# Master's degree thesis

**LOG950 Logistics**

**Speed optimization in maritime inventory routing**

Nataliia Evsikova

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# Contents

# 1.0  Introduction

Maritime transportation is the main method of goods transportation all over the world. According to the Review of Maritime Transport (2015), seaborne trade represents around 80% of global trade by volume and more than 70% of global trade by value. The importance of maritime transportation can be proved easily. On some routes, maritime transportation can replace transportation by road and rails from the point of time, cost, accessibility, distance, speed, and other constraints. In some places, the sea plays the role of a sole link between regions where maritime transportation is very important.

Michael and Noble (2008) state that economic growth and world trade development leads to an increase in the speed of ships. Higher vessel speed was achieved by significant technological improvements such as vessel hull design, engine efficiency improvement, and others. Both supply chain management and cargo handling systems made a huge contribution in increasing the delivery speed (Michael and Noble, 2008).

Speed is a flexible parameter, which can be used to optimize routes and schedules of delivery, decrease inventories in ports, and manage cost, profit, and emissions. Transportation process can be improved by applying proper speed management. First, cargo transportation by sea is connected with high level of uncertainty, for example, sailing times can be affected by weather conditions. In real life, unpredictable delays may occur and as a result deviations from the initial plan may be experienced. Such deviations can be compensated by higher speed if necessary.

Secondly, each company requires individual inventory policy. It depends on many features, which need to be taken into consideration such as storage capacity of the customer, the type of the transported product, whether the customer is ready to wait to get the product or not, and several others. In such a case inventory can be considered as a buffer when uncertainties appear. More than that, inventories have a strategic meaning, because maritime routing deals with long distances and as a result needs long term planning. Speed is required to make inventory policy more flexible and acts as windbreaker against unpredictable situations.

Thirdly, high vessel speed brings an increase in fuel consumption and air emission. Today, to be economically efficient vessels must be environment-friendly, which means less emission. Thus speed brings in new economic and environmental issues. The increase of fuel consumption leads to an increase in total cost of cargo deliveries as well as an

increase in air emission. As humanity is now concerned about the environment and air pollution such problems are now more significant than ever. According to Psaraftis and Kontovas (2013), at slower speed vessels emit much less compared to emissions observed at high speed, due to the non-linear dependence between fuel consumption and speed. In other words, higher vessel speed is not always the best solution when it comes to the optimization of delivery costs or amount of emission. Speed flexibility can help to find a trade-off between air emission and transportation cost.

Given all the features of speed in maritime transportation, Maritime Inventory Routing Problem (MIRP) with speed optimization is studied in this thesis. Maritime Inventory Routing Problem (MIRP) includes coordination between goods delivered to customers and the inventories. This research considers the influence of speed on the model performance, transportation costs and emission level with respect to inventories, the storage capacity of vessels and ports, and vessel speed.

The main objective of this thesis is to introduce speed as a key variable into the existing optimization tool to obtain better results. Speed optimization tool can be applied to sea transportation problems to minimize transportation cost and emission amount.

This research examines speed as a key variable in maritime transportation of goods, and consequently, speed optimization. Vessel speed is significant as it regulates transportation costs, fuel consumption, and air emission. Inventory is considered in terms of routing, thus customer stock level should also be taken into consideration. Each port has a finite storage capacity. When the speed of vessels is higher the deliveries can be more frequent and inventory level can be decreased. Inventory storage at a customer plays the role of a buffer and is a tool to hedge against various uncertainties such as demand, prices for fuel and goods, weather conditions.

This thesis considers three different models. In the first model, speed is fixed and given for each vessel. In the second model, fixed routes are introduced along with speed optimization. The optimization model chooses the most appropriate speed for each vessel which can vary between sailing legs. In the third model routes and the speed are optimized at the same time. The goal is to examine how these changes influence total expenses and the emissions. One of the main assumptions of this model is the non-linear dependence between fuel consumption and speed. According to Gkonis and Psaraftis (2012), fuel consumption and speed have non-linear dependence. Further analysis considers how these types of changes influence the system performance.

This thesis is structured as follows: section 2 is dedicated to literature relevant to this research topic and provides methodologies applied for this problem. Section 3 describes the problem itself and which initial assumptions are included into the model. Section 4 includes the description of the mathematical model with relevant definitions. Section 5 represents the experiments and computational evaluation of the presented models. Conclusions are described in section 6.

# 2.0 Literature review

The problem considered in this thesis can be described as a maritime inventory routing problem with speed optimization. Several research papers related to Maritime Inventory Routing Problems (MIRP) and speed optimization from the point of routing and emission minimization are relevant for this research.

Some research papers have been used as a source of primary data obtained from the authors. Another research paper served as a source of fuel consumption function for the project. Articles, describing modeling approaches and various models for MIRP are also relevant for this research.

## 2.1 Maritime inventory routing problems

First studies of IRP were variations of models created for vehicle routing problem and heuristics, where inventory costs were taken into consideration (Coelho, Cordeau, and Laporte, 2014).

According to Coelho, Cordeau, and Laporte (2014), the inventory routing problem is an integration of inventory management, vehicle routing, and delivery scheduling decisions. In a paper of Bell et al. (1983), published more than 30 years ago, the IRP appeared as a part of vendor-managed inventory (VMI). VMI is a business practice and its objective is cost reduction of logistics and business value addition. This is a situation where both vendors and buyers receive some benefits.

Bertazzi and Speranza (2012) studied the inventory routing problems (IRPs). In this research paper, instances, classification of IRP characteristics, different models, and customer service policies were presented. This problem involves serving customers from suppliers with capacitated vehicles and direct shipping in terms of IRP.

Agra, Christiansen, and Delgado (2012) examined a fuel oil distribution problem. Their research considers company coordinate routing and scheduling of vessels between ports in such a way that the demand for oil products is met over the planning horizon. Multiple time windows took place and inventories were considered only on the demand side. In this study, ships spend significant time in ports. That is why time in ports is modeled in detail. A mathematical model designed in this paper incorporated continuous and discrete time horizon. Several strategies were considered to improve this model. They were extended formulations, tightening bounds, and valid inequalities. The combination of these strategies gave an optimal solution in a reasonable timeframe.

Agra, Christiansen, and Delgado (2016) designed two alternative mixed integer formulations: discrete and continuous time formulations. Various extended formulations and inequalities are considered to allow linear gap reduction between two initial models. Experimental part compares models in terms of their size, and computational time based on real instances.

Agra et al. (2016a) studied a maritime inventory routing problem (MIRP) with single-product, production and consumption rates, which changes over the planning horizon. A heterogeneous fleet of vessels, production and consumption ports with given storage capacity are included into the problem.

De et al. (2017) also studied maritime inventory routing problem. The goal is to satisfy the demand in the number of ports over the planning horizon. The possibility of slow steaming policy integration is considered. The model in the research paper can be classified as mixed integer non-linear model including different capacity, loading/unloading, scheduling and routing constraints. Non-linearity of the model was based on the dependence between fuel consumption and vessel speed. Time windows and penalty costs for arrival delay were also included into the model. To solve this problem, heuristic commonly known as Particle Swarm Optimization for Composite Particle (PSO-CP) was applied.

Agra et al. (2015) discussed stochastic shipping problems. The company, which is considered in this paper, is responsible for product deliveries and inventory management. Routing and scheduling at sea are connected with unpredictability in weather conditions and imprecise waiting time at ports. In this paper, two-stage stochastic programming model was applied. The first stage includes routing, loading, and unloading and the second stage involves scheduling and inventories. Decomposition approach is used to solve the problem.

Jiang and Grossmann (2015) examined maritime inventory routing problem with a single product to study continuous and discrete time models. In this paper two models are presented: continuous time model where time slots were increased by changing time constraints and discrete time model. A computational study was conducted based on randomly generated instances for efficiency comparison.

## 2.2 Speed optimization problems

Panamarenka (2011) studied speed optimization from the point of emission minimization for the vessels' periodic supply planning problem. In this research paper, a

large heuristic neighborhood search with speed optimization was presented. The project showed that lower speed always means less fuel cost and emission. Total cost reduction from speed decrease depends on other types of expenses and from the algorithms and initial data. Computational experiments showed efficiency and applicability for larger instances.

Psaraftis and Kontovas (2014) considered issues regarding speed optimization and designed optimization models for a set of scenarios in a single ship setting. The work is supposed to incorporate those parameters. The paper gives examples to demonstrate the optimal solution and the possible trade-offs.

Psaraftis and Kontovas (2013) examined different speed models with speed as a decision variable. The speed is considered from the point of the fuel costs as major determinant, which is important in fleet sizing, ship sizing, and inventory costs.

Andersson, Fagerholt, and Hobbesland (2015) applied special approach for planning vessel routes that consist of two parts. First, it is assumed that vessels sail with fixed speed and later speed was optimized along the routes. Andersson, Fagerholt, and Hobbesland (2015) proposed a new modeling approach for integration of vessel speed when planning routes for vessels. Better solutions were achieved when speed optimization was incorporated into the routes' planning.

Norstad, Fagerholt and Laporte (2011) state that most the models for scheduling and routing problems use fixed speed and fixed amount of fuel consumed by each speed. Such an approach has nothing in common with real life problems. In such situations, vessel speed can vary inside this particular time window. The cubic function of speed can be used for fuel consumption approximation. This work presents speed optimization, where speed is defined as a decision variable on each arc. To solve this problem a multi-start local search heuristic is applied.

Wang and Meng (2012) considered a mixed-integer nonlinear model. In this paper, they studied the relationship between bunker consumption and sailing speed of container ships. Transshipment and container routing is considered in this work. The result of the research is optimal speed for each ship and each route in the network. According to Wang and Meng (2012), an efficient method was proposed depending on fuel consumption function which has properties such as convexity and non-negativity. The algorithm can be applied to real problems of a shipping company which was the object of research.

Fagerholt et al. (2015) studied the problem of cost minimization along with emissions. According to the paper, there are some limits on the maximum amount of

sulphur imposed in emission control areas. To satisfy such requirements ship operators should switch to another fuel which is more expensive but contains less sulphur. What now should be determined is speed and routes. Fagerholt et al. (2015) designed an optimization tool to determine routes to travel and optimum speed to minimize expenses. Experimental part is also presented for deeper evaluation about how does speed, routes, fuel consumption and costs are influencing ECA's establishment. Environmental impact is also studied. Computational results show that ECAs can be avoided by increasing traveling distances or ship operators can sail with higher speed outside ECAs and with lower speed inside these areas.

Wena et al. (2017) proposed routing and speed optimization problem, where time, expenses and environmental aspect are taken into consideration. To solve this problem a branch and price algorithm and a constraint programming model were designed. The Paper considers several objective functions which are trip duration, cost, and emissions minimization. Experimental part based on different scenarios are presented.

Aydin, Lee, and Mansouri (2017) consider speed optimization problem where stochastic time at ports and time windows are implemented. The purpose is fuel consumption minimization by schedule designing. In this paper, dynamic programming model is created along with deterministic model which is used to define a lower bound on the optimal expenses of the dynamic model. There is one more model which is presented in this paper. It is a dynamic programming model for bunkering problem to analyse bunker price effect. Speed decisions take unpredictable port times into account that results into fuel consumption cost reduction.

## 2.3  Methodology

This part considers existing solution methods for Inventory Routing Problems (IRP), Maritime Inventory Routing Problems (MIRP) and Speed Optimization Problems. There are exact approaches and heuristics used to solve such problems.

### 2.3.1  Approaches for inventory routing problems

Simple heuristics in early papers on IRP includes assignment heuristic (Dror, Ball, and Golden, 1985), interchanging algorithm (Dror and Levy, 1986), trade-offs based on approximate routing costs (Burns et al., 1985), and a clustering heuristic (Anily and Federgruen, 1990). Current heuristic algorithms can solve difficult optimization problems. They are based on the concept of metaheuristics, which apply local search procedures and

a strategy to eliminate local optimum (Gendreau and Potvin, 2010) (Coelho, Cordeau, and Laporte, 2014).

