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An econometric study of the Frequency elasticity on selected Air transport routes in Norway

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Preface

I wrote this thesis during the months January-May 2017, and I picked the topic because of an interest in doing some new research in the field of Air transport.

During my work on the thesis I had many helpful conversations with my supervisor, Svein Bråthen and I thus whish to thank him for his help during the process of writing this thesis.

I also wish to thank Falko Muller, because of his helpful insights on econometrics, which helped me construct my model.

Abstract

This thesis is a econometric analysis of the Frequency elasticity of air transport on three routes on the Norwegian air transport market, Bergen-Oslo, Trondheim-Oslo and Stavanger-Oslo, with two analyses, one for Bergen-Oslo and one for an aggregate of the three routes. Using 2SLS because of a possible endogeneity problem with Frequency, with income for airline as an instrument, I found that the Frequency seems to be exogenous in the two analyses. By estimating the same model using OLS I found that the Frequency elasticity is 0,63 on the Bergen-Oslo route and 0,67 on the aggregate of the three routes. I also find a long-term Frequency elasticity for the aggregate of the three routes analysis of 0,567, which indicates that the Frequency elasticity is lower in the long term. Besides this, I find that the aggregate of the three routes is a mature air transport market, with a GDP (Income) elasticity of 0,53. The route Bergen-Oslo on the other hand is much less mature, with a GDP (Income) elasticity of 0,84.

Abbreviations:

2SLS: Two stage least squares

OLS: Ordinary least squares

lf: Load factor

GDP: Gross domestic product

Pax= passengers

Freq= Frequency

Asiz= Aircraft size

Contents

1.0 Introduction and research topic

The demand for air transport has had a rapid growth over the last decades, and this growth is expected to increase (Boeing (2016)). This poses the question about which factors that drive this demand.

Studies on the factors that affect the demand for air travel often find that price elasticity is an important factor that affect air travel, with most air travel being significantly price elastic. There is also a lot of research done on what the price elasticity is for certain routes and sectors. (Brons et al. (2002)),

There are however another set of factors that could interact with the demand for air travel, namely service quality factors, with Flight frequency and aircraft size often showing a significant effect on demand. In the case of flight frequency, the theory is that an increase in it will lead to less delays for the traveler, since the traveler will have a flight which is closer in time to the traveler's preferred departure time, which would increase the utility for the traveler and thus increased the traveler's demand of air transport. In the case of aircraft size, the theory is that a larger plane will have more space and be more comfortable, which in theory would lead to an increase in demand. (Jorge-Calderón (1997))

1.1 Reason for choosing topic

As far as I know there have been done no previous empirical research on the frequency elasticity of demand on the air travel market in Norway. Because of this it would be interesting from a purely theoretical perspective to do such an empirical analysis. Such an analysis would also be interesting from a managerial and policymaker perspective, since knowing how demand and frequency interact could help the airlines, airports and policymakers in planning the right frequency for the demand they want. Examples of this could be airlines that want to maximize its benefits when faced with the tradeoff between having an increased revenue because of higher demand after a frequency increase and the costs associated with such a frequency increase. Knowing the frequency elasticity could also be interesting for policymakers that wish to reduce the demand for air travel to reduce the externalities of air travel, such as climate changing emissions, since it could be better to tax or regulate frequency rather than having taxes on the fare price of the tickets if the demand on the route responds more to a change in frequency than price. Thus, there is good reasons to do an econometric analysis of the frequency elasticity of demand for the air travel market in Norway. Doing such an analysis of the complete air transport market in

Norway would be beneficial, but because of limitations in time, resources and data, I have however chosen to focus my thesis on estimating the frequency elasticity of demand on 2-3 routes in Norway. But since air travel on different routes do share many common characteristics, the model developed in my thesis could also probably be applied to analyze the frequency elasticity of demand other routes on the air transport market in Norway or abroad, although this could possibly require some small adjustments to the model.
Given the above-mentioned reasons and questions, my research problem and research questions can be formulated as:

1.2 Research Questions

Research Problem: “Do changes in the flight frequency impact the demand for air travel on Norwegian routes?”

RQ1:

Is there a significant flight frequency elasticity of demand on the on the selected Norwegian air transport routes?

RQ2:

How frequency elastic or inelastic is the demand of air transport on the selected Norwegian routes?

RQ3:

Are there any other interesting findings after estimating the coefficients of the model used to estimate the frequency elasticity of demand on the selected routes of the Norwegian air transport market?

2.0 Literature review

2.1 Elasticity

The most basic definition of an elasticity is the change in variable X / the change in variable Y, which can be defined as how much variable X responds to a change in variable Y. The elasticity measurement can also be divided into three different types, own elasticities which measures the change in variable Y from the change in variable X itself, cross elasticities which measures the change in variable Y from another variable that is a complementary or substitute good to variable X, and lastly conditional elasticities, which is the change in variable Y from a symmetric change in variable X and a substitute or complementary good. These are the basic definitions of elasticities, but to measure the elasticity in practice three different methods can be used. The most basic one is the point elasticity, which measures the elasticity at a certain point on an unknown functional form curve. Thus, if elasticities are not constant, this point elasticity will not be accurate for other points on the curve. (Fearnley & Bekken (2005))

Another weakness of the point elasticity is that it measures marginal changes, thus large changes in the variables may cause problems. The point elasticity is measured as:
"

$$Point\ elasticity = \frac{\partial y}{\partial x}\frac{x}{y}$$

" (Fearnley & Bekken (2005)).

A better measurement of the elasticity given the unknowable functional form and the challenge with large changes in the variables is the Arc elasticity. The Arc elasticity measures the average elasticity between two points, and can be measured as.
"

$$Arc\ elasticity = \left(\frac{\ln y_2 - \ln y_1}{\ln x_2 - \ln x_1}\right)$$

" (Fearnley & Bekken (2005)).

The line elasticity is the third method of estimating the price elasticity, and it measures the elasticity as the average elasticity between two periods, same as the arc elasticity, but is measured without using the log from of the variables. Because of this its advantage is that it can measure a change from a value of 0, which the Arc elasticity can not do because taking the log form of 0, ln(0), will not produce a number.

"

$$Line\ elasticity = \frac{(y_2 - y_1) * (x_2 + x_1)}{(y_2 + y_1) * (x_2 - x_1)}$$

" (Fearnley & Bekken (2005)).

Fearnley & Bekken (2005) compares the three measurements and find that they produce the same values when then changes in the variables are small.

There are also some challenges with estimating elasticities with time series. With the exception of situations where all of the elasticity effects happens at once or when the data is non-stationary and cointegrated a static time series model can not estimate the long-term elasticity effects. Thus, the estimates of the elasticity in such a case will be neither long term nor short term, and will be biased because of overlooked dynamic effects. A way to correct for this, and to estimate the long-term elasticity is to include a lagged exogenous or endogenous variable. Both can be used to measure the long-term elasticity, but the lagged exogenous variable leads to serial correlation and multicollinearity problems, so the lagged endogenous variable is preferable. A model with a lagged endogenous variable, which is the lag of the dependent variable, can be states as the following equation adapted from ((Fearnley & Bekken (2005)) as:

$Y_t = B_0 + B_1 * B_1 X_{1t} + B_2 X_{2t} + B_3 Y_{t-1} + \varepsilon_t$"

With such a model, given that the variables are in the log form, the short term elasticity can be measured as B_1 for variable X1, and the long term elasticity for the same variable can be measured as "$B_1/(1 - B_3)$", since B3 measures the adjustment speed of the elasticity, so that if B3 is 0 all of the elasticity effects happens at once, while if it's between 0 and 1, the elasticity effect happen over time until it reaches a stable long term level (Fearnley & Bekken (2005)).

2.1.1 Determinants of air travel and price elasticities

The price elasticity of the demand of air travel depends on the availability and quality of the substitutes to air travel. Examples of such substitutes could be air travel to similar destinations, travel by another transport mode, or simply not travelling at all if the utility of spending the money on a non-travel good is higher than the utility from travelling. One factor that impacts the availability and quality of substitutes to air transport is the geography of a route, since if the route crosses areas with difficult terrain, such as mountains or seas, the alternative substitutes to air transport such as car or rail would be less available or have a lower utility than air transport because the trip by them takes a longer time. The distance of the air travel route also reduces the utility of substitute modes such as car or rail, since travelling by them can take a long time over such distances, thereby increasing the time cost of using such modes compared to air transport. Thus, in such a cases the price elasticity will be lower, as the substitutes either have a lower utility compared to air transport or sufficient substitutes to air transport are not available**.** Besides geographical factors and distance there are also economic and demographical factors that impact the quality and availability of substitutes, such when the characteristics in a city or destination determines the willingness of travelers to choose it over another comparable destination. (Brons et al (2002))

The price elasticity also depends on the type of passengers on the route, as leisure passengers and business passengers have different reasons for travelling. Leisure travelers travel by air transport for the utility of travelling to their destination itself, and also have other non-travel substitute goods to use their budget on. They are thus often price sensitive. Business travelers on the other hand travel as part of a business process, and thus compares the productivity gained from travelling by air with the productivity of not travelling, which means that the cost and profits of travelling by air is compared with other profits and costs of other business activities. Business travelers often have a higher value of time than leisure travelers, thus the total cost of a business travel is often made up of mainly the value of time costs, which means that an increase in ticket prices has less impact on the demand of air travel by business travelers than an increase in ticket prices has on the demand by leisure travelers. Since the trips of Business travelers are paid for by their companies, the business travelers have a higher budget than leisure travelers. Business travelers also want to be productive when travelling, and thus opt for tickets that are flexible and provide a high degree of service. This means that Business travelers are often less price sensitive than leisure traveler. Brons et al (2002))

There can also be a difference between long term elasticities and short term elasticities. The reason for this is that in the long term the traveler can adapt to changes in the price, such as moving business locations and so on. Thus, in the long term, the price elasticity is expected to be higher as more adjustments can be made, which should increase the impacts on demand. Brons et al (2002) does however also argue that since it may be hard to move business location in addition to the lack of substitutes with the same quality that air transport has, such as the speed of air travel, the long-term adjustments to a price change may actually be not that much higher than in the short term. They also argue that the inverse of a higher long term change might also be the case, such as when the short-term response to a price change are chaotic. In such a case the long-term response might be a more reasonable response to the price change than the initial chaotic response, in which case the long-term price elasticity would be lower than the short-term elasticity. Brons et al (2002) thus concludes that the long-term elasticity depends on a number of complex factors, so they argue that it is difficult to say whether it should be higher, lower or similar in the long term. (Brons et al (2002))

2.1.2 Frequency elasticity

A challenge with estimating the flight frequency elasticity of demand is that the flight frequency can be correlated with the demand for air travel, for example by the airline scheduling more flights because of an increase in demand, which then may lead to a higher demand because of an increase in flight frequency. (Wang et al (2014); Zou & Hansen (2014))

In econometric literature, such a correlation is called endogeneity, and poses some challenges when estimating a single function by OLS, since a correlation between the dependent and one or more independent variable means that the independent variable or variables would be correlated with the error term. This leads to biased estimates when using OLS, which means that another method has to be used to produce unbiased estimates. (Wooldridge (2015). I will get back to how this can be corrected for in the Methodology chapter.

So, what are some estimates of the flight frequency elasticity in the literature? One paper that estimates it is the paper Wang et al (2014), where they use the following Simultaneous equation model with two equations, one for Frequency and one Passengers:

“

$$lnFR_{k,t} = \alpha 0 + \alpha 1 lnPASS_{k,t} + \alpha 2 lnASIZ_{k,t} + \alpha 3 lnHHI_{k,t} + \alpha 4 lnDIS_{k,t} + \alpha 5 lnFUEL_{k,t} + \alpha 6 lnINC_{k,t} + \alpha 7 lnMinOUTPUT_{k,t} + \alpha 8 lnMaxATASIZ_{k,t} + \alpha 9 lnHUB_{k,t} + \alpha 8 lnT_{k,t} + \varepsilon_{k,t} \quad (1.1)$$

“

“

$$lnPASSk,t = \beta 0 + \beta 1lnFRk,t + \beta 2lnASIZk,t + \beta 3lnHHIk,t + \beta 4lnCOSTk,t + \beta 5lnDISk,t + \beta 6lnPOPk,t + \beta 7lnINCk,t + \beta 8TOURk,t + \varepsilon k,t \qquad (1.2)$$ “

Where, FR= Flights per year on a route, PASS= Number of passengers on a route, ASIZ= average aircraft size, HHI= HHI index, DIS= distance between airports on a route, FUEL= Jet fuel price on average, INC= income in the regions of airports on the route, MinOutput= minimum amount of airport passengers on a route, MaxATASIZ= the maximum size for an average plane on the route, HUB= dummy for a hub airport on the route, COST= operating costs on average for the major Chinese airlines, POP= population at the O-D regions of the route, TOUR= tourism dummy.

As seen most of the variables in the equations in the model includes are in log form. The benefit of this is that when the variables is in log form the estimated coefficient will be the elasticity of that variable (Wang et al (2014))

To estimate the endogenous variables in each equation Wang et al (2014) uses MinOUTPUT, FUEL, HUB and MaxATASIZ as instruments for the PASS equation, and COST, TOUR and POP which are used as instruments for the frequency equation. (Wang et al (2014))

To estimate these two equations, they use the 3SLS (three stage least square method), and find that the Frequency has either an elasticity of demand of 0,945 or an elasticity of demand of 0,679 if the lagged variable for demand is included. They also find that both the estimate with and without a lagged variable is significant. (Wang et al (2014))

Another paper that looks at service quality elasticities is the paper by Jorge-Calderón (1997), which runs a regression using 2SLS using a model that includes different drivers of demand, which includes frequency and aircraft size, using a dataset covering the entire European route network in 1989. He finds that the frequency elasticity of demand is 0,9396 assuming aircraft size is endogenous or 0,6506 if aircraft size is assumed to be exogenous. The model also has a good fit to the data, with an R^2 of 0,9543 when aircraft size is assumed to be endogenous and R^2 of 0,7224 when aircraft size is assumed to be exogenous. (Jorge-Calderón (1997)).

A paper by Tsekeris (2009) is also of interest as it looks at the frequency elasticity of air travel demand in a geographically remote market. The geographical remote market the paper looks at is air routes that serve the islands of Greece. The paper uses a dataset that covers the period 1968-2000 and which includes 18 routes between Athens and 7 of the

Greek islands. The model in the paper has passengers as the dependent variable and includes the following independent variables in logarithmic form: lagged variable for passengers, relative seat capacity, price of air travel relative to the price of sea travel by ferry, income measured as GDP, travel time on the route, population, the attractiveness of the route for tourism, and relative frequency of air travel compared to sea travel. The model is then estimated using System GMM and GMM with orthogonal deviations methods. After estimating the model using GMM with orthogonal deviations, Tsekeris (2009) finds that the demand for air travel on the geographical remote islands routes is inelastic to relative price changes, with the elasticity being -0,069 in February (which represent the winter season) and -0,109 in August (which represents the summer season). He also finds that the relative frequency elasticity is 0,183 in February and 0,135 in August. By using the System GMM method, Tsekeris (2009) finds that the elasticities for relative price is -0,102 and -0.135 in February and August respectively, and he also finds that the frequency elasticity is 0.119 and 0,070 respectively for February and August. The estimates from both methods indicate that the frequency change have a higher impact on demand than price in the winter month, but the estimates from the system GMM do also show a larger impact from price than frequency in the summer month, although price is still inelastic. (Tsekeris (2009)).

So, what can this tell us about the hypothetical elasticities of frequency on the air travel routes in Norway? Since the paper by Tsekeris (2009) shows that the demand of air travel to the geographical remote regions is relatively price inelastic when compared to the substitute of sea travel and that frequency has a higher effect on demand than price for such a market, as seen by the higher coefficient for frequency elasticity than price elasticity. It could be that all geographical remote regions have a price inelastic demand of air travel, and that the frequency elasticity has a higher coefficient than price elasticity for such a market. Norway being a geographical remote region might thus have price inelastic travelers on the air travel market, and the Norwegian air travel markets may be more affected by frequency changes than price changes. If this is the case is hard to say ex ante, but it could indicate that this might be the case. However, it is important to point out that there might be other factors that impact the price elasticity and frequency elasticity for the Norwegian and Greek markets respectively, so a generalization may not be possible based on the Greek results. Still, it will be interesting to compare the results.

A paper that looks at the determinants of flight frequency is the paper by Pai (2010), which looks at the factors that determine the flight frequency and aircraft size on US airline routes. By doing a regression analysis with flight frequency as the dependent variable and vectors of population demographics, time variables, route characteristics, airport characteristics, airline operational characteristics and hub characteristics as the independent variables, Pai (2010) finds that population and income has a positive effect on frequency. Pai (2010) also finds that the percentage of managers in the population has the largest effect on frequency out of the population variables, with a percentage increase in managers causing 20-24 more monthly flights. Pai (2010) argues that the positive effect of income and degree of managers is because the airline are concerned with the schedule delay cost to these passenger groups, and that the airlines thus increases the frequency because they know that these passenger groups have a high willingness to pay to reduce their schedule delay cost. Pai (2010) also finds that having other airports in the vicinity of the airports on the route leads to less frequencies on that route, with one extra airport within 75miles of an airport leading to 9 less monthly flights from that airport. Having a hub on the route is also related to a higher frequency, with one connection destination on routes from one of the airports on the route leading to 0,6 more monthly flights, and a 1% increase in connecting passengers as the percentage of travelers on a route being connected to 16 flights extra per month at the destination airport and 27 extra flights per month at the origin airport. Pai (2010) also finds that when the distance increases the number of departures per month decreases, with there being 62 fewer flights per month when the distance between origin and destination increases by 1000 miles. Other findings by Pai (2010) is that a low-cost carrier has higher frequencies, that slot constraints is connected to lower frequencies and that a ownership of regional airlines by a major airline leads to higher frequencies. (Pai (2010))

Another paper that looks at the relationship between demand and frequency of air travel is the paper by Zou & Hansen (2014). In their paper, Zou and Hansen (2014) reviews the theory on the frequency effects on air transport demand and the frequency planning for airlines and finds that airlines adapt to increases in passenger demand by either increasing capacity through aircraft size increases or frequency increases. Both options have their benefits, being economies of scale or a reduction in schedule delay for the passengers respectively. But the economies of scale from a lower unit costs from using larger aircraft may be offset a bit by larger aircraft requiring higher pilot salaries. Thus, airlines tend to employ smaller aircraft on short haul routes with a lot of traffic. Zou and Hansen (2014) also finds from industry outlooks and historical data that frequency looks like it will be the most used option by airlines to respond to an increase in demand, with only a small change in aircraft size. Zou and Hansen (2014) finds that there are three reasons for why this is the case, which are the Morhing effect, competitive pressures and a possibility to charge higher fares. Firstly, The Mohring effect is an effect first mentioned in a paper by Mohring (1972), which states that an increase in frequency will lead to an increase in demand which in turn will lead to an increase in frequency, creating a positive feedback relationship. This relationship is thus beneficial for airlines. Secondly, competitive pressures may lead to airlines increasing frequencies instead of increasing aircraft size, as frequencies are often tied to market shares when frequencies are over a certain threshold, an effect referred to as the S-shaped curve in literature. The number of competitors also increases the frequencies on routes. Thus, airlines want to increase frequencies to keep or increase their market share, forcing the airline to operate smaller aircraft at a higher frequency at such routes. Lastly, having more frequencies increases the passenger's willingness to pay, which increases the fares that can be charged by the airline. (Zou and Hansen (2014))

By running a 2SLS and OLS estimate of a frequency equation, Zou & Hansen (2014) finds that the majority of the growth in passengers is facilitated by an increase in frequency, since the coefficient for the elasticity of the frequency response to demand is around 0,65 (0,641 with OLS and 0,651 or 0,654 with the two 2SLS models). Thus, they find that the coefficient for the elasticity of the aircraft size response to demand is 0,35. They also find that increases in fuel costs leads to a lower frequency, with a 7,5% increase in fuel costs leading to a 1% reduction in frequency, and that longer routes tend to have lower frequencies, which Zou & Hansen (2014) argues is related to the lower degree of delay costs as part of total travel costs on longer routes. They also argue that longer routes have less competition from substitutes. Zou & Hansen (2014) also find that an increase in delay leads to a higher frequency, which they point out is not intuitive as airlines might want to reduce frequency when there are delays to reduce operating costs, but they argue that the reason for this response to delays, despite the added costs, is because airlines are willing to pay the extra cost to capture the high yield of market segments that are highly sensitive to delay costs. Zou & Hansen (2014) also points out, based on interviews with the air travel sector, that airlines are unwilling to cut departures, even when there is delay, since doing so could lead to a loss of slots, which they are not willing to do since doing so would give their competitor an advantage. The share of LCC on the route also affects frequency, with a 10% increase in the share of LCC on the route leading to 0,57% fewer flights. The total delay elasticity of frequency is found to be a 1,8% increase in flight traffic per 1 min delay for origin airports and a 0,28% increase in flight traffic per 1 min delay at arrival airports. (Zou & Hansen (2014))

The relationship between aircraft size, frequency and demand is also covered in the paper by Belobaba (2009). In it Belobaba (2009) also states that the reason why airlines increase frequencies is because such increases lead to an reduction in waiting time between flights and more departures at the preferred departure times of the passengers, which in turn lead to more demand for air travel and higher revenues for the airlines. In addition, he mentions that the airline is often being forced to increase frequencies to keep a market share when there is competition. Belobaba (2009) also mentions that business travelers are more sensitive to increases in frequencies, as they are more negatively affected by schedule delay and waiting time. Belobaba (2009) argues that increases in frequency are more important for short haul routes, since the waiting time between flights makes up a larger proportion of total flight time for short haul routes than for long haul routes. In addition to this Belobaba (2009) argues that the choice between increasing aircraft size or frequencies are closely related to each other. He uses an example of an airline wanting to transport 400 passengers, which can either be done by using 1 flight with an airplane with 400 seats or 4 flights with a plane with 100 seats. If there exists a competitor that operates 4 flights a day on the same route, the airline with 1 plane with a size of 400 seats will then only have a market share of 20%. Thus Belobaba (2009) argues that it is unlikely that the airline will have enough market share to fill its 400 seat plane to a profitable load factor. Thus, the airline would have to use a 100 seat size plane with 4 departures to keep its market share. Because of this example, Belobaba (2009) argues that airlines on short haul routes are forced to increase frequencies and keep aircraft sizes small. (Belobaba (2009))
An early paper that looks into the effects of frequency on demand of air transport the paper by Ippolito (1981). In it Ippolito (1981) states that while there had been some theoretical interest in the effect of the service quality on demand, few papers had investigated this relationship empirically. Ippolito (1981) points to De Vany(1975) as a notable exception to this, as he included flight frequency as a variable that affected demand in his model, but he also points out that this paper had a small sample.
To run an empirical analysis on the frequency elasticity of demand of air transport, Ippolito (1981) chooses to focus on monopoly routes, where at least 80% is non-stop traffic to avoid any network effects or oligopoly competition bias in the results. As local

routes had a higher fare price at the point of the papers publication, Ippolito (1981) also choose to include half local routes and half trunk routes in his sample.

