Master's degree thesis

LOG950 Logistics

An exact method approach for milk collection using trucks and trailers

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Preface

This master thesis concludes my two years at Molde University College – Specialized University in Logistics and represents my last academic work as a student. The research has been conducted in order to obtain an MSc degree in Logistics.

This work has been supervised by Professor Arild Hoff. I would like to thank him for giving me this case and express my deepest gratitude for all his good ideas, patience and for the continuous support of my thesis study.

Also I would like to thank the facility itself and its staff for entertaining both academic and social events.

Gulnara Shafiullina
Molde, Norway
May 2017

Contents

1.0 Introduction

Milk collection from farms to production factories is a challenging vehicle routing problem around the world. As it is well known, milk farms are located in rural areas often connected by narrow roads, making it difficult to reach farms by large trucks carrying trailers. This is especially evident in Norway, where some regions contain mountains, islands and fjords.

In accordance to that fact, that milk is a perishable product and that farms have a limited storage capacity, milk has to be collected and transported in a limited time. The production quantity of milk in the farms depends mainly on the number of cows.

In this thesis, we will consider a real-world milk collection problem in the western part of Norway earlier described in two PhD-thesis submitted to Molde University College and published as Hoff and Løkketangen (2006) and Pasha et al. (2015). In both works, the proposed methodology for constructing routes using trucks and trailers is the meta-heuristics tabu search. Compared with previous work, we will, in my study, develop mathematical models and use exact methods to find the optimal solution using "A Mathematical Programming Language" (AMPL), – Fourer et al. (2003). The given method should be possible to use in other geographical locations with different data and some small modifications, but only small instances can be solved to optimally, because large instances are usually too complex to solve using exact methods.

In the original milk collection problem, a company collects milk using a heterogeneous vehicle fleet of trucks with or without a trailer connected. The vehicles should visit a number of geographically distributed farms in different areas. The milk collection routes could be long and contain many farms, resulting in high transportation costs and collection times.

The purpose of this thesis is to create a model for determining routes for milk collection by minimizing total traveling distance. In addition, we will consider smaller parts of the huge problem to test the model and discuss some experiments related to the problem.

2.0 Problem description

2.1 Company overview

This thesis describes a real world problem where Norwegian dairy company TINE BA collects raw milk from farms and delivers it at the company's dairy plant. TINE BA is Norway's largest producer, distributor and exporter of dairy products with 11 400 owners and 9 000 cooperative farms (TINE, 2017). The core business of the company is producing dairy products from milk like consumer milk, cheese, cottage cheese, cream and yogurt. Trucks and trailers are owned by the company and their capacities can vary. For trucks from 5 000 to 18 500 liters and for trailers 11 000 – 19 000 liters.

In the original problem Hoff and Løkketangen (2006) describes the region with three dairy plants at different locations, covering a total of 990 farms in 20 different municipalities. The problem size is impressive because it includes a lot of data information, which makes it challenging to solve in an exact way to receive the optimal solution. That is why we need to reduce it and take into consideration just a part of the problem to make it solvable for this study.

Thus, we will focus on the largest dairy plant in the northern part of Møre & Romsdal Country in Elnesvågen and some suppliers attached to this plant. The plant has a certain number of vehicles with different capacities available.

2.2 The milk collection problem

In our problem the truck, carrying a trailer, cannot be driven to the farms, due to the varying nature in the area with mountains, valleys and fjords. The small farm roads are typically inaccessible for heavy truck/trailer combinations, and the trailer must be parked on the most convenient parking place while the single truck visits the farms and collect the milk. Then, the truck will return to the parking place and transfer milk to the trailer with an additional tank, before starting a new sub-tour. This could either start from the same parking place or eventually the trailer can be moved to a more convenient parking place for the new sub-tour. This will continue until both the truck tank and the trailer tank are filled up, and vehicle can drive back to the dairy plant. It is assumed that each supplier has a known demand and must be visited exactly once with one vehicle.

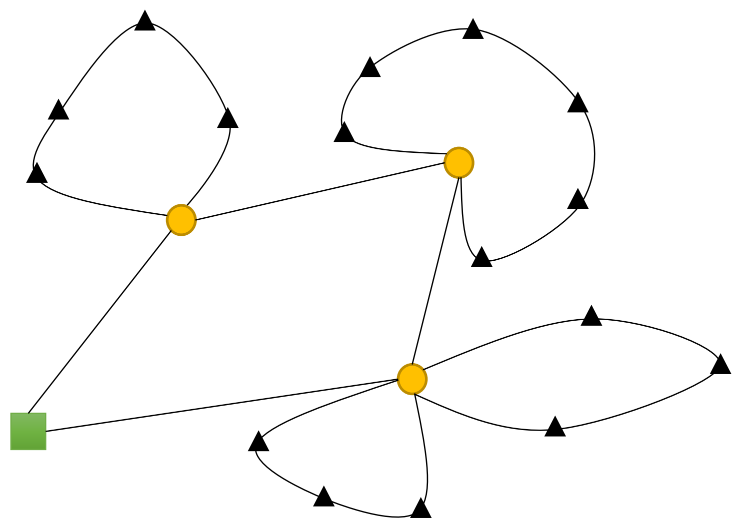


Figure 1. The route structure for the milk collection problem of TINE BA.

Figure 1 shows the structure of one route in the milk collection problem. The empty truck with a carried trailer starts from and return to the dairy plant, which is denoted by a square, then leaves the trailer at assigned parking places shown as circles and visit farms represented as triangles for milk collection.

A distance table between the subset of farms, parking places and dairy plant used in the thesis was created by using data from the Norwegian Mapping Authority as described in Hoff and Løkketangen (2006). (Appendix C).

2.3 Milk production and farms visiting frequency

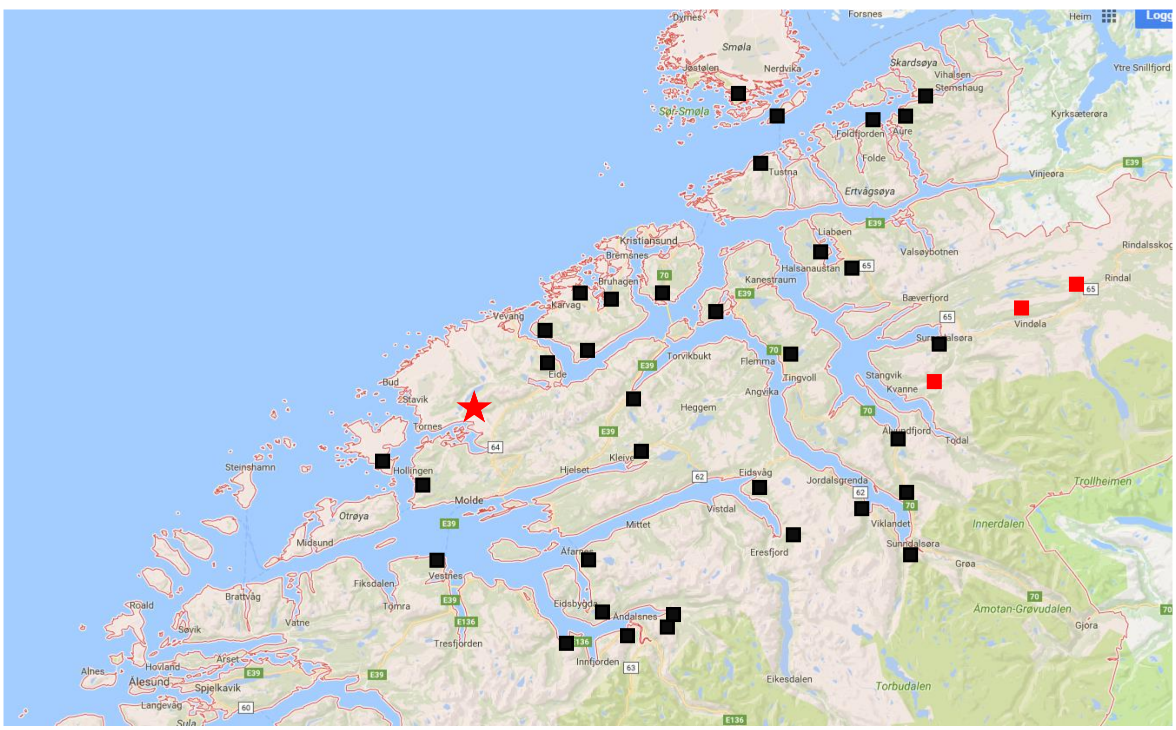
The raw milk is stored in a cooler tanks at the farms until the truck visits the farm and collects the milk. The milk can be stored maximum for three days at the farms, making one, two or three days the feasible visiting frequencies. Each farmer has a special tank for storing the milk. Since some suppliers produce organic milk and some produce traditional milk, the vehicle can collect the milk from different suppliers and keep it separated in the tank.

In the original problem Hoff and Løkketangen (2006), states that there are three different frequencies used by the company. The frequency codes: “7x2”, “7x3” and “6x2”. The code “7x2” means that the route will be driven every second day, the frequency code “7x3” means that the route will be driven every third day, and for both, all days are considered as working days. The code “6x2” means the route will be driven every second day except Sundays. “6x2” strategy requires the same capacity of tanks and vehicles as the 3-days frequency, since the Monday and Tuesday visits will collect 3 days production.

For solving the problem in this thesis we have chosen the “7x3” frequency code. This is also the strategy used by TINE in the area for the subset of farms in our test case. Both Hoff and Løkketangen (2006) and Pasha (2015) has shown that this is the most efficient collecting strategy using the maximum storage time of three days at the farms.

2.4 Parking places

As described in Hoff and Løkketangen (2006) the company rents 37 prearranged parking places typically located at parking lots and gas stations. The exact location of these parking places is represented by black and red squares in Picture 1 below. The dairy plant in Elnesvågen is represented by a star.



Picture 1. Parking places in the northern part of Møre and Romsdal and the dairy plant in Elnesvågen.

To solve the problem with exact methods using AMPL, we have chosen a subset including selected farms in the Surnadal and Rindal municipalities. This area contains three parking places, as shown by red squares in Picture 1.

3.0 Problem definition

The purpose of the thesis, as mentioned above, is to develop a model for minimizing the total traveling distance between the dairy plant and milk suppliers. By splitting the original problem into smaller sub-problems, solvable to optimality, we can provide good solutions for this real life problem. The finished model can provide answers on the following questions:

- What is the total traveling distance (optimal solution)?
- How many parking places will be used?
- Which parking places should be used as basis for the different sub-tours visiting farms?
- How many tours are necessary for visiting all farms?

