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**BØK950 Økonomi og administrasjon**

The Characteristics of the Swedish/Norwegian El-Certificate Market:  
An empirical impulse-response analysis

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# **The Characteristics of the Swedish/Norwegian El-Certificate Market: An empirical impulse-response analysis**

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## ***Abstract***

In the recent years the increasing threat of climate change has become a political issue all around the world. Looking for ways to minimize climate change has led to ideas of using renewable production of electricity instead of coal, oil and gas. In Sweden and Norway, the El-Certificate Scheme was introduced as a way to achieve a higher production of renewable energy, and with that achieve a lower amount of carbon emissions. This thesis analyses the conditional mean and volatility characteristics of the Spot and One-year forward price of the Swedish-Norwegian El-Certificate market (2005-2018). Based on statistical tests of the two weekly price movement series, a semi-nonparametric (SNP) model is used for a well-specified conditional mean and volatility. Moreover, from relatively new available model specification software, an impulse-response analysis is applied for extracting enhanced market characteristics. The market for Spot and One-Year Forward prices show similar results. Hence, for the impulse-response analysis both series show linear and asymmetric mean reversion. For both series, the responses are therefore greater for negative impulses. For the volatility of both series, the impulse-response analysis shows an increase in variance when the impulses increase, together with larger responses for negative impulses.

The impulse response is short, with a large standard derivation, indicating high uncertainty. The analysis shows large manifestations in the negative direction has a longer mean reversion time, the volatility is larger with a negative change.

Key words: Electricity certificates, Spot Price, One-Year Forward March, Trend, Conditional Mean, Volatility, Impulse-Response, Bootstrap, Semi-nonparametric (SNP), Asymmetry, Persistence.

## **Preface**

This thesis is a result of the completion of a Master of Science in Business Administration at Molde University College. The purpose of the thesis is to investigate the mean and volatility characteristics of the Swedish-Norwegian electricity certificate market and how the market operates under both normal and stressed conditions. The work has been an interesting challenge, and I have improved my understanding of programming (C/C++ in Linux-environment and Visual Basic in Windows environment) and of course insight of the importance of returns and risk (uncertainty) of financial commodity markets.

Microsoft Excel, Microsoft Word, EViews and Semi-Nonparametric (SNP, a C++ program) has been used in the analysis of the data and to present the results.

Molde, May 2018

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## 1.0 Introduction

El-Certificates is a subsidy for renewable power production. The power customers are financing the scheme by paying their power bill. The El-Certificate cost is incorporated in the power price. By the end of 2020 Sweden and Norway has a goal of increasing the renewable power production with 28.4 TWh, using El-Certificates to increase the profits and creating an incentive for investment. The subsidy of renewable energy sends a message of a desire to minimize the use on non-renewable energy sources. A power plant approved for el-certificates (renewable and covers the point in section 2.0) receives one el-certificate per megawatt hour (MWh) produced. The demand for el-certificates is formed by the state, making it mandatory for power suppliers and some power customers to cover a percentage of the power they sell with el-certificates. The producers of renewable energy receive the income for the sales of their el-certificates as a second income, next to the power price. This increases the profitability of green production. At the same time there is established a market for renewable energy which can be used for both speculation and reduction of risk (hedging).

In this thesis I have analyzed the conditional mean and variance for the Swedish/Norwegian El-certificate market for the period 2005 to 2018 (week 2), using a semi-nonparametric (SNP) model specification (Gallant and Tauchen 1990, 2017). The SNP model is an ARMA-GARCH model specification for the mean and volatility with an extension of Hermite functions to control for non-normal price movements. Features like ARMA/GARCH (Bollerslev 1986, Engle 1982), leptokurtosis(Clark 1973) and asymmetries(Nelson 1991) are of interest. The manuscripts and user manuals from Gallant, Rossi et al. is of great importance for the empirical results produced in this thesis. The framework form “Nonlinear Dynamic structures (Gallant, Rossi, and Tauchen 1993) is therefore used in the analysis. The analysis for the Swedish/Norwegian El-certificate Market, follows the structure of work done by Solibakke for the Nordic/Baltic Spot Electric Power System Price (Solibakke 2017).

The weekly prices for the period 2005 to 2018 are not stationary. For stationarity I therefore transformed the two series into logarithms and I calculate the price movement series. The SNP methodology follows the Schwartz Bayes information criterion (BIC) (E. Schwarz 1978) to obtain an preferred model specification for the mean, volatility and non-normality (Hermite functions). The residuals from the SNP model will be closely analyzed for misspecification.