To estimate the frequency elasticity, Ippolito (1981) first develops a model of the demand of air transport. This model includes Income, Population and Fares as independent variables. Ippolito (1981) argues that longer flights would be more price elastic than shorter flights, and since fares are higher with longer flights then the price elasticity should be higher when fares are high. To model this in the model, Ippolito (1981) decides to include the fare variable as the square of the natural units of the fare price.

Ippolito (1981) mentions that two ways that service quality increases lead to an improvement in demand is firstly by there being more flights, and secondly by there being a lower chance that a certain flight is full. The former happens when the flight frequency increases and the latter happens when the load factor increases. The reason why these two factors lead to an increase in demand is because having a higher flight frequency means that a passenger has less delay costs because the passenger has a flight closer to the passengers desired flight time, and having a lower load factor means that the passenger has a higher chance of getting a seat on the desired flight of the passenger, which in turn reduces the passengers potential waiting time for the next flight if the desired flight is full. Ippolito (1981) argues that this reduction in delay cost and waiting time leads to an increase in demand. To model this, Ippolito (1981) builds upon Dvany(1975) and assumes that plane size is given for certain segments, which means that the only variable that reduces delay cost and waiting time is the flight frequency. Ippolito (1981) argues that the reason for this is that by having aircraft size constant an increase in flight frequency will reduce the load factor. Thus Ippolito (1981) specifies the flight frequency as "Flight frequency = (Flights – load factor / 1 – load factor)". Ippolito (1981) mentions that if the flight frequency is < 1, then there is diminishing returns to increasing the flight frequency. Ippolito (1981 then includes the flight frequency and the load factor in the demand model, which also includes dummy variables for distance and certain locations, such as California and Florida. Ippolito (1981) also specifies a supply equation which includes variables such as enplaned passengers, fare, ramp to ramp time, proportion of O&D passengers and "through" passengers in addition to dummies for the size of the flight segment and the identity and type of the carriers. Ippolito (1981) then estimates the model using 2SLS, where flight frequency and load factor are endogenous in the demand equation, and finds a flight frequency elasticity of 0,864, which is significant at the 0.01 level. Ippolito (1981) also finds a price elasticity of -0,525, but also argues that results show that the price

elasticity is close to unity at a distance of 830 miles, since the price elasticity in the model was modelled to depend on the distance.
Another paper that looks at the flight frequency of air travel is a paper by Brueckner & Zhang (2001) which uses an economic analysis to look at what the flight frequency of air travel is in a hub and spoke network. They find that the Flight frequency is higher in a hub and spoke system than a direct flight network, and that while cost per passenger is lower of the hub and spoke system the fares are also higher for the non-connecting passengers compared to the direct flight network. Brueckner & Zhang (2001) argues that the reason for these effects is firstly that the added marginal revenue from connecting passengers leads the airline to increase the flight frequency to capture it. This increase in the flight frequency then leads to a lowering in the frequency delay for non-connecting passengers, causing the airline to be able to charge a higher price per flight since a higher frequency means that the market is more differentiated between passengers who value the utility of a certain departure times differently. It should however also be noted that Brueckner & Zhang (2001) points out that the cost per passenger and fares would be more closely linked in a competitive model compared to their model, which could impact their argument about higher fares. Brueckner & Zhang (2001) also argue that as long as cost per flight is low enough the airline operating in a hub and spoke network will increase flight frequency to serve both connecting and non-connecting traffic. However, if the cost per flight increases, the airline will increase the fare for connecting passengers, so that they choose not to travel, as connecting passengers are more price sensitive since they have a disutility from longer travel times. Thus, Brueckner & Zhang (2001) argues that the non-connecting passengers will always be fully served.

A paper that looks more directly at the Frequency elasticity of the demand of air travel is the paper by Pels & Nijkamp and Rietveld (2001), which looks at how the flight frequency of demand, airfares and airport tax interact in a multi airport region. Using a multinomial logit model, they derive a symmetric equilibrium analytically and find that assuming the load factor is constant there exists an equilibrium between airfare and frequency if the frequency elasticity of demand is less than 1. They argue that the reason for why the frequency elasticity of demand needs to be lower than 1 is because if the demand increases, given constant load factors, an airline will increase its frequency to accommodate this demand increase. This demand increase will then, if the frequency elasticity is higher than 1, lead to an even higher increase in demand, which in turn needs to be accommodated with a higher increase in frequency, and so on. Thus, they argue there that in this situation there is not an equilibrium between air fare and flight frequency. They also find that if the frequency elasticity of demand is less than 1 there exists a unique airfare-frequency equilibrium, so that given an optimal airfare-frequency equilibrium for one airline there exists a response from a competitor airline that is unique. They also find that there is also an equilibrium between airfare, frequency and airport taxes if it holds that frequency elasticity is < 1, as the model can be used to find the optimal airport tax by taking into account the optimal response by the airlines. Pels & Nijkamp and Rietveld (2001) finds that the equilibrium holds both for a symmetric analytical solution and a asymmetric numerical solution. Pels & Nijkamp and Rietveld (2001) also mentions that two earlier papers, Caves et al (1991) and Pels et al (1998) found the frequency elasticity of demand to be less than 1, but they also argue that more research is needed to see if the frequency elasticity of demand for air travel is actually less than 1 or not.

Another paper that covers the frequency elasticity in a hub and spoke network is the paper by Wei & Hansen (2006). In the paper Wei & Hansen (2006) develops an aggregated demand model that looks at the demand impacts of frequency, fare price, distance, aircraft size, the demographical data about the areas where the hub and spoke airports are located in addition to the demand impacts of the characteristics of the hub and spoke network, such as the number of spokes or the number and income of local passengers. In the model they divide the frequency elasticity of demand in the hub and spoke network into two parts, DFREQ and HFREQ, where DFREQ represents the frequency elasticity on the spoke to hub flight, and HFREQ representing the average frequency elasticity on the hub and spoke network as a whole. They estimate their model using a dataset covering most of the hubs in the US air transport market in the second quarter of 2000 and find that the Frequency elasticity of demand on the spoke to hub routes is 1,187, which is higher than 1 and thus against the argument that the frequency elasticity of demand has to be less than 1 stated in Pels & Nijkamp and Rietveld (1998). The average frequency elasticity on the whole hub and spoke network is however smaller, being only 0,265. Both estimates are also significant at the 0.01 significance level. Wei & Hansen (2006) argues the reason why the frequency elasticity is smaller for the whole network than for the spoke to hub route is that the passengers value the frequency on the first stage route higher than on connecting routes. Wei & Hansen (2006) also finds that the aircraft size elasticity of demand is 0.631, indicating that demand is more elastic to an increase in frequency than an increase in aircraft size. Besides this Wei & Hansen (2006) finds a price elasticity of -0.899 on the spoke to hub route.

Lastly, a paper that looks at how airlines set their Frequency is the paper by Richard(2003), which argues that airline adjust their frequency and passengers based on their costs, such as operating cost, cost per flight and fuel cost, so that they maximize their profits based on marginal cost and marginal revenue. (Richard (2003))

2.1.3 Income and price elasticity

Since the estimated model in this thesis will not only estimate the frequency elasticity but also the price and income elasticities, it will be good to cover some theory on income and price elasticities.
The ability of a passenger to travel is constrained by two factors, monetary and time constraints. The monetary constraints depend on the income and the price of the passenger. These two factors do not impact the willingness to travel the same, as the changes in price and income affect the decision to travel differently. Income effects which is also called the Income elasticity, is the percentage change in travel demand to a percentage change in income. The income elasticity depends on whether the passenger views the good in question as a normal or luxury good. If the good is a luxury good or service, then demand is expected to increase more than proportional to an increase in income, which means it has an income elasticity > 1. If, however the good is considered a normal good, then the income elasticity will be < 1, meaning that the demand of that good increases less than proportionally with an income increase. There is however a limit to the consumption of a good or service, and as income reaches a certain point there will be a saturation effect for the demand for the luxury good or service, leading to less growth in demand for the luxury good or service relative to the growth in income. This saturation effect is however lower for services, as higher quality services can be introduced, which sustains the demand and postpones the saturation effect. The demand for travel, being a service, could potentially experience a fall in income elasticities for travel demand, stopping short of an income elasticity of zero, as income grows, although the evidence of such a saturation effect is somewhat limited. Price elasticity on the other hand depend on two factors, income and substitution effects. The substitution effect depends on the availability and cost of substitutes to the good or service, with more substitutes leading to a higher substitute

effect. The income effect depends on the degree of consumer's income that is spent on the good in question, and the income elasticity of the consumer for that good or service. (Fouquet (2012))

To investigate how income and price elasticities evolve over time, Fouquet (2012) looks at the historical development in transportation demand in Britain, and finds that passenger demand for land transport increased 165 times in the period 1850-2000, or 220 times in the last 150 years if air travel was included as well. He points out that if the price and income elasticities were -1 and 1 respectively during this period, then demand for transport would have increased 144 times, something Fouquet (2012) argues is a sign that demand for transport were on average elastic in this period or possibly highly elastic at some points in time

To test the relationship between the demand for travel and income and price elasticities Fouquet (2012) uses a vector error correcting model that covers the travel demand in Britain over the last 150 years. He finds that income elasticity for travel demand has indeed decreased over the period, from as high as 3 around the 1850-1860 to 2,2 in 1890 and stabilizing at 1,2 in the period 1920-1980, before reaching an income elasticity for aggregate travel demand of 1 in 2000. He argues that this is an indication that travel moved from being a luxury service to a necessity in everyday life, as travel became a possibility for a larger part of the people, and people became depended on transport to commute to work because of factors such as urbanization and an increase in suburbanization. He also points out that had it not been for air travel having a higher elasticity, which increased the average elasticity of travel demand, the elasticity of aggregate travel demand would probably have decreased even more, which he shows by finding that the income elasticity of land transport reached an elasticity of 1 already in 1940.

Fouquet (2012) also shows that price elasticities have declined, from -1,5 in 1870s to -0,9 in 1920s and finally -0,6 in the period 2000-2010. He argues that the reason for this decline is a combination of travel making up less of total GDP and personal expenditure and because of the increase in substitution options to transportation, such as new forms of communication being invented. He argues that reduction in price elasticities in the period in question matches the theory, since the theory states that price elasticities will decline with a reduction in prices and an increase in income, both of which happened in the period in question. (Fouquet (2012))

It would however also be good to look at more detailed data about the income elasticities of air transport itself. One such study is a meta regressional analysis of the literature on the income elasticity of air travel by Gallet and Doucouliagos (2014), which uses a dataset containing 40 studies, which includes a total of 405 income elasticity estimates, published between 1972-2007. By calculating a simple average from the papers reviewed, they find that the income elasticity is 1,517, which they argue indicates that air travel is a luxury since an elasticity > 1 indicates the service or good is a luxury and an elasticity < 1 indicates that the good or service is a normal good. They also state that this indicates that air travel is an immature market, as an immature market would view air travel as a luxury, while a mature market would view it as a normal good. When they run the meta regression analysis they find that income elasticity is 1,186 for domestic air travel and 1,546 for international air travel. They did however also find that the income elasticity is reduced to 0,633 when the price of air transport is included in a dynamic specification of demand. By checking for selection bias on the part of the researchers they find it to be not significant, and they also find that regional differences do not have a significant impact on income elasticity of demand for air travel, although they also find that the income elasticities have increased in North America compared to the rest of the world, something they argue may have to do with better data or the focus of the studies on the North American market compared to the rest of the world. They also find that the type of measurement used to measure income, the estimation method of demand, data aggregation, the time horizon or the use of instrumental variables has little impact on the estimate of income elasticity. ((Gallet & Docucouliagos (2014))

2.1.4 Limits to growth of air travel

So why is it important to know the income elasticity for air travel? A paper that covers this is the paper by Graham (2000) which states that since there are certain limits to the growth of air travel, it would be beneficial to know if a market for air travel has reached that limit or if not how close the market is to reach it. Graham (2000) states that the growth of air travel can be divided into two segments, new travelers and additional travel by previous travelers. As air travel is becoming more and more common, with only about 5% of travelers being new travelers in the UK as of 2000, there comes a point where future growth must come from previous travelers travelling more. Graham (2000) cites from Graham (1995) that the point where no more new travelers is expected is when about 80% of the people in a nation already uses air travel, and she also cites James (1993) which puts the same limit to 75%. When such a limit is reached, the limits to growth would be the ability and desire of previous travelers to travel more. The ability of a traveler to travel more is stated by Graham (2000) to depend on the income of the traveler, and the desire to travel depends on the utility to the traveler of an addition flight, which she argues follows the law of diminishing returns for tourism related travel by air. Graham (2000) argues that together these factors can lead to demand maturity, which is a market with lower growth rates or demand saturation, which when a market is full and it stops growing.

So, given this, how can the growth potential or maturity of an air travel market be measured? Graham (2000) states that there are three ways to do this, either looking at the growth rates of the air travel market in question over time or by comparing the growth of air travel to the growth of GDP, based on the intuition that a lower growth in air travel than GDP indicates slowing growth and market maturity. A third way to measure it mentioned by Graham (2000) is to compare the growth in GDP with the growth in revenue for the airlines, with a higher growth in yield than GDP indicating an immature market with a potential for growth and the opposite indicating a mature market with lower growth. Lastly Graham (2000) develops a method to measure market maturity, based on his previous argument. This method consists of using the income elasticity to measure the maturity of the air travel market. She argues that the maturity of a market can be divided into five stages, Stage 5 which is a fully saturated market, Stage 4 which is a fully mature market, Stage 3 which is a market approaching maturity, Stage 2 which is a not a fully immature market and Stage 1 which is a fully immature market. She argues that the stage a market is in can be measured by the income elasticity with it being 0 indicates stage 5, 1 or < 1 indicating Stage 4, > 1 and approaching 1 being stage 2 and 3 and an income elasticity which is constant and way higher than 1 being Stage 1. To test this method, she applies it to data covering British long holidays for the two periods 1970-1998 and 1984-1998. By estimating the income elasticities for both international travel and leisure travel as a whole she finds that the income elasticities were 2,23 and 1,89 for international air travel for the periods 1970-1998 respectively and 1,30 and 1,28 for Total leisure travels in the same periods. She argues that this indicates that both markets are approaching maturity since both segments have decreasing income elasticities over time, but she also points out that the total leisure travel segment is closer to being fully mature, which is indicated by an income elasticity of 1. (Graham (2000)).

2.1.5 Summary of elasticities

Having reviewed the literature on relevant elasticities, I summarized the most important estimates in the papers covered above, in additions to other paper I reviewed not mentioned in the literature review, into Table 1, Table 2 Table 3 and Table 4 which shows a list of the different estimates of the Frequency, Income, Price and Load factor elasticities respectively. The first Table 1, shows the estimates for the Frequency elasticity in the different papers. I opted not to include the Frequency elasticities from Tsekeris (2009) in this Table 1, as they are relative Frequency elasticities, and may thus be hard to compare with the estimates in this thesis.

Frequency elasticity	Notes	Method	Paper
0,945	Static model	3SLS	Wang et al (2014
0,679	model with lag of demand	3SLS	Wang et al (2014
0,94	aircraft size assumed endogenous	2SLS	Jorge-Calderón (1997)
0,65	aircraft size assumed exogenous	2SLS	Jorge-Calderón (1997)
0,864		2SLS	Ippolito (1981)
1,187	Spoke-Hub frequency elasticity	OLS	Wei & Hansen (2006)
0,265	Whole network Frequency Elasticity	OLS	Wei & Hansen (2006)
0,79	with hub dummy	2SLS	Schipper, Rietveld and Nijkamp (2002)
0,77	without hub dummy	2SLS	Schipper, Rietveld and Nijkamp (2002)

Table 1. Frequency elasticities in reviewed papers

As seen from the summary of the Frequency elasticities in the papers reviewed, they tend to be in the range of 0,65-0.95, with an outlier by Wei & Hansen (2006). It is also interesting that most of the papers used 2SLS or 3SLS to estimate the Frequency elasticity, and that the only one that used OLS is the paper with the outlier estimates. That the Frequency elasticity estimates tend to be less than 1 also fits with the discussion in a few of the papers I reviwed on the theory behind the Frequency elasticity, which states that intuitively the Frequency elasticity should be less than 1 because of diminishing returns to demand by increasing Frequency. It also seems that having a lag of demand in the model reduces the Frequency elasticity a bit, as seen by the estimates of Wang et al (2014), and that assuming the aircraft size to be exogenous or endogenous also has the same effect on the Frequency elasticity.

The next table, Table 2, shows the price elasticity in different papers.

Price elasticity	Notes	Method	Paper
-0,542	Economy fare (aircraft size assumed endogenous)	2SLS	Jorge-Calderón (1997)
-0,948	Economy fare (aircraft size assumed exogenous)	2SLS	Jorge-Calderón (1997)
-0,525		2SLS	Ippolito (1981)
-0,899		OLS	Wei & Hansen (2006)
-0.8184	Short run	Praise Winsten Regression	Kopsch (2012)
-1.13	Long run	Praise Winsten Regression	Kopsch (2012)

Table 2. Estimates of the Price elasticity in different papers.

As seen from the elasticities in the summary, the price elasticity tends to be around -0,5 to -0,9 in the papers reviewed, and one paper Kopsch (2012) calculates the long-term price elasticity, and finds it to be -1,13, which is in line with the discussion of long term elasticities in Brons et al (2002). It is however important to point out that the papers reviewed estimated the Frequency elasticity for the US market, with the exception of Kopsch (2012) who estimated it for the Swedish market. The price elasticity values may thus not be representative of Norway, since Norway has a geographical situation that leads to there being less substitutions to air travel, which might have an impact on the price elasticity. Table 3 shows the income elasticity of the papers covered.

Income elasticity	Notes	Method	Paper
0,7967	Model without Frequency and Fare	OLS	Jorge-Calderón (1997)
2,35		2SLS	Ippolito (1981)
-0,361		OLS	Wei & Hansen (2006)
1.186	Domestic market	WLS (Meta analysis)	Gallet & Docucouliagos (2014
0.633	Dynamic equation, domestic	WLS (Meta analysis)	Gallet & Docucouliagos (2014
0,34	with hub dummy	2SLS	Schipper, Rietveld and Nijkamp (2002)
0,54	without hub dummy	2SLS	Schipper, Rietveld and Nijkamp (2002)

Table 3. Estimates of the Income elasticity in different papers.

Looking at the Income elasticity estimates in the papers reviewed, it seems that they differ quite a bit between the papers, ranging from -0,34 to 1,186, with outliers such as 2,35 and -0,361. Thus, it is hard to say that there is a common income elasticity in the papers reviewed. Given that the income elasticity is used as a measurement of maturity, the wide range of the estimates may be down to the different maturities of the markets analyzed, that the highest estimate is an old paper and an meta-analysis also seems to support this, as an older paper would have a higher elasticity if the theory that air transports markets mature over time holds, similarly a meta analysis would have a higher value as the estimate is an average between both mature and immature markets, which could inflate the value. However, it is hard to say for sure if this truly is the case or not.

Lastly Table 4 shows the load factor elasticity of one paper

Load factor elasticity	Notes	Method	Paper
-0,854		2SLS	Ippolito (1981)

Table 4. Load factor elasticity in the reviewed papers

As there was only one paper that estimated the load factor, it is hard to draw any general conclusions about it, but the estimate does fit with the theory that a high load factor should have a negative effect on demand.

