	620	620	590	670	5390
1	360	0	0	80	235	240	245	255	260	235	310	2220
2	360	0	0	80	235	240	245	255	260	235	310	2220
3	385	80	80	0	255	250	260	265	270	230	310	2385
4	590	235	235	255	0	5	45	60	60	65	155	1705
5	590	240	240	250	5	0	50	65	75	70	155	1740
6	605	245	245	260	45	50	0	15	15	30	110	1620
7	620	255	255	265	60	65	15	0	10	30	95	1670
8	620	260	260	270	60	75	15	10	0	30	95	1695
9	590	235	235	230	65	70	30	30	30	0	100	1615
10	670	310	310	310	155	155	110	95	95	100	0	2310

Table 38 Row Sum of Each Node in Example2

Example2:

	0	1	2	3	4	5	6	Row Sum
0	0	28	40	36	45	67	71	287
1	28	0	28	41	60	64	91	312
2	40	28	0	22	45	36	76	247
3	36	41	22	0	22	32	54	207
4	45	60	45	22	0	41	32	245
5	67	64	36	32	41	0	64	304
6	71	91	76	54	32	64	0	388

By using this indicator, all heuristics mentioned above need to be modified.

5.3.1 Heuristic 1

Step 1: setting the offshore installation that has the smallest row sum as a hub.

Step 2: assigning those adjacent nodes to the hub. The helicopter capacity can constrain the number of customer nodes assigned to the hub.

Step3: choosing another hub by using the same way and then going back to the step 2.
The solution of example1 is shown in table 39.

Table 39 The Solution of Heuristic 1 with Center Node Indicator in Example1

Example1 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub9 (8,7,10)	70	25060	**54615.7**
Hub6 (4,5)	63	23060	
Hub2 (1,3)	61	15240	
Sum	**194**	**63360**	

Conclusion 3: For two nodes that have the same row sum, the one that has largest total demand could be chosen as an offshore hub when we use center node indicator.

This method gives a smaller expected number of fatalities. The reason is that we choose the node that has larger total demand as a hub may leads decrease of the number of passenger landing.

In example1 under heuristic1, node 1 and node 2 have the same row sum. But node 2 has larger total demand (17) than node1 (7). Hence, the node 2 is offshore hub. The number of passenger landing is less than the number when node 1 is qualified as a hub (Table 40).

Table 40 The Comparison Based on The Number of Passenger Landings

Hub2(1,3)	PL: 61
Hub1(2,3)	PL: 71

The solution of example2 under this heuristic is indicated in table 41.

Table 41 The Solution of Heuristic 1 with Center Node Indicator in Example2

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub3(2,1)	63	2065	**3953.81**
Hub4(6,5)	58	2441	
Sum	**121**	**4506**	

Under all heuristics with center node indicator, during the process of assigning non-hub nodes to a chosen hub, if two or more non-hub nodes have the same distance to the hub, in this paper we decided that the node that has larger total demand would be assigned to the hub first.

5.3.2 Variety of Heuristic 1

First we sequence row sum of each node in increasing order, choosing the first *m* nodes, which is the same number of helicopters, as offshore hubs.

5.3.2.1 Variant 1

Assigning adjacent non-hub installations to the first offshore hub on the list that has the smallest row sum among all offshore nodes. It does not stop to assign nodes until adding one more node in this tour makes the total pickup demand or total delivery demand excess the capacity of a helicopter. Then it starts to assign spoke nodes to the offshore hub that has the second smallest row sum on the list. Assignment task is done when all non-hub nodes are assigned. The original solution and integrated solution of example1 are shown in table 42 and table 43, respectively. The solution of example2 is indicated in table 44.

Table 42 The Original Solution of Example1 under Variant1 of Heuristic1 with Center Node Indicator

Example1 Offshore Hub
Hub9 (8,4)
Hub6 (5,10,1)
Hub7 (2)
HP 0 (3)

Table 43 The Integrated Solution of Example1 under Variant1 of Heuristic1 with Center Node Indicator

Example1 Offshore Hub (Integrated Solution)	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub7 (2,3(6,6))	69	32315	**70398.88**
Hub9 (8,4,3(2,1))	66	24350	
Hub6 (5,10,1)	65	25043	
Sum	**200**	**81708**	

Table 44 The Original Solution of Example2 under Variant1 of Heuristic1 with Center Node Indicator

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub3 (2,1)	63	2065	**3953.81**
Hub4 (6,5)	58	2441	
Sum	**121**	**4506**	

5.3.2.2 Variant 2

There is no priority to offshore hubs. We give the priority in assignment process to the customer node that has the shortest distance to any chosen hubs. Capacity of a helicopter also works on limiting the number of customer nodes each hub could have. The original solution and integrated solution of example1 are shown in table 45 and 46, respectively. The solution of example2 is indicated in table 47.

Table 45 The Original Solution of Example1 under Variant2 of Heuristic1 with Center Node Indicator

Example1 Offshore Hub
Hub9 (3,1)
Hub6 (4,5)
Hub7 (8,10)
HP 0 (2)

Table 46 The Integrated Solution of Example1 under Variant2 of Heuristic1 with Center Node Indicator

Example1 Offshore Hub (Integrated Solution)	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities($\times 10^{-6}$)
Hub 7(8,10,2(3,7))	69	28220	**69815.8**
Hub 9(3,1,2(6,1))	68	29750	
Hub 6(4,5)	63	23060	
Sum	**200**	**81030**	

Table 47 The Original Solution of Example2 under Variant2 of Heuristic1 with Center Node Indicator

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub3 (2,1)	63	2065	**3953.81**
Hub4 (5,6)	58	2441	
Sum	**121**	**4506**	

5.3.3 Heuristic 2

It is a sweep heuristic. Collecting installations by using a line starting from HP and sweeping clockwise or counter clockwise to collect nodes. It does not stop until adding the next node will violate the capacity constraint. Repeating this collection step until several clusters can cover all nodes. Then choosing the node that has the smallest row sum as the offshore hub in each cluster. The original solution and integrated solution of example1 are shown in table 48 and 49, respectively.

Table 48 The Original Solution of Example1 under Heuristic2 with Center Node Indicator

Example1 Offshore Hub
HP 0 (3)
Hub9 (7,8)
Hub6 (4,5)
Hub2 (1,10)

Table 49 The Integrated Solution of Example1 under Heuristic2 with Center Node Indicator

Example1 Offshore Hub (Sequence of assigning single node is based on largest total demand of hubs)	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,10,3(4,4))	63	17520	**56765.7**
Hub9 (7,8,3(4,3))	68	25280	
Hub6 (4,5)	63	23060	
Sum	**194**	**65860**	

The sequence of integrating a single node into other clusters under all heuristics with center node indicator is based on the decreasing order of total demand of each hub instead of the increasing order of row sum of each hub.

In example1 under heuristic2, the increasing order of row sum of hub is hub9, hub6 and hub2. If we assign single node 3 in this sequence, the result is shown below (Table 50).

Table 50 Integration Solution using the way based on the smallest-row-sum sequence to assign the single node

Example1 Offshore Hub (Sequence of assigning single node is based on central-hub sequence)	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub9 (7,8,3(5,3))	70	26100	**58554.5**
Hub6 (4,5,3(2,2))	71	26520	
Hub2 (1,10,3(1,2))	53	15320	
Sum	**194**	**67940**	

Compared with table 50, the way of assigning single node based on largest total demand of hub could give a smaller value of objective function, although the indicator to choose hub is center node indicator.

The solution of example2 under heuristic2 with center node indicator is shown in table 51.

Table 51 The Solution under Heuristic2 with Center Node Indicator in Example2

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,5)	73	3463	**4924.6**
Hub3 (4,6)	61	2162	
Sum	**134**	**5625**	

5.3.4 Heuristic 3

It is Fisher-Jaikumars (FJ) heuristics. In helicopter routing problem, offshore hubs are viewed as seed nodes. The same objective, which is to minimize the expected number of fatalities in an accident, is used to select offshore hubs. Hence, m offshore installations with smallest row sum are selected as hubs. Then three possible approaches, which are Clark and Wright heuristic, transportation-work-related cost and distance or cost approach, could be used in assignment process. Solutions of example1 and example2 under transportation-work-related cost and distance or cost assigning approach are same as ones generated by Clark and Wright heuristic approach, respectively. Solutions of two examples corresponding to each approach are shown in table 52 and table 53.