Hence, we assume that the BIC specifications suggest an optimal semi-nonparametric GARCH model together with well-behaved residuals. This optimal SNP model is used for the impulse-response analysis. Moreover, the SNP methodology enable bootstrapping. The analysis uses a bootstrap methodology to report 95% confidence interval for the response results<sup>1</sup>.

For both the spot and the one-year forward price series, the main issue is to investigate market characteristics and possible optimal market positions from observed impulses. Do specific impulses produce impulses that increase our understanding of the el-certificate market? Can market participants establish optimal market positions by using observed market features from our impulse – response analysis? The thesis will use the SNP methodology proposed by Gallant, Rossi, and Tauchen (1993), Gallant and Tauchen (1990) to try and answer these questions.

The results show two well specified SNP specifications which reports residuals suggesting non-significant statistical model misspecification. The impulse response analysis indicates responses symmetric around the mean with a positive (negative) expected return response from positive (negative) price movement impulses. The analysis also suggests a more sensitive market for negative price movements. The negative price changes last for longer periods, and the volatility is higher for the negative impulses. This suggest a nervous market for the negative price movements. The holder of certificates wishes to sell when the price movement is negative because of an anticipation of larger negative price movements.

This thesis is systemized as follows;

part 2.0 gives a short overview of the Swedish/Norwegian El-certificate Market. The literature of relevance is presented in part 3.0. Part 4.0 shows the data for the Spot Price dataset and the One-Year Forward data set respectively, and the methodology used for the analysis. The empirical analysis and findings are presented in part 5.0, and part 6.0 concludes and summarizes the thesis. The appendix contains all the tables and figures, in full size, referred to

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<sup>1</sup> Sup-norm bands are constructed by bootstrapping, using simulation to consider the sampling variation in the estimation of. That is  $f(y/x)$ , changing the seed that generates densities and impulse response samples. The analysis applies 500 samples and 95% confidence intervals. A 95% sup-norm confidence band is a  $\epsilon$ -band around the profile  $f(y/x)$  that is just wide enough to contain 95% of the simulated profiles.

in the thesis. The page numbers in the List of tables and List of figures refers to illustrations in the appendix.

## 2.0 The Swedish-Norwegian EI-certificate market overview

The world is facing a change happening faster than anyone could foresee. Climate change poses a fundamental threat to the planet. Extreme weather threatening people's livelihood are seen every day. Warmer oceans, higher sea levels and droughts threatening freshwater supplies and crops (WWF 2018). To address this threat in an orderly manner the politicians have made it their mission to reduce carbon emissions and search for a natural adaptation to climate change. Therefore several actions towards reducing the emission of carbon dioxide. One of these schemes is the Swedish/Norwegian EI-certificate scheme.

The EI-certificate scheme, also known as the green certificate scheme, is a market-based instrument that aims to stimulate increased investments in renewable power production. The scheme is a policy being introduced in Sweden (2003) and Norway (2012) to handle emissions of carbon dioxide and other greenhouse gases. This was to fight against the threat of climate change. The scheme allows power producers, producing according to certain criteria, to receive electricity certificates that can be sold in the market. This will generate a higher revenue for the precipitants of certificates. The authorities order the electricity consumers to purchase certificates in line with their consumption, and the market arises. The suppliers of power add the cost of the mandatory certificates into the price of the electricity. The certificates are traded in a market and provides extra income to the producers. This makes it more profitable to invest in production of renewable energy. This way the certificates contributes to the global goal of more renewable energy. The criteria of receiving this type of subsidies are as follows, (Lovdata 2011):

1. Based on renewable energy sources, and construction start after 07.09.2009
2. Based on renewable energy sources and expanding their production, with construction start after 07.09.2009
3. New hydropower plants with construction start after 01.01.2004.

The market started in Sweden in 2003 and was implemented in Norway in 2012. The goal is to increase the power generation based on renewable power sources with 28,4 TWh from 2012 until 2020. In comparison Tafjord Kraft, a Norwegian producer of renewable power, produces approximately one twentieth of this amount.

The price of the certificates is determined by supply and demand. Demand is determined by the amount of electricity being used and the fixed energy certificate quota for each year. Producers and suppliers of electricity are forced by law to buy certificates for the amount of electricity they produce/supply. The supply is determined by the amount of renewable power produced. If there is many investing in renewable power production, there are many certificates in the market which leads to a lower price. If there are few investments in renewable power production, the price of the certificate increases to a point where investors invest.