2.2 Norwegian air transport market

Air transport is essential in Norway because of its large distances and challenging geography. An example of this is that it takes about 4-5 hours to longer to take a train from the largest cities in Southern Norway to Oslo than to travel the same distance by air travel. Because of this many, both business and leisure travelers prefer to travel by air. Norway also has a well-established air transportation infrastructure that covers most of the country. It built its main jet airports in the 50s and the 60s and later established a large number of regional airports that takes STOL aircraft. Because of this most areas in Norway have an airport within 1 hour travel time, with some coastal areas having a regional airport as close as within 30 min travel time. The type of travelers on the different routes differ a bit, with Northern Norway having a larger share of leisure travelers than Southern Norway, and Western Norway having a larger share of oil related traffic than the rest of the country. The number of air trips per year per person also increases the further north you get in Norway, something that is natural given the longer travel times and less substitutes to air travel in Northern Norway compared to Southern Norway. An example of this is that while it takes around 15 hours longer to travel from Bodø to Oslo by train than by air travel. Air travel is also essential for many businesses, and the demand for air travel follows the business cycle but with a bit more volatility. In the period 1980-200 the growth rate of air travel demand was about the double the growth rate of the GDP. (Lian et al (2005))

Because of the long distances in Norway and the decentralized population densities there is not only a demand for point to point services to Oslo from the other cities, like in Sweden where there are routes from the rest of the country to Stockholm. In Norway passengers would want a point to point route between their city and another city in Norway if they could, but such routes requires a sufficient market to be able to operate. As this is not the case for many routes, passengers on these routes have to travel on network flights with 2 or more flights. In Norway such network flights made up 28% of all domestic flights as of 2003 for purely domestic network flights or 42% of all non STOL domestic flights if network flights where an international flight is one of the stages in the network flight is included. Out of the non STOL domestic network flights in Norway, as of 2003 45% of it consists of network flights between the Southern Norway cities besides Oslo and Northern Norway, and about 32% consists of network flights between cities in Møre and Trondheim and Southern Norway besides Oslo. (Lian et al (2005))

The STOL routes that serve rural areas are also very dependent on network flights, as such routes are often between the rural area and the regional center. Thus, if the people in the rural area want to go to another large city in Norway, they are often forced to use network travel. Because of this, as of 2003, 65% of the STOL flights are part of a network flight. ((Lian et al (2005))

Out of the different regions in Norway, the regions with most network as a percentage of total travel from that region travel is Sogn og Fjordane, Northern Norway, and to a lesser degree, Møre og Romsdal, Trøndelag, Rogaland and Hordaland. (Lian et al (2005))

As of 2015 there are is about 15 million passengers annually on the domestic Norwegian air market. The two biggest players on the market are SAS and Norwegian, who have a market share of 46% and 37% respectively. SAS has had a decreasing market share compared to Norwegian the last 12 years, going from a respective market share of 72% and 12% for SAS and Norwegian in 2003 to a stabilization around 50-46% to SAS and 35-37% to Norwegian in 2009-2010. SAS has regained some market share the last years, but as of 2015 SAS still serves half a million passengers less compared to the number of passengers that flew with SAS in 2003. The increasing market share of SAS in the last years means that the market share of Norwegian is the same in 2015 as in 2009, but even with the lost market share Norwegian still has the same number of passengers as in 2011. (Thune-Larsen & Farstad (2016))

Of the routes out of Oslo, the most travelled routes are the routes serving Trondheim, Bergen and Stavanger, with 1,95 million, 1,81 million and 1,52 million passengers respectively as of 2015. The routes have a business travel share of 50%, 51% and 55% respectively as of 2015, and had a yearly growth of 3,6% 2,7% and 4,4% respectively in the period 2003-2013. On the individual routes the growth differ slightly, with Trondheim having continuing growth since 2009 but a stagnation in business travel, Bergen having growth in both business and leisure travel, while at the same time only having growth in the years 2011 and 2014 in the period after 2009. Lastly, Stavanger enjoyed strong growth until 2014 but then experienced a rapid drop in demand in 2015. Before the drop in the years 2013-2015 Stavanger had a growth in business travel and a drop in leisure travel on the route. Besides the growth these routes have a lot in common, while also having something unique per route. On all three routes SAS has the largest share of business travel while Norwegian has the largest share for leisure travel. Out of total travel on the route, SAS has a market share of over 50% between Oslo and Trondheim and between Oslo and Stavanger, while only 48% on Oslo and Bergen. All routes have about 50-52% business travel, but Stavanger has had a growth in business travel to 55% from 2013 to 2015. The route serving Stavanger also has the most oil related travel, with 1/6 of the demand being oil related. The routes serving Stavanger and Trondheim also has the most transfer traffic and network travel, mainly because of Trondheim's role as a hub for traffic coming from Northern Norway on its way to Oslo, and Stavanger because of the lack of direct flights between Stavanger and other Southern cities in Norway besides Oslo. All three routes also had about the same ticket price in the period, but the routes serving Trondheim and Stavanger had an increase in business fares in the period 2013-2015 compared to the route serving Bergen from Oslo. (Thune-Larsen & Farstad (2016))

3.0 Conceptual model

The model used to estimate the frequency elasticity of demand for the individual routes chosen have to be adapted based on the data availability and the characteristics of the data. Still it is good to first construct a conceptual model, which can then be adjusted to the individual analysis. As can be seen from my review of the theory of the determinants of the demand of air transport, the basic model for air transport demand can be divided into three main categories; Demographical factors, Service Quality factors and Geographical and airport factors. The demographical factors are variables such as Population size, Income of passengers and GDP for the area served by the route or routes in question. The service quality factors are variables such as the fare price for the flights on the route or routes, the flight frequency, aircraft size and load factor. Finally, the Geographical factors and airport factors are variables such as distance between airports on a route, and airport factors are factors such as the length of the runway, slots available and so on. To get a better overview of the usage of certain variables in the papers I have reviewed, I have compiled a list of the most common variables in Table 5 which lists the papers and total number of papers that have used this variable in their model. Table 5 is not a complete list of the variables used, and the list of variables used only once does not include the variables that are not relevant for a time series route level analysis. Examples of such variables are characteristics that do not change over time, such as airport distance or other variables less related to route level analysis is left out of the table. Dummy variables are also left out of the list.

Variables used in demand models	Articles						
Income / GDP	[1]	[2]	[3]	[4]	[5]	[6]	6
Population	[1]	[2]	[3]	[4]	[5]	[6]	6
Frequency	[1]	[2]	[3]	[4]	[5]		5
Fare		[2]	[3]	[4]	[5]	[6]	5
Aircraft size / Seat capaciaty	[1]	[2]	[3]		[5]		4
Tourism	[1]	[2]	[3]				3
Distance	[1]	[2]		[4]			3
HHI	[1]						1
Cost	[1]						1
Log of linear time trend	[1]						1
Lag of demand			[3]				1
Travel time			[3]				1
Load Factor				[4]			1

[1] Wang et al (2013)

[2] Jorge-Calderón (1997)

[3] Tsekeris (2009)

[4] Ippolito (1981)

[5] Wei & Hansen (2006)

[6] Kopsch (2012)

Table 5. Usage of variables in the demand models in the papers reviewed

Based on the variables in the papers I reviewed and my research questions, a basic conceptual model of air transport demand on the route level, with the inclusion of service quality variables such as load factor, Frequency can be developed. Because the data on route level are time series data, some of the variables, such as distance between airports cannot be included as they would be constant over the whole-time period, and it is therefore impossible to estimate its effect on the independent variable. Thus, the model, which includes the most common variables that impact demand of air transport without time constant variables can be stated as:

$$Demand = \beta_0 + \beta_1 Population + \beta_2 Income + \beta_3 GDP + \beta_4 Fare + \beta_5 + Frequency + \beta_6 Aircraft\ size + \beta_6 Tourism + \beta_7 Load\ factor$$

4.0 Methodology

To answer my research question, I have chosen to use econometrics to estimate the frequency elasticity of air transport demand, since this is the common way to do this in the theory I reviewed. Econometric consists of many methods, but in its most basic form is OLS, which consists of fitting an equation with a number of independent variables to the plot of the dependent variable so that the sum om squares of the distance between the line of the equation and the plots are as low as possible. I will not cover the basic theory on econometrics and OLS in more detail see Bjørnland and Thorsrud (2015) and Wooldridge (2015) for a more comprehensive discussion of how OLS works.

There are however, some requirements needed for an OLS regression using time series to be viable and unbiased, which I will cover here briefly. Woodelridge (2015) mentions that there are asymptotic properties of OLS are that Firstly, the variables in the regression need to be weakly dependent, stationary and follow a linear model. An example of when a variable is nonstationary is if the variable has unit root. If a variable has a unit root, then it will be difficult to estimate the OLS model, as it will not be weakly dependent. This can be corrected for using differencing, since a process with a unit root will be weakly dependent after differencing, also called difference-stationary. I will get back to this later if it is needed. Secondly there can be no perfect collinearity between the variables, which is the case if one of the variables in the model can be completely explained by a combination of two of the other variables. Thirdly, there can be no heteroskedastic and serial correlation. Lastly, the residuals need to have an unconditional mean of 0 and each of the repressors needs to be uncorrelated with it. (Woodelridge (2015))

However, since the theory I reviewed argues that Frequency is Endogenous with Demand, this need to be taken into account when running the regression. I will thus cover the theory on endogeneity and how to take it into account for it when running a regression in some more detail.

4.1.1 Endogeneity and Instrumental Variables and 2SLS

An endogenous variable is a variable that is correlated with the error term of regression in question. When this is the case, OLS will be biased, and an instrumental variable is needed to avoid this bias. An instrumental variable is a variable that is correlated with the both the independent variable and the endogenous dependent variable, but which does not have a direct effect itself on the independent variable, and which can thus be used to estimate the endogenous independent variable. These requirements can be formulated as two rules for instrumental variable regressions. (Wooldrige (2015))

1: That the Instrument does not suffer from the same problems as the endogenous independent variable, which means that the instrument cannot be correlated with the error term in the equation. This point is impossible to test, as the error term in a regression with an endogenous variable will be biased itself. Thus, whether the instrument is related to the error term or not has to be argued through logic and/or economic theory.

2: The Instrument must be correlated with the endogenous variable. This can be tested by seeing if the instrument is significantly different from 0 in the first stage regression of a 2SLS estimation. (Wooldrige (2015))

In the case of one endogenous independent variable only one such instrument is needed. When such an instrument, which satisfies the two rules, is identified, the model can be estimated using two stage least squares regression, shortened to 2SLS. 2SLS is also referred to as instrumental variable regression in the case where only one endogenous variable and one instrument is used, but 2SLS can also estimate models using more than one instrument for a single endogenous variable. Whether one or more instruments are used depends on the availability and strength of the instruments, since having more instruments I good if the instruments are strong. Estimating a model with 2SLS makes the estimates unbiased if the assumed endogenous variable is truly endogenous. If not, the estimates from 2SLS are still unbiased but OLS would be unbiased in this case and also more efficient. The reason for this is that while using 2SLS avoids the bias in the case where the model includes an endogenous independent variable, it also leads to a larger variance of the coefficients. Thus, if the variable is not endogenous, the 2SLS would be a method with a larger variance than OLS and thus less efficient. For a more comprehensive discussion of this, and 2SLS and instrumental variable regression itself, see Wooldrige (2015). (Wooldrige (2015))

Since the 2SLS makes the estimates unbiased in the case of an endogenous variable, it is expected that estimating a model with an endogenous variable with OLS and 2SLS will produce different results. To test this a test such as the Wu-Hausman test can be run, which tests whether the 2SLS regression is significantly different than the OLS regression. I will not go do deep into detail on this, but in short consists of running a regression of the reduced from equation of the endogenous variable (called first stage in 2SLS), predicting the residuals, and then regressing the residuals as a part of the equation with the endogenous variable. The residuals would in this case represent the bias from endogeneity, and if they are significantly different than 0 in the last regression, this would be an indication of endogeneity, and thus that 2SLS is more efficient. For a more comprehensive discussion of this see Wooldrige (2015). Wooldrige (2015) also mentions that based on the way the R^2 is calculated for the 2SLS it may be hard to interpret it as I may even be negative, I will thus not discuss the R^2 when running 2SLS analyses.

4.1.2 Cointegration

As mentioned in the literature review in Fearnley & Bekken (2005), a way to estimate the long-run elasticity is when the data is non-stationary and cointegrated. So how can this be done if the data is non-stationary and cointegrated?
A way to do this with cointegrated and nonstationary data is to use a ECM and the ADL model. Both models rely on the assumption that two variables have a long term cointegrated relationship, and that there is an error correcting term in the model that pushes both variables towards the long-term equilibrium, increasing the effect of one of the variables if it is lower than the equilibrium or increasing it if it is higher. An example of how this works used by Bjørnland and Thorsrud (2015) is Consumption and Income, which they argue have a long run equilibrium as there are limits to the share of income spent on consumption. They argue that in some periods the amount spent on consumption

may be larger than this, and at some time it may be lower, but in the long run it should end up at the long run equilibrium level. So, when can such a model be applied to measure the long-run and the short-run equilibrium? Since both the ECM and the ADL model relies on that the two variables are I(1) processes and that they are cointegrated. Whether the variable is integrated of I(1) or not can be tested with a ADF test. If it turns out that both variables in question follow a I(1) process, a test to check whether they are cointegrated or not has to be run. Bjørnland and Thorsrud (2015) mentions that such a test is the Engle-Granger test. Since two cointegrated I(1) processes should have a stationary error term, the Engle-Granger test consists of regressing the independent I(1) variable on the dependent I(1) variable, then checking if the residuals are stationary or not, which can be done by running a ADF test to check the residuals for a unit root. If the ADF fails to reject the H0 of a unit root, then the residuals are stationary and the two I(1) variables are cointegrated and a ECM or ADL model can be estimated. If not a static model has to be used. (Bjørnland and Thorsrud (2015))

4.2 Choice of Instrumental Variable

Because of the possible endogeneity of frequency, which I covered earlier in the literature review there is also a need to identify one or more instruments to frequency. As covered in the literature on instrumental variables covered above, such an instrument needs to a variable that affects flight frequency but not demand directly. As mentioned in Ippolito (1981) the service quality variables such as aircraft size, frequency, price is the variables affecting demand that an airline can change. Thus, any effects on demand from a cost change has to come through these variables. Doganis (2010) mentions that while prices are often determined by the market, they are also set with costs in mind, and Richard (2003) mentions that passengers and frequency are adjusted with costs I mind. Thus, the costs such as fuel costs, cost per flight or operating cost should be good instruments for Frequency, as they impact demand only through price, aircraft size or Frequency. So, based on the discussion about the requirements of instrumental regression, if price, aircraft size is also included in the model, then the instrument should not be correlated with the error term.

Given that papers such as Belobaba (2009), Zou and Hansen (2014) argue that airlines on short haul routes with competition, as is the case in Norway, tend to increase Frequency instead of aircraft size, I assume that changes in cost will be adjusted for by the airlines through only prices and frequency on the Norwegian routes, thus aircraft size could possibly be excluded from the model. Still I will test whether aircraft size is significant in the model to be sure.

5.0 Data

5.1 Selection of routes

To investigate the frequency elasticity on the Norwegian air market I have chosen to focus on three routes. Bergen-Oslo, Trondheim-Oslo and Stavanger-Oslo. The reason for this is twofold, Firstly, I wish to investigate the Frequency elasticity on the route level, and the three routes are the three largest routes in Norway. Secondly, the reason for why I choose to use the data from these three cities to Oslo, and not the other way, is because of the network effects and to get estimate the Frequency elasticity of the different regions of Norway. The network effects have to do with the fact that travelers in Bergen, Trondheim and Stavanger often has to travel to Oslo to transfer to foreign destinations or to other Norwegian cities. Since Wei & Hansen (2006) finds that the frequency increase in the first stage of a spoke to hub flight was most important for the passengers, I want to investigate the change in frequency on the demand for the spoke airports, which in this case are Bergen, Stavanger, Trondheim to Oslo. The added benefit is that estimating the frequency elasticity on the routes to Oslo and not from Oslo is that the Frequency elasticity can be interpreted as the Frequency elasticity of travelers from the region of the non-Oslo airport. This enables the estimation of the Frequency elasticity for a larger part of the Norwegian air market than just routes out of Oslo, which might only represent the Frequency elasticity of people in Oslo. It would be interesting to investigate the Frequency elasticity of travelers from Oslo to Bergen, Stavanger and Trondheim, but because of time constraints, I focused on only the Bergen, Stavanger and Trondheim to Oslo routes.
I also wanted to look at routes from Bodø, Tromsø and smaller airports to Oslo, but these turned out to be hard to estimate because the difficulty with estimating the frequency elasticity from small routes using national aggregates values for many of the repressors. I thus choose to only use the data from the three largest routes. The chosen routes can be seen in the map in Figure1, where the cities served, the counties where they are located, and the routes are plotted, with South-Trøndelag where Trondheim is located being

yellow, Hordaland where Bergen is located being Blue, and Rogaland where Stavanger is located is Green. Oslo is Orange. The airports are also marked with a red dot.

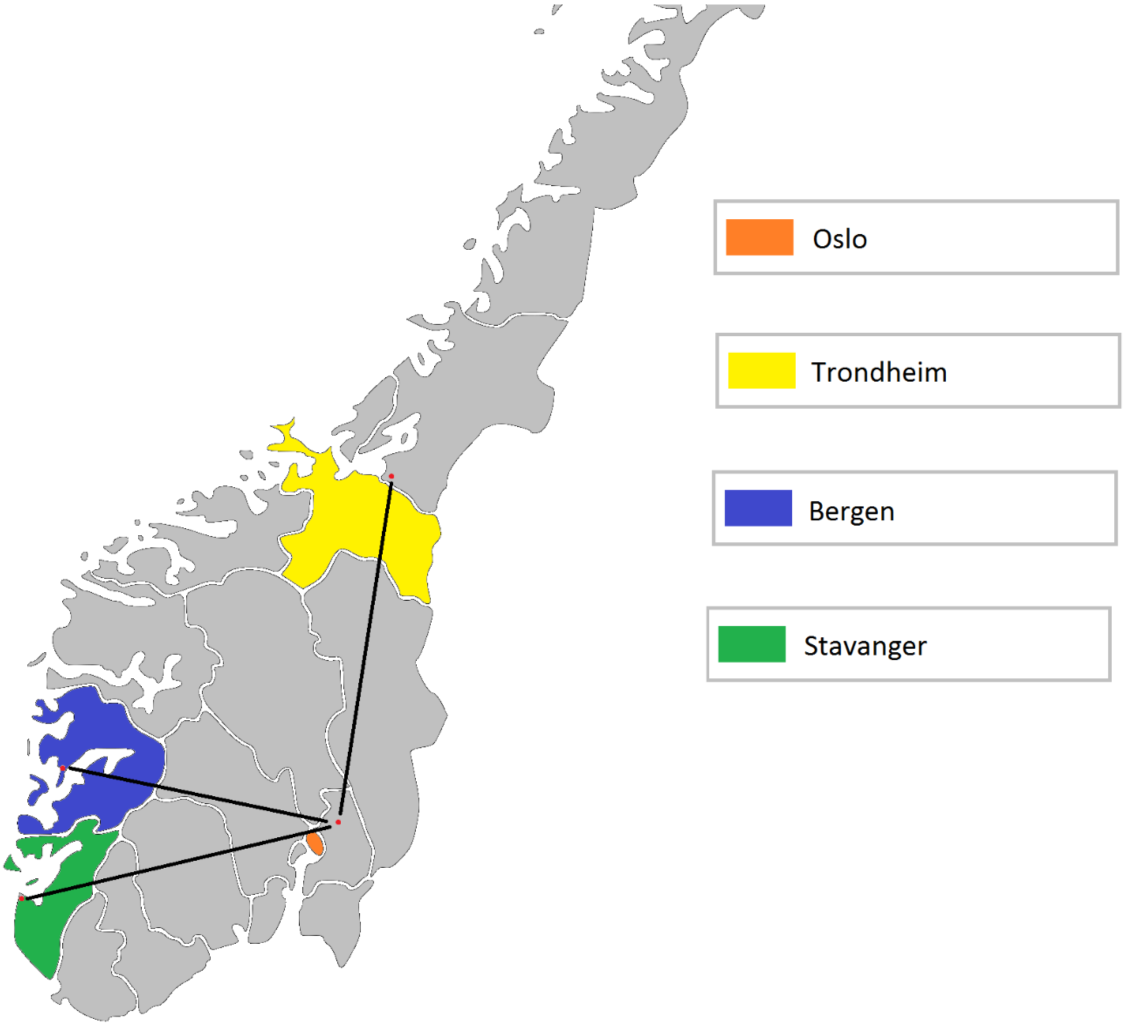

Figure 1. Map of routes and cities, with the counties where the cites are located colored.

5.2 Data collection

Having chosen the routes to investigate, I collected the data based on the conceptual model. The main challenge with this was to find data of sufficient quality and availability. For the routes; Bergen-Oslo, Trondheim-Oslo and Stavanger-Oslo the lack of route specific data for some variables led me to have to use national aggerate data for these variables. Another limitation was that the length of the time series, since the variable with the shortest available data time period put a limitation on the time period of the other time series. For the routes, the variable with the shortest time series was the number of passengers on the route, as I could only find data that covered the period 2009Q1-2016Q4 for this variable. Thus, the data covers this time period. The data I found for the routes comes from two sources, the first is SSB the Norwegian Statistic bureau, which had data on population, income, gdp, index for the fare price, income for airlines and the number of passengers travelling on the route. The data from SSB was all quarterly data. The second source is Capstat, which has monthly data on the average aircraft size and frequency on the routes. Since the time series from Capsat covered monthly periods, I had to aggregated the data to quarterly time series myself. Below is a short description of the variables and how they are measured.

