Master’s degree thesis

LOG950 Logistics

A heuristic approach to flexible liner shipping

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Preface

This thesis marks the final contribution to a master degree in logistics at Molde University College, specialized university in logistics, and yields 30 ECTS credits.

The road to this point has been long and difficult since the topic was chosen a long time ago. While it at times have been difficult to focus on the work required, it has also been an interesting journey, which has thought me a lot about myself, both good and bad.

There are several people that deserve extra mentions, but first and foremost I would like to thank my parents for their unrelenting love and support throughout my life. My father, Svein Bentsen, is sadly not alive any more, but my promise to one day finish this thesis has been an important source of inspiration during the tough periods. My mother, Margrethe Joten Bentsen, has been an incredible help on a personal level. Always supporting and encouraging me.

I would also like to thank all the employees at Molde University College, my supervisor Johan Oppen, and Lars Magnus Hvattum, who has helped me in the final stages.

Finally I would like to thank those that I have studied with during these years, a special mention to Astrid S. Otnes and Katrine Larsen, who has always been there if I have needed it.

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Summary

This thesis analyses a transport problem based on liner shipping, which uses intermodal transport, a combination of ship and road transport. The problem analyzed is one where frozen fish is transported from Murmansk through the Norwegian coastline, before going to the Netherlands and finally Grimsby in the United Kingdom.

An exact solution method is developed, and optimal solutions are found for all but the largest data sets. In addition a heuristic solution method is developed, which provides results for all the data sets. The results are compared between the different solution methods, and the heuristic is found to be performing around 1-4% worse than the exact solution method for most of the data sets.

The heuristic developed uses a basic local search algorithm to improve the constructed solution. The possible changes that are made are changes to the order that ports are visited. Removal or adding of ports to solution, switching demand that is not in the solution with demand that is part of it. In addition simple adding of demand if that is possible. In addition, it removes demand or ports if the time constraint is violated.

The results from the computational experiments of both the mathematical model and the heuristic, and comparisons of the results, showed that it is possible to find further improvements on the chosen heuristic. Several possible improvements were suggested.

Contents

1.0 Introduction

The shipping industry is growing internationally (Transportation 2010), and the Norwegian government has also expressed the need to utilize the sea better for cargo transport (Stensvold 2013). The cargo shipping industry is mainly divided into three subsets, those are industrial shipping, tramp shipping and liner shipping (Christiansen, Fagerholt, and Ronen 2004). In industrial shipping the cargo owner also controls the ships, tramp shipping is more comparable to a taxi service where the operator travels to the available cargo, and liner shipping is like a bus line where the stops are decided in advance of the voyage.

In liner shipping the decisions are usually performed at a strategic level where the routes stay the same for a substantial amount of time. Some examples of liner shipping routes for fish transport can be found in (Grønland 2011) and show time or date of arrival and departure time for each port. The routes provided in that article are strategic decisions that are meant to be used for the foreseeable future.

In this thesis, the focus will be on decisions made on an operational level. Volatile and seasonal demand can make it difficult to provide good routes on a strategic level. The element of mixing sea transport with road transport further increases the need to move the decisions to an operational level.

Intermodal transport has received increased attention recently (Caris, Macharis, and Janssens 2008), this is where different modes of transportation are combined. In this thesis, road transportation, both before and after the ship arrives at a given port, is used.

In order to solve this, two methods are used. A mathematical model is used to solve the problems to optimality. This will give the best solution for the stated problem, but suffers in terms of scalability as larger problems might not be solvable in an acceptable amount of time. Therefore a heuristic solution is also applied. This cannot guarantee an optimal solution, but will give a solution for larger problems much more quickly.

In this thesis an exact solution method will be used in order to find the optimal solution to as many problem instances as possible, in addition, a heuristic solution is used on all instances. The computational study will look at the difference in results, and analyze those.

The structure of the thesis will be as follows. First, the problem is described, then, the research questions that the report is to answer are put forward. An exact solution method is presented, followed by a heuristic solution method. The results from both are compared and analyzed.

2.0 Problem description

The problem that needs to be solved is a single journey starting in Murmansk, visiting some ports along the Norwegian coast, before going to the Netherlands and finally arriving in Grimsby in the United Kingdom. What happens after arrival in Grimsby is not within the scope of this thesis.

There are two main challenges which need to be solved in order to find the solution. The first is to find the route that the ship is to travel on its voyage. The second is to choose how much demand to bring between each of the ports. The combination of choices made will affect the need to use road transportation.

The demand is segregated into compulsory and optional demand. The problem is therefore to transport all the compulsory demand and the amount of optional demand that generates the largest profit. The routes produced will include arrival and departure time at each port, but will only be valid for that specific voyage, as opposed to the strategic decisions generally used in liner shipping.

The data is divided into four problem sizes that each are divided into four problems, one basic case, one where time is the main constraint, one where capacity is the main constraint, and finally one where both time and capacity are issues. The main differences between them are that the ship travels slower where time is an issue, and that there is more demand where capacity is an issue. The demand data is identical in the base case and the time constrained case, similarly the capacitated and the case with capacity and time constraints are identical in terms of demand.

The added ports as the problem size increases are all along the Norwegian coastline. Mostly fairly close to other ports, leading to an increase in road transportation, but there are also some ports added that have no road connections.

Road transportation can be divided into three parts. The first is the transport that is purely by road. The second is pre-road transport. In this case the goods are transported first by road from one port to another and then by ship from that port to the destination. This will typically be done if the ship does not visit the port from which the demand originates. The last road transport option is post-road transport. This works much in the same way as pre-road transport, however, here the ship transport happens first, and then road transport to the destination.

