Master's degree thesis

LOG950 Logistics

The Travelling Salesman Problem versus Transportation Work

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Preface

This master thesis is a final work of the Master of Science in Logistics program at Molde University College - Specialized University in Logistics. The research has been carried out from January 2017 to May 2017 in order to obtain an MSc in Logistics.

I would like to thank my supervisor professor Øyvind Halskau for the valuable help during the work with this thesis, all comments and remarks.

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Molde, Norway

May 2017

Summary

In this thesis, we introduce the new function, which is modification of Travelling Salesman Problem (TSP). Usually the main goal is to make a route with the smallest cost of travelling or shortest distance. The main purpose of the thesis is to minimize Transportation Work (TrW).

Accordingly, in order to conduct a study, we must compare models on example. Some heuristics cannot be applied to the TrW. Therefore, we have to figure it out and introduce heuristics, which are relevant for TrW. The new model brought changes in routes. Two different routes were build – one for TSP and one for TrW. The results of the calculations showed, that the route, profitable from the distance, not profitable for TrW minimization.

Therefore, we have provided experimental part. In experimental part, we have two experiments for 12 customers. The best routes for the TrW model and TSP models were found. Consequently, we have to show the difference between these models. The main measure of evaluation is a Fuel Consumption (FC).

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1.0 Introduction

The Vehicle Routing Problem (VRP) is a main problem in road transportation planning and is about routing a fleet of vehicles on a given set of roads in order to visit all customers. (T.Bektas, G.Laporte, 2011). It is a combinatorial optimization problem, which solves the problem of designing routes to be used by a set of vehicles to satisfy customers' demands. All vehicles start from the same depot and visit a subset of the customers' one and only one time. All vehicles have a given capacity. There are different types of problems such as:

- Capacitated VRP (CVRP) where the main constraint is the vehicle's capacities
- VRP with Time Windows (VRPTW) where customers should be served according to given time intervals and schedules.
- VRP with multiple depots (MVRP) when we have a few depots and serve customers according to cost of travelling or distance from the chosen depot.
- VRP with pickups and deliveries (VRPPD) where some of customers have goods to pick up and deliver. However, some of the customers may need only delivery or just a pick up.

The VRP is a generalization of the Traveling Salesman Problem (TSP), where a salesman has to visit a set of customers using a fleet of (homogeneous) vehicles, which belong to one depot (C.Zezzatti et.at, 2012, p. 209). The task is to find a route through a given set of cities with shortest possible length. In TSP, we have only one vehicle, which has to serve all customers. This trip called a Hamiltonian cycle. This means a sequence of nodes and edges. The vessel will start in any node and finish at the starting node. In TSP and VRP models, customers have a demand and a car has a given capacity. The cost of travelling is given in a cost matrix and may be symmetric or asymmetric. The objective is to minimize the cost or distance.

One of the main challenges for making decisions in transportation refers to different kinds of constraints. In this thesis, we will change the objective of a travelling salesman problem from classical objectives like distance, into an objective, that traditionally belongs to transportation economy, namely transportation work (TrW), which has some relation to the reduction of emissions. TrW is basically a concept in transportation economy. TrW is defined as the load on vehicle multiply by a distance of the route. The main idea is to minimize the TrW and through

this, reduce emissions from the vehicles. If we minimize the total transportation work, the total energy used may be less. It means that we may reduce fuel consumption (I. Kara, B.Y.Kara, M.K.Yetis, 2007).

From the Figure 1, we see that consumption of fuel changes with the vehicle load. Research shows, that reducing of the vehicle weight for 10 % will reduce fuel consumption by 4.5-8.0%. Other researchers describe the benefit in the improvement in fuel consumption from 0.15–0.70 L/100km for each 100 kg of weight reduction. (A.Bandivadekar et al., 2008).

It becomes obvious, that with a decrease of the transportation work, the fuel consumption goes down. It may help companies to save money. Therefore, it may be useful to minimize TrW to be competitive.

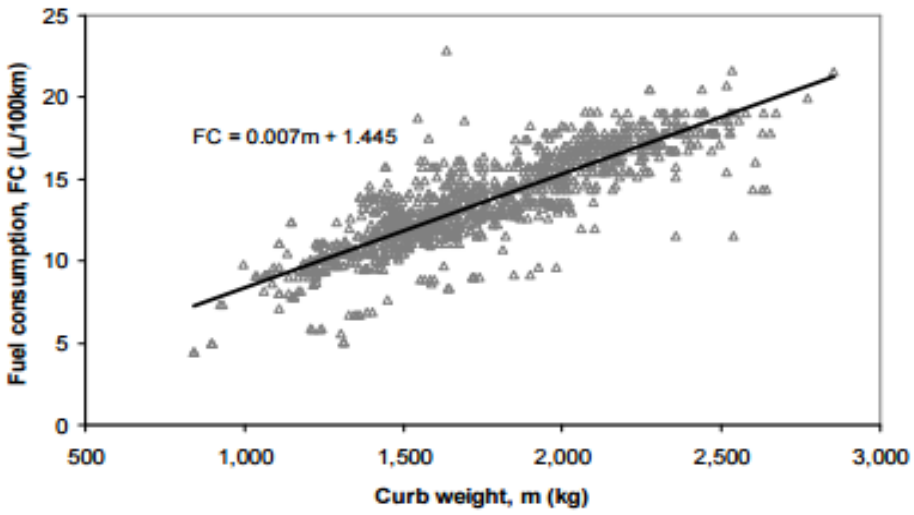


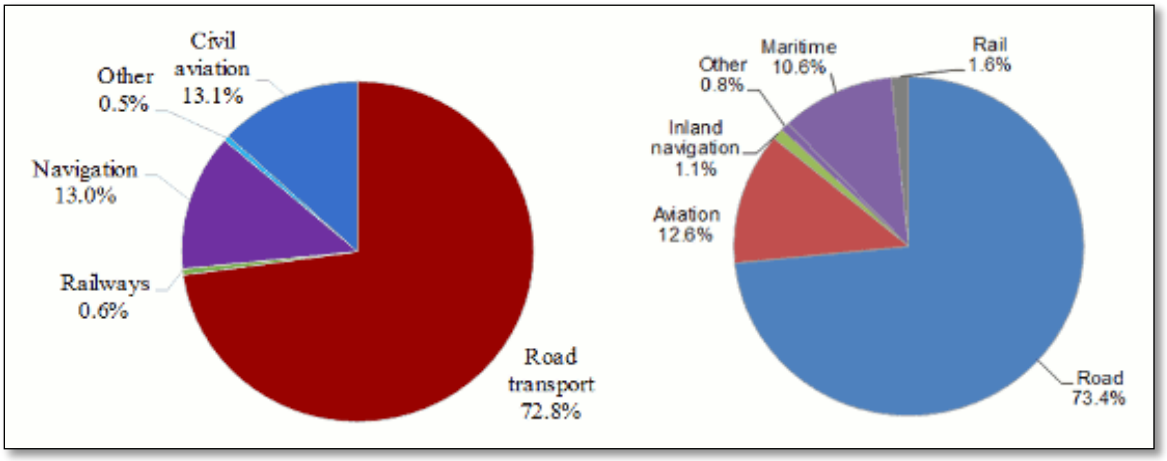
Figure 1. Curb weight and fuel consumption model

(Source: Bandivadekar, A, et al., 2008 p.45)

Nowadays, transport contributes to a growing amount of greenhouse gas emissions (GHG). GHG keeps heat and makes the planet warmer. The main sources of it are production of electricity by coal oil and gas, industry, agriculture, transportation. According to the United States Environmental Protection Agency transportation, cover 26% as a source of the GHG. It includes the usage of fossil fuel for vehicles, planes, vessels and trains.

Vehicle weight is one of the most important parameters that influence a vehicle's fuel consumption and hence, CO_2 emissions. In this sector, road transport takes more than 70% of

all GHG emissions from transport in 2014 (European Commission, ec.europa.eu), see Figure 2a and 2b below:



(a) *(b)*

Figure 2. (a): Greenhouse gas emissions from transport in 2014 (b): Percentages of transport energy demand by form in 2014

(*Source: European Commission website)*

Roads are the most common and popular mode of transportation. It takes 73.4% of the demand (Figure 2. (b)). It is obvious if roads are in demand, tracks, cars will create the largest percentages of pollution. Figure 2(a) shows that via all sources of transport, road transport contributes almost 73%.

After all the statistics above, we conclude that the problem of emissions has grown to large dimensions and have to be solved.

2.0 Problem description

A huge number of articles and books highlight problems of VRP and TSP. After a detailed description of VRP and TSP, it should be emphasized, that VRP is a generalization of TSP. In this thesis, we will consider one and only one vehicle that is the TSP –version of the problem.

The main idea is to make an optimal route for the travelling salesman, who has to visit all nodes once and return to the depot, but not minimizing costs, time or distance, but minimizing transportation work, that is the load on the vehicle multiplied by the driven distance between the nodes. Usually, in TSP we start from any node and end in the same node. The traveling salesman has to make the best route with the smallest cost of travelling. In our case –which is a variation of TSP – we have to specify the depot since the optimal tour will depend on the choice of the depot. We are going to use rules for a Vehicle Routing Problem and will start from the depot, but the vehicle will be just one.