Demand (pax):
Measures the number of passengers between the airports per quarter.

Population (*pop*) :
Population is measured as the number of people living in the municipality of choice at the start of the quarter. To measure the potential market of a route I have chosen to add up the population of the municipalities of the Origin airport and the destination airport. I decided against finding the catchment area of the airports, as the routes serves so many passengers that it is likely that the catchment area would be quite large, and instead only focused on the two municipalities served by the origin and destination airports. My assumption is that being an important business routes, many business travelers would be located in one of the two cities.

Income (*inc*) :
Income is measured as the disposable income for households per quarter.

GDP (*gdp*):
GDP is measured as the market value of GDP per quarter in 2014 prices.

Fare price (*price*) :
The fare price is a national aggregate price index of the price of air travel per quarter, which is reported by the airlines to SSB. This price is the price received by the airline for a trip without taxes or other fees that the airline collects as part of the ticket price. The index thus does not exactly reflect the price received by customers.

Income for airlines (*incair*) :

Income for airlines is measured as an index measuring the turnover of airlines in Norway. As I could not find any aggregate data on the fuel costs or costs of airline in Norway, I was forced to use income of airlines as a proxy for its costs. My assumption is that as the income of the airline shifts up or down, the airline will respond by changing its price, frequency or aircraft size to adapt to and to find a new profit maximizing equilibrium between income and costs. Similarly, to the discussion in chapter 4.2 of airline costs, the effects of a change in income for the airline is would be felt indirectly by the passengers, through changes in prices or the capacity offered. Thus I feel that the income of airlines fulfills the requirements of an instrumental variables in the same way was airline costs did, and that it thus can be used as an instrument for frequency.

Frequency (*freq*) :

Frequency is measured as the number of departures on a route per quarter. The data from Capstats was monthly, so I have aggregated the data myself into quarterly data.

Aircraft size (*asiz*) :

Aircraft size is measured as the average aircraft size on a route per quarter. The data from Capstats was monthly, so I have aggregated the data myself into quarterly data. I also identified some errors in the data for the three routes in some months of the two first quarters of 2016, as the aircraft size was much lower than the other months. By sorting the affected months by aircraft type and airline, I identified that some columns had values indicating an aircraft size of around 40 or 50 for a 737-700 or 737-800, I assumed that this was a simply mistake of omitting a 1, so that the correct aircraft sizes should have been 140 or 150. I updated the dataset with this in mind, and calculated the average aircraft size per month myself using the corrected columns and the columns without errors for the months in question. Doing this, gave me an average aircraft size for the corrected months that either were equal to or seemed reasonable given the previous and following months without any incorrect values. Based on this I assume that the cleaned dataset could be used.

Load factor (lf) :

I calculated the load factor myself using the equation: Load factor = (*pax / freq)* / *asiz).*

6.0 Analysis 1: Bergen-Oslo route

6.1.1 Descriptive

Having collected the data for the route Bergen-Oslo. I will first show the descriptive of the different variables over the time period in question, before moving on to the analysis.

Variable	Mean	St. dev	Change between 2009Q1-2016Q3
pax	214731	22889	23 %
freq	1940	163	3 %
pop	107	4	14 %
income	290285	34656	48 %
gdp	763870	32831	5 %
price	109	11	8 %
incair	134	33	56 %
asiz	157	8	6 %
lf	70	3	12 %

Table 6. Variables collected for route Bergen-Oslo for the time period 2009Q1-2016Q3

To get a better view of how the variables change over time, I will plot the route specific variables for the route in question, which are pax, freq, load factor and aircraft size. The other variables are national aggregate values, and are not route specific.

Pax:

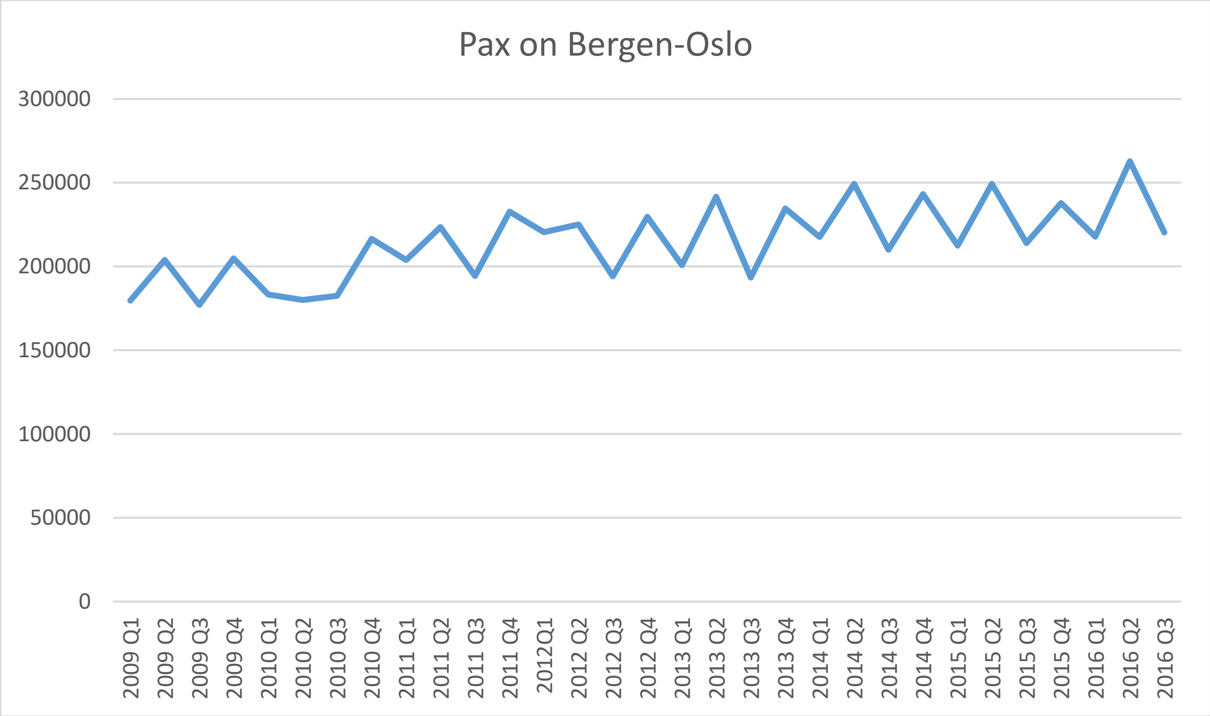

Figure 2. Pax on the route Bergen-Oslo plotted over time

As can be seen from the graph, the passengers on the route Bergen-Oslo has increased over the time period in question, and that the increase is more of a steady increase over time. The demand is also highly cyclical, with the largest peaks being often in the second quarter. There is also a slight downturn in the quarters Q1 20010 – Q3 2010.

Freq

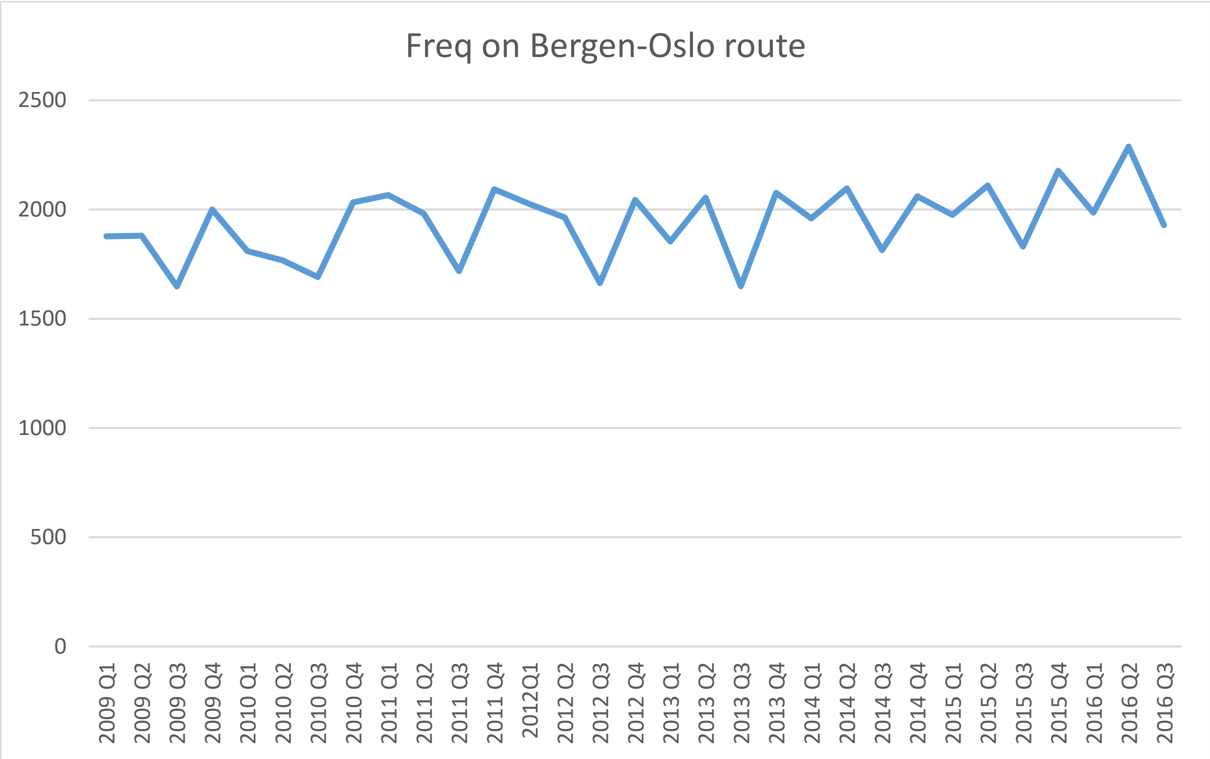

Figure 3. Freq on the route Bergen-Oslo plotted over time.

The frequency on the route Bergen-Oslo seem to also be highly cyclical, and seem to be slightly stationary between 2010Q4 - 2015 Q2, with values in the range between 1600 and 2100 , with some peaks in the 4 Quarter and 2 Quarter. There is also a slight increase at the end of the period and a slight increase in the first quarters until 2010 Q4.

Load factor:

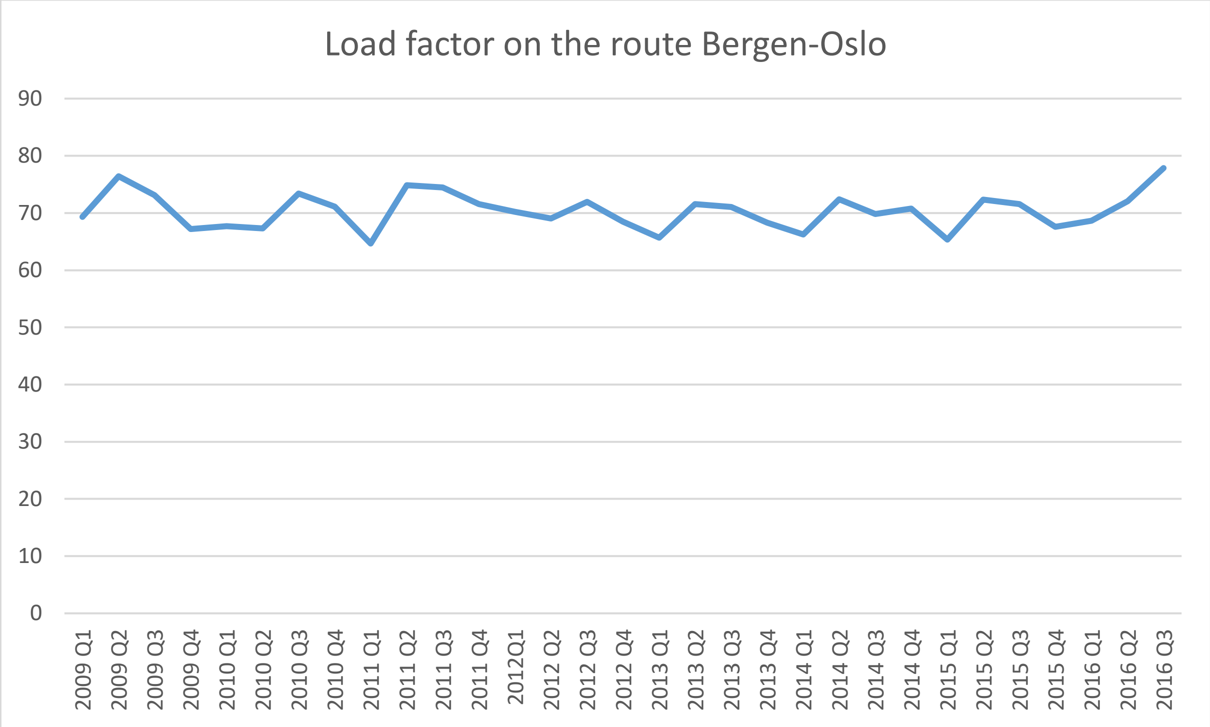

Figure 4. Load factor plotted over time on the route Bergen-Oslo

The load factor also seems to be cyclical, but not the same degree as frequency and demand. The load factor also starts high, then goes down until it regains some of the height in 2010 Q3 and Q2 and Q3 in 2011, before stabilizing around a load factor of 70 in Q3 of 2012. In the last three quarters, Q1, Q2 and Q3 2016 there is also a steady growth in the load factor

Aircraft size:

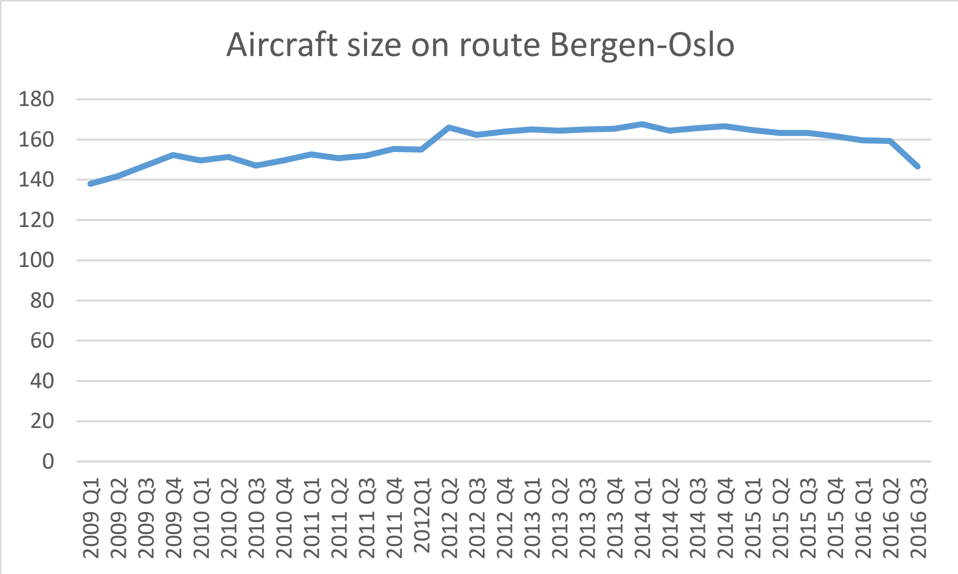

Figure 5. Average aircraft size on the route Bergen-Oslo.

The average aircraft size on the route, measured in available seats on average, is growing steady over the timer period, with a slight peak in 2012 Q2 and 2014 Q4. There is also a downturn in the last quarter of 2016. I investigate the data in more detail, and find that the decreasing average aircraft size on the route during Q3 2016 is due to the usage of a CRJ 900 and a Fokker 100 by SAS and Norwegian respectively on that route in that quarter, which has a seating capacity of 88 and 100 respectively. This pushes down the average aircraft size somewhat in that quarter.

6.2 Model

To estimate the elasticities, I will use the basic conceptual model in log form of the variables, as this is mentioned in Fearnley & Bekken (2005) as a method to do this. Givent he variables I was able to collect data for from the conceptual model, the complete model in log form the complete model can be stated as:

$$lnDemand = \beta_0 + \beta_1 lnPopulation + \beta_2 lnIncome + \beta_3 lnGDP + \beta_4 lnFare + \beta_5 + lnFrequency + \beta_6 lnAircraft\ size + \beta_7 lnLoad\ factor + \varepsilon_t$$

6.2.1 Tests of stationarity

But before building the model, I will first test the data for stationarity. The reason or this is as mentioned in the Methodology, is because of the challenges with estimating a model with non-stationary variables. Thus, it is important to test if the variables have to be differenced or not to correct for this non-stationary or not. With the test, I use the option *trend*, for variables that seem to be trending such as ln pax, ln pop, ln inc, ln gdp, ln price, ln incair and ln freq.

To do this I will use the Dicky Fuller test, which checks for a H0 of a unit root. For more detail on the test see Bjørnland and Thorsrud (2015); Wooldridge (2015) and (http://www.stata.com/manuals13/tsdfuller.pdf). The results from the test can be seen in Table 6, where I(x) denotes the number of time the data had to be differenced to become stationary, with a I(0) being a process without a unit root and a I(1) being a process that has a unit root, but which is differenced stationary after taking first differences. (Woodelridge (2015) As noted earlier in the conceptual model, the variables are in log form, as this helps in finding the elasticity.

ln pax	I(0)
ln pop	I(1)
ln inc	I(1)
ln gdp	I(0)
ln price	I(1)
ln incair	I(1)
ln freq	I(0)
ln lf	I(0)
ln asiz	I(1)

Table 7. Dicky Fuller test for route Bergen-Oslo

6.2.2 Model building

As seen by the stationary test, some of the variables in the model has to differenced once to make them stationary. I will thus use the log form and as first differences for the variables in the model to correct for this. Doing this also has the added benefit, as stated by (Wooldridge (2015) of making the coefficients interpretable as elasticities in addition to removing most of the serial correlation. Since the dependent variable follows a I(0) process, the cointegration analysis such as a ECM or ADL can also not be run, as such a cointegration analysis requires that the dependent variable is I(1), as mentioned in Fearnley & Bekken (2005).

Since the OLS estimates will be biased if the frequency is in fact an endogenous variable, I will start by building and estimating the model by using 2SLS. The reason I did this is because I have some concerns that since OLS will produce biased estimates if the endogenous variable is truly endogenous, then deciding what variables to include based on OLS might mean that I end up excluding important variables based on their biased test scores, which might lead to the wrong variables being omitted or included.

I will thus start to analyze the route using 2SLS, and then estimate the same model with OLS. Estimating the same model with OLS and 2SLS also facilitates a comparison of the results.

6.2.2.1 Model building for 2SLS

My main challenge with developing the model for the route was that without the load factor, the model did not seem to be good enough to estimate the frequency and to pass the diagnostic tests. I did however have some concerns about including the load factor because of its possible endogeneity with pax, or a possible linearity between freq, pax and load factor. I thus decided to lag the load factor, and use this as a variable in the model. By doing this, I assume the variable would become exogenous, while still being able to explain historic effects of a high load factor. Including such a variable would also be interesting, since it enables the estimation of whether the load factor in the previous quarter have an effect on demand in the current quarter.

Thus, I run the complete model, with load factor in time t-1 included and load factor in time t excluded.

Having run the complete model, I start by excluding the Income from the model. The reason for this is that there was collinearity between it and POP. I then run the remaining model with dummies for the quarters to see if there is any seasonality effects. I find that only the dummy for the second quarter is significant, so I exclude the other quarterly dummies. I also found that the POP was not significant, and removing it also gave the model a larger F value, so I also excluded the POP variable. I also excluded the aircraft size as it was not significant. I also had some concerns that aircraft size was endogenous in the model, so this is also a good reason to exclude it.

I choose to keep the Price variable included, even though it was not significant since the price elasticity it is of interest and the price variable show the right coefficient. Another reason for including the price variable is that it may be correlated with the *incair* instrument. The reason for this is that if the price variable was significant, and it was omitted, its variance would end up in the error term, and since I assume the incair instrument is correlated with price, this would mean that the incair instrument would be correlated with the error term after such as exclusion, thereby breaking the requirements of 2SLS of no endogeneity of the instrument. For a wider discussion on this see Woodelridge (2015). However, since the price variable is not significant, it is unlikely that dropping it would make the instrument *incair* correlated with the error term, but I still choose to keep it in out of interest and to be on the safe side.

I also tried to include a (t-1) lag of the pax variable (*lnpax1*), to see if historical effects impacted current demand since lagged dependent variables are stated by (Dynamic Models for Dynamic Theories: The Ins and Outs of Lagged Dependent Variables) to be able to capture such effects. The lagged dependent variable (*lnpax1*) turned out be not significant for demand, but it was however significant in the first stage regression for the frequency, so I kept it in the model. The plot of the demand also showed a downturn in period 5-7, which are the Quarter 1 2010 – Quarter 3 2010. To see if this had a significant effect on demand I constructed a dummy variable that covered this period, but found it to be not significant. I thus choose not to include it.

Lastly, Muller (2015) mentions in his thesis that the maximum variables in a model is restricted by the number of observations, as each variable requires a certain number of observations. He cites from Helgheim (2002) that a rule of thumb is that the optimal number of observations per variable is 5. He also uses this requirement when he construct his model. (Muller (2015)). Since I have 29 observations in my model, I can have 5,8 variables in my model. The variables I am left with after building my model is 6, and I thus feel that all of them can be used given the rule of thumb.