Table 52 The Solution of example1 under Heuristic 3 with Center Node Indicator

(Clark and Wright heuristic, transportation-work-related cost and distance or cost assigning approach)

Example1 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub6 (2,5)	69	28410	**70112.5**
Hub7 (1,8,10)	63	25595	
Hub9 (3,4)	68	27370	
Sum	**200**	**81375**	

Table 53 The Solution of Example2 under Heuristic 3 with Center Node Indicator

(Clark and Wright heuristic, transportation-work-related cost and distance or cost assigning approach)

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub3 (2,1)	63	2065	**3953.81**
Hub4 (5,6)	58	2441	
Sum	**121**	**4506**	

5.4 $\alpha = \frac{d_i + p_i}{c_{0i}}$ Indicator

We choose m nodes with the biggest ratio of total demand of the node to the distance between it and the heliport as offshore hubs. If a node has relatively larger total demand and relatively shorter distance to the heliport, the value of indicator would be maximized. It means it is a possible option to be an offshore hub, because the number of passenger landing can be minimized as the hub has the largest demand within a cluster. The expected number of fatalities in the first pair of takeoff and landing could be minimized likewise as the distance from the hub to the heliport is shorter. In this paper, we set the indicator as $\alpha = \frac{d_i + p_i}{c_{0i}}$.

Step1: Calculate the total demand of each offshore node.

Step2: Let total demand of each node divided by the distance from it to the heliport.

Step3: Sequence all value got from step 2 in decreasing orders (Table 54 and Table 55).

Table 54 The Sequence of Node of Example1 by Using Indicator α

Example1 Node	Total Demand $(d_i + p_i)$	Distance (c_{0i})	Indicator $\alpha = \frac{d_i+p_i}{c_{0i}}$	Sequence of Node Based on Decreasing Order of Indicator α
1	7	360	0.019444444	Node 2
2	17	360	0.047222222	Node 3
3	15	385	0.038961039	Node 4
4	14	590	0.023728814	Node 5
5	13	590	0.022033898	Node 1
6	9	605	0.014876033	Node 7
7	11	620	0.017741935	Node 8
8	11	620	0.017741935	Node 9
9	10	590	0.016949153	Node 6
10	8	670	0.011940299	Node 10

Table 55 The Sequence of Node of Example2 by Using Indicator α

Example2 Node	Total Demand $(\mathrm{d_i} + \mathrm{p_i})$	Distance $(\mathrm{c_{0i}})$	Indicator $\alpha = \frac{\mathrm{d_i+p_i}}{\mathrm{c_{0i}}}$	Sequence of Node Based on Decreasing Order of Indicator α
1	7	28	0.25	Node 2
2	17	40	0.425	Node 3
3	15	36	0.416666667	Node 4
4	14	45	0.311111111	Node 1
5	13	67	0.194029851	Node 5
6	9	71	0.126760563	Node 6

All heuristics mentioned above need to be modified as the new indicator α is utilized to filter offshore hubs from all nodes.

5.4.1 Heuristic 1

Step 1: setting an offshore installation that has the largest α as a hub.

Step 2: assigning those adjacent nodes to the hub. The helicopter capacity can constrain the number of customer nodes assigned to the hub.

Step3: choosing another hub by using the same way and then going back to the step 2.

The solution of example1 and example2 is shown in table 56 and table 57, respectively.

Table 56 The Solution of Example1 under Heuristic1 with indicator α

Example1 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,3)	61	15240	**54233.4**
Hub4 (5,6)	58	21710	
Hub7 (8,9,10)	69	25970	
Sum	**188**	**62920**	

Table 57 The Solution of Example2 under Heuristic1 with indicator α

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities($\times 10^{-6}$)
Hub2 (3,1)	61	2086	**3970.57**
Hub4 (5,6)	58	2441	
Sum	**119**	**4527**	

5.4.2 Variety of Heuristic 1

First sequencing indicator α in decreasing order, choosing the first m nodes, which is the same number of helicopter, as offshore hubs.

5.4.2.1 Variant 1

Assigning adjacent non-hub installations to the first offshore hub that has the largest α among all offshore nodes. It does not stop to assign nodes until adding one more node in this tour makes the total pickup demand or total delivery demand excess the capacity of a helicopter. Then it starts to assign spoke nodes to the offshore hub that has the second biggest α. Assignment task is done when all non-hub nodes are assigned. The original solution of example1 is shown in table 58 and the integrated solution is shown in table 59 and example2 is indicated in table 60.

Table 58 The Original Solution of Example1 under Variant1 of Heuristic1 with Indicator α

Example1 Offshore Hub
Hub2 (1,9)
Hub3 (5,6)
Hub4 (7,8)
HP 0 (10)

Table 59 The Integrated Solution of Example1 under Variant1 of Heuristic1 with Indicator α

Example1 Offshore Hub (Sequence of assigning single node is based on largest total demand of hubs)	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,9,10(5,0))	61	17940	**53886.8**
Hub3 (5,6,10(0,1))	61	20530	
Hub4 (7,8,10(0,2))	62	24050	
Sum	184	62520	

Table 60 The Solution of Example2 under Variant1 of Heuristic1 with Indicatorα

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,5)	57	2144	**3779.86**
Hub3 (4,6)	61	2162	
Sum	**118**	**4306**	

5.4.2.2 Variant 2

There is no priority to all offshore hubs. The priority in assignment process is given to the customer node that has the shortest distance to any chosen hubs. Capacity of a helicopter also works on limiting the number of customer nodes each hub could have. The original solution of example1 is shown in table 61 and the integrated solution is shown in table 62. Table 63 shows the solution of example2.

Table 61 The Original Solution of Example1 under Variant2 of Heuristic1 with Indicator α

Example1 Offshore Hub
Hub2 (1,7)
Hub3 (9,8)
Hub4 (5,6)
HP 0 (10)

Table 62 The Integrated Solution of Example1 under Variant2 of Heuristic1 with Indicator α

Example1 Offshore Hub (Sequence of assigning single node is based on largest total demand of hubs)	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,7,10(4,1))	63	18755	**53164.4**
Hub3 (9,8,10(1,2))	63	21215	
Hub4 (5,6)	58	21710	
Sum	**184**	**61680**	

Table 63 The Solution of Example2 under Variant2 of Heuristic1 with Indicatorα

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,5)	57	2144	**3779.86**
Hub3 (4,6)	61	2162	
Sum	**118**	**4306**	

5.4.3 Heuristic 2

It is a sweep heuristic. Collecting installations by using a line starting from HP and sweeping clockwise or count clockwise to collect nodes. It does not stop until adding the next node will violate the capacity constraint. Repeating this collection step until several clusters can cover all nodes. Then choosing the node that has the biggest α as the offshore hub in each cluster. The original solution of example1 is shown in table 64 and the integrated solution is indicated in table 65. Table 66 shows the solution of example2.

Table 64 The Original Solution of Example1 under Heuristic2 with Indicator α

Example1 Offshore Hub
HP 0 (3)
Hub2 (1,10)
Hub4 (5,6)
Hub7 (8,9)

Table 65 The Integrated Solution of Example1 under Heuristic2 with Indicator α

Example1 Offshore Hub (Sequence of assigning single node is based on largest total demand of hubs)	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,10,3(4,4))	63	17520	**56465.1**
Hub4 (5,6,3(2,2))	66	25090	
Hub7 (8,9,3(2,1))	59	22905	
Sum	**188**	**65515**	

Table 66 The Solution of Example2 under Heuristic2 with Indicator α

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,5)	57	2144	**3779.86**
Hub3 (4,6)	61	2162	
Sum	**118**	**4306**	

5.4.4 Heuristic 3

It is Fisher-Jaikumars (FJ) heuristics. In helicopter routing problem, offshore hubs are viewed as seed nodes. The same objective, which is to minimize the expected number of fatalities in an accident, is used to select offshore hubs. Hence, *m* offshore installations with largest α are selected as hubs. Then three possible approaches, which are Clark and Wright heuristic, transportation-work-related cost and distance or cost approach, could be used in assignment process. Three models in each approach do not change anything although the new indicator $\alpha = \frac{d_i+p_i}{c_{0i}}$ is used to select proper offshore hubs. For example1, Clark and Wright heuristic and transportation-work-related cost approach generated same solution (Table 67). But in distance or cost assigning approach the solution is different from those two (Table 68). For example2, three approaches gave the same solutions (Table 69)

Table 67 The Solution of Example1 under Heuristic3 with Indicator α

(Clark and Wright heuristic, transportation-work-related cost approach)

Example1 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (5,6)	61	19365	**56570**
Hub3 (8,9)	57	19130	
Hub4 (1,7,10)	66	27145	
Sum	**184**	**65640**	

Table 68 The Solution of Example1 under Heuristic3 with Indicator α

(cost or distance assigning approach)

Example1 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (6,8)	57	18385	**56630.2**
Hub3 (5,9)	61	20180	
Hub4 (1,7,10)	66	27145	
Sum	**184**	**65710**	

Table 69 The Solution of Example2 under Heuristic3 with Indicator α

(Clark and Wright heuristic, transportation-work-related cost and distance or cost assigning approach)

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,5)	57	2144	**3779.86**
Hub3 (4,6)	61	2162	
Sum	**118**	**4306**	

5.5 $\boldsymbol{\beta = \frac{d_i+p_i}{\sum_{j=0}^{n} c_{ij}}}$ *Indicator*

m offshore nodes with largest indicator $\frac{d_i+p_i}{\sum_{j=0}^{n} c_{ij}}$ would be possible options for being offshore hubs. We set, in this paper, indicator $\beta = \frac{d_i+p_i}{\sum_{j=0}^{n} c_{ij}}$. Larger value of indicator β shows the corresponding node might have relatively larger total demand as well as being central.