According to Coelho, Cordeau and Laporte (2014), recent IRP papers include:

- iterated local search (Ribeiro and Lourenco, 2003);
- variable neighborhood search (Zhao, Chen, and Zang, 2008);
- greedy randomized adaptive search (Campbell and Savelsbergh, 2004);
- memetic algorithms (Boudia and Prins, 2009);
- tabu search (Archetti et al., 2012);
- adaptive large neighborhood search (Coelho, Cordeau, and Laporte, 2012c).

Several heuristics are presented in the table below.

| Heuristics | Introduced by |
|---|---|
| Clustering heuristics | Anily and Federgruen (1990) and Campbell and Savelsbergh (2004) |
| Construction and improvement heuristics (heterogeneous fleet) | Chien, Balakrishnan, and Wong (1989) |
| Two-phase heuristic based on a linear programming model | Campbell et al. (1998) |
| Fast local search algorithm | Bertazzi, Paletta, and Speranza (2002) |
| Tabu search with the exact solution of mixed-integer linear programs (MILPs) | Archetti et al. (2012) |
| Adaptive large neighborhood search (ALNS) matheuristic | Coelho, Cordeau, and Laporte (2012c) |

Table 1 – IRP heuristics

*Source: Coelho, Cordeau and Laporte (2014)*

The first branch-and-cut algorithm for a single-vehicle IRP was developed by Archetti et al., (2007). This algorithm could solve two versions of IRP regarding different inventory policies: maximum level and order-up-to level. Archetti et al. (2007) improved the model by deriving some valid inequalities, which provided an opportunity to solve problems including up to 50 customers in a three-period horizon, and 30 customers in a six-period horizon (Coelho, Cordeau, and Laporte, 2014).

Solyalı and Süral (2011) used the formulation with shortest-path networks along with heuristics to give an initial upper bound to the branch-and-cut algorithm. They considered only order-up-to level policy and derived the solution for larger instances in comparison with Archetti et al. (2007) (Coelho, Cordeau, and Laporte, 2014).

Coelho and Laporte (2013b) and Adulyasak, Cordeau, and Jans (2013) further extended the Archetti et al. (2007) formulation considering multiple vehicles, and have solved it with the help of a branch-and-cut algorithm. Coelho and Laporte (2013a) extended previous models for a multiproduct version of the IRP to be solved (Coelho, Cordeau, and Laporte, 2014).

## 2.3.2 Approaches for maritime inventory routing problems

In this part of thesis, solution methods for maritime inventory routing problems are presented. Agra et al. (2016a) solved a specific product maritime inventory routing problem with a heterogeneous fleet of ships applying a MIP based local search heuristic.

Agra, Christiansen, and Delgado (2012) studied a short sea fuel oil distribution problem applying mixed integer formulations. The approach includes a mathematical model of the problem and several strategies to improve the model (tightening bounds, extended formulations, and valid inequalities).

Agra, Christiansen, and Delgado (2016) developed two alternative mixed integer formulations: discrete and continuous time formulations for fuel oil distribution problem. Extended formulations and inequalities are implemented to allow linear gap reduction between two initial models.

In the paper of Agra et al. (2015) stochastic shipping problem was considered. The company, which has been discussed in this paper, was responsible for product deliveries and inventory management. The two-stage stochastic programming model was applied in this paper. The decomposition approach was used to solve the problem.

Jiang and Grossmann (2015) examined maritime inventory routing problem with another specific product and presented continuous and discrete time model.

De et al. (2017) studied maritime inventory routing problem to satisfy the demand in the number of ports over the planning horizon. To solve this problem a heuristic called particle swarm optimization for composite particle (PSO-CP) is implemented.

## 2.3.3 Approaches for speed optimization problems

In this part of the thesis methods suitable for problems regarding speed optimization is discussed. According to Panamarenka (2011), a large neighborhood search heuristic for a periodic supply vessel planning problem (PSVPP) with speed optimization was first examined in a thesis by Alexander Shyshou (2010). Panamarenka (2011) discussed emission minimization through speed optimization using the same heuristic to

find a solution with the set of voyages for several vessels in her thesis (Panamarenka, 2011).

Psaraftis and Kontovas (2014) considered issues regarding speed optimization and designed optimization models for a set of scenarios in a single ship setting. The work incorporated those parameters and demonstrated the optimal solution and the possible trade-offs.

Psaraftis and Kontovas (2013) examined different speed models, considering speed as a decision variable. The speed is considered as fuel cost determinant, which is important in fleet sizing, ship sizing, and inventory costs.

Andersson, Fagerholt, and Hobbesland (2015) used special approach for planning the vessels' routes. This approach included two stages. The first stage was based on the fixed speed and on the second stage speed was optimized along the routes. Norstad, Fagerholt, and Laporte (2011) applied a multi-start local search heuristic for scheduling and routing problem. In this work, speed was defined as a decision variable of each arc.

Wang and Meng (2012) considered a mixed-integer nonlinear model. An efficient method was proposed with respect to characteristics of fuel consumption function. Fagerholt et al. (2015) studied the problem of cost and emission minimization. An optimization tool to determine routes to travel and speed is designed to minimize expenses. Wena et. al. (2017) examined routing and speed optimization problem and solved it by applying a branch and price algorithm and a constraint programming model.

Aydin, Lee, and Mansouri (2017) studied speed optimization problem. Dynamic programming model was designed along with deterministic model which was used to define a lower bound on the optimal expenses of the dynamic model.

The MIRP with speed optimization includes several goals to be achieved. The main research questions for this master thesis are the savings comparison of all three models and determining which one is the most efficient. The third model includes routing variables, which were used as input parameters in the second model, speed variables, and speed constraints. The optimal solution gives routes and speeds for the existing heterogeneous fleet of vessels in terms of minimizing transportation and handling cost, in each port during the planning horizon. The solution also shows the amount of product to be loaded or unloaded at each port with respect to the inventories. As metaheuristics require an advanced level of programming they are not applied in this thesis. Mathematical models are made in AMPL language and run in CPLEX. Solution comparison of different models is presented in the computational part.

# 3.0  Problem description

## 3.1  Problem description

This research considers a maritime inventory routing problem with speed optimization. This thesis considers a geographical region where maritime transportation of a single product takes place. Transportation is conducted with a heterogeneous fleet of vessels, which are different in size, capacity, and cost. Travelling distances are supposed to be included into the problem. There are several ports, where consumption or production facilities are placed. There are no ports where production and consumption are conducted simultaneously. Production ports have fixed production rate, consumption ports have fixed consumption rate. Both production and consumption rates are given.

There are inventories at production and consumption facilities. Each port has storage facilities with fixed lower and upper limits. Production facilities are not allowed to exceed their maximum level, consumption facilities are not allowed to have shortages. Ports can be visited several times by different vessels during the given planning horizon depending on the size of the storage facilities at the port, and the number of products to load or unload. Vessels have a given starting location. After the route is finished, vessels end up in a node called destination point and it is also given. The cost of reaching this point is equal to 0.
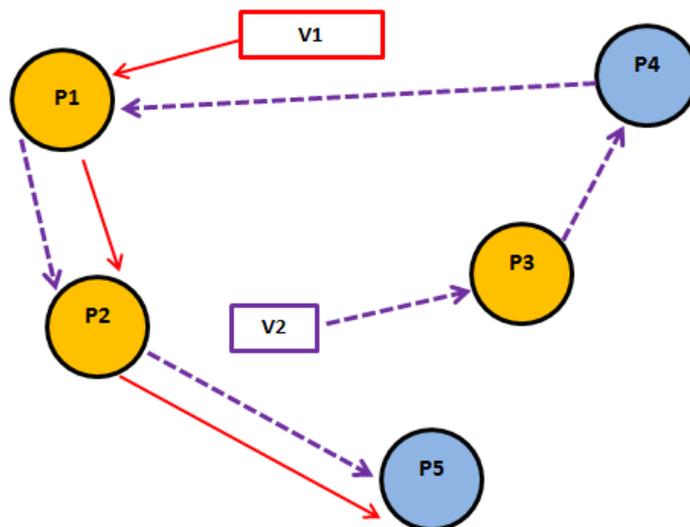
Figure 1 – Description of maritime inventory routing problem

*Source: made by the author*

From Figure 1, it is possible to see how two vessels travel their routes. The first vessel, which is red, goes to the port 1, where production facilities are located and fills up. Then it goes to the port 2, fills up again and sails to the port 5 to deliver. This is the route 1, which is shown by the thick red line in Figure 1. Another vessel goes first to the port 4, fills up; then continues to port 3 for unloading. After that it goes to port 1 to load, then sails to port 2 to load again and finishes at Port 5 with unloading. The route of the second vessel is represented by purple dashed line in the Figure 1.

## 3.2 Problem formulation

This research paper considers three different models to analyse how the speed influences the system performance.

The first model considers speed to be fixed and given for each vessel. In the second model, fixed routes are used along with a set of possible speeds for each vessel. Optimization tool can define speed for each vessel for each route from the set of given speeds. These decisions are made in terms of cost minimization. The main goal here is to examine how these changes influence the travel expenses and amount of the emission.

The third model considers routes and speed optimization simultaneously. Optimization tool is supposed to generate routes and choose the speed for each arc and vessel with respect to inventories and capacity of the vessel in terms of cost minimization.

In this part, mathematical formulations are introduced and considered in the first model. Maritime inventory routing problem, which is examined in this master thesis, is created in the same way as in Agra et al. (2016a). The same notation as in Agra et al. (2016a) is applied in this thesis. Speed optimization is introduced in a similar way as in Andersson, Fagerholt, and Hobbesland (2015).

Ports are supposed to have several nodes at a graph. This means that multiple nodes are allowed corresponding to each port. As a result, each port can have several visits during the planning horizon. For example, the vessel can start the route somewhere in the sea and end route in the port.
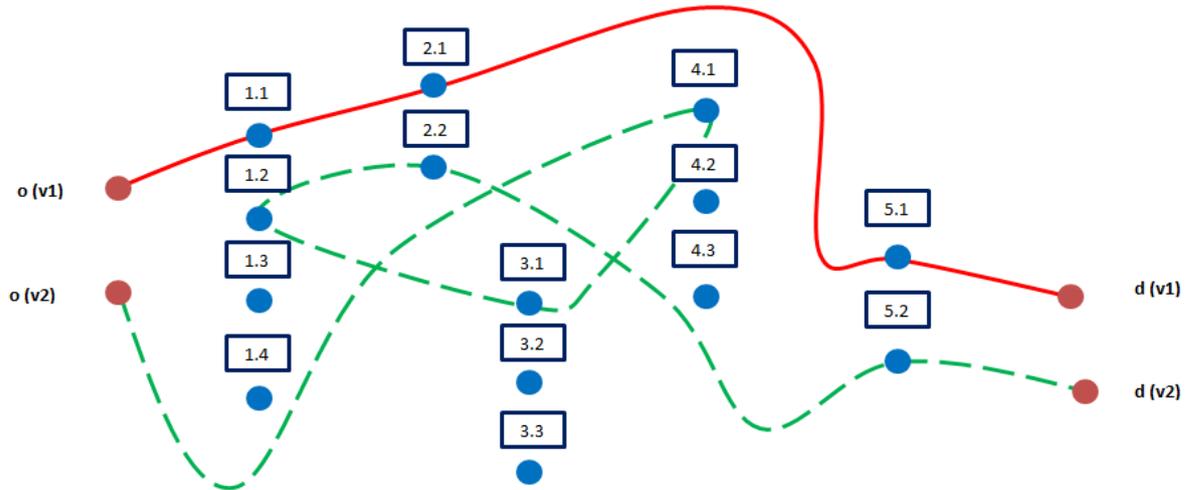
Figure 2 – Approach for ports: multiple nodes, representing visits to each port

*Source: made by the author*

The problem uses *V* to indicate a set of vessels and *N* to indicate a set of ports. Each vessel $v \in V$ has a starting point, which can be a point at sea. Each port is supposed to have several visits according to the time period.