Having completed the process of building the model, the final model can be stated as:

$$\Delta lnpax = B_0 + B_1\,\Delta lnfreq + B_2\,\Delta lngdp + B_3\,\Delta lnlft1 + B_4\,\Delta lnprice + B_5\,\Delta lnpax1 + q2 + \varepsilon_t$$

Where lnlf1 and lnpax1 is the lag of the load factor and the lag of the demand respectively.

Since the frequency (*lnfreq*) has to be estimated using the instrumental variable (*lnincair*) in the first stage of the 2SLS, the reduced from of l*nfreq* can be stated as:

$$\Delta lnfreq = B_0 + B_1\,\Delta lngdp + B_2\,\Delta lnlf1 + B_3\,\Delta incair + B_4\,\Delta lnprice + B_5\,\Delta lnpax1 + q2 + \varepsilon_t$$

6.3 Model estimation by 2SLS

		Number of obs =	29
		F(6, 22) =	92.51
		Prob > F =	0.0000
Total (centered) SS =	.567585974	Centered R2 =	0.9635
Total (uncentered) SS =	.5677912265	Uncentered R2 =	0.9635
Residual SS =	.0207198392	Root MSE =	.02673

D.lnpax	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lnfreq						
D1.	.5876323	.1490063	3.94	0.000	.2955852	.8796793
lngdp						
D1.	.9416919	.4330814	2.17	0.030	.0928679	1.790516
lnprice						
D1.	-.1264036	.1413411	-0.89	0.371	-.403427	.1506198
lnlf1						
D1.	-.3414517	.1510053	-2.26	0.024	-.6374167	-.0454868
lnpax1						
D1.	-.1110854	.0840746	-1.32	0.186	-.2758686	.0536977
q2	.1022682	.0200424	5.10	0.000	.0629858	.1415506
_cons	-.0233873	.0078404	-2.98	0.003	-.0387543	-.0080204

Figure 6. Demand model for Bergen-Oslo estimated by 2SLS.

Having estimate the model, it appears that there are four significant variables, the GDP (*lngdp*), Frequency (*lnfreq*), lag of the load factor (*lnlf1*) and the quarterly dummy for the second quarter (*q2*). The coefficients of the different variables fit well with the theory I reviewed on price elasticity, income elasticity and frequency elasticity, and load factor elasticity. I will review each one in turn, starting with my main focus, the Frequency elasticity. As can be seen from the results, the frequency elasticity is significant and is 0.588, or 0.59 rounded up. This shows that the route has a slightly lower frequency elasticity than what is previously reported in the literature. Drawing from the discussions about maturity from the literature on price elasticities, since I could not find similar discussions about this on the frequency elasticity, a lower frequency elasticity can be a sign that the route Bergen-Oslo is experiencing some maturity with regards to the frequency compared to the other Non-Norwegian routes covered in the literature I reviewed. The literature I reviewed, such as Ippolito (1981) did indicate that a frequency elasticity less than 1 meant that there were diminishing returns from increasing frequency, and a frequency elasticity of 0,59 compared to a frequency elasticity of 0,65 or 0,96 means that the returns diminishes faster given the same growth rate in frequency over time. The inclusion of a lagged dependent variable in the model also facilitates the estimation of the long-term Frequency elasticity, as mentioned in Fearnley & Bekken (2005), but since the lagged dependent variable is not significant, I feel such long-term estimates would be unreliable. I have thus chosen not to calculate the long-term Frequency elasticity for this route.

The next significant variable is the GDP, which has an elasticity of 0,94. I assume the GDP can be interpreted as the Income, as the papers I reviewed often used GDP as the variable for the income elasticity. Given the wide range of income elasticity in the papers reviewed it is hard to compare the elasticity of 0,94 with the other income elasticities, but it is at least in the range of the other estimates. That the GDP elasticity is 0,94 shows that it is inelastic and that the market has reached maturity based on the arguments in Graham (2000). The next significant variable is the lag of the load factor, which has an elasticity of -0,34 that is significant at the 5% significance level. This fits with the theory in that a high load factor is expected in the literature to lead to discomfort or the risk of not finding a seat as argued in Ippolito (1981) and Calderon (1997). I assume that the reason for why the lag of the load factor is significant can be because of lower or higher demand in the current quarter because of such negative or positive experiences connected to the load factor by passengers from in previous quarter. The best would be to measure this the un-lagged load factor, but because of concerns about endogeneity and linear dependence I was forced to use the lag of the load factor. So, the coefficient should be interpreted with some care as it does not represent the impact of a load factor in the current period, but only the lagged response to it.

The last significant variable is the quarterly dummy for the second quarter, which also seems to be intuitively correct, as the second quarter is the months April, May and June, which are months with public holydays and June is start of the summer vacation for many, which are commons reasons for trips to visit friends and family or to go abroad, in which case I assume the travelers on the routes often have to fly to Oslo to transfer to an international flight, transferring at Oslo may also be needed for trips to friends and family if there is no direct flight between Bergen and the other city in Norway. Thus having a higher demand in these months seems reasonable. I also checked the plot of the demand over time, and the peaks in demand is often on the second quarter.

Lastly, while it is not significant the price elasticity show the right coefficient and has a reasonable value of -0.126. As mentioned in Brons et al (2002), less substitutes to air travel and more business travelers and difficult terrain on the route should lead to a lower price elasticity. Thus as mentioned in Lian et al (2005) and Thune-Larsen & Farstad (2016) that there is around 50% of business traveler on the route, and that it crosses a mountain range and that few substitutes of the same quality are available, then this price elasticity seems reasonable according to the theory. But it should be pointed out that the price elasticity it is not significant, and that the price data is the national aggregate price for air travel. Thus, the true price elasticity on the route may differ from this value.

6.3.1 Model Diagnostics

After running the 2SLS estimate of the model, there are a few diagnostics tests that need to be run to see if the results are unbiased.

Firstly, I will test the strength of the instruments, since a weak instrument leads to inconsistent results. A test for whether the instruments are weak or not is to run a F test on the first stage of the 2SLS which test whether the instrument has a significant impact on the endogenous variable or not. If the F statistics in this test is higher than 10, then as a rule of thumb the instruments are considered strong. (Bjørnland & Thorsrud(2015)).

I run this test for the 2SLS estimates of the route Bergen-Oslo and find the F statistic to be 12,598 which is larger than 10. Thus, the instruments can be considered strong.
As mentioned above in the methodology, another requirement for an instrument to be used is that it is exogenous in the equation. Since I assume that an airline only adjusts its frequency and price to an increase in their income on the short term, the instrument incair should not be related with the error term in the equation, and thus be exogenous. However, the airline could increase its aircraft size in response to an increase in income, which would put such a change in the error term if aircraft size was excluded from the equation, and thus make the instrument endogenous. To be on the safe side, I estimate the model again with the aircraft size, and find it to be not significant at the 5% significance level.

The instrument should thus not be correlated with the error term, since aircraft size does not have a significant effect on demand.

Thus, the instrument appears to be a strong and exogenous instrument, which satisfies the requirements that an instrument is correlated with the endogenous independent variable and exogenous in the equation, as mentioned in Wooldridge (2015).
Having tested the instrumental variable, I will then test whether the model is homogenous, lacks serial correlation, has a linear functional form, as these are stated as important assumptions for 2SLS to be consistent in Wooldridge (2015). Wooldridge (2015) also states that if the requirements hold then 2SLS should be asymptotically normal, thus the residuals should be normal. But, to be on the safe side I also test the normality of the residuals.

Firstly, I run the Cumby-Huizinga test for serial correlation in the errors, which has as its H0 that there is no serial correlation. The Cumby-Huizinga test has as its advantage that it can be used when the model includes an IV (Baum & Schaffer (2013)). Based on the test I fail to reject the H0, thus it appears that there is no serial correlation in the model.

The next test I run is the Pagan-Hall of heteroskedasticity. The test tests for whether the equation being estimated by IV regression has heteroskedasticity or not, against the H0 that there is no heteroskedasticity in the equation. (Baum & Schaffer and Stillman (2003)). Given the test results, I fail to reject the H0, thus the model does not appear to have heteroskedasticity.

Another test I will run is the Ramsey RESET test, which test whether there are any nonlinearities in the functional form, with a H0 that there are none. This also means that under the H0 the model has the correct functional form. (Wooldridge (2015)). Running the test I fail to reject the H0, thus the model does appear to have the right functional form.

Next, I predict the residuals, and test them for normality using the Shapiro Wilk test, which has as its H0 that variable is normally distributed (Stata (n.d.d.)). Having run the test I fail to reject the H0 of normality at the 5% significance level. Thus the residuals appear to be normally distributed.

Thus, the model appears to fulfill the conditions 2SLS. But, Kopsch (2012) also mentions in his paper that for an unbalanced regression to be well preforming it needs to pass three tests. Since the model has a I(0) dependent variable and more than two I(1) independent variables, it is a unbalanced regression, and thus need to fulfill these requirements. These requirements is firstly the fitted values of the dependent variable needs to be a I(0) process, secondly the error term also needs to be a I(0) process, and lastly the variance ratio between the fitted and observed values of the dependent variable needs to be not significantly different from 1. There also needs to be more than two I(1) repressors, but the model already fulfills this, so there is no need to test this further.

Thus I predict the fitted values of $\Delta lnpax$ and the residuals, and test whether they are a I(0) process with the Dicky Fuller test, and then run a variance ratio test between the fitted and the observed values of $\Delta lnpax$. The Dicky fuller test fails to reject the H0 of no unit root (Stata (n.d.a.) thus the fitted values of $\Delta lnpax$ appear to be a I(0) process. The variance ratio test also fails to reject the H0 that the ratio is 1. And lastly the Dicky Fuller test fails to reject the H0 that the residuals follow a I(0) process.

Thus, the model seems to pass the requirements for an unbalanced regression to be well preforming.

Thus, the model estimated by 2SLS seems to be unbiased.

Lastly, it is interesting to see if the frequency was an endogenous variable or not. A test for this is the Wu-Hausman test, as discussed earlier in the methodology chapter. Stata reports both it and the Durbin-Wu-hausman test, and both test have the H0 that the variable is exogenous (Stata (n.d.b.). I run the test and fail to reject the H0 that the variable lnfreq is in fact an endogenous variable. It is important to remember what the test does, which is to compare the OLS regression with the instrumental regression, and a failure to reject thus indicates that any significant difference between the two cannot be identified. This only means that the exogeneity of the variable cannot be rejected, not that it is proved that the variable is not endogenous. However, if it *lnfreq* is truly exogenous, then OLS will produce a more efficient estimation than 2SLS. Because of this, I will run the OLS estimate of the model, and compare it with the 2SLS estimate. If *lnfreq* is exogenous then the estimates should be similar, although the 2SLS will have higher variances because of its estimation method. For a wider discussion of this, see Wooldridge (2015) and the methodology chapter above.

A summary of the diagnostic tests with their p values can be seen in Table 7, with the exception of the weak instrument test, as it did not report a p-value.

Diagnostic tests	p- value
Cumby-Huizinga test	0.894
Pagan-Hall test	0.6602
Ramsey/Pesaran-Taylor RESET test	0.9549
Shapiro-Wilk test for residuals	0.20327
Wu hausman F test	0.74657
Durbin-Wu-Hausman	0.70111

Dicky-Fuller test:	p- value
Residuals	0,0000
Fitted	0,0000
Fitted, trend	0,0000

Variance ratio test:	p- value
ratio != 1	0.9243
ratio < 1	0.4622
ratio > 1	0.5378

Table 8. Summary of diagnostic tests for 2SLS estimate for the route Bergen-Oslo

6.4 Model estimation by OLS

Source	SS	df	MS
Model	.547048326	6	.091174721
Residual	.020537648	22	.000933529
Total	.567585974	28	.020270928

Number of obs =	29
F(6, 22) =	97.67
Prob > F =	0.0000
R-squared =	0.9638
Adj R-squared =	0.9539
Root MSE =	.03055

D.lnpax	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnfreq						
D1.	.6330368	.102778	6.16	0.000	.4198883	.8461854
lngdp						
D1.	.8359083	.3807034	2.20	0.039	.0463778	1.625439
lnprice						
D1.	-.1215789	.1609157	-0.76	0.458	-.4552977	.2121399
lnlf1						
D1.	-.3083089	.141297	-2.18	0.040	-.6013409	-.0152768
lnpax1						
D1.	-.0992705	.0893679	-1.11	0.279	-.2846082	.0860672
q2	.1007135	.0224327	4.49	0.000	.0541909	.147236
_cons	-.0229068	.0088461	-2.59	0.017	-.0412524	-.0045612

Figure 7. OLS estimate of the model for the route Bergen-Oslo.

As seen from the OLS estimate of the model, the same variables are significant as with the 2SLS estimate, and the coefficients differ only slightly. Since coefficients of the variables are so similar to the OLS I will not discuss their fit to the theory in detail, as the conclusions would be the same as for the 2SLS estimate. Thus, the coefficients of the OLS estimate seem to fit the theory. The model also seems to fit the data well, with a R^2 of 0,9638.

There are some differences however in the size of the coefficients, which I will cover in turn. The first the frequency elasticity, is 0,63, which is slightly larger than the frequency elasticity estimated with 2SLS. This Frequency elasticity is closer to the Frequency elasticities in the papers I reviewed, but it is still a bit lower than the lowest estimate of 0,65 in Jorge-Calderón (1997).So, the possibility that Bergen-Oslo is more mature with regards to the Frequency elasticity might still hold, although the difference between the estimate for the Bergen-Oslo route and the other papers is lower.

The next variable, GDP has a lower elasticity than under 2SLS, being only 0.836. Interpreting the GDP as the income elasticity means that the GDP elasticity indicates that the market is inelastic and thus a mature market, since the elasticity is less than unity. The route, being one of the largest in Norway, and Norway being a western country means that this seems like a reasonable finding.

The lag of the load factor and the quarterly dummy for the second quarter only differ slightly from the 2SLS estimate, so the coefficients appear to show the same effects as with the 2SLS estimate.

6.4.1 Model Diagnostics

Having run the OLS estimate of the model, I will run some diagnostics test on it, before comparing the OLS and 2SLS estimates. Based on the asymptotically conditions for unbiased OLS given in Wooldridge (2015), the model needs to lack serial correlation, have a linear functional form and lack heteroskedasticity. In addition the residuals need to be normality distributed. According to Wooldridge (2015), under the asymptotically conditions for OLS, the residuals will be asymptotically normal if the other conditions hold. I will however also test the normality of the residuals to be on the safe side. Thus tests needs to be run to control if these conditions hold or not.

Firstly, the model has to be controlled for serial correlation. A common test for this is the Durbin Watson test. This test can however not be used to test my model as it cannot be applied to test a model with a lagged dependent variable (Wooldridge (2015)). An alternative test that can be used however is Durbin's alternative test, which has as its H0 that the model does not have serial correlation. This test also has the added benefit that it works even when not all repressors are strictly exogenous. (Wooldridge (2015); Stata (n.d.c.)). Having run the Durbin's alternative test on the model for a small sample size, I fail to reject the H0, and the model thus does not seem to have serial correlation.

The lack of serial correlation in the residuals is good since Keele & Kelly (2006) states that a model with a lagged dependent variable would be biased when estimated with OLS in the presence of serial correlation in the residuals. They also state that for an OLS regression of a model with a lagged dependent variable to be unbiased the dependent variable also needs to be stationary, which is also the case for this model. In addition to this, they state that a LaGrange Multiplier test should be run after estimating a model with a lagged dependent variable, to see if the residuals are white noise, i.e. do not have residual serial correlation. (Wooldridge (2015)). I run the Breusch-Godfrey LM test for autocorrelation, which has as its H0 that there is not serial correlation in the residuals (Stata (n.d.c.)). Doing this I fail to reject the H0, and the model appears to not have any serial correlation in the residuals, and a model with a lagged dependent variable is thus not biased when estimated with OLS.

I also run two test to see if there is heteroskedasticity, Whites test and the Breusch-Pagan / Cook-Weisberg test, which both has at its H0 that the residuals are homogeneous. (UCLA. (2017)). With both tests I fail to reject the H0. Thus the model do not appear to have heteroskedasticity.

Next, I test the normality of the residuals, I use the Shapiro–Wilk normality test, which has as its H0 that the variable has normality. (Stata (n.d.c.)). I predict the residuals from the regression and run the Shapiro-Wilk normality test on the predicted residuals, and fail to reject the H0. Thus, the model seems to have normal distributed residuals.

Lastly, I fail to reject the H0 of omitted variables with the Ramsey RESET test. It is important here to point out that the Ramsey RESET test actually tests whether the model has nonlinearities in its functional form against the H0 that I does not. Thus, the RESET test that the functional form is correctly specified, not that there are omitted variables, since if the omitted variables are linear, then the test will not detect them. (Wooldridge (2015))

Thus, the model satisfies the OLS conditions of no heteroskedasticity, no serial correlation, normality in the residuals and a linear functional form.

Since the model is a unbalanced regressions, as mentioned in the 2SLS analysis, there are a few more tests that need to be run. These are as cited by Kopsch (2012) from Baffes (1997) that a unbalanced model, with at last two I(1) regessors and a I(0) dependent variable, can still perform well if three requirements are satisfied. Firstly, the fitted dependent variable needs to also be I(0), the error term needs to also be I(0) process, and lastly the variance ratio between the fitted and observed dependent variable needs to be 1. Kopsch (2012))

First I run the Dicky Fuller test on the fitted values of $\Delta lnpax$, and I fail to reject the H0 the fitted values of $\Delta lnpax$ is a I(0) process. Thus both the fitted and observed values of $\Delta lnpax$ seem to follow a I(0) process.
Next I run the Dicky-Fuller test on the residuals of the regression, and fail to reject the H0 that it is a I(0) process.
I fail to reject the H0 that the variance ratio between the fitted and observed values of $\Delta lnpax$ is 1.
Thus the model fulfills the requirements by Kopsch (2012) for the model to be well preforming when the regression is unbalanced.
Thus, the OLS estimate is well performing and unbiased, assuming $lnfreq$ is exogenous.

A summary of the diagnostic test can be found in Table 12:

Diagnostic tests	p- value
Durbin's alternative test	0.4193
Breusch-Godfrey LM test	0.35
Breusch-Pagan / Cook-Weisberg test	0.8562
White's test	0.3119
Shapiro-Wilk test for residuals	0.1538
Ramsey RESET test	0.9079

Dicky-Fuller test:	p- value
Residuals	0,0000
Fitted	0,0000
Fitted, trend	0,0000

Variance ratio test:	p- value
ratio != 1	0.9432
ratio < 1	0.4716
ratio > 1	0.5284

Table 9. Diagnostic tests for OLS estimate for Bergen-Oslo

6.5 Comparison and Findings

Having estimated the model by both 2SLS and OLS, I will compare the results, discuss whether frequency can be assumed to be exogenous, and summarize the findings.

	Variables	2SLS	Sig.	OLS	Sig.
Δ	lnfreq	0.588	***	0.633	***
Δ	lngdp	0.942	**	0.836	**
Δ	lnlf1	-0.341	**	-0.308	**
Δ	lnprice	-0.126		-0.122	
Δ	lnpax1	-0.111		-0.099	
	q2	0.102	***	0.101	***
	F	93,75		98.93	
	R^2	0.9639		0.9643	

Table 10. Comparison between 2SLS estimates and OLS estimates for the route Bergen-Oslo

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As can be seen from the table, the results from the 2SLS and OLS are similar. The tests for endogeneity for the 2SLS estimates failed to reject the exogeneity of *lnfreq*, and according to Wooldridge (2015) the estimates from 2SLS and OLS should be similar if the *lnfreq* is exogenous. That the estimates from 2SLS and OLS do differ slightly can be down to the fact that OLS is more efficient if the presumed endogenous variable is in fact exogenous.

Thus, the this seem to support that *lnfreq* is exogenous. Given this I will base my discussion on the OLS results, as if *lnfreq* is truly exogenous then OLS is more efficient. I will however point out that a failure to reject the exogeneity of *lnfreq* is not a proof that it is exogenous. Thus, I recommend further research to determine if the frequency on the route can be considered endogenous or not.

So, what can be said about the route Bergen-Oslo. Firstly, the low price elasticity of 0.12 points to that there are few substitutes to air travel on the route and or many business travelers, the inelastic Income (GDP) elasticity of 0.84 indicates that it is a mature air transport market. The negative elasticity of -0.31 for the lag of the load factor and the frequency elasticity of 0.63 show however that it is a market that responds positively to a reduction in delay costs, both in the same quarter as seen by the frequency elasticity and in the previous period as seen by the lag of the load factor. The frequency elasticity of 0,63 show that the passengers react to an increase in departures, and they can thus be assumed to value delay costs, as a decrease in such costs from an increase in departures leads to significant increase in demand all else equal. The negative elasticity of -0.31 for the lag of load factor also shows that passengers on the route respond negatively to a high load factor in the previous quarter, or positively to a lower load factor in the previous period, which I assume based on the theory I reviewed, such as Ippolito (1981) to be because of historic effects because of delay costs and discomfort because of sold out planes or cramped flights respectively in the previous quarter. The significant quarterly dummy shows that demand for air travel on the route is expected to be 10,1% higher in the second quarter. There I also a significant increase in demand in the second quarter on the route, being 10,1% higher in that quarter than the rest of the year.

6.5.1 Limitations

The main limitation of my analysis of the route Bergen-Oslo is that I did not have route specific data for the price variable. Not having such data forced me to use the lag of the load factor to get a working model, so it is likely that having the price data for the route would make the results better by making the model stronger. So, the results have to be interpreted with this in mind.