Consequently, we will make some changes in the model of TSP. The model will be made to calculate transportation work. With a small number of customers, it is possible to make a calculation by hand. The number of possible solutions is $(n-1)!$, where n is a number of customers. Therefore, if the number of customers increases, it is impossible to do the calculations by hand. Changing from a classical TSP-model to a model minimizing TrW may lead to larger driven distances than in the TSP solution. On the other hand, the TSP can have a larger TrW, than the optimal one. There are some reasons why we are going to change our objective function and why it is so important to calculate transportation work.

The reason is an emission problem. Transport fabricates approximately 25-30% of CO2 emissions in the world and the percentage is growing. From Figure 3 we see that the transport sector brings the largest share of pollution. This again proves the need for reducing pollution from transport.

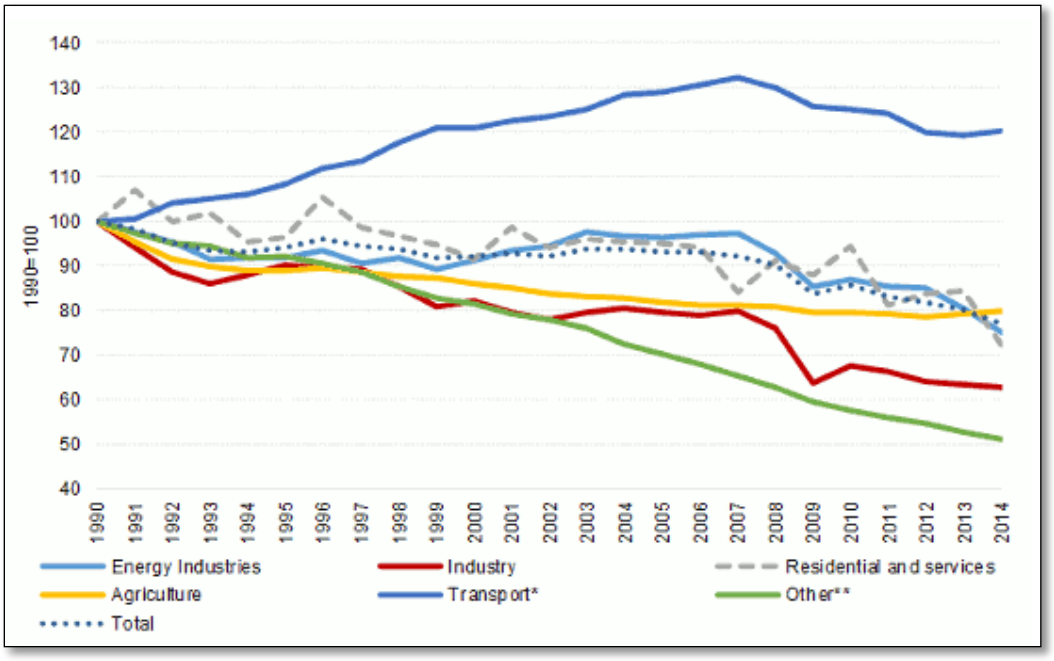


Figure 3. CO2 emission by the sector

(Source: European Commission website)

Transport contributes a large amount of greenhouse gas emissions. Hence, there will exist a route that minimize the transportation work and as a consequence possibly reducing the emission from the vehicle, but at a higher cost in terms of travelled distance. Consumption of the fossil fuel could be reduced. There are three ways to reduce C02 emission:

1) Reduce the distances of traveling by vehicle

2) Change to another fuel with lower greenhouse gas emission

3) Improve vehicles standards

A smaller amount of transportation work may give less fuel consumption and as a consequence less pollution. The price for fuel seems to grow. This is a problem for logistics companies, which have sets of vehicles. An empty vehicle needs less fuel, compared to the fully loaded vehicle. Companies are interested in reducing the amount of fuel consumption.

Therefore, the question is: how are we going to reduce it? For a given vehicle we have a given fuel consumption per km. It depends on the given engine and type of the vehicle. As we know, fuel consumption will depend on the load on the vehicle, so we might build a route, which might

make it smaller. Therefore, the point is to minimize the TrW. Does it bring a good result? By minimizing the TrW we may reduce the fuel consumption. If we reduce the fuel consumption, we will reduce the money, spent on fuel. We have to analyses and see, what benefit will come from the reduction of the TrW.

3.0 Literature Review

3.1 The two main concepts

C. Barnhart et al define (2006) "The Classical Vehicle Routing Problem (VRP) is one of the most researched problems in combinatorial optimization, and its study has given rise to several exact and heuristic solution techniques of general applicability".

The first article in VRP was written by Dantzig and Ramser (1959).The original name of the article is "The Truck Dispatching Problem". The problem depicts delivery of gasoline to service stations. The task formulates as a generalization of TSP, namely Capacitated Vehicle Routing Problem.

Traveling Salesman Problem (TSP) is a well-known combinatorial optimization problem. In this kind of problems, we are trying to minimize total distance or cost. The term TSP has been around from in 1931-32 (E.LLawler et.al, 1986)

Authors like Croes in 1958 in the paper "A method for solving travelling salesman problem" shade a light on it.

Roberts and Flores published in 1966 the article "An engineering approach to the travelling salesman problem" where they work with the problem by excluding links, which were not optimal.

In 1976, Cristofides presented a heuristic method, based on the minimal spanning tree. The algorithm helps to find approximate solutions. Nowadays, it is the heuristics with the best worst-case result.

Most of our entire problem concerns Capacitated Vehicle Routing Problem (CVRP). CVRP is the most common type of problem, where each vehicle has capacity constraints, on the amount of load it can carry. CVRP generalizes the TSP. The problem consists of searching of *K* circuits, where each refers to a vehicle route. A collection of circuits has to be with minimum cost.

Initially, the vehicles are located at a single depot and capacity restriction is the main measure. (P.Toth, D.Vigo, 2002)

In the last 40 years, VRP and TSP heuristics has been developed. Some of the heuristics became classical. Examples can be Clarke and Wright saving heuristic, Sweep algorithm, the Fisher and Jaikumar algorithm.

The Clarke and Wright saving heuristic is one of the well-known. The algorithm was proposed in the 1964 year by Clarke. G and Wright. J.R. The algorithm based on the merging two routes into one with the best saving.

Fisher and Jaikumar algorithm was proposed in 1981. The algorithm works in two steps. Firstly, we should make feasible clusters, by solving a generalized assignment problem (GAP) then we shall make a route for each cluster, like a TSP problem.

The sweep algorithm. Several different authors have worked on the algorithm. Here we should mention the research for the vehicle dispatch problem (Gillett.BE and Miller.LR, 1974).The algorithm works as creating feasible routes by adding customers in the vehicle route.

More details about the heuristics are given in chapter 4.

3.2 Transportation Work

The concept of transportation work is important in transportation economy. Some problems have been highlighted, mostly, emission questions.

In 2007, the article "Energy minimizing vehicle routing problem" by Kara et.al was published. The problem represents Capacitated Vehicle Routing Problem (CVRP). Then the problem has been expanded by a transportation work (load multiplied by distance). The new problem is called Energy Minimizing Vehicle Routing Problem (EMVRP). The result shows the difference between solutions with load objective and distance –minimization. This research is closely related to our topic.

Studies, related to reducing emission in shipping (Fagerholt et al., 2010) and emission in freight transport (Bauer et al., 2010) have appeared. The first article describes route for the ship, where such parameters as time windows and speed play an important role. Then several alternative mathematical models with a different primary decision variable were presented.

Resent research “The Pollution –Routing Problem” by Bektas and Laporte (2011) represents Pollution Routing Problem (PRP). PRP is an extension of VRP model. This study is more extensive because it includes different aspects of transportation. The article addresses new mathematical models with different extensions, such as the amount of greenhouse emission, fuel consumption, travel time and their costs. The mathematical models with different parameters were included in the model and then a comparative analysis was performed. The effect of time windows, speed, cost considerations, the number of vehicles was evaluated and analyzed.

4.0 Heuristics

VRP is a generalization of TSP and m-TSP. The problem is referring to making routes with minimum total costs. The model input: depot, the number of customers to serve, and customers’ demands. Each vehicle has limited capacity. Vehicles dispatched from a depot, serve a subset of customers, and then go back to the depot. The customers are visited exactly once. The purpose is to find an optimal set of routes, which will satisfy requirements.

The traveling salesman problem (TSP) is to find a routing for a salesman, who starts from a depot, visits a prescribed set of cities and returns to the original location in such a way that the total distance travelled is minimum and each city is visited exactly once. See Gutin et.at, 2007.

Two finite sets V and E are contained in a graph G. The elements of V consist of all vertex. The elements of E consist of all edges. For example, the set V could be {a, b, c, d, e, f, g, h}, and E may be {{a, d}, {a, e}, {b, c}, {b, e}, {b, g}, {c, f}, {d, f}, {d, g}, {g, h}}. (J.M. Harris et al., 2008).