In this thesis, I will use data from the biggest trading site in Sweden, [www.skm.se](http://www.skm.se). “SKM is the largest and most liquid marketplace for trading in El-certificates and they have the only public price quotation. The customers are both quota-liable electricity suppliers (buyers) and producers (sellers) and trading companies.” (Kraftmekling 2018) Using this site, I redeemed weekly data from 2005 until the second week of 2018 (about 680 weekly observations).

### 3.0 Literature

The methodology used in this thesis is the semi-nonparametric time series analysis (SNP) and was first introduced by Gallant and Nychka (1987), Gallant, Rossi, and Tauchen (1993), Gallant and Tauchen (1990). There is not a massive amount of research attempting to capture the dynamics of El-certificate prices, this thesis will be a contribution to the research of the scheme and the renewable power production.

For the purpose of this thesis I will use the work of Solibakke (2002) and follow the methodology of Gallant, Rossi, and Tauchen (1993). I will use Solibakke (2017) as an important building stone in the process and perform a similar analysis on the Swedish/Norwegian El-certificate market. Solibakke (2017) analyzed the Nordic/Baltic Spot Electric power system using the SNP model, by analyzing nonlinear impulse-response features, and using bootstrap.

The Impulse response analysis uses methodologies outlined by Gallant, Rossi, and Tauchen (1993). Gallant, Rossi, and Tauchen (1993) developed an approach for “analyzing the dynamics of a nonlinear time series that is represented by a nonparametric estimate of its one-step ahead conditional density”. The paper studies time series given shocks and comparing them to baseline series. In this thesis we use the same methodology and implements shock to the El-certificate market.

Methodology for forecasting in the Swedish/Norwegian market for El-Certificates (Wolfgang, Jaehnert, and Mo 2015) is one of the papers written to obtain a better understanding of the el-certificate market. The paper describes a methodology for forecasting in the market, integrating it in the electricity-market model EMPS. EMPS or EFI’s Multi-Area Power market Simulator is a computer tool for the optimization and simulation of the operation of power systems. The paper has a goal of forecasting the price of the el-certificates for the future. The paper presented the resulting performance of the market as a case study. The paper concludes with a stable unconditional expected value of the certificates, with the condition of few occurrences of deficit in the case. In this case the penalty cost and the electricity market are incorporated to achieve an analysis of the market, to obtain an understanding of the future prices of the certificates.

“Optimal management of green certificates in the Swedish-Norwegian market” (Fred Espen Benth 2014). The purpose is to investigate a valuation model for the income of selling tradeable green certificates in the Swedish-Norwegian Market. This is formulated as a single stochastic control problem. For optimization of the model they have implemented the production rate of renewable energy from a “typical plant”, the dynamics market price of TGCs and the cumulative number of certificates sold. The model finds optimal decision rules and a closed form solution to the control problem. In addition, the paper performed an empirical analysis of the log returns based on data from 2009 to 2013. The empirical analysis indicates a normal inverse Gaussian distributed Levy process. The paper provides the optimal selling strategy for producers of renewable energy (receivers of certificates).

“The role of regulatory uncertainty in certificate markets: A case study of the Swedish/Norwegian market” is written as an analysis of the impact of regulatory changes on the el-certificate price volatility. The paper takes into consideration the regulatory risks the investors is faced with. Regulatory risk means that it can be a change in the regulation which will impact the price. The analysis indicates a higher volatility caused by the changes in regulation. The paper analysis the change in volatility after implementing a joint Swedish-Norwegian el-certificate market. Results show an increased volatility in the period 2010 and 2011. This shows the impact of regulatory changes, in this case evidence of its negative impact leading to increased volatility. Given the strongly regulated el-certificate market, this is important to have in mind when analyzing the volatility. The economic analysis is based on a GARCH model, using a sample of weekly observations from January 1<sup>st</sup>, 2007 to March 25<sup>th</sup>, 2013.

My thesis will contribute with an empirical impulse-response analysis to obtain data on the characteristics of the data for the entire period 2005-2018. This to give el-certificate sellers/buyers a better understanding of the market and the possibility to obtain an optimal market position.