Step1: calculate the total demand of each offshore node.

Step2: calculate row sum for each node that is the sum of distance from each node to other nodes and the heliport.

Step3: total demand of each node is divided by its own row sum.

Step4: sequence all value got from step 3 in decreasing order (Table 70 and Table 71).

Table 70 The Sequence of Node of Example1 by Using Indicator β

Example1 Node	Total Demand $(d_i + p_i)$	Row Sum $(\sum_{j=0}^{n} c_{ij})$	Indicator $\beta = \frac{d_i+p_i}{\sum_{j=0}^{n} c_{ij}}$	Sequence of Node Based on Decreasing Order of Indicator β
1	7	2220	0.003153153	Node 4
2	17	2220	0.007657658	Node 2
3	15	2385	0.006289308	Node 5
4	14	1705	0.008211144	Node 7
5	13	1740	0.007471264	Node 8
6	9	1620	0.005555556	Node 3
7	11	1670	0.006586826	Node 9
8	11	1695	0.006489676	Node 6
9	10	1615	0.00619195	Node 10
10	8	2310	0.003463203	Node 1

Table 71 The Sequence of Node of Example2 by Using Indicator β

Example2 Node	Total Demand $(d_i + p_i)$	Row Sum $(\sum_{j=0}^{n} c_{ij})$	Indicator $\beta = \frac{d_i+p_i}{\sum_{j=0}^{n} c_{ij}}$	Sequence of Node Based on Decreasing Order of Indicator β
1	7	312	0.022435897	Node 3
2	17	247	0.068825911	Node 2
3	15	207	0.072463768	Node 4
4	14	245	0.057142857	Node 5
5	13	304	0.042763158	Node 6
6	9	388	0.023195876	Node 1

All heuristics mentioned above need to be modified as the new indicator β is used to choose offshore hubs from all nodes.

5.5.1 Heuristic 1

Step 1: setting an offshore installation that has the largest β as a hub.

Step 2: assigning those adjacent nodes to the hub. The helicopter capacity can constrain the number of customer nodes assigned to the hub.

Step 3: choosing another hub by using the same way and then going back to the step 2.

Table 72 and table 73 show the solutions of example1 and example2 given by this heuristic, respectively.

Table 72 The Solution of Example1 under Heuristic1 with indicator β

Example1 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub4 (5,6)	58	21710	**54233.4**
Hub2 (1,3)	61	15240	
Hub7 (8,9,10)	69	25970	
Sum	**188**	**62920**	

Table 73 The Solution of Example2 under Heuristic1 with indicator β

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub3(2,1)	63	2065	**3953.81**
Hub4(5,6)	58	2441	
Sum	**121**	**4506**	

5.5.2 Variety of Heuristic 1

First we sequence indicator β in decreasing order, choosing the first m nodes, which is the same number of helicopter, as offshore hubs.

5.5.2.1 Variant 1

Assigning adjacent non-hub installations to the first offshore hub that has the largest β among all offshore nodes. It does not stop to assign nodes until adding one more node in this tour makes the total pickup demand or total delivery demand excess the capacity of a helicopter. Then it starts to assign spoke nodes to the offshore hub that has the second biggest β. Assignment task is done when all non-hub nodes are assigned. The original solution of example1 is shown in table 74 and the integrated solution is shown in table 75.

Table 74 The Original Solution of Example1 under Variant1 of Heuristic1 with Indicator β

Example1 Offshore Hub
Hub4 (6,7)
Hub2 (1,3)
Hub5 (8,9)
HP 0 (10)

Table 75 The Integrated Solution of Example1 under Variant1 of Heuristic1 with Indicator β

Example1 Offshore Hub (Sequence of assigning single node is based on largest total demand of hubs)	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,3,10 (1,0))	63	15910	**55019**
Hub4 (6,7,10(4,1))	64	24850	
Hub5 (8,9,10(0,2))	59	23075	
Sum	**186**	**63835**	

The solution of example2 given by variant1 of heuristic1 with Indicator β is shown below (Table 76).

Table 76 The Solution of Example2 under Variant1 of Heuristic1 with Indicatorβ

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub3 (4,1)	57	1891	**4035.28**
Hub2 (5,6)	61	2712	
Sum	**118**	**4603**	

5.5.2.2 Variant 2

There is no priority to all offshore hubs. The priority in assignment process is given to the customer node that has the shortest distance to any chosen hubs. Capacity of a helicopter also works on limiting the number of customer nodes each hub could have. The original solution of example1 is shown in table 77 and the integrated solution is shown in table 78. The solution of example2 is given in table 79.

Table 77 The Original Solution of Example1 under Variant2 of Heuristic1 with Indicator β

Example1 Offshore Hub
Hub2 (1,3)
Hub4 (6,7)
Hub5 (9,8)
HP 0 (10)

Table 78 The Integrated Solution of Example1 under Variant2 of Heuristic1 with Indicatorβ

Example1 Offshore Hub (Sequence of assigning single node is based on largest total demand of hubs)	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,3,10 (1,0))	63	15910	**55019**
Hub4 (6,7,10(4,1))	64	24850	
Hub5 (8,9,10(0,2))	59	23075	
Sum	**186**	**63835**	

Table 79 The Solution of Example2 under Variant2 of Heuristic1 with Indicatorβ

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub3(4,6)	61	2162	**4924.6**
Hub2(1,5)	73	3463	
Sum	**134**	**5625**	

5.5.3 Heuristic 2

It is a sweep heuristic. Collecting installations by using a line starting from HP and sweeping clockwise or count clockwise to collect nodes. It does not stop until adding the next node will violate the capacity constraint. Repeating this collection step until several clusters can cover all nodes. Then choosing the node that has the biggest β as the offshore hub in each cluster. The original solution of example1 is shown in table 80 and the integrated solution is indicated in table 81. Table 82 shows the solution of example2.

Table 80 The Original Solution of Example1 under Heuristic2 with Indicator β

Example1 Offshore Hub
HP 0 (3)
Hub4 (5,6)
Hub2 (1,10)
Hub7 (8,9)

Table 81 The Integrated Solution of Example1 under Heuristic2 with Indicator β

Example1 Offshore Hub (Sequence of assigning single node is based on largest total demand of hubs)	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,10,3(4,4))	63	17520	**56465.1**
Hub4 (5,6,3(2,2))	66	25090	
Hub7 (8,9,3(2,1))	59	22905	
Sum	**188**	**65515**	

Table 82 The Solution of Example2 under Heuristic2 with Indicator β

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (1,5)	73	3463	**4924.6**
Hub3 (4,6)	61	2162	
Sum	**134**	**5625**	

5.5.4 Heuristic 3

It is Fisher-Jaikumars (FJ) heuristics. In helicopter routing problem, offshore hubs are viewed as seed nodes. The same objective, which is to minimize the expected number of fatalities in an accident, is used to select offshore hubs. Hence, m offshore installations with largest β are selected as hubs. Then three possible approaches, which are Clark and Wright heuristic, transportation-work-related cost and distance or cost approach, could be used in assignment process. Solutions of example1 and example2 using transportation-work-related cost and distance or cost assigning approach are same as ones generated by Clark and Wright heuristic approach, respectively. Solutions of two examples corresponding to each approach are shown in table 83 and table 84. Three models in each approach do not change anything although the new indicator $\beta = \frac{d_i + p_i}{\sum_{j=0}^{n} c_{ij}}$ is used to select proper offshore hubs.

Table 83 The Solution of Example1 under Heuristic3 with Indicator β

(Clark and Wright heuristic assigning approach)

Example1 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub2 (8,9)	59	18890	**62096.8**
Hub4 (1,7,10)	66	27145	
Hub5 (3,6)	61	26030	
Sum	**186**	**72065**	

Table 84 The Solution of Example2 under Heuristic 3 with Indicator β

(Clark and Wright heuristic assigning approach)

Example2 Offshore Hub	Number of Passenger Landings (PL)	Transportation Work (PC)	The Expected Number of Fatalities ($\times 10^{-6}$)
Hub3 (4,6)	61	2162	**4924.6**
Hub2 (1,5)	73	3463	
Sum	**134**	**5625**	

6.0 The comparison of solutions given by using five different choose-hub indicators

There are two tables showing value of objective function under each heuristic with each indicator both in example1 and example2, respectively (Table 85 and Table 86).