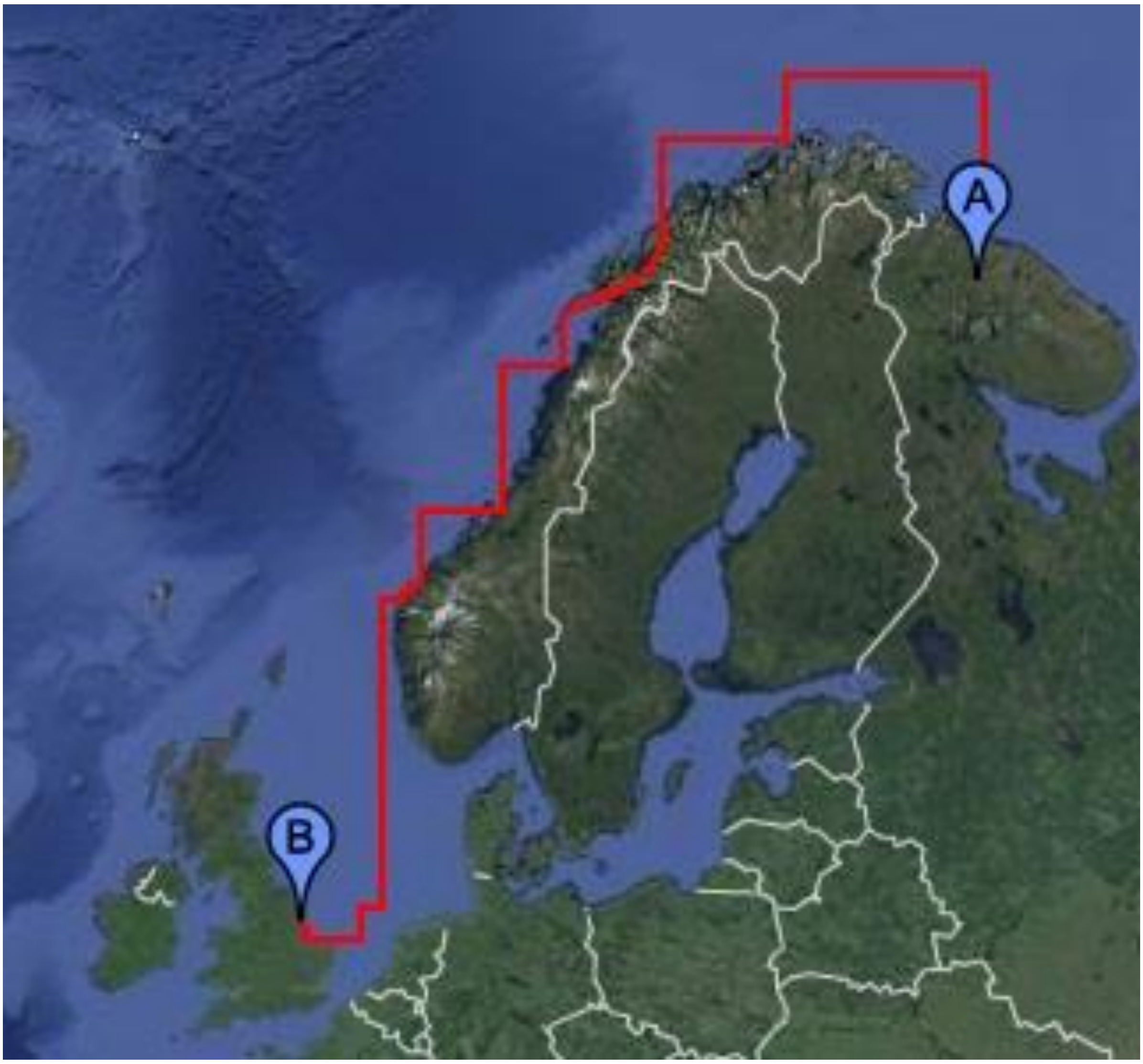

Figure 1, possible route

Figure 1, possible route, shows a possible route from the start port in Murmansk to the end port in Grimsby.(ports.com 2016) was used to create the figure.

2.1 *Assumptions*

In a problem like this, there are several assumptions that needs to be made. Some concern what data is collected, others what is left out. All those decisions will impact the validity of the solutions found.

2.1.1 Deterministic data

One of the main assumptions made is that the data can be considered deterministic. In this case, the data is secondary data collected from a real world instance. Travel distances are clearly very close to deterministic. There might be some inaccuracies concerning road transport where some roads might close on short notice due to accidents, but both the chances of that happening and the impact of such events does seem minimal.

Vessel speed is another matter, the speed indicated in the data will have to be considered as the average speed. Weather conditions might have an impact on the travel speed, making it slightly risky to consider it deterministic, at least if exact arrival times at ports are considered important. The time at each port is given as a fixed time plus a variable time based on the amount of loading and unloading that needs to be done, this seems fair.

It is assumed that the cargo can be handled immediately no matter when the ship arrives, this might be true for some ports, but most likely not for all of them. Introducing time windows would give the solution better validity but also increases complexity.

It is also not certain that the demand can be considered known for all arcs before the start of the voyage. The total travel time from Murmansk to Grimsby is around 10 days, so it does seem plausible that the demands might change in that time. However, there are some assumptions that need to be made in order to make the problem size manageable.

2.1.2 Handling times

On the other hand, assumptions are also made that increase the complexity. One of those is the use of fixed and variable port handling times. Given the assumption of no time windows at each port, the added accuracy provided by those calculations might not significantly improve the solution. However, including the calculations does make it easier to include time windows at some point, and also improves the accuracy of the arrival and departure times from the ports.

3.0 Research questions

There are several questions that this paper is meant to answer, both in terms of the exact solution method and the heuristic. Finding these answers will be the main part of the paper. They vary in difficulty and to what degree a single answer can be found, but all of them will be addressed at various points.

3.1 Which existing models can be used for this problem?

This research question is important in order to find what other research is done in this area earlier, and what models that research has found in order to solve similar problems. This is mostly relevant in terms of the exact solution method, however, finding elements from earlier work can also be very helpful for the heuristic solving method.

3.2 Which parts of the real life problem can be disregarded?

As with all modelling, the model that is created here can never include every factor that is relevant in the real world. Therefore it is important to find out which elements are the most significant in order to make the model as realistic as possible. There is no point in creating a model that solves a problem no one is interested in, therefore the elements that are eliminated need to be of as little value to the overall solution as possible.

3.3 How big problems can be solved exactly?

Finding the limit of the problem size that the exact solution can solve within a reasonable amount of time is also important as that provides the starting point for the heuristic solution method. An exact optimal solution will always be the best option, but it is not likely that the biggest problems can be solved to optimality.

3.4 Which heuristic is best used to find good solutions?

There are several different heuristics available, and this research question is therefore similar to the first one as it deals with how similar problems have been handled previously. In this case it is the heuristic solutions that are looked at in order to find the heuristic best suited for this problem.

3.5 How are heuristic solutions evaluated?

An important element is to evaluate the solution found. If no optimal solution can be found, it is important to find a way of evaluating the heuristic solution.

3.6 Is additional data and testing needed?

When the original data sets are solved and tested, there might be some unanswered questions that can only be resolved through additional data or further testing.

4.0 Theory Review

There are several kinds of literature that is relevant to this problem. The first is to find an article that looks at ship routing in general in order to obtain a better understanding of how this field is handled earlier in the literature, and seeing what sorts of problems are looked at in the most detail. Here the article (Christiansen, Fagerholt, and Ronen 2004) is a good starting point as it looks at the three main categories of freight shipping, industrial, tramp and liner. Further it looks into planning at a more strategic level than what is necessary for this research, as the strategic level decision of how many vessels to have in the fleet to a large degree is given here. It also looks into the operational level of planning for each of the disciplines of shipping mentioned. For this research the most relevant is the part about operational level decisions for liner shipping. There is also an updated version of this article available (Christiansen et al. 2013), which includes references to newer articles. Both articles are relevant to look at as a starting point.

4.1 Review articles

The article (Christiansen, Fagerholt, and Ronen 2004) divides the planning problem in liner shipping into four different planning levels. The first of those is route and schedule design, followed by deciding the fleet size and mix, fleet deployment and finally cargo booking. The route and schedule design contains deciding which routes and schedules to be chosen, this decision is already taken in this problem. The fleet size and mix is to a certain extent decided, however there are options when considering land transport as an option, so this could be relevant, however, this is only to a small degree as no long term decisions for number of available land transport trucks will be made, it is assumed that the necessary number will be available.

Fleet deployment works on a similar level, however, here the land transport element is more relevant. In the article this level is compared to aircraft scheduling. Cargo booking is considered an operational planning problem, and is highly relevant for this research.

This article is important in order to better the understanding of shipping in general, and to classify the problem as a liner shipping problem. The next step is to look more closely at liner shipping, and the different aspects it contains. For this the article "Classification of ship routing and scheduling problems in liner shipping" by (Kjeldsen 2011) is an excellent starting point. This article both classifies the liner shipping problems with regards to several factors, and also provides a review of the literature available. A good start to the research is therefore to go through the article step by step in order to classify the given problem in terms of each of the available criteria. Further, the articles are classified by each of those criteria, making it possible to get a good overview of which articles are most closely related to the problem at hand.

The first step is to look at how many starting points there are on the route. Starting points are defined as ports that the ship is forced to call. The options are one, none or many. Looking further from that it is possible to see which previous articles have dealt with problems of the different types. For this research several starting point is most relevant, which is dealt with in only three of the articles mentioned. One of those, (Rana and Vickson 1991) provides a fairly similar problem, however, it uses several ships.

By going through each of the classification elements in the article it is possible to get a fairly clear picture of which articles has handled problems that are most similar to the research topic. In addition, on the points where there are differences between the most interesting articles and the research topic, it is also possible to find how this specific topic is handled in other literature.