## 4.0 Methodology and dataset

### 4.1 The methodology

The estimation of SNP models entails using a standard maximum likelihood procedure together with a model selection strategy that determines the appropriate degree of the expansion. The tasks of model fitting and specification testing is, in a large extent, automated within the program. This makes the method, for a given data set, no more demanding in the terms of total computational effort than those of a typical nonlinear method. (Gallant and Tauchen 1990). The program incorporates many features related to prediction, residual analysis, plotting and simulation. Predicted values and residuals, for example, are of central importance for running diagnostic analysis and calculation measures of fit. Density plots illustrates the key characteristics of the process, such as asymmetry and heavier tails. The simulation has numerous useful application, one is the bootstrapped confidence intervals described in Gallant, Rossi, and Tauchen (1992).

The model is chosen because of the simplicity of expansion, and the possibility of using several datasets at the same time. In this thesis I only use one dataset, but the possibility of the methodology can be further expanded later, or in an expansion of the work done here. The Hermite function describes the density in the model instead of a pre-decided estimate. This means that I will have no obligation of describing an estimate of the density before starting the modelling. The Hermite functions can also expand in to infinite dimensions, I can use bivariate, trivariates and use several data sets at the same time

The methodology is used widely, and for both univariate and bivariate solutions the papers has been published internationally. See especially the work of Gallant and Tauchen. (Gallant and Tauchen 1998)

#### 4.1.1 The SNP model

The SNP model, used in this thesis, is a semi-nonparametric model. This indicates that the model is in between parametric and nonparametric methods. The user guide and the code for the program can be viewed at <http://www.aronaldg.org>. The SNP is based on an expansion in Hermite functions, for estimation of the conditional density (Gallant and Tauchen 1990). The



optimized model is chosen by using the Schwartz Bayes information criterion, (E. Schwarz 1978)<sup>2</sup>.

$$\text{BIC} = S_n(\theta) + \left(\frac{1}{2}\right) \left(\frac{p_p}{n}\right) \log(n)$$

The criterion is defined to choose the model with the best fit, with a low possibility of overfitting the model. The model is fitted using maximum likelihood function, with a penalty term growing for each parameter. The model with the lowest BIC is preferred. I therefore continue optimizing the model until the BIC is no longer reducing, when I reach this point the model with the lowest BIC is preferred.

The theoretical foundation of the model is the Hermite series expansion<sup>3</sup>. For time series data this is particularly interesting based on both modeling and computational considerations. In terms of modeling, the Gaussian component of the Hermite expansion makes it easy to subsume into the leading term familiar time series models, including ARMA, and GARCH used in this paper. (Engle 1982, Bollerslev 1986). In terms of computation a Hermite density is easy to evaluate and differentiate. Finally, a Hermite function is very practicable to sample form, which facilitates simulation. (Gallant and Tauchen 1990).

Using the C++ code for implementing the SNP methodology makes it easy to retrieve residuals, prediction of conditional means, and prediction of conditional variances. The fitting strategy entails moving upward along an expansion path. The movement along the path stops when the best model under the Schwartz criterion (E. Schwarz 1978) has been obtained. When this is reached, the residual tests are performed to specify the first and second moments. If this test shows a well specified model, the optimization is complete. If not,

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<sup>2</sup> The criterion rewards good fits by small

$$s_n(\hat{\theta}) = -\left(\frac{1}{n}\right) \sum_{i=1}^n \log[f(y_i | x_{i-1}, \theta)]$$

and uses the term

$$\left(\frac{1}{2}\right) \left(\frac{p_p}{n}\right) \log(n)$$

to penalize good fits gotten by means of excessively rich parameterization.

<sup>3</sup> The SNP model is based on the notion that Hermite expansion can be used as a general purpose of approximation to a density function. Letting  $z$  denote the  $M$ -vector, we can write the Hermite density as  $h(z) \propto [P(z)]^2 \vartheta(z)$  where  $P(z)$  denotes a multivariate polynomial of degree  $K_z$  and  $\vartheta(z)$  denotes the density function of the Gaussian distribution with mean zero and the identity as its variance-covariance matrix.

The constant of proportionality is  $1 / \int [p(s)]^2 \vartheta(s) ds$  which makes  $h(z)$  integrate to one. As seen from the expression that results, namely;

$$h(z) = \frac{[P(z)]^2 \vartheta(z)}{\int [p(s)]^2 \vartheta(s) ds}$$

further expansion of the model is needed in order to achieve satisfactory performance on the diagnostics.

This fitted model is the basis of the further analysis. The next analysis will be based on the SNP model, and given impulses to receive response, and analyze it. The SNP model will be optimized, one for the Spot Price dataset and one for the One Year Forward dataset. The two datasets will be presented in section 4.2. This the only model I know of that gives us the possibility to analyze the mean and the volatility in the market.