Table 85 The Value of Objective Function of Example1

Example1	Value of Objective Function ($\times10^{-6}$)				
Optimal Solution	**52988.35**				
Heuristic	Largest Demand Indicator	**Shortest Distance Indicator**	Center Node Indicator	Indicator $\alpha = \frac{d_i+p_i}{c_{0i}}$	Indicator $\beta = \frac{d_i+p_i}{\sum_{j=0}^{n} c_{ij}}$
H1	54233.4	53457.95	54615.7	54233.4	54233.4
H1v1	53886.8	53211.95	70398.88	53886.8	55019
H1v2	**53164.4**	**53108.75**	69815.8	**53164.4**	55019
H2	56465.1	55687.45	56765.7	56465.1	56465.1
H3v1	56570	**53048.55**	70112.5	56570	62096.8
H3v2	56570	**52988.35**	70112.5	56570	62096.8
H3v3	56630.2	**53048.55**	70112.5	56630.2	62096.8

Table 86 The Value of Objective Function of Example2

Example2	Value of Objective Function ($\times10^{-6}$)				
Optimal Solution	3779.86				
Heuristic	Largest Demand Indicator	**Shortest Distance Indicator**	Center Node Indicator	Indicator $\alpha = \frac{d_i+p_i}{c_{0i}}$	Indicator $\beta = \frac{d_i+p_i}{\sum_{j=0}^{n} c_{ij}}$
H1	3970.57	4060.49	3953.81	3970.57	3953.81
H1v1	3779.86	4051.24	3953.81	3779.86	4035.28
H1v2	**3779.86**	**3958.36**	3953.81	**3779.86**	4924.6
H2	3779.86	3958.36	4924.6	3779.86	4924.6
H3v1	3779.86	**3958.36**	3953.81	3779.86	4924.6
H3v2	3779.86	**3958.36**	3953.81	3779.86	4924.6
H3v3	3779.86	**3958.36**	3953.81	3779.86	4924.6

For the center node indicator, it does not work well in both examples. One possible reason is that in both cases the number of offshore hubs is more than one. Under the situation that only one node could be an offshore hub; the indicator working with those heuristics might offer some better solutions.

Based on table 85, the value of objective function given by the heuristics3, which uses shortest distance indicator to select offshore hubs and treats the minimized the transportation-work-related cost as the objective to assign non-hub nodes (H3v2), is the same as optimal value. Heuristics3 that uses Clark and Wright heuristic assigning approach (H3v1) or distance or cost approach (H3v3) working with shortest distance indicator generates solution (53048.55×10^{-6}) that is first closest to the optimal one (52988.35×10^{-6}). Shortest distance indicator under variant2 of heuristic1 (H1v2) generates second closest solution (53108.75×10^{-6}) to the optimal value. Moreover, the expected number of fatalities given by H1v2 working with largest demand indicator or indicator α is 53164.4×10^{-6} that is the third closest value to the optimal one.

From table 86, we find that H3v2 with shortest distance indicator did not give optimal solution in example2. This combination is not stable that means it could not always find optimal solution, but it generates the second closest solution (3958.36×10^{-6}) to optimal value. For H3v1, H3v3 and H1v2, when each of them works with shortest distance indicator in example2, same value (3958.36×10^{-6}) is made. In example2 H1v2 working with largest demand indicator or indicator α gives optimal solution although they do not generate optimal solution in example1.

Generally, Heuristic3 using any assigning approach may generate optimal solution or closer value to optimal one if it works with shortest distance indicator. It also has high possibility to generate optimal solution or solution that is closer to optimal value if variant2 of heuristic1 (H1v2) works with largest demand indicator, indicator α or shortest distance indicator. In the light of these two examples, shortest distance indicator may have relatively higher quality than others.

7.0 Lifeboat-related constraint

In order to handle some emergencies led by external factors, such as bad weather, or human errors, each offshore installation has at least one lifeboat. The number of seats in hub lifeboats could be a limit for choosing offshore hubs and assigning customer nodes to chosen offshore hubs. In this paper, we assume each offshore installation has the same type and some number of lifeboat seats. It means the number of lifeboat seats does not influence on the process of filtering offshore hub(s) from all installations, but constrain which customer installation could be assigned to which chosen offshore hub. Before we go into details of this new constraint, we first state that there are three stages to serve customer nodes.

Stage1 is a helicopter lands on a hub. The number of employees on an offshore hub when a helicopter lands on it consists of delivery demand and pickup demand of a hub, the delivery demand of all customer nodes served by the hub as well as fixed number of workers who still working on the offshore hub after helicopter returns back to heliport. Stage2 refers to the process of transferring people from the hub to customer nodes. The number of workers on an offshore hub, in this stage, is dynamic. When a helicopter departs from the hub with delivery demand of the node that it is serving, the number of people on the hub contains the delivery demand and pickup demand of the hub, the total delivery demand of all customer nodes assigned to this hub excepting one node that the helicopter is serving and the fixed number of workers on the hub. When the helicopter lands on the hub with pickup demand of the served node, besides all people we mentioned above, the number of workers just picked up from the served node also need to be involved in the total number of people on the hub currently. Stage3 refers to the moment before the helicopter takes off from the offshore hub with pickup demand collected from the hub and all customer nodes served by the hub, the total number of workers on the hub are delivery demand and pickup demand of the hub, the pickup demand of all customer nodes served by the hub as well as fixed number of workers who still work on the offshore hub after helicopter goes back to heliport.

Hence, the total number of workers on the hub should not violate the capacity of lifeboats the hub has at any stage. More exactly, the capacity of lifeboats should always be larger or equal to the total number of workers on the hub in each stage. Hence, we introduce worst case to make this issue being concrete. Worst case refers to the possible largest total number of workers on a hub in each stage. If it is less or equal to the total number of seats of all lifeboats on the hub, then lifeboat-related constraint would not be violated during process of transporting workers.

7.1 Random service method to customer nodes

We classify offshore nodes into two categories. If delivery demand of a node is less than its pickup demand, then the node is in the first category. In the second category, delivery demand of a node is equal or larger than its pickup demand (Table 87).

Table 87 Category of Customer Node

Category	Characteristics of Customer Node k	$\Delta = p_k - d_k$
Category 1	$d_k < p_k$	Positive
Category 2	$d_k \geq p_k$	0 or Negative

In this situation, a helicopter serves customer nodes within a cluster randomly. We assume a cluster contains one or more category1 nodes and at least one node classified in category2. If a helicopter serves category1 nodes first, then the worst case exists and occurs in stage2, which is the process of serving customer installations. More exactly, after finishing serving all category1 nodes in a cluster, the number of people on the hub would be maximized at that moment. For instance, we have a cluster containing one hub and three customer nodes (Table 88).

Table 88 A Small Example to Randomly Serving Situation

<table>
<tr><td>Cluster</td><td>Delivery Demand (d_k)</td><td>Pickup Demand (p_k)</td><td>Category of Customer Node k</td><td>$\Delta = p_k - d_k$of Customer Node</td></tr>
<tr><td>Hub 1</td><td>1</td><td>2</td><td></td><td></td></tr>
<tr><td>Node 2</td><td>3</td><td>4</td><td>Category 1</td><td>1</td></tr>
<tr><td>Node 3</td><td>2</td><td>6</td><td>Category 1</td><td>4</td></tr>
<tr><td>Node 4</td><td>5</td><td>2</td><td>Category 2</td><td>-3</td></tr>
<tr><td colspan="5">Worst Case: It exits if a helicopter serves node 2 and node 3 first and happens in stage2</td></tr>
<tr><td colspan="5">Worst Case (does not contain the number of people who still work on the hub after helicopter returns back to heliport with all pickup workers in a cluster.)
$= (d_1 + p_1 + d_2 + d_3 + d_4) + \Delta_2 + \Delta_3 = 18$</td></tr>
</table>

Mathematical formula for worst case limited by the number of lifeboat seats

(10)

$$\sum_{\substack{k=1 \\ k \neq i}}^{n} d_k W_{ik} + (d_i + p_i)U_i + g + \sum_{k:\,(p_k - d_k)>0} (p_k - d_k)W_{ik} \leq L,\ \forall i = 1 \ldots n$$

g = the number of people who still work on the hub after helicopter returns back to heliport with all pickup workers in a cluster.
L= the total capacity of lifeboats

Worst case of a cluster in this situation consists of the delivery demand of all customer nodes, the delivery demand and pickup demand of a hub of this cluster, the number of people who still work on the hub after helicopter returns back to heliport with all pickup workers, the sum of positive difference between pickup demand and delivery demand of each customer node. So the total number of people on the hub (worst case) should be less or equal to the total capacity of lifeboats.

7.2 Sequential service method to customer nodes

In this situation, if a cluster has one or more category2 nodes and at least one category1 node, then we enforce a helicopter to serve category2 customer nodes first. After finishing serving all category2 customer nodes, the helicopter starts to deliver and pick up people to and from those customer nodes classified in category1.

Mathematical formula for worst case constrained by the number of lifeboat seats

(11)

$$\sum_{\substack{k=1\\k\neq i}}^{n} d_k W_{ik} + (d_i + p_i)U_i + g + \sum_{\substack{k=1\\k\neq i}}^{n} (p_k - d_k)W_{ik} \leq L,\ \forall i = 1 \dots n$$

(12)

$$\sum_{\substack{k=1\\k\neq i}}^{n} (p_k - d_k)W_{ik} \geq 0,\ \forall i = 1 \dots n$$

We explain formula (12) under three sub-situations.