The strategic decisions regarding transportation planning are considered with the total milk production for three days. Since there is no need to collect milk every day, the same vehicle can be used on different routes with a three-day recurring frequency.

We can describe the problem by the following rules:

- Each supplier can be served by only one vehicle
- Suppliers cannot be visited with a vehicle carrying a trailer, so the trailer needs to be parked before the truck visits the farms. A full tour for a vehicle with a trailer includes a tour starting and ending at the same depot, visiting one or more parking places and driving sub-tours with only the truck from those.
- Parking places are not attached to any suppliers and do not have any demand They can be visited if convenient, but do not need to be visited if not.
- Time windows are not taken into consideration. Drivers are supposed to make an agreement with each farmer regarding exact time for the visit. In the real-world problem ferry times could be relevant. For the tour between the dairy plant and our subset of customer, exactly one ferry trip is needed in each direction. These ferries will depart every half hour, which means that the vehicles do not need to wait very long. In other more distant areas, ferries are not so frequent and time windows for ferries would also matter.

- Extra costs are not taken into consideration as well. The model minimizes the traveling distance, and does not consider ferry/toll road costs, drivers' wages and different fuel consumption for different vehicles.
- The model is based on collection of milk every third day no matter holidays and weekends.
- The model considers only minimizing distance (km unit). It is assumed that the cost is proportional to the distance.
- A more detailed problem definition will be explained in Section 5. Mathematical model development.

4.0 Literature review

This chapter outlines the relevant theory. In order to introduce the theories and methods used, we explain earlier defined research problems related to this particular real-world problem such as the truck and trailer routing problem and the two-echelon vehicle routing problem. We sum up much of the research done so far, and give a brief explanation on some of the more relevant theories.

4.1 The vehicle routing problem

According to Toth and Vigo (2002) in the vehicle routing problem (VRP), a set of homogeneous vehicles located at a depot must be routed in order to serve geographically distributed customers. The objective of the VRP is to minimize the fleet size and the total routing cost. Each customer has a known demand and must be visited only once by exactly one vehicle. Each route starts and ends at a depot, and capacity of a vehicle cannot be exceeded. The VRP is NP-hard, since it is not solvable in polynomial time, and is usually solved by using heuristics/metaheuristics, but small instances can also be solved by exact methods. Toth and Vigo (2002) proposed three main classes of exact methods for VRP: Branch and Bound, Branch and Cut and Set Covering based algorithm.

4.2 The fleet size and mix vehicle routing problem

The fleet size and mix vehicle routing problem (FSMVRP) is a problem that determines fleet size composition and routing of a heterogeneous fleet of vehicles in order to service a set of customers with known demand from a depot. Golden et al. (1984) presented several efficient heuristic solution procedures for generating a lower bound and an underestimate of the optimal solution. The objective of the FSMVRP is to determine the optimal fleet composition by minimizing a total cost function, including both fixed vehicle cost and variable routing cost components.

In our problem, the company has a certain number of heterogeneous vehicles with trailers to collect the milk from the suppliers. The milk collection problem is concerned with assigning a fleet of vehicles to serve suppliers at various districts, and considers a routing problem with the chosen fleet. TINE has access to a huge number of trucks and trailers which can be put together in different combinations. The vehicles are not necessary assigned to one plant, and to some extent, we can assume that we can select the most convenient truck/trailer combination to the routes. Thus the real-world problem will be similar to a FSMVRP; first to find the best fleet composition and then to find the best routing for it. In the Rindal/Surnadal area, which is a large agricultural area with many farms, the largest possible combination 18500/15000 is the one used by the company. Restrictions by the Norwegian road authorities, states that this corresponds to the largest weight of vehicles allowed driving on the main roads.

The FSMVRP involves the design of a set of minimum cost routes starting and finishing at a central depot, for a fleet of heterogeneous vehicles with different capacities, fixed costs and variable costs to serve a set of customers with known demands.

Gheysens et al. (1986) proposed a two-stage general assignment based heuristic, which was based on – their previous mathematical programming formulation for the FSMVRP (Gheysens et al. 1984).

Salhi and Rand (1993) constructed a seven phase improvement heuristics. The aim was to match the total demand on a given route with applicable vehicle capacity and improve the utilization of the vehicles.

The papers mentioned describe constructive heuristics for finding a solution to the FSMVRP. Later papers concerns metaheuristics for improving an already constructed solution.

Gendreau et al. (1999) introduced a tabu search algorithm, which was based on their previous GENI (generalized insertion) and post optimization US (unstringing and stringing) algorithm. Osman and Wassan (2002) developed variants of tabu search for FSMVRP, with reactive tabu search concepts, variable neighborhoods and special data structures.

Brandão (2009) created and implemented a more advanced tabu search heuristics, and later, Brandão (2011) used that algorithm with additional features to solve the VRP with heterogeneous fixed fleet. Subramanian et al. (2012) developed an iterated local search with seven neighborhoods and they have attained the best known solutions for the standard FSMVRP test instances so far. Their heuristic was a hybrid algorithm using exact methods for solving smaller sub-problems (routing part) of the larger FSMVRP.

4.3 The truck and trailer routing problem

Our problem can be classified as a special case of the truck and trailer routing problem (TTRP). TTRP is a variant of the vehicle routing problem, where a fleet of trucks and trailers serves a set of customers. It is an extended version of VRP, and in the standard version some customers can be served by a truck with trailer, while others can be served by a truck alone. According to Chao (2002) there are three types of routes in a solution to the problem:

1) a pure truck route traveled by a truck alone,
2) a pure vehicle route without any sub-tours traveled by a truck with trailer
3) a truck and trailer route consisting of a main tour traveled by a truck with the trailer, and one or more sub-tours traveled by the truck alone.

For our particular problem, only the third type of routes is suitable.

The objective for these types of problems is to minimize the total traveling distance or total cost incurred by the fleet. The problem is more difficult to solve than the basic vehicle routing problem, but it is considered closer to many real life situations.

The term “truck and trailer routing problem” was first mentioned by Chao (2002). Since then, there has been presented a lot of work for different TTRP variants. The most of the studies published on TTRP are using metaheuristics algorithms. Chao (2002) and Scheuerer (2006) introduced tabu search algorithms. Lin et al. (2011) proposed a simulated annealing (SA) heuristics for solving the truck and trailer routing problem with time windows (TTRPTW), both at customers and the depot. Caramia and Guerriero (2010) proposed an approach based on mathematical programming and local search to cope with the truck and trailer vehicle routing problem. They combine heuristics and exact concepts in one hybrid metaheuristics.

Furthermore, Drexl (2011) proposed the generalized truck and trailer routing problem (GTTRP), where he performs a general consumption of TTRPs. In this variant he considers transshipment (decoupling) locations, heterogeneous fleet of vehicles and time windows. A vehicle fleet consisting of single lorries and lorry-trailer combination. Some customers can be visited by a lorry only or by a lorry with a trailer. On a transshipment location the trailers can be parked, and goods from the lorry can be transferred to the trailer. He presents two mixed-integer programming (MIP) formulations for the GTTRP. For an exact solution procedure, he describes a branch-and-price algorithm and heuristics variants. He solves instances with up to ten lorry customers, ten trailer customers and ten transshipment locations to optimally.

Belenguer et al. (2015) introduced the single truck and trailer routing problem with satellite depots (STTRPSD). In this problem a truck with a trailer based at a main depot must serve a given set of customers only by one single truck. Thus it is important to park the trailer at the prearranged parking place, before serving the customers. They present an integer programming formulation of the STTRPSD and propose a branch-and-cut algorithm, which solves instances with up to 50 customers and ten parking places to optimality.

Recently, Rothenbächer et al. (2016) proposed a new branch-and-price-and-cut algorithm to solve TTRPTW and two real-world extensions. This algorithm uses predefined routes and chooses among them, thus, it does not consider all possible combinations. In the first extension, there is two days planning horizon and customers can be visited either on both days or only on one. The second extension integrates load transfer times depending on the amount moved from a truck to its trailer. The model also includes a heterogeneous fleet of vehicles. Computational results presented on single-day instances. Compared to the results of Drexl (2011), 36 new optimal solutions were found.

In comparison, in a general VRP customers can always be reached by all vehicles, but in TTRP not all customers are accessible by trucks when carrying a trailer. In the general TTRP it is possible to have two types of customers, where some can be visited by a truck carrying a trailer and some only by the truck alone. In our special case, however, no customers can be visited by a truck with a trailer, and the trailer has to be uncoupled from the truck before visiting the farms. After visiting the farms, the truck need to drive back to the parked trailer and couple it again before moving it to another location or driving back to the dairy plant.

4.4 The two-echelon vehicle routing problem

Truck and trailer routing problems are also related to two-echelon vehicle routing problems (2E-VRP). 2E-VRP aims to deliver the freight from the depot to the customers by consolidating the freight through the satellites – intermediate depots while minimizing the overall transportation cost. I.e. trucks are delivering goods to the satellites, and then the final delivery to customers are usually performed from the satellites by using smaller vans.

The direct shipping from the depot to the customers is not possible. Perboli et al. (2011) propose a math-heuristic, where the depot-to-satellite transfer and satellite-to-customer delivery is solved as two routing sub-problems. Computational results are shown on a wide set of instances with up to 50 customers and five satellites.

There also exists multi-echelon vehicle routing problem, where several intermediate levels of satellites are presented, but the most common version is two-echelon vehicle routing. The reason for calling it the multi-echelon VRP is that the overall transportation network can be decomposed into $k \geq 2$ levels:

- the first level, which connects the depots to the first level satellites
- $k - 2$ intermediate levels interconnecting all the satellites
- the last level, where the freight is delivered from the satellites to the customers.

Figure 2 illustrates the 2E-VRP.

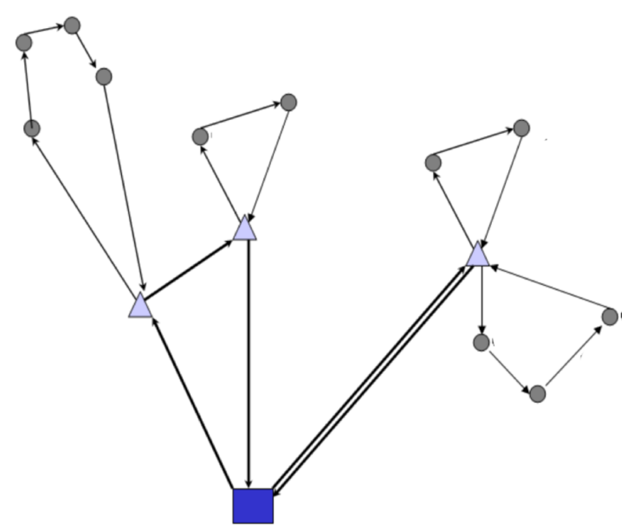


Figure 2. Example of two-echelon vehicle routing problem.