Nodes in a network are indicated by a pair *(i, m)*, where *i* is a port and *m* is the number of visits to port *i*. Arcs of direct vessel movements from node *(i, m)* to the node *(j, n)* are indicated by *(i, m, j, n)*.

Figure 2 represents a part of an approach which is applied to ports. Figure 2 shows origins and destinations for two vessels. There are four visits for Port 1, two visits for Port 2 and three visits for Port 3. In the extended version, it looks like there is port 1 - visit 1, port 1 - visit 2, port 1 - visit 3, port 1 - visit 4; port 2 - visit 1, port 2 - visit 2; port 3 - visit 1, port 3 - visit 2, port 3 - visit 3; port 4 - visit 1, port 4 - visit 2, port 4 - visit 3; port 5 - visit 1, port 5 - visit 2.

Vessel 1 starts at the origin designated for vessel 1; goes to port 1 for visit 1; then sails to port 2 for visit 1 followed by port 5 for the visit 1 and the route is over. The route is shown in Figure 3 by the red line. Vessel 2 starts in its origin, sails to port 4 for the visit 1, moves to the port 3 for visit 1, sails to port 1 for visit 2, goes to port 2 for visit 2, moves to port 5 for visit 2 and finishes its route there. This route is shown by dashed green line.

The main objective is total routing cost minimization, which includes transportation and operating costs for all the routes conducted by vessels.

According to Andersson, Fagerholt and Hobbesland (2015) consumption of bunker for the vessel is a non-linear function of speed. This fact is represented in the Figure 3. This is the main assumption of the second and third model. Vessels must perform a certain speed for safety reasons to achieve minimum cost. Maximum speed can be achieved only in perfect weather conditions, which happens rarely. So, the maximum speed used for route planning should be somewhat lower. Andersson, Fagerholt, and Hobbesland (2015) decided to approximate the non-linearity in bunker consumption by discretizing speed alternative to deal with non-linearity computationally. Such an approach, presented by Andersson, Fagerholt, and Hobbesland (2015) states that linearization of the fuel consumption turns into cost overestimation because of convexity. The approach is illustrated in the Figure 3 for three speed options defined by red arrows.
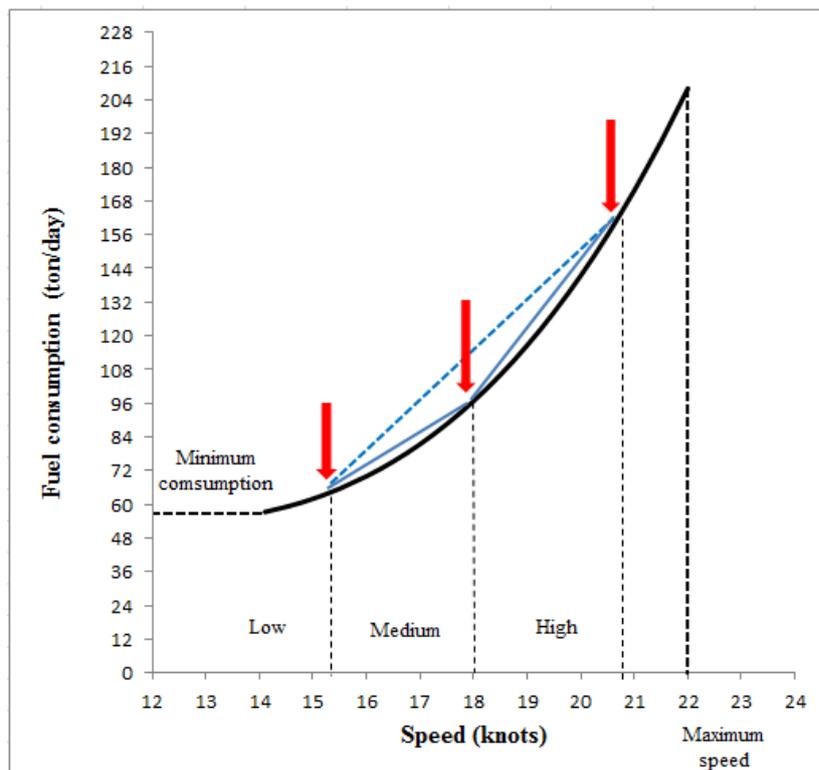


Figure 3 – The non-linear function of fuel consumption per time unit as a function of speed
*Source: made by the author based on Andersson, Fagerholt and Hobbesland (2015) and Norstad, Fagerholt, and Laporte (2011)*

In the second model, predefined routes are used as input parameters. This case also includes some other changes, concerning an additional set of speeds and speed variables. Input data include several speeds for each vessel. Since data include three different speeds

for each vessel, two of them were defined according to the corresponding ranges for vessels with general cargo and different capacities (GlobalSecurity.org, 2017). The lowest speed for each vessel was calculated according to the slow steaming policy, which was discussed in Faber et al. (2012) where speed is supposed to be reduced by 10% from the average.

According to Andersson, Fagerholt, and Hobbesland (2015), the non-linear dependence between travel time and speed of the vessel gives additional overestimation. In other words, the linearization of the travel time also means that the true costs are overestimated. The approach is illustrated in Figure 4. The same speed curve as for the Figure 3 was used to buid this graph. Breakpoints in Figure 4 are different from those, which were chosen for Figure 3. Other breakpoints were selected for visualization purposes. Travel time obtained from the solution is higher than the actual time corresponding to a speed chosen from the speed alternatives. Higher speed reduces traveling time and *vice versa*.
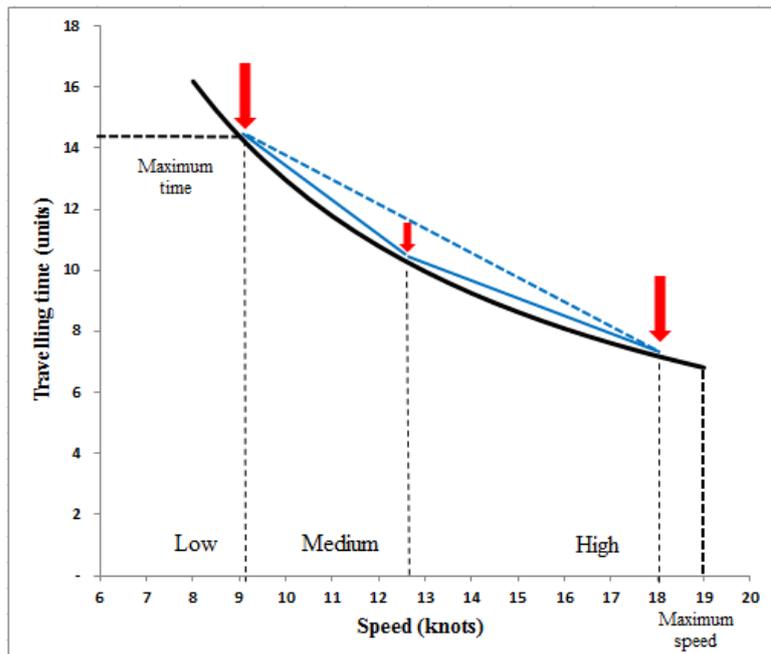


Figure 4 – The non-linear function of travel time as a function of speed
*Source: made by the author based on Bialystocki and Konovessis (2016)*

There are two speed variables, which are introduced into the special case of the first model. Same speed variables are also included into the model 3. These variables can be equal to one, zero or fractional value. The first one is $g_{imvs}^{o}$ that shows in which

proportions particular speeds are chosen or not chosen on the route between the origin and the port; the second is $g_{imjnvs}$ that shows in which proportions particular speeds are chosen or not chosen along the route between ports. The meaning of these variables is presented in Figure 5.
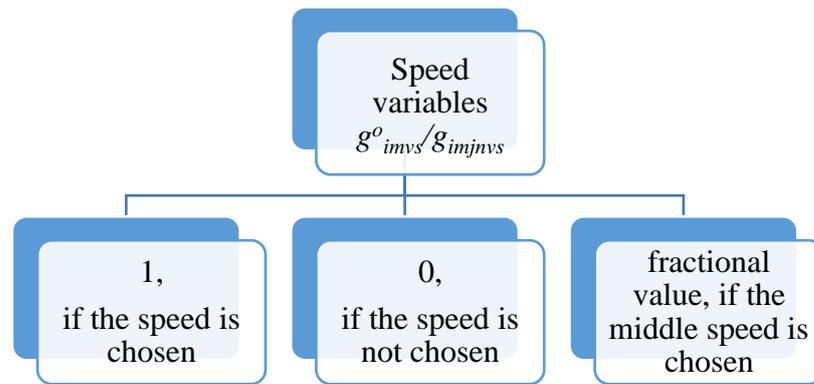


Figure 5 – The meaning of the speed variables, introduced into the models 2 and 3
*Source: made by the author*

Daily sailing cost is considered as the cost of fuel consumed while sailing. The amount of fuel consumption were calculated according to the Bialystocki and Konovessis (2016), where the function of fuel consumption looks as *Fuel Cons = 0.1727×Speed$^2$ – 0.217×Speed*. Figure 6 shows non-linear dependence between vessel speed and the amount of fuel consumption.
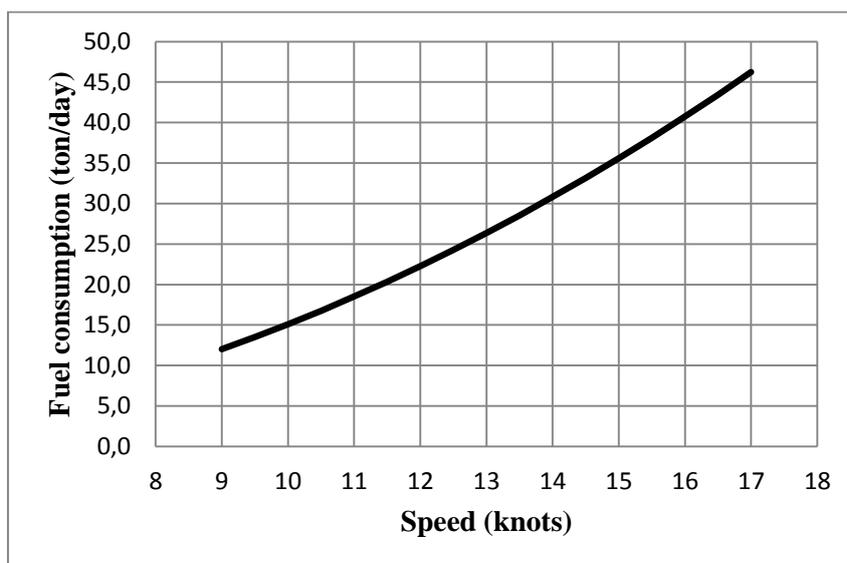


Figure 6 – Fuel consumption per time unit as a function of speed
*Source: made by the author based on Bialystocki and Konovessis (2016)*

The daily sailing cost for each speed and vessel are calculated proportionally, according to the given data. Applying the given data, it is possible to state that speed of vessels and daily sailing costs have non-linear dependence as shown in Figure 7. The graph was created based on one of the instances of the three vessels.
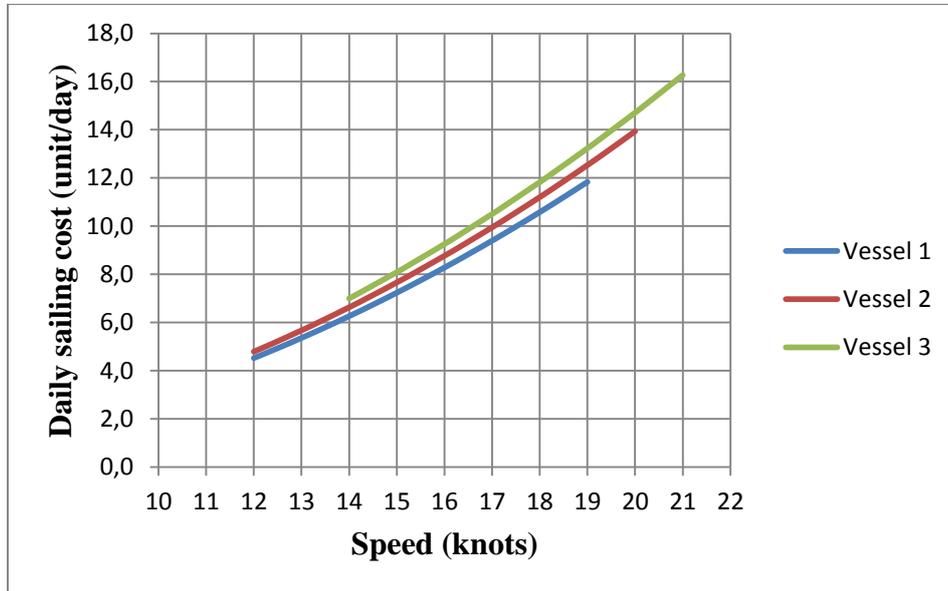


Figure 7 – Dependence between the daily sailing cost and speed for different vessels

*Source: made by the author based on Bialystocki and Konovessis (2016) and Agra et al. (2016a)*

# 4.0 Mathematical model

Maritime inventory routing problem is modeled in the same way as in Agra et al. (2016a) and with the same notation as in Agra et al. (2016a). Model 1 is presented first, then model 3 and finally model 2. Model 2 is a special case of model 1, the reason model 3 being presented before model 2.