When

$$\sum_{\substack{k=1\\k\neq i}}^{n} (p_k - d_k)W_{ik} > 0,\ \forall i = 1 \dots n$$

This first sub-situation under sequential service method means in a cluster the sum of difference between pickup demand and delivery demand of each customer node is positive. Worse case on a hub occurs at the moment when pickup demand of all customer nodes is collected to the hub (stage3). We use the same example to make it being concrete (Table 89).

Table 89 A Small Example to First Sub-situation under Sequential Service Method

Cluster	Delivery Demand (d_k)	Pickup Demand (p_k)	Category of Customer Node k	$\Delta_k = p_k - d_k$ of Customer Node	Service Sequence for Customer Node
Hub 1	1	2			Alternative 1
Node 2	3	4	Category 1	1	Hub 1→ Node 4; Hub 1→ Node 2; Hub 1→ Node 3.
Node 3	2	6	Category 1	4	
Node 4	5	2	Category 2	-3	Alternative 2
Worst Case: It exits and occurs in stage3					Hub 1→ Node 4; Hub 1→ Node 3; Hub 1→ Node 2.
Worst Case (does not include the number of people who still work on the hub after helicopter returns back to heliport with all pickup workers in a cluster.) $= (d_1 + p_1 + d_2 + d_3 + d_4) + (\Delta_2 + \Delta_3 + \Delta_4)$ = 15					

In this sub-situation, the sequence of serving category1 nodes does not take any influence on the worse case.

Compared with worst case in table 88, the worst case under sequential service method is reduced to 15. We could get:

Conclusion 4: Sequential service method could reduce the possible largest number of people on the hub.

When

$$\sum_{\substack{k=1 \\ k \neq i}}^{n} (p_k - d_k) W_{ik} = 0, \ \forall i = 1 \dots n$$

The second sub-situation under sequential service method means in a cluster the sum of difference between pickup demand and delivery demand of each customer node is zero.

The number of workers on the hub is not changed in stage1 and stage3. Because the sum of pickup demands of all customer nodes is the same as the sum of delivery demand of them. The number of workers on a hub in stage 1 or stage 3 could be defined as worse case as it is largest in entire process of doing worker transportation. We, in this paper, think that worse case occurs in stage 3 instead of stage 1. We use a simple example to show it (Table 90).

Table 90 A Simple Example to Second Sub-situation under Sequential Service Method

Cluster	Delivery Demand (d_k)	Pickup Demand (p_k)	Category of Customer Node k	$\Delta_k = p_k - d_k$ of Customer Node	Service Sequence for Customer Node
Hub 1	1	2			Alternative 1
Node 2	3	4	Category 1	1	Hub 1→ Node 4; Hub 1→ Node 2; Hub 1→ Node 3.
Node 3	2	6	Category 1	4	
Node 4	7	2	Category 2	-5	Alternative 2
Worst Case: It exits and occurs in stage3					Hub 1→ Node 4; Hub 1→ Node 3; Hub 1→ Node 2.
Worst Case (does not include the fixed number of workers) = $(d_1 + p_1 + d_2 + d_3 + d_4) + (\Delta_2 + \Delta_3 + \Delta_4)$ = 15					

When

$$\sum_{\substack{k=1 \\ k \neq i}}^{n} (p_k - d_k) W_{ik} < 0, \ \forall i = 1 \dots n$$

The third sub-situation is possible, but we treat the negative value as 0 in mathematical model. For a cluster in the third sub-situation, worst case generally occurs in stage1 as when a helicopter lands on the hub with delivery demand of all customer nodes and all demand of the hub, at that moment the number of people on the hub is the largest.

7.3 Test two updated mathematical model

In this section, we used AMPL programming to test updated mathematical models. We would like to find answers about how lifeboat-related constraints affect the way of assigning customer nodes to hubs and the expected number of fatalities. We also compared random service method to customer nodes with sequential service method and find which way is better.

We first tested a new model that contains lifeboat-related constraint made under random service situation (Table 91). After testing it, we tested the second updated model with lifeboat-related constraint that works under the sequential service situation (Table 92). Data for testing the updated models is from example1 we used in previous sections.

Table 91 Results Given by Model with Lifeboat-related Constrain under Random Service Method

Example1 Random service for customer nodes				
The number of seat of lifeboats	Solutions	The number of passenger landing	Transportation work	Value of objective function ($\times10^{-6}$)
60-65	Infeasible			
66	Hub 1 (4,7 and 10)	204	76405	65840.9
	Hub 6 (2 and 5)			
	Hub 9 (3 and 8)			
67	Hub 1 (2 and 3)	204	63695	54910.3
	Hub 6 (7,8 and 10)			
	Hub 9 (4 and 5)			
71	Hub 1 (4,7 and 10)	191	61540	53048.5
	Hub 2 (6 and 8)			
	Hub 3 (5 and 9)			
72	Hub 1 (4,7 and 10)	191	61470	52988.3
	Hub 2 (5 and 6)			
	Hub 3 (8 and 9)			

Table 92 Results Given by Model with Lifeboat-related Constrain under Sequential Service Method

<table>
<tr><td colspan="5">Example1
Sequential service for customer nodes</td></tr>
<tr><td>The number of seat of lifeboats</td><td>Solutions</td><td>The number of passenger landing</td><td>Transportation work</td><td>Value of objective function $(\times10^{-6})$</td></tr>
<tr><td>60-64</td><td>Infeasible</td><td></td><td></td><td></td></tr>
<tr><td>66</td><td>Hub 1 (2 and 3)</td><td rowspan="3">198</td><td rowspan="3">62970</td><td rowspan="3">54282.9</td></tr>
<tr><td></td><td>Hub 4 (5 and 9)</td></tr>
<tr><td></td><td>Hub 7 (6,8 and 10)</td></tr>
<tr><td>68</td><td>Hub 1 (4,7 and 10)</td><td rowspan="3">191</td><td rowspan="3">61540</td><td rowspan="3">53048.5</td></tr>
<tr><td></td><td>Hub 2 (6 and 8)</td></tr>
<tr><td></td><td>Hub 3 (5 and 9)</td></tr>
<tr><td>69</td><td>Hub 1 (4,7 and 10)</td><td rowspan="3">191</td><td rowspan="3">61470</td><td rowspan="3">52988.3</td></tr>
<tr><td></td><td>Hub 2 (5 and 6)</td></tr>
<tr><td></td><td>Hub 3 (8 and 9)</td></tr>
</table>

Conclusion 5: Smaller number of lifeboats seat takes more negative effect on the expected number of fatalities.

Both tables show the larger number of lifeboat seats an offshore installation has, the lower value objective function has. This conclusion is available to both random service method and sequential service method.

We compared table 91 with table 92 afterwards, we found sequential service for customer nodes is better than random service method. For example, we assumed that a node has 68 lifeboat-seats totally; in the light of two tables the objective function has lower value by using sequential service method to customer nodes. On the other hand, the solution given by 71 lifeboat-seats under random service method is same as the situation with 68 lifeboat-seats under sequential service method, which means we might get the same expected number of fatalities with fewer lifeboats only if we use sequential service method. Moreover, in order to achieve optimal expected number of fatalities, under random service method an offshore hub need to have at least 72 lifeboat-seats totally whereas only 69 under sequential service method lifeboat-seats could make the objective function get an optimal value.

8.0 Conclusion

In this paper, after testing five different indicators that works with several heuristics to two examples, we conclude variant2 of heuristic1 working with largest demand indicator, indicator α or shortest distance indicator and heuristic3 working with shortest distance indicator are more likely to generate optimal solution or closer solution to optimal value than other combinations of heuristic and indicator. Shortest distance indicator works better than other indicators in both two examples in general. For the lifeboat-related constrain section, under the situation when they have same number of seat of lifeboats, sequential service method is better than serving customer nodes randomly within a cluster. Sequential service method could make the value of possible worst case smaller. Moreover, fewer number of lifeboat seats under situation of using sequential service method could let expected number of fatalities get optimal than using random service method.

9.0 Limitation

In the mathematical model, every node could be visited only once. In further research, this constraint could be loosed. Moreover, all indicators are tested under heuristics from Halskau (2012). For further work, these indicators could be tested by combining with other heuristics. Then, the relatively higher quality indicator for filtering offshore hubs may be not the one we concluded in this paper as judgment of optimization of a indicator depends on what type of heuristic is used somewhat. Furthermore, as the number of helicopter limits the number of offshore hubs, in exmaple1 each solution only has three offshore hubs. It leads no difference between solutions generated by largest demand indicator and by indicator α working with all heuristics in both examples. But the order of all offshore nodes based on indicator α and largest demand indicator are different. The distinction of two lists starts from the fifth node. Hence, in further research people could find that one of them might work better than another.

In this paper, the number and type of lifeboats are supported to be the same for each offshore node. But in real life, they could be different. Hence, it may affect the process of choosing offshore hub(s) which is only influenced by a hub-selected indicator in this paper.