4.2 Mathematical modelling

In order to find an exact solution to the problem a mathematical model will be developed. As mentioned, there are several different mathematical models which in some aspects incorporate the elements that are present in this problem. In order to properly review the literature in this field it is therefore important to start by mapping out what elements are crucial to include, and start by finding the best ways of modelling those. When this is done, a basic working model can be created, and the task of expanding the model can begin.

This will include looking at models used in similar problems, and finding the best ways of implementing those that are needed in order to make the solution incorporate all the needed elements, while not including elements that only deteriorates the solution time, without adding value to the solution in terms of improved realism.

The profitable arc tour problem (PATP) is presented in (Feillet, Dejax, and Gendreau 2005) and is relevant to this problem. This type of problem has profits and costs related to each arc, in much the same way as this problem, but also several differences. In that problem the goal is find a set of cycles through the graph that maximizes the profit, with a set of constraints about the length of the cycles and number of visits on each arc.

4.3 Heuristic solution method

Another interesting article is (Agarwal and Ergun 2008). In this article the task is to create service routes as efficiently as possible giving certain demands along possible ports. Here a mixed-integer solving method is used in addition to a heuristic solving method. The use of a heuristic is interesting in reference to the research topic as this will have to be used.

The last step is to research the possible heuristic solutions in order to find the ones that resemble the problem the most closely. The task is than to find heuristics used to solve vehicle routing problems (VRP). Looking at the research on this one of the articles that stand out is (Derigs and Reuter 2009) which uses a tabu search heuristic to solve VRPs.

It is also possible to combine different methods when using a heuristic solution method. An article by (Belhaiza, Hansen, and Laporte 2014) shows a hybrid of tabu search and variable neighborhood search to solve VRP with multiple time windows, which could be similar to this problem.

4.4 Intermodal transport

The most noticeable difference between the problems described above and the research problem handled in this thesis is the opportunity to combine different modes of transportation, with both sea and road transport used. This field is called intermodal transport and is described in (Macharis and Bontekoning 2004) as a combination of at least two different transport types in a single transport chain. This article gives an overview of the previous research in this field. While the focus is mainly on the combination of rail and road transport in the article, the principle should be fairly easy to convert into the problem of combining sea and land transport. As with the article classifying the liner shipping problem (Kjeldsen 2011) this article also points to different articles for each type of problem, based on if it uses a tactical, strategic or operational approach. Using those could be very helpful in finding similar problems, and see how the previous literature has handled those.

5.0 Solution Methodology

Two solutions methods for the given problem are developed. An exact solution is found using mathematical modelling and a heuristic is used in order to find good solutions for all problem sizes. The solutions found using the two methods have to be comparable, therefore some assumptions will have to be clarified before specifics for each solution method is decided.

There are several reasons to use an exact method as part of the solution. If an answer is found within a reasonable time using an exact method this will always be the optimal solution. It also makes it possible to test the heuristic on a smaller scale when the optimal solution is known.

The main weaknesses are issues related to scaling. Large problem instances will increase the solving time exponentially, and there will therefore be a limit to the size of the problems that can be solved using this method.

Another weakness comes from the assumptions that will have to be made in order to make the problem size manageable. It is very important to make the correct assumptions in order to ensure that the solution stays valid for the given problem. For instance, an assumption is made with regards to travel speed, where this is kept deterministic and stays the same regardless of where the ship is, and other factors such as weather conditions.

This can be solved by making the model more complex, but this will lead to an increase in solving time, including several of these elements can lead to the exact solution method being too slow to provide solutions to real world problem sizes.

The main advantages of the heuristic method are solving speed and the possible problem sizes that can be solved. A simple heuristic can give answers very quickly, within a reasonable degree of accuracy.

On the other hand, the main disadvantage is the fact that there is no guarantee that the solution that is found is optimal, and it is not necessarily possible to know how good the solution is. Even if the solution found by the heuristic is close to optimal for the small problems, the discrepancies could be large for big problems.

5.1 Data needs

The data needed to emulate the real world problem to a satisfying degree can be divided into three categories to be used in the models. Those are sets, which are collections of entities used to index parameters, variables and constraints. Parameters are values that does not change. They are defined before executing the solution method, and will stay the same until a solution is found.

Variables are values that vary in search for the best possible solution. In this problem there are both binary variables, that are either 0 or 1, and continuous variables that can hold any value, within the confines of the constraints.

Arcs are, in this problem, connections between two ports. There are parameters connected to the arcs. For instance a demand arc between two ports will have parameters for compulsory demand, optional demand and price.

5.1.1 Sets

There are two main types of sets required both for the exact and the heuristic solution methods, those are the ports and the arcs. The port set lists all the ports that are accessible in the given data set. A subset of those are the required ports, which are those pre-determined to be visited.

The main arcs are the demand arcs. Those represent every connection where there can be demand between the given ports. There are two subsets of those arcs. Those are the sea arcs and the road arcs. Those represent the possible direct travel routes by sea and road. In addition there are pre-road and post-road arcs, which combine the two.

5.1.2 Parameters

Several parameters are needed. There are travel distances for each of the sea and road arcs. For the ports, each port has a given handling time. In addition there are parameters for different kinds of costs, the capacity of the ship, the speed at which it travels, and the latest arrival time at the end port.

Some of those parameters stay the same throughout all of the different data sets that are tested, while others may vary. The demand will for instance vary from data set to data set, while the capacity will always stay the same. The travel distance from one port to another will also always stay the same, but changing the number of ports will also change the number of arcs that have a travel distance attached to it. All parameters stay the same throughout the solution process.

5.1.3 Variables

There are two main types of variables, binary and continuous. The binary variables indicate if an arc is used or not. As there are four types of arcs, there are also four binary variables. One each for sea arcs, road arcs, pre-road arcs and post-road arcs.

Similarly there are also a continuous variable for each of those, in addition to one for each of the demand arcs. These variables represent the amounts being carried on the arcs.

In addition to those, there are also variables that are calculated based on the previous variables. The load when leaving each port is dependent on the amounts shipped, while the time variables depend on the route selected and also the amounts shipped.

5.2 Objective

The objective is to maximize the profit from the voyage. This means that every income and cost contributions needs to be included. The income comes purely from the amount of goods transported with the given price paid from the customer. The costs can be split into four parts. There are the sailing costs, which vary depending on the amount of nautical miles travelled by the ship. In addition there are costs connected to road transport, one each for pure road transport, pre-road transport and post-road transport.

5.3 Constraints

There are a number of constraints needed in order to make sure that the solution that is found is feasible. There are constraints connecting the binary and continuous variables, for instance to make sure that goods cannot be transported directly to a port that is not visited by the ship. There are also capacity constraints, and time constraints. All constraints are explained further below.

5.4 Data assumptions

There are several assumptions that need to be made in order to solve a problem like this. Those are related to the possible arcs, the road transport, the demand, and the handling time at the ports. Some are made to decrease the problem size, while others are simply choices made where both options are reasonable, and some do both.

In order to make the problem size more manageable some of the theoretically possible arcs have been removed. For instance it does not make much sense to turn the ship around and return to Kirkenes in the northeastern part of Norway after reaching ports much further south. However, the decision to remove some of the possible arcs does open up the possibility of removing some that in fact could have been part of the optimal solution.

One assumption that is more likely to have an impact on the chosen solution is the decision to make each road trip different. An example of this could be that there is demand from two different ports going to a port where the ship does not stop, but where post-transport is used. It is in those cases assumed that there are two different trucks used for the final road transport, even though there is no capacity constraint on the road transport. The road costs are split into three parts, fixed costs, loading costs and a cost per kilometer travelled. This assumption doubles the fixed costs and the cost per kilometer in the cases where this happens, while the loading costs are unaffected as the amount transported stays the same. Avoiding this problem would demand more binary variables, and would therefore increase the solving time.