A sequence of nodes and edges, starting in any node and go through all the other nodes and returning to the start node is called a Hamilton – cycle (H-cycle). Below are some variations of TSP, which have been described from situations in real life.

The cost of moving from one node to another could take different costs. According to this, there are two types of matrices – symmetric and asymmetric matrices. We make a route, and start from the depot and go to one node then to the second one, and then we might go back. In the

case of symmetric matrices, we choose the same arcs (roads) to go back, so the distance will be the same. In case of asymmetric matrices, the cost may be different, either smaller or bigger.

Symmetric matrices.

The cost of travelling from one node to another is:

c_{ij} ,if $c_{ij} = c_{ji}$ for all i, j, the cost matrix is said to be symmetric

In an asymmetric matrix, it will be different for some i and j

$c_{ij} \neq c_{ji}$ for at least one pair of nodes

A common method for solving TSP is using a heuristic. Heuristics for TSP can be divided into different categories:

• Tour construction methods

• Tour improvement methods.

• Composite methods

• Mathematically based methods

Tour construction method. In these heuristics, we construct an H-cycle step by step – usually node by node – using some rules or criteria. Some of the main algorithms can be:

The sweep-algorithm, nearest neighbor, Clarke and Wright's saving algorithm.

4.1 Clarke and Wright's saving algorithm

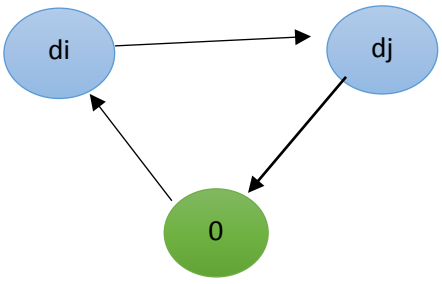


Figure 4. Location of the nodes in Clarke and Wrights algorithm

This algorithm allows merging 2 routes into 1 route.

$c_1 = 2c_{0i} + 2c_{0j}$ -visiting 2 nodes separately

$c_2 = c_{0i} + c_{ij} + c_{j0}$ -visiting 2 nodes directly

$s_{ij} = c_1 - c_2$

After that one calculates a distance, called saving $s_{ij} = c_{0i} + c_{0j} - c_{ij}$

The algorithm works as follow:

Step 1: We calculate savings for all pairs of nodes.

Step 2: Make saving matrix.

Step 3: Find the largest savings, connect the two nodes.

Step 4: Find the next largest savings, connect the two nodes.

Step 5: Make a route, which should not include sub-cycles and nodes cannot get degree 3 or larger.

Clarke and Wright's saving algorithm applied to a TrW situation

$TrW_1 = d_i c_{0i} + d_j c_{0j}$ -visiting 2 nodes separately

$TrW_2 = (d_i + d_j)c_{0i} + d_j c_{ij}$ -visiting 2 nodes directly

$TrW_1 - TrW_2 = d_i c_{0i} + d_j c_{0j} - (d_i + d_j)c_{0i} - d_j c_{ij}$

$TrW_1 - TrW_2 = d_j c_{0j} - d_j c_{0i} - d_j c_{ij}$

$TrW_1 - TrW_2 = d_j(\underbrace{c_{0j} - (c_{0i} + c_{ij})}_{\leq 0})$

$TrW_1 < TrW_2$

From the results, we conclude that algorithm, applied for TrW situation does not bring a good solution. Saving gives a negative number in the brackets. We refer to the triangle inequality: the length of one side in triangle should be less than the sum of lengths of the two other sides. In this case, according to TrW minimization, it is better to visit two customers separately.

That is, the optimal solution is to visit all customers separately creating a hub and spoke system, where the depot is the hub.

In the previous example, we tried to go 0-*i*-*j*-0.We can try to go 0-*j*-*i*-0 now

$TrW_1 = d_j c_{0j} + d_i c_{0i}$ -visiting 2 nodes separately

$TrW_2 = (d_j + d_i)c_{0j} + d_i c_{ji}$ -visiting 2 nodes directly

$TrW_1 - TrW_2 = d_j c_{0j} + d_i c_{0i} - (d_j + d_i)c_{0j} - d_i c_{ji}$

$TrW_1 - TrW_2 = d_i c_{0i} - d_i c_{0j} - d_i c_{ji}$

$TrW_1 - TrW_2 = d_i(c_{0i} - (c_{0j} + c_{ji}))$

≤ 0

$TrW_1 < TrW_2$

The situation is the same and we refer to the triangle inequality again.

4.2 Sweep algorithm

Step 1 Choose one node randomly.

Step 2 Then put an arrow, the end of arrow locates at the chosen node, and head of the arrow at another node.

Step 3 After that we should choose a direction of turning the arrow - clockwise or counter clock wise. The cycle is created step by step as the arrow reaches the other nodes as it turns. Then we make a route, all nodes have to be covered.

We have no possibility to change the algorithm. For the TrW situation, it works the same as for TSP.

4.3 Nearest neighbor

Step 1: Choose one node randomly.

Step 2: Find the nearest node to the first node, which we picked at step 1, and add to the route. If 2 nodes have the same distance, we should make 2 alternative routes.

Step 3: Repeat steps until all nodes will be covered, and then calculate the cost of routes.

Nearest neighbor applied at a TrW situation

Step 1: Choose one node randomly.

Step 2: Then make pairs of the depot and each node and calculate transportation work for each pair.

Step 3: Compare results and pick the smallest number.

Step 4: Repeat steps until all nodes will be covered, and then calculate the total transportation work.

4.4 2-exchange algorithm

This is an improvement method

2-exchange for TSP

Step 0: Start with any Hamilton-cycle.

Step 1: Choose 2 edges, not neighbors.

Step 2: Delete these edges.

Step 3: Use the two edges that make a new Hamiltonian cycle H_2 .

Step 4: Calculate the new distance H_2 .

2-exchange applied at a TrW situation

Step 0: Start with any Hamilton-cycle, the direction of the edges given.

Step 1: Choose 2 edges, not neighbors.

Step 2: Delete this edges.

Step 3: Use the two edges that make a new Hamiltonian cycle H_2 .

Step 4: Calculate the new TrW on the new H_2. The calculations must me made clockwise and counter clock wise. Choose the best and compare with the original.

4.5 Special constructed heuristics for the TrW

So far, we have tried to adopt classical heuristics for TSP. Below we offer a heuristics special made for TrW:

$$TrW = L_0 c_{oi_1} + \left(L_0 - d_{i_1}\right)c_{i_1 i_2} + \left(L_0 - \left(d_{i_1} + d_{i_2}\right)\right) c_{i_2 i_3}$$

$$+. \ . \ .+(L_0 - \left(d_{i_1} + d_{i_2} +. \ . \ .+ d_{i_k}\right))c_{r_{k-1}} c_{i_r}$$

$$+. \ . \ .+(L_0 - (d_{i_1} + d_{i_2}+. \ . \ .+ d_{i_k} +. \ . \ .+ d_{i_{n-1}})c_{i_{n-1,} i_n}$$

The expression above can be reorganized to the following:

$$TrW = \ L_0 \ (c_{oi_1}+ c_{i_1 i_2} + c_{i_{k-1,} i_k} +. \ . \ .+ c_{i_{n-1,} i_n})$$

$$- d_{i_1} \ (c_{i_1 i_2} + c_{i_2 i_{13}} +. \ . \ .+ c_{i_{k-1,} i_k} +. \ . \ .+ c_{i_{n-1,} i_n})$$

$$-d_{i2} \ (c_{i2} c_{i3} +. \ . \ .+ \ c_{i(n-1),in} \ )$$

$$-d_{i_n} c_{i_{n-1,} i_n}$$

From the last expression, it is easy to see, that the TrW may become small if we go to the node with the largest demand first, then to the next largest.

5.0 Methodology

The methodology includes exact methods and heuristics. For solving combinatorial optimizations problem we will use heuristic methods. Since the research includes the work with one vehicle, it can be such heuristics as was described above:

- Clark and Wright savings heuristic
- Sweep algorithm
- Nearest neighbor heuristic
- 2-exchange heuristics

The research will start with a small example to show the algorithm. Initially, we will try heuristics for the TSP problem. Then we will try to implement the same heuristics for a TrW

problem. So, a part of the master thesis will be to check which of the classical heuristics can work in the TrW situation, which ones that can be modified. We have already mentioned that Clark and Wright, Sweep does not work with TrW. We may increase the number of nodes and try to implement exact methods. If it does not bring a result, we will go back to heuristics methods.

Exact methods include branch and bound, dynamic programming, linear and integer programming, enumerative method. In an exact solution, we will change objective function from the classical TSP to the new one, namely TrW model. Since our research is about the TrW, we conclude, that the weight of the vehicle takes an important part in a model. Therefore, we should include the weight of the vehicle make a route.

The most important part is an analysis of results. We have to compare results, which we will have from the experimental part. The first way is a building two routes –TSP and TrW. Then its calculation of the TrW and the distance of the travelling for both routes. As we work with new objective function, it should bring some changes in the new routes. From our research, fuel consumption is one of the main indicators. If we will have less fuel consumption in the new model, then it means that minimization of TrW is playing a significant role in routing. Therefore, we should make a comparative analysis.