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11.0 Appendixes

11.1 AMPL program for mathematical model

11.1.1 Model

Table 93 Mathematic Formulation and its Corresponding Name in AMPL

Mathematic Formulation	AMPL Name
$\min(\pi_L PL + \pi_c PC)$	Minimize Total_ExpectedNumberofFatalities:
St:	
$PL = 2(D+P) - \sum_{i=1}^{n}(d_i + p_i)U_i$	subject to NumberofPassengerLanding
$PC = \sum_{i=1}^{n} c_{0,i}(d_i + p_i)U_i + \sum_{i=1}^{n}\sum_{k=1}^{n} c_{0,i}\,(d_k + p_k)W_{ik} + \sum_{i=1}^{n}\sum_{k=1}^{n} c_{i,k}\,(d_k + p_k)W_{ik}$	subject to TransportationWork
$\sum_{i=1}^{n} U_i = m$	subject to NumberofHelicopters
$W_{ik} \le U_i, \forall i, k = 1,2,\dots,n$	subject to Service{i in NODE diff {0}, k in NODE diff {0}: i<>k}
$\sum_{\substack{k=1 \\ k\neq i}}^{n} d_k\,W_{ik} + d_i U_i \le Q, \forall i = 1,2,\dots n$	subject to DeliveryCapacity{i in NODE diff {0}}
$\sum_{\substack{k=1 \\ k\neq i}}^{n} p_k\,W_{ik} + p_i U_i \le Q, \forall i = 1,2,\dots n$	subject to PickupCapacity{i in NODE diff {0}}
$\sum_{i=1}^{n}\sum_{\substack{k=1 \\ k\neq i}}^{n} d_k\,W_{ik} + d_i U_i = D$	subject to TotalDemand
$\sum_{i=1}^{n}\sum_{\substack{k=1 \\ k\neq i}}^{n} p_k\,W_{ik} + p_i U_i = P$	subject to TotalPickup
$\sum_{i=1}^{n} W_{ik} = 1, \forall k = 1,2,\dots,n, k \neq i$	subject to OneNodeServedOnce

Table 94 The Meaning of Each Set, Parameter and Variable and its Corresponding Name in AMPL

Set	AMPL Name
NODE = a set of offshore nodes	1 2 3 4 5 6 7 8 9 10
Parameters:	AMPL Name
m= the number of helicopters	helicopter
Q= the capacity of a helicopter	capacity
D= total delivery demand	total_D
P= total pickup demand	total_P
π_L= the probability of a fatal accident during takeoff and landings	prob_L
π_C= the probability of a fatal accident during cruising	prob_C
d_k /d_i= the delivery demand of node k or node i, *i,k = 1,2,…,n*	demand{NODE diff {0}}
p_k /p_i= the pickup demand of node k or node i, *i,k = 1,2,…,n*	pickup{NODE diff {0}}
$c_{0,i}/c_{i,k}$= the distance between the heliport and a node or the distance between two nodes	cost{NODE,NODE}
Variables	AMPL Name
PL = the number of passenger landings	PL
PC = the transportation work	PC
U_i = 1 if the node i is selected to be an offshore hub ; U_i = 0 otherwise, *i = 1,2,…,n*	OffshoreNode{i in NODE diff {0}}
W_{ik} = 1 if the customer node k is served by the offshore hub i ; W_{ik} = 0 otherwise, *i,k = 1,2,…,n*	NodeServed{i in NODE diff {0},k in NODE diff {0}: i<>k}

```
set NODE;
param helicopter;
param capacity ;
param total_D ;
param total_P;
param prob_L;
param prob_C;
param demand{NODE diff {0}};
param pickup{NODE diff {0}};
param cost{NODE,NODE} >=0;
var PL;
var PC;
var OffshoreNode{i in NODE diff {0}} binary >=0, <=1;
var NodeServed{i in NODE diff {0},k in NODE diff {0}: i<>k} binary >=0, <=1;

minimize Total_ExpectedNumberofFatalities:
prob_L * PL + prob_C * PC;

subject to NumberofPassengerLanding:
PL = 2*(total_D + total_P) - sum {i in NODE diff {0}}(demand[i] + pickup[i])*
OffshoreNode[i];

subject to TransportationWork:
PC = sum{i in NODE diff {0}} cost[0,i]* (demand[i]+pickup[i])* OffshoreNode[i] +
sum{i in NODE diff {0} ,k in NODE diff {0}: i <>k} cost[0,i]*(demand[k]+pickup[k])*
NodeServed[i,k] +sum{i in NODE diff {0},k in NODE diff {0}: i<>k}
cost[i,k]*(demand[k]+pickup[k])*NodeServed[i,k];
```

```
subject to NumberofHelicopters:

sum{i in NODE diff {0}} OffshoreNode[i]= helicopter;


subject to Service{i in NODE diff {0}, k in NODE diff {0}: i<>k}:

NodeServed[i,k]<= OffshoreNode[i];


subject to DeliveryCapacity{i in NODE diff {0}}:

sum{k in NODE diff {0}:k<>i} demand[k]*NodeServed[i,k] + demand[i]*
OffshoreNode[i] <= capacity;


subject to PickupCapacity{i in NODE diff {0}}:

sum{k in NODE diff {0}: k<>i} pickup[k] * NodeServed[i,k] + pickup[i] *
OffshoreNode[i] <= capacity;


subject to TotalDemand:

sum{i in NODE diff {0}} (sum{k in NODE diff {0}: i<>k} demand[k]* NodeServed[i,k]+
demand[i]* OffshoreNode[i]) = total_D;


subject to TotalPickup:

sum{i in NODE diff {0}} (sum{k in NODE diff {0}: i<>k} pickup[k]* NodeServed[i,k]+
pickup[i]* OffshoreNode[i]) = total_P;


subject to OneNodeServedOnce {k in NODE diff{0,1,2,3}}:

sum{i in NODE diff {0,4,5,6,7,8,9,10}: i<>k} NodeServed[i,k] = 1;
```

11.1.2 Data file

```
set NODE := 0 1 2 3 4 5 6 7 8 9 10;
```

```
param helicopter := 3;

param capacity := 20 ;

param total_D:= 57 ;

param total_P:= 58;

param prob_L:= 0.65;

param prob_C:= 0.86;

param demand:= 1 2  2 9  3 8  4 8  5 8  6 2  7 5  8 7  9 3  10 5;

param pickup:= 1 5  2 8  3 7  4 6  5 5  6 7  7 6  8 4  9 7  10 3;


param cost:   0    1    2    3    4    5    6    7    8    9    10 :=

           0  0    360  360  385  590  590  605  620  620  590  670

           1  360  0    0    80   235  240  245  255  260  235  310

           2  360  0    0    80   235  240  245  255  260  235  310

           3  385  80   80   0    255  250  260  265  270  230  310

           4  590  235  235  255  0    5    45   60   60   65   155

           5  590  240  240  250  5    0    50   65   75   70   155

           6  605  245  245  260  45   50   0    15   15   30   110

           7  620  255  255  265  60   65   15   0    10   30   95

           8  620  260  260  270  60   75   15   10   0    30   95

           9  590  235  235  230  65   70   30   30   30   0    100

          10  670  310  310  310  155  155  110  95   95   100  0  ;
```

11.1.3 Run file

```
model model.mod;

data  model.dat;

option solver cplexamp;

option cplex_options 'sensitivity';
```

```
solve;
display  Total_ExpectedNumberofFatalities > model.sol;
display  OffshoreNode > model.sol;
display  NodeServed > model.sol;
display  PL > model.sol;
display  PC > model.sol;
```

11.1.4 Solution File

```
Total_ExpectedNumberofFatalities = 52988.3

OffshoreNode [*] :=
 1  1
 2  1
 3  1
 4  0
 5  0
 6  0
 7  0
 8  0
 9  0
10  0
;

NodeServed [*,*]
:   1   2   3   4   5   6   7   8   9  10   :=
1   .   0   0   1   0   0   1   0   0   1
2   0   .   0   0   1   1   0   0   0   0
```

```
3   0  0  .  0  0  0  0  1  1  0
4   0  0  0  .  0  0  0  0  0  0
5   0  0  0  0  .  0  0  0  0  0
6   0  0  0  0  0  .  0  0  0  0
7   0  0  0  0  0  0  .  0  0  0
8   0  0  0  0  0  0  0  .  0  0
9   0  0  0  0  0  0  0  0  .  0
10  0  0  0  0  0  0  0  0  0  .
;
```

PL = 191

PC = 61470

11.1.5 How to perform the AMPL mode

In the first step, running the mode without the last constraint, which is

subject to OneNodeServedOnce {k in NODE diff{0,1,2,3}}:

sum{i in NODE diff {0,4,5,6,7,8,9,10}: i<>k} NodeServed[i,k] = 1;

After running the mode, the result shows us which offshore nodes could be chosen as offshore hubs.

In the second step, we put the name of each offshore hub and the heliport (Here the name of each offshore hub and heliport is a number) into “*k in NODE diff {…}*” and put the name of non-hub nodes and the heliport into “*sum {i in NODE diff {…}: i<>k}*”. We run the mode with this constraint afterward. Finally the new result solves the issue about how to assign non-hub nodes to offshore hubs.