7.0 Analysis 2 : Aggregate of the routes: Trondheim-Oslo, Bergen-Oslo and Stavanger-Oslo.

My initial intention with the analysis was to compare the different routes against each other, to see if there were any differences in the frequency elasticity between them. I tried to estimate the same model as used on the Bergen-Oslo route, with adjustments, using the data from the Trondheim-Oslo and Stavanger-Oslo routes, but did not get satisfactory results. There were either non significant variables, variables with the wrong coefficient, serial correlation or non-liniarities included in the functional form. My assumption was that the reason for this was that the demand on these routes could not be sufficiently explained by the national aggregated values I used for GDP, price, incair, and that the model thus became too poor to get meaningful results. To counter this, I tried to aggregate the pax and freq values for the three routes and to calculate the average aircraft size and load factor for the three routes, since I assumed that the aggregated values, covering all three largest routes in Norway, would produce a better model when combined with the national aggregated values of GDP, price, incair, and that the model thus would perform better. This turned out to be the case, so I decided to run the analysis for the aggregate data from the routes Trondheim-Oslo, Bergen-Oslo and Stavanger-Oslo. This does not enable me to compare Bergen-Oslo directly with the other routes, but should give me an opportunity to compare Bergen-Oslo to aggregate of the three routes, and thus an opportunity to see if Bergen-Oslo differs from the three routes combined.

7.1.1 Descriptive

As with the route Bergen-Oslo I will also plot the route specific variable for the aggregate of the routes Trondheim, Stavanger, Bergen to Oslo. These, are the same route specific variables as in the case of Bergen-Oslo, which is the passengers (pax) on the route, the frequency (freq), the average aircraft size, and the average load factor. The mean, standard. Dev and the change in the period can be seen in Table 5.

Variable	Mean	St. dev	Change between 2009Q1-2016Q3
pax	621660.871	67575.93162	21 %
freq	5715.16129	507.4276746	3 %
pop	106.8483313	4.016487592	12.88 %
income	290284.7419	34092.21569	5 %
gdp	763869.6129	32830.69185	5 %
price	109.2	10.90709249	8 %
incair	133.5741935	32.79233618	56 %
asiz	157.3545806	7.740989858	7 %
lf	69.0005	2.602418608	10 %

Table 11. Descriptives for the aggregate of the three routes

Having shown the descriptive for the variables, I will plot the routes specific variables, Frequency (*freq*), aircraft size (*asiz*) and load factor (*lf*), and comment on them.

Pax:

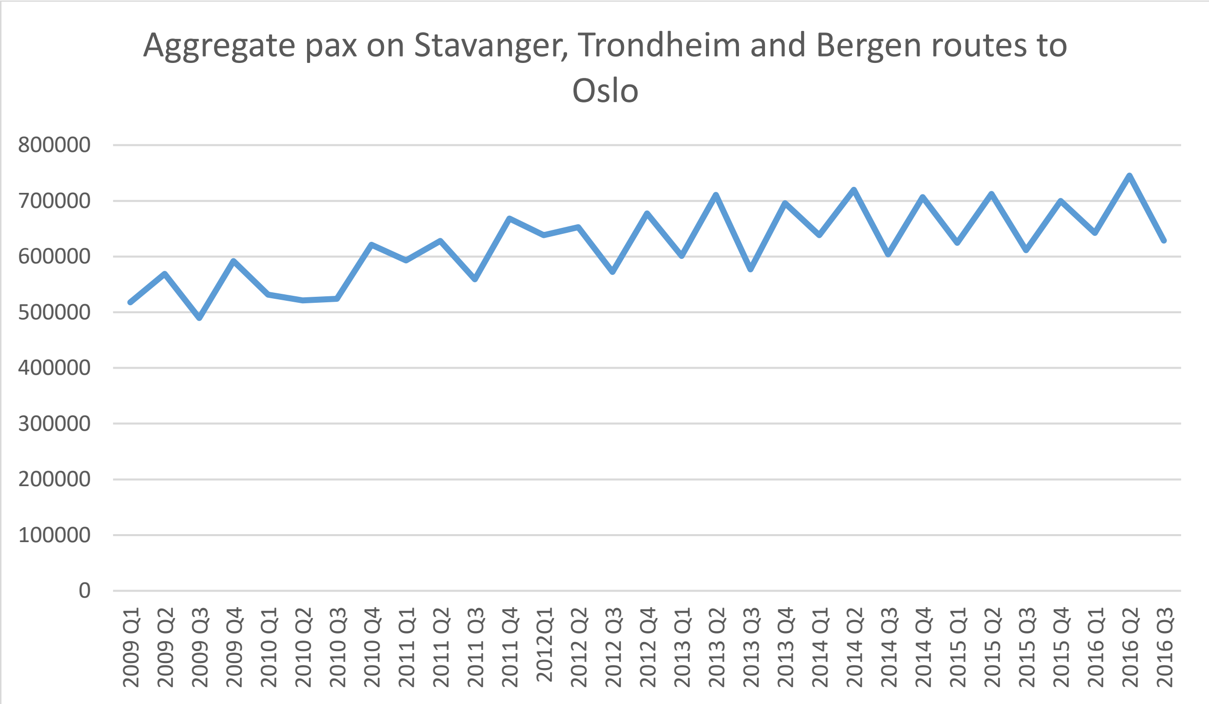


Figure 8. Plot of aggregate pax of the routes Trondheim, Stavanger and Bergen to Oslo.

The pax plotted over time of the aggregate of the three routes follow a very similar curve to the one of the Bergen-Oslo route. To investigate this further, I plotted the pax of the Bergen-Oslo route, the aggregate pax on the Trondheim and Stavanger to Oslo routes, and compared it with the aggregate of all three routes.

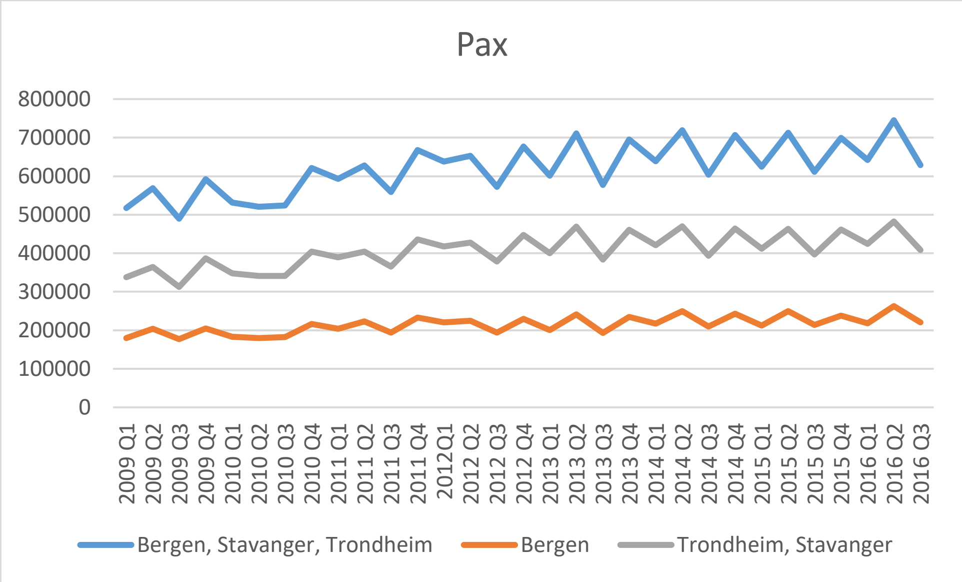


Figure 9. Plot of pax on Bergen-Oslo and aggregate pax of Trondheim, Stavanger to Oslo and aggregate pax on the routes Trondheim, Stavanger and Bergen to Oslo.

The comparison plot shows that there is a common development in the passengers on the three routes but the cycles in the aggregates are larger than in the single route Bergen-Oslo. Thus, my comments on the route Bergen-Oslo also applies to the aggregate of the routes Bergen-Oslo, Trondheim-Oslo and Stavanger-Oslo. I will thus not discuss it further, and refer the reader to my discussion on this in the descriptive of the Bergen-Oslo route analysis.

Freq:

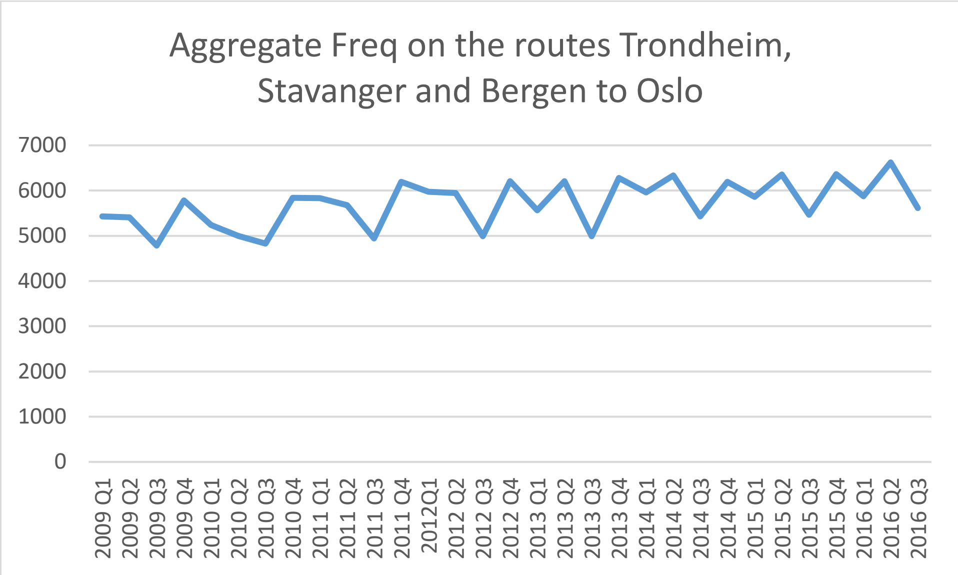


Figure 10. Plot of the aggregate freq of the routes Trondheim-Oslo, Bergen-Oslo and Stavanger-Oslo.

The plot of the frequency per quarter over time also show some similarities to the Bergen-Oslo route, so I choose to plot both in the same graph to compare them.

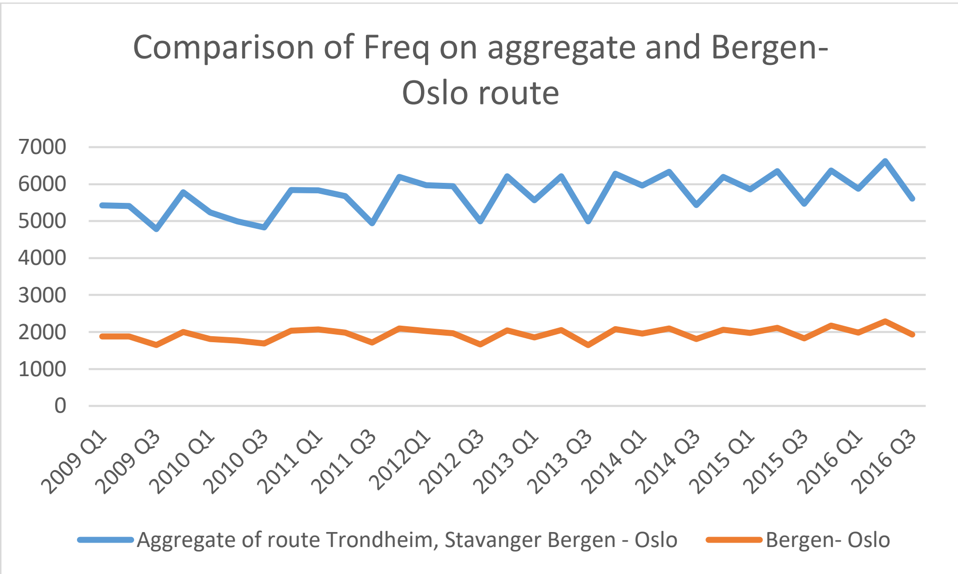


Figure 11. Comparison Plot of the aggregate of the three routes and Bergen-Oslo

The plot shows that similar to the pax plot, the aggregated frequency of the three routes follow a similar curve to the one in Bergen-Oslo, although with larger cycles. Because of this, I refer the reader to my discussion on the curve in the analysis of Bergen-Oslo for a more detailed discussion of the curve.

Load factor:

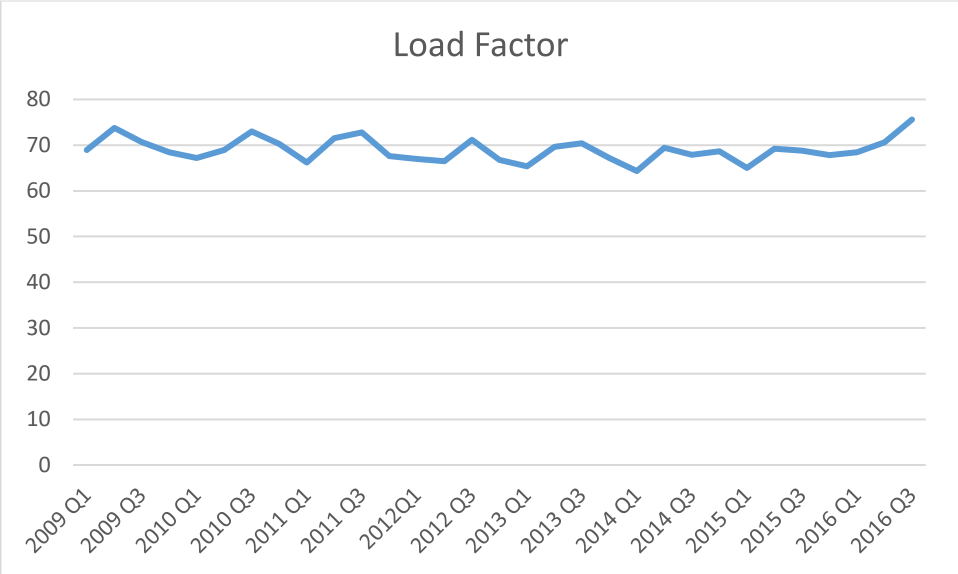


Figure 12. Average load factor of the three routes, Bergen, Stavanger, Trondheim- Oslo.

The average load factor does not follow exactly the same curve as in the case of Bergen-Oslo, but it is similar, with it being cyclical and slightly stable round 65-70, with some peaks in Q2 in 2009 and 2012, and the third quarter in 2010, 2011, 2012 and 2013, and a slight increase in the last quarters. There is also a slight decrease in the period 2011 Q3 – 2015 Q4.

Aircraft size:

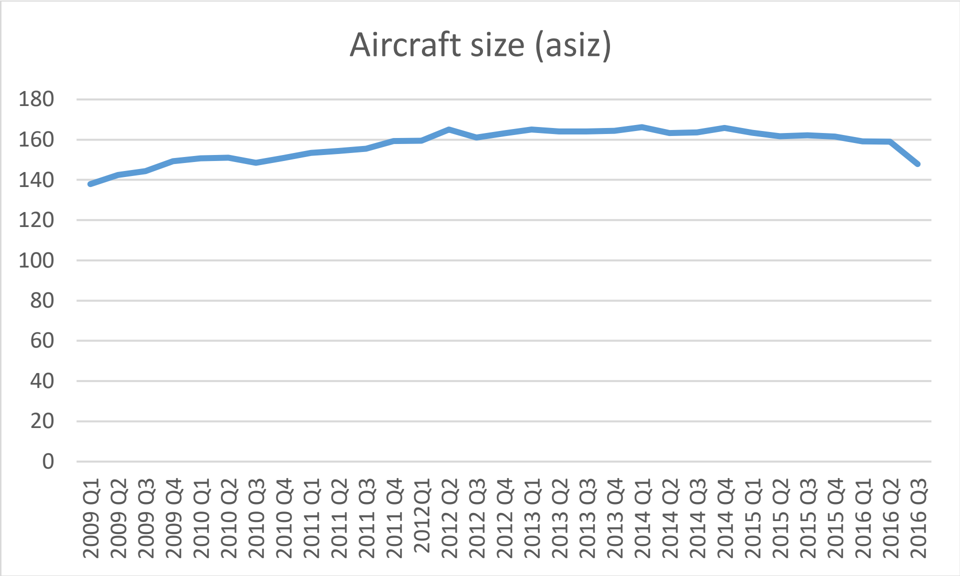


Figure 13. Average aircraft size on the three routes, Trondheim-Oslo, Stavanger-Oslo and Bergen-Oslo.

The average aircraft size on the three routes is also similar to the average aircraft size on the route Bergen-Oslo, with a slight peak in 2012 Q2, a general growth over the period, and a decrease in average aircraft size in the 2016 Q3. I investigated the dataset for the aircraft size for the routes Trondheim-Oslo and Stavanger-Oslo, and found that in the same way as with the route Bergen-Oslo, the usage of CRJ900, Fokker 100 and the 39 seat Dash 8 in this quarter seem to explain the decrease in the average aircraft size.

7.2 Model

To estimate the elasticities, I will use the basic conceptual model in log form of the variables, as this is mentioned in Fearnley & Bekken (2005) as a method to do this. Given the variables I was able to collect data for from the conceptual model, the complete model in log form the complete model can be stated as:

$$lnDemand = \beta_0 + \beta_1 lnPopulation + \beta_2 lnIncome + \beta_3 lnGDP + \beta_4 lnFare + \beta_5 + lnFrequency + \beta_6 lnAircraft\ size + \beta_7 lnLoad\ factor + \varepsilon_t$$

7.2.1 Tests of stationarity

Before building the model, I will first test the data for stationarity. The reason or this is as mentioned in the Methodology, is because of the challenges with estimating a model with non-stationary variables. Thus, it is important to test if the variables have to be differenced or not to correct for this non-stationary or not.

To do this I will use the Dikcy Fuller tests, which checks for a H0 of a unit root. For more detail on the test see Bjørnland & Thorsrud(2015), Wooldridge (2015) and Stata (n.d.a.). With the test, I use the option *trend*, for variables that seem to be trending such as ln pax, ln pop, ln inc, ln gdp, ln price, ln incair and ln freq.

The results from the test can be seen in Table 6, where I(x) denotes the number of time the data had to be differenced to become stationary, with a I(0) being a process without a unit root and a I(1) being a process that has a unit root, but which is differenced stationary after taking first differences. (Wooldridge (2015)) As noted earlier in the conceptual model, the variables are in log form, as this helps in finding the elasticity.

ln pax	I(0)
ln pop	I(1)
ln inc	I(1)
ln gdp	I(0)
ln price	I(1)
ln incair	I(1)
ln freq	I(0)
ln lf	I(0)
ln asiz	I(1)

Table 12. Dicky Fuller test of the variables for the aggregate of the three routes

7.2.2 Model Building

Since the stationary test shows that the dependent variable follows a I(0) process, the cointegration analysis such as a ECM or ADL can also not be run, as such a cointegration analysis requires that the dependent variable is I(1), as mentioned in Fearnley & Bekken

(2005). Since some of the repressors follow a I(1) process, the model do however need be first differenced as mentioned in the methodology.
Given this, I will build the model for the aggregate of the routes Trondheim-Oslo, Stavanger-Oslo, Bergen – Oslo, using the same approach as when I developed the model for the Bergen-Oslo route, which is to first develop a model to be estimated by 2SLS and then compare the results with the same model estimated by OLS. For a more discussion of why I choose to do this, see the analysis of the Bergen-Oslo route. I also use the log form and first difference of the variable in the model, as according to Wooldridge (2015) this both makes the variables that follow a I(1) process weakly dependent, removes most of the serial correlation and enables the interpretation of the variables as elasticities.

7.2.2.1 Model building for 2SLS

Because of similar challenges with the load factor as with the Bergen-Oslo route, I choose to also use the lag of the load factor in the model for the aggregate of the three routes, and to exclude the load factor variable itself. Having done this, I start by estimating the complete model with quarterly dummies to take seasonality into account. Because of collinearity between the Pop and the Inc variable, I start by excluding the Inc variable. Of the three quarterly dummies, only the dummy for the second quarter is significant, so I exclude the two others. Having done this, I also exclude the pop and aircraft size variable they were not significant. This leaves *lngdp*, *lnprice*, lag of the load factor (*lnlf1*), frequency (*lnfreq*) and the dummy for the second quarter in the model. I also test whether including the lag of the dependent variable leads to a better model, and find that the lag of demand (*lnpax1*) is both significant and makes the model stronger. Lastly, since there was a downturn in the descriptive of the demand on the route in Quarter 1 2010 – Quarter 3 2010, I include a dummy variable for this period to see if it has a significant effect on demand, but found it to be not significant. I thus choose not to include it.

Thus, I include pax1. This process left with me 6 variables in the model, which satisfies the requirement of having 5 observations per variable as mentioned in Muller(2015).

