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Alternative models for routing of aircraft

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Preface

This thesis is conducted as a part of the Master of Science in Logistics at Molde University College, Specialized University in Logistics. This paper marks the end of our studies in Operations Management.

This research was conducted between November 2015 and May 2016.

We will like to thank our supervisor Professor Johan Oppen at Møreforskning Molde AS for helpful advice and insightful comments during our work with this thesis.

This thesis concerns the topic of routing of aircraft in northern Norway.

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Molde, Norway
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Abstract

This thesis concerns the topic of routing of aircraft in northern Norway. The purpose of the thesis was to make a relatively small model that could optimize the routes, when minimizing the total distance travelled.

The focus of the thesis was on the regional flight routes in the northern part of Norway. This is a special area when it comes to air transportation, there are long distances, few people, many small airports and chained trips with two or more legs. The regional air transport is regulated by PSO, which means that some legs have to be traversed even though it is not profitable. The focus of the thesis was on 28 airports in northern Norway and Trøndelag.

Due to the size of the problem, we decided to use a two-phase approach. The first step was to use the Fisher and Jaikumar algorithm and a model for the General Assignment Problem to divide the airports into clusters. Each cluster was built around a depot.

The second step was to make routes for each cluster using our modified model for a Vehicle Routing Problem. The model we made, takes into account the number of landings at each airport, the maximum duration of a roundtrip, the number of landings per roundtrip and the arrival and departure time at the depot and airports.

We have tested our model using six different scenarios. The scenarios contain different number of depots and different depots. The different depots were chosen based on the geographical location and the size of the airports. We compared the different scenarios based on the total distance travelled, the total cost and the total travel time.

The scenario that gave us the best solution, have Trondheim, Bodø, Tromsø and Kirkenes as depots. The depots are evenly spread among the area, and are located in different regions. The two-phase approach gave a reasonable solution, but in order to use the model on real life instances more extensions need to be implemented.

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1.0 Introduction

Norway has an unwelcoming nature with mountains, rivers, fjords, glaciers and moors, this combined with a rough climate make parts of the country less reachable. The topography results in large distances between settlements. The key factor to connect these remote areas to the rest of the world is to have a good air transportation network. Without good air transportation, these large areas would be more or less isolated. Air transport services are very important and it allows natural and human resources to be used efficiently in the society (Williams and Bråthen 2010).

Air transportation is very important when looking at the beneficial outcome in the society. "The benefit of high-speed travel mode like air transportation may be reflected in many ways: industries are better off, income levels are higher, population development is more favorable and the feeling of being remote is lower than if there are no air service." (Williams and Bråthen 2010, p. 61).

To ensure coverage of remote airports, the European Union established Public Service Obligations (PSO). The purpose of the PSO regulations is to ensure a minimum level of air service to the areas that depend on air transportation when considering the economic development of the regions.

Northern Norway has a special air transportation network, where many of the routes are chained air trips with two or more legs and most of the airports are small. These types of airports are called short take-off and landing (STOL) airports and they have a runway that is about 800 meters long. The focus in this thesis will be on 28 airports located in northern Norway and in Trøndelag.

Our supervisor Johan Oppen introduced us to this topic. It was brought to his attention when Møreforsking was writing a report for the Norwegian Ministry of Transportation and Communications. The topic of the report was the tendering arrangements for regional flights in northern Norway (Bråthen et al. 2015).

Routing of aircraft is an area that is not much explored when considering the model building part. Different models have been made, most of these are large and complex, which makes them difficult to solve.

2.0 Problem description

Today, there is one big actor operating in the aviation market in northern Norway. This actor is Widerøe, they have been operating routes in this area since 1955. Widerøe was one of the actors that were fighting a political battle to make the authorities open a STOL network in Norway. During the1960s and 1970s the authorities finally decided to establish small regional airports. The building of the regional airports on Helgeland was the first step in building a STOL network in Norway (Widerøe 2016). The already existing routes will not be analyzed in this thesis. Instead, we will explore if it is possible to make a mathematical model that can find the most efficient routes, considering different factors and objectives.

2.1 The geography and the airports in the north of Norway

As mentioned before, Norway is an elongated country with large distances between populations, mainly because of obstacles like mountains, rivers, moors and fjords. In the north there are additional factors that can affect the living conditions, one of these factors could for example be the extreme winter season with cold weather and few hours of daylight. These factors can also affect the transportations network in terms of closed roads, and railways, and bad landing conditions for aircraft.

There are different transportation methods that can be used when travelling in northern Norway. The options are to go by air, road, railway or sea. The last option might not be that favorable since it is both very time consuming and weather dependent. To go by road might be a good option if the distances are short. The Norwegian railway system does not go further north than Bodø, so this is often not an alternative. Ofotbanen goes between Narvik and the Swedish border, this will not be included as an alternative since it is not connected to the rest of the Norwegian railway system. Travelling by air seems to be the best option, since it is possible to travel large distances in relatively short time.

Only about 11% of the total population in Norway lives in northern part. Still, more than 50% of Avinors airports are located in this region (Store Norske Leksikon 2009).

Figure 1: Map of all Avinors airports

Figure 1 presents all Avinors airports in Norway (Avinor 2016a). In this thesis, the focus will be on the airports in northern Norway as well as the airports Trondheim, Rørvik and Namsos. Based on the length of the runway, the airports can be divided into three different categories, those are small airports – where the runway is less than 1000 meters, medium-sized airports – 1000 to 1799 meters and large airports – longer than 1799 meters. In table 1, we present the airports and their runway lengths.

Small airports		Medium airports	
Vardø	905	Kirkenes	1605
Vadsø	830	Sandnessjøen	1199
Mehamn	830	Brønnøysund	1199
Leknes	830		
Rørvik	830	**Large airports**	
Berlevåg	829	Bodø	2794
Mosjøen	829	Evenes	2716
Hammerfest	824	Lakselv	2604
Båtsfjord	810	Andenes	2468
Mo i Rana	801	Trondheim	2446
Honningsvåg	800	Tromsø	2003
Narvik	800	Bardufoss	1970
Hasvik	799	Alta	1907
Storslett/Sørkjosen	799		
Stokmarknes	799		
Namsos	798		
Svolvær	776		

Table 1: Airports and their runway lengths

(Avinor 2016a)

2.2 PSO – Public service regulations

> In order to enable governments to maintain essential air services, Article 16, 17 and 18 of Regulation (EEC) No. 1008/2008 define a system of public service obligations (PSOs) which can impose on carriers operating on designated routes. In essence, the legislation allows Member States to impose a public service obligations in respect of scheduling air service between any airport in the Community and an airport serving a peripheral or development region within its territory or on a thin route to any airport in its territory considered vital for the economic and social development of the region served by the airport. If no airline is willing to provide a service under the conditions imposed, the government may restrict access to the route to a single carrier and award financial compensation to the carrier in return for compliance with the PSO. (Williams 2010,p. 99)

Looking at the list of Public Service Obligations provided by the European Commission in December 2015, Norway is the country with the largest number of PSO routes (51), followed by France (45) (European Commission 2015). One of the reasons why Norway has so many PSO routes is because of the large distances and the relatively small number of people living in the districts. The small number of people living there results in low revenue for the airlines and makes the routes unprofitable in the free market. The Norwegian Ministry of Transport and Communications carries out the PSO tendering process. Before the PSO tendering process is carried out, the Ministry sends out an "Invitation to tender". This document consists of obligations applied to the individual routes, which says something about the minimum frequencies, seating capacity, routing and the timetables.

2.2.1 PSO routes in northern Norway

The Ministry tenders the PSO routes out for a period of about 4 years each time. The PSO routes in northern Norway were last out for tendering in 2012, and this agreement lasts until 2017. Below we see an overview over the routes that were included in the PSO tendering process in 2012 (Northern Norway) and 2013 (Finmark and North-Troms).

Northern Norway:

1. Lakselv – Tromsø v.v.
2. Andenes – Bodø v.v.
3. Svolvær – Bodø v.v.
4. Leknes – Bodø v.v.
5. Narvik (Framnes) – Bodø v.v.
6. Brønnøysund – Bodø v.v., Brønnøysund – Trondheim v.v.
7. Sandnessjøen – Bodø v.v., Sandnessjøen – Trondheim v.v.
8. Mo i Rana – Bodø v.v., Mo i Rana – Trondheim v.v.
9. Namsos – Trondheim v.v., Rørvik – Trondheim v.v.

(The Ministry of Transport and Communications 2012a)

Finnmark and North-Troms:

1. Routes between Kirkenes, Vadsø, Vardø, Båtsfjord, Berlevåg, Mehamn, Honningsvåg, Hammerfest and Alta.
2. Hasvik – Tromsø v.v., Hasvik – Hammerfest v.v., Sørkjosen – Tromsø v.v."

(The Ministry of Transport and Communications 2012b)

Examples of obligations applying to the route Lakselv – Tromsø:

	Weekdays (Mon-Fri)	**Weekends (Sat–Sun)**
Frequencies	Min. 3 daily return services	Min. 3 return services combined
Seating capacity	690 seats in both direction	135 seats in both directions
Routing	In both directions at least 2 of the required daily services should be non-stop	In both directions, at least 2 of the required services combined shall be non-stop.
Timetables	First arrival Tromsø no later than 08.30. Last departure from Tromsø no earlier than 19.30. First departure from Tromsø should be no later than 11.30. Last departure from Lakselv no earlier than 17.00	
Aircraft	Minimum 30 seats	

Table 2: PSO applying to the route Lakselv – Tromsø

These obligations apply throughout the year. There are different requirements for each route included in the tendering process.

2.3 Laws and regulations concerning air traffic

When conducting a flight plan, many law and regulations should be followed.
Presented below are the regulation we consider most important for our problem:

- Home base – Each aircraft needs to have a permanent home.
- Flight duty period (FDP) – this is the time during which a person operates in the aircraft as a member of its crew. For a flight that starts between 17.00 and 05.00 the maximum flight duty period is 10 hours, for short-haul flights starting between 06.00 and 13.29 the maximum duration is 13 hours. In some cases the FDP can be maximum 14 hours, this is only if the flights starts between 07.00 and 13.29 and the resting period before and after the flight is extended (EASA European Aviation Safety Agency 2016b). Between 13.30 and 16.59 the maximum duration decreases 15 minutes for each half hour, starting at 12 hours and 45 minutes at 13.30 and ending at 11 hours and 15 minutes at 16.59 (EASA European Aviation Safety Agency 2016a).

There are also regulations regarding maintenance on the aircraft. There are two types of maintenance:

- Ongoing maintenance, a regular inspection and correction of minor errors.
- Heavier maintenance that occurs in regular intervals, this could be based on number of landings, flight time or a specific number of weeks, months, etc..

2.4 What we want to achieve with our research

During our research, we want to make a model that will give us a reasonable and feasible solution. The goal is to make the model such as it takes into account all the deterministic factors, like the PSO regulations and the laws and regulations concerning air traffic. We will use different objectives like minimizing total distance, total cost and total travel time when testing our model. In addition, will we analyze the different routes given by the different objectives and try to combine the solutions into one optimal route.

2.5 Research questions

The questions we want to answer throughout the work on this thesis will be presented in this section.

2.5.1 Main research questions

1: Is it possible to make a model that is solvable in reasonable time and also gives a feasible solution?

2: What is the best combination of routes?

2.5.2 Sub-questions

1.1: What research has been done in this area?

1.2: Is the solution possible to implement in real life?

1.3: What happens to the computation time when we add one more airport?

1.4: How many variables and constraints can the model contain, and still be solvable?

1.5: Can we combine different existing models to make one that fits our problem?

2.1: Which combination of clusters gives the best solution?

2.2: Are the PSO regulations and the laws considered in these routes?

3.0 Literature review

The relevant theory and literature for our thesis will be presented in this chapter.
This includes the Vehicle Routing Problem, exact solution methods, heuristics and previously work in the field of VRP and aircraft routing. The VRP is a well-covered topic, the more specific problem of aircraft routing is less researched.

3.1 VRP

The Vehicle Routing Problem (VRP) is a combinatorial optimization and integer programming problem that seeks to service a number of customers with a fleet of vehicles. The main goal of the VRP is to find a set of routes at a minimum cost (time, travel distance, number of trucks, etc.), beginning and ending in the same node, and at the same time fulfilling the demands of all the nodes. The vehicles have a limited capacity and each node can only be visited once. Laporte defined VRP as the problem of "designing least-cost delivery routes from a depot to a set of geographically scattered customers, subject to side constraints" (Laporte 2009, p. 408).

Dantzig and Ramser first introduced the VRP in the article "The Truck Dispatching Problem" from 1959. In the paper, they dealt with the problem of finding the optimum routes for a fleet of gasoline delivery trucks, between a depot and a number of stations. They tried to find a method for assigning stations to trucks, such that the demand is fulfilled and the total distance travelled is minimized. In this article they formulated the problem as a generalization of the "Travelling-Salesman Problem" (Dantzig and Ramser 1959).

Figure 2 shows an example of a typical input for a VRP. In this example there are 16 customers represented by the blue points and one depot in the middle. Figure 3 shows one of the possible solutions for the instance in figure 2.

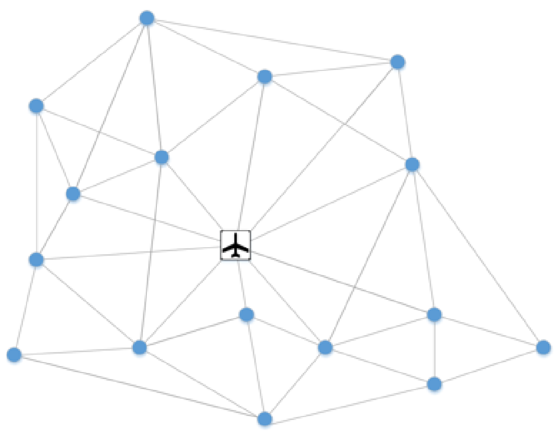

Figure 2:An instance of a VRP

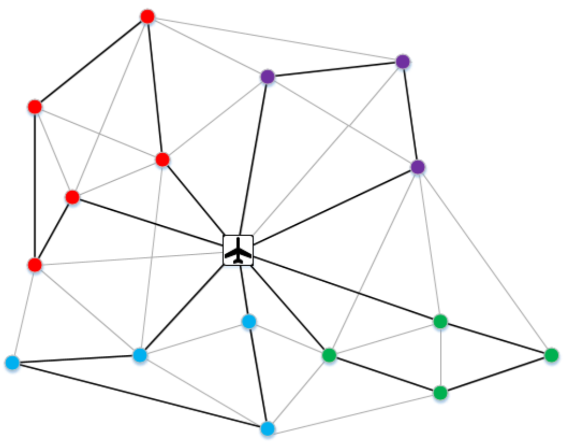

Figure 3: Feasible solution of a VRP instance

3.1.1 TSP

The "Travelling Salesman Problem" is defined as the problem of "finding a route of a salesman who starts from a home location, visits a prescribed set of cities and returns to the original location in such a way that the total distance travelled is minimum and each city is visited exactly once" (Gutin and Punnen 2007, p. 1). The TSP is a hard combinatorial optimization problem, but an optimal solution can be found for problems with up to several thousand of nodes. The VRP is a much harder problem.

The TSP can be visualized using a complete graph *G*, which can be directed or undirected. Each edge has a cost associated with it. The objective of the TSP is to find a tour (Hamiltonian cycle) in *G,* such that the cost is minimized (the sum of costs of all edges in the tour). A Hamiltonian cycle is a route that visits each node exactly once (Gutin and Punnen 2007).

3.1.2 The basic VRP

The basic version of the VRP is the capacitated VRP (CVRP). Here, all the customers correspond to deliveries. The demand from the customers is deterministic, which means that it is known in advance. In addition, the demand cannot be split. There is a set of identical vehicles with a given capacity, and one depot from where the vehicles depart and arrive. The objective of the CVRP is to minimize the total cost, while serving all the customers (Toth and Vigo 2002).

The CVRP is defined on a complete graph $G = (V, A)$ where $V = \{0, .., n\}$ is a set of nodes and *A* is the arc set. The vertices $i = 1, .., n$ correspond to the customers. Node 0 represents the depot. The node $(n + 1)$ can also represent the depot. There is a travel cost, c_{ij}, associated with each arc $(i, j) \in A$, which is the cost of travelling from node *i* to node *j*. It is not allowed to use loop arcs, (*i*,*i*) (Toth and Vigo 2002).

The CVRP can be asymmetric or symmetric. When the cost matrix is asymmetric, it means that the cost of travelling between two nodes *i* and *j* is different based on which direction you travel. This problem is called the asymmetric CVRP (ACVRP). In the symmetric version (SCVRP), the cost matrix is symmetric $c_{ij} = c_{ji}$ for all$(i, j) \in A$ (Toth and Vigo 2002).

The customers have a nonnegative demand, d_i, that should be delivered. The depot does not have a demand. At the depot, a set of *K* vehicles are available to serve the customers. Each vehicle has a given capacity *C*. To ensure that the problem is feasible, the demand from any customer should be less than the vehicle's capacity. A vehicle can only drive one route. The minimum number of vehicles needed to serve the customer set S is denoted by r(S) (Toth and Vigo 2002).

The CVRP aims to find a set of exactly *K* routes with a minimum cost (sum of the costs of all arcs included in the route). These routes have to satisfy the constraints such that each route visits the depot, each customer are visited by exactly one route and the sum of the demand from the customers visited on a route does not exceed the vehicle's capacity (Toth and Vigo 2002).

Next, the basic model for the VRP will be presented.

Objective function

$$min \sum_{i \in V} \sum_{j \in V} c_{ij} * X_{ij} \qquad (1)$$

Subject to

$$\sum_{i \in V} X_{ij} = 1 \qquad \forall j \in V \backslash \{0\}, \qquad (2)$$

$$\sum_{j \in V} X_{ij} = 1 \qquad \forall\, i \in V \backslash \{0\} \qquad (3)$$

$$\sum_{i \in V} X_{i0} = K \qquad (4)$$

$$\sum_{j \in V} X_{0j} = K \qquad (5)$$

$$\sum_{i \notin S} \sum_{j \in S} X_{ij} \geq r(S) \qquad \forall S \subseteq V \backslash \{0\}, S \neq \emptyset \qquad (6)$$

$$X_{ij} \in \{0,1\} \qquad \forall\, (i,j) \in V \qquad (7)$$

This model is a two-index vehicle flow formulation; the binary variable X is used to indicate which arcs that are travelled in the optimal solution. The objective of the model is to minimize total cost of the routes. Constraint (2) says that exactly one arc has to enter each node *j*, and constraint (3) ensures that exactly one arc has to leave each node *i*. Constraints (4) ensures that the number of arcs into the depot is equal to the number of vehicles, and constraint (5) ensures that the number of arcs out from the depot should be equal to the number of vehicles. Constraint (6) is a capacity cut constraint that imposes both the vehicle capacity requirement and the connectivity of the solution. The variables domain are given in constraint (7) (Toth and Vigo 2002. P. 12).

3.1.3 VRP Extensions

There exist several variants of the VRP. One extension is where the number of available vehicles is higher than the minimum number of vehicles needed. In this case, it is normal that each vehicle has a cost associated with using it and a new constraint will be to minimize the number of routes driven. Another version will be when the vehicles have different capacities (Toth and Vigo 2002).

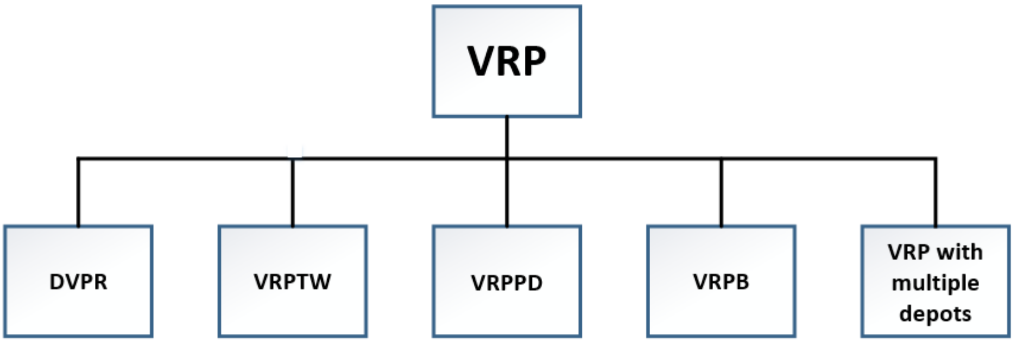

Figure 4: Shows the VRP extensions mentioned in this thesis

The version **Distance-Constrained VRP (DVRP)** is when the capacity constraint is replaced by a maximum length or time constraint. Each arc is associated with a nonnegative length. The total length of a route cannot exceed the maximum length of a trip. When the length of an arc is given in travel time, a service time at each customer may be given (number of time periods the vehicle must stop at the customer). The objective is to minimize the total length or duration of the routes, when service time is included (Toth and Vigo 2002).