## 4.1 Model 1

**Notation:**

**Sets:**

$V$ - set of vessels

$N$ - set of production and consumption ports

$S^A$ - set of possible nodes *(i, m)*

$S_v^A$ - set of nodes that can be visited by vessel $v$

$S_v^X$ - set of all movements *(i, m, j, n)* of vessel $v$

**Variables:**

$x_{imjnv}$ $-$ 1 if vessel $v \in V$ moves directly between nodes *(i, m)* and *(j, n)*, 0 otherwise, $v \in V$, $(i,m,j,n) \in S_v^X$

$x_{imv}^o$ $-$ 1 if vessel $v$ departs from its initial position to node *(i, m)*, 0; otherwise, $v \in V$, $(i,m) \in S_v^A$

$w_{imv}$ $-$ 1 if vessel $v$ visits node *(i, m)*, 0; otherwise, $v \in V$, $(i,m) \in S_v^A$

$z_{imv}$ $-$ 1 if vessel $v$ finishes its route at node *(i, m)*, 0; otherwise, $v \in V$, $(i,m) \in S_v^A$

$z_v^o$ $-$ 1 if origin of vessel $v$ is the same as destination (ship does not make any moves), $v \in V$

$q_{imv}$ $-$ the amount of product loaded/unloaded from vessel $v$ at node *(i, m)*, $v \in V$, $(i,m) \in S_v^A$

$f_{imjnv}$ $-$ the amount of product that vessel $v$ transports from node *(i, m)* to the node *(j, n)*, $v \in V$, $(i,m,j,n) \in S_v^X$

$f^o_{imv}$ – the amount of product that vessel $v$ transports from the origin to the node $(i, m)$, $v \in V$, $(i,m) \in S^A_v$

$f^d_{imv}$ – the amount of product that vessel $v$ transports from the origin to destination node $(i, m)$, $v \in V$, $(i,m) \in S^A_v$

$s_{im}$ – stock levels at ports at the start of the visit $m$, to port $i$, $(i,m) \in S^A$

$y_{im}$ – 1 if there is a visit $(i, m)$ to port $i$, 0 otherwise, $i \in N$, $(i,m) \in S^A$

$b_{imv}$ – 1 if vessel $v$ operates in port $(i, m)$, 0 otherwise, $v \in V$, $(i,m) \in S^A_v$

$t_{im}$ – starting time of the $m^{th}$ visit to port $i$, $(i,m) \in S^A_v$


**Parameters:**

$T$ – number of days in the planning horizon

$H_i$ – minimum number of visits to port $i \in N$

$M_i$ – maximum number of visits to port $i \in N$

$D_i$ – consumption or demand at port $i \in N$ in period $t$

$J_i$ – 1 if production facilities are located in port $i$, -1 if consumption facilities are located in port $i$, $i \in N$

$P_{iv}$ – port cost at port $i \in N$ for ship $v \in V$ per time period

$C_v$ – capacity of ship $v \in V$

$L_v$ – initial load onboard ship $v \in V$ when leaving port $i$

$\underline{S}_i$ – lower bound on the inventory level at port $i \in N$ in time period $t$

$\overline{S}_i$ – upper bound on the inventory level at port $i \in N$ in time period $t$

$S^o_i$ – the initial stock level in port $i \in N$ at the beginning of the planning horizon

$A_{im}$ – earliest time for starting visit $m$ to port $i$, $(i,m) \in S^A_v$

$B_{im}$ – latest time for starting visit $m$ to port $i$, $(i,m) \in S^A_v$

$K_i$ – time between two consecutive visits to port $i \in N$

$\overline{Q}_i$ – maximum load/unload quantity in port $i \in N$

$\underline{Q}_i$ – minimum load/unload quantity in port $i \in N$

$U_{im}$ – latest time for starting visit $m$ to port $i$, $(i,m) \in S^A_v$

$T^Q_v$ – time for unloading/load each unit by ship $v \in V$

$C^{PP}_{ijv}$ – sailing cost from port $i \in N$ to port $j \in N$ with ship $v \in V$

$C^{OP}_{iv}$ – sailing cost from origin to port $i \in N$ by ship $v \in V$

$T^{PP}_{ijv}$ – time required by ship $v \in V$ to sail from port $i \in N$ to port $j \in N$

$T^{OP}_{iv}$ – time required by ship $v \in V$ to sail from its origin to port $i \in N$

**Formulation:**

$$\min \sum_{v \in V} \sum_{(i,m,j,n) \in S_v^X} C^{PP}_{ijv} x_{imjnv} + \sum_{v \in V} \sum_{(i,m) \in S_v^A} C^{OP}_{iv} x^o_{imv} + \sum_{v \in V} \sum_{(i,m) \in S_v^A} P_{iv} b_{imv} \tag{1}$$

subject to

$$\sum_{(j,n) \in S_v^A} x^o_{jnv} + z^o_v = 1, \forall v \in V \tag{2}$$

$$w_{imv} - \sum_{(j,n,i,m) \in S_v^X} x_{jnimv} - x^o_{imv} = 0, \forall v \in V, (i,m) \in S_v^A \tag{3}$$

$$w_{imv} - \sum_{(i,m,j,n) \in S_v^X} x_{imjnv} - z_{imv} = 0, \forall v \in V, (i,m) \in S_v^A \tag{4}$$

$$\sum_{v \in V} w_{imv} = y_{im}, (i,m) \in S^A \tag{5}$$

$$y_{i(m-1)} - y_{im} \geq 0, (i,m) \in S^A : H_i + 1 \leq m \leq M_i \tag{6}$$

$$y_{im} = 1, (i,m) \in S^A : m \in \{1..H_i\} \tag{7}$$

$$b_{imv} \geq q_{imv} / C_v, \forall v \in V, (i,m) \in S_v^A \tag{8}$$

$$q_{imv} \leq \min\{C_v, \overline{Q}_i\} w_{imv}, \forall v \in V, (i,m) \in S_v^A \tag{9}$$

$$\underline{Q}_i w_{imv} \leq q_{imv}, \forall v \in V, (i,m) \in S_v^A \tag{10}$$

$$f^o_{imv} = L_v x^o_{imv}, \forall v \in V, (i,m) \in S_v^A \tag{11}$$

$$f^o_{jnv} + \sum_{(i,m,j,n) \in S_v^X} f_{imjnv} + J_i q_{jnv} = \sum_{(j,n,i,m) \in S_v^X} f_{jnimv} + f^d_{jnv}, \forall v \in V, (j,n) \in S_v^A \tag{12}$$

$$f_{imjnv} \leq (C_v - \underline{Q}_i) x_{imjnv}, \forall v \in V, (i,m), (j,n) \in S^A \tag{13}$$

$$f^d_{jnv} \leq C_v z_{jnv}, \forall v \in V, (j,n) \in S_v^A \tag{14}$$

$$t_{im} + \sum_{v \in V} T^Q_v q_{imv} - t_{jn} + \sum_{v \in V} \max\{U_{im} + T^{PP}_{ijv} - A_{jn}, 0\} x_{imjnv} \leq U_{im} - A_{jn},$$

$$\forall v \in V, (i,m), (j,n) \in S^A \tag{15}$$

$$t_{im} - t_{i(m-1)} - \sum_{v \in V} T_v^Q q_{i(m-1)v} - K_i y_{im} \geq 0, (i,m) \in S^A : m > 1 \tag{16}$$

$$\sum_{v \in V} T_{iv}^{OP} x_{imv}^o \leq t_{im}, (i,m) \in S^A \tag{17}$$

$$t_{im} \geq A_{im}, (i,m) \in S^A \tag{18}$$

$$t_{im} \leq B_{im}, (i,m) \in S^A \tag{19}$$

$$s_{i1} = S_i^o + J_i D_i t_{i1}, \forall i \in N \tag{20}$$

$$s_{im} = s_{i(m-1)} - J_i \sum_{v \in V} q_{i(m-1)v} + J_i D_i (t_{im} - t_{i(m-1)}), (i,m) \in S^A : m > 1 \tag{21}$$

$$s_{im} + \sum_{v \in V} q_{imv} - D_i \sum_{v \in V} T_v^Q q_{imv} \leq \overline{S}_i, (i,m) \in S^A : J_i = -1 \tag{22}$$

$$s_{im} - \sum_{v \in V} q_{imv} + D_i \sum_{v \in V} T_v^Q q_{imv} \geq \underline{S}_i, (i,m) \in S^A : J_i = 1 \tag{23}$$

$$s_{iM_i} + \sum_{v \in V} q_{iM_i v} - D_i (T - t_{iM_i}) \geq \underline{S}_i, \forall i \in N : J_i = -1 \tag{24}$$

$$s_{iM_i} - \sum_{v \in V} q_{iM_i v} + D_i (T - t_{iM_i}) \leq \overline{S}_i, \forall i \in N : J_i = 1 \tag{25}$$

$$s_{im} \geq \underline{S}_i, (i,m) \in S^A : J_i = -1 \tag{26}$$

$$s_{im} \leq \overline{S}_i, (i,m) \in S^A : J_i = 1 \tag{27}$$

$$x_{imjnv} \in \{0,1\}, v \in V, (i,m,j,n) \in S_v^X \tag{28}$$

$$x_{imv}^o \in \{0,1\}, v \in V, (i,m) \in S_v^A \tag{29}$$

$$w_{imv} \in \{0,1\}, v \in V, (i,m) \in S_v^A \tag{30}$$

$$z_{imv} \in \{0,1\}, v \in V, (i,m) \in S_v^A \tag{31}$$

$$z_v^o \in \{0,1\}, v \in V \tag{32}$$

$$y_{im} \in \{0,1\}, i \in N, (i,m) \in S^A \tag{33}$$

$$b_{imv} \in \{0,1\}, v \in V, (i,m) \in S_v^A \tag{34}$$

$$q_{imv} \geq 0, \; v \in V, \; (i,m) \in S_v^A \tag{35}$$

$$f_{imjnv} \geq 0, \; v \in V, \; (i,m,j,n) \in S_v^X \tag{36}$$

$$f_{imv}^o \geq 0, \; v \in V, \; (i,m) \in S_v^A \tag{37}$$

$$f_{imv}^d \geq 0, \; v \in V, \; (i,m) \in S_v^A \tag{38}$$

$$s_{im} \geq 0, \; (i,m) \in S^A \tag{39}$$

$$t_{im} \geq 0, \; (i,m) \in S^A \tag{40}$$

The objective function (1) expresses the minimization of the sum of traveling costs between ports and operational costs in each port. Constraints (2)−(7) are routing constraints. Constraint (2) shows that each ship travels from origin to the port or from origin to the destination. Constraint (3) ensures that the node is visited by the ship on the route from port to port or on the route from the origin to port or ends its route. Constraint (4) shows that if the vessel arrives at the node it also leaves it and finishes the route. Constraint (5) defines that the node *(i, m)* is visited by vessel only if $y_{im}$ is equal to one. Constraint (6) shows that there are mandatory visits. Equation (7) guarantees that if a port is visited *m* times, then visit *m-1* also took place.