The depot is denoted as a square. The satellites represented by triangles, is a set of intermediate depots. The customers are denoted as circles. The depot is the starting point of the freight, and the transit points are capacitated (However, if the second level vehicles are ready and waiting when the first level vehicle arrives, there is no need for storage capacity at the transit points). The destination of the freight is customers, and each customer has a demand – a quantity of goods that has to be delivered to that customer. Each first level vehicle can deliver the freight of one or more customers and serve more than one transit point in the same route. This is classic example of 2E-VRP. The freight must be consolidated from the depot and shipped to transit points and later to the desired customers. The second level vehicles can be reused on other transit points if there is no need at the current transit point.

As we can see from the Figure 2, the 2E-VRP is very similar to TTRP (Figure 1), which makes both problems related to the location routing problem.

According to Perboli et al. (2011), there are three main groups of two-echelon routing problem:

- Basic variant with no time dependence:
  - Two-echelon capacitated VRP (2E-CVRP). Each level has vehicles with the same fixed vehicle capacity and number of vehicles is not known. There is one depot and a fixed number of transit points. All the customers' demands are known and should be satisfied. Demand of each customer must be smaller than the capacity of the vehicle that serve that customer. There are no time windows for deliveries. The objective is to serve the customers by minimizing traveling cost while satisfying all capacity constraints. This is the variant most related to our milk-collection problem.
- Basic variant with time dependence:
  - Two-echelon VRP with time windows (2E-VRP-TW). This variant is an extension of 2E-CVRP where time windows – arrival and departure time at the transit points – are added.
  - Two-echelon VRP with satellites synchronization (2E-VRP-SS). Time constraints at transit points are considered here as well. The vehicle arrive at a transit point and unload the freight, which must be immediately loaded into city freight (second level vehicle). These constraints can be of two types: hard and soft. In the hard case, every time a first-level vehicle unloads its freight, the second-level vehicle must be ready to load it (very small hard window). In the soft case, the demand is lost and a penalty must be paid if the city freighters are not available for loading.
- Other variants 2E-CVRP variants are:
  - Multi-depot single-delivery problem. In this case there are several depots on the first level. On the second level there is just one vehicle considered.
  - 2E-CVRP with pickups and deliveries (2E-CVRP-PD). In this case, transit points can be considered as intermediate warehouses (depots), where the freight can be stored. Some

customers have freight to be delivered. Others can have freight for pickup, while some might have both.

Baldacci et al. (2013) performed an exact algorithm, where a mathematical formulation of the 2E-CVRP were used to derive valid lower bounds and an exact method decomposing the two-echelon capacitated vehicle routing problem into several multi-depot vehicle routing problems with side constraints. They solved instances with 50 customers and up to five satellites.

Cuda et al. (2015) present a recent survey, where they declare that TTRPs can be classified as two-echelon routing problems. In a feasible solution, a two-level route may be present with these characteristics: the first level route is traveled by a truck with a trailer, whereas the second level route, starting and ending at a vertex visited in the first level tour, is traveled by the truck alone. One difference is that unlike 2E-VRPs, in some TTRPs on the first or (and) on the second level route the truck can be driven alone or as a complete vehicle. The levels in TTRP are in general not as strict as in the 2E-VRPs, and that makes our problem closer to a 2E-VRP since no farms can be visited by the full truck carrying a trailer. However, in the real-world problem, not all trucks are carrying a trailer when leaving the depot, and they are driving sub-tours directly from the depot. Due to that, our problem also can be considered as a special case of 2E-VRP, but with certain assumptions.

5.0 Mathematical model development

5.1 Model description

The model of the problem presented in this paper can be described by the definitions below:

- The problem consists of a set of plants P, a set of parking places S, a set of farms Z, a fleet of complete vehicles (truck with trailer) R and a set of a single trucks V (the same number as fleet of complete vehicles but disconnected from trailers).
- Each plant $p \in P$ should have delivered a minimum daily amount of milk u_p.
- Each farm $z \in Z$ has a certain amount of daily production h_i
- The model does not consider the single truck leaving the dairy plant. Only complete vehicles can leave the dairy plant and park their trailers at the prearranged parking places.
- A solution to the problem consists: 1) Trucks with trailers $t \in R$ starting and ending their routes at plant p and visiting the prearranged parking places $s \in S$. 2) A single truck $v \in V$, after disconnecting from the trailer, starting and ending routes at parking place s and visit all the farms assigned to that parking place.
- The trucks and trailers can be of different sizes. The capacity of the complete vehicle e_t must not be exceed on a full tour, and the capacity of a single truck c_v must not be exceeded on a sub-tour.
- If parking place $s \in S$, is set, it has to be assigned to exactly one complete vehicle. It means that a parking place can be visited only by one truck with the trailer. We made this assumption as a simplification of the real world problem to make it solvable.
- The distances between all the locations in our sub-problem are given in the distance matrix in Appendix C.

5.2 Model development

Most of the works proposed on this type of problems have been solved with metaheuristics approaches. However, in this paper we will use exact methods as a way of generating solutions for the problem. The exact methods have to be presented mathematically in order to present the methods in a descriptive way.

The idea of using precedence constraints in this model was inspired by Dondo (2007).

The model to minimize the total cost of the system may be formulated as follows:

Mathematical formulation.

Sets:

Z – Set of farms

P – Set of plants

S – Set of possible parking places

L – Set of locations (L = $Z \cup P \cup S$)

V – Set of sub-tours driven by trucks

R – Set of complete vehicles (trucks with trailers)

Parameters:

u_p – Demand of the plants, $p \in P$

e_t – Vehicle capacity, $t \in R$

c_v – Truck capacity, $v \in V$

h_z – Milk production at farms, $z \in Z$

T_{ij} – Distance between all nodes, $(i, j) \in L$

M – Big-M, $M = 99\,999$

Variables:

a_{zv} – 1 if farm *z* is assigned to the truck *v*, 0 otherwise, $z \in Z$, $v \in V$

b_{sv} – 1 if sub-tour v with a single truck is assigned to parking place s, 0 otherwise, $s \in S$, $v \in V$

x_{ij} – 1 if farm i is visited before farm j, 0 otherwise, $(i, j) \in Z$, $i \neq j$

m_{st} – 1 if truck and trailer t is assigned to parking place s, 0 otherwise, $s \in S$, $t \in R$

n_{pt} – 1 if truck and trailer t is assigned to the plant p, 0 otherwise, $p \in P$, $t \in R$

r_{ij} – 1 if parking place i is visited before parking place j, 0 otherwise, $(i, j) \in S$, $i \neq j$

w_{zs} – 1 if farm z is visited from a parking place s, 0 otherwise, $z \in Z$, $s \in S$

y_s – 1 if parking place s is used, 0 otherwise, $s \in S$

f_{tsp} – Load of the vehicle t sent from parking place s to plant p, $t \in R$, $s \in S$, $p \in P$

v_v – Total traveling routing distance of a sub-tour v, $v \in V$ driven by a truck.

t_t – Total traveling routing distance for a truck with trailer t, $t \in R$

s_s – Distance driven from plant p to a parking place s, $s \in S$

z_z – Distance driven from parking place s to a farm z, $z \in Z$

Formulation:

Minimize

$$\sum_{v \in V} v_v + \sum_{t \in R} t_t \tag{1}$$

The objective function (1) minimizes the total routing distance of complete vehicles and single trucks from the dairy plant to the farms through the parking places and the way back.

S.t.

The first group of constraints is presented for a truck with trailer, when the complete vehicle leaves a dairy plant and drives a route visiting the prearranged parking places.

Assignment constraints:

$$\sum_{t \in R} m_{st} = y_s \qquad \forall \quad s \in S \qquad (2)$$

$$\sum_{p \in P} n_{pt} \leq 1 \qquad \forall \quad t \in R \qquad (3)$$

Constraints (2) insures that if parking place s, $s \in S$ is used, then it must be assigned to exactly one truck with trailer t, $t \in R$. This constraint is a simplification from the real-world problem, where the same parking place can be used by several vehicles although not at the same time. We add this assumption and simplify the model to make problem solvable. Constraints (3) states that each truck with trailer t, $t \in R$ must be assigned to maximum one dairy plant p.

Precedence constraints:

$$s_i \geq T_{ji}(n_{jt} + m_{it} - 1) \qquad \forall \quad i \in S, j \in P, t \in R \qquad (4)$$

$$s_j \geq s_i + T_{ji} - M(1 - r_{ij}) - M(2 - m_{jt} - m_{it}) \qquad \forall \quad t \in R, i \in S, j \in S, i \neq j \qquad (5)$$

$$s_i \geq s_j + T_{ij} - M(r_{ij}) - M(2 - m_{jt} - m_{it}) \qquad \forall \quad t \in R, i \in S, j \in S, i \neq j \qquad (6)$$

$$t_t \geq s_i + T_{ip} - M(2 - n_{pt} - m_{it}) \qquad \forall \quad p \in P\ i \in S, t \in R \qquad (7)$$

Constraints (4) states that the routing distance from dairy plant $p, p \in P$ to parking place $i, i \in S$ must be at least as long as the direct distance from this plant to this parking place. Constraints (5) and (6) are mutually excluding constraints. They define the relationship between the traveling distances up to nodes $i, j \in S$ on the same tour. If parking place $j \in S$ is served after parking place $i \in S$ by truck with trailer t, $t \in R$, then traveling distance to parking place $j \in S$ must be larger than to parking place $i \in S$. Vice versa if node j is visited earlier than node i. Constraints (7) insures that the total routing distance of truck with trailer t, $t \in R$, must be larger than or equal to the the routing distance from a dairy plant $p, p \in P$, to parking place $i, i \in S$, plus the transportation distance from this parking place to the dairy plant. Constraints (5) – (7) take into the use "big-M" method. M is a positive number sufficiently large to ensure that the right hand side will take a zero or negative value if a node is not included in the path.

The second group of constraints is for a single truck, when a truck disconnects from a trailer and leave it at a parking place to collect milk from farms.