In the **VRP with Time Windows (VRPTW)** each customer has a time window, which decides when the service at the customer should start and end. The travel time for each arc and the service time at each customer is also given. If the vehicle arrives at the customer before the time window opens, it must wait before starting the service. The VRPTW aims to find a set of routes that minimizes the total costs, and satisfy the constraints such that each route visits the depot, each customer is visited by exactly one route, the demand on the route does not exceed the vehicle's capacity and each customer is served within their time window (Toth and Vigo 2002).

The **VRP with Backhauls (VRPB)** is an extension of the CVRP. Here, the customers are divided into two subsets, linehaul and backhaul customers. With linehaul customers the delivery demand is higher than the pickup demand, for backhaul customers it is opposite. In VRPB, there is a precedence constraint between the customers; all linehaul customers have to be served before backhaul customers on a route. In the VRPB, the aim is to find a set of routes, with a minimum cost, that will satisfy a number of constraints. Each route visits the depot, each customer is visited by one route, the total demand of the customers on the route does not exceed the capacity of the vehicle and in each route, the linehaul customers are served before the backhaul customers. Routes that contain only backhaul customers are not allowed. (Toth and Vigo 2002).

In **VRP with Pickup and Delivery (VRPPD),** each customer has both a pickup and delivery demand (d and p). The demand can sometimes be presented with only one quantity, the net demand (positive or negative). In the VRPPD, it is assumed that delivery is performed before the pickup. The load before arriving at a customer is calculated as the starting load from the depot minus all delivered demand plus all that is picked up from the customers already visited. The aim of the VRPPD is to find a set of routes, that minimizes the total cost and satisfies the constraints. Each route has to visit the depot, each customer is visited by only one route and the load along the route can neither be negative nor exceed the capacity of the vehicle (Toth and Vigo 2002).

In **VRP with multiple depots**, there are multiple depots where the vehicles can be scheduled to leave from.

3.2 Exact solution methods

The next chapter is going to present the exact methods that can be used to solve our problem. Exact methods in general are not able to find the optimal solution in reasonable time for problems with more than 50 customers (Oppen and Løkketangen 2006). The disadvantages of using an exact method is that it usually have long computation time, on the other hand these methods always give a globally optimal solution. The most used exact methods are branch-and-bound and the branch-and-cut algorithms.

3.2.1 Branch-and-Bound

The Branch-and-Bound algorithm is defined in (Winston 2003) as a method which implicitly enumerates all possible solutions to an integer problem. By solving a sub problem, many possible solutions may be eliminated from consideration. Branch-and-bound is also well described in (Laporte 2009).

3.2.2 Cutting plane

The Cutting plane method starts with finding a solution to a linear problem. If the solution to the problem is fractional, it can be solved by creating a set of constraints that can cut off the fractional solution. The new solution is optimal if it is integer. If the solution is not integer, then continue to add new constraints until an integer solution is found (Toth and Vigo 2002).

3.2.3 Branch-and-cut

The Branch-and-Cut algorithm is presented in (Mitchell 2002) as a method that guarantees optimality. The algorithm is a combination of a cutting plane method and a branch-and-bound algorithm. The algorithm consists of solving the linear relaxation to get an integral solution, and then proceeds with a classical branch-and-bound method (Toth and Vigo 2002).

3.3 Heuristics

A heuristic is a method for solving a problem. This method can give a good solution faster than the exact methods. However, it does not give any guarantee for finding the optimal solution. The heuristics used for the VRP problem can be classified into two main classes, these are classical heuristics which was developed between year 1960 and 1970 and metaheuristics (modern heuristics) (Toth and Vigo 2002).

3.3.1 Classical heuristics

The classical heuristics produce typically good solutions within relatively short computation time. This is when performing a relatively limited exploration of the search space. Classical heuristics can be divided into three different categories; constructive heuristics, improvement heuristics and two-phase heuristics. The constructive heuristics keep an eye on the solution cost when gradually building a feasible solution. An example of a constructive heuristic is Clark and Wright algorithm (savings algorithm), this is one of the best known heuristics for the VRP (Toth and Vigo 2002). Improvement heuristics for the VRP use search mechanism to try to improve a feasible solution, this can be done on a single-route or on a multiroute (Toth and Vigo 2002). Two-phase heuristics is a combination of finding a solution in two different phases, clustering and routing. Some of the algorithms are Fisher and Jaikumar algorithm (which will be explained in more detail below) and Christifides, Mingozzi and Toth algorithm.

Fisher and Jaikumar algorithm

Fisher and Jaikumar presented the cluster-first, route-second heuristic in 1981 (Fisher and Jaikumar 1981). The Fisher and Jaikumar heuristic will always find a feasible solution if one exists. It is also easy to adapt the heuristic to handle additional problems like multiple depots, multiple time periods, capacity constraints and constraints on duration of the routes. The solution quality outperforms heuristics like Clark and Wright and Christofides. In tests done by Fisher and Jaikumar, their heuristic found the best solution in 9 out of 12 problems, and on average it provided the best solution value. (Fisher and Jaikumar 1981).

The heuristic consists of two parts, the first part is to locate the seeds and construct clusters around them. This is done in order to minimize the distance between the customers and the seeds and at the same time it has to satisfy the capacity constraint. The second part is to use the TSP to determine a route for each cluster (Laporte 2009). Fisher and Jaikumar solve a Generalized Assignment Problem (GPA) to form the clusters.

A seed is a specific customer node that needs to be visited by a specific vehicle. There will be the same number of seed nodes as vehicles. The seed nodes can be chosen randomly, but to get a good solution, some sense should be used when deciding the nodes. This can be done by selecting nodes that probably not would be served by the same vehicle in the optimal solution. That could be nodes located geographically far from each other, or nodes that have large demands, so that it would violate the capacity if they were served by the same vehicle.

The steps of the algorithm are:

Step 1: The seed selection. Choose seed vertices $j_k \in V$ to initialize each cluster *k*.

Step 2: Allocation of customers to seeds. Compute the cost d_{ik} of allocating each customer *i* to each cluster *k* as $d_{ik} = \min\left\{c_{0i} + c_{ij_k} + c_{j_k0}, c_{0j_k} + c_{j_ki} + c_{i0}\right\} - \left(c_{0j_k} + c_{j_k0}\right)$.

Step 3: Generalized assignment. Solve a GAP with costs d_{ij}, customer weights q_i, and vehicle capacity Q.

Step 4: TSP solution. Solve a TSP for each cluster corresponding to the GAP solution. (Toth and Vigo 2002, p.117).

The Generalized Assignment Problem

Let n be the number of tasks to assign to m agents and define $N=\{1,2,\ldots,n\}$ and $M=\{1,2,\ldots,m\}$. The parameter c_{ij} is the cost of assigning task j to agent i.
The parameter r_{ij} is the amount of resource required for task j by agent i. Let b_i be the resource units available to agent i.

The decision variable, X_{ij} , is equal to one if task j is assigned to agent i.

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot X_{ij} \tag{8}$$

St.

$$\sum_{j=1}^{n} r_{ij} \cdot X_{ij} \leq b_i \qquad \forall\, i \in M \tag{9}$$

$$\sum_{i=1}^{m} X_{ij} = 1 \qquad \forall\, j \in N \tag{10}$$

$$X_{ij} = 0 \text{ or } 1, \qquad \forall\, i \in M, j \in N \tag{11}$$

The main objective (8) is to minimize the total cost of the assignment. Constraint (9) enforces resource limitation and constraint (10) ensures that each agent gets exactly one task. The variables domain are displayed in constraint (11) (Nauss 2006).

3.3.2 Metaheuristics

Metaheuristics can be seen as a natural improvement of the classical heuristics, they perform a deep exploration of the most promising regions of the solution space. The quality of the solution is higher when using metaheuristics, at the same time the computation time increase (Toth and Vigo 2002). Metaheuristics can be divided into local search, population search and learning mechanisms. Tabu search and simulated annealing are local search algorithms. These algorithms start the search from an initial solution and move to another solution in the neighborhood (Laporte 2009). Population search, like Genetic algorithms, mimics the process of the natural selection (Toth and Vigo 2002).

3.4 Aircraft routing

In this section of the paper, some of the articles that are written on the subject of aircraft routing will be presented.

In an article written by Desaulniers et al. (1997), the authors consider the daily aircraft routing and scheduling problem (DARSP). The objective is to maximize the profit derived from the aircraft of a heterogeneous fleet when determining daily schedules. In this article they use two different models; in the first they define a binary variable for each possible schedule for an aircraft type giving rise to a large Set Partitioning-type problem. In the second model they present a binary variable representing the possible connection between two flight legs performed by a particular aircraft. This is a time constrained multicommodity network flow formulation. Both models are integer-programming models and they are solved by branch-and-bound. They use this definition of DARSP: "*Given a heterogeneous aircraft fleet, a set of operational flight legs over a one-day horizon, departure time windows, duration and costs/revenues according to the aircraft type for each flight leg, find a fleet schedule that maximizes profits and satisfies certain additional constraints.*" (Desaulniers et al. 1997, p. 842).

Pita, Adler and Antunes (2014) present a socially oriented flight scheduling and fleet assignment optimization model (SFSFA). The objective function is to minimize the total social costs. The aim of the paper is to assist "*the public authorities in the design of subsidized air transport network, with specific analysis of the requirements such network should meet with respect to the level of service offered to passengers"* (Pita, Adler, and Antunes 2014,17). They analyze the different results obtained from the model from the perspectives of passengers, airline, airport and government to compare the service levels and the funding. The model considers airport cost and revenues, travel time, passenger demand and social welfare. They also consider the PSO requirements. Their main research question is: *"How should an air transport network that is operated as a monopolistic public service be organized such that network costs are minimized?"* (Pita, Adler, and Antunes 2014,18). The SFSFA-model is used on a single day that is divided into time-periods, and on routes with up to two intermediate stops. The authors have used their model on the PSO network in Norway (Pita, Adler, and Antunes 2014).

3.5 Other relevant articles

In an article written by Daniel Karapetyan and Abraham P. Punnen (2013) , the authors present an integer programming model for the ferry scheduling problem (FSP). The aim of the FSP is to find a route and a schedule for the ferries, so that the demand at the ferry ports is satisfied, while minimizing operations cost and passenger dissatisfaction. In the model, they are given a set of ports, a set of ferries and a planning horizon. In their model, they also include new constraints such as passenger transfers, crew scheduling and loading/unloading. They were able to make a model that gives a good solution in 12 hours, when using CPLEX 12.4 (Karapetyan and Punnen 2013).

In a report conducted by Møreforskning about the tendering arrangements in northern Norway, there is also a part about modeling (Bråthen et al. 2015). This part was called "A mathematical model for planning of aviation routes" and is written by Johan Oppen. The model that is presented is based on an extension of the Vehicle Routing Problem, where they take into consideration factors like cost, time and capacity related to the aircraft and the flight. The objective function is to minimize the total cost, this includes the sum of variable costs for all legs flown and the fixed cost for using the aircraft.

4.0 Analysis

Before building the model, it is important to get an overview over all the factors that can affect a flight. The factors we consider as relevant for our problem will be analyzed in this chapter.

4.1 Modeling choices and assumptions

This section will present the choices and assumptions taken in this thesis. Routing of aircraft is a complex problem, and the routes can be influenced by many factors. To make the problem easier to handle, all these factors needs to be evaluated and the problem needs to be limited.

4.1.1 Uncertainty

The weather, human errors and mechanical errors are all different types of uncertainty. A flight can be affected by any of these, some more often than others. The weather is a big uncertainty factor, especially in northern Norway where the weather often changes. Snow, wind and freezing temperatures can all delay a planned flight. We have only mentioned some of the uncertainty factors, the reason is that we will not include any of these in our model. If the uncertainty were to be included in the model, both the computation time and the complexity would increase. That is why the model in this thesis are going to be deterministic.

4.1.2 Planning horizon

The planning horizon is set to be one day, and it is assumed that the routes are the same for each day. More specific, the planning horizon is going to be between 05:00 and 24:00. The reason for choosing this horizon is that most of the flights happen during these hours. In addition, limiting the planning horizon will make the problem easier to handle.

This means that an aircraft cannot leave the depot before 05:00 in the morning, and it has to be back at the depot before 24:00 in the evening.

4.1.3 Airports

We wanted to explore how our model would handle routes with multiple short legs. Since most of the chained air trips in Norway are carried out in the northern parts, this will be the focus area of this thesis. The area consists of 28 airports, which includes three airports in Trøndelag. In addition, are some of the smallest airports excluded, this is because they usually only handle helicopter traffic. Table 3 shows the airports included in this thesis.

The depots are where the aircraft will stay overnight and where the maintenance will happen. The depots are chosen based on size and location of the airports. Why and how the specific depots are chosen will be explained in more detail in chapter 6.2.

Trondheim	TRD	Stokmarknes	SKN	Lakselv	LKL
Namsos	OSY	Narvik	NVK	Honningsvåg	HVG
Rørvik	RVK	Evenes	EVE	Mehamn	MEH
Brønnøysund	BNN	Andenes	ANX	Berlevåg	BVG
Mosjøen	MJF	Bardufoss	BDU	Båtsfjord	BJF
Sandnessjøen	SSJ	Tromsø	TOS	Vardø	VAW
Mo i Rana	MQN	Storslett	SOJ	Vadsø	VDS
Bodø	BOO	Alta	ALF	Kirkenes	KKN
Leknes	LKN	Hasvik	HAA		
Svolvær	SVJ	Hammerfest	HFT		

Table 3: List of airports

4.1.4 Aircraft

There will be a number of available aircraft, and the model will decide how many aircraft to use to cover the visit frequency. The size of the aircraft is not important since the visit frequency is used instead of the passengers demand.

4.1.5 Visit frequency

Visit frequency will be used instead of passenger demand. The visit frequency is the number of landings on a specific airport during the time horizon, and this frequency will be based on the size of the airports. The visit frequency does not take into consideration where the passengers are travelling to and from.

4.1.6 Cost

The fixed and variable cost will be considered in this thesis. The variable costs are the cost of flying, only the fuel costs will be included here. In addition, there are costs of using an airport and the services that the airport provides. Those are included in the fixed costs. The fixed cost of using an airport consist of: take-off charge, terminal fee, air navigation fee, passenger and security charges (Avinor 2016b). These costs are fees that the airlines have to pay to Avinor, when using an airport owned by Avinor. The Norwegian Ministry of Transport and Communications regulates these fees.

4.1.7 Roundtrips

The aircraft will have a maximum number of times they can fly out of the depot to service a route. This is implemented to allow an aircraft to fly more than one roundtrip. Each of the roundtrips have a time limit of 32 time periods, which is equal to 8 hours. The reason for choosing this number was that the flight duty period for short haul flights are maximum 10 hours. Because our maximum time for a roundtrip does not include the time the aircraft is on the ground between flights, it is calculated some slack into the time limit. Based on this, we have chosen to limit the number of roundtrips per aircraft to a maximum of three trips per time horizon. If an aircraft flies the maximum duration of a roundtrip, it only has time to travel two roundtrips. The possibility that an aircraft uses 32 time periods on a roundtrip is small, as it will be a limit on how many airports an aircraft can visit during a roundtrip.

4.2 Parameters

The parameters that will be used in the model will be presented in this section. The parameters will be described in more detail later in this thesis.
As mentioned before all our parameters are deterministic.

- Visit frequency
- Distance
- Travel time
- Time periods
- Number of roundtrips
- A big number
- Service time at the depot
- Maximum time of one roundtrip
- Maximum number of landings per roundtrip

5.0 Model

In this chapter the model will be presented and explained in detail. It is used a two-phased approach to solve the problem. The reason for this is that it is difficult to make a model that uses exact methods when handling a multiple-depot VRP with 28 airports.

In the first phase it will be used an algorithm to divided the airports into different clusters. Each cluster will have a depot. The second phase will be to make routes for the clusters, this will be done by using our modified model for the VRP.

5.1 First phase mathematical formulation

The first phase will present the method used to divide the airports into different clusters.

To make clusters out of the airports, the algorithm by Fisher and Jaikumar is used. In this thesis the objective of this algorithm is to minimize the total distance travelled between the depot and the airports. Another possibility would be to minimize the travel time or the cost. The goal of the clustering model is to connect the airports to the depots.

The first step is to choose seed-nodes, one for each cluster. The next step is to calculate the added distance when connecting the airports to the seed nodes. The model described in 5.5.1 will be used to minimize the added distance and find the clusters. This model is based on the model for the generalized assignment problem, presented in 3.3. When the clusters are found, the VRP model presented in 5.2 will be used to make the routes for each of the clusters.

The Fisher and Jaikumar heuristic is originally meant for problems with one visit to each customer, but it is possible to use in a problem that allows multiple visits to a customer on a route. The GAP will connect each node to a seed node and the model for the VRP will determine the routes and the number of visits.

5.1.1 F&J model

Let N be a set of nodes and P a set of aircraft. The parameter ad_{ip} is the added distance of connecting node *i* to aircraft *p*. The binary variable Y_{ip} is equal to 1 if node *i* is connected to aircraft *p*.

In order to calculate the added distance from each depot, we used a version of the calculation presented in 3.3. The formula presented in the theory can be written as $ad_{ip} = c_{ij_k} + c_{i0} - c_{0j_k}$. In our specific problem, the seed nodes also are the depots, so we can change the formula to $ad_{ip} = 2 * c_{ij_k}$. That means the added distance of connecting an airport to a depot is the distance from the depot to the airport and back. This way, we ensure that all airports will be connected to the nearest depot.

$$\min \sum_{i \in N} \sum_{P \in P} ad_{ip} \cdot Y_{ip} \qquad (12)$$

St.

$$\sum_{p \in P} Y_{ip} = 1 \qquad i \in 1..N \qquad (13)$$

$$Y_{ip} \in \{0,1\} \qquad i \in 1..N, p \in P \qquad (14)$$

The main objective (12) is to minimize the total added distance. Constraint (13) ensures that each node is linked together with only one aircraft. The variables domain are displayed in constraint (14).

5.2 Second phase mathematical formulation

In this part, the mathematical model for the modified VRP will be presented. The notation used in the model will be presented first, then the model and the constraints, and at last the different extensions.

Notation:

Let $\mathcal{A}$ be a set of airports, and $\mathcal{P}$ be a set of aircraft. As mentioned before, we only use one aircraft size. In addition, all the aircraft needs to have a “home-base” that they operate from. We have decided that the depot should be the base.

Let $\mathcal{N} = 0,1,\ldots,N$ be a set of nodes, where $0 \in \mathcal{N}$ is the depot and $N \in \mathcal{N}$ is the copy of the depot. Let *ARC* be the set of arcs $(i,j) \in ARC$, which represents all the arcs in the network. The distance is given by d_{ij} and the flying time is tt_{ij}.

Let T be the time periods, each time period is equal to 15 minutes and there are 76 time periods. Each aircraft have to stay at an airport for one period between arrival and departure. Let R be the number of roundtrips. When the aircraft has finished one roundtrip it has to stay at least a given number of time periods *sd* in the depot before it leaves for the next roundtrip.

The parameter *Tmax* gives the travel time between the two nodes that are located furthest apart from each other. This parameter is used to make sure that the routes end before the time horizon is over. We have the parameter vf_j , which is the visit frequency for airport *j*. The parameter M represents a large number.

Let *l* be the maximum number of landings during one roundtrip. The maximum number of allowed landings at each airport for each aircraft per roundtrip is represented by la.

We have decided to operate with two types of binary routing variables. X_{ijptr} is equal to 1 if aircraft *p* leaves airport *i* to go to airport *j* in time period *t* on roundtrip *r*, 0 otherwise. Y_{ijptr} will take the value 1 if aircraft *p* arrives at airport *j* from airport *i* in time period *t* on roundtrip *r*, 0 otherwise.