#### 4.1.2 Impulse response analysis of nonlinear models.

A definition of the impulse response on nonlinear data sets are the total effect of the impulse. This is collected by comparing the profile of the shock, with the baseline profile (profile without shocks). The difference between the baseline profile and the profile of the shock will conclude the effect of the shock. When talking about the effect, both the mean and volatility effect are included.

The methodologies below are outlined by Gallant, Rossi, and Tauchen (1993); (Gallant, Rossi, and Tauchen 1993) defines step-ahead forecast of the mean conditioned on the history of process as  $E(y_{t+1}/(y_{t-k})_{k=0}^{\infty})$  in general or  $E(y_{t+1}/(y_{t-k})_{k=0}^{L-1})$  for a Markovian Process. Similarly, the one-step variance, also called the volatility, is the one-step ahead forecast of the variance conditioned on history;

$$Var(y_{t+1}/(y_{t-k})_{k=0}^{\infty}) = E\{[y_{t+1} - E(y_{t+1}/(y_{t-k})_{k=0}^{\infty})][y_{t+1} - E(y_{t+1}/(y_{t-k})_{k=0}^{\infty})] / \{y_{t-k}\}_{k=0}^{\infty}\} \text{ or } Var(y_{t+1}/(y_{t-k})_{k=0}^{L-1})$$

for a Markovian process. Remembering the conditional moment profile

$$E[g(y_{t+j-j}, \dots, y_{t+j}) / \{y_{t-k}\}_{k=0}^{L-1}] \text{ where } (j = 0, 1, 2, \dots, 5).$$

The word moment in the moment profile refers to the time-invariant function  $g(y_{-f}, \dots, y_0)$ .

This means that the conditional mean profile is short for “the conditional moment profile of the one-step mean”, and similar for the conditional volatility profile; “the conditional moment profile of the one step variance”. To trace out the effect of a shock on subsequent mean we put;  $y_j(x) = E(g(y_{t-L+j}, \dots, y_{t+j})/x_t = x) = E(E(y_{t+j}/y_{t-L+j}, \dots, y_{t+j})/x_t = x)$  and  $y_j^i$  for impulse ranges  $i = -30\%, \dots, +30\%$ . In this thesis I use five steps-ahead (five days)  $j = 0, 1, \dots, 5$ , and shocks ranging from -30% to +30% (maximum and minimum values of the

data set). The interpretation of  $x$  is written as  $x = (y_{-L+1}, y_{-L+2}, \dots, y_0)$  where  $L$  represents lags.

To trace out the effect of a shock on subsequent volatility we put;  $\psi_j(x) =$

$E(g(y_{t-L+j}, \dots, y_{t+j})/x_t = x) = E(Var(y_{t+j}/x_{t+j-j})/x_t = x)$  using five steps-ahead (five days)  $j = 0, 1, \dots, 5$ . The interpretation of  $x$  is written as  $x = (y_{-L+1}, y_{-L+2}, \dots, y_0)$  and  $\psi_j(x)$  is the forecast of the one step ahead variance  $j$  steps ahead, conditional on  $x_t = x$ .

#### 4.1.3 Bootstrapping

By applying bootstrapping to the analysis, it is possible to construct a 95% confidence interval. Sup-norm bands are constructed using SNP with 500 simulations. This allows us to consider the variation in the sample. Bootstrapping means using random sampling to generate 500 new datasets for use in the impulse-response analysis. This will conclude with a 95% confidence, which will include 95% of the profiles from the random sampling. The bootstrapping method will derive estimates of the confidence interval, and it will be used to check the stability of the results. Using bootstrap is more accurate to conclude with a confidence interval than using sample variance. On the other hand, bootstrapping never provide guarantees that the sample is on point. The number of bootstrapping samples is important when using the method and availability to computer programs has in the latter years increased the recommended samples. In this thesis I use 500 bootstrapped samples to achieve a wide range of data sets. The impulse-response analysis used on the samples will include shocks of negative 10% and positive 10%, both for Spot Price and for One Year Forward.

#### 4.1.4 Persistence

One of the prospects of a stationary time series is the assumption that it will return to its mean after time (mean reverting). This means that today's information does not influence the estimates for the long run. Persistence is widely used examining a shock to a time series because of the knowledge that if today's return has a large impact on the expected variance in the future, the volatility should be persistent. To measure the persistence, it is of the used halftime of the shock. This means; the half-life of a shock measures the time of which the volatility has moved halfway back to its unconditional mean.