Thus, the final model can be specified as:

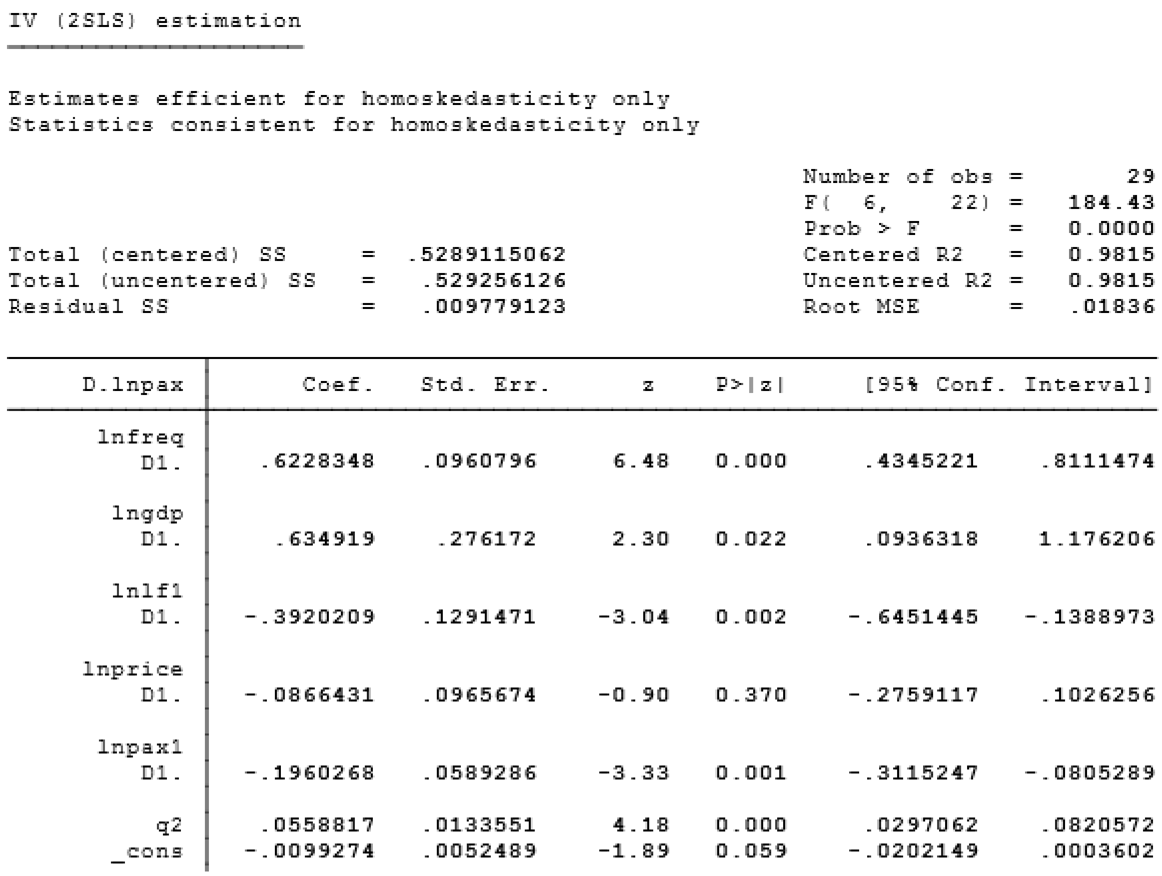
$$\Delta lnpax = B_0 + B_1\,\Delta lnfreq + B_2\,\Delta lngdp + B_3\,\Delta lnlft1 + B_4\,\Delta lnprice + B_5\,\Delta lnpax1 + q2 + \varepsilon_t$$

Since the frequency (*lnfreq*) has to be estimated using the instrumental variable (*lnincair*) in the first stage of the 2SLS, the reduced from of l*nfreq* can be stated as:

$$\Delta lnfreq = B_0 + B_1\,\Delta lngdp + B_2\,\Delta lnlf1 + B_3\,\Delta incair + B_4\,\Delta lnprice + B_5\,\Delta lnpax1 + q2 + \varepsilon_t$$

7.3 Model estimation by 2SLS

Having built the model, I estimate it by 2SLS.



IV (2SLS) estimation

Estimates efficient for homoskedasticity only
Statistics consistent for homoskedasticity only

Total (centered) SS	= .5289115062	Number of obs =	29
Total (uncentered) SS	= .529256126	F(6, 22) =	184.43
Residual SS	= .009779123	Prob > F =	0.0000
		Centered R2 =	0.9815
		Uncentered R2 =	0.9815
		Root MSE =	.01836

D.lnpax	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lnfreq D1.	.6228348	.0960796	6.48	0.000	.4345221	.8111474
lngdp D1.	.634919	.276172	2.30	0.022	.0936318	1.176206
lnlf1 D1.	-.3920209	.1291471	-3.04	0.002	-.6451445	-.1388973
lnprice D1.	-.0866431	.0965674	-0.90	0.370	-.2759117	.1026256
lnpax1 D1.	-.1960268	.0589286	-3.33	0.001	-.3115247	-.0805289
q2	.0558817	.0133551	4.18	0.000	.0297062	.0820572
_cons	-.0099274	.0052489	-1.89	0.059	-.0202149	.0003602

Figure 14. Model estimated by 2SLS for aggregate of routes

As can be seen from the estimates, there are 5 significant variables, the Frequency (*lnfreq),* GDP(*lngdp*), lag of the load factor (*lnlf1),* the lag of the pax (*lnpax1*), and the quarterly dummy for the second quarter *q2.*

lnfreq, which indicates the frequency elasticity has a significant elasticity of 0,62. This means that 10% increase in frequency will lead to a 6,2% increase in demand, all else equal. Thus, there seems that increased number of departures on the three routes, which represent a large part of Norwegian air market, will lead to a reduced delay cost for passengers, which in turn will induce demand. However, since the elasticity is inelastic, by being less than 1, there will be diminishing returns to an increase in frequency on the route. The literature I reviewed on Frequency elasticities also found inelastic frequency elasticities, with frequency elasticities often being in the range 0,65-0,95. Pels & Nijkamp and Rietveld (2001), also argues that the frequency elasticity should be less than 1. Thus, the estimates of the frequency elasticity seem to be similar to what is found and argued in the literature. I should however be pointed out that none of the literature reviewed covered the Norwegian air market, so it is interesting that the estimates for the selected Norwegian routes so closely match the results in the reviewed literature. As I discussed in the analysis of the route Bergen-Oslo, it is also interesting that the Frequency elasticity of the aggregate of the three routes is close to the lowest estimate in the literature, something that might indicate some form of maturity. But, the lag of demand may play a part here, as one of the papers that had a similar Frequency elasticity coefficient, the paper Wang et al (2014) which had a Frequency elasticity of 0,679, also had a lag of demand in the model. According to the theory on price elasticities, Fearnley & Bekken (2005) the inclusion of a lag of demand means that the estimated elasticity is a short-term elasticity, thus this might explain why the elasticity is low in my results and the paper by Wang et al (2014) when compared to the other papers who do not have a lag of the demand, and thus do not estimate either the long term or short term elasticity. Having the lag of the demand included also enables me to calculate the long-term Frequency elasticity, using the equation in Fearnley & Bekken (2005) which is the
“$(Coeficient\ for\ Frequency\ elasticity/(1 - Coeficient\ for\ lag\ of\ demand\ )$". Using this I calculate a long-term Frequency elasticity of 0.52 for the aggregate of the three routes. It is interesting that the long Frequency elasticity is lower than the short-term elasticity. As there is no discussion on the long-term Frequency elasticity in the literature I reviewed, I draw on the discussion of the long-term elasticity in Brons et al (2002), where

they argue that a smaller long-term elasticity might be a sign of a chaotic response in the short-term and then a more reasonable response in the long-term. Thus, it seems that this might be the case for the Frequency elasticity on these three routes. Since the Frequency elasticity was lower in the long term, I also find it unlikely that the inclusion of the lag of the demand is the reason for the low Frequency elasticity for the three routes, which leaves the possible maturity of the Frequency elasticity for these three routes compared to the Frequency elasticity in the other papers reviewed. However, it should be pointed out that further research is needed, especially on comparable markets to Norway, to say if this is the case for sure.

The next significant variable is the *lngdp*, which measures the GDP elasticity. Since some of the papers I revived measured Income elasticity by using the GDP, I will compare the estimates of the *lngdp* with the other income elasticities reviewed. The variable itself show that when GDP increased by 10%, demand will increase by 6,3%. This shows that demand for air travel follows the business cycle of the GDP to a certain degree, which fits with what is findings in Lian et al (2005) since they show that the air travel demand in Norway follows the business cycle. The Income elasticities mentioned in the papers reviewed previously have a larger range than the frequency elasticities reviewed, and ranges from as low as 0,30 to 2,3. However the model estimated is a dynamic model, since it includes the lag of the demand, so it would be good to compare to a paper with a similar model. Once such paper is the paper Gallet & Docucouliagos (2014), which finds an income elasticity of 0,633 for a dynamic model for a domestic market. Thus, the estimates of *lngdp* seems to be quite close to the estimate in this paper. Theory I reviewed on income elasticities, such as Graham (2000) also argues that an inelastic income elasticity, which is when the income elasticity is less than 1, indicates a mature air market. An income elasticity of 0,633 thus appears to show that the routes Bergen-Oslo, Stavanger-Oslo and Trondheim-Oslo is a mature market when aggregated. This is not unreasonable considering Norway is a country with a large degree of air travel.

Another significant variable is the lag of the load factor, which has an elasticity of -0,39, being significant at the 5% significance level. Theory on the load factor I reviewed such as Ippolito (1981), as mentioned in the analysis of the route Bergen-Oslo, argues that a reduced load factor would lead to a higher chance of getting a seat on a flight, thus possibly reducing the waiting time between the desired flight that was full and the next

flight, in addition to reducing the discomfort of a cramped plane. Thus, a negative coefficient should be expected for the load factor elasticity. That the lag of the load factor is significant indicates that there may be some historical effects of bad experiences with a high load factor, and historical good experienced with a low load factor in the previous quarter that drives up demand. The best would be to also estimate the load factor in the current quarter also, but because of endogeneity and linearity constraints mentioned in the analysis of the route Bergen-Oslo, this was not possible to include in the model. Because of this, and since I could not find any previous papers that used the lag of the load factor in the model, I have some concerns with comparing my estimates with estimates of the un-lagged load factor in the papers I have reviewed. One such paper, by Ippolito (1981) do find a load factor elasticity of -0,854. So, there are some support in the literature reviewed for the load factor having a negative coefficient, although as discussed above, caution should be taken when directly comparing the estimates of the load factor elasticity with the estimates of the elasticity of the lag of the load factor.
The next variable, *lnprice* which is the price elasticity is not significant, and has a very low coefficient of -0,086. Previous papers reviewed have a much higher price elasticity than this, but these are either papers that are from the 70-80s, or papers that do not cover the Norwegian market. Since the literature reviewed on the price elasticity supports that countries with large distances, geographical features such as mountains and a lack of substitutes have lower price elasticity, such a low elasticity for the three air routes aggregated does not seem unreasonable. Still, since the variable is not significant and because of the lack of route specific data on the price, it is hard to say whether it is in fact so low or not.

The next variable, the lag of the demand, *lnpax1*, shows that a reduction of air travel demand in the previous quarter of 10% leads to an increase in demand of 1,9% in the current quarter. Why this is the case is hard to say without further investigation, but since Keele & Kelly (2006) states that a lag of the dependent variable captures historical effects on the route, the variable shows that there are some historical effects that impact on the current demand.

Lastly, the significant dummy variable for the second quarter *q2*, of 0,058, shows that air travel demand is 5,8% higher in the second quarter than the rest of the year. As discussed

in the analysis above, this seems reasonable as the second quarter is a month with many vacations and holydays.
Thus, the model seems to fit the literature reviwed on the demand of air transport and frequency and load factor elasticities.
Having estimated the model by 2SLS and commented on the results, I will run a diagnostic test of the model estimated by 2SLS.

7.3.1 Model diagnostics

Since I have discussed what the tests do and why they are done above in the previous analysis , I will only report the results here.

First I start by testing whether the instrument used for frequency, *incair* is a strong instrument or not. I find that the F statistic of the weak identification test is 14,231, which is larger than 10. The instrument can thus be considered strong.

To test whether the instrument, *incair*, is correlated with the error terms in the equation, I run the 2SLS regression again with *lnasiz* included as an independent variable. Since *lnasiz* is not significant, *incair* should not be correlated with the error term. Thus, the instrument can be considered strong and not endogenous.
Next I test for serial correlation, by running the Cumby-Huizinga test. I fail to reject the H0 of no serial correlation, thus the model does not appear to have serial correlation.
Next, I test for heteroskedasticity in the model by running the Pagan-Hall test with a H0 of the model having homoskedasticity. I fail to reject the H0, thus the model does not appear to have heteroskedasticity.

I then run the Ramsey RESET test, to see whether there are nonlinearities in the functional form. I fail to reject the H0 of no nonlinearities in the functional form, thus the model appears to have the correct functional form.

Lastly, I predict the residuals of the regression, and run the Shapiro-Wilk test of normality on them. I fail to reject the H0 of normality, thus the residuals appear to be normal distributed.

Thus, the model not no appear to have heteroskedasticity, serial correlation, non-normal residuals, and it has the right functional form. Thus, the regression should not be unbiased. There are however three extra test that need to be run because $lnpax$ is a I(0) process and the some of the repressors are I(1) processes, leading to a unbalanced regression as mentioned in Kopsch (2012). Kopsch (2012) mentions in his paper, that the requirement of a unbalanced regression being well preforming is that the fitted values of $\Delta lnpax$ is I(0) process, that the error term follows a I(0) process, and that the variance ration between the fitted and observed variables of $\Delta lnpax$ is not significantly different from 1.

To test tthis I start by running the Dicky Fuller on the fitted values of $\Delta lnpax$, And find that I fail to reject the H0 that it follows a I(0) process.
I then run the Dicky Fuller test on the residuals of the regression, and fail to reject the H0 that the residuals follow a I(0) process.
I also run the variance ratio test, and find that the H0 that it is 1 fails to be rejected. Thus according to Kopsch (2012), the model should be well preforming, even though the regression is unbalanced. Kopsch (2012) also states that more than two repressors need to be I(1) processes, which is the case with the model.

Lastly, I test whether the variable *lnfreq* is endogenous or not. To do this, I run the Durbin-Hausman and Durbin-Wu-Hausman tests, which has as their H0 that the variable is exogenous. I fail to reject the H0 in both tests, and *lnfreq* thus seems to be exogenous. Since the tests indicate that *lnfreq* is exogenous, the 2SLS should be a less efficient regression method than OLS. Because of this, I will also run the OLS regression of the model estimated by 2SLS, and then compare the results. According to Wooldridge (2015) the results should be similar with both 2SLS and OLS if *lnfreq* is exogenous, but 2SLS will be less efficient because of higher variances.

A summary of the diagnostic test with their p values can be found in Table 13, with the exception of the weak instrument test as no p value was reported for it:

Diagnostic tests	p- value
Cumby-Huizinga test	0.7381
Pagan-Hall test	0.5225
Ramsey/Pesaran-Taylor RESET test	0.4167
Shapiro-Wilk test for residuals	0.05712
Wu hausman F test	0.28364
Durbin-Wu-Hausman	0.38647

Dicky-Fuller test:	p- value
Residuals	0,0000
Fitted	0,0000
Fitted, trend	0,0000

Variance ratio test:	p- value
ratio != 1	0.8127
ratio < 1	0.4063
ratio > 1	0.5937

Table 13. Diagnostic tests for the 2SLS estimate for the aggregate of the three routes

7.4 Model estimation by OLS

```
. regress D.lnpax D.lnfreq D.lngdp D.lnlf1 D.lnprice D.lnpax1 q2

      Source |       SS       df       MS              Number of obs =      29
-------------+------------------------------           F(  6,    22) =  198.73
       Model |  .519329783     6  .086554964           Prob > F      =  0.0000
    Residual |  .009581723    22  .000435533           R-squared     =  0.9819
-------------+------------------------------           Adj R-squared =  0.9769
       Total |  .528911506    28  .018889697           Root MSE      =  .02087

------------------------------------------------------------------------------
     D.lnpax |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      lnfreq |
         D1. |   .6689063   .0684337     9.77   0.000     .5269835    .8108291
             |
       lngdp |
         D1. |   .5303812   .2474585     2.14   0.043     .0171837    1.043579
             |
       lnlf1 |
         D1. |  -.3466568   .1205114    -2.88   0.009    -.5965821   -.0967315
             |
     lnprice |
         D1. |   -.083299   .1095725    -0.76   0.455    -.3105384    .1439404
             |
      lnpax1 |
         D1. |  -.1802789   .0603259    -2.99   0.007    -.3053872   -.0551706
             |
          q2 |    .054867   .0150616     3.64   0.001     .0236312    .0861028
       _cons |  -.0096301   .0059399    -1.62   0.119    -.0219487    .0026884
------------------------------------------------------------------------------
```

Figure 15. Estimate of the model using OLS for the aggregate of the routes Bergen, Stavanger and Trondheim – Oslo.

The coefficients in the model estimated by OLS are very similar to when the model was estimated by 2SLS, and the signs are the same. I will thus not discuss whether the coefficients fit the theory, as the same comments that applied to the estimate using 2SLS also applies to the estimate using OLS. The coefficients thus have reasonable values and signs when compared to previous research and theory. The model does however seem to have a good fit to the data, with a R^2 of 0,9819.

Given the coefficients the long-term Frequency elasticity can also be calculated based on the coefficients of the *lnfreq* and *lnpax1* as discussed in the 2SLS analysis. The long-term Frequency elasticity is thus, 0.5667. This is also similar to the long-term Frequency elasticity found for the 2SLS analysis, so I refer the reader to that analysis for a discussion of this result.

As the remaining discussions of the results are very similar as in the 2SLS analysis, I will go directly to the diagnostics of the model estimated by OLS, before comparing the coefficients from the model estimated by 2SLS and OLS.

7.4.1 Model diagnostics

As mentioned above in the previous analysis, there are some tests that need to be passed for OLS to be considered unbiased, namely that there is no heterogeneity, that there is no serial correlation and that the functional form is correctly specified. I will also test whether the residuals are normally distiributed. I addition, as mentioned in Kopsch (2012), since the regression is unbalanced, there is a need to test whether the fitted values of the dependent variable follows a I(0) process, that the residuals of the regression follows a I(0) process, and that the variance ratio between the fitted and observed values of the dependent variable is not significantly different than 1.

To run these tests, I first start by checking for serial correlation. As mentioned above, since the model includes a lagged dependent variable, it is also recommended to run the Breusch-Godfrey LM test for serial correlation in addition to durbin's alternative test.
I run both tests, which has as their H0 that there is no serial correlation, and fail to reject the H0 of no serial correlation with both tests.

Next I run the White's test and the Breusch-Pagan / Cook-Weisberg for heterogeneity, which has as their H0 that there is no heterogeneity. I fail to reject the H0 with both tests, thus here does not appear to be any heterogeneity in the model.

I then run the Ramsey RESET test, which has as its H0 that there are no nonlinearities in the functional form. And I fail to reject the H0. The functional form of the model thus appears to be correct.

Next, I predict the residuals and run the Shapiro-Wilk test on them. The test has as its H0 that the variable is normally distributed, and I fail to reject the H0. Thus the residuals appear to be normally distributed.

As mentioned above, there are also three tests that need to be run because the regression is unbalanced, to see whether the model is well preforming even when the regression is unbalanced. First, I test the residuals with Dickey Fuller, and find that it is a I(0) process. I then test the fitted values of $\Delta lnpax$ with the Dickey Fuller test, and find them to be a I(0) process. I also run a variance ratio test between the fitted and observed values of $\Delta lnpax$, and I fail to reject the H0 that the variance ratio is 1.

Thus, the model estimated by OLS can be considered unbiased and well preforming.

A summary of the diagnostic test can be found in Table 14.

Diagnostic tests	p- value
Durbin's alternative test	0.4185
Breusch-Godfrey LM test	0.3492
Breusch-Pagan / Cook-Weisberg test	0.9864
White's test	0.3425
Shapiro-Wilk test for residuals	0.36335
Ramsey RESET test	0.3419

Dicky-Fuller test:	p- value
Residuals	0,0000
Fitted	0,0000
Fitted, trend	0,0000

Variance ratio test:	p- value
ratio != 1	0.8347
ratio < 1	0.4173
ratio > 1	0.5827

Table 14. Diagnostic tests for the OLS estimate for the aggregate of the three routes

7.5 Comparison and Findings

Having estimated the model by both 2SLS and OLS, I will compare the results, discuss whether frequency can be assumed to be exogenous, and summarize the findings.

	Variables	2SLS	Sig.	OLS	Sig.
Δ	lnfreq	0.623	***	0.669	***
Δ	lngdp	0.635	**	0.530	**
Δ	lnlf1	-0.392	***	-0.347	***
Δ	lnprice	-0.087		-0.083	
Δ	lnpax1	-0.196	***	-0.180	***
	q2	0.056	***	0.055	***
	F	184,43		198,73	
	R^2	0.9815		0.9819	

Table 15. Comparison between 2SLS and OLS estimates for the aggregate of the routes.

	2SLS	OLS
Long-term Frequency elasticity	0.521	0.567

Table 16. Long-Run elasticities for the aggregate of the three routes, from 2SLS and OLS

Looking at the results, it seems that the estimates from the 2SLS and the OLS are quite similar. Since the theory on instrumental variables, as mentioned in Woodelridge (2015) states that if the presumed endogenous variable is in fact exogenous, then the two methods will give similar results, although 2SLS will be less efficient, this means that the results point to the fact that frequency (*lnfreq*) is in fact exogenous in the equation. The failure to reject the exogeneity of the frequency *(lnfreq*) in the diagnostics of the regression with 2SLS also points towards this. I will thus base my discussion of the results on OLS, since it appears that frequency is exogenous in the equation, and OLS will be more efficient than 2SLS when this is the case.