Assignment constraints:

$$\sum_{v \in V} a_{zv} = 1 \qquad \forall \quad z \in Z \qquad (8)$$

$$\sum_{s \in S} b_{sv} \leq 1 \qquad \forall \quad v \in V \qquad (9)$$

Constraints (8) ensures that every farm z, $z \in Z$, is visited by exactly one truck v, $v \in V$. Constraints (9) states that each sub-tour $v \in V$, has to be assigned to maximum one parking place s, $s \in S$, i.e. parking place, where a truck left its trailer.

Precedence constraints:

$$z_i \geq T_{si}(b_{sv} + a_{iv} - 1) \qquad \forall \quad s \in S, i \in Z, v \in V \qquad (10)$$

$$z_j \geq z_i + T_{ji} - M\left(1 - x_{ij}\right) - M(2 - a_{jv} - a_{iv}) \qquad \forall \quad v \in V, i \in Z, j \in Z, i \neq j \qquad (11)$$

$$z_i \geq z_j + T_{ij} - M\left(x_{ij}\right) - M(2 - a_{jv} - a_{iv}) \qquad \forall \quad v \in V, i \in Z, j \in Z, i \neq j \qquad (12)$$

$$v_v \geq z_i + T_{is} - M(2 - b_{sv} - a_{iv}) \qquad \forall \quad i \in Z, s \in S, v \in V \qquad (13)$$

Constraints (10) – (13) are similar to constraints (4) – (7), which are described above. Constraints (10) is responsible for the selecting the first farm while taking into consideration constraints: (11), (12), (13). Constraints (10) insures that routing distance from parking place s, $s \in S$, to farm $i, i \in Z$ must be at least equal to the traveling distance from this parking place to this farm. Constraints (11) and (12) are mutually excluding constraints. If farm j , $j \in Z$, is visited after farm $i, i \in Z$, by a truck in a sub-tour v, $v \in V$, then the routing distance to farm j, must be larger than the routing distance to farm i, and vice versa. Constraints (13) states that the total traveling distance of a sub-tour v, $v \in V$ driven by a truck, must be larger or equal to the routing distance from a parking place to farm $i \in Z$ plus the routing distance from this farm to the parking place. Constraints (11) – (13) use the “big-M” value in the same way as described in constraints (5) – (7).

Flow constraints

$$\sum_{p\in P}\sum_{t\in R} f_{tsp} = \sum_{z\in Z} h_z w_{zs} \qquad \forall \quad s \in S \qquad (14)$$

Constraints (14) make sure that the total load collected from the visited farms, which are assigned to the parking place $s, s \in S$, must be equal to the load transferred from this parking place to the dairy plant *p*.

Capacity constrains

$$\sum_{s\in S}\sum_{t\in R} f_{tsp} - u_p \leq 0 \qquad \forall \quad p \in P \qquad (15)$$

$$\sum_{z\in Z} h_z a_{zv} \leq c_v \sum_{s\in S} b_{sv} \qquad \forall \quad v \in V \qquad (16)$$

$$\sum_{s\in S} f_{tsp} - e_t n_{pt} \leq 0 \qquad \forall \quad t \in R, p \in P \qquad (17)$$

Constraints (15) insures that the total load from parking place *s* to the plant $p, p \in P$ should not exceed the dairy plant demand. Constraints (16) states that total load from all farms visited in a sub-tour $v, v \in V$ by a truck should not exceed the truck capacity. Constraints (17) insures that total load of a truck with a trailer $t,\ t \in R$ from the assigned parking places *s* to plant $p, p \in P$ should not exceed the truck and trailer capacity.

Consistency constraints

$$a_{iv} + b_{sv} - w_{is} \leq 1 \qquad \forall \quad i \in Z, s \in S, v \in V \qquad (18)$$

$$e_t m_{st} - f_{tsp} \geq 0 \qquad \forall \quad t \in R, s \in S, p \in P \qquad (19)$$

Constraints (18) are consistency constraints between assignment variables. It insures that if farm $i, i \in Z$ and parking place $s, s \in S$ are assigned to a sub-tour $v, v \in V$, then this farm should be assigned to the same parking place. Constraints (19) are flow conservation constraints between flow and assignment variables. It states that if truck and trailer $t, t \in R$ is assigned to parking place $s, s \in S$ then the load sent by the truck with trailer from parking place *s* to plant $p, p \in P$ should not exceed the truck and trailer capacity.

Integrality and non-negativity requirements.

$$a_{zv} \in \{0,1\} \quad \forall \quad z \in Z, v \in V \tag{20}$$

$$b_{sv} \in \{0,1\} \quad \forall \quad s \in S, v \in V \tag{21}$$

$$x_{ij} \in \{0,1\} \quad \forall \quad (i,j) \in Z \tag{22}$$

$$m_{st} \in \{0,1\} \quad \forall \quad s \in S, t \in R \tag{23}$$

$$n_{pt} \in \{0,1\} \quad \forall \quad p \in P, t \in R \tag{24}$$

$$r_{ij} \in \{0,1\} \quad \forall \quad (i,j) \in S \tag{25}$$

$$w_{zs} \in \{0,1\} \quad \forall \quad z \in Z, s \in S \tag{26}$$

$$y_s \in \{0,1\} \quad \forall \quad s \in S \tag{27}$$

$$f_{tsp} \geq 0 \quad \forall \quad t \in R, s \in S, p \in P \tag{28}$$

In addition, the restrictions, (20) – (28), state the attributes of the variables.

6.0 Experimental analyses

In this section, we will present the solutions we obtained by using our algorithm implemented in the AMPL language using solver CPLEX 12.7.0. We will start our experiments with small instances and will enlarge the size it with each experiment to test our algorithm.

Tables 1-8 show input data used for the experiments. All volumes in the tables are expressed in liters.

Input and output data of all experiments is available in Appendix B, and travel distances between all nodes are available in Appendix C.

6.1 Experiment 1

In the first experiment, we assume that TINE BA in Elnesvågen collects milk from ten farmers with one complete vehicle with the biggest possible capacity (Table 1). The truck can leave its trailer on three possible parking places. Table 2 shows the three-day rate production quantity of milk in each of the farms considered.

ID vehicle	Truck capacity	Trailer capacity	Capacity (complete vehicle)
R_1	18 500	15 000	33 500

Table 1. Vehicle characteristics.

Farm	1	2	3	4	5	6	7	8	9	10	∑
Milk production	2730	1266	1338	1437	1524	3342	2319	1947	1071	1362	18336

Table 2. Farm characteristics.

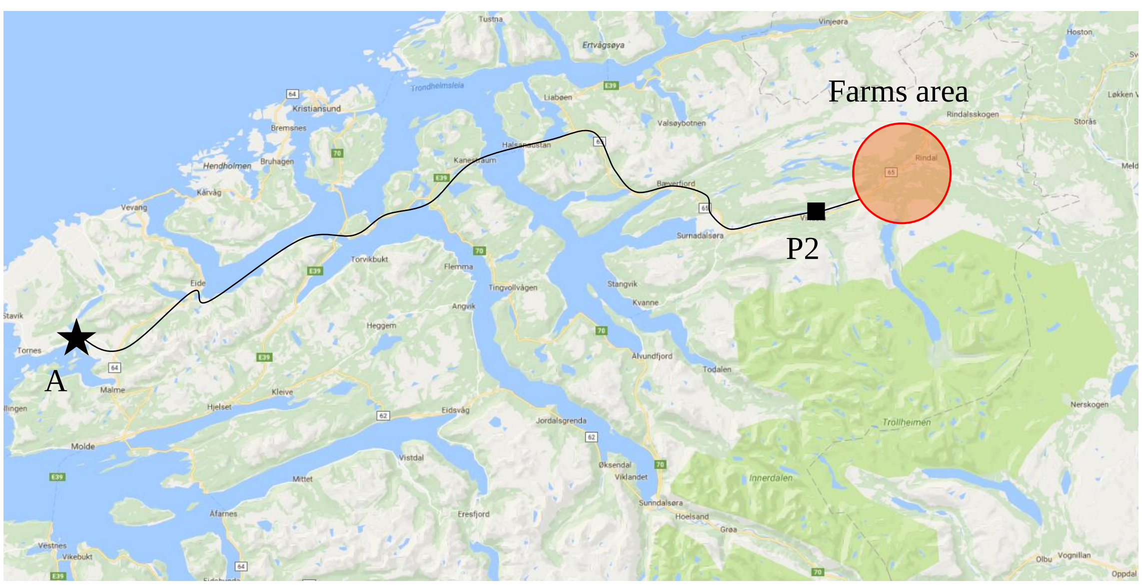
6.1.1 Computational results

The computational results demonstrate that the proposed algorithm find the optimal solution for this small instance. In case of this experiment, the truck with a trailer start its tour from dairy plant, leave the trailer at one prearranged parking place and collects milk from the 10 farms within one tour. The model chose from three possible parking places, and find the most convenient one, which is P2.

In Pictures 2 and 3, we present visual illustrations of the locations and the routing solution we have found. Coordinates of the locations are given in Table 3.