This is given by;

$$\tau = k : |h_{t+k|t} - \sigma^2| = \frac{1}{2} |h_{t+1|t} - \sigma^2|$$

where  $h_{t+1|t}$  is the expected value of the variance in return k periods in the future, and  $\sigma^2$  is the long-term volatility (Engle and Patton 2007).

## 4.2 The dataset

The analysis covers the period from January 2005 until February 2018, with weekly numbers. This gives us a total of 13 consecutive years with 52 weeks, and approximately 680 price observations of spot price and 680 of One-year forward. The data sets are long and gives us sufficient information about the distribution of returns. The data sets also give a broad range in the composition of volatility. Figure 1 and 2 illustrates the prices from 2005 to 2018, Figure 1 for the Spot Price and Figure 2 for the One Year Forward March. Both figures show a clear negative trend from 2008 to 2018, and the prices has been unpredictable with large variations. The Spot Price and the One Year Forward March are similar, this is illustrated in Figure 3 where the price changes from 2005 until 2018 for One Year Forward and for Spot Prices are illustrated in the same chart. Here we see the same changes, with a barely noticeable higher price for the Spot Price. This is expected because of the risk aversion obtained by locking the price One-Year forward.

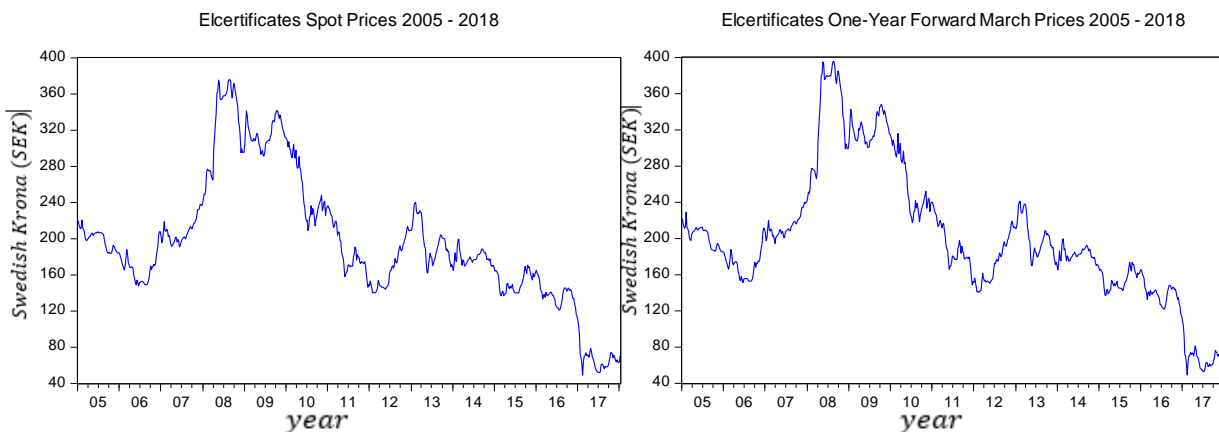


Figure 1; Spot Prices, 2005-2018

Figure 2; One-Year Forward Prices, 2005-2018

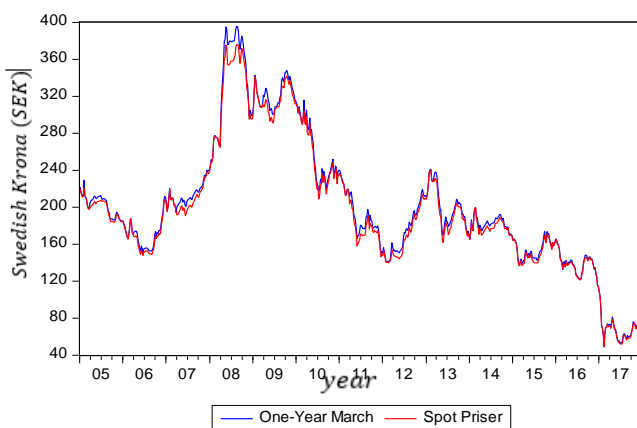


Figure 3, Spot and One-Year Prices, 2005-2018

To obtain a stationary data set I apply the daily percentage change to the spot price. The daily percentage change (logarithmic) of the data sets from 2005 to 2018 is  $y_i, i = 1, \dots, 680$ . The SNP model requires a stationary model. A time series has stationarity if the shape of the distribution doesn't change following a shift in time. Basic properties of the distribution, such as mean, variance, autocorrelations are all constant over time. The price of the certificates has a negative trend and are non-stationary, as discussed earlier. I therefore calculate the logarithmic return, to create the price movement series;