It is also assumed that the demand can be split into any amount. This might be correct, but it does seem to at least be a simplification. As mentioned, the goods transported are frozen fish, and while it does seem reasonable to be able to split the demand somewhat, having the amount transported as a continuous variable might not be entirely accurate. However, the alternatives do add to the complexity of the problem. It could be possible to split the demand, but only into integer values, or it could vary depending on where the demand originated from and was headed to. Both of those options add to the complexity.

The time factor is one of the assumptions that does not really add or take away from the complexity of the problem. It is assumed that the voyage starts at time 0, regardless of the amount loaded at the start port, and the latest arrival time at the end port is the same regardless of the amount to be unloaded there. A consequence of this is that the demand originating from the start port or ending at the end port is more valuable when the time is a critical constraint. While demand originating and arriving at ports in between will take away from the available time at both ports, demand that goes all the way from the start to the end port will not add time at all. Again, it is possible that this is realistic, but an alternative could have been to include the time spent at both those ports, but to increase the latest arrival time accordingly.

6.0 Exact solution method

The exact model chosen to solve this problem is based on (Oppen 2016). The exact model is notated below with sets, parameters and variables, followed by the mathematical formulation.

Sets:

N – Set of ports

R – Set of required ports $R \subset N$

A^D – Set of demand arcs $A^D \subset (NxN)$

A^R – Set of road arcs $A^R \subset A^D$

A^S – Set of sea arcs $A^S \subset A^D$

A^{PRE} – Set of pre-road arcs $A^{PRE} \subset (A^R x N)$

A^{POST} – Set of post-road arcs $A^{POST} \subset (N x A^R)$

Parameters:

M – Big number

D_{ij}^C – Compulsory demand from port i to j $(i,j) \in A^D$

D_{ij}^{O} – Optional demand from port i to j $(i, j) \in A^D$

P_{ij} – Price of goods transported from port i to j $(i, j) \in A^D$

S_{ij} – Travel distance by sea from port i to j $(i, j) \in A^S$

R_{ij} – Travel distance by road from port i to j $(i, j) \in A^R$

U^S – Starting port $S \in N$

U^E – End port $E \in N$

C^S – Cost of travelling by sea per nautical mile

C^R – Cost of travelling by road per kilometer

C^L – Cost of loading and unloading for road transport

C^F - Fixed cost of using truck

T_i^H - Handling time per ton in port i $i \in N$

T^A – Latest arrival time on end port

T^F – Fixed time at each visited port

V – Vessel speed

Q – Vessel capacity

Variables:

a_{ij} – Total tons from port i to j $(i, j) \in A^D$

q_{ij} – Total tons by ship from port i to j $(i, j) \in A^S$

r_{ij} – Total tons by road from port i to j $(i, j) \in A^R$

w_{ijk} – Tons by road from i to j, followed by ship from j to k $(i, j, k) \in A^{PRE}$

z_{ijk} – Tons by ship from i to j, followed by road from j to k $(i, j, k) \in A^{POST}$

l_i – Load when leaving port i $\quad i \in N$

t_i^s – Time spent at port i $\quad i \in N$

t_i^a – Arrival time at port i $\quad i \in N$

t_i^d – Departure time from port i $\quad i \in N$

ρ_{ij} – 1 if sea arc is used, 0 otherwise $\quad (i,j) \in A^S$

φ_{ij} – 1 if road arc is used, 0 otherwise $\quad (i,j) \in A^R$

σ_{ijk} – 1 if pre-link is used, 0 otherwise $\quad (i,j,k) \in A^{PRE}$

γ_{ijk} – 1 if post-link is used, 0 otherwise $\quad (i,j,k) \in A^{POST}$

Mathematical model formulation:

Max $\sum_{(i,j)\in L^D} a_{ij} * P_{ij}$

$$- \sum_{(i,j)\in L^S} \rho_{ij} * C^S * S_{ij}$$

$$- \sum_{(i,j)\in L^R} (\varphi_{ij} * \left(C^F + \left(R_{ij} * C^R\right)\right) + C^L * r_{ij})$$

$$- \sum_{(i,j,k)\in L^{PRE}} (\sigma_{ijk} * \left(C^F + \left(R_{ij} * C^R\right)\right) + C^L * w_{ijk})$$

$$- \sum_{(i,j,k)\in L^{POST}} (\gamma_{ijk} * \left(C^F + \left(R_{jk} * C^R\right)\right) + C^L * z_{ijk}) \qquad (1)$$

Subject to:

$$a_{ij} \le D_{ij}^C + D_{ij}^O \qquad \forall (i,j) \in A^D \qquad (2)$$

$$a_{ij} \ge D_{ij}^C \qquad \forall (i,j) \in A^D \qquad (3)$$

$$q_{ij} = a_{ij} - \sum_{(i,k,j)\in L^{PRE}} w_{ikj} + \sum_{(k,i,j)\in L^{PRE}} w_{kij} - \sum_{(i,k,j)\in L^{POST}} z_{ikj} + \sum_{(i,j,k)\in L^{POST}} z_{ijk} \quad \forall (i,j) \in A^D \backslash A^R \quad (4)$$

$$q_{ij} = a_{ij} - \sum_{(i,k,j)\in L^{PRE}} w_{ikj} + \sum_{(k,i,j)\in L^{PRE}} w_{kij} - \sum_{(i,k,j)\in L^{POST}} z_{ikj} + \sum_{(i,j,k)\in L^{POST}} z_{ijk} - r_{ij} \quad \forall (i,j) \in A^R \quad (5)$$

$$\sum_{(i,j)\in L^S} M * \rho_{ij} \geq \sum_{(i,j)\in L^D} q_{ij} \quad \forall j \in N \backslash U^S \quad (6)$$

$$\sum_{(i,j)\in L^S} M * \rho_{ij} \geq \sum_{(i,j)\in L^D} q_{ij} \quad \forall i \in N \backslash U^E \quad (7)$$

$$M * \varphi_{ij} \geq r_{ij} \quad \forall (i,j) \in L^R \quad (8)$$

$$M * \sigma_{ijk} \geq w_{ijk} \quad \forall (i,j,k) \in L^{PRE} \quad (9)$$

$$M * \gamma_{ijk} \geq z_{ijk} \quad \forall (i,j,k) \in L^{POST} \quad (10)$$

$$l_{U^S} = \sum_{(U^S,j)\in A^D} q_{(U^S,j)} \quad (11)$$

$$l_i \leq Q \quad \forall i \in N \quad (12)$$

$$l_i + \sum_{(j,k)\in A^D} q_{jk} - \sum_{(k,j)\in A^D} q_{kj} \leq M * \left(1 - \rho_{ij}\right) + l_j, \forall (i,j) \in A^S \quad (13)$$

$$\sum_{(i,j)\in A^S} \rho_{ij} = 1 \quad \forall i \in R, U^S \quad (14)$$

$$\sum_{(i,j)\in A^S} \rho_{ij} = \sum_{(j,i)\in A^S} \rho_{ji} \quad \forall i \in N \backslash U^S, U^E \quad (15)$$

$$t_i^d + \frac{S_{ij}}{V} \leq M * (1 - \rho) + t_j^a \quad \forall (i,j) \in L^S \quad (16)$$

$$t_i^s \geq T^F + \left(\sum_{(i,j)\in A^D} q_{ij} + \sum_{(j,i)\in A^D} q_{ji}\right) * T_i^H, \forall i \in N \backslash U^E, U^S \quad (17)$$

$$t_i^d = t_i^a + t_i^s \quad \forall i \in N \backslash U^S, U^E \quad (18)$$

$$t_{U^E}^a \leq T^A \quad (19)$$

$$\rho_{ij}, \varphi_{ij}, \sigma_{ijk}, \gamma_{ijk} \in \{0,1\} \quad (20)$$

$$a_{ij}, q_{ij}, r_{ij}, w_{ijk}, z_{ijk}, l_i, t_i^s, t_i^a, t_i^d \geq 0 \quad (21)$$

The objective function (1) maximizes the total profit, calculated as the amount shipped on each demand arc multiplied by the price. Subtracted from this are the costs associated with the transport. There are sailing costs, which vary according to the chosen route. In addition there are costs associated with road transport. Pure road costs occur if demand is shipped directly using road transport. Pre-transport costs occurs for demand shipped first by road, followed by sea transport, and post-transport costs occurs for demand shipped by sea first, followed by road.