Sets:	
$\mathcal{N}$	set of nodes
$\mathcal{A}$	set of airports
$\mathcal{P}$	set of aircraft
ARC	set of arcs $(i,j) \in ARC, i \in \mathcal{N} \setminus \{N\}, j \in \mathcal{N} \setminus \{0\}$
Parameters:	
T	Number of time periods
R	Number of roundtrips
Tmax	The longest travel time between the nodes
M	Big number
Rmax	Maximum duration of the route in time periods
vf_j	Visit frequency for node *j* $j \in \mathcal{A}$
d_{ij}	Distance from node *i* to node *j* $i \in \mathcal{N}, j \in \mathcal{N}$
tt_{ij}	Travel time from node *i* to node *j* $i \in \mathcal{N}, j \in \mathcal{N}$
sd	Service time at the depot in time periods
l	The maximum allowed number of landings during one roundtrip
la	The maximum allowed number of landings at each airport for each aircraft on each roundtrip
Decision variables:	
X_{ijptr}	1 if aircraft *p* leaves airport *i* in time period *t* to go to airport *j* on roundtrip *r*, 0 otherwise $(i,j) \in ARC, p \in \mathcal{P}, r \in 1..R, t \in 1..T$
Y_{ijptr}	1 if aircraft *p* arrives at airport *j* in time period *t* from airport *i* on roundtrip *r*, 0 otherwise $(i,j) \in ARC, p \in \mathcal{P}, r \in 1..R, t \in 1..T$

Table 4: Notation - sets, parameters and variables

Mathematical model:

$$min \sum_{(i,j)\in ARC} \sum_{p\in \mathcal{P}} \sum_{t=1}^{T} \sum_{r=1}^{R} X_{ijptr} * \ d_{ij} \tag{15}$$

$$\sum_{(0,j)\in ARC} \sum_{t=1}^{T} X_{0jptr} \geq \sum_{t=1}^{T} X_{ikptr} \qquad p \in \mathcal{P}, r \in 1..R, (i,k) \in ARC \tag{16}$$

$$\sum_{(0,j)\in ARC} \sum_{t=1}^{T} X_{0jptr} = \sum_{(i,6)\in ARC} \sum_{t=1}^{T} X_{i6ptr} \qquad p \in \mathcal{P}, r \in 1..R \tag{17}$$

$$\sum_{i\in \mathcal{N}} \sum_{t=1}^{T} X_{ijptr} = \sum_{k\in \mathcal{N}} \sum_{t=1}^{T} X_{jkptr} \qquad j \in \mathcal{A}, p \in \mathcal{P}, r \in 1..R \tag{18}$$

$$\sum_{(i,j)\in ARC} \sum_{p\in \mathcal{P}} \sum_{t=1}^{T} \sum_{r=1}^{R} X_{ijptr} \geq \ vf_j \qquad j \in \mathcal{A} \tag{19}$$

$$\sum_{t=1}^{T} \sum_{i\in \mathcal{A}} X_{0iptr} \geq \sum_{t=1}^{T} \sum_{i\in \mathcal{A}} X_{0ipt(r+1)} \qquad p \in \mathcal{P}, r \in 1..R-1 \tag{20}$$

$$\sum_{(0,j)\in ARC} \sum_{t=1}^{T} X_{0jptr} \leq 1 \qquad r \in 1..R, p \in \mathcal{P} \tag{21}$$

$$X_{ijptr} = \ Y_{ijp(t+tt_{ij})r} \qquad (i,j) \in ARC, p \in \mathcal{P}, \ t \in 1..T - Tmax, r \in 1..R \tag{22}$$

$$\sum_{i\in \mathcal{N}} X_{ijptr} = \sum_{k\in \mathcal{N}} X_{jkp(t+1)r} \qquad j \in \mathcal{A}, p \in \mathcal{P}, t \in 1..T-1, r \in R \tag{23}$$

$$t + \ X_{ijptr} * \ tt_{ij} \leq T \qquad (i,j) \in ARC, p \in \mathcal{P}, t \in 1..T, \ r \in 1..R \tag{24}$$

$$\sum_{t=T-T_{max}}^{T} X_{ijptr} = 0 \qquad (i,j) \in ARC, p \in \mathcal{P}, r \in 1..R \tag{25}$$

$$M * \left(1 - \sum_{j \in \mathcal{A}} \sum_{u=1}^{T} X_{0jpu(r+1)}\right) + \left(\sum_{j \in \mathcal{A}} \sum_{u=1}^{T} X_{0jpu(r+1)} * u\right) \geq \sum_{i \in \mathcal{A}} Y_{i6ptr} * (t + sd) \qquad t \in 1..T, p \in \mathcal{P}, r \in 1..R-1 \qquad (26)$$

$$X_{ijptr} \in \{0,1\} \qquad (i,j) \in ARC \qquad (27)$$

$$Y_{ijptr} \in \{0,1\} \qquad (i,j) \in ARC \qquad (28)$$

5.2.1 Model description

The objective function (15) is to minimize the total distance travelled for all aircraft during the time horizon.

Constraint (16) says that in order to leave node *i*, the aircraft has to start the trip by leaving the depot and go to some node *j*. Constraint (17) is a continuity constraint for the depot. The constraint ensures that the number of aircraft leaving the depot is equal to the number of aircraft arriving at the depot at the end of a roundtrip. Continuity constraint (18) make sure that the number of aircraft arriving at a node is the same number as aircraft leaving that exact node. It ensures balance for all nodes, aircraft and roundtrips. Constraint (19) says that the number of visits at an airport should be larger or equal to the required visit frequency.

Constraint (20) is implemented to ensure the right order of the roundtrips, which says that if the aircraft are to fly roundtrip two, then roundtrip one have to be flown first. Constraint (21) prevents the same aircraft from leaving the depot more than one time during each roundtrip.

Constraints (22) and (23) connects the two types of binary routing variables. Constraint (22) ensures that if the aircraft leaves node *i* then it has to arrive at node *j* a given number of time periods after leaving node *i*. Constraint (23) make sure that if an aircraft arrives at an airport, the same aircraft has to leave that airport one time period later. Constraint (24) is a time constraint, ensuring that no arcs are travelled after the time horizon is over.

Constraint (25) says that no aircraft can travel on any of the arcs (*i*,*j*) during the last time periods. When saying that the time for starting the last leg cannot be after T-Tmax, the constraint makes sure that the last leg travelled will end before the time period is over. Tmax is the longest travel time between two airports in each cluster. This means that the legs with shorter travel time could have flown later than T-Tmax, and the aircraft would still have made it back to the depot before the end of the time horizon.

Constraint (26) is a "Big M"-constraint, the main goal is to ensure that the time of the roundtrips are correct. The constraint says that the time of departure for the next roundtrip should be later than the time of arrival included the service time at the depot.

The variables domain are given in constraints (27) and (28).

5.2.2 Extensions

This part will describe the different extensions of the model.

The first extension is a time-constraint.

$$\sum_{(i,j)\in ARC} \sum_{t=1}^{T} X_{ijpt} * \; tt_{ij} \leq Rmax \qquad p \in P, r \in 1..R \qquad (29)$$

Constraint (29) ensures that the active flight time does not exceed a given number of time periods. The active flight time is the time the aircraft is in the air. In the first chapter we presented the different regulations concerning the flight duty period (FDP), we included this constraint to make the model more realistic.

$$\sum_{t=1}^{T} \sum_{(i,j)\in ARC} X_{ijptr} \leq la \qquad p \in P, r \in 1..R, j \in A \qquad (30)$$

This constraint (30) ensures that during one roundtrip, the aircraft can only land a given number of times at each airport.

$$\sum_{(i,j)\in ARC}\sum_{p\in P}\sum_{r=1}^{R} X_{ijptr} + \sum_{(i,j)\in ARC}\sum_{p\in P}\sum_{r=1}^{R} Y_{ijptr} \leq 1 \qquad t \in 1..T, j \in A \tag{31}$$

Constraint (31) ensures that no aircraft lands and take off in the same time period at the same airport.

$$\sum_{(0,j)\in ARC}\sum_{p\in P}\sum_{r=1}^{R} X_{0jptr} + \sum_{(i,N)}\sum_{p\in P}\sum_{r=1}^{R} Y_{i(N)ptr} \leq 1 \qquad t \in 1..T \tag{32}$$

Constraint (32) ensures that no aircraft lands and take off at the depot in the same time period.

$$\sum_{t=1}^{T}\sum_{(i,j)\in ARC} X_{ijptr} \leq l \qquad p \in P, r \in 1..R \tag{33}$$

This constraint (33) limits the total number of landings for each aircraft on each roundtrip.

$$\sum_{t=1}^{20}\sum_{(i,j)\in ARC}\sum_{p\in P}\sum_{r=1}^{R} X_{ijptr} \geq 1 \qquad j \in A \tag{34}$$

Constraint (34) ensures that at least one aircraft leaves each airport before 10:00 in the morning (which is equal to time period 20).

$$\sum_{t=48}^{T}\sum_{(i,j)\in ARC}\sum_{p\in P}\sum_{r=1}^{R} X_{ijptr} \geq 1 \qquad j \in A \tag{35}$$

Constraint (35) ensures that there is at least one aircraft leaving each airport after 17:00 in the afternoon (equal to time period 48).

5.3 Data collection

The data used in this thesis is secondary data collected from different sources. This section will present the data that will be used when solving the model. The data presented is based on the factors we analyzed earlier in the thesis. The data consists of visit frequency, service time, costs, geographical distance, and the time it takes to travel between two particular airports.

5.3.1 Distances

The distances between the airports are based on a direct line measured in kilometers. The data was collected from a website called Distance24. This website gave the direct distance between all the airports included in the thesis (Distance24 2016). Table 5 show the distance used when testing the model.

Distance							
		1	2	3	4	5	6
Trondheim	1		127	165	247	299	307
Namsos	2	127		46	120	172	181
Rørvik	3	165	46		85	142	144
Brønnøysund	4	247	120	85		58	61
Mosjøen	5	299	172	142	58		33
Sandnessjøen	6	307	181	144	61	33	

Table 5: Distance data used when testing the first model

The overview over the rest of the distances can be found in Appendix A.

5.3.2 Travel time

The travel time and distance of the routes that are operated today was collected from Widerøe. The different routes are divided into four groups according to the length: from 0 to 99 kilometers, 100 to 249 kilometers, 250 to 399 kilometers and routes that are longer than 400 kilometers. From these four groups and the real data provided by Widerøe it was calculated a factor, this factor is the average number of kilometers travelled per minute. To get the right travel time on the different legs, the real distance between the airports was divided by the right factor according to the length of the route. After finding the travel time in minute this was changed into time periods of 15 minutes. The groups and the average number of kilometers per minute can be found in table 6, and the calculations can be found in appendix B. The overview over all the distances can be found in appendix A. Example: lets say that there is a route that is 299 kilometers, to find the time it takes to travel the route we took the distance 299 and divide this on the factor 5,7 which is equal to 53 minutes.

Distance (km)	**Average km/min**	Median	σ
400 ->	**6,4**	5,8	1,2
250 - 399	**5,7**	5,9	0,7
100 - 249	**4,6**	4,7	0,6
0 - 99	**3,0**	3,0	0,7

Table 6: Kilometers per minute

In our model, it is used discrete time and one time period is equal to 15 minutes. The time horizon is from 05:00 to 24:00, which is equal to 76 time periods.

Time periods							
		1	2	3	4	5	6
Trondheim	1		1	2	3	4	4
Namsos	2	1		1	1	2	2
Rørvik	3	2	1		2	2	2
Brønnøysund	4	3	1	2		1	1
Mosjøen	5	4	2	2	1		1
Sandnessjøen	6	4	2	2	1	1	

Table 7: Travel time in periods

Overview over all the travel times can be found in appendix C.

5.3.3 Service time

The service time is defined as the time from an aircraft lands on an airport until it leaves the same airport. Included in this time is the unloading and loading of passengers and baggage, as well as cleaning and document handling.

We have analyzed the flights in northern Norway, and found that an aircraft on average uses between 10 to 25 minutes on the ground in between two flights. Based on that analysis, we have decided that the service time (time on the ground) should be one time period. When it comes to the service time at the depot, we have decided that it should be at least three periods. Meaning that the aircraft have to stay in the depot for at least three time periods before leaving for the next roundtrip.

5.3.4 Visit frequency

The visit frequency indicates how many times an airport should be visited during the time horizon. The visit frequency will be based on the size of the airports. Large airports should be visited more times than smaller airports. The visit frequency will be used as an indicator to find the distribution in landings between the airports.

The visit frequency can be found in appendix D.

5.3.5 Costs

The cost is divided into variable and fixed costs. The variable cost will consist of the fuel cost. This cost is calculated from the fuel consumption and the fuel price as shown below.

Fuel consumption: 2.3 liter per kilometer (FlightRun 2015).
Fuel price: 2.4 NOK per kilometer (index mundi 2016).
Fuel cost: 2.3 liter/km*2.4 NOK/liter = 5.52 NOK/km

The fixed cost includes the costs concerning the use of an airport, the costs of handling the aircraft, as well as the safety cost. The fees included in our calculations are: take-off charge, terminal fee, air navigation fee, passenger charge and security charge. The take-off, terminal and air navigation fees are all based upon the size of the aircraft. The take-off charge is also based on the size of the airport from where the aircraft is leaving. From the international and national airports, the cost of take-off is 64 NOK * MTOW, which means the maximum takeoff weight. The regional airports have a 30% discount. A Dash 8 aircraft is used as a basis in the calculations. This aircraft has a maximum take-off weight of 17 tons. The Dash 8 is the same aircraft as Widerøe operate on some of the routes today.

The terminal fee is based on the number of service units and the size of the airport. A service unit is calculated as (MTOW/50)ˆ0,7. For Trondheim the cost is 1787.43 NOK per service unit, for all the other airports it is 1251.20 NOK per unit. The air navigation fee is 381.42 NOK per service unit (Avinor 2016b).

The safety and the passenger cost are both based on the number of passengers in the aircraft. The safety charges are 56 NOK per passenger and the passenger charge is 54

NOK per passenger (Avinor 2016b). To calculate the passenger and safety charges per trip, we have assumed a 60% coverage, as it is not normal that every aircraft is fully booked.

Total cost will consist of: take-off fee + safety fee + passenger fee + terminal fee + air navigation fee + fuel cost.

The safety, passenger and the air navigation fee are the same for each airport and can be excluded from the calculation. Then the calculations of the costs of travelling between airports, based on which airport you are travelling from, are as following:

Total costs	
Trondheim	1927.97+5.52*km
Tromsø and Bodø	1675.98+5.52*km
Other	1349.58+5.52*km

Table 8: Total cost of travelling from the different airports.

A detail explanation of the calculation can be found in appendix E, and an overview of travelling cost between airports can be found in Appendix F.

6.0 Computational experiments

In this part, the method used to solve the problem will be presented, as well as the different scenarios used. Since the problem consist of 28 airports with multiple landings on each airport, the Fisher and Jaikumar algorithm will be used to divide the different scenarios into clusters. The results of the clustering will be presented and explained later in this chapter.

There will also be a part where the model will be tested and analyzed, to see if the model needs to be changed or if more extensions need to be implemented. After the test the new modified model will be used on the different clusters.

6.1 Method

To solve the problem the AMPL modeling language will be used and the problem will be solved through a Gurobi solver, which are provided by the NEOS server. The reason why an Internet based solver is used is because the solver on the school computers have a relatively low capacity, and the NEOS server gives us a shorter computation time.

AMPL is an algebraic modeling language used for large-scale optimization and mathematical programming problems (Fourer, Gay, and Kernighan 2003). We have learned this language during our master program.

"The NEOS Server is a free internet-based service for solving numerical optimization problems" (Wisconsin Institutes for Discovery 2016a). NEOS can handle relatively large problems, but it will terminate jobs that are not finished in 8 hours without giving out any results. This is why we have chosen to use a time limit of maximum 7 hours when testing the model. The research organization that operate the website, Morgridge Institute of Research (MIR) does not guarantee that the output of the service is the correct result or that it will be completed (Wisconsin Institutes for Discovery 2016b). The NEOS server does also have a memory limitation of 3 GB RAM. In table 9 we have provided an overview over the hardware specifications for the NEOS server used in this thesis.

neos-7 is a Dell PowerEdge R430 server with the following configurations: • CPU – 2x Intel Xeon E5-2698 @ 2.3 Hz (32 cores total), HT Enabled • Memory – 192 GB RAM • Disk – 4x 300G SAS drivers setup in RAID5 • Network – 1 Gb/s Ethernet

Table 9: Hardware specifications for the NEOS solver

6.2 Scenarios

We have decided to use different numbers of seed nodes, these are divided into three different groups: one with three seed nodes, one with four seed nodes and the last one with five seed nodes. For each of these groups we will have two different scenarios, meaning that there will be different combination of seed nodes in each of the two scenarios. One of the scenarios in each group is based only on the location of the seed nodes. The other scenario in the group will be a more realistic combination of seed nodes. This is done to compare the different outcomes.

In the beginning of the clustering chapter, it was mentioned that the clustering could be based on minimizing either the total distance, total time or total cost. It is not necessary to use anything else than distance since both the time and the cost are dependent on the distance, and the clusters would be the same. Instead, the total cost and time are displayed for each of the clusters and the scenarios used in the model.

The different scenarios will be presented in the next part of this chapter. In addition, will it be explained why these specific seed nodes are chosen.

6.2.1 Scenario 1: Trondheim, Bodø, Tromsø

Trondheim, Bodø and Tromsø are natural to choose when only choosing three seed nodes in the northern part of Norway. This is based on the geographical location and the number of people living in the area surrounding the airports. These three towns are also the capital of their region, and there are institutions like hospitals and universities located there.

6.2.2 Scenario 2: Bodø, Tromsø, Kirkenes

In this scenario, Kirkenes replaces Trondheim. The reason for this is that Kirkenes is in Finnmark, and by placing one of the seed nodes there we can reach many of the smaller airports located in the area. We excluded Trondheim as a seed node because in our data set we only had three airports in Trøndelag. There was no need of having Trondheim as a seed node when considering the total distance travelled. In a real life situation, this would not be an optimal choice since Trondheim is connected to the rest of Norway, and the demand in the region is high.

Even thought the county administration in Finnmark is located in Vadsø, it is more natural to choose Kirkenes as the seed node in this area. The reason for this is that Kirkenes is a much larger airport and it have direct connections to Oslo.

6.2.3 Scenario 3: Trondheim, Bodø, Tromsø, Kirkenes

In this scenario there are four different seed nodes. These are Trondheim, Bodø, Tromsø and Kirkenes. These seed nodes are chosen based on the average domestic demand at the airports and the distances between the chosen seed nodes. In addition, the seed nodes are located in different regions: Trøndelag, Nordland, Troms and Finnmark.

This scenario is similar to the situation today, where all the seed nodes are located relatively far from each other. In addition, based on the demand, they are the four largest airports in the northern part of Norway.

6.2.4 Scenario 4: Trondheim, Brønnøysund, Tromsø, Lakselv

In this scenario we have only considered the location of the airports, but this would most likely not work in the real life. Trondheim is chosen as a seed node since this airport covers the airports furthest south in the data area. Brønnøysund is chosen because it is a medium sized airport in between Trondheim and Tromsø. Tromsø is included as a seed node since it is natural to choose when considering the location. Lakselv is chosen as a seed node because it is located in the middle of Finnmark.

6.2.5 Scenario 5: Trondheim, Mosjøen, Bodø, Tromsø, Lakselv

In this scenario we have used the same strategy as in scenario 4, where all the seed nodes are chosen based only on the geographical location. The size of the airports are not considered, neither are the domestic demand of the different airports. Trondheim, Tromsø and Lakselv are chosen for the same reason as mentioned in scenario 4. In this scenario we have chosen to have two seed nodes between Tromsø and Trondheim, the best choice based on the distance was then to choose Bodø and Mosjøen.

6.2.6 Scenario 6: Trondheim, Bodø, Evenes, Tromsø, Kirkenes

When choosing the seed nodes for this scenario factors like size of the airports, number of visits each day and the geographically location was taken into account. In this scenario, it is only Evenes that has not been chosen as a seed node in any of the previous scenarios. Evenes is chosen because it is a large sized airport located in between Tromsø and Bodø.