11.2 AMPL program for model of Clark and Wright heuristic assignment method

11.2.1 Model file

```
set OFFSHOREHUBS;
set NODES;

param savingcost{NODES,OFFSHOREHUBS};
param demand{NODES};
param pickup{NODES};
param capacity;
param demandhub{OFFSHOREHUBS};
param pickuphub{OFFSHOREHUBS};

var NodeServed{i in NODES, s in OFFSHOREHUBS} binary >=0,<=1;

maximize Total_SavingCost:
sum{i in NODES, s in OFFSHOREHUBS} savingcost[i,s]* NodeServed[i,s];

subject to NodeServedOnce{i in NODES}:
sum{s in OFFSHOREHUBS}NodeServed[i,s]=1;

subject to DemandLessThanCapacity{s in OFFSHOREHUBS}:
sum{i in NODES} demand[i]* NodeServed[i,s]<= capacity-demandhub[s];

subject to PickLessThanCapacity{s in OFFSHOREHUBS}:
sum{i in NODES} pickup[i]* NodeServed[i,s]<= capacity-pickuphub[s];
```

11.2.2 Data File

```
set OFFSHOREHUBS := 2 3 4;

set NODES := 1 5 6 7 8 9 10;


param capacity := 20 ;
```

```
param demand:= 1 2    5 8  6 2  7 5  8 7  9 3  10 5;

param pickup:= 1 5    5 5  6 7  7 6  8 4  9 7  10 3;


param demandhub:= 2 9  3 8  4 8;

param pickuphub:= 2 8  3 7  4 6;


param savingcost:     2       3        4  :=

              1          720     665      715

              5          710     725      1175

              6          720     730      1150

              7          725     740      1150

              8          720     735      1150

              9          715     745      1115

              10         720     745      1105;
```

11.2.3 Run file

```
model H3v1.mod;

data  H3v1.dat;

option solver cplexamp;

option cplex_options 'sensitivity';

solve;

display  Total_SavingCost > H3v1.sol;


display  NodeServed > H3v1.sol;
```

11.2.4 Solution file

```
Total_SavingCost = 5880
```

```
NodeServed [*,*]

:   2  3  4   :=

1   0  0  1

5   1  0  0

6   1  0  0

7   0  0  1

8   0  1  0

9   0  1  0

10  0  0  1

;
```

11.3 AMPL program for model of transportation-work-related cost assignment method

11.3.1 Model file

```
set OFFSHOREHUBS;
set NODES;

param transportationwork{NODES,OFFSHOREHUBS};
param demand{NODES};
param pickup{NODES};
param capacity;
param demandhub{OFFSHOREHUBS};
param pickuphub{OFFSHOREHUBS};

var NodeServed{i in NODES, s in OFFSHOREHUBS} binary >=0,<=1;

minimize Total_TransportationWork:
sum{i in NODES, s in OFFSHOREHUBS} transportationwork[i,s]* NodeServed[i,s];

subject to NodeServedOnce{i in NODES}:
```

```
sum{s in OFFSHOREHUBS}NodeServed[i,s]=1;

subject to DemandLessThanCapacity{s in OFFSHOREHUBS}:
sum{i in NODES} demand[i]* NodeServed[i,s]<= capacity-demandhub[s];

subject to PickLessThanCapacity{s in OFFSHOREHUBS}:
sum{i in NODES} pickup[i]* NodeServed[i,s]<= capacity-pickuphub[s];
```

11.3.2 Data file

```
set OFFSHOREHUBS := 2 3 4;
set NODES := 1 5 6 7 8 9 10;

param capacity := 20 ;

param demand:= 1 2    5 8  6 2  7 5  8 7  9 3  10 5;
param pickup:= 1 5    5 5  6 7  7 6  8 4  9 7  10 3;

param demandhub:= 2 9  3 8  4 8;
param pickuphub:= 2 8  3 7  4 6;

param transportationwork:    2      3      4:=
            1       0       560     1645
            5       3120    3250    65
            6       2205    2340    405
            7       2805    2915    660
            8       2860    2970    660
            9       2350    2300    650
            10      2480    2480    1240;
```

11.3.3 Run file

```
model H3v2.mod;
```

```
data  H3v2.dat;
option solver cplexamp;
option cplex_options 'sensitivity';
solve;
display  Total_TransportationWork > H3v2.sol;

display  NodeServed > H3v2.sol;
```

11.3.4 Solution file

```
Total_TransportationWork = 14140

NodeServed [*,*]
:   2   3   4   :=
1    0   0   1
5    1   0   0
6    1   0   0
7    0   0   1
8    0   1   0
9    0   1   0
10   0   0   1
;
```

11.4 AMPL program for model of cost or distance assignment method

11.4.1 Model file

```
set OFFSHOREHUBS;
set NODES;


param cost{NODES,OFFSHOREHUBS};
param demand{NODES};
```

```
param pickup{NODES};
param capacity;
param demandhub{OFFSHOREHUBS};
param pickuphub{OFFSHOREHUBS};

var NodeServed{i in NODES, s in OFFSHOREHUBS} binary >=0,<=1;

minimize Total_Cost:
sum{i in NODES, s in OFFSHOREHUBS} cost[i,s]* NodeServed[i,s];

subject to NodeServedOnce{i in NODES}:
sum{s in OFFSHOREHUBS}NodeServed[i,s]=1;

subject to DemandLessThanCapacity{s in OFFSHOREHUBS}:
sum{i in NODES} demand[i]* NodeServed[i,s]<= capacity-demandhub[s];

subject to PickLessThanCapacity{s in OFFSHOREHUBS}:
sum{i in NODES} pickup[i]* NodeServed[i,s]<= capacity-pickuphub[s];
```

11.4.2 Data file

```
set OFFSHOREHUBS := 2 3 4;
set NODES := 1 5 6 7 8 9 10;

param capacity := 20 ;

param demand:= 1 2    5 8  6 2  7 5  8 7  9 3  10 5;
param pickup:= 1 5    5 5  6 7  7 6  8 4  9 7  10 3;
```

```
param demandhub:= 2 9  3 8  4 8;
param pickuphub:= 2 8  3 7  4 6;

param cost:     2       3       4:=
        1       0       80      235
        5       240     250     5
        6       245     260     45
        7       255     265     60
        8       260     270     60
        9       235     230     65
        10      310     310     155;
```

11.4.3 Run file

```
model H3v3.mod;
data  H3v3.dat;
option solver cplexamp;
option cplex_options 'sensitivity';
solve;
display  Total_Cost > H3v3.sol;

display  NodeServed > H3v3.sol;
```

11.4.4 Solution file

```
Total_Cost = 1435

NodeServed [*,*]
```

```
:   2  3  4   :=

1   0  0  1

5   0  1  0

6   1  0  0

7   0  0  1

8   1  0  0

9   0  1  0

10  0  0  1

;
```

11.5 AMPL program for mathematical model with lifeboat-related constraint of random service method

11.5.1 Model file

Table 95 Mathematic Formulation of lifeboat constraint of random service method, its related parameters and their Corresponding Name in AMPL

Mathematic Formulation	**AMPL Name**
$\sum_{\substack{k=1\\k\neq i}}^{n} d_k W_{ik} + (d_i + p_i)U_i + g + \sum_{k:\,(p_k - d_k)>0} (p_k - d_k)W_{ik} \leq L,\ \forall i = 1 \dots n$	subject to TheProcessofTransferringPPL{i in NODE diff {0}}:
g	param g
$\boldsymbol{L}$	Param lifeboatcapacity

```
set NODE;

param helicopter;

param capacity ;

param total_D ;

param total_P;

param prob_L;
```

```
param prob_C;

param demand{NODE diff {0}};

param pickup{NODE diff {0}};

param cost{NODE,NODE} >=0;


param g;

param lifeboatcapacity;


var PL;

var PC;

var OffshoreNode{i in NODE diff {0}} binary >=0, <=1;

var NodeServed{i in NODE diff {0},k in NODE diff {0}: i<>k} binary >=0, <=1;



minimize Total_ExpectedNumberofFatalities:

prob_L * PL + prob_C * PC;


subject to NumberofPassengerLanding:

PL = 2*(total_D + total_P) - sum {i in NODE diff {0}}(demand[i] + pickup[i])*
OffshoreNode[i];


subject to TransportationWork:

PC = sum{i in NODE diff {0}} cost[0,i]* (demand[i]+pickup[i])* OffshoreNode[i] +
sum{i in NODE diff {0} ,k in NODE diff {0}: i<>k} cost[0,i]*
(demand[k]+pickup[k])*NodeServed[i,k]+ sum{i in NODE diff {0},k in NODE diff {0}:
i<>k} cost[i,k]*(demand[k]+pickup[k])*NodeServed[i,k];


subject to NumberofHelicopters:

sum{i in NODE diff {0}} OffshoreNode[i]= helicopter;
```

```
subject to Service{i in NODE diff {0}, k in NODE diff {0}: i<>k}:

NodeServed[i,k]<= OffshoreNode[i];


subject to DeliveryCapacity{i in NODE diff {0}}:

sum{k in NODE diff {0}:k<>i} demand[k]*NodeServed[i,k] + demand[i]*
OffshoreNode[i] <= capacity;


subject to PickupCapacity{i in NODE diff {0}}:

sum{k in NODE diff {0}: k<>i} pickup[k] * NodeServed[i,k] + pickup[i] *
OffshoreNode[i] <= capacity;


subject to TotalDemand:

sum{i in NODE diff {0}} (sum{k in NODE diff {0}: i<>k} demand[k]* NodeServed[i,k]+
demand[i]* OffshoreNode[i]) = total_D;


subject to TotalPickup:

sum{i in NODE diff {0}} (sum{k in NODE diff {0}: i<>k} pickup[k]* NodeServed[i,k]+
pickup[i]* OffshoreNode[i]) = total_P;


subject to TheProcessofTransferringPPL{i in NODE diff {0}}:

sum{k in NODE diff {0}: k<>i} demand[k]*NodeServed[i,k]+ demand[i] *
OffshoreNode[i] + pickup[i]* OffshoreNode[i] + g + sum{k in NODE diff{0}: k<>i and
pickup[k]>demand[k] } (pickup[k]-demand[k])*NodeServed[i,k] <= lifeboatcapacity;




subject to OneNodeServedOnce {k in NODE diff{0,1,6,9}}:

sum{i in NODE diff {0,4,2,5,3,7,8,10}: i<>k} NodeServed[i,k] = 1;
```