6.0 An illustrative example

We provide a small example to illustrate the differences between TSP and TrW. We have one vehicle, which should visit 4 customers. The vehicle load is 220 and it is equal to the customer demand. The purpose is to make an optimal route for a vehicle. In our case, the route with the smallest Transportation Work and compare it with TSP-solution.

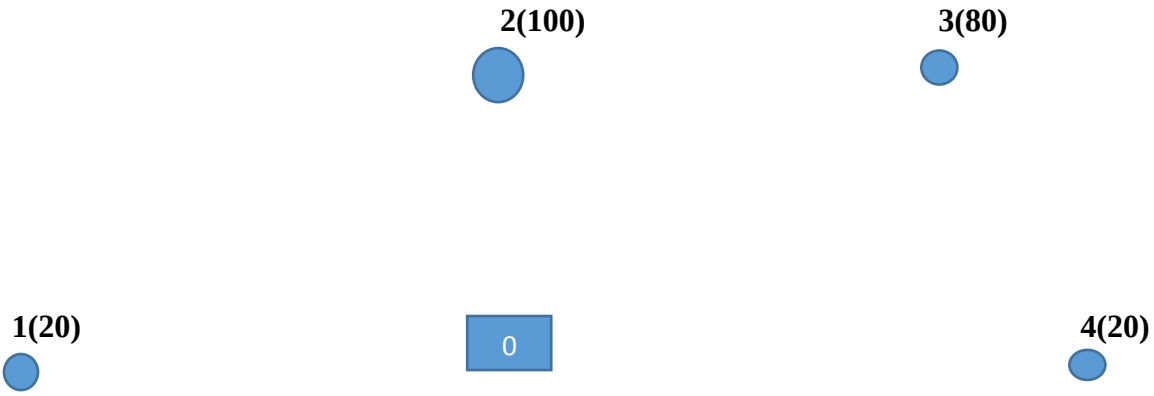


Figure 5 . Position of the customers with a demand

(Customers demand in brackets, 0 is the depot)

Table 1 . Distance matrix for traveling from one node to another

	0	**1**	**2**	**3**	**4**
0	-	70	25	50	85
1		-	75	110	155
2			-	35	90
3				-	55
4					-

The number of H-cycles will be *(n-1)!* We have 5 nodes, so in our case (5-1)! = 24 possible solution. In table 2 all H-cycles are given together with the distance and the associated transportation work. In a fourth column the weight of the vehicle was included and the corresponding TrW. Therefore, we got a different solution. It is assumed, that the weight of the vehicle is equal to 25.

Table 2. The calculations of the distance and transportation work of H-cycles

Tour	**Dist (km)**	**Weight of the vehicle not included**	**Weight of the vehicle included**
		TrW	**TrW**
1)0-1-2-3-4-0 70+75+35+55+55	**320**	70*220+75*200+35*100+55*20= 35000 *	42250
2)0-1-2-4-3-0 70+75+90+55+50	**340**	70*220+75*200+90*100+55*80= 43800	52300
3)0-1-3-2-4-0 70+110+35+90+85	**390**	70*220+110*200+35*120+90*20= 45600	53150
4)0-1-3-4-2-0 70+110+55+90+25	**350**	70*220+110*200+55*120+90*100= 53000	61750
5)0-2-1-3-4-0 25+75+110+55+85	**350**	25*220+75*120+110*100+55*80= 29900	38650
6)0-2-1-4-3-0 25+75+155+55+50	**360**	25*220+75*120+155*100+55*80= 34400	43400
7)0-2-3-1-4-0 25+35+110+155+85	**410**	25*220+35*120+110*40+155*20= 17200	27450
8)0-2-3-4-1-0 25+35+55+155+70	**340**	25*220+35*120+55*40+155*20= 15000 *	23500
9)0-1-4-2-3-0 70+155+90+35+50	**400**	70*220+155*200+90*180+35*80= 65400	75400
10)0-1-4-3-2-0 70+155+55+35+25	**340**	70*220+155*200+55*180+35*100=	

		59800	68300
11)0-2-4-1-3-0 25+90+155+110+50	**430**	25*220+90*120+155*100+110*80= 30600	51350
12)0-2-4-3-1-0 25+90+55+110+70	**350**	25*220+90*120+55*100+110*20= 24000	32750
13)0-3-1-2-4-0 50+110+75+90+85	**410**	50*220+110*140+75*120+90*20= 37200	47450
14)0-3-1-4-2-0 50+110+155+90+25	**430**	50*220+110*140+155*120+90*100= 54000	64750
15)0-3-2-1-4-0 50+35+75+155+85	**400**	50*220+35*140+75*40+155*20= 22000	32000
16)0-3-2-4-1-0 50+35+90+155+70	**400**	50*220+35*140+90*40+155*20= 22600	32600
17)0-3-4-1-2-0 50+55+155+75+25	**360**	50*220+55*140+155*120+75*100= 44500	53800
18)0-3-4-2-1-0 50+55+90+75+70	**340**	50*220+55*140+90*120+75*20= 31000	39500
19)0-4-1-2-3-0 85+155+75+35+50	**400**	85*220+155*200+75*180+35*80= 66000	76000
20)0-4-1-3-2-0 85+155+110+35+25	**410**	85*220+155*200+110*180+35*100= 73000	83250
21)0-4-2-1-3-0			

85+90+75+110+50	**410**	85*220+90*200+75*100+110*80= 53000	63250
22)0-4-2-3-1-0 85+90+35+110+70	**390**	85*220+90*200+35*100+110*20= 42400	52150
23)0-4-3-1-2-0 85+55+110+75+25	**350**	85*220+55*200+110*120+75*100= 50400	59150
24)0-4-3-2-1-0 85+55+35+75+70	**320**	85*220+55*200+35*120+75*20= 35400	43400

From the table, we see that there is huge difference both- in the distance and transportation work. From the transportation work route 8, vehicle load minimizing tour **0-2-3-4-1-0**, TrW is15000. The total distance is 340 km.

If we look to the tour 10, with the same distance 340 km, transportation work has changed significantly. The combination of nodes has been changed to **0-1-4-3-2-0**, but transportation work grew up almost 4 times.

There is one more interesting example. Tours with the same distance 350 km give us different numbers in a transportation work. In the first case, it is 50400, in the second is 34400, in another one 24000. The last one could be a candidate for the solution with medium distance and optimal transportation work.

From the table 2, the distance minimizing route is 0-**1-2-3-4-0** (total distance is 320 km).The transportation work is 35000.

So, from this example, we see, that in order to minimize the TrW, we get a tour that is slightly longer than the TSP-tour. Two most important cycles are illustrated below.