6.3 *Result of clustering*

The different clusters and scenarios are presented in table 10. In the table, the seed nodes are displayed with bold font. The model created the clusters by minimizing the added distance connecting the airports and the depot.

	Scenario 1	**Scenario 2**	**Scenario 3**	**Scenario 4**	**Scenario 5**	**Scenario 6**
Cluster 1	**Trondheim**, Namsos, Rørvik	Trondheim, Namsos, Rørvik, Brønnøysund, Mosjøen, Sandnessjøen, Mo i Rana, **Bodø**, Leknes, Svolvær, Stokmarknes	**Trondheim**, Namsos, Rørvik	**Trondheim**	**Trondheim**, Namsos	**Trondheim**, Namsos, Rørvik
Cluster 2	Brønnøysund, Mosjøen, Sandnessjøen, Mo i Rana, **Bodø**, Leknes, Svolvær, Stokmarknes	Narvik, Evenes, Andøya, Bardufoss, **Tromsø**, Sørkjosen, Alta, Hasvik, Hammerfest	Brønnøysund, Mosjøen, Sandnessjøen, Mo i Rana, **Bodø**, Leknes, Svolvær, Stokmarknes	Namsos, Rørvik, **Brønnøysund**, Mosjøen, Sandnessjøen, Mo i Rana, Bodø	Rørvik, Brønnøysund, **Mosjøen**, Sandnessjøen, Mo i Rana	Brønnøysund, Mosjøen, Sandnessjøen, Mo i Rana, **Bodø**, Leknes
Cluster 3	Narvik, Evenes, Andøya, Bardufoss, **Tromsø**, Sørkjosen, Alta, Hasvik, Hammerfest, Lakselv, Honningsvåg, Mehamn, Berlevåg, Båtsfjord, Vardø, Vadsø, Kirkenes	Lakselv, Honningsvåg, Mehamn, Berlevåg, Båtsfjord, Vardø, Vadsø, **Kirkenes**	Narvik, Evenes, Andøya, Bardufoss, **Tromsø**, Sørkjosen, Alta, Hasvik, Hammerfest	Leknes, Svolvær, Stokmarknes, Narvik, Evenes, Andøya, Bardufoss, **Tromsø**, Sørkjosen	**Bodø**, Leknes, Svolvær, Stokmarknes	Svolvær, Stokmarknes, Narvik, **Evenes**, Andøya
Cluster 4			Lakselv, Honningsvåg, Mehamn, Berlevåg, Båtsfjord, Vardø, Vadsø, **Kirkenes**	Alta, Hasvik, Hammerfest, **Lakselv**, Honningsvåg, Mehamn, Berlevåg, Båtsfjord, Vardø, Vadsø, Kirkenes	Narvik, Evenes, Andøya, Bardufoss, **Tromsø**, Sørkjosen	Bardufoss, **Tromsø**, Sørkjosen, Alta, Hasvik, Hammerfest
Cluster 5					Alta, Hasvik, Hammerfest, **Lakselv**, Honningsvåg, Mehamn, Berlevåg, Båtsfjord, Vardø, Vadsø, Kirkenes	Lakselv, Honningsvåg, Mehamn, Berlevåg, Båtsfjord, Vardø, Vadsø, **Kirkenes**

Table 10: Overview over the scenarios and the clusters

By analyzing table 10, we see that the difference between scenario 1 and 2 is that the airports in scenario 2 is more evenly spread among the seed nodes in the clusters. Cluster 3 in scenario 1 contains 17 airports and cluster 1 in the same scenario only contains three airports. This imbalance can make it harder to solve the modified VRP for the clusters that contains many airports.

Both scenario 3 and 4 does have four seed nodes. When Trondheim and Brønnøysund both are chosen as seed nodes same as in scenario 4, Trondheim ends up alone. The reason for this is that the other airports nearest to Trondheim are located closer to Brønnøysund than Trondheim.

In the scenarios with five seed nodes, the airports are more evenly spread among the seeds. There are still some clusters that are bigger than others, one example of this is that in both scenario 5 and 6 cluster 5 is the biggest cluster. These clusters does both have a seed node located in Finnmark. In our data, Finnmark is the area with the most airports located relatively close to each other.

When analyzing the results from the clustering and the routes provided by the modified model, it is important to evaluate if it is possible to implement these routes in the real life. It is also important that the seed nodes are located in a large city, which houses institutions like hospital, university and the county administration.

6.4 Testing the modified VRP model

To explore how the model behaves and if it gives a feasible solution, we have to start with a relatively small problem. It is easier to find the mistakes when using a smaller amount of data. The test will be used to figure out which combination of constraints that fits our problem the best. The results from this test will be used to build the model that we will use throughout the thesis.

First, the data used to test the model will be presented, after that the model will be tested with different constraints.

Visit frequence:	
Namsos	3
Rørvik	2
Brønnøysund	4
Mosjøen	6
Sandnessjøen	4

Table 11: Visit frequency

In the process of testing we focused on six airports. These airports were Mosjøen, Brønnøysund, Sandnessjøen, Rørvik, Namsos and Trondheim (which represented the depot). The distance data and the time period data are the same as presented in chapter 4. The visit frequency used in these tests are shown in table 11. We also specified that each aircraft only could travel maximum three roundtrips during the time horizon. The data presented in this part will be used on all the different tests conducted in this chapter.

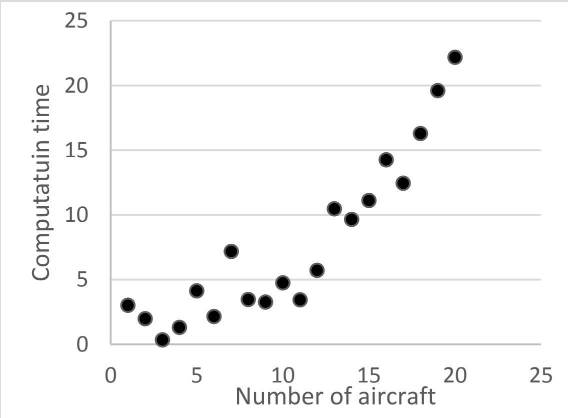

Figure 5: Results from NEOS

We wanted to test the computation time of NEOS. To do this we used constraints (15) to (23) and changed the number of aircraft one by one. The reason why we did not use all constraints is that the computation time of the whole model is too long. To get a clearer picture of which number of aircraft that has the best computation time, we have decided that each aircraft only can fly one roundtrip. The results from this test are shown in figure 5.

Figure 5 displays the computation time from NEOS. In this figure, we can see that it is a pattern. The computation time increases when more aircraft are added, but there are still some decreasing that cannot be explained. Example, the computation time when using seven aircraft is 7.19, but when we use eight aircraft the time decreases to 3.47.

From this test, the conclusion is that it is important to think about how many aircraft that will be used in the data. If there are excessively many aircraft available, the runtime of the model will increase. That is why we need to adjust the number of available aircraft in relation to the size of the clusters.

6.4.1 Test 1

In this test, the constraints from (15) to (28) is used. The extensions is not included. This test is done to explore how the output of the model is, and to see if there are any constraints missing. Table 12 presents the routes made by this model.

Test 1	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TRD	6	OSY	8
	OSY	9	MJF	12
	MJF	13	SSJ	14
	SSJ	15	BNN	16
	BNN	17	MJF	18
	MJF	19	BNN	20
	BNN	21	SSJ	22
	SSJ	23	MJF	24
	MJF	25	OSY	28
	OSY	29	TRD	31
Aircraft 3 Roundtrip 1	TRD	24	OSY	23
	OSY	27	RVK	28
	RVK	29	MJF	31
	MJF	32	BNN	33
	BNN	34	SSJ	35
	SSJ	36	MJF	37
	MJF	38	SSJ	39
	SSJ	40	BNN	41
	BNN	42	MJF	43
	MJF	44	RVK	46
	RVK	47	TRD	50

Table 12: Routes from test model 1

The output from test 1 shows that the model only chooses to use two aircraft and not any roundtrips. This is because the model does not have any limitations regarding the number of time periods used on each trip. Each aircraft visits at least nine airports. From the passengers perspective, this is not an optimal route. For example, if someone is supposed to travel from Namsos (OSY) to Trondheim, they have three options. They can take the aircraft leaving Namsos 07:15 (time period 9) or they can take the aircraft leaving Namsos 12:15 (time period 29). The catch is that those two options are the same aircraft on the same roundtrip and they will not arrive in Trondheim before 12:45. That is why the morning flight most likely will not be that popular, unless they are travelling to Mosjøen or Sandnessjøen. The last option is to travel by aircraft 3 which leaves Namsos 11:45 (time period 27) and arrives in Trondheim 17:30 (time period 50), after visiting nine other airports first.

When running the model and the data file in NEOS, it was discovered that the computation time was long. In order to find the reason for this, the model was tested with and without constraints (24), (25) and (26). These exact constraints were chosen since it was after implementing these constraints that the models computation time increased. In table 13, the different combinations of constraints are presented.

Model:		Computation time (seconds) :	Objective:	Comments :	NEOS	
					Job:	Password:
4Mod A	Without 24,25,26	180,51	2532		4493505	twFqNWSD
4ModA-1	Without 25,26	171,36	2532		4492775	alcHoGCS
4ModA-2	Without 24,25	117,37	2532		4492781	QvNgLFHq
4ModA-3	Without 24,26	Time limit: 10800	3064	No basis	4492817	ilpvXsWN
4ModA-4	Without 24	Time limit: 10800	3064	No basis	4492825	dcWvJsjy
4ModA-5	Without 25	154,16	2532		4492842	GlOwQPev
4ModA-6	Without 26	Time limit: 10800	3064	No basis	4492855	ULknCVTS

Table 13: Testing the computation time

From these tests, we can see that it is a connection between the increased computation time and constraint (25). NEOS have terminated all the tests including constraint (25), when running the model with a time limit of 3 hours. The results came back with no basis, which means that the solver have not found enough proof to say that the solution is the optimal one. To see if it the model could find an optimal solution after a longer time, the time limit was increased to 7 hours. The result was still the same after 7 hours as it was after 3 hours.

Constraint (25) says that the aircraft cannot fly any of the arcs during the last time periods.

Constraint (25) cannot be excluded from the model, because this constraint make sure that there are no aircraft flying during the last periods of the time horizon. If the constraint were to be removed, aircraft would be flying out from airports and not reaching back to the depot before the time horizon was ending.

Conclusion from this test is that the constraint that increases the computation time, cannot be removed. This constraint is necessary in order to ensure that all the roundtrips will be finished and the aircraft are at their depot at the end of the time horizon. In the next test, a new constraint concerning the active flight time, which is the actual time the aircraft is in the air, will be implemented. This to make sure that each flight is no longer than it is supposed to, when considering the regulations.

6.4.2 Test 2

In this test, a new constraint (29), which is a time-constraint that says that the active flying time should not be more than a given number of time periods, will be included

$$\sum_{(i,j)\in ARC} \sum_{t=1}^{T} X_{ijpt} * \; tt_{ij} \leq Rmax \qquad p \in P, r \in 1..R \qquad (29)$$

Table 14 show the routes from the model used in test 2. The result is almost the same as in test 1. This time aircraft 3 and 4 are used. In this test, another problem occurs. The problem is that two aircraft seems to be at the exact same airports at the exact same time. Moreover, two aircraft cannot land or take-off at the same airport at the same time. Therefore, a new constraint that can prevent this from happening is implemented in the next test.

Test 2	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 3 Roundtrip 1	TRD	5	RVK	8
	RVK	9	MJF	11
	MJF	12	BNN	13
	BNN	14	SSJ	15
	SSJ	16	MJF	17
	MJF	18	SSJ	19
	SSJ	20	BNN	21
	BNN	22	MJF	23
	MJF	24	OSY	27
	OSY	28	TRD	30
Aircraft 4 Roundtrip 1	TRD	4	OSY	6
	OSY	7	RVK	8
	RVK	9	MJF	11
	MJF	12	BNN	13
	BNN	14	SSJ	15
	SSJ	16	MJF	17
	MJF	18	SSJ	19
	SSJ	20	BNN	21
	BNN	22	MJF	23
	MJF	24	OSY	27
	OSY	28	TRD	30

Table 14: Routes from test 2

The result shows that the constraint that was implemented regarding the active flight time did not make any difference. A possible reason could be that the maximum active flight time was set to be 8 hours. This is because the flight duty period for short-haul flights are maximum 10 hours. There is some slack, since the constraint only restricts the time the aircraft is in the air and not including the time an aircraft is at an airport. For example, when looking at the total time from aircraft 4 leaves the depot in time period 4, and until it arrives back at the depot in time period 30, it has been 6 hours and 30 minutes. This do not exceed the time limit, and there is no reason to decrease this limit either, since the FTD is 10-12 hours.

6.4.3 Test 3

The next step is to implement constraints that prevents the aircraft to land more than one time at the same airport during one roundtrip. It is also implemented constraints saying that the aircraft cannot arrive or depart from the same airport or depot in the same time period. When testing these constraints, constraint (29) was removed and only the basic model and constraints (30), (31) and (32) was used.

$$\sum_{t=1}^{T} \sum_{(i,j)\in ARC} X_{ijptr} \leq la \qquad p \in P, r \in 1..R, j \in A \qquad (30)$$

$$\sum_{(i,j)\in ARC} \sum_{p\in P} \sum_{r=1}^{R} X_{ijptr} + \sum_{(i,j)\in ARC} \sum_{p\in P} \sum_{r=1}^{R} Y_{ijptr} \leq 1 \qquad t \in 1..T, j \in A \qquad (31)$$

$$\sum_{(0,j)\in ARC} \sum_{p\in P} \sum_{r=1}^{R} X_{0jptr} + \sum_{(i,N)} \sum_{p\in P} \sum_{r=1}^{R} Y_{i(N)ptr} \leq 1 \qquad t \in 1..T \qquad (32)$$

Constraint (30) limits the number of visits at each airport to one, for each aircraft per roundtrip. Constraint (31) ensures that no aircraft arrives or departs at the same airport in the same time period. Constraint (32) ensures that no aircraft arrives to or departs from the depot in the same time period.

Test 3	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TRD	4	OSY	6
	OSY	7	MJF	10
	MJF	11	SSJ	12
	SSJ	13	BNN	14
	BNN	15	RVK	16
	RVK	17	TRD	20
Aircraft 1 Roundtrip 2	TRD	35	MJF	40
	MJF	41	TRD	46
Aircraft 2 Roundtrip 1	TRD	62	MJF	67
	MJF	68	TRD	73
Aircraft 3 Roundtrip 1	TRD	1	MJF	6
	MJF	7	SSJ	8
	SSJ	9	BNN	10
	BNN	11	OSY	13
	OSY	14	TRD	16
Aircraft 3 Roundtrip 2	TRD	41	MJF	46
	MJF	47	SSJ	48
	SSJ	49	BNN	50
	BNN	51	TRD	55
Aircraft 4 Roundtrip 1	TRD	26	OSY	28
	OSY	29	MJF	32
	MJF	33	SSJ	34
	SSJ	35	BNN	36
	BNN	37	RVK	38
	RVK	39	TRD	42

Table 15: Routes from test 3

Presented in table 15, are the routes provided by this test. More aircraft have been used, since the one-landing constraint has been included. This time the model choose to use aircraft 1,2,3 and 4, in addition does aircraft 1 and 3 have two roundtrips.

When implementing these constraints, the time limit is not a concern anymore. Still, constraint (29) will be included in the model that will be used in the next chapter. The reason for this is that the clusters might be bigger, and that can result in longer flights and the time limit might be exceeded.

The constraints work, since the aircraft no longer are at the same airports in the same time periods.
In addition, the aircraft does not land more than one time on the same airport during one roundtrip.

6.4.4 Test 4

The constraint tested in test 4 is constraint (33), which says that an aircraft on a roundtrip can maximum land four times. This constraint have been implemented to prevent too many stops during a trip, as we could see in test 1 and 2, where each aircraft visits over nine airports. When testing this constraint, only the basic model and this extension will be used. This is to see how this extension will affect the solution.

$$\sum_{t=1}^{T} \sum_{(i,j)\in ARC} X_{ijptr} \leq l \qquad p \in P, r \in 1..R \qquad (33)$$

Test 4	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TRD	18	BNN	22
	BNN	23	SSJ	24
	SSJ	25	MJF	26
	MJF	27	TRD	32
Aircraft 1 Roundtrip 2	TRD	60	MJF	65
	MJF	66	SSJ	67
	SSJ	68	MJF	69
	MJF	70	TRD	71
Aircraft 2 Roundtrip 1	TRD	17	OSY	19
	OSY	20	TRD	22
Aircraft 2 Roundtrip 2	TRD	53	OSY	55
	OSY	56	RVK	57
	RVK	58	OSY	59
	OSY	60	TRD	62
Aircraft 3 Roundtrip 1	TRD	34	MJF	39
	MJF	40	BNN	41
	BNN	42	RVK	43
	RVK	44	TRD	46
Aircraft 4 Roundtrip 1	TRD	1	MJF	6
	MJF	7	SSJ	8
	SSJ	9	MJF	10
	MJF	11	TRD	16
Aircraft 4 Roundtrip 2	TRD	26	BNN	30
	BNN	31	SSJ	32
	SSJ	33	BNN	34
	BNN	35	TRD	39

Table 16: Routes from test 4

As shown in table 16, the constraint works as it is supposed to do. This time aircraft 1,2,3 and 4 is used, in addition does aircraft 1,2 and 4 have two roundtrips each.

The constraint concerning the start time of the next roundtrip is correct.

Constraint (33) is working; none of the aircraft visits more than three towns, not including the depot.

6.4.5 Test 5

In this test all the constraints and the extensions are implemented. In addition, there are two new constraints. The constraint maximum one landing per airport (30) is included, a limit concerning the maximum active flight time (29), maximum four landings per roundtrip (33) and constraint (31) and (32) preventing the aircraft to arrive and depart from the same depot or airport in the same time period.

$$\sum_{t=1}^{20} \sum_{(i,j)\in ARC} \sum_{p\in P} \sum_{r=1}^{R} X_{ijptr} \geq 1 \qquad j \in A \qquad (34)$$

$$\sum_{t=48}^{T} \sum_{(i,j)\in ARC} \sum_{p\in P} \sum_{r=1}^{R} X_{ijptr} \geq 1 \qquad j \in A \qquad (35)$$

Test 5	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TRD	5	RVK	8
	RVK	9	SSJ	11
	SSJ	12	MJF	13
	MJF	14	TRD	19
Aircraft 1 Roundtrip 2	TRD	49	MJF	54
	MJF	55	BNN	56
	BNN	57	OSY	59
	OSY	60	TRD	62
Aircraft 2 Roundtri 1	TRD	1	OSY	3
	OSY	4	TRD	6
Aircraft 2 Roundtri 2	TRD	13	BNN	17
	BNN	18	SSJ	19
	SSJ	20	MJF	21
	MJF	22	TRD	27
Aircraft 3 Roundtrip 1	TRD	4	MJF	9
	MJF	10	BNN	11
	BNN	12	OSY	14
	OSY	15	TRD	17
Aircraft 3 Roundtrip 2	TRD	44	BNN	48
	BNN	49	SSJ	50
	SSJ	51	MJF	52
	MJF	53	TRD	58
Aircraft 4 Roundtrip 1	TRD	60	RVK	63
	RVK	64	SSJ	66
	SSJ	67	MJF	68
	MJF	69	TRD	74

Table 17: Routes from test 5.

The first of the new constraints are constraint number (34), which ensures that at each airport, at least one aircraft have to leave the airport before 10:00 (time period 20). The next constraint is number (35), which says that at least one aircraft have to land at each airport after 17:00 (timer period 48).

Table 17 presents the routes from test 5. The results shows that all the airports have been visited at least one time in the morning and one time in the afternoon. This time, aircraft 1,2,3 and 4 are used, aircraft 1,2 and 3 also have two roundtrips each. Figure 5 and 6 shows how the aircraft are flying. Figure 5 shows the routes between time period 1 and 38, and figure 6 shows the route between time period 39 and 76.