Constraints (2) and (3) makes sure that the amount shipped is at least the compulsory demand, but no more than the sum of compulsory and optional demand.

Constraints (4) and (5) determines the amount shipped using sea transport as the total amount shipped on an arc, subtracting the amount shipped by road.

Constraints (6) and (7) ensures that a port must be visited in order to be able to ship goods, similarly constraints (8), (9) and (10) ensures that the accompanying binary variable must be set to 1 in order to be able to ship goods by road.

Constraint (11) sets the initial load variable as the amount shipped using sea transport from the starting port. Constraints (12) makes sure that the load at each port is less than the capacity of the ship. The last load constraints (13) determines that the load for each visited port.

Constraints (14) makes sure that each of the required ports and the start port is visited. Constraints (15) are the continuity constraints which makes sure that each port, except for the start and end port, have an incoming arc used as often as an outgoing arc.

Constraints (16), (17), (18) and (19) are time constraints. (16) determines that the arrival time at any port must be at least the departure time at the previous port, plus the sailing time between the ports. (17) calculates the time spent at each port, as the fixed time, added with the total loading and unloading at the port multiplied by the handling time. Constraints (18) sets the departure time at a port as the arrival time added with the time spent, and finally constraint (19) sets the latest arrival time at the end port.

(20) and (21) sets variables as binary and continuous.

6.1 Further explanations

In order to clarify the variables the following example can be shown.

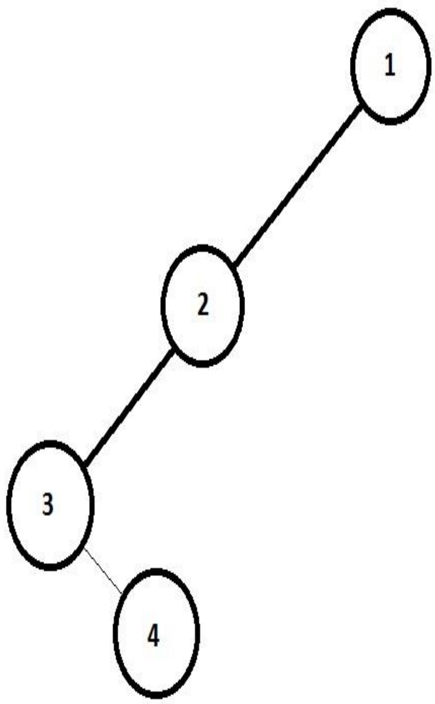

Figure 2

Figure 2 shows a possible situation where there are four ports, the ship travels from port 1, to port 2, and ending in port 3, illustrated by thick lines. There is a road connection between ports 3 and 4. There is demand of 100 tons from port 1, both to ports 3 and 4, which is met in the solution. The variables connected with this will then be as follows.

$a_{13} = 100$

$a_{14} = 100$

Those are the variables that show what is shipped where, without considering the method of transport.

$z_{134} = 100$

The z variable is used for post-transport. This means that 100 tons are transported by ship from 1 to 3, and by road from 3 to 4.

$q_{13} = 200$

The q variable shows what is transported by ship, this becomes 200 tons because it includes both the demand directly to the port, but also the demand which arrives at port 4 after using post-transport. This can be calculated from constraints 4 in the model. This also shows that the value of q_{14} will be 0, as it subtracts the value of z_{ikj} but adds the value of z_{ijk} from the a_{ij} value in order to calculate the q_{ij} value.

6.2 The size of M

The parameter M is described in the model as a big number, in order to cut the solution speed as much as possible it would be an advantage to get this value as close to optimality as possible. It is used in constraints 6 through 10, and the first demand is that it needs to be large enough to make sure that it does not influence the optimal solution. For the constraints 8 through 10 this only means that it needs to be larger than the largest amount shipped by the different forms of road transport.

The largest number will generally be coming from constraints 6 and 7. In those, the value of M needs to be at least as large as the sum of goods incoming to any port, or outgoing from any port. This value is more difficult to find an exact formula for. It is clear that the upper limit will be the capacity of the vessel, but that would indicate that the ship is full when arriving at a port, and delivers all of its cargo to that port. In practice it seems possible to use smaller numbers. It could be possible to use the sum of demand to or from the port where this is largest, but as shown above, the q value can be larger than the demand to that particular port, if it is connected by road to other ports.

As it is difficult to find an exact formula a number is chosen, and the results of the computational experiments are examined to see if the values are close to the M value. If that is the case another test is made with a larger M value to see if this influences the optimal solution.

6.3 Known issues

There is a known issue with the mathematical formulation used, as there are no variables that handle the order of visits by the ship, thereby making it possible to send goods by ship between any ports as long as they are both visited by the ship, regardless of which port was visited first.

This follows from constraints 6 and 7, which are the ones that constrain the value of q_{ij}. These constraints show that the value of q_{ij} must be 0 if the sum of ρ_{ij} and ρ_{ji} is zero. When those values are not zero, the q value can hold any value. Similarly the a_{ij} value is only constrained by the demand, given in constraints 2 and 3. This means that, looking back at Figure 2, if the ship travels from 1 to 2, and then to 3, the value for ρ_{12} and ρ_{23}will be 1. This means that the q value will not be constrained, and if there were a demand of 50 from 3 to 2, the value of a_{32} could be 50, and the value of q_{32} would be given by its formula given in constraints 6 and 7 to also be 50.

At the time this issue was discovered there was not enough time to correct it and make new computational experiments. Solving this issue would require introducing new binary variables which dealt with the precedence issue. Constraints would have to be made in order to make sure that direct sea transport could only be used if the arrival time at the destination port for the demand was larger than the arrival time at the port of origin.

This means that there will be more difficult to compare the results found using the exact method with the results found using the heuristic method, as this issue was handled in the heuristic solution. Even though the cost of switching to road transport is known, it is not certain that this will represent the new optimal solution. When comparing the solution the current optimal solution can be seen as an upper bound on the optimal solution, while switching to road transport on all the affected arcs can be seen as a lower bound.

The nature of the problem makes this more difficult to spot, as most of the demand will follow a natural route from north to south. It was therefore only when comparing the results from the heuristic with the exact solution that this problem was found.

7.0 Heuristic solution method

When selecting a heuristic solution method, the first step is to choose whether to use a local search heuristic, or a population search. According to (Cordeau et al. 2002) a local search will at each iteration move from the current solution to promising solutions in its neighborhood. While a population search maintains a pool of parent solutions which are combined to produce offspring. (Cordeau et al. 2002) also points to there being four criteria to which the performance of a heuristic should be evaluated. Those are accuracy, speed, simplicity and flexibility. It is very difficult to create a heuristic that satisfy all of these criteria, as there are tradeoffs between them. A heuristic that is fast and accurate will often be tailored to the specific problem at hand, and will therefore be less flexible. Flexibility in this context is the ability to use the heuristic to solve different kinds of problems.

A heuristic solution method will, to a larger degree than the exact solution method, be based on the specific problem at hand. Some simplifications are made to narrow the search for the best solution based on the available data. This might make it more difficult to find the optimal solution in some cases.