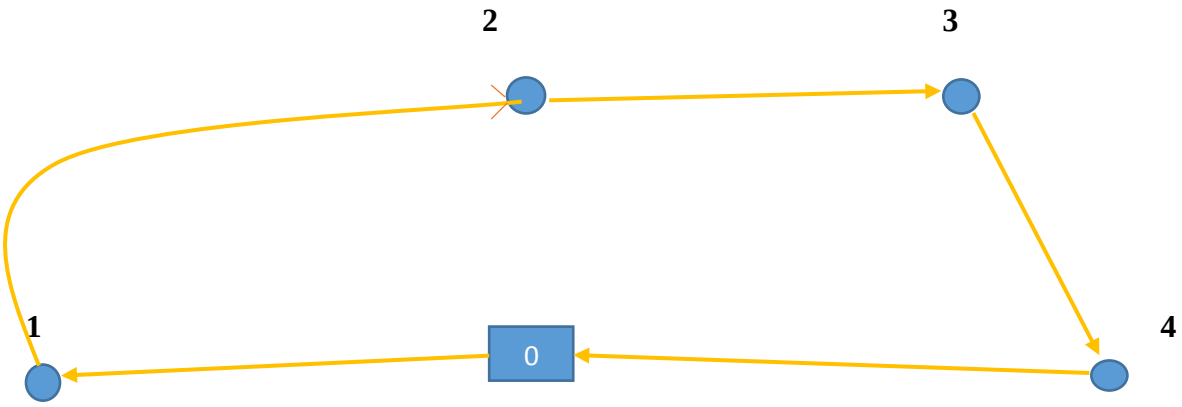


Figure 6. The distance minimizing tour

Tour: 0-1-2-3-4-0

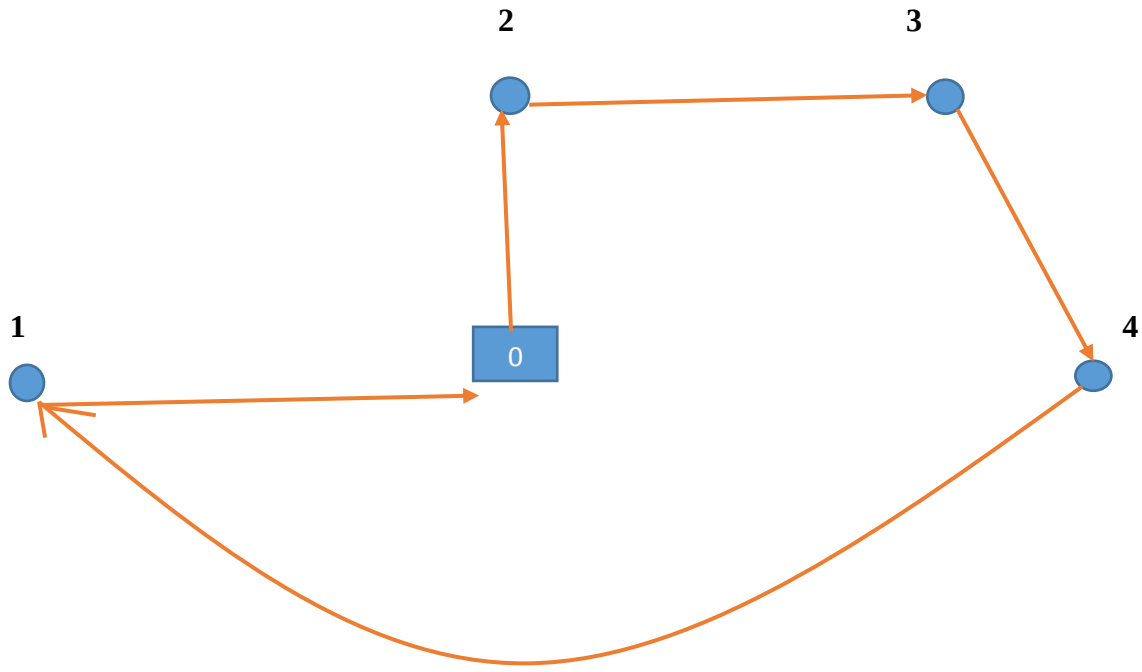


Figure 7. The transportation work minimizing tour

Tour: 0-2-3-4-1-0

Now we are going to show on example how heuristics work for TrW.

Nearest neighbor

In the classical algorithm, we start from the depot and add the nearest node to the depot. In our example, it is node 2. Then we found the next closest node – node 3.We repeat all steps until we have included all customers on the route.

Nearest neighbor route: 0-2-3-4-1-0 (starting from the depot), which is not the optimal TSP-tour.

Nearest neighbor (Transportation work)

We will now use this small example to illustrate different heuristics for TrW.

On the first step, we have to make pairs of the depot and all customers. We show with an asterisk the smallest TrW.

1)0-1: 70*20=1400 (*)

0-2:25*100=2500

0-3: 50*80=4000

0-4: 85*20=1700

The smallest number, which we got, belongs to the edge 0-1, so in the second step, we will continue our route from the node 1.

2) We make pairs of the node 1 and other nodes. In the next steps, we will make the same calculations.

1-2: 75*100=7500

1-3: 110*80=8800

1-4: 155*20=3100(*)

We continue the path with the node 4

3) 4-2: 90*100=9000

4-3: 55*80=4400 (*)

4) 3-2: 35*100 =3500

2-0: 25

The route for the transportation work: 0-1-4-3-2-0.

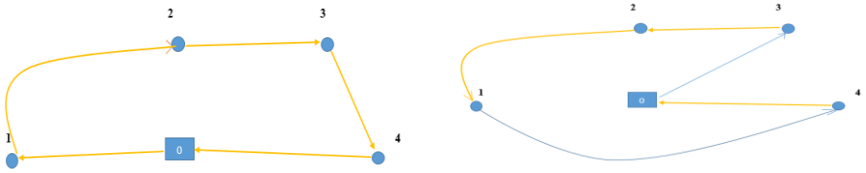
TrW respectively 68300(weight of the vehicle is included) and 59800(weight of the vehicle is not included).

The route the other way around: 0-2-3-4-1-0.The best result of TrW is 23500(weight of the vehicle is included) and 15000(weight of the vehicle is not included).

Hence, the nearest neighbor finds the optimal solution in the TrW case, but not in the TSP-case.

2 –exchange algorithm

We start with the H-cycle to the left.



We remove edges 3-4 and 1-0, change it to 3-0 and 1-4.

The route now is 0-3-2-1-4-0, and then we will calculate new TrW. We assume that the weight of the vehicle is 25.

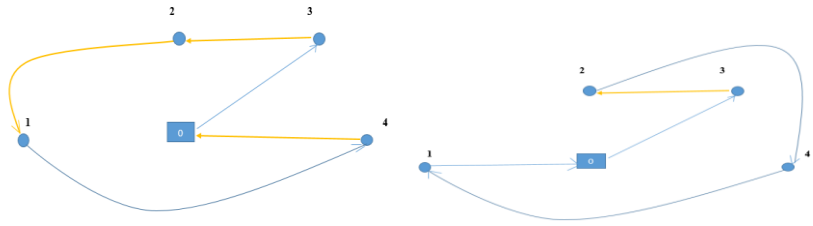
TrW = 50*245+35*165+75*65+155*45+85 *25 = 32000

The vehicle might move in the opposite direction 0-4-1-2-3-0, therefore we should calculate TrW in this case as well, giving

TrW = 85*245+155*225+75*205+35*105+50*25=76000

Comparison shows, that the first variant is much better, so the tour is 0-3-2-1-4-0.

Then we might make new changes, so we delete edges 4-0 and 1-2, use new edges 2-4 and 1-0, the route is 0-3-2-4-1-0.

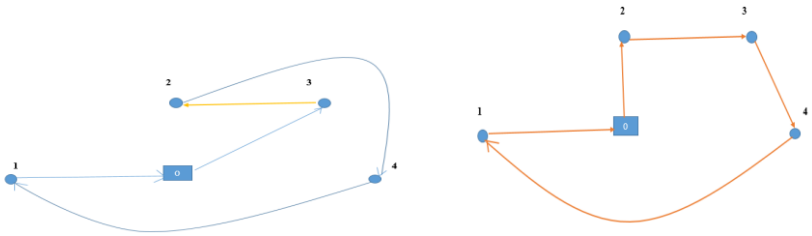


TrW = 32600 (the same c method for a calculation like in a previous example).

The route the other way around: 0-1-4-2-3-0.

TrW = 75400

In the next attempt we remove edges 2-4 and 0-3 and change to 0-2 and 3-4.



The route is 0-2-3-4-1-0, TrW = 23500, which is the optimal TrW solution.

The route is 0-1-4-3-2-0, TrW = 68300

From the example, we conclude that it is possible to use heuristics in the TrW situation.

Special constructed heuristics for the TrW

In special constructed heuristics we are going to the customers with the biggest demand first, then serve next customer with the largest demand. In our case, the route is 0-2-3-4-1-0.

TrW (weight of the vehicle is not included)= 220*25+(220-100)35*+(220-(100+80))*55+(220-(100+80+20)) *155 = 15000

TrW (weight of the vehicle is included)=245*25+(245-100)35*+(245-(100+80))*55+(245-(100+80+20)) *155+(245-(100+80+20+20))*70 = 23500

With the special constructed heuristics, we immediately found the best solution.

7.0 Mathematical models

Here we will describe the mathematical models, which we are going to use. There are two models: a TSP model and a mathematical model for the transportation work, which based on the classical model of TSP. The goal of the classical model is to minimize a total distance or cost. In our case, it is the total distance. The goal of the second one is to minimize transportation work.

7.1 A standard model for TSP

We assume a set of nodes denoted *i=0, 1, 2...n* where *i=0* is the depot

i= (0, 1...n) depot and nodes

c_{ij} − Distance between node *i* and *j*

X_{ij} *=1,* binary variable, means if arc (*ij*) is used, otherwise *0*

$$(1) min \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} \; X_{ij}$$

Subject to

$$(2) \qquad \sum_{i=0}^{n} X_{ij} = 1, \; \forall \, i, j$$

$$(3) \qquad \sum_{i=0}^{n} X_{ji} = 1 \, , \forall \, i, j$$

$$(4) \qquad X_{ij} \in \{0,1\}$$

(1) The objective function, the goal is to minimize the total cost or distance

Constraints (2) and (3) mean, that the number of arcs enters the node is equal to the number of arcs that exits the node. Hence, each node is visited exactly 1 time

(4) Binary variable means if arc (*ij*) is used the value is *1*, otherwise *0*

The main idea of this thesis is to change the objective of classical Traveling Salesman Problem into the Transportation Work. In this case, we must specify the customer's demand, the vehicle capacity and which node that is chosen as the depot. We also have to introduce a new variable, giving the load on the vehicle.

Parameters

d_i -number of units, needed for a customer *i*

$d_o = 0$ - Depot

$c_{ij} -$ Distance between node *i* and *j*

Q – Capacity of the vehicle

$$Q \geq \sum_{i=1}^{n} d_i$$

w - the weight of the vehicle (if we include it)

Variable

X_{ij} *=1,* if arc from *i* to *j* is used, otherwise *0*

L_i- load on the vehicle, when vehicle leaves the node *i*

$L_0 = \sum_{i=1}^{n} d_i$ -load on the vehicle is equal to the customer demand, when the vehicle leaves the depot

7.2 Mathematical model for the transportation work

$$(4) \quad min \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} \; L_i X_{ij}$$

Subject to

$$(5) \quad \sum_{i=0}^{n} X_{ij} = 1, \; \forall \, i, j$$

$$(6) \quad \sum_{i=0}^{n} X_{ji} = 1 \, , \forall \, i, j$$

$$(7) \quad L_j \leq L_i - d_j + Q(1 - X_{ij})$$

$$(8) \quad L_j \geq L_i - d_j - Q(1 - X_{ij})$$

$$(9) \quad X_{ij} \in \{0,1\} \text{and } L_i \geq 0$$

(4) The objective is to minimize transportation work

Constraints (5) and (6) mean, that the number of arcs enters the node is equal to the number of arcs that exits the node. Hence, each node is visited exactly 1 time.