However, given all the literature on the endogeneity of the frequency, I am still cautious about rejecting that frequency is endogenous outright, since a failure to reject the exogeneity is not a proof of exogeneity. So I recommend future research to investigate the question of endogeneity of the Frequency on the three routes in more depth.

So, based on the OLS estimates, what are the findings from estimating the model for the aggregate of the routes Bergen-Oslo, Trondheim-Oslo and Stavanger-Oslo. Firstly, the results show that the short -term frequency elasticity on the routes is 0,67. This means that when the operators on one of these routes increases the frequency by 10%, they can expect to have an increase in demand by 6,7% on average. The coefficient for the frequency elasticity is also the highest of the coefficients in the model, which shows that travelers on these routes value an increase in frequency, and by extension a reduced schedule delay, most when deciding to travel by air or not. However, since the frequency elasticity is inelastic, there are diminishing returns to scale for a frequency increase over time on these routes. The estimate of the long-term Frequency elasticity, which is 0.567, also indicates that the short term response to a Frequency increase is higher than the long term response to such an increase. An explanation for this could be that some people are excited by the introduction of a new flight time and thus choose to travel on this departure, but in the long term find out that they could do without it and thus go back to travel on their old departure time or not travel at all.

The results also show that the demand on the routes is affected by the GDP, with a 10% increase in the GDP leading to 5,3% increase in demand. This shows that the demand for air travel on these routes is connected with the business cycle in Norway. The coefficients for the lag of demand and the lag of the load factor also shows that historical factor have an impact on current demand, either because of a possible bad experience with not getting tickets on a flight or cramped flights, or other factors that have a historic effect on the demand on these routes. That the price elasticity is not significant and has such a low coefficient may indicate that the price does not matter to the decision to travel on these route on average. However, it is important to point out that the price variable (*lnprice*) is the national index for air fares, and that it thus may not be able to estimate the price elasticity on these routes. Because of this, it is hard to say whether the price is not significant or if the data is not route specific enough to estimate it correctly. Lastly, the significant second quarter dummy *q2* shows that the demand for air travel is expected to be 5,5% higher in the months April, May, June than the rest of the year.

7.5.1 Limitations

Same as with the route Bergen-Oslo analysis, the main limitation with my analysis was the lack of route specific price data. Not having such data put some doubts to how well the results fit the real-life price elasticities, as I assume the price has a significant effect in reality. This probably has an effect on the estimated Frequency elasticity, but it is hard to say how large this is without doing a new study with accurate price data, so the estimates can be compared.

8.0 Comparison of Analysis 1 and 2

Having estimated the same model for both the route Bergen-Oslo and the aggregate of the routes Bergen-Oslo, Trondheim-Oslo and Stavanger-Oslo, it would be good to compare the results and to discuss what the results tells about the air transport market that the routes cover.

Since the diagnostics tests for both analyses pointed to Frequency being exogenous, I will report the OLS results from both analysis's, and compare them.

		Bergen-Oslo		Aggregate	
	Variables	OLS	Sig.	OLS	Sig.
Δ	lnfreq	0.633	***	0.669	***
Δ	lngdp	0.836	**	0.53	**
Δ	lnlf1	-0.308	**	-0.347	***
Δ	lnprice	-0.122		-0.083	
Δ	lnpax1	-0.099		-0.18	***
	q2	0.101	***	0.055	***
	F	98.93		198,73	
	R^2	0.9643		0.9819	

Table 17. Comparison between the OLS estimate of the model for Bergen-Oslo and the Aggregate of the three routes.

As seen from the table the Bergen-Oslo route is very similar to the aggregate of it and the two other routes, Stavanger-Oslo and Trondheim-Oslo. From this the conclusion can be drawn that the factors that determine demand on the three routes do not differ too much with the exception of the GDP, as if this was the case I would expect the aggregate to differ somewhat from the Bergen-Oslo estimate, because if one of the routes differed enough it would probably skew the results. Thus, since this is not the case, it seems that the determinants of air travel are somewhat similar for the three routes. This is especially the case with the Frequency elasticity and the elasticity of the lag of the load factor, so it seems that the service quality factors impact the travelers on the three routes very similarly. Besides GDP, which I will comment on after, there are some variables that differ a somewhat between the aggregate and the Bergen-Oslo route, which is the impact of the second quarter dummy, the lag of the demand and the price variable. This seems to indicate that the price elasticity may be more important for the Bergen-Oslo route and that the increase of travel in the second quarter is larger for the Bergen-Oslo than the aggregate of the three routes. It is however important to remember that the price elasticity is not significant in both analyses, so any differences should be interpreted with caution. The lag of the demand variable also differs, but interestingly it is also not significant for the Bergen-Oslo route and significant for the aggregate of the three routes. This indicates that historic factors that impact the demand is more important and has a significant impact on the aggregate of the three routes, while having a low non-significant impact on the Bergen-Oslo route. Lastly, GDP has a much higher impact on air travel demand on the route Bergen-Oslo than on the aggregate demand for the three routes, which shows that travel on the Bergen-Oslo is more tied to the business cycle than the three routes are aggregated.

So, what findings can be drawn from this. Firstly, it seems that air travel on the three largest routes in Norway have a similar Frequency elasticity, and that it is between 0.63-0-67, or 6,6% on average. This shows that the increase in frequency on these routes have large impact on demand, with a 10% increase in frequency leading to a demand increase of around 6,6% on average. That the Frequency elasticity is less than 1, also indicates that there is diminishing returns to scale of frequency increases on these routes. That the value is around 0.6 also shows that the frequency elasticity of the Norwegian routes is close to previous studies on Frequency elasticity.

Secondly, the GDP if interpreted as an income elasticity, shows that the three routes are mature air markets, since the GDP elasticity is less than 1, however the Bergen-Oslo route seems to be less mature since it has a GDP elasticity of 0,84 compared to the GDP elasticity on the aggregate of the three routes of 0.53.

The load factor in the previous quarter seems to have similar effects on the Bergen-Oslo route than the aggregate of the three routes, although this difference is small. The estimates show that the load factor has an elasticity of -0,3 or -0,34, which shows that an 10% increase in load factor the last quarter leads to a 3% decrease in demand. This seems to indicate that the travelers on the three routes have a negative response to a high load factor, that transfers over to the next quarter, which can be because of discomfort or higher chance of fully booked flights according to the papers Ippolito (1981) and Calderon (1997).

Lastly, the demand on the Bergen-Oslo route is 10,1% higher during the second quarter months than the aggregate of the three routes. Why this is the case is hard to say, but it may be that the passengers on the Bergen-Oslo route travel more when on vacation than the other routes, or that they simply travel more abroad, thereby possibly needing to travel to Oslo to transfer to an international flight. Bergen is also a famous tourist destination, so this may also be a reason for this increase in demand, as tourists would possibly have to go back to Oslo after having visited Bergen, especially if they themselves come on international flights from abroad.
It is also interesting that the lag of demand is significant for the aggregate of the three routes but not for Bergen-Oslo, but it is hard to say why this is the case besides the fact that historical factors possibly play a larger part of explaining demand on the aggregate of the three routes than explaining demand on the Bergen-Oslo route.

9.0 Conclusions

By revisiting my research questions, some conclusions about the estimates can be drawn. Firstly, the estimates do show that there is a significant Frequency elasticity on the selected routes on the Norwegian air transport market, and that this Frequency elasticity is 0.63 or 0,67 for the Bergen-Oslo route and the aggregate of the three routes respectively. The difference in the Frequency elasticity between the three routes also does not seem to be big, based on the similar Frequency elasticity in the Bergen-Oslo analysis and the analysis of the three aggregate routes. This shows that the Norwegian travelers on these routes value a frequency increase because of delay costs connected to missed flights or no flight close to their desired flight time. The Frequency elasticity was also lower for the two analyses than the papers I reviewed, which might indicate that the Norwegian air transport market has a bit more maturity with regards to Frequency increases compared to the other non-Norwegian markets analyzed in the other papers, such as the US.

I also find that the long-term elasticity calculated for the aggregate of the three routes is 0,567, which indicates that the long-term elasticity is lower than the short-term elasticity for the Frequency. This could be because of chaotic responses to a frequency increase in the short term.

Secondly, there were some interesting findings besides the Frequency elasticity, namely that the lag of the load factor had an impact on demand, with an elasticity of -0,3 or -0,34 for the route Bergen-Oslo and the aggregate of the three routes respectively. This might indicate that delay costs because of a full plane or discomfort carries over to the next quarter.

Another interesting finding was that the Norwegian air market on the routes selected seemed to be mature, with a GDP(Income) elasticity of 0,84 and 0,53 for the Bergen-Oslo route and the Aggregate of the three routes respectively. That Bergen-Oslo is so much more tied to the GDP than the aggregate of the three routes is also interesting, given the coefficient it may be because Bergen-Oslo is a more immature market than the rest of the routes, but it is hard to say if this is the case or if some other factor is affecting the GDP elasticity without further studies.

It is also interesting that the price elasticity for both analyses is not significant, and quite small. But, as I will get back to in the limitations chapter, this can be because of a lack of sufficient data.

Lastly, both the analysis of the route Bergen-Oslo and the aggregate of the three routes indicated that the Frequency elasticity was in fact exogenous in the equation. This seemed like an interesting finding, as all literature I reviewed pointed to Frequency being endogenous. Still, I failure to reject a H0 of exogeneity is not the same as proving it is exogenous, so more research would have to be done on this for the Norwegian routes to test whether the Frequency is in fact exogenous or not.

9.1 Limitations

However, there are some limitations with these estimates, which is that the data on the price elasticities is not route specific enough. Thus, it is hard to say whether the price elasticity estimates are sufficiently good enough, and thus whether the price elasticity is truly not significant on the routes analyzed. The lack of route specific price data also puts the estimation of the other coefficients in the two analyses into doubt. That the Frequency elasticity is close to other papers on the Frequency elasticity is a promising sign, but it is hard to say for sure if the Frequency elasticity would change if more route specific price data were included in the analysis. Another limitation of this thesis is that the analysis only covers a few routes on the Norwegian air market, as more routes covered would be better. However, because of the lack of route specific data and time constraints, I had to focus the thesis on the three routes.

9.2 Recommendations for Future research

This bring me to my recommendation for future research, which is to run the same analysis I have done here with better route specific price data, if such data is available, for the same or more routes on the Norwegian air transport market. Another interesting idea for future research would be to run the analysis for routes in a comparable market to Norway, such as Sweden, to see if there is any difference in the Frequency elasticity. Lastly, it would also be interesting to run a route level analysis of the Frequency elasticity for the PSO network.

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Appendix1: Diagnostics 2SLS Bergen-Oslo

```
Weak identification test (Cragg-Donald Wald F statistic):                12.598
Stock-Yogo weak ID test critical values: 10% maximal IV size             16.38
                                         15% maximal IV size              8.96
                                         20% maximal IV size              6.66
                                         25% maximal IV size              5.53
Source: Stock-Yogo (2005).  Reproduced by permission.
```

```
. actest

Cumby-Huizinga test for autocorrelation
  H0: variable is MA process up to order q
  HA: serial correlation present at specified lags >q
```

H0: q=0 (serially uncorrelated) HA: s.c. present at range specified				H0: q=specified lag-1 HA: s.c. present at lag specified			
lags	chi2	df	p-val	lag	chi2	df	p-val
1 - 1	0.018	1	0.8940	1	0.018	1	0.8940

```
  Test allows predetermined regressors/instruments
  Test requires conditional homoskedasticity

. ivhettest
IV heteroskedasticity test(s) using levels of IVs only
Ho: Disturbance is homoskedastic
    Pagan-Hall general test statistic   :   4.122  Chi-sq(6)  P-value = 0.6602

. ivreset
Ramsey/Pesaran-Taylor RESET test
Test uses square of fitted value of y (X-hat*beta-hat)
Ho: E(y|X)  is linear in X
Wald test statistic:                 Chi-sq(1) =   0.00    P-value = 0.9549

.

. swilk residuals
```

Shapiro-Wilk W test for normal data

Variable	Obs	W	V	z	Prob>z
residuals	29	0.95176	1.495	0.830	0.20327

```
. dfuller fitted

Dickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -25.435            -3.730            -2.992            -2.626
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

. dfuller fitted, trend

Dickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -24.936            -4.352            -3.588            -3.233
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

. sdtest fitted == dlnpax

Variance ratio test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
  fitted |      29    .0026604    .0258394    .1391492   -.0502691    .0555899
  dlnpax |      30    .0068043    .0258761     .141729   -.0461181    .0597268
---------+--------------------------------------------------------------------
combined |      59    .0047675     .018131    .1392671   -.0315257    .0410607
------------------------------------------------------------------------------
    ratio = sd(fitted) / sd(dlnpax)                               f =   0.9639
Ho: ratio = 1                                  degrees of freedom =   28, 29

    Ha: ratio < 1               Ha: ratio != 1                 Ha: ratio > 1
  Pr(F < f) = 0.4622         2*Pr(F < f) = 0.9243           Pr(F > f) = 0.5378

.

. dfuller residuals

Dickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -5.180            -3.730            -2.992            -2.626
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000
```

```
. ivendog D.lnfreq

Tests of endogeneity of: D.lnfreq
H0: Regressor is exogenous
    Wu-Hausman F test:                 0.10722   F(1,21)     P-value = 0.74657
    Durbin-Wu-Hausman chi-sq test:     0.14732   Chi-sq(1)   P-value = 0.70111
```

Appendix 2: Diagnostics OLS Bergen-Oslo

```
. estat durbinalt
```

Durbin's alternative test for autocorrelation

lags(p)	chi2	df	Prob > chi2
1	0.652	1	0.4193

H0: no serial correlation

```
. estat bgodfrey
```

Breusch-Godfrey LM test for autocorrelation

lags(p)	chi2	df	Prob > chi2
1	0.874	1	0.3500

H0: no serial correlation

```
. estat hettest

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of D.lnpax

         chi2(1)      =     0.03
         Prob > chi2  =   0.8562

. estat imtest, white

White's test for Ho: homoskedasticity
         against Ha: unrestricted heteroskedasticity

         chi2(26)     =    28.98
         Prob > chi2  =    0.3119

Cameron & Trivedi's decomposition of IM-test
```

Source	chi2	df	p
Heteroskedasticity	28.98	26	0.3119
Skewness	9.74	6	0.1360
Kurtosis	1.94	1	0.1638
Total	40.66	33	0.1687

```
. swilk residuals

                  Shapiro-Wilk W test for normal data
```

Variable	Obs	W	V	z	Prob>z
residuals	29	0.94710	1.640	1.020	0.15380

```
. ovtest

Ramsey RESET test using powers of the fitted values of D.lnpax
       Ho:  model has no omitted variables
                 F(3, 19) =      0.18
                 Prob > F =      0.9079

.
```

```
. dfuller fitted

Dickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -24.649            -3.730            -2.992            -2.626
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

. dfuller fitted, trend

Dickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -24.142            -4.352            -3.588            -3.233
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000
```

```
. dfuller residuals

Dickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -5.571            -3.730            -2.992            -2.626
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000
```

```
. sdtest fitted == dlnpax

Variance ratio test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
  fitted |      29    .0026604    .0259558    .1397764   -.0505077    .0558285
  dlnpax |      30    .0068043    .0258761     .141729   -.0461181    .0597268
---------+--------------------------------------------------------------------
combined |      59    .0047675    .0181705      .13957   -.0316047    .0411396
------------------------------------------------------------------------------
    ratio = sd(fitted) / sd(dlnpax)                               f =   0.9726
Ho: ratio = 1                                  degrees of freedom =   28, 29

    Ha: ratio < 1               Ha: ratio != 1                 Ha: ratio > 1
  Pr(F < f) = 0.4716         2*Pr(F < f) = 0.9432           Pr(F > f) = 0.5284

.
```

Appendix3: Diagnostics 2SLS aggregate of the three routes

```
------------------------------------------------------------------------------
Weak identification test (Cragg-Donald Wald F statistic):               14.231
Stock-Yogo weak ID test critical values: 10% maximal IV size             16.38
                                         15% maximal IV size              8.96
                                         20% maximal IV size              6.66
                                         25% maximal IV size              5.53
Source: Stock-Yogo (2005).  Reproduced by permission.
------------------------------------------------------------------------------

. actest

Cumby-Huizinga test for autocorrelation
  H0: variable is MA process up to order q
  HA: serial correlation present at specified lags >q
-----------------------------------------------------------------------------
  H0: q=0 (serially uncorrelated)        |  H0: q=specified lag-1
  HA: s.c. present at range specified    |  HA: s.c. present at lag specified
-----------------------------------------+-----------------------------------
    lags   |      chi2      df     p-val | lag |      chi2      df     p-val
-----------+-----------------------------+-----+-----------------------------
   1 -  1  |      0.112      1    0.7381 |   1 |      0.112      1    0.7381
-----------------------------------------------------------------------------
  Test allows predetermined regressors/instruments
  Test requires conditional homoskedasticity


. ivhettest
IV heteroskedasticity test(s) using levels of IVs only
Ho: Disturbance is homoskedastic
    Pagan-Hall general test statistic    :   5.167  Chi-sq(6) P-value = 0.5225


. ivreset
Ramsey/Pesaran-Taylor RESET test
Test uses square of fitted value of y (X-hat*beta-hat)
Ho: E(y|X) is linear in X
Wald test statistic:             Chi-sq(1) =   0.66    P-value = 0.4167
```

```
. swilk residuals

                   Shapiro-Wilk W test for normal data

    Variable |    Obs       W           V         z       Prob>z
-------------+--------------------------------------------------
   residuals |     29    0.93063      2.150     1.579    0.05712
```

```
. dfuller fitted

Dickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -21.166            -3.730            -2.992            -2.626
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000
```

```
 dfuller fitted, trend

ickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
Z(t)             -20.881            -4.352            -3.588            -3.233
------------------------------------------------------------------------------
acKinnon approximate p-value for Z(t) = 0.0000
```

```
. dfuller residuals

Dickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -5.477            -3.730            -2.992            -2.626
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000
```

```
. sdtest fitted == dlnpax

Variance ratio test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
  fitted |      29    .0034472    .0251546    .1354615   -.0480796     .054974
  dlnpax |      30    .0068043    .0258761     .141729   -.0461181    .0597268
---------+--------------------------------------------------------------------
combined |      59    .0051542    .0179004    .1374953   -.0306772    .0409857
------------------------------------------------------------------------------
    ratio = sd(fitted) / sd(dlnpax)                               f =   0.9135
Ho: ratio = 1                                    degrees of freedom =   28, 29

    Ha: ratio < 1               Ha: ratio != 1                 Ha: ratio > 1
  Pr(F < f) = 0.4063         2*Pr(F < f) = 0.8127           Pr(F > f) = 0.5937
```

```
. ivendog D.lnfreq

Tests of endogeneity of: D.lnfreq
H0: Regressor is exogenous
    Wu-Hausman F test:                    0.28364  F(1,21)     P-value = 0.59992
    Durbin-Wu-Hausman chi-sq test:        0.38647  Chi-sq(1)   P-value = 0.53416
```

Appendix 4: Diagnostics OLS aggregate of the three routes

```
. estat durbinalt
```

Durbin's alternative test for autocorrelation

lags(p)	chi2	df	Prob > chi2
1	0.654	1	0.4185

H0: no serial correlation

```
. estat bgodfrey
```

Breusch-Godfrey LM test for autocorrelation

lags(p)	chi2	df	Prob > chi2
1	0.876	1	0.3492

H0: no serial correlation

```
. estat imtest, white

White's test for Ho: homoskedasticity
         against Ha: unrestricted heteroskedasticity

         chi2(26)      =     28.33
         Prob > chi2   =    0.3425
```

Cameron & Trivedi's decomposition of IM-test

Source	chi2	df	p
Heteroskedasticity	28.33	26	0.3425
Skewness	12.63	6	0.0494
Kurtosis	1.86	1	0.1721
Total	42.82	33	0.1177

```
. estat hettest

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of D.lnpax

         chi2(1)      =     0.00
         Prob > chi2  =   0.9864
```

```
. ovtest

Ramsey RESET test using powers of the fitted values of D.lnpax
       Ho:  model has no omitted variables
                  F(3, 19) =      1.18
                  Prob > F =      0.3419
```

```
. swilk residuals

                   Shapiro-Wilk W test for normal data

    Variable |    Obs       W           V         z       Prob>z
-------------+--------------------------------------------------
   residuals |     29    0.96178      1.185     0.350    0.36335

. predict fitted, xb
(14 missing values generated)

. dfuller residuals

Dickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -5.866            -3.730            -2.992            -2.626
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000
```

```
. dfuller fitted

Dickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -20.630            -3.730            -2.992            -2.626
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

. dfuller fitted, trend

Dickey-Fuller test for unit root                   Number of obs   =        28

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -20.324            -4.352            -3.588            -3.233
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

. sdtest fitted == dlnpax

Variance ratio test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
  fitted |      29    .0034472    .0252897    .1361892   -.0483564    .0552508
  dlnpax |      30    .0068043    .0258761     .141729   -.0461181    .0597268
---------+--------------------------------------------------------------------
combined |      59    .0051542    .0179455    .1378419   -.0307675     .041076
------------------------------------------------------------------------------
    ratio = sd(fitted) / sd(dlnpax)                               f =   0.9234
Ho: ratio = 1                                    degrees of freedom =   28, 29

    Ha: ratio < 1               Ha: ratio != 1                 Ha: ratio > 1
  Pr(F < f) = 0.4173         2*Pr(F < f) = 0.8347           Pr(F > f) = 0.5827
```