11.5.2 Data file

```
set NODE := 0 1 2 3 4 5 6 7 8 9 10;


param helicopter := 3;

param capacity := 20 ;

param total_D:= 57 ;

param total_P:= 58;

param prob_L:= 0.65;

param prob_C:= 0.86;

param demand:= 1 2  2 9  3 8  4 8  5 8  6 2  7 5  8 7  9 3  10 5;

param pickup:= 1 5  2 8  3 7  4 6  5 5  6 7  7 6  8 4  9 7  10 3;


param g := 40;

param lifeboatcapacity:= 66;


param cost: 0    1    2    3    4    5    6    7    8    9    10 :=

      0  0    360  360  385  590  590  605  620  620  590  670

      1  360  0    0    80   235  240  245  255  260  235  310

      2  360  0    0    80   235  240  245  255  260  235  310

      3  385  80   80   0    255  250  260  265  270  230  310

      4  590  235  235  255   0   5    45   60   60   65   155

      5  590  240  240  250   5   0    50   65   75   70   155

      6  605  245  245  260   45  50   0    15   15   30   110

      7  620  255  255  265   60  65   15   0    10   30   95

      8  620  260  260  270   60  75   15   10   0    30   95

      9  590  235  235  230   65  70   30   30   30   0    100

      10 670  310  310  310   155 155  110  95   95   100  0  ;
```

11.5.3 Run file

```
model model-lifeboat.mod;

data  model-lifeboat.dat;

option solver cplexamp;

option cplex_options 'sensitivity';

solve;

display  Total_ExpectedNumberofFatalities > model-lifeboat.sol;

display  OffshoreNode > model-lifeboat.sol;

display  NodeServed > model-lifeboat.sol;

display  PL > model-lifeboat.sol;

display  PC > model-lifeboat.sol;
```

11.6 AMPL program for mathematical model with lifeboat-related constraint of sequential service method

11.6.1 Model file

Table 96 Mathematic Formulation of lifeboat constraint of sequential service method, its related parameters and their Corresponding Name in AMPL

Mathematic Formulation	**AMPL Name**
$\sum_{\substack{k=1\\k\neq i}}^{n} d_k W_{ik} + (d_i + p_i)U_i + g + \sum_{\substack{k=1\\k\neq i}}^{n} (p_k - d_k)W_{ik} \le L,$ $\forall i = 1 \dots n$	subject to TheProcessofTransferringPPL{i in NODE diff {0}}
$\sum_{\substack{k=1\\k\neq i}}^{n} (p_k - d_k)W_{ik} \ge 0,\ \forall i = 1 \dots n$	subject to SS{i in NODE diff {0}
g	param g
L	Param lifeboatcapacity

```
set NODE;

param helicopter;
param capacity ;
param total_D ;
param total_P;
param prob_L;
param prob_C;
param demand{NODE diff {0}};
param pickup{NODE diff {0}};
param cost{NODE,NODE} >=0;

param g;
param lifeboatcapacity;

var PL;
var PC;
var OffshoreNode{i in NODE diff {0}} binary >=0, <=1;
var NodeServed{i in NODE diff {0},k in NODE diff {0}: i<>k} binary >=0, <=1;

var SequenceService >= 0;

minimize Total_ExpectedNumberofFatalities:
prob_L * PL + prob_C * PC;

subject to NumberofPassengerLanding:
```

```
PL = 2*(total_D + total_P) - sum {i in NODE diff {0}}(demand[i] + pickup[i])*
OffshoreNode[i];


subject to TransportationWork:

PC = sum{i in NODE diff {0}} cost[0,i]* (demand[i]+pickup[i])* OffshoreNode[i] +
sum{i in NODE diff {0} ,k in NODE diff {0}: i<>k} cost[0,i]*
(demand[k]+pickup[k])*NodeServed[i,k]+ sum{i in NODE diff {0},k in NODE diff {0}:
i<>k} cost[i,k]*(demand[k]+pickup[k])*NodeServed[i,k];


subject to NumberofHelicopters:

sum{i in NODE diff {0}} OffshoreNode[i]= helicopter;


subject to Service{i in NODE diff {0}, k in NODE diff {0}: i<>k}:

NodeServed[i,k]<= OffshoreNode[i];


subject to DeliveryCapacity{i in NODE diff {0}}:

sum{k in NODE diff {0}:k<>i} demand[k]*NodeServed[i,k] + demand[i]*
OffshoreNode[i] <= capacity;


subject to PickupCapacity{i in NODE diff {0}}:

sum{k in NODE diff {0}: k<>i} pickup[k] * NodeServed[i,k] + pickup[i] *
OffshoreNode[i] <= capacity;


subject to TotalDemand:

sum{i in NODE diff {0}} (sum{k in NODE diff {0}: i<>k} demand[k]* NodeServed[i,k]+
demand[i]* OffshoreNode[i]) = total_D;


subject to TotalPickup:

sum{i in NODE diff {0}} (sum{k in NODE diff {0}: i<>k} pickup[k]* NodeServed[i,k]+
pickup[i]* OffshoreNode[i]) = total_P;
```

```
subject to SS{i in NODE diff {0}}:

SequenceService >= sum {k in NODE diff{0}: k<>i} (pickup[k] - demand[k]) *
NodeServed[i,k];


subject to TheProcessofTransferringPPL{i in NODE diff {0}}:

sum{k in NODE diff {0}: k<>i} demand[k]*NodeServed[i,k]+ demand[i] *
OffshoreNode[i] + pickup[i]* OffshoreNode[i] + g + SequenceService <=
lifeboatcapacity;




subject to OneNodeServedOnce {k in NODE diff{0,1,2,3}}:

sum{i in NODE diff {0,8,4,5,9,6,7,10}: i<>k} NodeServed[i,k] = 1;
```

11.6.2 Data file

```
set NODE := 0 1 2 3 4 5 6 7 8 9 10;


param helicopter := 3;

param capacity := 20 ;

param total_D:= 57 ;

param total_P:= 58;

param prob_L:= 0.65;

param prob_C:= 0.86;

param demand:= 1 2  2 9  3 8  4 8  5 8  6 2  7 5  8 7  9 3  10 5;

param pickup:= 1 5  2 8  3 7  4 6  5 5  6 7  7 6  8 4  9 7  10 3;


param g := 40;

param lifeboatcapacity:= 70;
```

```
param cost: 0    1    2    3    4    5    6    7    8    9    10 :=
        0  0    360  360  385  590  590  605  620  620  590  670
        1  360  0    0    80   235  240  245  255  260  235  310
        2  360  0    0    80   235  240  245  255  260  235  310
        3  385  80   80   0    255  250  260  265  270  230  310
        4  590  235  235  255   0   5    45   60   60   65   155
        5  590  240  240  250   5   0    50   65   75   70   155
        6  605  245  245  260   45  50   0    15   15   30   110
        7  620  255  255  265   60  65   15   0    10   30   95
        8  620  260  260  270   60  75   15   10   0    30   95
        9  590  235  235  230   65  70   30   30   30   0    100
        10 670  310  310  310   155 155  110  95   95   100  0  ;
```

11.6.3 Run file

```
model model-lifeboatsequence.mod;
data  model-lifeboatsequence.dat;
option solver cplexamp;
option cplex_options 'sensitivity';
solve;
display  Total_ExpectedNumberofFatalities > model-lifeboatsequence.sol;
display  OffshoreNode > model-lifeboatsequence.sol;
display  NodeServed > model-lifeboatsequence.sol;
display  PL > model-lifeboatsequence.sol;
display  PC > model-lifeboatsequence.sol;
display  SequenceService > model-lifeboatsequence.sol;
```