Figure 5: The routes between time period 1-38 test 5

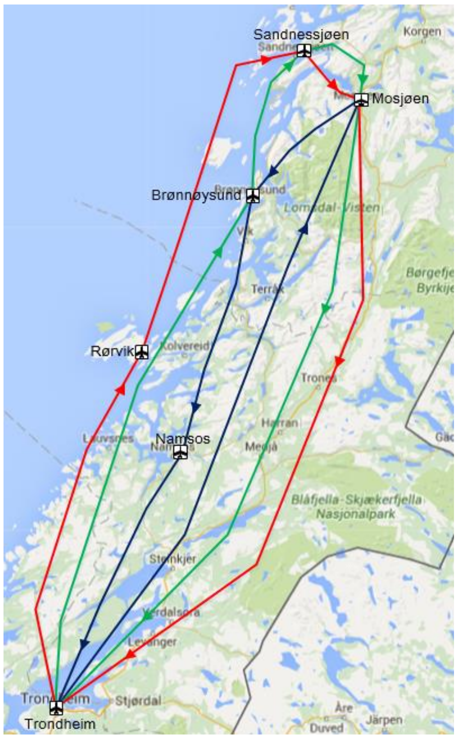

Figure 6: The routes between time period 39-76 test 5

6.4.6 Conclusion

The model that will be used in the next part of this thesis includes the basic model and all the extensions tested in this chapter. The problem includes in total 28 airports, this makes it too big to run as one, which is why the two-phase solution approach is going to be used. First the Fisher and Jaikumar algorithm will be used to divide into clusters, after that the modified VRP will be used on each of the clusters.

As seen in the tests, the model uses a long computation time when the data file only consists of six airports. This is one of the reasons for dividing into clusters.The result from the testing in NEOS also came back with “no basis”, which means that the program cannot be 100 percent sure that the solution found is actually the optimal one, even though it most likely is. This is because the program has not explored all the possible options yet.
We tested the model using time limits of 3 and 7 hours. We could not use more time than that since NEOS automatically terminates the job after 8 hours. The results from the same model tested in 3 and 7 hours, was exactly the same. That is why we have decided to use a time limit of 3 hours when running the model in the next part of the thesis. If there is no solution after 3 hours, we will extend the time limit.

During these tests, we also discovered that constraint (24) is not necessary to include. This is because we have constraint (25) saying that no aircraft can fly in the last periods. Therefore, it will not be included in the model.

In order to display total flight time used in one cluster, the total cost and the distance per aircraft per route, we have included three new sets of variables, a new parameter and three constraints. Variable D_{pr} is the distance flied by aircraft *p* in roundtrip *r*, variable *TC* is the total cost of all the routes flied during the time horizon and variable *FT* is the total flight time for all the legs flied during the time horizon. The parameter c_{ij} is the cost of flying from node *i* to node *j*. Constraint (36) calculates the distance flied by each aircraft on each roundtrip. The total flight time for all the legs flied in hours are calculated by constraint (37), and the total cost for all the routes are calculated by constraint (38). The variables domains are displayed in (39).

Decision variables:	
D_{pr}	The distance flied by aircraft *p* on roundtrip *r*
TC	The total cost of all the routes flied during the time horizon
FT	Total flight time for all the legs flied during the time horizon.
Parameters:	
c_{ij}	The cost of flying from node *i* to node *j* $i \in \mathcal{N}, j \in \mathcal{N}$

Table 18: Notation – variable and parameters

$$\sum_{t=1}^{T} \sum_{(i,j)\in ARC} X_{ijptr} \,.\, d_{ij} = D_{pr} \qquad p \in P, r \in 1..R \qquad (36)$$

$$\sum_{(i,j)\in ARC} \sum_{t=1}^{T} \sum_{p\in P} \sum_{r=1}^{R} X_{ijptr} * \frac{tt_{ij} * 15}{60} = FT \qquad (37)$$

$$\sum_{(i,j)\in ARC} \sum_{t=1}^{T} \sum_{p=P} \sum_{r=1}^{R} X_{ijptr} * C_{ij} = TC \qquad (38)$$

$$TC, D_{pr}, FT_{pr} \geq 0 \qquad (39)$$

7.0 Results

In this section, the results obtained by using the modified VRP on the different clusters found in section 6.3 will be presented . The model and the extensions used are also displayed in appendix H. The different routes, total distance, the total travel time (in hours) and the total cost for the different scenarios will also be presented. In addition, the routes for scenario 3 will be presented in detail. The routes for the rest of the scenarios can be found in appendix G.

The data used for solving the scenarios are the same as presented in chapter 4. In addition, we have decided that the maximum number of landings for each aircraft per roundtrip should be equal to five, including the landing at the depot. The maximum number of landings at each airport is two for each aircraft per roundtrip. The aircraft should not go back and forth between airports multiple times, which is the reason why we have restricted the number of visits. These numbers are fixed for all the scenarios, unless something else is stated in the text.

	Total distance	Total cost	Total time
Scenario 1	14 061	250 407	50
Scenario 2	13 646	253 478	49,5
Scenario 3	9 602	212 172	38,5
Scenario 4	9 022	175 137	33,75
Scenario 5	10 194	221 662	44,5
Scenario 6	9 602	213 123	37,5

Table 19: Total distance, cost and time for each scenario

Table 19 shows the total cost, distance and time for each of the scenarios. Scenario 4 is the best solution. This scenario has the shortest distance, uses least time and cost the least. This is because the cost and the time are related to the distance travelled. The seed nodes in scenario 4 are Trondheim, Brønnøysund, Tromsø and Lakselv. The seed nodes are chosen based only on the distance, the total distance travelled are 9022 kilometers. The seed nodes in scenario 5 were chosen based on the location, but the total distance travelled here was 1172 kilometers longer than scenario 4. Scenario 5 have the following seed

Figure 7: Seed nodes in scenario 5

nodes: Trondheim, Mosjøen, Bodø, Tromsø and Lakselv. These are shown in figure 7, the seed nodes are evenly spread around the country. In theory, scenario 5 should provide a better solution than scenario 4, since it consists of more seed nodes. This will minimize the distances from the depot to the airports. The reason why scenario 4 gives a better solution in terms of distance, could be because cluster 1 in scenario 4 only consist of Trondheim, which means that there are no aircraft arriving or departing from this seed node. It is not a good solution to have one seed node alone with no airports connected to it.

Scenario 1 and 2 consist only of three seed nodes, which made them harder to solve than the other scenarios. Since there were fewer seed nodes, each cluster became bigger, and the largest cluster consisted of 17 airports including the depot. The challenges we had while trying to solve it are presented in the last part of this chapter.

In the next part the solutions from scenario 3 will be presented.

7.1 Scenario 3

This scenario has Trondheim, Bodø, Tromsø and Kirkenes as seed nodes. The reason why this scenario is presented instead of any of the others, is that this scenario is the most likely to be implemented in real life. This scenario is most realistic because each of the seed nodes are located in one of the four regions. The seed nodes chosen are also one of the largest airports in their region.

Cluster 1	Cluster 2	Cluster 3	Cluster 4
TRD	**BOO**	**TOS**	**KKN**
OSY	BNN	NVK	LKN
RVK	MJF	EVE	HVG
	SSJ	ANX	MEH
	MQN	BDU	BVG
	LKN	SOJ	BJF
	SVJ	ALF	VAW
	SKN	HAA	VDS
		HFT	

Table 20: Clusters for scenario 3

Table 20 presents an overview over the different clusters in scenario 3, the seed nodes are displayed in bold font. We can see that three out of the four clusters are similar in size. The exception is cluster 1 which only consist of Trondheim, Namsos and Rørvik. This is because Trondheim is located in the south, and the majority of the airports in our data set are located in the northern part.

Table 21 provides an overview over the total distance travelled in each cluster, the total cost and the total time used in hours. It also presents the number of variable and constraints the model uses. In addition, the solution time is presented in seconds. This is the time used when the solver, which uses a cutting plane approach, finds the best solution. Not the time when it is finished exploring all the possible options.

Scenario 3	Distance	Cost	Time	Best sol. sec	Variables	Constraints
Cluster 1	768	18 256	2,5	1,14	6 116	4 492
Cluster 2	3 262	75 866	13	3 822	94 034	53 549
Cluster 3	3 165	66 977	13,75	2 423	90 290	50 795
Cluster 4	2 407	51 073	9,25	4 027	97 202	55 675
Total:	**9 602**	**212 172**	**39**			

Table 21: Data from all the clusters in scenario 3

Next, each of the clusters in scenario 3 will be presented.

Cluster 1

Cluster 1 consist of three airports. Those are Trondheim (which is the depot), Namsos and Rørvik. These airports are all located in Trøndelag.

S3C1	Departure		Arrival			
	Airport	Time	Airport	Time	Distance	768
Aircraft 1 Roundtrip 1	TRD	4	OSY	5	Cost	18 256
	OSY	6	RVK	7	Time	3
	RVK	8	OSY	9	Best sol. sec	1
	OSY	10	TRD	11	Variables	6 116
Aircraft 2 Roundtrip 1	TRD	49	RVK	58	Constraints	4 492
	RVK	52	OSY	55		
	OSY	54	RVK	53		
	RVK	56	TRD	51		

Table 22: Results from Scenario 3 Cluster 1

The results from this cluster are presented in table 22. The output shows that the model chooses to use two aircraft, one aircraft in the morning and one aircraft in the afternoon. Aircraft 1 leaves the depot at 06:00 in the morning, then it flies to Namsos, Rørvik, back to Namsos and ends up back at the depot 07:45. Aircraft 2 starts from the depot at 17:15 then it flies Rørvik, Namsos, back to Rørvik and is back at the depot 19:30.

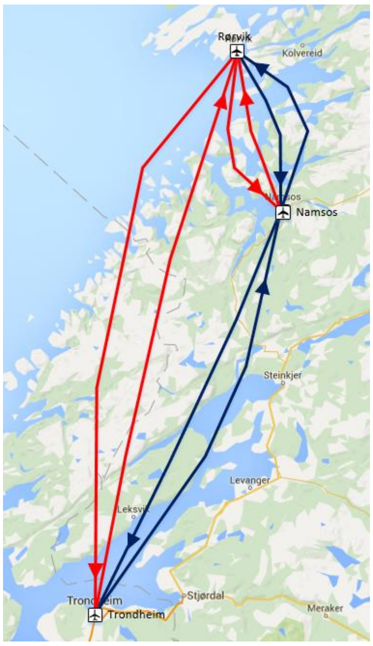

Figure 8: Routes from Scenario 3 Cluster 1

The aircraft visits both Namsos and Rørvik three times, which is equal to the visit frequency. Realistically would this route not be optimal, since it is not necessary to go from Rørvik to Namsos and then back to Rørvik and the other way around. This is because both airports are relatively small and there is not a very high demand on the flight leg between them. In addition, these two airports are close to each other, and it only takes about two hours to travel between them by car. As mentioned in the beginning of this chapter some of the parameters are fixed, for example the number of landings per airport is two for each aircraft per roundtrip.

For cluster 1 we decided to decreased the maximum number of landings on each airport to be one for each aircraft per roundtrip. This is done to remove the option of traveling from Namsos to Rørvik and back to Namsos. The result from this change is shown in table 23.

S3C1	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TRD	19	RVK	21
	RVK	22	OSY	23
	OSY	24	TRD	25
Aircraft 1 Roundtrip 2	TRD	57	OSY	58
	OSY	59	RVK	60
	RVK	61	TRD	63
Aircraft 2 Roundtrip 1	TRD	10	OSY	11
	OSY	12	RVK	13
	RVK	14	TRD	16

Distance	1 014
Cost	22 572
Time	3
Best sol. sec	4,4
Variables	6 116
Constraints	4 492

Table 23: Alternative solution for Scenario 3 Cluster 1

When comparing the two alternative solutions, the second solution seems more realistic. Table 23 shows that the model chooses to use two aircraft, aircraft 1 and aircraft 2. In addition, does aircraft 1 travel two roundtrips.

The routes are as following: Aircraft 1 leaves from the depot at 9:45, goes to Rørvik then to Namsos and lands back at the depot at 11:15. The same aircraft leaves for trip two at 19:15 goes first to Namsos, then to Rørvik and is back at the depot at 20:45. Aircraft 2 leaves from the depot at 07:30, goes to Namsos, then Rørvik and is back at the depot 09:00.

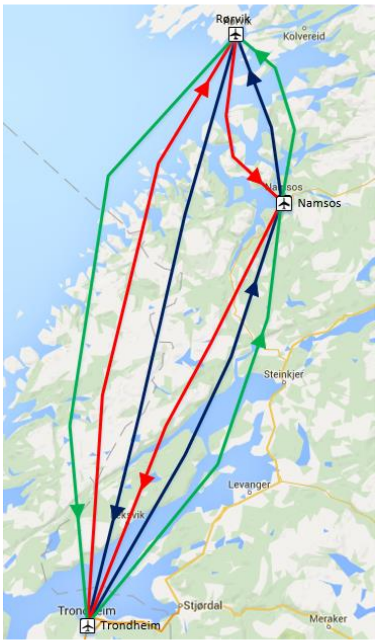

Figure 9: Alternative routes from Scenario 3 Cluster 1

If we compare the distance, cost and time from the two different solutions, we see that when we limit the number of landings per airport per aircraft on each roundtrip the distance increases by 246 kilometers. This is natural since the aircraft can no longer visit an airport two times during the same roundtrip. Since both the time and the cost is linked to the distance of the aircraft, they increases as well.

Cluster 2

This cluster consist of eight airports, those are Bodø (which is the depot), Brønnøysund, Mosjøen, Sandnessjøen, Mo i Rana, Leknes, Svolvær and Stokmarknes. The routes are represented in figure 10, here we can see that the solution is divided into two different groups. One group consist of Lofoten (which is Leknes, Stokmarknes and Svolvær) and the other group is Mo i Rana, Mosjøen, Bønnøysund and Sandnessjøen. There are three routes going to Lofoten and there are three routes visiting the other airports. In figure 10, the pink dashed line represents a route that is travelled in both directions each day. The morning flights from Leknes, Stokmarknes and Svolvær lands in Bodø before 08:30. The morning flight from Sandnessjøen, Brønnøysund, Mosjøen and Mo i Rana lands in Bodø by 08:00.

S3C2	Departure Airport	Departure Time	Arrival Airport	Arrival Time
Aircraft 1 Roundtrip 1	BOO	4	LKN	5
	LKN	6	SKN	8
	SKN	9	SVJ	10
	SVJ	11	LKN	12
	LKN	13	BOO	14
Aircraft 1 Roundtrip 2	BOO	29	SSJ	31
	SSJ	32	BNN	33
	BNN	34	MJF	35
	MJF	36	MQN	38
	MQN	39	BOO	40
Aircraft 1 Roundtrip 3	BOO	47	SSJ	49
	SSJ	50	MJF	51
	MJF	52	SSJ	53
	SSJ	54	MQN	56
	MQN	57	BOO	58
Aircraft 2 Roundtrip 1	BOO	5	MQN	6
	MQN	7	MJF	9
	MJF	10	BNN	11
	BNN	12	SSJ	13
	SSJ	14	BOO	16
Aircraft 3 Roundtrip 1	BOO	25	LKN	26
	LKN	27	SKN	29
	SKN	30	LKN	32
	LKN	33	BOO	34
Aircraft 3 Roundtrip 2	BOO	51	MQN	52
	MQN	53	MJF	55
	MJF	56	BNN	57
	BNN	58	SSJ	59
	SSJ	60	BOO	62
Aircraft 4 Roundtrip 1	BOO	1	MQN	2
	MQN	3	MJF	5
	MJF	6	BNN	7
	BNN	8	SSJ	9
	SSJ	10	BOO	12
Aircraft 4 Roundtrip 2	BOO	48	SVJ	49
	SVJ	50	SKN	51
	SKN	52	SVJ	53
	SVJ	54	LKN	55
	LKN	56	BOO	57

Distance	3 262
Cost	75 866
Time	13
Best sol. sec	3 822
Variables	94 034
Constraints	53 549

Table 24: Results from Scenario 3 Cluster 2

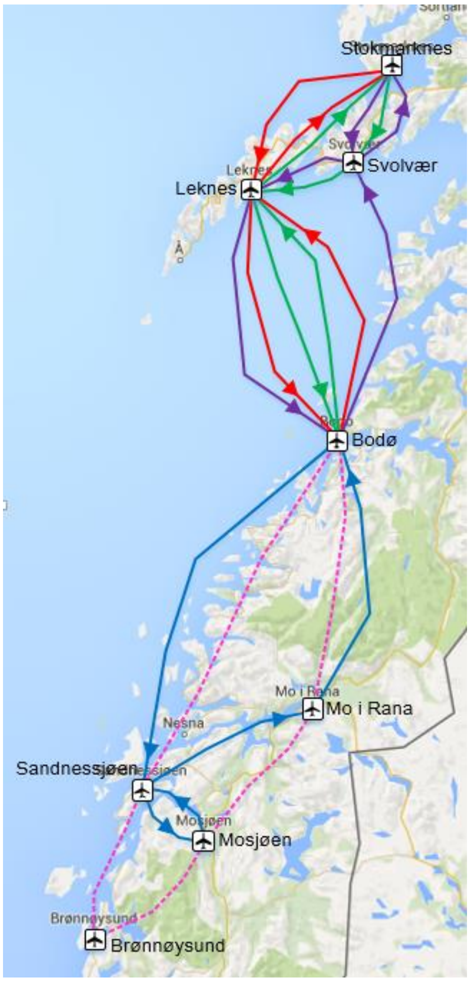

Figure 10: Routes from Scenario 3 Cluster 2

The latest arrival times at the different airports in the evening are as following: Stokmarknes 18:15, Leknes 18:45, Mosjøen 18:45, Brønnøysund 19:15, Svolvær 19:45, Sandnessjøen 19:45 and Mo i Rana 19:00. This is in line with the time constraint. Table 24 shows the flight times and figure 10 displays the routes.

When analyzing the solution, we see that it could have been better to use maximum one landing on each airport for each aircraft per roundtrip, as we did in cluster 1. This is because the aircraft travels from Sandnessjøen to Mosjøen and back again to Sandnessjøen during one flight. The travel time by car from Sandnessjøen to Mosjøen is only about one hour, so it is not necessary to go back and forth between these two airports.

Cluster 3

In cluster 3, Tromsø is the seed node, and the cluster consists of nine airports. Those are Narvik, Evenes, Andenes, Bardufoss, Storslett, Alta, Hasvik and Hammerfest. In the solution, the routes are divided into two different groups. The first group consists of Andenes, Evenes, Narvik and Bardufoss. In this group, there are two different routes. One goes from Tromsø to Evenes then to Narvik and back the same way, which is shown as the blue line in figure 11. It is not necessary to travel back and forth between Evenes and Narvik. It is only a one-hour drive between these two airports. The other route in this group is represented with the pink dashed line in figure 11. This route travels from the depot to Andenes, Evenes, Narvik and then back to the depot. The route is travelled two times each day, both times in the same direction. This is a reasonable route that could be implemented in real life, but either Narvik or Evenes needs to be excluded.

S3C3	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TOS	3	SOJ	5
	SOJ	6	ALF	8
	ALF	9	HFT	11
	HFT	12	ALF	14
	ALF	15	TOS	17
Aircraft 1 Roundtrip 2	TOS	20	HAA	22
	HAA	23	HFT	24
	HFT	25	ALF	27
	ALF	28	SOJ	30
	SOJ	31	TOS	33
Aircraft 1 Roundtrip 3	TOS	63	HFT	65
	HFT	66	HAA	67
	HAA	68	HFT	69
	HFT	70	TOS	72
Aircraft 2 Roundtrip 1	TOS	52	ALF	54
	ALF	55	HFT	57
	HFT	58	ALF	60
	ALF	61	SOJ	63
	SOJ	64	TOS	66
Aircraft 3 Roundtrip 1	TOS	5	ANX	6
	ANX	7	EVE	9
	EVE	10	NVK	11
	NVK	12	BDU	14
	BDU	15	TOS	16
Aircraft 3 Roundtrip 2	TOS	32	EVE	34
	EVE	35	NVK	36
	NVK	37	EVE	38
	EVE	39	TOS	41
Aircraft 3 Roundtrip 3	TOS	58	ANX	59
	ANX	60	EVE	62
	EVE	63	NVK	64
	NVK	65	BDU	67
	BDU	68	TOS	69

Distance	3 165
Cost	66 977
Time	13,75
Best sol. sec	2 423
Variables	90 290
Constraints	50 795

Table 25: Results from Sceanrio 3 Cluster 3

The other group consist of Storslett, Alta, Hammerfest and Hasvik. In this group there are three different routes. One goes from Tromsø to Hammerfest, Hasvik, back to Hammerfest and then to Tromsø. This is not an optimal route since the aircraft travels back and forth between two small airports, Hammerfest and Hasvik. Both the airports have a low demand.The next route is more reasonable since it goes from Tromsø to Storslett, Alta, Hammerfest, Hasvik and back to Tromsø. The last route in this group travels from Tromsø to Storslett, Alta, Hammerfest and then back through Alta and to Tromsø. This route is travelled twice each day, each time in different directions. The red dashed line in figure 11 represents this route.