Constraints (8)−(11) are loading and unloading constraints. Constraints (8)−(9) define lower and upper bounds on the loading or unloading quantities at each node. Constraint (10) ensures that if there is a visit to the node *(i, m)* then loaded/unloaded amount should be more or equal to the minimum quantity. Constraint (11) guarantees that if vessel travels from the initial position then transported amount is equal to the initial load of the vessel.

Constraints (12)−(14) are arc-flow constraints. Constraints (12) show that the sum of the incoming flow of product to port and the amount loaded/unloaded should be equal to outgoing flow. Constraint (13) defines that if the ship travels between ports then the flow from port to port should not exceed the difference between ship capacity and minimum loaded/unloaded quantity. Constraint (14) guarantees that if ship travels to the destination then the flow to the destination should not exceed ship capacity.

Constraints (15)−(19) are time constraints. Constraints (15) connect the start time at node *(i, m)* to the start time at node *(j, n)* when the ship travels between ports. Constraint (16) defines the minimum interval between two sequential visits to port. Constraints (17) guarantee that if ship travels from the origin then traveling time should not exceed start

time of the visit to the port. Constraints (18)−(19) are time windows for the start and end of the visits to ports.

Constraints (20)−(27) are inventory constraints. Equation (20) defines the stock level at the start time of the first visit to the particular port. Equation (21) connects the stock level of $m^{th}$ visit to the stock level of the previous visit. Constraints (22)−(23) guarantee that inventory is within the limits at the end of the visits. Constraints (24)–(25) define lower and upper bounds at time T for consumption and production ports. Constraints (26)–(27) ensure that inventory is within the limits at the start of each visit to the port.

Constraints (28)–(34) state that variables are binary. Equations (35)–(40) ensure that variables are nonnegative.

## 4.2 Model 3

Model 3 is the final model which is supposed to give both optimal routes and speeds for each vessel at once. A modelling approach for speed optimization as in the research paper of Andersson, Fagerholt and Hobbesland (2015) was used.

New set of speeds $S_v^S$ is introduced into the model. Each vessel supposed to have several options of speed, depending on their capacity.

The objective function (41) now expresses minimization of the sum of traveling costs between ports depending on the chosen speed and operational costs in each port. The final model includes routing variables (28)–(34) from the model 1, time variable (40), arc-flow variables (36)–(38), inventory variable (39), loading and unloading variable (35) from the first model and speed variables (46)–(47), which are new.

The model includes routing constraints (2)–(7), loading and unloading constraints (8)–(11), arc-flow constraints (12)–(14), time constraints (16), (18), (19), inventory constraints (20)–(27), (28)–(34) non-negative restrictions on the variables from the first model, speed constraints (42)–(43) and (46)–(47) restrictions on the variables.

A new block of constraints was added to the model. Constraints (42)–(43) are speed constraints. Constraints (42)–(43) state that speed for an arc from origin to port and from port to port exists only if vessel travels along this arc.

Constraints (44) and (45) have the same meaning as constraints (15) and (17) from the model 1, but $x_{imv}^o$ and $x_{imjnv}$ were replaced by $g_{imvs}^o$ and $g_{imjnvs}$. Traveling time from origin to port and from port to port now depends on the speed. Constraints (46)–(47) ensures that variables are between zero and one.

According to the model, the solution gives routes for each vessel with appropriate speed to minimize total traveling and operating costs. The model also defines which nodes are visited and where each ship ends its route, the amount of product carried by each ship along the routes, starting time for each visit to ports and stock level at the start of each visit to ports.

**Notation:**

**Sets:**

$S_v^S$ - set of speeds which can be used by vessel $v$

**Variables:**

$g_{imjnvs}$ − the proportion of the speeds $s$ used by vessel $v$ to travel on the route $(i, m, j, n)$, $v \in V$, $(i, m, j, n) \in S_v^X$, $s \in S_v^S$

$g_{imvs}^o$ − the proportion of the speeds $s$ used by vessel $v$ to travel from origin to node $(i, m)$, $v \in V$, $(i, m) \in S_v^A$, $s \in S_v^S$

**Parameters:**

$R_{vs}$ − speed $s \in S_v^S$ of ship $v \in V$

$c^{PP}_{ijvs}$ − sailing cost from port $i \in N$ to port $j \in N$ with ship $v \in V$ with speed $s \in S_v^S$

$c^{OP}_{ivs}$ − sailing cost from origin to port $i \in N$ by ship $v \in V$ with speed $s \in S_v^S$

$T^{PP}_{ijvs}$ − time required by ship $v \in V$ to sail from port $i \in N$ to port $j \in N$ using speed $s \in S_v^S$

$T^{OP}_{ivs}$ − time required by ship $v \in V$ to sail from its origin to port $i \in N$ with the speed $s \in S_v^S$

**Formulation:**

$$\min \sum_{v \in V} \sum_{(i,m,j,n) \in S_v^X} \sum_{s \in S_v^S} C^{PP}_{ijvs} g_{imjnvs} + \sum_{v \in V} \sum_{(i,m) \in S_v^A} \sum_{s \in S_v^S} C^{OP}_{ivs} g_{imvs}^o + \sum_{v \in V} \sum_{(i,m) \in S_v^A} P_{iv} b_{imv} \quad (41)$$

subject to

(2) – (14), 16, (18) – (40),

$$\sum_{s \in S_v^S} g_{imjnvs} = x_{imjnv}, \forall v \in V, (i, m, j, n) \in S_v^X \tag{42}$$

$$\sum_{s \in S_v^S} g_{imvs}^o = x_{imv}^o, \forall v \in V, (i, m) \in S_v^A \tag{43}$$

$$t_{im} + \sum_{v \in V} T_v^Q q_{imv} - t_{jn} + \sum_{v \in V} \sum_{s \in S_v^S} \max\left\{U_{im} + T_{ijvs}^{PP} - A_{jn}, 0\right\} g_{imjnvs} \leq U_{im} - A_{jn},$$

$$\forall v \in V, (i, m), (j, n) \in S^A \tag{44}$$

$$\sum_{v \in V} \sum_{s \in S_v^S} T_{ivs}^{\mathbf{OP}} g_{imvs}^o \leq t_{im}, (i, m) \in S^A \tag{45}$$

$$0 \leq g_{imjnvs} \leq 1, \ v \in V, \ (i, m, j, n) \in S_v^X, \ s \in S_v^S \tag{46}$$

$$0 \leq g_{imvs}^o \leq 1, \ v \in V, \ (i, m) \in S_v^A, \ s \in S_v^S \tag{47}$$

## 4.3 Model 2

This section examines the second model which is the special variant of the model 1. The second model has some changes compared to the first one and considered as a middle phase between first and final model. The model gives the speed for each vessel according to predefined routes. These routes are generated by the model 1. The main goal of model 2 is to determine more economic solutions by only speed optimization or route generation and speed optimization simultaneously. The modelling approach is applied for speed optimization as in Andersson, Fagerholt and Hobbesland (2015). Transportation cost is supposed to include the cost of fuel consumption.

The first model gives routes for vessels, which are used as input parameters for model 2. They are the following routing variables that become routing parameters in the second model: $x_{imjnv}$, $x_{imv}^o$, $w_{imv}$, $z_{imv}$, $z_v^o$, $y_{im}$, $b_{imv}$. Routing constraints (2)–(7) from the first model are eliminated.

There is also a possibility to fix arc-flow variables (36)–(38) in the second model but has not been considered in this thesis. The decision was to fix routing variables and optimize speed and quantities transported along the predefined routes.

Model 2 includes all sets from model 1 and new set of speeds, which was introduced into the model 3. The objective function (41) expresses the minimization of the sum of traveling costs between ports depending on the chosen speed and operational costs in each port.

Loading and unloading constraints (8)–(11), arc-flow constraints (12)–(14), time constraints (16), (18), (19), inventory constraints (20)–(27), non-negative constraints on variables (35)–(40) stays the same as in model 1. Also, this model includes speed constraints (42)–(43), restrictions on the variables (46)–(47) and time constraints (44)–(45) from the third model.

# 5.0 Computational study

The experimental part of the thesis was run on a personal computer with a 2.50 GHz Intel Core i5-6500T CPU processor and 16 GB of RAM under Windows 10. Models for speed optimization are coded in AMPL language and run in CPLEX 12.7.0.0.

## 5.1 Test instances description

Computational evaluation of the different instances is presented in this section. The models were tested on the artificial data which was obtained from Agra et al. (2016a). All data instances are presented in Table 2. Data is structured according to the number of vessels in specific instance with the speed ranges applied in each of those instances.

There are seven main instances, which differ from each other in several parameters. The title of each instance begins with the letter A, B, C, D, E, F, and G which corresponds to the instances from Agra et al. (2016a). Main instances are different in different ports, ships, and length of the planning horizon. That is why the name of instances includes:

- the number of ports;
- the number of vessels;
- the length of the planning horizon;
- index number.

| Number of instance | Number of ports | Number of ships | Planning horizon, days | Speed ranges of vessels, knots | | |
|---|---|---|---|---|---|---|
| | | | | 13.5-15.0-19.0 | 14.4-16.0-20.0 | 16.2-18.0-21.0 |
| A-4-1-30-1 | 4 | 1 | 30 | | | ✓ |
| A-4-1-60-1 | 4 | 1 | 60 | | | ✓ |
| A-4-1-60-2 | 4 | 1 | 60 | | | ✓ |
| B-3-2-30-1 | 3 | 2 | 30 | ✓ | ✓ | |
| B-3-2-60-1 | 3 | 2 | 60 | ✓ | ✓ | |
| B-3-2-60-2 | 3 | 2 | 60 | ✓ | ✓ | |
| C-4-2-30-1 | 4 | 2 | 30 | ✓ | ✓ | |
| C-4-2-60-1 | 4 | 2 | 60 | ✓ | ✓ | |
| C-4-2-60-2 | 4 | 2 | 60 | ✓ | ✓ | |
| D-5-2-30-1 | 5 | 2 | 30 | ✓ | | ✓ |
| D-5-2-60-1 | 5 | 2 | 60 | ✓ | | ✓ |
| D-5-2-60-2 | 5 | 2 | 60 | ✓ | | ✓ |

| | | | | | | |
|---|---|---|---|---|---|---|
| E-5-2-30-1 | 5 | 2 | 30 | ✓ | | ✓ |
| E-5-2-60-1 | 5 | 2 | 60 | ✓ | | ✓ |
| E-5-2-60-2 | 5 | 2 | 60 | ✓ | | ✓ |
| F-4-3-30-1 | 4 | 3 | 30 | ✓ | ✓ | ✓ |
| F-4-3-60-1 | 4 | 3 | 60 | ✓ | ✓ | ✓ |
| F-4-3-60-2 | 4 | 3 | 60 | ✓ | ✓ | ✓ |
| G-6-5-30-1 | 6 | 5 | 30 | ✓ | ✓ | ✓ |
| G-6-5-60-1 | 6 | 5 | 60 | ✓ | ✓ | ✓ |
| G-6-5-60-2 | 6 | 5 | 60 | ✓ | ✓ | ✓ |

Table 2 – Data instances for computational evaluation of the models

*Source: made by the author*

Index number shows that instances include internal parameters, which are different for different instances. They are demand, initial stock level, upper bound for stock level, daily sailing cost, the distance between ports and fleet of vessels.