Name	Description	Longitude	Latitude
A	Elnesvågen dairy plant	7.13429	62.85357
P2	Parking place #2 in Surnadal	8.95172	62.99303
1	Farms	9.10527	63.02479
2		9.09748	63.03192
3		9.09692	63.02783
4		9.08533	63.02715
5		9.09702	63.03513
6		9.18047	63.04762
7		9.15693	63.05148
8		9.19217	63.06112
9		9.19477	63.06242
10		9.19609	63.06624

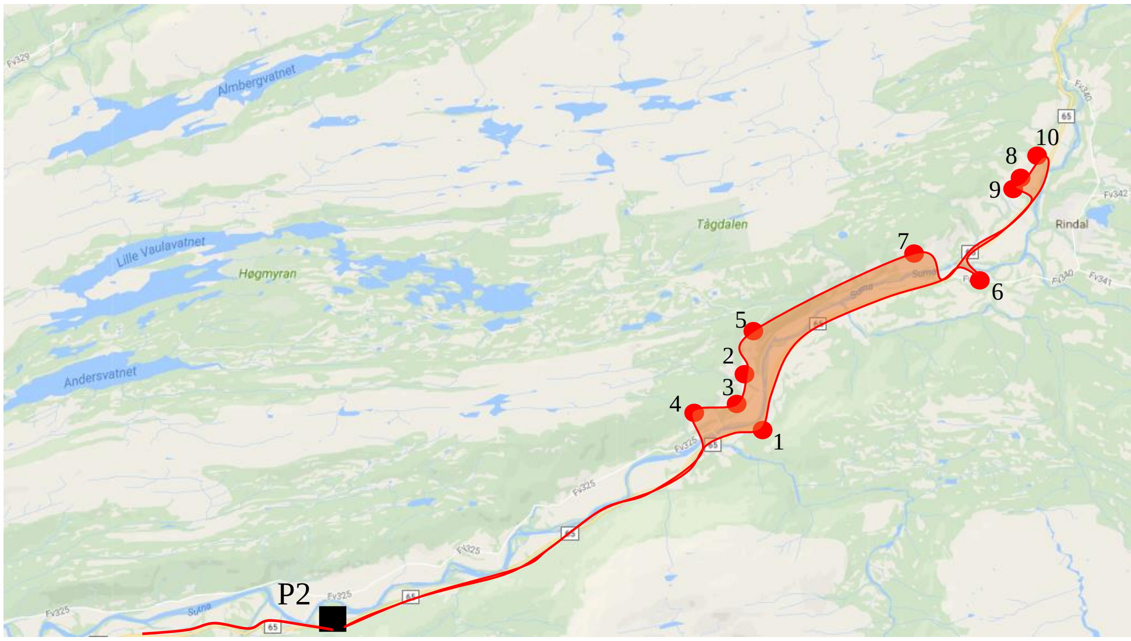
Table 3. Coordinates of the locations.



Picture 2. Routing solution for Experiment 1. Route from dairy plant to parking place.

In Picture 2 we can see the routing solutions from the dairy plant through the parking place (P2) and further to the area with the farms. The dairy plant is represented by star (A), the parking place is denoted by square (P2) and area with the farms by a red circle.

In Picture 3 we can see the more detailed route from the parking place to the farms. The farms are denoted by red circles from 1 to 10 and the parking place by a black square (P2).



Picture 3. Routing solution for Experiment 1 from parking place to farms.

The optimal route from the dairy plant and back is: A – P2 – 1 – 6 – 10 – 8 – 9 – 7 – 5 – 2 – 3 – 4 – P2 – A, with total traveling distance 279.927 km. In this case, the capacity of the single truck is enough to collect milk from all the farms, and the truck makes just one tour from the parking place. It means that the truck does not have to transfer milk to its trailer, and for the instance with this input data, it would be enough to have just a single truck. In the real-world situation, with these conditions a truck would drive alone without the trailer. This experiment was performed to check whether the algorithm worked properly with the parking of trailers and with sub-tours from the parking place.

The solution was found in 0.17 sec.

6.2 Experiment 2

Here we use the same farm characteristics as in Example 1, but consider a truck with a smaller capacity (Table 4).

ID vehicle	Truck capacity	Trailer capacity	Capacity (complete vehicle)
R_1	9 500	15 000	24 500

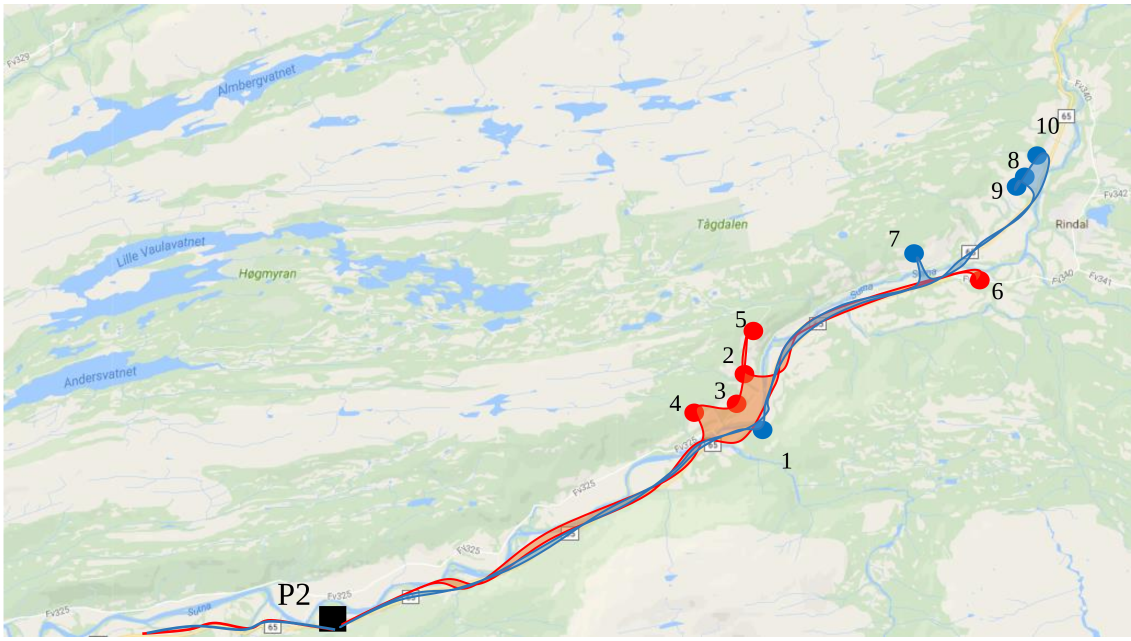
Table 4. Vehicle characteristics.

6.2.1 Computational results

In this case, the truck with a smaller capacity cannot collect all the milk within one tour and has to return to the parking place, where it left its trailer, transfer milk to it and then collect milk from the remaining farms. In this case, the truck alone makes two sub-tours from the parking place. The model choses parking place (P2), so the route from the dairy plant to the parking place (P2) will be the same as in Picture 2.

To show the locations on the map we have used coordinates from Table 3.

In Picture 4 we can see the route structure for this case, where the parking place is shown by a black square (P2), customers from one sub-tour is represented by blue circles and customers from another sub-tour is represented by red circles.



Picture 4. Routing solution for Experiment 2 from parking place to farms.

The optimal route performance for this case is: A – P2 – 1 – 7 – 8 – 9 – 10 – P2 – 6 – 2 – 5 – 3 – 4 – P2 – A, with total travelling distance 306.819 km.

As we can see from the solution, in the first sub-tour, which consists farms: 1, 7, 8, 9, 10, the truck tank is almost full. If we summarize the milk gathered from these farms it will be 9 429 liters, and the capacity of the truck is 9 500 liters. On the second sub-tour, milk collected from the rest of the farms will be 8 907 liters. It is obvious that the truck had to make two sub-tours to collect milk from all the farms due to the limited capacity of the truck tank. The total load of the complete vehicle is 18 336, which is far within the total capacity of the vehicle which is 24 500 liters. In contrary to the first experiment, this experiment is relevant for a real-world situation, where both truck and trailer used for milk transportation.

It took 0.69 sec. to find a solution.

6.3 Experiment 3

In this case, we will consider a situation, where one complete vehicle uses two parking places to visit farms.

ID vehicle	Truck capacity	Trailer capacity	Capacity (complete vehicle)

R_1	6 500	12 500	19 000

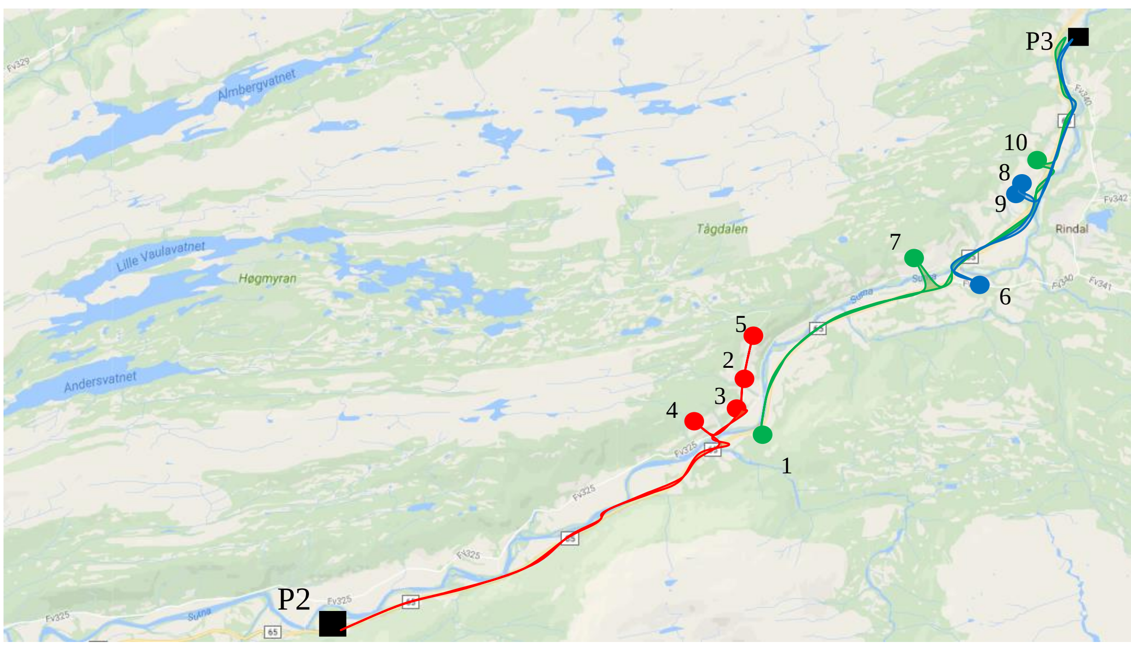
Table 5. Vehicle characteristics.

Table 5 shows vehicle's characteristics for this experiment Farm characteristics are the same as in Table 2. We consider a vehicle with smaller capacity than in the previous experiments and we use the same number of customers.

Coordinates for all locations presented in Picture 5 below is given in Table 9.

6.3.1 Computational results

The optimal route solution is presented in Picture 5. The sub-tour from parking place (P2) is represented by red color, and farms that belong to that sub-tour by red circles. The sub-tours from parking place (P3) are represented by green and blue colors, similar as the farms.



Picture 5. Routing solution for Experiment 3 with sub-tours from parking places to farms.

The optimal route for this case is: A – P3 – 7 – 1 – 10 – P3 – 9 – 8 – 6 – P3 – P2 – 3 – 2 – 5 – 4 – P2 – A, with a total traveling distance 326.25 km. The complete vehicle R_1 leaves the dairy plant (A) and drives to the first prearranged parking place (P3), leaves the trailer there and makes two sub-tours from that parking place. The first sub-tour includes farms 7, 1 and 10 and after collecting milk from them, the truck drives back to the parking place (P3) to transfer the milk to the trailer. Then, the single truck makes a second sub-tour to collect milk from the rest of the farms assigned to that parking place. After that, the vehicle (R_1) continues the route and moves the trailer to the next prearranged parking place (P2), parks it there and collects milk from the rest of the farms. Finally, the truck comes back to the parking place (P2), connects the trailer and drives back to the dairy plant (A).