In this cluster there are 3 aircraft used, those are aircraft 1, 2 and 3. In addition, does aircraft 1 and 3 fly three roundtrips each. The morning flight from all the different airports reaches Tromsø before 09:30, and the evening flight visits each of the airports between 19:45 and 22:15.

The routes are shown in table 25.

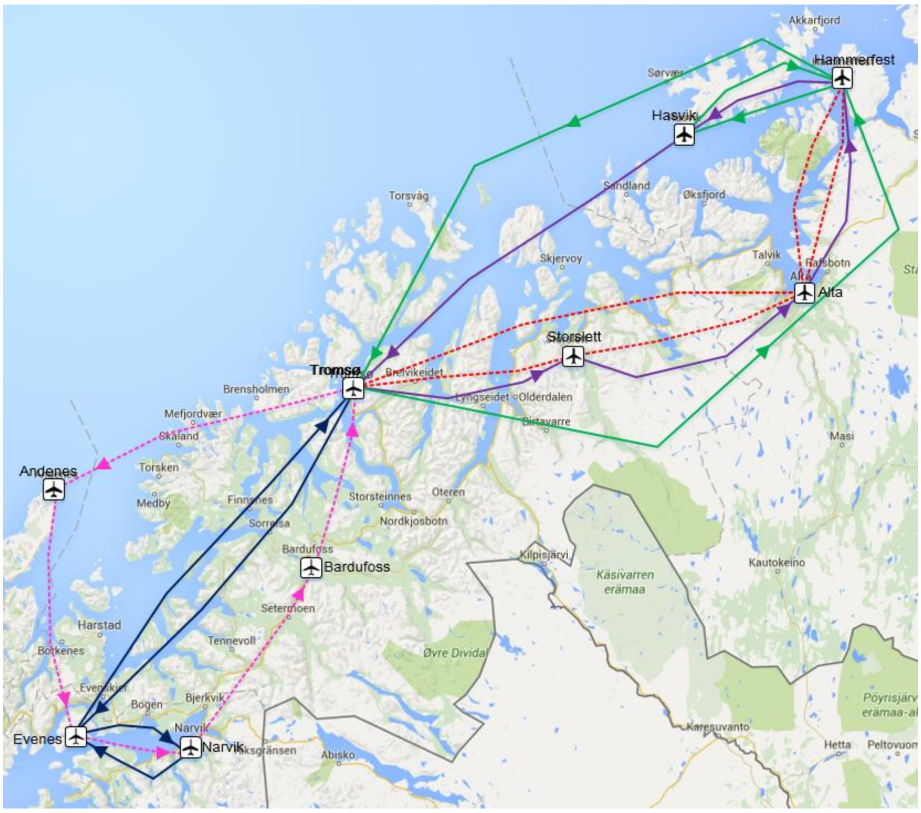

Figure 11: Routes from Scenario 3 Cluster 3

Cluster 4

This last cluster consists of seven airports plus the depot. The airports are Vardø, Vadsø, Båtsfjord, Berlevåg, Honningsvåg, Mehamn, Lakselv and Kirkenes (which is the seed node). This cluster consist of five different routes, three longer ones and two shorter routes. The routes are shown in figure 12. All the longer routes visit both Honningsvåg and Lakselv before going back to Kirkenes. The morning flight reaches Kirkenes before 8:15 from each of the airports. The last flight in the evening lands at Båtsfjord 17:15, Berlevåg 17:45, Mehamn 18:15, Honningsvåg 19:00, Lakselv 19:30, Vardø 21:45 and Vadsø at 22:15.

S3C4	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	KKN	45	VAW	47
	VAW	48	BJF	49
	BJF	50	HVG	52
	HVG	53	LKL	54
	LKL	55	KKN	57
Aircraft 1 Roundtrip 2	KKN	64	VDS	65
	VDS	66	VAW	67
	VAW	68	VDS	69
	VDS	70	KKN	71
Aircraft 2 Roundtrip 1	KKN	2	VDS	3
	VDS	4	MEH	5
	MEH	6	HVG	8
	HVG	9	LKL	10
	LKL	11	KKN	13
Aircraft 2 Roundtrip 2	KKN	49	BVG	51
	BVG	52	MEH	53
	MEH	54	HVG	56
	HVG	57	LKL	58
	LKL	59	KKN	61
Aircraft 4 Roundtrip 1	KKN	1	VAW	3
	VAW	4	BJF	5
	BJF	6	BVG	7
	BVG	8	BJF	9
	BJF	10	KKN	11
Aircraft 4 Roundtrip 2	KKN	52	VDS	53
	VDS	54	VAW	55
	VAW	56	VDS	57
	VDS	58	KKN	59

Distance	2 407
Cost	51 073
Time	9,25
Best sol. sec	4 027
Variables	97 202
Constraints	55 675

Table 26: Results from Scenario 3 Cluster 4

In this cluster, most of the routes are reasonable. There is only one route that should have been changed, this route is from Kirkenes, to Vadsø, Vardø, back to Vadsø and then back to Kirkenes. It would have been better if this route was Kirkenes, Vadsø, Vardø and then back to Kirkenes.

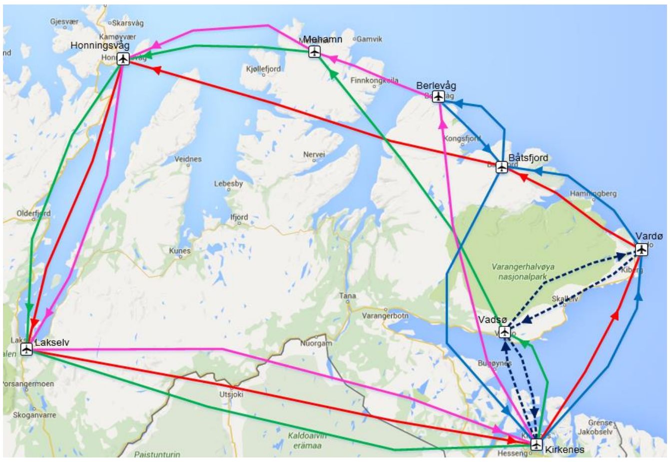

Figure 12: Routes from Scenario 3 Cluster 4

7.2 Conclusion

The overall conclusion is that scenario 3 seems to be the best solution. The routes are realistic, but there are also small changes that we could have done to make the model better. One of those changes would be that for each airport the maximum number of landings should be one for each aircraft per roundtrip. Then we would not have to travel the small distances between for example Rørvik and Namsos more than one time on each flight. Another alternative could be to regulate the number of landings regarding the size of the airport. Lets say that on all medium sized and larger airports the aircraft can land maximum two times on each roundtrip, and on the smaller airports this could be limited to one. This would reduce the number of unnecessary travelling back and forth between small airports with low demands, and at the same time, it would allow the larger airports with high demand to be visited multiple times.

We chose to use visit frequency on each airport instead of the demand. We used this approach because it was not possible for us find the demand on legs not travelled today. When deciding the visit frequency we did not consider the size of the clusters, the size of the airports or which airport the aircraft was arriving from or which airport is was going to visiting next. This makes the visit frequency less realistic. An airport might have a higher visit frequency when it is included in a large cluster, than it would if it was included in a smaller cluster. This is because the airport would then have more options to travel in terms of arcs. A better way would probably be to look at the demand on each arc instead of using visit frequency. We did not use this approach because of the difficulties to collect the necessary data, regarding the demand on each arc. The visit frequency works for our purpose, and it still provides a reasonable solution.

In this thesis, we have only mentioned the PSO regulations to show that there are many factors that need to be considered when deciding the routes. To make the model more realistic, the PSO regulations should been considered more. To do that we could have implemented fixed variables, saying that the leg between two specific airports should be travelled. One example of a PSO regulated route is between Lakselv and Tromsø, this leg should be travelled at least three times daily, and two of them should be non-stop.

7.3 Challenges

Some of the clusters consisted of many airports, this lead to challenges when solving the model. Scenario 1 cluster 3 was challenging to solve, because it consists of 16 airports not including the depot. When trying to solve this problem we ended up with 1 099 742 variables and 576 400 constraints. Because of the size of this problem, NEOS ran out of memory before it could find a feasible solution. In order to get a solution for this cluster it was necessary to limit the dataset. This can be done in different ways; one option is to try to limit the model by remove one of the indexes. Another option is to divide the airports into two new clusters, when still keeping the same depot (by either county or north and south of the depot). The third option is to decide some of the flight legs that have to be traversed or some that is not allowed to be flied.

The first option we tried was to remove the roundtrips and instead increase the number of available aircraft. This reduced the problem to 544 832 variables and 286 865 constraints, but it was still too large to be solved by NEOS. We also tried to limit the problem by fixing some variables, which removed the option of travelling on specific legs. This did not reduce the problem enough to make the model solvable for NEOS.

In order to get a solution for this cluster, we decided to split the cluster into two groups. Both groups would be connected to the same depot. In addition, we made sure that aircraft was not departing or arriving at the depot in the same time period. We split cluster 3 into two equal groups, each consisting of eight airports plus the depot. This reduced the problem to consist of 120386 + 120878 = 241 264 variables and 67506 + 67950 = 135 456 constraints. Both groups were small enough to be solved by the NEOS solver. To prevent the aircraft from using the depot in the same time period, we solved one of the groups first and analyzed the output to see which time periods the depot was used. Then, when solving the next group, we used fixed variables saying that it was not allowed to arrive or depart from the depot in the same time periods as they did in the first group.

We got the same problem with Scenario 2 Cluster 1. This cluster consists of 10 airports plus the depot. To get a solution we divided the airports into two groups, based on their position to the depot. Group 1 consists of airports south of the depot, and group 2 of airports north of the depot. This way of dividing into two groups could not be done for

Scenario 1 Cluster 3, as the cluster north of the depot would still be too large to get a solution.

8.0 Conclusion

The main purpose of this thesis was to make a model for routing of aircraft, with focus on the northern Norway. The northern Norway is a special area because many of the routes there are regulated by the PSO. The area consists of many small airports. In addition, many of the flights are chained air trips with multiple landings on a roundtrip.

The task was to make a relatively easy model using an exact method. We used the basic VRP model as a starting point, and modified this to fit our problem. When modifying the VRP model, we decided to use two routing variables indexed by the arc, aircraft, time and roundtrip. In addition did we extend the model by including time-constraints, constraints regarding roundtrips and constraints regarding time of landing and departure as well as the number of landings. To test the model and see if there was any constraints missing we tested the model on a small instance with six airports. We found that when we added the constraint restricting the aircraft from flying in the last periods, then the computation time increased a lot.

Due to the size of the problem, which consists of in total 28 airports, it became necessary to us a two phase approach to the problem. That is why we implemented a cluster first, route second approach. We used Fisher and Jaikumar to divide the 28 airports into different clusters, and then we used the model we built on these different clusters. Originally, we wanted to try different objective like minimizing total distance, total cost and total travel time to solve the problem. This would not be necessary to do, since the cost and the travel time, is dependent on the distance travelled and the answer would most likely be the same.

We divided the airports into clusters, because the problem was too complex to solve as one. This is not an optimal solution method when considering air traffic. This is because it is harder to see the whole picture, there can be legs that should be traversed between airports in different clusters. To get a realistic solution the whole problem should be solved by using a model for the multi depot VRP.

Our goal was to make a model that gave us feasible solutions that could be implemented in real life. The routes made in some of the scenarios are realistic when comparing it to the routes that are travelled today. There are still a lot more to consider when conducting flight routes. In our model, we only used the PSO regulations as a guide on what to implement regarding time of the flights. If the model is going to conduct routes that could be used in real life all the PSO regulations needs to be taken into consideration.

Further research could be to extend the model regarding more regulations, and implement passenger demand on the legs instead of visits frequency. In order to further develop the model uncertainty could also be implemented.

9.0 References

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10.0 Appendices

Appendix A: Distance

Distance		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Trondheim	1		127	165	247	299	307	367	467	545	567	607	642	628	703	725	788	845	919	935	986	972	1070	1124	1147	1152	1180	1119	1109
Namsos	2	127		46	120	172	181	239	340	420	441	481	515	501	576	598	661	718	793	808	859	847	944	999	1023	1029	1058	997	988
Rørvik	3	165	46		85	142	144	210	305	381	403	443	483	467	539	566	627	687	764	776	829	820	914	971	997	1004	1034	974	967
Brønnøysund	4	247	120	85		58	61	125	221	301	321	361	398	382	456	481	542	602	679	691	744	735	829	886	912	920	951	891	885
Mosjøen	5	299	172	142	58		33	68	169	258	274	313	343	329	407	426	489	547	623	636	688	678	733	830	855	862	894	833	827
Sandnessjøen	6	307	181	144	61	33		75	161	240	260	300	339	323	395	422	483	544	623	632	686	680	772	830	858	866	899	839	835
Mo i Rana	7	367	239	210	125	68	75		108	205	215	253	275	263	345	359	422	479	555	568	620	610	705	761	787	794	827	766	761
Bodø	8	467	340	305	221	169	161	108		102	106	145	181	163	237	263	323	386	467	473	527	527	615	675	704	715	752	692	691
Leknes	9	545	420	381	301	258	240	205	102		41	71	161	132	165	225	272	346	434	425	483	498	573	637	672	687	730	674	679
Svolvær	10	567	441	403	321	274	260	215	106	41		39	120	91	136	186	236	307	395	389	447	459	537	600	634	648	690	634	638
Stokmarknes	11	607	481	443	361	313	300	253	145	71	39		104	74	97	156	201	275	364	354	412	428	503	567	602	618	662	606	612
Narvik	12	642	515	483	398	343	339	275	181	161	120	104		30	111	83	148	205	287	293	347	347	434	494	525	436	576	518	521
Evenes	13	628	501	467	382	329	323	263	163	132	91	74	30		98	101	160	224	309	310	366	371	454	516	548	560	601	544	548
Andøya/Ander	14	703	576	539	456	407	395	345	237	165	136	97	111	98		99	116	195	286	265	324	351	415	481	520	538	585	532	543
Bardufoss	15	725	598	566	481	426	422	359	263	225	186	156	83	101	99		67	125	211	211	266	274	355	416	449	463	506	450	457
Tromsø	16	788	661	627	542	489	483	422	323	272	236	201	148	160	116	67		78	170	153	211	235	302	367	404	421	469	416	428
Sørkjosen/Stor	17	845	718	687	602	547	544	479	386	346	307	275	205	224	195	125	78		91	90	142	156	230	292	326	343	390	337	350
Alta	18	919	793	764	679	623	623	555	467	434	395	364	287	309	286	211	170	91		71	79	65	151	207	239	253	299	249	261
Hasvik	19	935	808	776	691	636	632	568	473	425	389	354	293	310	265	211	153	90	71		60	116	150	217	259	281	334	289	310
Hammerfest	20	986	859	829	744	688	686	620	527	483	447	412	347	366	324	266	211	142	79	60		84	91	158	200	223	278	236	262
Lakselv	21	972	847	820	735	678	680	610	527	489	459	428	347	371	351	274	235	156	65	116	84		110	153	178	190	234	181	198
Honningsvåg	22	1070	944	914	829	773	772	705	615	573	537	503	434	454	415	355	302	230	151	150	91	110		68	114	143	201	173	207
Mehamn	23	1124	999	971	886	830	830	761	675	637	600	567	494	516	481	416	367	292	207	217	158	153	68		49	82	141	128	167
Berlevåg	24	1147	1023	997	912	855	858	787	704	672	634	602	525	548	520	449	404	326	239	259	200	178	114	49		34	92	91	131
Båtsfjord	25	1152	1029	1004	920	862	866	794	715	687	648	618	536	560	538	463	421	343	253	281	223	190	143	82	34		59	63	102
Vardø	26	1180	1058	1034	951	894	899	827	752	730	690	662	576	601	585	506	469	390	299	334	278	234	201	141	92	59		61	82
Vadsø	27	1119	997	974	891	833	839	766	692	674	634	606	518	544	532	450	416	337	249	289	236	181	173	128	91	63	61		40
Kirkenes	28	1109	988	967	885	827	835	761	691	679	638	612	521	548	543	457	428	350	261	310	262	198	207	167	131	102	82	40	

Appendix B: Travel times between airports

The table below shows the calculation used to find the travel times between each airport.

The travel times are based on the distance between the airports.

From:	To:	Time (min):	Distance (km):	km/min:
Bodø	Trondheim	60	467	7,8
Kirkenes	Tromsø	74	428	5,8
Vadsø	Tromsø	72	416	5,8
			Average	**6,4**
			Median	5,8
			Standardeviation	1,2
Mo i Rana	Trondheim	63	367	5,8
Tromsø	Bodø	50	323	6,5
Sandnessjøen	Trondheim	50	307	6,1
Honningsvåg	Tromsø	75	302	4,0
Mosjøen	Trondheim	50	299	6,0
Kirkenes	Alta	47	261	5,6
			Average	**5,7**
			Median	5,9
			Standardeviation	0,7
Vadsø	Alta	46	249	5,4
Brønnøysund	Trondheim	43	247	5,7
Andøya	Bodø	45	237	5,3
Vadsø	Hammerfest	50	236	4,7
Lakselv	Tromsø	45	235	5,2
Bodø	Brønnøysund	45	221	4,9
Mo i Rana	Rørvik	45	210	4,7
Tromsø	Stokmarknes	40	201	5,0
Berlevåg	Hammerfest	39	200	5,1
Narvik	Bodø	39	181	4,6
Vadsø	Honningsvåg	55	173	3,1
Mosjøen	Namsos	36	172	4,8
Alta	Tromsø	35	170	4,9
Bodø	Mosjøen	43	169	3,9
Rørvik	Trondheim	34	165	4,9
Evenes	Bodø	38	163	4,3
Bodø	Sandnessjøen	35	161	4,6
Tromsø	Evenes	35	160	4,6
Hasvik	Tromsø	33	153	4,6
Stokmarknes	Bodø	33	145	4,4
Hammerfest	Sørkjosen	30	142	4,7
Vadsø	Mehamn	32	128	4,0
Namsos	Trondheim	30	127	4,2
Tromsø	Andøya	30	116	3,9
Bodø	Mo i Rana	30	108	3,6
Svolvær	Bodø	26	106	4,1
			Average	**4,6**
			Median	4,7
			Standardeviation	0,6
Andøya	Evenes	24	98	4,1
Andøya	Stokmarknes	25	97	3,9
Vadsø	Berlevåg	25	91	3,6
Honningsvåg	Hammerfest	25	91	3,6
Hammerfest	Alta	23	79	3,4
Sørkjosen	Tromsø	25	78	3,1
Mehamn	Honningsvåg	20	68	3,4
Mo i Rana	Mosjøen	28	68	2,4
Lakselv	Alta	22	65	3,0
Vadsø	Båtsfjord	21	63	3,0
Vadsø	Vardø	21	61	2,9
Sandnessjøen	Brønnøysund	17	61	3,6
Hammerfest	Hasvik	20	60	3,0
Berlevåg	Mehamn	17	49	2,9
Rørvik	Namsos	18	46	2,6
Kirkenes	Vadsø	17	40	2,4
Stokmarknes	Svolvær	20	39	2,0
Sandnessjøen	Mosjøen	20	33	1,7
			Average	**3,0**
			Median	3,0
			Standardeviation	0,7