Constraint (7) means that load on the vehicle when we visit customer j should be less or equal to the load from the previous customer i minus demand of the customer j

Constraint (8) works the same as constraint (7)

In order to get a new function of the quadratic terms in (4)

From the model, we see that our objective is quadratic, but we want a linear model. Then, we need to make some changes in the model.

We introduce a set of variable $Y_{ij} = L_i * X_{IJ}$

$$\text{if } X_{ij} = 0, \text{then } Y_{ij} = 0 \text{ and if}$$

$$X_{\boldsymbol{ij}} = 1, \text{then } Y_{ij} = L_i$$

We also should add 3 new constraints (linearization constraints)

$$(10) \quad Y_{ij} \leq L_i$$

$$(11) \quad \mathrm{Y_{ij}} \leq \mathrm{MX_{ij}}$$

$$(12) \quad L_i - Y_{ij} + MX_{ij} \leq M,$$

Where M is a large number

$$(13) \quad \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} Y_{ij},$$

Hence, by changing the objective (4) to (13) and adding the restrictions (5)-(12) we have a linear model, minimizing the transportation work.

However, the above model does not include the weight w of the vehicle and the corresponding transportation work. If we want to include this, (13) must be changed to (14).

$$(14) \quad \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} Y_{ij} + w \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} X_{ij,}$$

8.0 Experimental part

The experimental part of the thesis consists of two parts. In the first part, we will work with the mathematical tool and build up two models. First one is to minimize the travelling distance (TSP). The second one is based on the first one, but the objective will be to minimization of the transportation work. All data for the practical part was taken from the Operation Research Group Bologna, Italy (http://or.dei.unibo.it/). After that, we shall bring some description and explanation if the new model is better or not. We cannot just make a conclusion based on the AMPL model. The second part of the experiment will bring the next step of the research. We will take the data, which we have from the first part and analyze it. The analysis might be carried out from different sides. With a given data, we can take into consideration fuel consumption.

The problem of fuel consumption is not new, some authors have done researchers and brought result. Xiao, Y., Zhao, Q., Kaku, I., & Xu, Y. (2012) address to the CVRP. They include FCR in the extended model of CVRP and show on the example, that new model helps to reduce fuel consumption. They also pointed, that the fuel cost for the vehicle, which travels for the 1000 km is equal to 60% of the total transportation cost.

Rizet, C., Cruz, C., & Mbacké, M. (2012) have described the case of decreasing CO2. They found a connection between loads on the vehicle, energy efficiency. As the result, they suggest to improve the load factor, according to the reduction of emission.

We will calculate additional fuel consumption for the 2 routes. Then we will make comments about the models, fuel consumption model, results of the research and give some recommendation.

8.1 First experiment

Initially, we have the set of coordinates for the nodes (see table 3(a) below)). From this table, all distances from one node to another can be calculated. A complete distance matrix can be found in the appendix.

Table 3. Node coordinate section (a) and customers demand (b)

node	X	Y
0	30	40
1	37	52
2	49	49
3	52	64
4	20	26
5	40	30
6	21	47
7	17	63
8	31	62
9	52	33
10	51	21
11	42	41
12	31	32

(a)

Customer	Demand
1	7
2	30
3	16
4	9
5	21
6	15
7	19
8	23
9	11
10	5
11	19
12	29
Sum	∑ =204

(b)

Figure 8 below presents the relative positions of the nodes. The red node is the depot, and green nodes are customers.

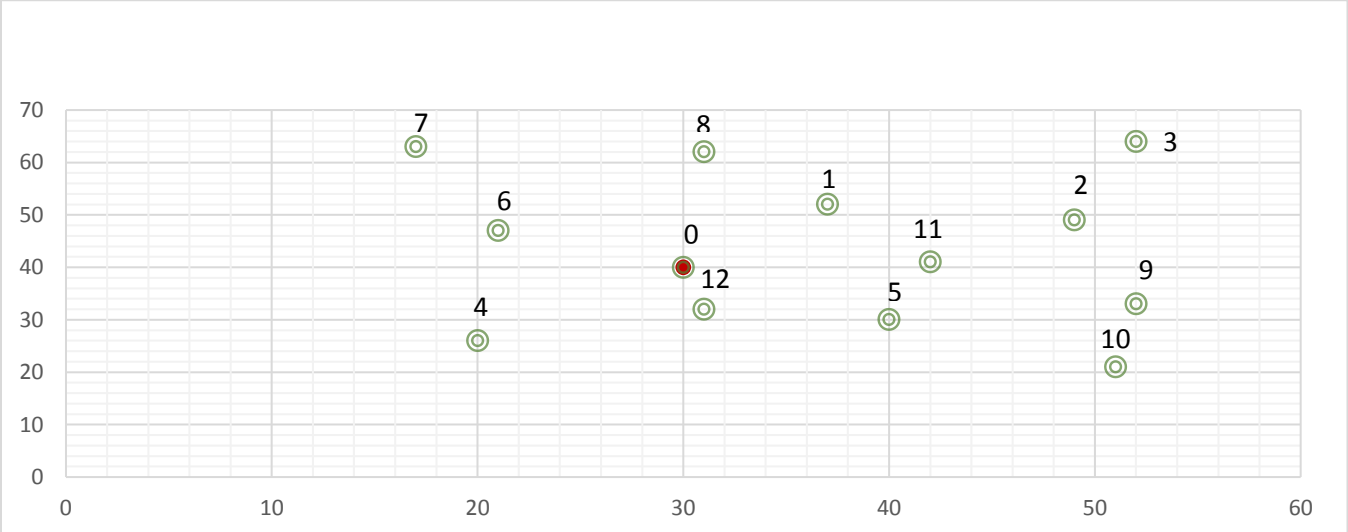


Figure 8. Location of the depot and customers

Figure 9 with arrows is represented the vehicle movement of the model with the smallest total distance. The route is 0-4-12-5-10-9-11-2-3-1-8-7-6-0.Total distance: 176.72. Transportation work: 33795.79

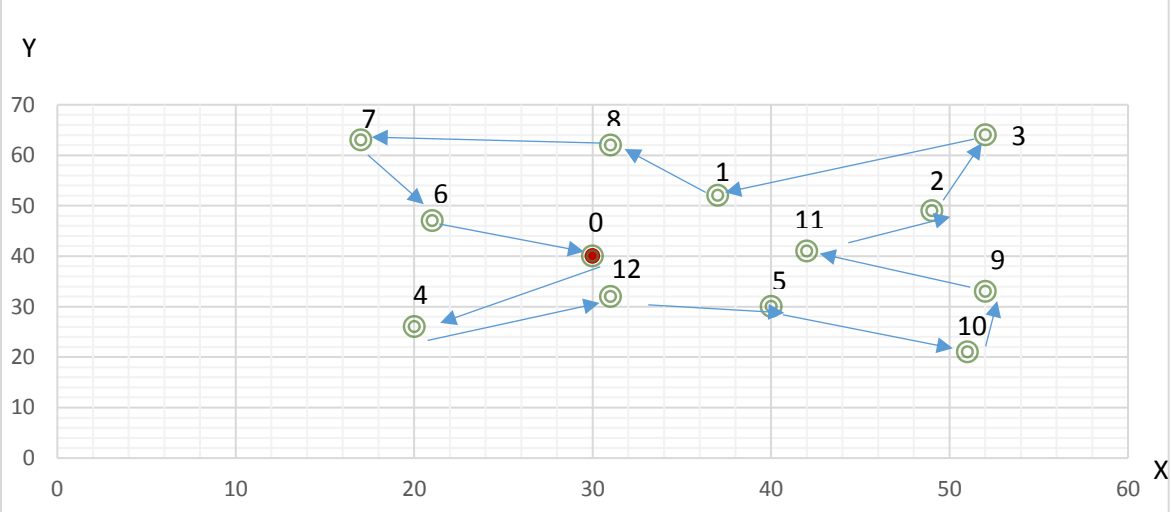


Figure 9. The distance minimizing tour

Figure 10 with arrows represents the vehicle routing of the model, which minimize transportation work. The route is 0-12-5-10-9-11-2-3-1-8-7-6-4-0. Total distance: 191.42. Transportation work: 31363.8