A solution is here defined as any combination of variables that create a feasible solution. The solution found from the construction step is named S_0, and any changes made will have to increase the total profit from the one found in S_0.

The first step in the construction of the solution is to choose which ports to visit and in which order. In order to maximize profits it is essential to be able to pick up as much of the demand as possible. When choosing ports the most important factor is therefore to add the ports that make it possible to deliver more goods.

7.1 Clustering

In order to achieve this, the first step is to divide each of the ports into different clusters. Each cluster can be defined as ports that are connected by a road network. This means that the first port, Murmansk, is defined as cluster 1, any port that is connected by road to Murmansk is also defined as part of cluster 1. The closest port not connected by road is than defined as cluster 2, and this goes on until all ports have a defined cluster.

After this was done the next step was to choose what ports are visited and in what order. Two approaches were considered here. The first was to start with the bare minimum in order to construct a feasible solution, and then add ports as long as it was profitable in the improvement part of the heuristic. The other way was to attempt to find a good solution right away.

Both approaches have their advantages. The main advantage of starting with the bare minimum is that it is easier. The main disadvantage of this approach is that the initial solution construction becomes fairly unbalanced, meaning that the heuristic will favor the demand from the selected ports, and increasing the calculation time for the improvement heuristic.

It is therefore decided to choose the ports at this stage. The difference in the final outcome should not be significant either way. There are, as mentioned, four different types of data sets. The base case and the capacity case are both using a vessel speed of 13 knots, while the time case and the time-capacity case use a vessel speed of 9 knots.

This difference is quite important when deciding how many ports to visit. It is possible to calculate the upper limit on the time spent at each port, as the handling time at each of the ports are known. This means that it is possible to assume that if by ignoring the capacity constraint, a solution which carries all demand is found that is feasible with regards to the time constraint, it can be assumed that it is safe to visit all the ports. It is, however, not impossible to have a situation where it is possible to visit all the ports without violating the time constraint, but where there are better solution that ignore one or more ports. The initial construction will use all ports if that is possible.

The choice is therefore only for the data sets where time is an issue. As time is important here, the choice is made to initially look at the handling time at the ports as one of the factors. While the difference is fairly small between the different ports, the fastest being 0.011 hours per ton, and the slowest being 0.019 h/t. This difference is still large enough that if the total loading and unloading at a port is 500 tons, which is fairly common, this still makes a difference of 4 hours on the total travel time. It the initial construction it is decided that at least one port from each cluster is chosen, and additional ports are added based on total supply and demand, and handling time.

7.2 Construction

With the ports now selected the next step is to construct the route that is travelled. For the time-constrained data sets this is now fairly straight forward as it will usually just be one port in each cluster. The choices are made for the other data sets, where there can be several options. Here time is not a critical constraint, it is therefore the capacity that is most critical. At this point it is not known where the bottlenecks will be in terms of capacity, every port can therefore be seen as a potential bottleneck. As the total supply and demand of each port is available through the demand data, the decision variable for selecting the order of visits in a cluster is decided to be to start with the port where the demand is largest compared to the supply. This is to minimize the amount of ports where the capacity is critical.

7.3 New variables and parameters

Some new variables and parameters are created in order to make the heuristic work as well as possible. These variables and parameters will generally be possible to calculate directly using the known values, and are included in order to increase the speed of the solving process.

These include the modes of transport possible for each demand arc. The possibility of using direct road transport will be a parameter, while the rest are variables, determined by the selected route. For each of these values there are also costs and time associated. The time that is added to the arrival time at the end port is calculated by the handling times at the port of origin and arrival for the demand arcs. This value will change if pre-road or post-road transport is used, and the handling time at that port is different from the ones using direct sea transport.

Similarly there is a cost associated with using any form of road transport. This is also visualized in the heuristic in order to more easily be able to pick the best option directly instead of having to calculate these values several times.

There are also constructed tables which show for each port how much each of the demand arcs contributes to the load when leaving each port. Similarly tables are constructed to show how much extra capacity is needed when leaving each port for the demand that is not currently met. This makes it easier when the improvement heuristic is reached, as those values can be compared to each other, easily showing if improvements are possible.

In addition a variable is also created to calculate the income contribution per hour spent for each of the arcs. This is done by multiplying the amount shipped by the price, and dividing it by the amount of hours that are added by including this demand. This is done in order to make the best possible changes when time is the critical constraint.

7.3.1 Construction steps

Now that the route is decided, and the new variables and constraints are calculated, the next step is to create an initial solution.

When this is done the actual construction of the initial solution starts. This takes place by the following steps.

1. For each demand arc where there is compulsory demand, if direct sea transport is possible then:
    a. Find the port where the demand originates, add the amount to the load outgoing from that port.
    b. Continue adding this amount to the load of each port until the destination is reached.
    c. Set the values of a_{ij} and q_{ij} to the given amount.
    d. Set mode of transport to ship.
    e. Update contribution table.
2. Else, if direct road transport is possible:
    a. Set values of a_{ij} and r_{ij} to the given amount.
    b. Set mode of transport to pure road.
3. Else, if pre-road transport is possible:
    a. Update origin port from set of pre-road arcs.
    b. Follow 1a-b.
    c. Set the values of a_{ij} and w_{ikj} to given amount.
    d. Set mode of transport to pre-road.
4. Else, if post-road transport is possible:
    a. Update destination port from set of post-road arcs.
    b. Follow 1a-b.
    c. Set the values of a_{ij} and z_{ikj} to the given amounts.
    d. Set mode of transport to post-road.

5. For each arc where there is optional demand, if direct sea transport is available:
    a. For each port, check if the contribution from adding the demand is higher than the amount of available capacity.
    b. If there is not enough capacity to transport entire amount, update the amount to ship to the highest possible value that does not exceed capacity at any port.
    c. If that amount is larger than 0, follow steps 1a-e
6. Similar for other modes of transport, check capacity, transport as much as possible
7. Check time, if time at arrival at end port is larger than the latest arrival time then:
    a. Switch to direct road transport where possible
    b. Sort demand in current solution by contribution to income per hour, from lowest to highest.
    c. Choose demand with lowest contribution to income per hour
    d. If time saved by removing demand is less than the time needed to cut, remove demand from solution and repeat 7b-c
    e. If time saved is equal or larger than the time needed to cut, remove as much as needed in order to make the solution feasible.

This ensures that the initial solution is feasible both in terms of time and capacity.

7.3.2 Further explanations

As can be seen from the steps above, this initial construction phase will start by taking all the compulsory demand. It is assumed that all compulsory demand can be transported by ship without the need for capacity checks.

Furthermore, it is decided that sea transport is the preferred method for all demand, road transport is only used in the cases where it is necessary because of a lack of capacity or if the ship does not visit one of the ports.

There were two options for how to handle the time constraint. The one that was chosen was to first ignore it, and later make the solution feasible by removing the demand that contributed the least to the total profit per hour of increased time. Another option would have been to simply add another check, for instance at the step where capacity is checked, and choose not to include any demand that makes the time of arrival at the end port exceed the constraint. Both options have their advantages. The choice was made to fix the solution if it was infeasible.

7.4 Improvement

The next step is to make an improvement heuristic to make the solution better, as this construction basically just takes as much demand as possible without considering factors such as price. The solution can still be good, even optimal, but this is very rare.

If the total profit found in the construction phase of the heuristic is defined as S_0, an improvement to this can be defined as any changes that creates a new total profit, S_1, where $S_1 > S_0$. This means that any change to the solution will need to create an increase in the total profit in order to be performed.

The improvement step consists of two main stages at which changes can happen. There can be changes to the route, for instance by switching the order in which two ports are visited, adding a port that is not currently visited, or removing a port that was previously in the solution.