Appendix C: Travel time in time periods

Time periods		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Trondheim	1		1	2	3	4	4	5	5	6	6	6	7	7	7	8	8	9	10	10	10	10	11	12	12	12	12	12	12
Namsos	2	1		1	1	2	2	3	5	4	5	5	5	5	6	6	7	7	8	8	9	9	10	10	11	11	11	10	10
Rørvik	3	2	1		2	2	2	2	4	6	4	5	5	5	6	6	7	7	8	8	9	9	10	10	10	10	11	10	10
Brønnøysund	4	3	1	2		1	1	1	3	4	5	5	6	6	5	5	6	6	7	7	8	8	9	9	10	10	10	9	9
Mosjøen	5	4	2	2	1		1	2	2	4	4	5	5	5	4	4	5	6	6	7	7	7	8	9	9	9	9	9	9
Sandnessjøen	6	4	2	2	1	1		2	2	3	4	4	5	5	6	4	5	6	6	7	7	7	8	9	9	9	9	9	9
Mo i Rana	7	5	3	2	1	2	2		1	2	3	4	4	4	5	5	4	5	6	6	6	6	7	8	8	8	9	8	8
Bodø	8	5	5	4	3	2	2	1		1	1	2	2	2	3	4	5	6	5	5	5	5	6	7	7	7	8	7	7
Leknes	9	6	4	6	4	4	3	2	1		1	2	2	2	2	3	4	5	5	4	5	5	6	7	7	7	8	7	7
Svolvær	10	6	5	4	5	4	4	3	1	1		1	1	2	2	2	3	4	6	6	5	5	6	6	7	7	7	7	7
Stokmarknes	11	6	5	5	5	5	4	4	2	2	1		1	2	2	2	3	4	5	5	4	4	5	6	6	6	7	6	6
Narvik	12	7	5	5	6	5	5	4	2	2	1	1		1	1	2	2	2	4	4	5	5	5	5	5	5	6	5	5
Evenes	13	7	5	5	6	5	5	4	2	2	2	2	1		2	1	2	3	4	4	5	5	5	5	6	6	6	6	6
Andøya/Ander	14	7	6	6	5	4	6	5	3	2	2	2	1	2		2	1	2	4	4	5	5	4	5	5	6	6	6	6
Bardufoss	15	8	6	6	5	4	4	5	4	3	2	2	2	1	2		1	1	2	2	4	4	5	4	5	5	5	5	5
Tromsø	16	8	7	7	6	5	5	4	5	4	3	2	2	2	1	1		2	2	2	2	3	4	5	4	4	5	4	4
Sørkjosen/Stor	17	9	7	7	6	6	6	5	6	5	4	4	2	3	2	1	2		2	2	2	2	3	4	5	5	6	5	5
Alta	18	10	8	8	7	6	6	6	5	5	6	5	4	4	4	2	2	2		2	2	1	2	2	3	4	4	4	4
Hasvik	19	10	8	8	7	7	7	6	5	4	6	5	4	4	4	2	2	2	2		1	1	2	3	4	4	5	4	4
Hammerfest	20	10	9	9	8	7	7	6	5	5	5	4	5	5	5	4	2	2	2	1		2	2	2	2	3	4	3	4
Lakselv	21	10	9	9	8	7	7	6	5	5	5	4	5	5	5	4	3	2	1	1	2		1	2	2	2	3	2	2
Honningsvåg	22	11	10	10	9	8	8	7	6	6	6	5	5	5	4	5	4	3	2	2	2	1		2	1	2	2	2	2
Mehamn	23	12	10	10	9	9	9	8	7	7	6	6	5	5	5	4	5	4	2	3	2	2	2		1	2	2	1	2
Berlevåg	24	12	11	10	10	9	9	8	7	7	7	6	5	6	5	5	4	5	3	4	2	2	1	1		1	2	2	2
Båtsfjord	25	12	11	10	10	9	9	8	7	7	7	6	6	6	6	5	4	5	4	4	3	2	2	2	1		1	1	1
Vardø	26	12	11	11	10	9	9	9	8	8	7	7	6	6	6	5	5	6	4	5	4	3	2	2	2	1		1	2
Vadsø	27	12	10	10	9	9	9	8	7	7	7	6	5	6	6	5	4	5	4	4	3	2	2	1	2	1	1		1
Kirkenes	28	12	10	10	9	9	9	8	7	7	7	6	5	6	6	5	4	5	4	4	4	2	2	2	2	1	2	1	

Appendix D: Visit frequency

Visit frequence	
Trondheim	7
Namsos	3
Rørvik	3
Brønnøysund	4
Mosjøen	5
Sandnessjøen	6
Mo i Rana	5
Bodø	8
Leknes	5
Svolvær	3
Stokmarknes	3
Narvik	3
Evenes	4
Andøya	2
Bardufoss	2
Tromsø	12
Sørkjosen	3
Alta	5
Hasvik	2
Hammerfest	5
Lakselv	3
Honningsvåg	3
Mehamn	2
Berlevåg	2
Båtsfjord	3
Vardø	4
Vardsø	5
Kirkenes	6

Appendix E : Calculation of cost of traveling between airports

Total costs	
Trondheim	64*17 + 56*39*0,6+ 54*39*0,6 + 1787,43*(17/50)^0,7 + 381,42*(17/50)^0,7+ 5,52*km =
	1088+1310,4+1263,6+839,97+179,24+5,52*km
Tromsø and Bodø	64*17* + 56*39*0,6+ 54*39*0,6 + 1251,20*(17/50)^0,7 + 381,42*(17/50)^0,7+ 5,52*km =
	1088+1310,4+1263,6+587,98+179,24+5,52*km
Other	64*17 *0,7 + 56*39*0,6+ 54*39*0,6 + 1251,20*(17/50)^0,7 + 381,42*(17/50)^0,7+ 5,52*km =
	761,6+1310,4+1263,6+587,98+179,24+5,52*km

Appendix F: Cost of travelling between nodes

Cost		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Trondheim	1		2629	2839	3291	3578	3623	3954	4506	4936	5058	5279	5472	5395	5809	5930	6278	6592	7001	7089	7371	7293	7834	8132	8259	8287	8442	8105	8050
Namsos	2	2629		1604	2012	2299	2349	2669	3553	3668	3784	4005	4192	4115	4529	4651	5325	5313	5727	5810	6091	6025	6560	6864	6997	7030	7190	6853	6803
Rørvik	3	2839	1604		1819	2133	2144	2509	3360	3453	3574	3795	4016	3927	4325	4474	5137	5142	5567	5633	5926	5876	6395	6710	6853	6892	7057	6726	6687
Brønnøysund	4	3291	2012	1819		1670	1686	2040	2896	3011	3122	3342	3547	3458	3867	4005	4668	4673	5098	5164	5456	5407	5926	6240	6384	6428	6599	6268	6235
Mosjøen	5	3578	2299	2133	1670		1532	1725	2609	2774	2862	3077	3243	3166	3596	3701	4375	4369	4789	4860	5147	5092	5396	5931	6069	6108	6284	5948	5915
Sandnessjøen	6	3623	2349	2144	1686	1532		1764	2565	2674	2785	3006	3221	3133	3530	3679	4342	4352	4789	4838	5136	5103	5611	5931	6086	6130	6312	5981	5959
Mo i Rana	7	3954	2669	2509	2040	1725	1764		2272	2481	2536	2746	2868	2801	3254	3331	4005	3994	4413	4485	4772	4717	5241	5550	5694	5732	5915	5578	5550
Bodø	8	4506	3553	3360	2896	2609	2565	2272		2239	2261	2476	2675	2576	2984	3128	3459	3807	4254	4287	4585	4585	5071	5402	5562	5623	5827	5496	5490
Leknes	9	4936	3668	3453	3011	2774	2674	2481	2239		1576	1742	2238	2078	2260	2592	3177	3260	3745	3696	4016	4099	4513	4866	5059	5142	5379	5070	5098
Svolvær	10	5058	3784	3574	3122	2862	2785	2536	2261	1576		1565	2012	1852	2100	2376	2979	3044	3530	3497	3817	3883	4314	4662	4849	4927	5158	4849	4871
Stokmarknes	11	5279	4005	3795	3342	3077	3006	2746	2476	1742	1565		1924	1758	1885	2211	2786	2868	3359	3304	3624	3712	4126	4479	4673	4761	5004	4695	4728
Narvik	12	5472	4192	4016	3547	3243	3221	2868	2675	2238	2012	1924		1515	1962	1808	2493	2481	2934	2967	3265	3265	3745	4076	4248	3756	4529	4209	4226
Evenes	13	5395	4115	3927	3458	3166	3133	2801	2576	2078	1852	1758	1515		1891	1907	2559	2586	3055	3061	3370	3398	3856	4198	4375	4441	4667	4352	4375
Andøya/Andenes	14	5809	4529	4325	3867	3596	3530	3254	2984	2260	2100	1885	1962	1891		1896	2316	2426	2928	2812	3138	3287	3640	4005	4220	4319	4579	4286	4347
Bardufoss	15	5930	4651	4474	4005	3701	3679	3331	3128	2592	2376	2211	1808	1907	1896		2046	2040	2514	2514	2818	2862	3309	3646	3828	3905	4143	3834	3872
Tromsø	16	6278	5325	5137	4668	4375	4342	4005	3459	3177	2979	2786	2493	2559	2316	2046		2107	2614	2521	2841	2973	3343	3702	3906	4000	4265	3972	4039
Sørkjosen/Storslett	17	6592	5313	5142	4673	4369	4352	3994	3807	3260	3044	2868	2481	2586	2426	2040	2107		1852	1846	2133	2211	2619	2961	3149	3243	3502	3210	3282
Alta	18	7001	5727	5567	5098	4789	4789	4413	4254	3745	3530	3359	2934	3055	2928	2514	2614	1852		1742	1786	1708	2183	2492	2669	2746	3000	2724	2790
Hasvik	19	7089	5810	5633	5164	4860	4838	4485	4287	3696	3497	3304	2967	3061	2812	2514	2521	1846	1742		1681	1990	2178	2547	2779	2901	3193	2945	3061
Hammerfest	20	7371	6091	5926	5456	5147	5136	4772	4585	4016	3817	3624	3265	3370	3138	2818	2841	2133	1786	1681		1813	1852	2222	2454	2581	2884	2652	2796
Lakselv	21	7293	6025	5876	5407	5092	5103	4717	4585	4049	3883	3712	3265	3398	3287	2862	2973	2211	1708	1990	1813		1957	2194	2332	2398	2641	2349	2443
Honningsvåg	22	7834	6560	6395	5926	5617	5611	5241	5071	4513	4314	4126	3745	3856	3640	3309	3343	2619	2183	2178	1852	1957		1725	1979	2139	2459	2305	2492
Mehamn	23	8132	1896	6710	6240	5931	5931	5550	5402	4866	4662	4479	4076	4198	4005	3646	3702	2961	2492	2547	2222	2194	1725		1620	1802	2128	2056	2271
Berlevåg	24	8259	6997	6853	6384	6069	6086	5694	5562	5059	4849	4673	4248	4375	4220	3828	3906	3149	2669	2779	2454	2332	1979	1620		1537	1857	1852	2073
Båtsfjord	25	8287	7030	6892	6428	6108	6130	5732	5623	5142	4927	4761	4308	4441	4319	3905	4000	3243	2746	2901	2581	2398	2139	1802	1537		1675	1697	1913
Vardø	26	8442	7190	7057	6599	6284	6312	5915	5827	5379	5158	5004	4529	4667	4579	4143	4265	3502	3000	3193	2884	2641	2459	2128	1857	1675		1686	1802
Vadsø	27	8105	6853	6726	6268	5948	5981	5578	5496	5070	4849	4695	4209	4352	4286	3834	3972	3210	2724	2945	2652	2349	2305	2056	1852	1697	1686		1570
Kirkenes	28	8050	6803	6687	6235	5915	5959	5550	5490	5098	4871	4728	4226	4375	4347	3872	4039	3282	2790	3061	2796	2443	2492	2271	2073	1913	1802	1570	

Appendix G: Results from the modified VRP

Scenario 1

Cluster 1:

S1C1	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TRD	4	OSY	5
	OSY	6	RVK	7
	RVK	8	OSY	9
	OSY	10	TRD	11
Aircraft 2 Roundtrip 1	TRD	49	RVK	51
	RVK	52	OSY	53
	OSY	54	RVK	55
	RVK	56	TRD	58

Distance	768
Cost	18256
Time	2,5
Best sol. sec	1,08
Variables	6116
Constraints	4492

Cluster 2:

S1C2	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	BOO	4	LKN	5
	LKN	6	SKN	8
	SKN	9	SVJ	10
	SVJ	11	LKN	12
	LKN	13	BOO	14
Aircraft 1 Roundtrip 2	BOO	29	SSJ	31
	SSJ	32	BNN	33
	BNN	34	MJF	35
	MJF	36	MQN	38
	MQN	39	BOO	40
Aircraft 1 Roundtrip 3	BOO	47	SSJ	49
	SSJ	50	MJF	51
	MJF	52	SSJ	53
	SSJ	54	MQN	56
	MQN	57	BOO	58
Aircraft 2 Roundtrip 1	BOO	5	MQN	6
	MQN	7	MJF	9
	MJF	10	BNN	11
	BNN	12	SSJ	13
	SSJ	14	BOO	16
Aircraft 3 Roundtrip 1	BOO	25	LKN	26
	LKN	27	SKN	29
	SKN	30	LKN	32
	LKN	33	BOO	34
Aircraft 3 Roundtrip 2	BOO	51	MQN	52
	MQN	53	MJF	55
	MJF	56	BNN	57
	BNN	58	SSJ	59
	SSJ	60	BOO	62
Aircraft 4 Roundtrip 1	BOO	1	MQN	2
	MQN	3	MJF	5
	MJF	6	BNN	7
	BNN	8	SSJ	9
	SSJ	10	BOO	12
Aircraft 4 Roundtrip 2	BOO	48	SVJ	49
	SVJ	50	SKN	51
	SKN	52	SVJ	53
	SVJ	54	LKN	55
	LKN	56	BOO	57

Distance	3 262
Cost	75 866
Time	13
Best sol. sec	10 163
Variables	94 034
Constraints	53 549

Cluster 3:

Part 1:

S1C3-1	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TOS	8	ANX	9
	ANX	10	EVE	12
	EVE	13	NVK	14
	NVK	15	BDU	17
	BDU	18	TOS	19
Aircraft 1 Roundtrip 2	TOS	57	SOJ	59
	SOJ	60	ALF	62
	SLF	63	HFT	65
	HFT	66	HAA	67
	HAA	68	TOS	70
Aircraft 2 Roundtrip 1	TOS	29	ALF	31
	ALF	32	HFT	34
	HFT	35	ALF	37
	ALF	38	SOJ	40
	SOJ	41	TOS	43
Aircraft 3 Roundtrip 1	TOS	5	ALF	7
	ALF	8	HFT	10
	HFT	11	HAA	12
	HAA	13	HFT	14
	HFT	15	TOS	17
Aircraft 3 Roundtrip 2	TOS	42	EVE	44
	EVE	45	NVK	46
	NVK	47	EVE	48
	EVE	49	ANX	51
	ANX	52	TOS	53
Aircraft 4 Roundtrip 1	TOS	11	SOJ	13
	SOJ	14	ALF	16
	ALF	17	HFT	19
	HFT	20	TOS	22
Aircraft 4 Roundtrip 2	TOS	62	EVE	64
	EVE	65	NVK	66
	NVK	67	BDU	69
	BDU	70	TOS	71

Distance	3165
Cost	66581
Time	13,75
Best sol. sec	2270
Variables	120386
Constraints	67506

Part 2:

S1C3-2	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TOS	3	LKL	6
	LKL	7	BVG	9
	BVG	10	KKN	12
	KKN	13	BJF	14
	BJF	15	TOS	19
Aircraft 1 Roundtrip 2	TOS	54	MEH	59
	MEH	60	BJF	62
	BJF	63	HVG	65
	HVG	66	LKL	67
	LKL	68	TOS	71
Aircraft 2 Roundtrip 1	TOS	24	KKN	28
	KKN	29	VDS	30
	VDS	31	VAW	32
	VAW	33	KKN	35
	KKN	36	TOS	40
Aircraft 3 Roundtrip 1	TOS	13	VDS	17
	VDS	18	VAW	19
	VAW	20	VDS	21
	VDS	22	KKN	23
	KKN	24	TOS	28
Aircraft 3 Roundtrip 2	TOS	57	BVG	61
	BVG	62	VAW	64
	VAW	65	VDS	66
	VDS	67	KKN	68
	KKN	69	TOS	73
Aircraft 4 Roundtrip 1	TOS	5	HVG	9
	HVG	10	MEH	12
	MEH	13	HVG	15
	HVG	16	LKL	17
	LKL	18	TOS	21
Aircraft 4 Roundtrip 2	TOS	45	BJF	49
	BJF	50	VAW	51
	VAW	52	VDS	53
	VDS	54	KKN	55
	KKN	56	TOS	60

Distance	6866
Cost	89704
Time	20,75
Best sol. sec	3908
Variables	120878
Constraints	67950

Scenario 2

Cluster 1:

Part 1 :

S2C1-1	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	BOO	6	RVK	10
	RVK	11	SSJ	13
	SSJ	14	MJF	15
	MJF	16	MQN	18
	MQN	19	BOO	20
Aircraft 1 Roundtrip 2	BOO	25	SSJ	27
	SSJ	28	BNN	29
	BNN	30	TRD	33
	TRD	34	RVK	36
	RVK	37	BOO	41
Aircraft 1 Roundtrip 3	BOO	49	MQN	50
	MQN	51	TRD	56
	TRD	57	MJF	61
	MJF	62	MQN	64
	MQN	65	BOO	66
Aircraft 2 Roundtrip 1	BOO	14	TRD	19
	TRD	20	OSY	21
	OSY	22	TRD	23
	TRD	24	MQN	29
	MQN	30	BOO	31
Aircraft 2 Roundtrip 2	BOO	34	OSY	39
	OSY	40	TRD	41
	TRD	42	BNN	45
	BNN	46	MJF	47
	MJF	48	BOO	50
Aircraft 2 Roundtrip 3	BOO	56	RVK	60
	RVK	61	TRD	63
	TRD	64	OSY	65
	OSY	66	TRD	67
	TRD	68	BOO	73
Aircraft 3 Roundtrip 1	BOO	2	MQN	3
	MQN	4	BOO	5
Aircraft 3 Roundtrip 2	BOO	8	SSJ	10
	SSJ	11	MJF	12
	MJF	13	BNN	14
	BNN	15	SSJ	16
	SSJ	17	BOO	19
Aircraft 3 Roundtrip 3	BOO	60	BNN	63
	BNN	64	SSJ	65
	SSJ	66	MJF	67
	MJF	68	SSJ	69
	SSJ	70	BOO	72

Distance	7074
Cost	109053
Time	22,5
Best sol. sec	10800
Variables	70724
Constraints	40489

Part 2:

S2C1-2	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	BOO	7	LKN	8
	LKN	9	SKN	11
	SKN	12	SVJ	13
	SVJ	14	LKN	15
	LKN	15	BOO	17
Aircraft 1 Roundtrip 2	BOO	32	LKN	33
	LKN	34	SVJ	35
	SVJ	36	SKN	37
	SKN	38	LKN	40
	LKN	41	BOO	42
Aircraft 2 Roundtrip 1	BOO	53	LKN	54
	LKN	55	SKN	57
	SKN	58	SVJ	59
	SVJ	60	BOO	61

Distance	1028
Cost	26529
Time	4,25
Best sol. sec	4
Variables	11300
Constraints	7598

Cluster 2:

S2C2	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TOS	3	SOJ	5
	SOJ	6	ALF	8
	ALF	9	HFT	11
	HFT	12	ALF	14
	ALF	15	TOS	17
Aircraft 1 Roundtrip 2	TOS	20	HAA	22
	HAA	23	HFT	24
	HFT	25	ALF	27
	ALF	28	SOJ	30
	SOJ	31	TOS	33
Aircraft 1 Roundtrip 3	TOS	63	HFT	65
	HFT	66	HAA	67
	HAA	68	HFT	69
	HFT	70	TOS	72
Aircraft 2 Roundtrip 1	TOS	52	ALF	54
	ALF	55	HFT	57
	HFT	58	ALF	60
	ALF	61	SOJ	63
	SOJ	64	TOS	66
Aircraft 3 Roundtrip 1	TOS	5	ANX	6
	ANX	7	EVE	9
	EVE	10	NVK	11
	NVK	12	BDU	14
	BDU	15	TOS	16
Aircraft 3 Roundtrip 2	TOS	32	EVE	34
	EVE	35	NVK	36
	NVK	37	EVE	38
	EVE	39	TOS	41
Aircraft 3 Roundtrip 3	TOS	58	ANX	59
	ANX	60	EVE	62
	EVE	63	NVK	64
	NVK	65	SOJ	67
	SOJ	68	TOS	69

Distance	3 165
Cost	66 977
Time	14
Best sol. sec	2 423
Variablers	90 290
Constraints	50 795

Cluster 3:

S2C3	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	KKN	5	BVG	7
	BVG	8	MEH	9
	MEH	10	HVG	12
	HVG	13	LKL	14
	LKL	15	KKN	17
Aircraft 1 Roundtrip 2	KKN	64	VDS	65
	VDS	66	VAW	67
	VAW	68	VDS	69
	VDS	70	KKN	71
Aircraft 3 Roundtrip 3	KKN	61	LKL	63
	LKL	64	HVG	65
	HVG	66	MEH	68
	MEH	69	BJF	71
	BJF	72	KKN	73
Aircraft 4 Roundtrip 1	KKN	3	VDS	4
	VDS	5	VAW	6
	VAW	7	BJF	8
	BJF	9	VAW	10
	VAW	11	KKN	13
Aircraft 4 Roundtrip 2	KKN	20	VDS	21
	VDS	22	VAW	23
	VAW	24	VDS	25
	VDS	26	KKN	27
Aircraft 4 Roundtrip 3	KKN	46	BJF	47
	BJF	48	BVG	49
	BVG	50	HVG	51
	HVG	52	LKL	53
	LKL	54	KKN	56

Distance	2 379
Cost	50 919
Time	9
Best sol. sec	7 024
Variables	97 202
Constraints	55 675

Scenario 4:

Cluster 1: Only Trondheim, that is why distance travelled.