Each vessel displays following operational characteristics:

- the capacity of the vessel (ton);
- the initial load of the vessel (ton);
- set of possible speeds (knots/hour);
- the daily sailing cost for each speed option (unit/day).

According to the chosen approach, there is a set of speeds with three options for each vessel. Two of them were determined according to the corresponding ranges for vessels with general cargo and different capacities (GlobalSecurity.org, 2017). The lowest speed for each vessel was defined in terms of slow steaming policy, which was presented in Faber et al. (2012), where speed is reduced by 10% from the average value. It is assumed that each vessel performs in a speed exactly equal to one from the set or speed which is between two of previously chosen as a set. Speed ranges of different vessels are presented in Table 3. Operating speed of vessels in model 1 is 16.2 knots.

| Vessels' capacity, units | Speed ranges of vessels, knots | | |
|---|---|---|---|
| | 13.5-15.0-19.0 | 14.4-16.0-20.0 | 16.2-18.0-21.0 |
| 100 | ✓ | | |
| 120 | ✓ | | |
| 130 | | ✓ | |
| 140 | | ✓ | |
| 150 | | | ✓ |
| 160 | | | ✓ |

Table 3 – Speed ranges of different vessels depending on capacity (basic curve)
*Source: made by the author*

The main approach while introducing speed into the existing tool for speed optimization is linearization for sailing cost calculation. Daily sailing costs are determined by using the curve where the amount of fuel consumption depends on the speed of the vessel.

There are two curves which are used to test data instances. The first curve was obtained from Bialystocki and Konovessis (2016) where the function of fuel consumption in tons per day is presented as *Fuel Cons = 0.1727× Speed$^2$ – 0.217×Speed*. This curve is a basic curve for conducting all computational tests. The second curve was taken from Norstad, Fagerholt, and Laporte (2011), an expression which shows the amount of fuel consumed per travelled nautical mile as following: *Fuel Cons = 0.0036×Speed$^2$ – 0.1015×Speed + 0.8848*. This expression can be applied for speed range of 14.1–22 knots. This curve was modified to achieve the amount of fuel consumed per day. New curve looks as *Fuel Cons = 0.0864×Speed$^3$ – 2.436×Speed$^2$ + 21.2352×Speed.* The second curve is non-basic and is used for error and computational time calculation and presented in a separate section.

This section includes analysis of computational time, the amount of expenses for different models and average speed evaluation. Error calculation from linearization and actual cost for different curves are also presented in this section.

## 5.2 Assessment of computational results for basic curve

### 5.2.1 Computational time

In this part, comparisons of computational time for all three models are provided. Table 4 gives computational time for all the models and instances. The first model always shows low computational time when compared to model 3. This happens because of the sample size of the problem; the last model has more variables.

| Number of instance | Computational time model 1, sec | Computational time model 2, sec | Computational time model 3, sec |
|---|---|---|---|
| A-4-1-30-1 | < 1 | < 1 | < 1 |
| A-4-1-60-1 | 1 | < 1 | 2 |
| A-4-1-60-2 | 1 | < 1 | 4 |
| B-3-2-30-1 | 1 | < 1 | 15 |
| B-3-2-60-1 | 5 | < 1 | 20 |
| B-3-2-60-2 | 5 | < 1 | 16 |
| C-4-2-30-1 | 2 | < 1 | 31 |
| C-4-2-60-1 | 11 | < 1 | 50 |
| C-4-2-60-2 | 40 | < 1 | 672 |
| D-5-2-30-1 | 20 | < 1 | 133 |
| D-5-2-60-1 | 274 | < 1 | 1198 |
| D-5-2-60-2 | 25 | < 1 | 16 |
| E-5-2-30-1 | 65 | < 1 | 95 |
| E-5-2-60-1 | 177 | < 1 | 408 |
| E-5-2-60-2 | 48 | < 1 | 167 |
| F-4-3-30-1 | 61 | < 1 | 494 |
| F-4-3-60-1 | 39 | < 1 | 125 |
| F-4-3-60-2 | 28 | < 1 | 308 |
| G-6-5-30-1 | 104 | < 1 | 976 |
| G-6-5-60-1 | 1973 | < 1 | 7898 |
| G-6-5-60-2 | 2509 | < 1 | 11849 |

Table 4 – Computational time of different models (basic curve)

*Source: made by the author*

It is worth mentioning that calculation time of the model 2 is less than 1 sec, the solution appears almost immediately. It should be noticed that lower level of demand in ports and higher planning horizon increases computational time.

Even though G instance includes 6 ports and 5 ships, computational time is the highest. Computational time, for instance, G-6-5-60-2 of model 3 equals to 11849 seconds. The most influencing things are big fleet of vessels and the length of the planning horizon. The model requires a long time to select the best result among all possible alternatives.

## 5.2.2 Sailing cost and savings

In this part analysis of sailing cost and savings are presented. Table 5 provides the amount of savings of model 2 and model 3 in comparison to model 1. At most instances, speed optimization gives better results in terms of cost savings, than the approach with constant speed and fixed routes. A number of ships in the instance influence the complexity of the decision-making process. The more ships are included in the instance, the more alternatives are supposed to be examined by the solver to find the optimal solution in terms of sailing cost minimization.

| Number of instances | Sailing cost of model 1, units | Savings of model 2, % | Savings of model 3, % |
|---|---|---|---|
| A-4-1-30-1 | 130.65 | 0.0 | 0.0 |
| A-4-1-60-1 | 331.29 | 0.0 | 0.0 |
| A-4-1-60-2 | 331.29 | 0.0 | 0.0 |
| B-3-2-30-1 | 364.81 | -8.4 | -12.8 |
| B-3-2-60-1 | 490.98 | -7.6 | -10.4 |
| B-3-2-60-2 | 529.07 | -14.8 | -14.8 |
| C-4-2-30-1 | 391.49 | -14.6 | -14.6 |
| C-4-2-60-1 | 528.78 | -7.0 | -10.4 |
| C-4-2-60-2 | 545.60 | -5.4 | -5.8 |
| D-5-2-30-1 | 347.14 | -1.5 | -13.0 |
| D-5-2-60-1 | 379.37 | -4.2 | -4.2 |
| D-5-2-60-2 | 288.61 | -5.6 | -5.6 |
| E-5-2-30-1 | 347.14 | +0.1 | -11.1 |
| E-5-2-60-1 | 287.17 | -4.4 | -8.4 |

| | | | |
|---|---|---|---|
| E-5-2-60-2 | 274.78 | -0.7 | -8.2 |
| F-4-3-30-1 | 460.89 | -10.6 | -10.6 |
| F-4-3-60-1 | 387.18 | -14.2 | -14.2 |
| F-4-3-60-2 | 419.08 | -0.9 | -2.7 |
| G-6-5-30-1 | 645.81 | -3.2 | -3.2 |
| G-6-5-60-1 | 672.10 | -3.9 | -9.8 |
| G-6-5-60-2 | 648.05 | -4.1 | -5.9 |

Table 5 – Cost savings of models 2 and 3 in comparison with initial model

*Source: made by the author*

Figure 8 represents a graphic comparison of sailing costs for different models. The analysis shows that first model almost always shows higher total cost in comparison with the other models. The graph also indicates that cost of the model with speed optimization is the lowest or at least as good as in the model 2.
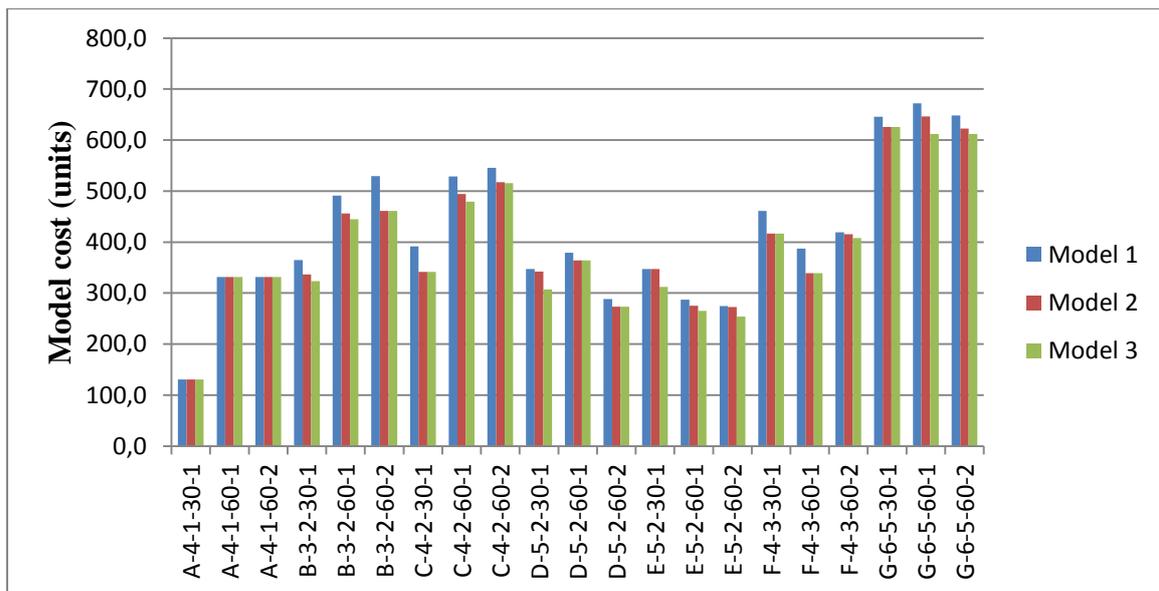


Figure 8 – Cost comparison for different models (basic curve)

*Source: made by the author*

### 5.2.3 Speed and emissions amount

Table 6 provides a comparison of average speed reduction for different models in comparison with the model 1. Average speed is calculated as average of the change in speed for each leg. If percentage speed reduction of model 2 and model 3 is compared, model 2 in some instances performs better and as a result emission amount is lower. Model

2 optimizes speed on the predefined routes, which are the cheapest. Model 2 is not allowed to change these routes, only to choose the optimal speed. Model 3 optimizes routes and speeds simultaneously and in some legs, perform higher speed to satisfy all the constraints. That is why the average speed of model 3 can be higher than the average speed of model 2. When it comes to analysis it is easy to conclude that in terms of fuel consumption and emission minimization, speed optimization always gives better results.

| Number of instances | Average speed of model 1, knots | Speed reduction for model 2, % | Speed reduction for model 3, % |
|---|---|---|---|
| A-4-1-30-1 | 16.2 | 0.0 | 0.0 |
| A-4-1-60-1 | 16.2 | 0.0 | 0.0 |
| A-4-1-60-2 | 16.2 | 0.0 | 0.0 |
| B-3-2-30-1 | 16.2 | -10.4 | -10.1 |
| B-3-2-60-1 | 16.2 | -10.1 | -12.2 |
| B-3-2-60-2 | 16.2 | -16.1 | -15.7 |
| C-4-2-30-1 | 16.2 | -16.1 | -15.6 |
| C-4-2-60-1 | 16.2 | -14.4 | -15.3 |
| C-4-2-60-2 | 16.2 | -7.4 | -9.2 |
| D-5-2-30-1 | 16.2 | -3.4 | -0.1 |
| D-5-2-60-1 | 16.2 | -5.3 | -5.3 |
| D-5-2-60-2 | 16.2 | -6.6 | -6.6 |
| E-5-2-30-1 | 16.2 | -1.8 | -0.7 |
| E-5-2-60-1 | 16.2 | -5.9 | -9.1 |
| E-5-2-60-2 | 16.2 | -3.2 | -2.3 |
| F-4-3-30-1 | 16.2 | -11.1 | -11.1 |
| F-4-3-60-1 | 16.2 | -13.9 | -13.9 |
| F-4-3-60-2 | 16.2 | -3.2 | -13.7 |
| G-6-5-30-1 | 16.2 | -4.8 | -4.8 |
| G-6-5-60-1 | 16.2 | -5.0 | -13.0 |
| G-6-5-60-2 | 16.2 | -6.5 | -8.7 |

Table 6 – Speed reduction of models 2 and 3 in comparison with initial model

*Source: made by the author*

Further comparison shows that in most of the instances with speed optimization, 60 days planning horizon performs higher speed reduction than the same instances with 30 days planning horizon. When speed optimization is applied, longer planning horizon gives an opportunity to reduce fuel consumption along with emission amount. As a result, fuel consumption costs are lower with low speed in comparison with fuel consumption for the high speed.