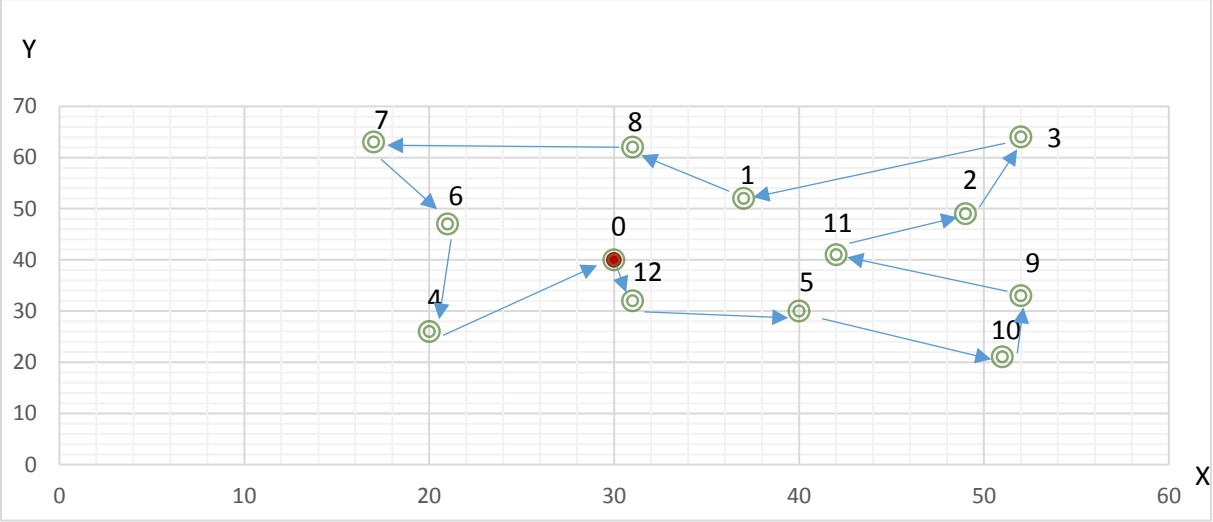


Figure 10. Transportation work minimizing tour

Now we have established the optimal TSP and the optimal TrW sequences. The question will be: is there any substantial differences in fuel consumption in the two cases. In order to we will use the formula, given in Figure 11.

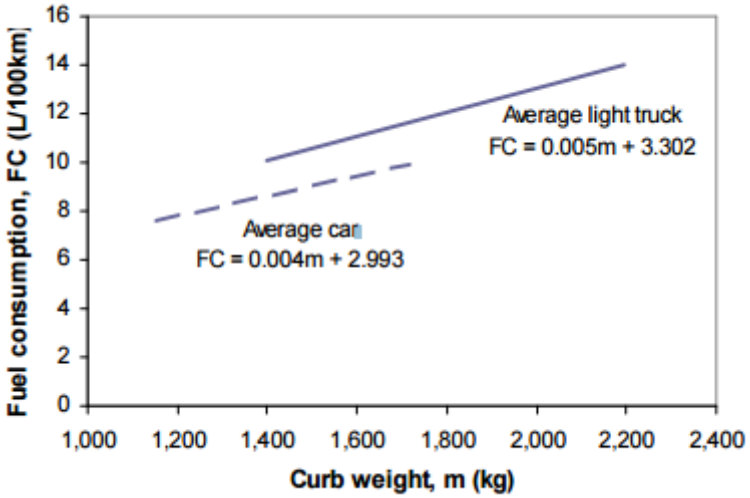


Figure 11. Vehicle fuel consumption

(Bandivadekar, A, et al. 2008 p.46)

There are two formulas for the Fuel Consumption calculation.

The first one is for an average car and second one for the average light truck. We are going to use the second one. On the next step, we will calculate the fuel consumption for the route with the smallest transportation work and for the route with the smallest distance. This part is one of the significant because just based on these results we may answer all the questions. From the Figure 11, we need the second formula: FC=0.005*m* + 3.302, where *m* is the load on the vehicle and FC is fuel consumption. First, we will start with the sequence for the smallest distance (TSP). We should know the weight on the vehicle on each node. According to it, we need to have a demand of the customers. Then we will try with smallest transportation work.

Route in the TSP-case: 0-4-12-5-10-9-11-2-3-1-8-7-6-0

We start from the depot and have a load on the vehicle is equal to 204, then from the node to node, the load on the vehicle is getting smaller.

We go from 0 to 4: FC = 0.005* 204+3.302; FC = 4.322(L/100 km)

We need to know the fuel consumption for the distance from 0 to 4.The distance is equal to 17.20, so $\frac{4.322}{100} * 17.20 = 0.743384$ liter

Then we take the fuel consumption along edge 4-12 etc.

The result is given in the second column in table 4. In a similar fashion, we calculate the fuel consumption for the best TrW that is 0-12-5-10-9-11-2-3-1-8-7-6-4-0. The results are given in column 4 in table 4.

Table 4. Fuel consumption for TSP and TrW

Nodes	TSP	Nodes	TrW
0-4	0.743384	0-12	0.3483532
4-12	0.5354804	12-5	0.3851194
12-5	0.3809704	5-10	0.5786312
5-10	0.5722367	10-9	0.4872588
10-9	0.4818408	9-11	0.5113752
9-11	0.5056107	11-2	0.4142511
11-2	0.4094676	2-3	0.573291
2-3	0.566406	3-1	0.7044307
3-1	0.6957862	1-8	0.42334912
1-8	0.4182442	8-7	0.4934351
8-7	0.4871216	7-6	0.5642878
7-6	0.5568673	6-4	0.7035394
6-0	0.376428	4-0	0.567944
Sum	$\sum = 6.7298439$	Sum	$\sum = 6.7554081$

From the results of the calculations, we got almost the same fuel consumption rate. On the other hand, we got good results. Firstly, let us compare the distance. The smallest total distance 176.72 with the load on the vehicle gives additional fuel consumption 6.7298439. The second route with the smallest transportation work has the distance 191.42. The additional fuel consumption is 6.7554081. With the transportation work model the driving distance for 14 km longer, but additional fuel consumption is the same.

As we see, the fuel consumption will depend on the number of customers

Their relative positions, the distances between nodes also play an important role.

The customers' demands effect on the choice of road and route construction.

8.2 Second experiment

In the second experiment, we will change some of the customers demand, but the total demand for all customers will be the same as we had before. From the data, given in Table 5, we conclude that customers 1, 2, 3 have quite big demand, compare to other customers.

Table 5. Customers demand

Customer	Demand
1	40
2	30
3	50
4	8
5	10
6	11
7	5
8	9
9	11
10	5
11	9
12	16
Sum	∑=204

In TSP model, the route will be the same, since we minimize total distance.

The route is 0-4-12-5-10-9-11-2-3-1-8-7-6-0, the distance 176.72 and the TrW is now 35149.03.

TrW model brings changes to our experiment.

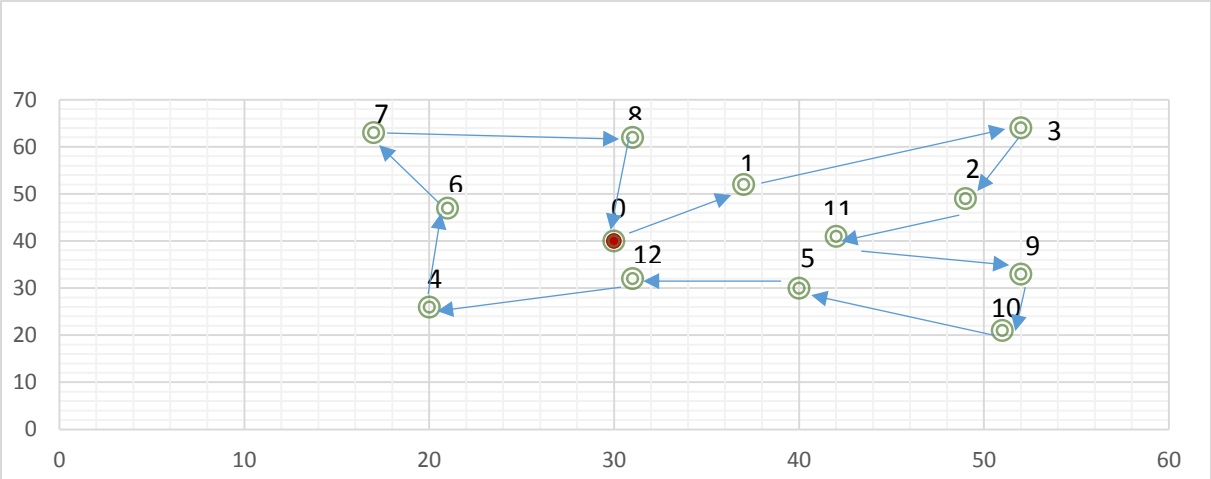


Figure 12. Transportation work minimizing tour

Now the route become 0-1-3-2-11-9-10-5-12-4-6-7-8-0.Transportation work is 29445.3, and it is less than in the first experiment. The distance is 183.41. In the next step, we should compare two results, which we got. According to it, we have to calculate fuel consumption rate. The results can be found in table 6.