Cluster 2:

S4C2	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	BNN	48	MJF	49
	MJF	50	MQN	52
	MQN	53	BOO	54
	BOO	55	SSJ	57
	SSJ	58	BNN	59
Aircraft 2 Roundtrip 1	BNN	10	SSJ	11
	SSJ	12	BOO	14
	BOO	15	MQN	16
	MQN	17	MJF	19
	MJF	20	BNN	21
Aircraft 2 Roundtrip 2	BNN	60	OYS	61
	OYS	62	EVK	63
	EVK	64	OYS	65
	OYS	66	BNN	67
Aircraft 3 Roundtrip 1	BNN	6	EVK	8
	EVK	9	OYS	10
	OYS	11	EVK	12
	EVK	13	BNN	15
Aircraft 3 Roundtrip 2	BNN	45	MJF	45
	MJF	47	MQN	49
	MQN	50	BOO	51
	BOO	52	SSJ	54
	SSJ	55	BNN	56
Aircraft 3 Roundtrip 3	BNN	61	MJF	62
	MJF	63	BOO	65
	BOO	66	MQN	67
	MQN	68	BOO	69
	BOO	70	BNN	73
Aircraft 4 Roundtrip 1	BNN	4	BOO	7
	BOO	8	MQN	9
	MQN	10	BOO	11
	BOO	12	SSJ	14
	SSJ	15	BNN	16
Aircraft 4 Roundtrip 2	BNN	22	SSJ	23
	SSJ	24	BOO	26
	BOO	27	SSJ	29
	SSJ	30	MJF	31
	MJF	32	BNN	33

Distance	3 759
Cost	77 260
Time	13,5
Best sol. sec	1 481
Variables	89 147
Constraints	51 776

Cluster 3:

S4C3	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TOS	8	BDU	9
	BDU	10	ANX	12
	ANX	13	LKN	15
	LKN	16	EVE	18
	EVE	19	TOS	21
Aircraft 1 Roundtrip 2	TOS	27	ANX	28
	ANX	29	LKN	31
	LKN	32	SKN	34
	SKN	35	EVE	37
	EVE	38	TOS	40
Aircraft 1 Roundtrip 3	TOS	53	SKN	55
	SKN	56	NVK	57
	NVK	58	BDU	60
	BDU	61	ANX	63
	ANX	64	TOS	65
Aircraft 2 Roundtrip 1	TOS	2	SKN	4
	SKN	5	LKN	7
	LKN	8	SKN	10
	SKN	11	TOS	13
Aircraft 2 Roundtrip 2	TOS	35	SVJ	38
	SVJ	39	SKN	40
	SKN	41	LKN	43
	LKN	44	SOJ	49
	SOJ	50	TOS	52
Aircraft 2 Roundtrip 3	TOS	60	LKN	64
	LKN	65	SVJ	66
	SVJ	67	SKN	68
	SKN	69	TOS	71
Aircraft 3 Roundtrip 1	TOS	9	SOJ	11
	SOJ	12	NVK	14
	NVK	15	SKN	16
	SKN	17	SOJ	21
	SOJ	22	TOS	24
Aircraft 3 Roundtrip 2	TOS	59	SOJ	61
	SOJ	62	TOS	64
Aircraft 3 Roundtrip 3	TOS	67	EVE	69
	EVE	70	TOS	72
Aircraft 4 Roundtrip 1	TOS	17	NVK	19
	NVK	20	SVJ	21
	SVJ	22	TOS	25
Aircraft 4 Roundtrip 2	TOS	36	EVE	38
	EVE	39	TOS	41

Distance	5719
Cost	95438
Time	21
Best sol. sec	127
Variables	120830
Constraints	67902

Cluster 4:

S4C4	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	LKL	2	BVG	4
	BVG	5	BJF	6
	BJF	7	VAW	8
	VAW	9	VDS	10
	VDS	11	KKN	12
	KKN	13	LKL	15
Aircraft 1 Roundtrip 2	LKL	36	VAW	39
	VAW	40	VDS	41
	VDS	42	LKL	44
Aircraft 1 Roundtrip 3	LKL	54	HFT	56
	HFT	57	KKN	61
	KKN	62	VAW	64
	VAW	65	HVG	67
	HVG	68	LKL	69
Aircraft 2 Roundtrip 1	LKL	1	KKN	3
	KKN	4	ALF	8
	ALF	9	LKL	10
Aircraft 2 Roundtrip 2	LKL	14	ALF	15
	ALF	16	HAA	18
	HAA	19	HFT	20
	HFT	21	LKL	23
Aircraft 2 Roundtrip 3	LKL	45	KKN	47
	KKN	48	VDS	49
	VDS	50	BJF	51
	BJF	52	MEH	54
	MEH	55	HVG	57
	HVG	58	LKL	59
Aircraft 3 Roundtrip 1	LKL	5	HVG	6
	HVG	7	MEH	9
	MEH	10	HVG	12
	HVG	13	HFT	15
	HFT	16	ALF	18
	ALF	19	LKL	20
Aircraft 3 Roundtrip 2	LKL	60	ALF	61
	ALF	62	HFT	64
	HFT	65	HAA	66
	HAA	67	HFT	68
	HFT	69	LKL	71
Aircraft 4 Roundtrip 1	LKL	13	KKN	15
	KKN	16	VDS	17
	VDS	18	KKN	19
	KKN	20	HVG	22
	HVG	23	LKL	24
Aircraft 4 Roundtrip 2	LKL	40	ALF	41
	ALF	42	LKL	43
Aircraft 4 Roundtrip 3	LKL	52	BJF	54
	BJF	55	BVG	56
	BVG	57	BJF	58
	BJF	59	VAW	60
	VAW	61	VDS	62
	VDS	63	LKL	65

Distance	5 263
Cost	97 877
Time	20,25
Best sol. sec	4 500
Variables	183 146
Constraints	100 460

Scenario 5:

Cluster 1:

S5C1	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TRD	1	OSY	3
	OSY	4	TRD	6
Aircraft 2 Roundtrip 1	TRD	5	OSY	7
	OSY	8	TRD	10
Aircraft 2 Roundtrip 2	TRD	61	OSY	63
	OSY	64	TRD	66

Distance	762
Cost	15 774
Time	3
Best sol. sec	0
Variables	2 636
Constraints	2 237

Cluster 2:

S5C2	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	MJF	2	MQN	4
	MQN	5	MJF	7
Aircraft 1 Roundtrip 2	MJF	21	MQN	23
	MQN	24	SSJ	26
	SSJ	27	MQN	29
	MQN	30	MJF	32
Aircraft 1 Roundtrip 3	MJF	50	MQN	52
	MQN	53	MJF	55
Aircraft 2 Roundtrip 1	MJF	13	SSJ	14
	SSJ	15	BNN	16
	BNN	17	RVK	19
	RVK	20	SSJ	22
	SSJ	23	MJF	24
Aircraft 2 Roundtrip 2	MJF	28	SSJ	29
	SSJ	30	BNN	31
	BNN	32	RVK	34
	RVK	35	BNN	37
	BNN	38	MJF	39
Aircraft 2 Roundtrip 3	MJF	46	MQN	48
	MQN	49	MJF	51
Aircraft 3 Roundtrip 1	MJF	47	SSJ	48
	SSJ	49	BNN	50
	BNN	51	RVK	53
	RVK	54	SSJ	56
	SSJ	57	MJF	58

Distance	1657
Cost	43280
Time	10,25
Best sol. sec	24
Variables	27 227
Constraints	17 406

Cluster 3:

S5C3	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	BOO	1	LKN	2
	LKN	3	SKN	5
	SKN	6	SVJ	7
	SVJ	8	BOO	9
Aircraft 2 Roundtrip 1	BOO	44	LKN	45
	LKN	46	SVJ	47
	SVJ	48	SKN	49
	SKN	50	LKN	52
	LKN	53	BOO	54
Aircraft 3 Roundtrip 1	BOO	53	LKN	54
	LKN	55	SKN	57
	SKN	58	SVJ	59
	SVJ	60	LKN	61
	LKN	62	BOO	63

Distance	1 028
Cost	26 529
Time	4,25
Best sol. sec	21
Variables	16 949
Constraints	11 244

Cluster 4:

S5C4	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	TOS	4	SOJ	6
	SOJ	7	EVE	10
	EVE	11	NVK	12
	NVK	13	EVE	14
	EVE	15	TOS	17
Aircraft 2 Roundtrip 1	TOS	1	SOJ	3
	SOJ	4	TOS	6
Aircraft 2 Roundtrip 2	TOS	9	BDU	10
	BDU	11	EVE	12
	EVE	13	NVK	14
	NVK	15	ANX	16
	ANX	17	TOS	18
Aircraft 2 Roundtrip 3	TOS	56	SOJ	58
	SOJ	59	TOS	61
Aircraft 3 Roundtrip 1	TOS	49	BDU	50
	BDU	51	EVE	52
	EVE	53	NVK	54
	NVK	55	ANX	56
	ANX	57	TOS	58

Distance	1484
Cost	38202
Time	6,75
Best sol. sec	234
Variables	39728
Constraints	23955

Cluster 5:

S5C5	Departure		Arrival	
	Airport	Time	Airport	Time
Aircraft 1 Roundtrip 1	LKL	2	BVG	4
	BVG	5	BJF	6
	BJF	7	VAW	8
	VAW	9	VDS	10
	VDS	11	KKN	12
	KKN	13	LKL	15
Aircraft 1 Roundtrip 2	LKL	36	VAW	39
	VAW	40	VDS	41
	VDS	42	LKL	44
Aircraft 1 Roundtrip 3	LKL	54	HFT	56
	HFT	57	KKN	61
	KKN	62	VAW	64
	VAW	65	HVG	67
	HVG	68	LKL	69
Aircraft 2 Roundtrip 1	LKL	1	KKN	3
	KKN	4	ALF	8
	ALF	9	LKL	10
Aircraft 2 Roundtrip 2	LKL	14	ALF	15
	ALF	16	HAA	18
	HAA	19	HFT	20
	HFT	21	LKL	23
Aircraft 2 Roundtrip 3	LKL	45	KKN	47
	KKN	48	VDS	49
	VDS	50	BJF	51
	BJF	52	MEH	54
	MEH	55	HVG	57
	HVG	58	LKL	59
Aircraft 3 Roundtrip 1	LKL	5	HVG	6
	HVG	7	MEH	9
	MEH	10	HVG	12
	HVG	13	HFT	15
	HFT	16	ALF	18
	ALF	19	LKL	20
Aircraft 3 Roundtrip 2	LKL	60	ALF	61
	ALF	62	HFT	64
	HFT	65	HAA	66
	HAA	67	HFT	68
	HFT	69	LKL	71
Aircraft 4 Roundtrip 1	LKL	13	KKN	15
	KKN	16	VDS	17
	VDS	18	KKN	19
	KKN	20	HVG	22
	HVG	23	LKL	24
Aircraft 4 Roundtrip 2	LKL	40	ALF	41
	ALF	42	LKL	43
Aircraft 4 Roundtrip 3	LKL	52	BJF	54
	BJF	55	BVG	56
	BVG	57	BJF	58
	BJF	59	VAW	60
	VAW	61	VDS	62
	VDS	63	LKL	65

Distance	5 263
Cost	97 877
Time	20,25
Best sol. sec	4 500
Variables	183 146
Constraints	100 460

Appendix H: The mathematical model used and the notations

The whole model used including the extensions, and excluded the unnecessary constraints.

Sets:	
$\mathcal{N}$	set of nodes
$\mathcal{A}$	set of airports
$\mathcal{P}$	set of aircraft
ARC	set of arcs $(i,j) \in ARC, i \in \mathcal{N} \setminus \{N\}, j \in \mathcal{N} \setminus \{0\}$
Parameters:	
T	Number of time periods
R	Number of roundtrips
Tmax	The longest travel time between the nodes
M	Big number
Rmax	Maximum duration of the route in time periods
vf_j	Visit frequency for node *j* $j \in \mathcal{A}$
d_{ij}	Distance from node *i* to node *j* $i \in \mathcal{N}, j \in \mathcal{N}$
tt_{ij}	Travel time from node *i* to node *j* $i \in \mathcal{N}, j \in \mathcal{N}$
sd	Service time at the depot in time periods
l	The maximum allowed number of landings during one roundtrip
la	The maximum allowed number of landings at each airport for each aircraft on each roundtrip
c_{ij}	The cost of flying from node *i* to node *j* $i \in \mathcal{N}, j \in \mathcal{N}$
Decision variables:	
X_{ijptr}	1 if aircraft *p* leaves airport *i* in time period *t* to go to airport *j* on roundtrip *r*, 0 otherwise $(i,j) \in ARC, p \in \mathcal{P}, r \in 1..R, t \in 1..T$
Y_{ijptr}	1 if aircraft *p* arrives at airport *j* in time period *t* from airport *i* on roundtrip *r*, 0 otherwise $(i,j) \in ARC, p \in \mathcal{P}, r \in 1..R, t \in 1..T$
D_{pr}	The distance flied by aircraft *p* on roundtrip *r*
TC	The total cost of all the routes flied during the time horizon
FT	Total flight time for all the legs flied during the time horizon.

Mathematical model

$$min \sum_{(i,j)\in ARC} \sum_{p\in\mathcal{P}} \sum_{t=1}^{T} \sum_{r=1}^{R} X_{ijptr} * d_{ij} \tag{15}$$

$$\sum_{(0,j)\in ARC} \sum_{t=1}^{T} X_{0jptr} \geq \sum_{t=1}^{T} X_{ikptr} \qquad p \in \mathcal{P}, r \in 1..R, (i,k) \in ARC \tag{16}$$

$$\sum_{(0,j)\in ARC} \sum_{t=1}^{T} X_{0jptr} = \sum_{(i,6)\in ARC} \sum_{t=1}^{T} X_{i6ptr} \qquad p \in \mathcal{P}, r \in 1..R \tag{17}$$

$$\sum_{i\in\mathcal{N}} \sum_{t=1}^{T} X_{ijptr} = \sum_{k\in\mathcal{N}} \sum_{t=1}^{T} X_{jkptr} \qquad j \in \mathcal{A}, p \in \mathcal{P}, r \in 1..R \tag{18}$$

$$\sum_{(i,j)\in ARC} \sum_{p\in\mathcal{P}} \sum_{t=1}^{T} \sum_{r=1}^{R} X_{ijptr} \geq vf_j \qquad j \in \mathcal{A} \tag{19}$$

$$\sum_{t=1}^{T}\sum_{i\in\mathcal{A}} X_{0iptr} \geq \sum_{t=1}^{T}\sum_{i\in\mathcal{A}} X_{0ipt(r+1)} \qquad p \in \mathcal{P}, r \in 1..R-1 \qquad (20)$$

$$\sum_{(0,j)\in ARC}\sum_{t=1}^{T} X_{0jptr} \leq 1 \qquad r \in 1..R, p \in \mathcal{P} \qquad (21)$$

$$X_{ijptr} = Y_{ijp(t+tt_{ij})r} \qquad (i,j) \in ARC, p \in \mathcal{P}, t \in 1..T - Tmax, r \in 1..R \qquad (22)$$

$$\sum_{i\in\mathcal{N}} X_{ijptr} = \sum_{k\in\mathcal{N}} X_{jkp(t+1)r} \qquad j \in \mathcal{A}, p \in \mathcal{P}, t \in 1..T-1, r \in R \qquad (23)$$

$$\sum_{t=T-T_{max}}^{T} X_{ijptr} = 0 \qquad (i,j) \in ARC, p \in \mathcal{P}, r \in 1..R \qquad (25)$$

$$M * \left(1 - \sum_{j\in\mathcal{A}}\sum_{u=1}^{T} X_{0jpu(r+1)}\right) + \left(\sum_{j\in\mathcal{A}}\sum_{u=1}^{T} X_{0jpu(r+1)} * u\right) \geq \sum_{i\in\mathcal{A}} Y_{i6ptr} * (t + sd) \qquad t \in 1..T, p \in \mathcal{P}, r \in 1..R-1 \qquad (26)$$

$$X_{ijptr} \in \{0,1\} \qquad (i,j) \in ARC \qquad (27)$$

$$Y_{ijptr} \in \{0,1\} \qquad (i,j) \in ARC \qquad (28)$$

$$\sum_{(i,j)\in ARC}\sum_{t=1}^{T} X_{ijpt} * tt_{ij} \leq Rmax \qquad p \in P, r \in 1..R \qquad (29)$$

$$\sum_{t=1}^{T}\sum_{(i,j)\in ARC} X_{ijptr} \leq la \qquad p \in P, r \in 1..R, j \in A \qquad (30)$$

$$\sum_{(i,j)\in ARC}\sum_{p\in P}\sum_{r=1}^{R} X_{ijptr} + \sum_{(i,j)\in ARC}\sum_{p\in P}\sum_{r=1}^{R} Y_{ijptr} \leq 1 \qquad t \in 1..T, j \in A \qquad (31)$$

$$\sum_{(0,j)\in ARC}\sum_{p\in P}\sum_{r=1}^{R} X_{0jptr} + \sum_{(i,N)}\sum_{p\in P}\sum_{r=1}^{R} Y_{i(N)ptr} \leq 1 \qquad t \in 1..T \qquad (32)$$

$$\sum_{t=1}^{T}\sum_{(i,j)\in ARC} X_{ijptr} \leq l \qquad p \in P, r \in 1..R \qquad (33)$$

$$\sum_{t=1}^{20}\sum_{(i,j)\in ARC}\sum_{p\in P}\sum_{r=1}^{R} X_{ijptr} \geq 1 \qquad j \in A \qquad (34)$$

$$\sum_{t=48}^{T}\sum_{(i,j)\in ARC}\sum_{p\in P}\sum_{r=1}^{R} X_{ijptr} \geq 1 \qquad j \in A \qquad (35)$$

$$\sum_{t=1}^{T}\sum_{(i,j)\in ARC} X_{ijptr} . d_{ij} = D_{pr} \qquad p \in P, r \in 1..R \qquad (36)$$

$$\sum_{(i,j)\in ARC}\sum_{t=1}^{T}\sum_{p\in P}\sum_{r=1}^{R}X_{ijptr}*\frac{tt_{ij}*15}{60}=FT \quad (37)$$

$$\sum_{(i,j)\in ARC}\sum_{t=1}^{T}\sum_{p=P}\sum_{r=1}^{R}X_{ijptr}*C_{ij}=TC \quad (38)$$

$$TC, D_{pr}, FT_{pr} \geq 0 \quad (39)$$