$$y_t = 100 \cdot \left( \frac{P_t}{P_{t-1}} \right).$$

Notice that if a process is covariance-stationary, the covariance between  $y_t$  and  $y_{t-k}$  depends only on  $k$ , the length of time separating the observation, and not on  $t$ , the date of the observation. It follows that for a covariance-stationary process,  $y_k$  and  $y_{-k}$  would represent the same magnitude (Hamilton 1994). In this way we impose weak stationarity, and the means, variances and covariances are independent of times. That is, a process  $\{y_t\}$  is weakly stationary if for all  $t$ , it holds that  $E\{y_t\} = \mu \leq \infty, V\{y_t\} = E\{(y_t - \mu)^2\} = \gamma_0 < \infty$  and  $cov\{y_t, y_{t-k}\} = E\{(y_t - \mu)(y_{t-k} - \mu)\} = \gamma_k, k = 1, 2, 3, \dots$ . A shock to a autoregressive process of order 1 (AR(1)) effects all future observations, with diminishing effect.

The logarithmic returns for the El-certificate Spot price, together with a kernel distribution on the left is reported in Figure 4, and the same for One Year Forward prices in Figure 5. There are both advantages and disadvantages in using log-returns. A log operation constitutes of a non-linear transformation. The SNP model allows the user to avoid the problem of non-linearity and therefor avoid the disadvantages of using log-returns.

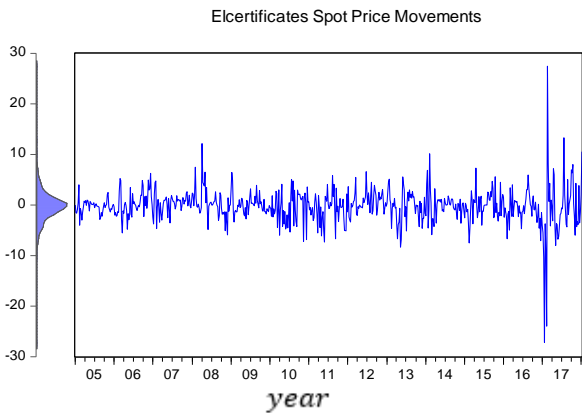


Figure 4; Spot Price movements, 2005-2018

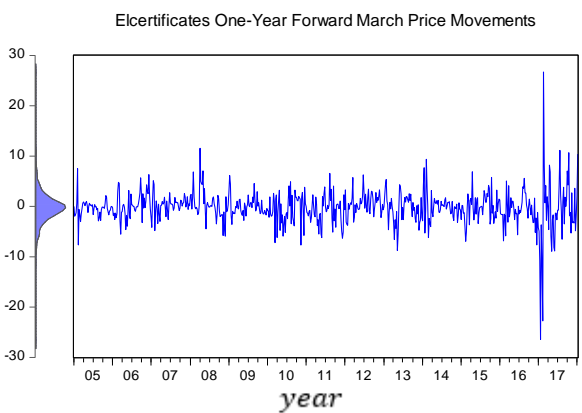


Figure 5; One-Year Forward March movements, 2005-2018

The Spot Price statistics is presented first, then the One-Year Forward March statistics.

#### 4.2.1 Spot price Statistics

**Table 1 Statistics for Spot Price Elcertificate Market 2005-2018**

**Panel A: Spot Price Series**

	Mean (all)/	Median	Maximum /	Moment	Quantile	Quantile	Cramer-	Serial dependence	VaR	
<b>Returns</b>	M (-drop)	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	von-Mises	Q(12)	Q <sup>2</sup> (12)	(1%; 2,5%)
	-0.16549	-0.09988	27.4355	17.76170	0.48125	6.8609	2.5439	62.0	291.64	-7.586%
	█ -0.16189	3.29699	-27.1800	-0.31074	-0.05136	{0.0324}	{0.0000}	{0.0000}	{0.0000}	-14.596%
	BDS-Z-statistic (e = 1)				KPSS (Stationary)		Augmented	ARCH	RESET	CVaR
	m=2	m=3	m=4	m=5	Intercept	I + Trend	DF-test	█ (12)	(12;6)	(1%; 2,5%)
	9.5262	10.6936	12.0988	13.6935	0.20068	0.04632	-20.1549	18.65	16.02314	-14.596%
	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.7390}	{0.2160}	{3.9716}	{0.0000}	{0.0000}	-9.613%