In the first sub-tour from the parking place (P3) the single trucks collects 6 411 liters of milk, which makes the truck tank almost full, due to the capacity of 6 500 liters. After transferring the milk from the first sub-tour to the trailer, on the second sub-tour from the parking place (P3) the truck collected almost the same amount of milk – 6 360 liters. It means that we have a total amount of 12771 liters from the two first sub-tours and most of the amount from the second sub-tour must be transferred to the trailer, but the capacity is not large enough for all. Thus, a small amount of milk has to remain in the truck tank before starting on the last sub-tour. In the last sub-tour from the parking place (P2) the load of the truck alone was 5 565 liters. The total load of milk from three sub-tours and two parking places is 18 425, which is within the total vehicle capacity of 19 000. This case shows that the vehicle will utilize almost all its capacity. This experiment is relevant for a real-world situation. The model finds it optimal to use two parking places in one route to minimize the total routing distance.

The solver completed the calculations in 1.27 sec.

6.4 Experiment 4

In this experiment, we extend the number of farms to 20 with demand shown in Table 7 and use two vehicles with bigger capacity as shown in Table 6 to collect milk from all the farms.

ID vehicle	**Truck capacity**	**Trailer capacity**	**Capacity (complete vehicle)**
R_1	14 000	11 000	25 000
R_2	18 500	15 000	33 500

Table 6. Vehicle characteristics.

Farm	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Milk production	2730	1266	1338	1437	1524	3342	2319	1947	1071	1362

Farm	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	∑
Milk production	2889	3246	2484	2874	3273	2625	2616	1092	2844	3585	45864

Table 7. Farm characteristics.

6.4.1 Computational results

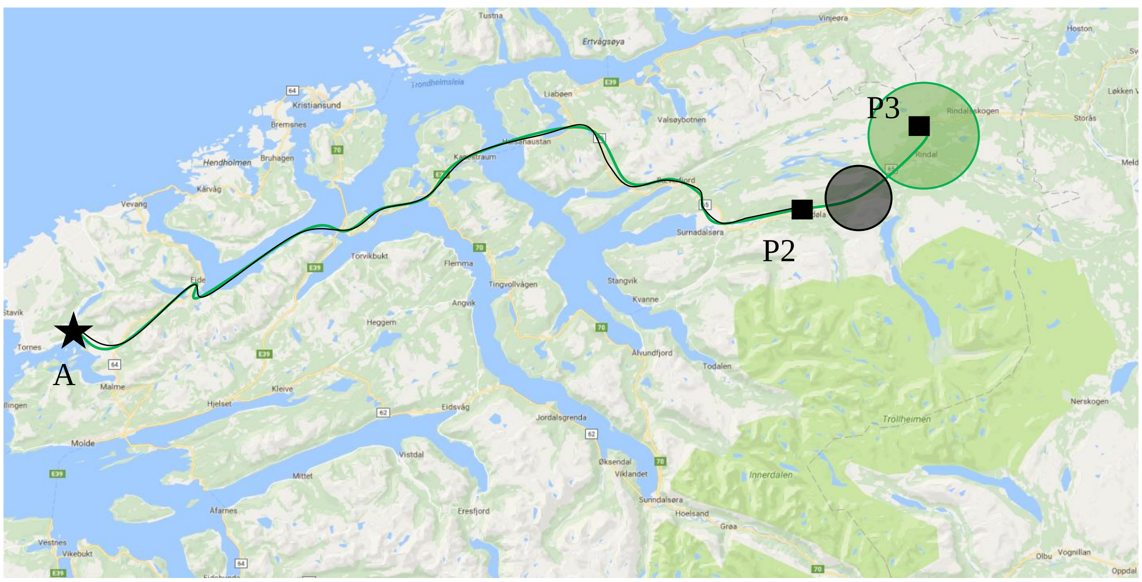
Coordinates of all locations shown in Pictures 6 and 7 are given in Table 8.

Name	Description	Longitude	Latitude
A	Elnesvågen dairy plant	7.13429	62.85357
P2	Parking place #2 in Surnadal	8.95172	62.99303
P3	Parking place #3 in Rindal	9.20766	63.07098
1	Farms	9.10527	63.02479
2		9.09748	63.03192
3		9.09692	63.02783
4		9.08533	63.02715
5		9.09702	63.03513
6		9.18047	63.04762
7		9.15693	63.05148
8		9.19217	63.06112
9		9.19477	63.06242
10		9.19609	63.06624
11		9.2009	63.07214
12		9.2022	63.07271
13		9.29493	63.10318
14		9.20066	63.07995
15		9.20347	63.06867

16		9.21665	63.09245
17		9.23278	63.09573
18		9.24923	63.09672
19		9.25258	63.10087
20		9.27443	63.09337

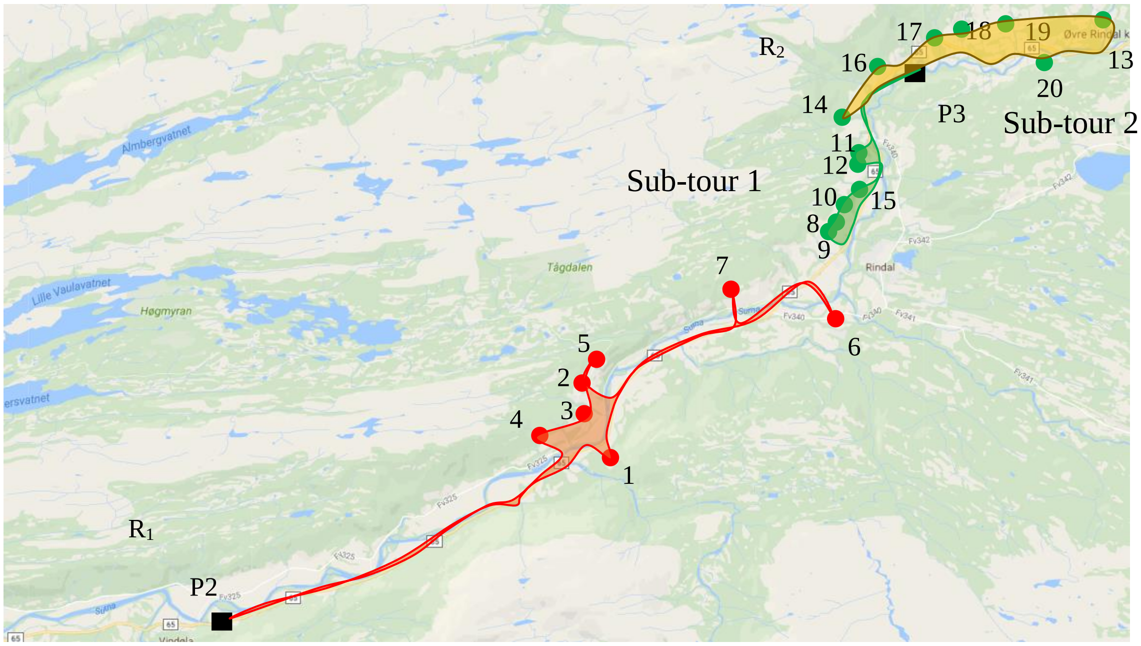
Table 8. Coordinates of locations used in Experiment 4.

In this case, we have two complete vehicles with different capacities. In Picture 6 the visiting area of farms and route driven by vehicle R_1 represented by black color, and green color represents route and farms area that belongs to vehicle R_2. From three possible parking places the model chooses parking place (P2) for the route of vehicle R_1 and parking place (P3) for the route of vehicle R_2.



Picture 6. Routing solution for Experiment 4 from dairy plant to parking places and farms.

In Picture 7, red color represents the farms and route visited from parking place (P2). Farms that are assigned to the other truck are represented by green color and the sub-tours from parking place (P3) are represented by green and yellow colors.



Picture 7. Routing solution for Experiment 4 from parking places to farms.

In the Picture 7, we can see the optimal route structure for R_1 which is: A – P2 – 4 – 3 – 5 – 2 – 7 – 6 – 1 – P2 – A. The truck with the trailer (R_1) drives to parking place (P2) and parks the trailer there. After that, the single truck collects milk from the farms 1-7 in one tour. The situation for this vehicle is similar to the situation in Experiment 1, where the capacity of a single trick was enough to collect milk from all the farms without using the trailer of this truck.

The optimal route solution for vehicle R_2 is: A – P3 – 15 – 10 – 8 – 9 – 12 – 11 – P3 – 14 – 16 – 17 – 18 – 19 – 13 – 20 – P3 – A. The truck with the trailer drives to parking place (P3), leave its trailer there and then the truck alone makes two sub-tours to collect the milk from the farms. The first sub-tour includes five farms and the second sub-tour includes six farms (Picture 7).

The total traveling distance for both vehicles is 578.759.

The calculations were completed in 3854.51 sec., corresponding to more than one hour computing time.

The solution shows that the truck disconnected from the complete vehicle R_2 has made two sub-tours. In the first sub-tour it collected 16 272 liters of milk and in the second 15 246 liters which both are within the capacity of 18 500 for the truck alone. Then the total load of the truck with the trailer is 31 518, while the total capacity of this complete vehicle is 33 500, making it almost full.

The truck disconnected from vehicle R_1 makes one tour and collect 13 956 liters of milk, when its capacity is 14 000. If the truck's capacity was not sufficient to collect milk from these 7 farms, than it would have to drive more sub-tours. As mentioned above, in a real-world situation with this data the truck would not need to carry a trailer, since the truck's capacity is enough to collect milk from the assigned farms. However, in our model we made an assumption that all the trucks have to leave the dairy plant with a trailer connected. We can conclude that in this experiment, the total capacity of both vehicles (R_1 - R_2) was much bigger than the total milk production of the farms and that is the reason why one of the vehicles made just one sub-tour. In a real-life situation for this case, one would only have to drive one complete vehicle and one single truck.

This experiment also shows that in the optimal solution, the vehicle capacities will be utilized as much as possible to reduce the number of sub-tours. Thus, even with the assumptions in our model about using a parking place, the solution will identify the routes which can be performed by a single truck when we have surplus capacity.