### 5.2.4 Structural analysis of the solutions

After cost comparison, it is important to evaluate the structure of the solutions for models. The most typical instances were chosen for such analysis. Some instances performed the same costs, some of them have decreasing costs from model to model, and others have same costs for model 1 and model 2 or for model 2 and model 3.

Structural analysis is presented in Table 7. Model 1 and model 2 always perform the same number of visits and the same travelling distance since model 2 includes routes from the model 1. Average speed for model 1 is not mentioned in Table 7 because vessels travel with the constant speed of 16.2 knots.

However, sailing costs for the second model in comparison with the initial one are lower in most instances. This happens because cost calculation of model 2 is based on the curve from Bialystocki and Konovessis (2016) and depends on fuel consumption and speed. Application of this curve allows speed and consequently, sailing cost reduction.

First instance (A-4-1-60-1) was chosen because all three models showed the same amount of sailing costs. All three models have the same number of visits, same travelling distance, same routes and the same average speed. In the model 3, average speed is 16.2 knots and it does not change for any of the models. This speed is the lowest in the performed set for that ship, which is enough to satisfy demand in each port in time and minimize sailing cost.

Instances B-3-2-30-1 and B-3-2-60-1 perform fewer visits and less traveling distance in the model 3. Speed optimization approach gives an opportunity to decrease sailing distance and number of visits. As a result, average speed of the ship is lower for model 3 in comparison with the model 1. Both instances B-3-2-30-1 and B-3-2-60-1 for model 3 perform one leg where speed is more than the average. These legs belong to the longest routes. The increase in speed gives an advantage in savings. It allows achieving speed reduction in other travelled legs. As a result, average speed and emissions amount

are reduced. Examined instances show that speed optimization approach gives the best performance.

| Name of instances | Model 1 / Model 2 | | | Model 3 | | |
|---|---|---|---|---|---|---|
| | Sailing distance, units | Number of visits | Average speed of model 2, knots | Sailing distance reduction, units | Number of visits | Average speed, knots |
| A-4-1-60-1 | 11 271 | 7 | 16.20 | 0 | 7 | 16.20 |
| B-3-2-30-1 | 15 159 | 6 | 14.67 | -3 109 | 4 | 14.72 |
| B-3-2-60-1 | 20 212 | 8 | 14.71 | -2 332 | 7 | 14.44 |
| C-4-2-30-1 | 12 049 | 5 | 13.95 | 0 | 5 | 14.01 |
| D-5-2-30-1 | 15 544 | 6 | 15.67 | -1 554 | 6 | 16.18 |
| E-5-2-30-1 | 13 601 | 6 | 15.91 | -1 554 | 6 | 16.08 |

Table 7 – Structural analysis of the solutions of different models

*Source: made by the author*

Instance C-4-2-30-1 shows the same costs, travelling distance and routes for model 2 and model 3. Approximate average speed is the same in these models. Instance D-5-2-30-1 shows the same number of visits for all models. Reduction of sailing cost occurred in model 3 because of less travelling distance in comparison with previous models. The reason is the different routes of model 2 and model 3. Average speed in model 3 is higher than in model 2, but travelling distance reduction gives cost savings.

Instance E-5-2-30-1 gives the same number of visits in all models. The average speed of model 2 is lower than in model 1, travelling distance and routes stay the same. Model 2 is more expensive than model 1, which is not typical for experimental results. Model 1 gives a speed of 16.2 knots in each leg, but model 2 performs in lower speed in several legs. There is one leg, which performs in higher speed than average speed of model 1. The reason for such not typical results is that the speed of model 1 is not one of the breakpoints for second vessel in model 2. Operational speed of model 1 is 16.2 knots, while speed ranges for second vessel are 13.5-15-19 knots. Speeds chosen by model 2 are different from 16.2 knots, because of possible rounding error. Routes generated by model

1 are infeasible for model 2 when using the same speed. Model 3 includes different routes and higher average speed so speed optimization approach gives cost savings.

Speed optimization approach allows increasing cost savings in comparison to the other models. There are three main alternatives of cost savings. First is distance reduction, because of route generation, second is visit reduction and travel distance reduction. The third alternative is speed reduction or increase, which goes in pair with traveling distance changes.

### 5.2.5 Computational error

As it was mentioned earlier the approach applied for speed optimization of vessels gives some estimation error in total cost. This approach was implemented according to the curve presented in from Bialystocki and Konovessis (2016). The presence of estimation error is based on the linearization approach. There are several sets of speeds given, one for each vessel. The solution includes one speed for each arc, where this speed is chosen from a given set; or it is a 'middle' speed decided by the solver. That is the reason behind errors in calculation of fuel consumption cost.

| Number of instance | Total cost of voyage, units | Actual cost of voyage, units | Cost decline, units | Percentage cost decline, % |
|---|---|---|---|---|
| B-3-2-30-1 | 323.41 | 323.25 | 0.167 | 0.052 |
| B-3-2-60-1 | 444.57 | 444.44 | 0.131 | 0.029 |
| C-4-2-60-2 | 515.63 | 515.61 | 0.022 | 0.004 |
| D-5-2-30-1 | 307.23 | 307.19 | 0.037 | 0.012 |
| E-5-2-30-1 | 312.50 | 312.42 | 0.074 | 0.024 |
| E-5-2-60-2 | 253.98 | 253.96 | 0.017 | 0.007 |
| F-4-3-60-2 | 407.94 | 407.89 | 0.053 | 0.013 |
| G-6-5-30-1 | 610.29 | 610.16 | 0.129 | 0.021 |
| G-6-5-60-2 | 611.67 | 611.56 | 0.112 | 0.018 |

Table 8 – Computational error of model 3 as a result of linearization (basic curve)

*Source: made by the author*

Table 8 shows all the instances from model 3 where such an error has occurred. Calculation error is presented for instances where fractional speed was assigned for vessels. Percentage cost decline is very low, between 0.004 and 0.052 %. This happened

because solution includes maximum two arcs where fractional speed is chosen and several (to be precise three) options of speed for each vessel. Low calculation error proves that linearization approach and speed curve give results which are very close to the real optimal solution.

## 5.3 Assessment of computational results for non-basic curve

Data instances were tested on the basic and non-basic curves. The non-basic curve was obtained from Norstad, Fagerholt, and Laporte (2011). The non-basic curve can be used within speed ranges of 14.1 and 22 knots. Speed ranges of different vessels are presented in Table 9. Operating speed of vessels in model 1 is 16.2 knots.

This part describes computational time and computational error for all the instances with 30 days planning horizon. Table 9 shows speed ranges which were applied for this curve for vessels with different capacity.

| Vessels' capacity, units | Speed ranges of vessels, knots | | |
|---|---|---|---|
| | 14.4-16.0-20.0 | 16.2-18-21 | 15.0-17.5-22.0 |
| 100 | ✓ | | |
| 120 | ✓ | | |
| 130 | | ✓ | |
| 140 | | ✓ | |
| 150 | | | ✓ |
| 160 | | | ✓ |

Table 9 – Speed ranges of different vessels depending on capacity (non-basic curve)
*Source: made by the author*

### 5.3.1 Computational time

Table 10 provides computational time for instances with 30 days planning horizon, where the non-basic curves are the basis of tests. Model 2 still gives the lowest computational time, which is less than 1 second. The complexity of the decision-making process increases along with the number of ships. The instance with 6 ports and 5 ships in model 3 gives the highest computational time, which is approximately 40 minutes.

| Number of instance | Computational time model 1, sec | Computational time model 2, sec | Computational time model 3, sec |
|---|---|---|---|
| A-4-1-30-1 | <1 | <1 | <1 |
| B-3-2-30-1 | 1 | <1 | 10 |
| C-4-2-30-1 | 2 | <1 | 24 |
| D-5-2-30-1 | 20 | <1 | 147 |
| E-5-2-30-1 | 64 | <1 | 928 |
| F-4-3-30-1 | 58 | <1 | 500 |
| G-6-5-30-1 | 105 | <1 | 2393 |

Table 10 – Computational time of different models (non-basic curve)

*Source: made by the author*

## 5.3.2 Computational error

Calculation of computational error for the non-basic curves is presented for instances from model 3 where fractional speed is chosen. The results are presented in Table 11. The curve from Norstad, Fagerholt, and Laporte (2011) also gives estimation error from linearization approach which is between 0.03 and 0.06 %. The reason stays the same. There are maximum two routes where 'middle' speed is chosen and more than two speed options. These results can be applied to practical problems because of proximity to optimum value.

| Number of instance | Total cost of voyage, units | Actual cost of voyage, units | Cost decline, units | Percentage cost decline, % |
|---|---|---|---|---|
| B-3-2-30-1 | 323.41 | 323.22 | 0.189 | 0.06 |
| D-5-2-30-1 | 331.25 | 331.17 | 0.084 | 0.03 |
| E-5-2-30-1 | 335.02 | 334.84 | 0.180 | 0.05 |
| G-6-5-30-1 | 629.29 | 629.05 | 0.238 | 0.04 |

Table 11 – Computational error of model 3 as a result of linearization (non-basic curve)

*Source: made by the author*

The results of computational part show that speed optimization approach is applicable for problems of different sizes when cost savings are considered. In terms of emission minimization, this approach performs better for instances with long planning horizon.

The length of planning horizon directly influences the average speed reduction. Higher planning horizon reduces average speed of vessels during the voyage. Lower sailing speed gives less fuel consumption and sailing cost as a result. Emissions are supposed to be less with lower sailing speed.

The difference between model 2 and model 3 is its applicability which depends on the main purpose. Model 3 performs better when minimization of sailing cost was considered. As a result, cost savings are significant according to the computational results. Emissions amount is also minimized in a long-term perspective. Model 2 can be applied in a short-term perspective for emissions minimization. An additional advantage of model 2 is low computational time.

# 6.0 Conclusion

Maritime transportation has significant value in the area of goods transportation. This type of transportation relates to high level of uncertainties, which should be minimized to obtain better results. The problem considered in this thesis, includes finding speed and schedules that satisfies demand and requires planning. More than that nowadays the society is concerned about emissions from vessels. As a result, this thesis can also be considered important not only from the point of optimal cost and speed but also from the point of green logistics.

Different methods were studied and analysed in this thesis to develop a tool for speed optimization implemented step by step. Three different models are considered in this thesis. The first model is a deterministic model, where speed is a fixed parameter. The second model, as a special case of model 1, includes fixed routes and speeds for each ship as a set. The third model generates routes and optimal speeds for each vessel at the same time along with cost optimization.

If the speed is considered as a decision variable, several objectives can be achieved simultaneously. Firstly, there is a possibility to improve total cost of the voyage by means of speed and sailing cost reduction. Secondly, a certain level of inventories can be performed at the end of the planning horizon. Finally, lower speed allows reduction of emission.

Considering speed as a variable allows hedging from unpredictable situations such as delays, weather conditions, low level of inventory and high level of demand. It should be highlighted that models which include speed optimization are much closer to practice in comparison with an initial model where speed is fixed. Applied approach gives an opportunity to control traveling time, costs, fuel consumption, emissions and speed of the vessels.