Table 6. Fuel consumption for TSP and TrW, based on demand from table 5

Nodes	TSP	Nodes	TrW
0-4	0.743384	0-1	0.6003258
4-12	0.5361064	1-3	0.7918362
12-5	0.3874244	3-2	0.592416
5-10	0.5899992	2-11	0.3956486
10-9	0.4968908	11-9	0.4710237
9-11	0.5216232	9-10	0.4360888
11-2	0.4280701	10-5	0.5111337
2-3	0.5933181	5-12	0.3270334
3-1	0.6967467	12-4	0.4340684
1-8	0.3995882	4-6	0.7203554
8-7	0.4744946	6-7	0.5560428
7-6	0.5535693	7-8	0.46958413
6-0	0.376428	8-0	0.3975608
Sum	∑=6.7975059	Sum	∑=6.70311773

With a new data, we have a quite good result. The fuel consumption is smaller with TrW objective, but the route is 6, 69 km longer. We save 0.09438817 liters. We might have a lot of cargo in the car, so the savings of gasoline could be significant. So, we see from the example, how TrW works. It will help plan routes for a fleet of vehicles or just for one vehicle.

9.0 Analysis

The section presents results of analyses of experiments. We have shown two experiments with the different demand of the customers. Experiments were conducted 12 customers and 1 vehicle, which has to serve all customers. The first experiment gave us two similar routes. This is explained by the distance and customer demand. From the table 4 just two customers have a big demand. Nodes were quite far from each other. Perhaps, according to TrW algorithm, a vehicle have to make a cluster. Then, it was not possible, so it went almost the same tour. We got almost the same fuel consumption results. So, we conclude that the new objective function did not bring changes to the model. The distance in the model is longer, but fuel consumption is the same.

In the second experiment, we changed demand of the customers. Customers with the biggest demands were located in one area, close to each other. This is logically and mathematically explained the choice of the vehicle. The distance is shorter between these customers, so for the TrW model is better to serve firstly customers with the biggest demand. In this experiment, two routes are built in different ways. We have a better solution for the model. Below are presented charts of the load of the vehicle, according to the route from customer to the customer from the second experiment.

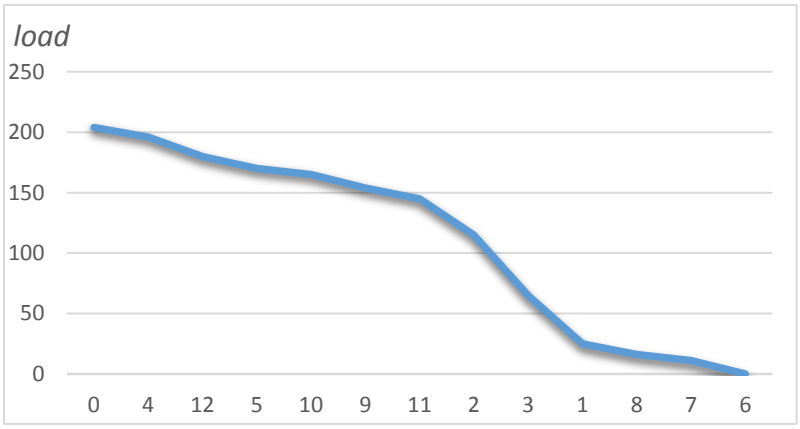


Figure 13. Load on the vehicle (TSP model)

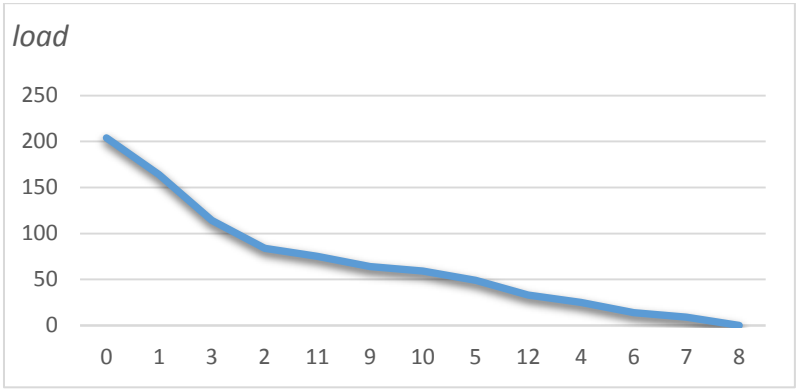


Figure 14. Load on the vehicle (TrW model)

Figure 13 and Figure 14 clearly show how customers were served.

In Figure 13 the slope of TSP is quite low in the beginning. Customers with the highest demand were served in the middle of the route. It means that during the first part of the trip the vessel was more loaded. It leads to additional fuel consumption due to the load on the vehicle. Then the slope falls sharply, fuel consumption becomes smaller.

On Figure 14 (TrW model) customers with a high demand were served first, that is the reason why the slope goes down directly after the visiting first customer. It means that the vehicle did not carry huge cargo during the rest of the trip. After that, the slope smoothly goes down. It brought a good result, fuel consumption has become less and during the trip is gradually becomes smaller.

We should look deeper now. The TSP problem always makes the shortest way of travelling, even with the fully loaded car. So, it does not matter if we have a heavy vehicle, or not, the point is to make the route shorter. In some cases, it's good, but in some others, it may be bad.

Therefore, our model keep vehicle always with the smallest transportation work, and it also brings saving in FC, but it did not give huge savings in FC. Anyway, it leads to thoughts, that TrW may bring better result. In which cases then?

It depends of course on the size of the vehicle. The huge vehicle will bring a large proportion of TrW. The weight of the vehicle plays an important role. Our model includes the weight of the vehicle. Therefore, we observe a tendency, that improving in a load factor will reduce CO2 emissions. Therefore, it is important for companies use the full capacity of the vehicles.

The next measure is visited frequency. The vehicle may travel every day to serve customers during a year. Therefore, route planning for a long time plays an important role and can save money for the company.

The length of travelling. Customers may be located in different cities. So, vehicles have to drive long distance. In this case, the calculation of TrW might be also useful.

10.0 Conclusion and further research

Usually, in TSP and VRP the main goal is to minimize the distance of travelling or cost. However, it might not be an optimal solution, because some factors are not included in the model. We introduced the new model, which based on TrW minimization. Therefore, the shortest distance may not be an optimal solution in case of TrW minimization. Our goal was calculation load on the vehicle *distance. Why is the new model better? According to state this, we have to give reasons. In our case, we calculated fuel consumption. Usually, just calculation of the fuel consumption during the trip plays an important role. With the increase in cargo, we need to use more gasoline. It means that we have additional fuel consumption, which depends on the load. So it's possible to choose a route with the smallest TrW. We described examples and has shown experiments, where TrW bring good results. One of the main reason is a clean fresh air. With the growing number of cars with harmful emissions, the environment is getting worse. The new model will help to reduce TrW, so it will decrease the percentage of the world fuel consumption. We recommend using a new model for companies and save not only money but also the world.

Further researchers might include TrW minimization but it may be implemented onVRP. Therefore, we shall work with several vessels. Firstly, vehicles may have different capacities.

For example, locations of some customers are close to the depot. In this case, a vehicle with a small capacity can be used. A vehicle with normal capacity visit the customers, who are further from the depot.

Secondary, time windows. Some customers want to be served within a specific time window.

Thirdly, tour duration. The time a vehicle uses on a trip may be restricted.

Also, a fleet size that is the number of available vehicles.

In addition, vehicles might travel in different conditions and landscapes. When the vehicle travels uphill, more gasoline required than when it goes downhill. We think that further research might consider it.

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Appendix

param dis :	0	1	2	3	4	5	6	7	8	9	10	11	12	:=
0	.	13.89	21.02	32.55	17.20	14.14	11.40	26.42	12.04	23.09	28.31	12.04	8.06	
1	13.89	.	12.37	19.21	31.66	22.20	16.76	22.82	11.66	24.20	34.01	12.08	20.88	
2	21.02	12.37	.	15.30	37.01	21.02	28.07	34.93	22.20	16.28	28.07	10.63	24.76	
3	32.55	19.21	15.30	.	49.68	36.05	35.35	35.01	21.09	31	43.01	25.08	38.27	
4	17.20	31.66	37.01	49.68	.	20.40	21.02	37.12	37.64	32.76	31.40	26.62	12.52	
5	14.14	22.20	21.02	36.05	20.40	.	25.50	40.22	33.24	12.37	14.21	11.18	9.22	
6	11.40	16.76	28.07	35.35	21.02	25.50	.	16.49	18.03	34.01	39.70	21.84	18.03	
7	26.42	22.82	34.93	35.01	37.12	40.22	16.49	.	14.03	46.10	54.03	33.30	31	
8	12.04	11.66	22.20	21.09	37.64	33.24	18.03	14.03	.	35.80	45.62	23.71	30	
9	23.09	24.20	16.28	31	32.76	12.37	34.01	46.10	35.80	.	12.04	12.81	21.02	
10	28.31	34.01	28.07	43.01	31.40	14.21	39.70	54.03	45.62	12.04	.	21.93	22.82	
11	12.04	12.08	10.63	25.08	26.62	11.18	21.84	33.30	23.71	12.81	21.93	.	14.21	
12	8.06	20.88	24.76	38.27	12.52	9.22	18.03	31	30	21.02	22.82	14.21	.	;

Table A – The distance between all customers and depot