6.5 Experiment 5

Here we will consider a case very similar to Experiment 4, but with reduced vehicles capacities. Two vehicles with the same capacity are presented in Table 9. The farm characteristics are shown in Table 7, and the total demand is close to the combined capacity of the two vehicles.

ID vehicle	Truck capacity	Trailer capacity	Capacity (complete vehicle)
R_1	12 500	12 000	24 500
R_2	12 500	12 000	24 500

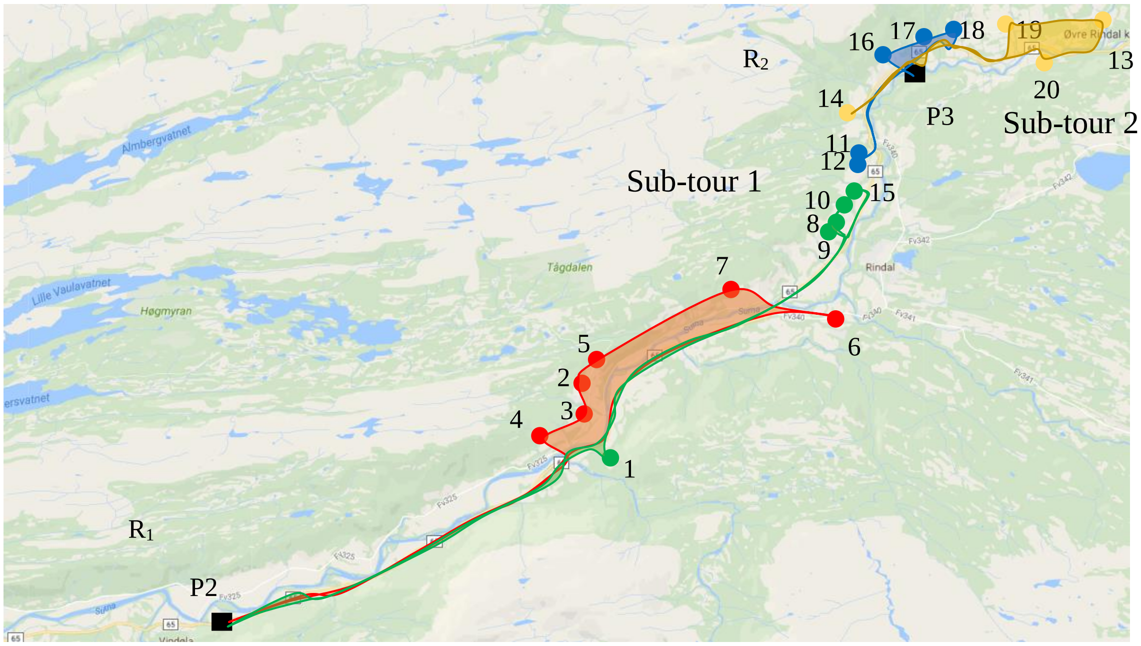
Table 9. Vehicle characteristics.

6.5.1 Computational results

Coordinates for all locations shown in Pictures 8 are given in Table 8.

The route performance from the dairy plant (A) to the parking places will be the same as in the previous Experiment (Picture 6).

In Picture 8, red and green colors represent two sub-tour driven from parking place (P2) by vehicle R_1. Blue and yellow colors represent the sub-tours from parking place (P3) driven by vehicle R_2.



Picture 8. Routing solution for Experiment 5 from parking places to farms.

The route solution for vehicle R_1 it will be: A – P2 – 4 – 3 – 2 – 5 – 7 – 6 – P2 – 15 – 10 – 8 – 9 – 1 – P2 – A. The complete vehicle R_1 drives from the dairy plant to parking place (P2), parks its trailer there and the single truck collects milk from farms 2 – 7. After transferring milk to the trailer, the truck makes a second sub-tour where it collects milk from the rest of the farms assigned to that parking place. The total load of both sub-tors from parking place (P2) is 21 609 liters.

The route structure for R_2 is: A – P3 – 11 – 12 – 18 – 17 – 16 – P3 – 20 – 13 – 19 –14 – P3 – A. The vehicle R_2 will drive from the plant (A) to the parking place (P3) from where the truck will make two sub-tours as well. The total load of vehicle R_2 after collecting milk from the farms is 24 255 liters and the total vehicle capacity is 24 500, which makes the capacity of the truck tank almost fully utilized.

In this experiment, the truck tank and trailer tank of both vehicles R_1 and R_2 are used. The trucks both had to make two sub-tours because of the decreased capacities compared to the previous example.

The total traveling distance for both vehicles is 612.6416.

The calculations were completed in 73715.92 sec., which is around 20 hours and 30 minutes of computing time.

The model itself is quite complex. The computing time depends on the size of the model's sets and parameters, on the numbers of variables and constraints, and on the number and complexity of the terms in the constraints. Thus, larger instances will require more time to solve. We can clearly see with the increased searching time in Experiment 5, that this is on the upper limit of how large instances can be that are solvable with our model.

As we saw in Experiment 4 and 5, the computing time do not only depend on the number of nodes, but also the size of the vehicles. A solution with many sub-tours is more complex than a solution with fewer, and we saw that the searching time increased considerably when the second vehicle was forced to drive one more sub-tour due to the capacity restrictions.

7.0 Conclusion and further research

In this research, we have created a model for finding the exact solution. The model is general for the full milk collection problem, but only solvable for small instances of the problem. We have tested the model using real-world data, and we have shown that it works successfully for a small subset of farms taken from the real-world problem. However, it is not perfect for real world situations, due to the complexity and size of the problem. Our calculations showed that to utilize the complete vehicles optimally, their total capacity has to be as close as possible to the total production of milk in the farms assigned to the vehicle. If it is smaller, the solution will be infeasible. Moreover, the bigger capacity of the truck, fewer sub-tours for collecting milk before transferring it to the trailer, has to be driven. This will decrease the total traveling distance of the truck, and thus using a large truck carrying a small trailer will be the best truck-trailer combination for this problem.

To solve a real-world problem with our model we can split the problem into manageable instances. Thus, we need to solve a partitioning problem aligning farms to vehicles and vehicles to plants. Then the routes for each vehicle could be solvable to optimality. Still the total solution might not be optimal, since we do not know for certain if the partitions are the correct ones.

For routes with trucks carrying trailers and subsets of the farms assigned to the vehicles, this method is able to identify a good exact solution for collecting milk in this area. However, by using hybrid methods for example using heuristics for partitioning farms into clusters where demands equals vehicle capacities, and exact solution of routes for each vehicle, we could probably find good solutions for the full problem.

Further research in this area can consider:

- the different visiting frequencies for milk collection,
- including time windows both at farms and for ferries,
- selecting the most proper vehicles on each tour,
- stochastic demand at farms,
- including total costs instead of distance,
- including costs for ferries and toll-roads,
- different costs for large vehicles than for small,
- drivers wage different on small tours from large tours etc.

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9.0 Appendices

Appendix A: Mathematical model formulation

Minimize

$$\sum_{v\in V} v_v + \sum_{t\in R} t_t \tag{1}$$

S.t.

$$\sum_{t\in R} m_{st} = y_s \qquad \forall \quad s \in S \tag{2}$$

$$\sum_{p\in P} n_{pt} \leq 1 \qquad \forall \quad t \in R \tag{3}$$

$$s_i \geq T_{ji}(n_{jt} + m_{it} - 1) \qquad \forall \quad i \in S, j \in P, t \in R \tag{4}$$

$$s_j \geq s_i + T_{ji} - M\left(1 - r_{ij}\right) - M(2 - m_{jt} - m_{it}) \qquad \forall \quad t \in R, i \in S, j \in S, i \neq j \tag{5}$$

$$s_i \geq s_j + T_{ij} - M\left(r_{ij}\right) - M(2 - m_{jt} - m_{it}) \qquad \forall \quad t \in R, i \in S, j \in S, i \neq j \tag{6}$$

$$t_t \geq s_i + T_{ip} - M(2 - n_{pt} - m_{it}) \qquad \forall \quad p \in P\ i \in S, t \in R \tag{7}$$

$$\sum_{v\in V} a_{zv} = 1 \qquad \forall \quad z \in Z \tag{8}$$

$$\sum_{s\in S} b_{sv} \leq 1 \qquad \forall \quad v \in V \tag{9}$$

$$z_i \geq T_{si}(b_{sv} + a_{iv} - 1) \qquad \forall \quad s \in S, i \in Z, v \in V \tag{10}$$

$$z_j \geq z_i + T_{ji} - M\left(1 - x_{ij}\right) - M(2 - a_{jv} - a_{iv}) \qquad \forall \quad v \in V, (i,j) \in Z, i \neq j \tag{11}$$

$$z_i \geq z_j + T_{ij} - M\left(x_{ij}\right) - M(2 - a_{jv} - a_{iv}) \qquad \forall \quad v \in V, i \in Z, j \in Z, i \neq j \tag{12}$$

$$v_v \geq z_i + T_{is} - M(2 - b_{sv} - a_{iv}) \qquad \forall \quad i \in Z, s \in S, v \in V \tag{13}$$

$$\sum_{p\in P}\sum_{t\in R} f_{tsp} = \sum_{z\in Z} h_z w_{zs} \qquad \forall \quad s \in S \qquad (14)$$

$$\sum_{s\in S}\sum_{t\in R} f_{tsp} - u_p \leq 0 \qquad \forall \quad p \in P \qquad (15)$$

$$\sum_{z\in Z} h_z a_{zv} \leq c_v \sum_{s\in S} b_{sv} \qquad \forall \quad v \in V \qquad (16)$$

$$\sum_{s\in S} f_{tsp} - e_t n_{pt} \leq 0 \qquad \forall \quad t \in R, p \in P \qquad (17)$$

$$a_{iv} + b_{sv} - w_{is} \leq 1 \qquad \forall \quad i \in Z, s \in S, v \in V \qquad (18)$$

$$e_t m_{st} - f_{tsp} \geq 0 \qquad \forall \quad t \in R, s \in S, p \in P \qquad (19)$$