Computational experiments conducted on the data, obtained from one of the scientific articles, show that speed optimization tool is quite efficient in terms of cost savings for different size of problems and emissions reduction for problems of a large size. Computational studies also prove that linearization approach for non-linear dependence between speed and fuel consumption generate low level of computational error as there are several close data points to be considered on the curve. This approach can help to improve route planning and speed optimization. This thesis can be applied to the real-life problems to achieve these goals in practice.

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# Appendix

This section includes AMPL code for the model 3. Code is separated into sets, parameters, variables, objective function and constraints according to their type for better overview of the model's structure. To run the model files with .dat and .run extensions are required. Solution is transferred into the file with .sol extension.

```
################SETS################
set PORTS;
set SHIPS;
set SPEEDS1;
set SPEEDS2;
set SPEEDS3;
set SPEEDS4;
set SPEEDS5;

set SPEEDS:= SPEEDS1 union SPEEDS2 union SPEEDS3 union SPEEDS4 union SPEEDS5;
set OPTIONS {SHIPS};
param T>=0;
param Nmin {i in PORTS} >=0;
param maxvisits {i in PORTS} >=0;
```

```
################PARAMETERS################
param DemandRate {i in PORTS} >=0;
param J {i in PORTS};
param ShipCap {v in SHIPS} >=0;
param InitialLoad {v in SHIPS} >=0;
param LoadRate {v in SHIPS} >=0;
param UpperStock {i in PORTS} >=0;
param LowerStock {i in PORTS} >=0;
param InitialStock {i in PORTS} >=0;
param DistOrig {i in PORTS, v in SHIPS} >=0;
param Distance {i in PORTS, j in PORTS} >=0;
param Speed {v in SHIPS, s in OPTIONS[v]} >=0;
param DailySailCost {v in SHIPS, s in OPTIONS[v]} >=0;
param PortCost {i in PORTS, v in SHIPS} >=0;
param Qmax {i in PORTS}:=UpperStock[i];
param Qmin {i in PORTS} >=0;
param TB {i in PORTS}>=0;
```

param A {i in PORTS, m in 1..maxvisits[i]}:=0;

param B {i in PORTS, m in 1..maxvisits[i]}:=T;

param U {i in PORTS, m in 1..maxvisits[i]}:=min(T, T+B[i,m]*Qmax[i]);

param TL {v in SHIPS}:=1/LoadRate[v];


param TravelCost {i in PORTS, j in PORTS, v in SHIPS, s in OPTIONS[v]}:=DailySailCost[v,s]*Distance[i,j]/(24*Speed[v,s]);

param TravelTime {i in PORTS, j in PORTS, v in SHIPS, s in OPTIONS[v]}:=Distance[i,j]/(24*Speed[v,s]);

param TravelTimeOrig {i in PORTS, v in SHIPS, s in OPTIONS[v]}:=DistOrig[i,v]/(24*Speed[v,s]);

param TravelCostOrig {i in PORTS, v in SHIPS, s in OPTIONS[v]}:=DailySailCost[v,s]*DistOrig[i,v]/(24*Speed[v,s]);


################VARIABLES###############

###ROUTING VARIABLES

var X {i in PORTS, m in 1..maxvisits[i], j in PORTS, n in 1..maxvisits[j], v in SHIPS: i<>j} binary;

var Xo {i in PORTS, m in 1..maxvisits[i], v in SHIPS} binary;

var Z {i in PORTS, m in 1..maxvisits[i], v in SHIPS} binary;

var Zo {v in SHIPS} binary;

var W {i in PORTS, m in 1..maxvisits[i], v in SHIPS} binary;

var Y {i in PORTS, m in 1..maxvisits[i]} binary;

var O {i in PORTS, m in 1..maxvisits[i], v in SHIPS} binary;


###SPEED VARIABLES

var G {i in PORTS, m in 1..maxvisits[i], j in PORTS, n in 1..maxvisits[j], v in SHIPS, s in OPTIONS[v]: i<>j} >=0, <=1;

var Go {i in PORTS, m in 1..maxvisits[i], v in SHIPS, s in OPTIONS[v]} >=0, <=1;


###LOADING/UNLOADING VARIABLES

var Q {i in PORTS, m in 1..maxvisits[i], v in SHIPS} >=0;


###FLOW VARIABLES

var F {i in PORTS, m in 1..maxvisits[i], j in PORTS, n in 1..maxvisits[j], v in SHIPS: i<>j} >=0;

var Fo {i in PORTS, m in 1..maxvisits[i], v in SHIPS} >=0;

var Fd {i in PORTS, m in 1..maxvisits[i], v in SHIPS} >=0;


###TIME VARIABLES

var t {i in PORTS, m in 1..maxvisits[i]} >=0, <=T;

### STOCK VARIABLES
var S {i in PORTS, m in 1..maxvisits[i]}>=0;


################OBJECTIVE FUNCTION###############
minimize Total_Cost:
sum {i in PORTS, m in 1..maxvisits[i], j in PORTS, n in 1..maxvisits[j], v in SHIPS, s in OPTIONS[v]:
i<>j} TravelCost[i,j,v,s] * G[i,m,j,n,v,s] +
sum {v in SHIPS, i in PORTS, m in 1..maxvisits[i], s in OPTIONS[v]} TravelCostOrig[i,v,s] * Go[i,m,v,s] +
sum {v in SHIPS, i in PORTS, m in 1..maxvisits[i]} PortCost[i,v] * O[i,m,v];


### SPEED CONSTRAINTS
subject to SPEED_CHOICE1 {i in PORTS, m in 1..maxvisits[i], j in PORTS, n in 1..maxvisits[j], v in
SHIPS: i<>j}:
sum {s in OPTIONS[v]} G[i,m,j,n,v,s] = X[i,m,j,n,v];


subject to SPEED_CHOICE2 {i in PORTS, m in 1..maxvisits[i], v in SHIPS}:
sum {s in OPTIONS[v]} Go[i,m,v,s] = Xo[i,m,v];


### ROUTING CONSTRAINTS
subject to FLOW1 {v in SHIPS}:
sum {j in PORTS, n in 1..maxvisits[j]} Xo[j,n,v] + Zo[v]=1;


subject to FLOW2 {v in SHIPS, i in PORTS, m in 1..maxvisits[i]}:
W[i,m,v] - sum {j in PORTS, n in 1..maxvisits[j]: i<>j} X[j,n,i,m,v] - Xo[i,m,v]=0;


subject to FLOW3 {v in SHIPS, i in PORTS, m in 1..maxvisits[i]}:
W[i,m,v] - sum {j in PORTS, n in 1..maxvisits[j]: i<>j} X[i,m,j,n,v] - Z[i,m,v]=0;


subject to SHIP_VISIT {i in PORTS, m in 1..maxvisits[i]}:
sum {v in SHIPS} W[i,m,v] = Y[i,m];


subject to PORT_VISIT {i in PORTS, m in 2..maxvisits[i]}:
Y[i,m-1] - Y[i,m] >= 0;


subject to MANDATORY_VISITS {i in PORTS, m in 1..Nmin[i]}:
Y[i,m] = 1;

### ###LOADING AND UNLOADING CONSTRAINTS

subject to CONSTRAINT1 {i in PORTS, m in 1..maxvisits[i], v in SHIPS}:

$O[i,m,v] >= Q[i,m,v]/ShipCap[v];$

subject to CONSTRAINT2 {i in PORTS, m in 1..maxvisits[i], v in SHIPS}:

$Q[i,m,v] <= \min(ShipCap[v], Qmax[i]) * W[i,m,v];$

subject to CONSTRAINT3 {i in PORTS, m in 1..maxvisits[i], v in SHIPS}:

$Qmin[i] * W[i,m,v] <= Q[i,m,v];$

subject to CONSTRAINT4 {v in SHIPS, i in PORTS, m in 1..maxvisits[i]}:

$Fo[i,m,v] = InitialLoad[v] * Xo[i,m,v];$

### ###ARC - FLOW MODEL

subject to CONSTRAINT5 {v in SHIPS, j in PORTS, n in 1..maxvisits[j]}:

$Fo[j,n,v] + \sum \{i \text{ in PORTS, m in 1..maxvisits}[i]: i<>j\} F[i,m,j,n,v] + J[j] * Q[j,n,v] =$

$\sum \{i \text{ in PORTS, m in 1..maxvisits}[i]: j<>i\} F[j,n,i,m,v] + Fd[j,n,v];$

subject to CONSTRAINT6 {i in PORTS, j in PORTS, m in 1..maxvisits[i], n in 1..maxvisits[j], v in SHIPS: i<>j}:

$F[i,m,j,n,v] <= (ShipCap[v] - Qmin[i]) * X[i,m,j,n,v];$

subject to CONSTRAINT7 {j in PORTS, n in 1..maxvisits[j], v in SHIPS}:

$Fd[j,n,v] <= ShipCap[v] * Z[j,n,v];$

### ###TIME CONSTRAINTS

subject to START_TIME {i in PORTS, j in PORTS, m in 1..maxvisits[i], n in 1..maxvisits[j]: i<>j}:

$t[i,m] + \sum \{v \text{ in SHIPS}\} TL[v] * Q[i,m,v] - t[j,n] +$

$\sum \{v \text{ in SHIPS, s in OPTIONS}[v]\} \max(U[i,m] + TravelTime[i,j,v,s] - A[j,n],0) * G[i,m,j,n,v,s] <= U[i,m] - A[j,n];$

subject to MIN_INTERVAL {i in PORTS, m in 1..maxvisits[i]:m>1}:

$t[i,m] - t[i,m-1] - \sum \{v \text{ in SHIPS}\} TL[v] * Q[i,m-1,v] - TB[i] * Y[i,m] >=0;$

subject to CONSTRAINT {i in PORTS, m in 1..maxvisits[i]}:

$\sum \{v \text{ in SHIPS, s in OPTIONS}[v]\} TravelTimeOrig[i,v,s] * Go[i,m,v,s] <= t[i,m];$

subject to TIME_WINDOW1 {i in PORTS, m in 1..maxvisits[i]}:

$t[i,m] >= A[i,m];$

subject to TIME_WINDOW2 {i in PORTS, m in 1..maxvisits[i]}:

t[i,m] <= B[i,m];


### INVENTORY CONSTRAINTS

subject to STOCK_START {i in PORTS}:

S[i,1] = InitialStock[i] + J[i] * DemandRate[i] * t[i,1];


subject to RELATE_STOCK {i in PORTS, m in 1..maxvisits[i]:m>1}:

S[i,m] = S[i,m-1] - J[i] * sum{v in SHIPS} Q[i,m-1,v] + J[i] * DemandRate[i] * (t[i,m] - t[i,m-1]);


subject to STOCK_LIMIT1 {i in PORTS, m in 1..maxvisits[i]:J[i]=-1}:

S[i,m] + sum{v in SHIPS} Q[i,m,v] - DemandRate[i] * sum{v in SHIPS} TL[v] * Q[i,m,v] <= UpperStock[i];


subject to STOCK_LIMIT2 {i in PORTS, m in 1..maxvisits[i]:J[i]=1}:

S[i,m] - sum{v in SHIPS} Q[i,m,v] + DemandRate [i] * sum{v in SHIPS} TL[v] * Q[i,m,v] >= LowerStock[i];


subject to LBOUND {i in PORTS:J[i]=-1}:

S[i, maxvisits[i]] + sum{v in SHIPS} Q[i, maxvisits[i],v] - DemandRate[i] * (T-t[i, maxvisits[i]]) >= LowerStock[i];


subject to UBOUND {i in PORTS:J[i]=1}:

S[i, maxvisits[i]] - sum{v in SHIPS} Q[i, maxvisits[i],v] + DemandRate[i] * (T-t[i, maxvisits[i]]) <= UpperStock[i];


subject to LIMIT1 {i in PORTS, m in 1..maxvisits[i]: J[i]=-1}:

S[i,m] >= LowerStock[i];


subject to LIMIT2 {i in PORTS, m in 1..maxvisits[i]: J[i]=1}:

S[i,m] <= UpperStock[i];