Similarly changes can happen with regards to the demand. This will typically be to insert unmet demand into the solution, and taking some out. Simply adding unmet demand is also possible if the capacity constraints allow it.

7.4.1 Improvement steps

Similarly to the construction heuristic, the improvement heuristics can also be explained as a step by step process.

1. For each demand arc where demand is not met in S_0
    a. Save price and amount of goods
    b. Save, for each port, how much more capacity is needed
    c. If demand arc is tested previously and S_0 is unchanged, go to step 5
2. For each demand arc where goods are sent, if demand is optional, compare
    a. If price of goods in solution is higher or equal than price of goods found in step 1, return to step 2 and try next arc
    b. If price is lower, compare contribution to load for each port to amount needed to include demand in solution
    c. Reduce amount to send if amount needed is larger than contribution for ports in current solution

    d. If amount to send is 0, go to step 4

3. Perform switch
    a. Add amount to a_{ij} and q_{ij} for the demand to include
    b. Remove similarly from demand that is taken out, reducing a_{ij} and the appropriate other variables depending on whether direct sea transport or road transport options were used
    c. Update tables for contribution to load for each arc, and capacity needs
    d. Set S_0 to new solution and return to step 1
4. If amount to send is 0, try removing an additional demand arc from solution
    a. Add price of demand arcs to be removed, if total price is larger or equal to price of demand trying to insert into solution, return to step 4
    b. Add contribution tables for demand arcs, follow steps 2b-c
    c. If amount is 0, return to step 4 and try another, if all possible combinations of two arcs are tried, return to step 1
    d. If amount is larger than 0, perform step 3
5. For each port in current solution
    a. Compare to other ports in same cluster in current solution, if none exists return to step 5 and try the next port
    b. If a switch is infeasible with regards to capacity, return to step 5
    c. Check if demand is currently shipped directly between the ports, either by ship or by pure road transport, if it is, save the cost of pure road transport
    d. Calculate change to sailing cost if switching ports
    e. If switching reduces sailing cost, and there is no demand directly between the ports, perform switch, return to step 5
    f. If pure road transport is currently used, and the cost of this is larger than the potential increased cost of switching the order of visits, perform the switch and change mode of transport to direct sea travel for demand, return to step 5
    g. If sea transport is currently used, but the reduced sailing cost of switching the order of visits is larger than the cost of pure road transport, perform the switch, change mode of transport from sea to pure road transport, return to step 5
    h. Return to step 5 if there are untested ports, else go to step 6

6. For ports not in current solution, calculate added time by inserting it to the solution, save previous port of choice with lowest added cost
    a. If the added cost of inserting the port to the solution is higher than the savings from reduced road transport, return to step 6
    b. If the added cost of inserting the port is smaller than the potential savings, check if new solution is feasible with regards to time and capacity constraints, if it is, insert the port to the solution
7. If the solution is infeasible because of the time constraint
    a. Calculate number of hours that is needed to reduce in order to make the solution feasible
    b. Calculate the added cost per hour of each demand arc
    c. Calculate the potential time saving and lost profits of removing any port from the solution that is possible to remove, calculate profit lost per hour
    d. Calculate the total profit lost from removing all needed demand without removing any ports, calculate total profit lost per hour
    e. If removing a port removes less profit per hour than removing demand, remove port, return to step 1
    f. Otherwise, remove demand to make solution feasible, return to step 1, repeat until no improvement is made

7.4.2 Further specifications

The improvement heuristic is basically divided into two fairly independent parts. First, the demand not included in the solution is compared to what is in the constructed solution. The solution is updated whenever a change is found that improves the solution. This step is repeated until all demand that is not included in the solution is tested, and no possible improvements are found.

When this is done, the next step is to look at the ports, and try to make changes to improve the solution. These changes can be to switch the order in which two ports are visited, or, especially if the time constraint is binding, to remove a port from the solution. Only moves that improve the solution is considered.

7.5 Weaknesses in the chosen method

There are some weaknesses in the chosen method, which makes it clear that it is possible to improve on the current solution.

7.5.1 Non-improving moves

One of the things that could have been done to slightly improve the heuristic is to include moves that neither improves nor worsens the solution. This is especially relevant to the steps 1-4 in the improvement heuristic. It could have been possible to identify changes to the solution that does not improve the total profit, but that decreases the load when leaving certain ports. Such moves could open up the possibility of being able to perform other moves later which improves the solution.

7.5.2 Multi-start

One of the most significant improvements that could have been made would have been to introduce multi-starts. (Martí 2003) claims that diversification is required in hard combinatorial optimization problems. A way of achieving this could be to randomize the construction heuristic in order to create several starting points, and then running the improvement heuristic for each of them. This would most likely lead to better solutions, as it gives the heuristic different starting points from which it can find other solutions.

7.5.3 Multiple in or out moves

The improvement heuristic includes the possibility of removing two shipments while inserting one. This could have been expanded upon, as there are a number of possibilities. Increasing these options will increase the likelihood of reaching an optimal solution, but it will also increase the time spent on running the heuristic rapidly.

In the method chosen only the combinations of two arcs which in total has a lower price than the demand that is attempted to input is considered, this is not particularly many, at least not on the smaller data sets. Considering inserting two new demand arcs, while removing three, would require a lot more calculations. It is, however, clear that incorporating this would lead to a better chance of reaching the optimal solution.

7.5.4 Tabu search

Another option is to choose to use tabu search instead of the local search used in this thesis. A tabu search works much in the same way as the local search, however, it also allows for moves that worsen the solution, in order to move away from the local optimum which is quickly reached when using a simple local search. As mentioned in 4.3 Heuristic solution method (Derigs and Reuter 2009) provides a tabu search algorithm that solves an open vehicle routing problem. Using elements from this it could have been possible to create a tabu search algorithm for this problem.
An option here could have been to use the tabu search algorithm for one part of the heuristic, for instance the choice of which ports to include in the solution, as it is fairly rare to be able to make changes to the selected ports which render immediate improvement to the solution.

8.0 Computational experiments

This part of the thesis will present the computational experiments that have been made in order to test the models. Both the mathematical model and the heuristic have been tested using 16 different data sets, divided into four problem sizes, each with four different scenarios.

The problem sizes differ in terms of the number of ports that are available. The smallest having 10 possible ports, then 16, 20 and 25 ports respectively.

8.1 Testing the exact solving method

The exact solution method was coded in AMPL and tested using CPLEX 9.0 solver. For the instances where no solution was found (NEOS 2016) was also tried, but no solution was found within the maximum time of 8 hours.

The time used in order to find an optimal solution can be found in the following Table 1, solving time.

Table 1, solving time

Type/size	Basic	Time	Capacity	Time-capacity
10 ports	0.1s	0.1s	0.1s	0.1s
16 ports	102s	242s	433s	630s
20 ports	119s	6651s (1h50m)	3897s (1h5m)	8799s (2h26m)
25 ports	9078s(2h31m)	No solution(8h)	No solution(8h)	No solution(8h)

Table 1, solving time, shows that the basic data sets are solved fairly quickly up to large sizes, but the others start to slow down significantly already at 16 ports. Even for the basic data set, the difference in solution time when going from 20 to 25 ports is significant. It should be noted that while all the tests were run on identical hardware, some tasks running in the background might have been different, which might influence the running time.

8.2 Testing the heuristic solving method

The heuristic solving method was tested using Visual Basic Application in Microsoft Excel. The same data sets were tested. The time spent in order to finish the improvement step of the solving varied from almost instantly for the smallest sizes up to around 60 seconds on the largest problem sizes.