$$a_{zv} \in \{0,1\} \qquad \forall \quad z \in Z, v \in V \qquad (20)$$

$$b_{sv} \in \{0,1\} \qquad \forall \quad s \in S, v \in V \qquad (21)$$

$$x_{ij} \in \{0,1\} \qquad \forall \quad (i,j) \in Z, i \neq j \qquad (22)$$

$$m_{st} \in \{0,1\} \qquad \forall \quad s \in S, t \in R \qquad (23)$$

$$n_{pt} \in \{0,1\} \qquad \forall \quad p \in P, t \in R \qquad (24)$$

$$r_{ij} \in \{0,1\} \qquad \forall \quad (i,j) \in S, i \neq j \qquad (25)$$

$$w_{zs} \in \{0,1\} \qquad \forall \quad z \in Z, s \in S \qquad (26)$$

$$y_s \in \{0,1\} \qquad \forall \quad s \in S \qquad (27)$$

$$f_{tsp} \geq 0 \qquad \forall \quad t \in R, s \in S, p \in P \qquad (28)$$

Appendix B: Route performance

<table>
<tr><td rowspan="12">Route performance</td><td rowspan="8">Input</td><td>Experiment #</td><td>Experiment 1</td><td>Experiment 2</td><td colspan="2">Experiment 3</td><td colspan="2">Experiment 4</td><td colspan="2">Experiment 5</td></tr>
<tr><td>Plant</td><td>1</td><td>1</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td></tr>
<tr><td>Parking places</td><td>#1 #2 #3</td><td>#1 #2 #3</td><td colspan="2">#1 #2 #3</td><td colspan="2">#1 #2 #3</td><td colspan="2">#1 #2 #3</td></tr>
<tr><td>Farms</td><td>10</td><td>10</td><td colspan="2">10</td><td colspan="2">20</td><td colspan="2">20</td></tr>
<tr><td>Number of complete vehicles</td><td>1</td><td>1</td><td colspan="2">1</td><td colspan="2">2</td><td colspan="2">2</td></tr>
<tr><td>Truck capacity</td><td>18 500</td><td>9 500</td><td colspan="2">6500</td><td>14 000</td><td>18 500</td><td colspan="2">12 500</td></tr>
<tr><td>Trailer capacity</td><td>15 000</td><td>15 000</td><td colspan="2">12500</td><td>11 000</td><td>15 000</td><td colspan="2">12 000</td></tr>
<tr><td>Complete Vehicle’s capacity</td><td>33 500</td><td>24 500</td><td colspan="2">19 000</td><td>25 000</td><td>33500</td><td colspan="2">24 500</td></tr>
<tr><td rowspan="4">Output</td><td>Total traveling distance (km)</td><td>281,927</td><td>309,819</td><td colspan="2">326,25</td><td colspan="2">580,759</td><td colspan="2">612.6416</td></tr>
<tr><td>Parking places chosen</td><td>#2</td><td>#2</td><td>#2</td><td>#3</td><td>#2</td><td>#3</td><td>#2</td><td>#3</td></tr>
<tr><td>Number of sub-tours</td><td>1</td><td>2</td><td>1</td><td>2</td><td>1</td><td>2</td><td>2</td><td>2</td></tr>
<tr><td>Computing time (sec.)</td><td>0.17</td><td>0.69</td><td colspan="2">1.27</td><td colspan="2">3854.51</td><td colspan="2">73715.92</td></tr>
</table>

Table 10. Input data and results for Experiments 1- 5.

Appendix C: distance matrix

The symmetric distance matrix in km between 20 farms, three parking places (P1-P3) and one plant (A) in both directions.

	1	2	3	4	5	6
1	.	2.3044	1.8742	2.4695	2.7138	5.2008
2	2.3044	.	0.6343	1.4425	0.4256	6.437
3	1.8742	0.6343	.	1.0123	1.0437	6.0349
4	2.4695	1.4425	1.0123	.	1.8519	6.8431
5	2.7138	0.4256	1.0437	1.8519	.	6.8464
6	5.2008	6.437	6.0349	6.8431	6.8464	.
7	5.622	6.3622	5.9601	6.7683	6.7716	1.5818
8	6.936	7.793	7.3909	8.1991	8.2024	2.8958
9	6.9913	7.8483	7.4462	8.2544	8.2577	2.9511
10	7.496	8.353	7.9509	8.7591	8.7624	3.4558
11	9.2831	10.1401	9.7380	10.5462	10.5495	5.2429
12	9.1810	10.0380	9.6359	10.4441	10.4474	5.1408
13	15.2151	16.0721	15.6700	16.4782	16.4815	10.8618
14	9.7013	10.5583	10.1562	10.9644	10.9677	5.6611
15	7.6785	8.5355	8.1334	8.9416	8.9449	3.6383
16	10.7873	11.6443	11.2422	12.0504	12.0537	6.7471
17	11.7270	12.5840	12.1819	12.9901	12.9934	7.6868
18	12.7779	13.6349	13.2328	14.0410	14.0443	8.7377
19	13.3587	14.2157	13.8136	14.6218	14.6251	9.3185
20	14.0081	15.1102	14.7081	15.5163	15.5196	8.8749
A	128.8812	129.2613	128.9252	129.5205	128.9053	133.9246
P1	21.2908	21.765	21.3348	21.9301	22.1744	26.3342
P2	8.7429	9.2171	8.7869	9.3822	9.6265	13.7863
P3	8.4356	9.2926	8.8905	9.6987	9.702	4.3954

	7	8	9	10	11	12	13	14	15
1	5.622	6.936	6.9913	7.496	9.2831	9.1810	15.2151	9.7013	7.6785
2	6.3622	7.793	7.8483	8.353	10.1401	10.0380	16.0721	10.5583	8.5355
3	5.9601	7.3909	7.4462	7.9509	9.7380	9.6359	15.6700	10.1562	8.1334
4	6.7683	8.1991	8.2544	8.7591	10.5462	10.4441	16.4782	10.9644	8.9416
5	6.7716	8.2024	8.2577	8.7624	10.5495	10.4474	16.4815	10.9677	8.9449
6	1.5818	2.8958	2.9511	3.4558	5.2429	5.1408	10.8618	5.6611	3.6383
7	.	2.8587	2.914	3.4187	5.2058	5.1037	11.1378	5.6240	3.6012
8	2.8587	.	0.3032	0.7984	3.4357	3.3336	9.3677	3.8539	1.8311
9	2.914	0.3032	.	0.8632	3.4910	3.3889	9.4230	3.9092	1.8864
10	3.4187	0.7984	0.8632	.	3.9957	3.8936	9.9277	4.4139	2.3911
11	5.2058	3.4357	3.4910	3.9957	.	0.1245	7.3866	1.8728	1.8062
12	5.1037	3.3336	3.3889	3.8936	0.1245	.	7.2845	1.7707	1.7041
13	11.1378	9.3677	9.4230	9.9277	7.3866	7.2845	.	6.3246	7.7382
14	5.6240	3.8539	3.9092	4.4139	1.8728	1.7707	6.3246	.	2.2244
15	3.6012	1.8311	1.8864	2.3911	1.8062	1.7041	7.7382	2.2244	.
16	6.7100	4.9399	4.9952	5.4999	2.9588	2.8567	4.6408	1.8968	3.3104
17	7.6497	5.8796	5.9349	6.4396	3.8985	3.7964	3.6999	2.8365	4.2501
18	8.7006	6.9305	6.9858	7.4905	4.9494	4.8473	2.7874	3.8874	5.3010
19	9.2814	7.5113	7.5666	8.0713	5.5302	5.4281	2.6780	4.4682	5.8818
20	10.1759	8.4058	8.4611	8.9658	6.4247	6.3226	2.0923	5.8471	6.7763
A	134.3458	135.6598	135.7151	136.2198	138.0069	137.9048	143.9389	138.4251	136.4023
P1	26.7554	28.0694	28.1247	28.6294	30.4165	30.3144	36.3485	30.8347	28.8119
P2	14.2075	15.5215	15.5768	16.0815	17.8686	17.7665	23.8006	18.2868	16.2640
P3	4.3583	2.5882	2.6435	3.1482	1.3585	1.2564	7.2905	1.7767	0.9587

	16	17	18	19	20	A	P1	P2	P3
1	10.7873	11.7270	12.7779	13.3587	14.0081	128.8812	21.2908	8.7429	8.4356
2	11.6443	12.5840	13.6349	14.2157	15.1102	129.2613	21.765	9.2171	9.2926
3	11.2422	12.1819	13.2328	13.8136	14.7081	128.9252	21.3348	8.7869	8.8905
4	12.0504	12.9901	14.0410	14.6218	15.5163	129.5205	21.9301	9.3822	9.6987
5	12.0537	12.9934	14.0443	14.6251	15.5196	128.9053	22.1744	9.6265	9.702
6	6.7471	7.6868	8.7377	9.3185	8.8749	133.9246	26.3342	13.7863	4.3954
7	6.7100	7.6497	8.7006	9.2814	10.1759	134.3458	26.7554	14.2075	4.3583
8	4.9399	5.8796	6.9305	7.5113	8.4058	135.6598	28.0694	15.5215	2.5882
9	4.9952	5.9349	6.9858	7.5666	8.4611	135.7151	28.1247	15.5768	2.6435
10	5.4999	6.4396	7.4905	8.0713	8.9658	136.2198	28.6294	16.0815	3.1482
11	2.9588	3.8985	4.9494	5.5302	6.4247	138.0069	30.4165	17.8686	1.3585
12	2.8567	3.7964	4.8473	5.4281	6.3226	137.9048	30.3144	17.7665	1.2564
13	4.6408	3.6999	2.7874	2.6780	2.0923	143.9389	36.3485	23.8006	7.2905
14	1.8968	2.8365	3.8874	4.4682	5.8471	138.4251	30.8347	18.2868	1.7767
15	3.3104	4.2501	5.3010	5.8818	6.7763	136.4023	28.8119	16.2640	0.9587
16	.	1.1527	2.2036	2.7844	4.3178	139.5111	31.9207	19.3728	2.8627
17	1.1527	.	1.2627	1.8435	3.3769	140.4508	32.8604	20.3125	3.8024
18	2.2036	1.2627	.	0.9310	2.9410	141.5017	33.9113	21.3634	4.8533
19	2.7844	1.8435	0.9310	.	2.8316	142.0825	34.4921	21.9442	5.4341
20	4.3178	3.3769	2.9410	2.8316	.	142.7319	35.1415	22.5936	6.3286
A	139.5111	140.4508	141.5017	142.0825	142.7319	.	109.2007	120.1428	137.1594
P1	31.9207	32.8604	33.9113	34.4921	35.1415	109.2007	.	12.5524	29.569
P2	19.3728	20.3125	21.3634	21.9442	22.5936	120.1428	12.5524	.	17.0211
P3	2.8627	3.8024	4.8533	5.4341	6.3286	137.1594	29.569	17.0211	.