8.3 Results

Table 2, results

Case	Exact	Constructed	Improved	worse
10_base	5342518,50	5342518,50	5342518,50	0,00 %
10_cap	5812518,50	5606389,00	5802551,00	0,17 %
10_time	5248596,00	5173563,73	5241009,80	0,14 %
10_captime	5696105,00	5031420,26	5600860,76	1,67 %
16_base	5281220,00	5125215,50	5210215,50	1,34 %
16_cap	5629170,00	5486372,00	5543450,00	1,52 %
16_time	4837960,00	3733091,00	4713563,60	2,57 %
16_captime	5294620,00	3891224,34	5194451,00	1,89 %
20_base	5503990,00	5020307,50	5348216,50	2,83 %
20_cap	6038880,00	5618275,50	5853275,50	3,07 %
20_time	4958600,00	4310309,94	4776995,22	3,66 %
20_captime	5199930,00	4687677,20	4841408,24	6,89 %
25_base	5770460,00	5578626,00	5678626,00	1,59 %
25_cap	5857370,50*	5465170,50	5715170,50	2,43%*
25_time	4603410,00*	4252180,17	4342147,01	5,67%*
25_captime	4709036,17*	4509402,95	4569787,63	2,96%*

Table 2, results shows the results from the computational experiments. As can be seen from these results, the heuristic performs gradually worse the larger the problem size becomes, but with the exception of the case with 20 ports and both time and capacity issues, all results from known cases show less than a 4% loss of profit compared to the

optimal solution. The worse value is calculated by
$(exact\ solution - heuristic\ solution)/exact\ solution.$

8.4 Analysis

As the results do seem to get progressively worse with added complexity, it is likely that the results from the three data sets where no optimal solution was found is worse than those with 20 ports. By looking at the optimal solutions there is no clear trend that increasing the number of ports influences the value of the optimal solution. Therefore it is not possible to conclude based purely on those values. In all cases the optimal solution for the cases with capacity constraints have shown a larger total profit than those in the base case, this is also the case here, although the difference is smaller than it is in the smaller data sets.

Another option is to find the bounds of the optimal solution, but this is also difficult. It is possible to find the cheapest possible route through the ports, and assume that all the demand is met, without considering the capacity, and use that as the upper bound, but this serves very little value as this bound would far exceed any realistic solution.

The best option might be to abort the computational experiment at a given time limit, and use the solution found there as the lower bound on the optimal solution. The results where this is done is marked in Table 2, results, by a *. The worse stats for those cases will be the lower bound of this value, as it is possible that the optimal solution is better than the one found using this method. In order to obtain those results the exact solution method was aborted after about two hours for each of the three cases. It is difficult to know how close to the optimal solution this was, but it gives a slight indication. To further test this, the case with 20 ports and both time and capacity challenges was tested running for only two minutes, with the result being only about 0.5% worse than the optimal solution, which might indicate that it fairly quickly reaches a solution close to optimality.

As mentioned in 6.3 Known issues there is an issue with the exact solution which makes it likely that the results from some of the data sets found in the exact solution is infeasible. Going through all the data sets in order to find the ones where there are problems would be very time consuming with a fairly limited value. Instead, the solution found in the base case with 20 ports will be examined.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	.	50	50	0	0	0	0	0	50	0	0	0	0	0	0	0	0	0	50	0
2		.	30	30	30	50	50	20	100	0	0	0	0	120	0	0	20	0	0	100
3			.	30	50	0	0	20	0	0	0	20	0	60	0	0	0	0	40	50
4				.	20	10	10	10	10	10	0	0	0	0	0	0	0	0	30	0
5				30	.	20	20	0	0	0	0	0	0	0	0	0	0	0	30	30
6						.	0	20	0	0	0	0	20	0	0	0	20	0	120	150
7						20	.	0	0	0	0	0	0	20	0	20	0	0	0	150
8								.	0	20	0	0	0	30	0	0	0	0	20	0
9									.	20	20	20	0	0	0	0	20	0	0	100
10										.	10	0	0	0	0	0	0	0	0	50
11											.	0	0	0	0	0	0	0	40	20
12												.	0	0	0	0	0	20	30	0
13													.	0	0	0	0	0	0	30
14												20		.	0	0	0	50	0	50
15															.	0	0	0	0	30
16																.	0	0	30	30
17																	.	0	50	20
18																		.	50	50
19																			.	0
20																				.

Table 3

Table 3 shows the values of q_{ij} where the ports are sorted by the order in which they are visited. The values in red are therefore the amounts shipped that violate the precedence requirement that is used in the heuristic. As can be seen, there are three values in red, but it is very difficult to conclude in any way with regards to what is the actual optimal solution.

The values for q_{45} and q_{54} are both larger than 0, at 20 and 30 respectively. It is therefore obvious that road transport is needed in order to fulfill both of those demands, but it is not known if keeping the same route is optimal. For the other demand that violate the precedence constraints there are no demand going the other way, but it is still not clear if a change of route would give a better solution than simply using road transport.

What is known is that the lower bound of the actual optimal solution can be seen as the current optimal solution, subtracting the costs associated with changing the violating demand to pure road transport. The sum of costs for doing this is 12872, which means that the lower bound of the optimal solution is $5503990 - 12872 = 5491118$, which means that the solution found from the heuristic solution method is 2.6% worse than the lower bound of the optimal solution, and 2.83% worse than the upper bound.

It is difficult to determine what makes a heuristic good or bad. As mentioned the four criteria used to analyze a heuristic is the speed, accuracy, simplicity and flexibility. This heuristic does seem to perform well in terms of speed. It is also relatively simple, but it does seem to lack in accuracy and flexibility. The heuristic is made to fit this particular problem, and for it to be useful for other problems, quite a few changes would have to be made.

Accuracy is in some ways both very easy and very difficult to analyze. The results are available, but there is no information about what is done at present to make any conclusions in regards to whether any savings are possible for the real life entities that deal with these routes.

In retrospect it seems likely that incorporating more elements into the heuristic in order to increase the accuracy would have given a better result. Choosing a route for this sort of problem does not seem to be particularly time-sensitive. Based on this, it seems that adding multi-start would have been an important first step in improving the heuristic. It also seems likely that using a tabu search based heuristic would lead to an improvement, at least for the time sensitive data sets, as not all ports are visited in those.

The decision to not include any handling time at the start and end ports does seem to give some rather strange consequences on the time sensitive data sets. In most cases nearly all demand originating from the start port or ending at the end port is picked up, as that demand adds less time to the arrival time at the end port than the demand that goes between other ports.

The data sets that are evaluated do provide varied problems, so it does not seem that further data sets are needed. However, it is difficult to make any clear conclusions about the effectiveness of the heuristic with only 16 instances in which to test it. Additional data sets could have been generated for the purposes of further testing the heuristic.

All in all, the results from the computational experiments show that the heuristic is able to produce close to optimal solutions for three of the four small data sets, but for all the others it is in the region of 1-4% worse than the exact solution method, and slightly worse than that for a couple of the bigger ones with time as a binding constraint.

9.0 Conclusion and further research

In this thesis a mathematical model has been developed based on an existing model in order to create an exact solution method. This solution method was shown to provide an optimal solution for all but the largest data sets available.

The model used incorporates most of the elements of the real world problem, although elements like weather conditions and time windows at certain ports are disregarded.

Further, a heuristic solution method was developed, using a local search algorithm. The results from the computational experiments comparing the two methods are made, which show that for most of the data sets the heuristic performs about 1-4% worse than the optimal solution found from the exact solution method.

When looking at what further research is needed it seems that implementing a better heuristic should be possible. Using multi-start would be a good starting point in order to find an improved heuristic. Implementing something like a tabu search is also an option.

In addition, the move strategies used in the heuristic could be improved.

Time windows could also be included to increase the complexity. There is a known issue with regards to the exact solution method that also would need to be addressed.

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