Table 1; Statistics for Spot Price El-certificate Market

Table 1 reports the characteristics of the Spot price data set. The mean is negative, and the standard deviation is quite high (3.3). The minimum value is relatively low (-27.18), and the maximum value relatively high (27.43). The kurtosis is high and positive (17.76), indicating heavier tails than a normal distribution (more data in the tails). The skewness measures the symmetry in the dataset. In this case the skewness is -0.31 and indicates a symmetrical data set with a bias to the left of the distribution. Figure 6 illustrates the One-Day ahead conditional returns density for unconditional mean. The chart shows negative tails and a higher, slimmer distribution than the normal distribution. The higher tails are compatible with the data for the kurtosis. It is often said that financial asset returns do not follow a normal distribution. The distribution is almost always fat-tailed, which means that a higher number of data is in the tails of the distribution. Our data is no exception to these observations.

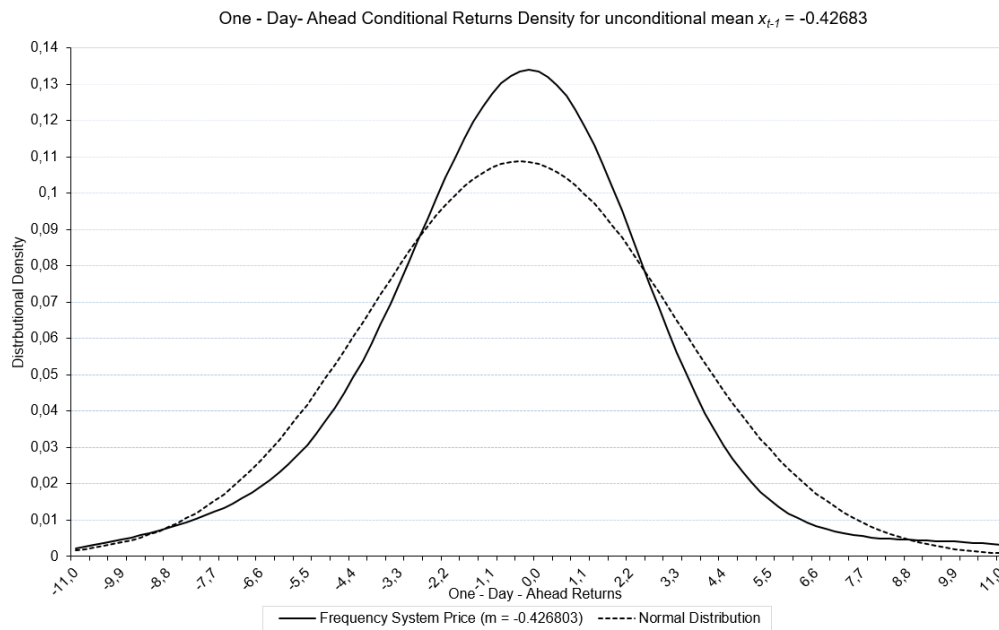


Figure 6; One-Day-Ahead Conditional returns density for unconditional mean

Before using the ARMA-GARCH modeling on the data set, it is important to check for autocorrelation in the series. Autocorrelation is tested for by the use of the Ljung Box statistics,  $Q$ -statistic (Ljung and Box 1978). The test is used as a part of the process of fitting the model. If the test indicates autocorrelation, the statistic suggests some form of data dependency in the data series. Under the null hypothesis of no autocorrelation, the  $Q$ -test statistics is asymptotically Chi-Square distributed (Box 1994). The test is run in EViews and tested for both normal and squared data. The Ljung Box test statistics is:

$$Q^* = T(T + 2) \sum_{k=12}^h (T - k)^{-1} r_k^2$$

Where  $T$  is the sample size. We use 12 lags ( $k$ ). In the spot price statistics, the  $Q$ -statistics confirms significant autocorrelation in our data. This means that the price for one week, has an correlation with the price from the previous week. First lag correlation can often occur because of the market factor and thin trading of smaller companies (Taylor 2008). The companies act on the prices from the week before, with a certain amount of time delay. A small amount of first-lag autocorrelation can therefore be explained.

The BDS test (Broock et al. 1996, Brock and Dechert 2001) detects nonlinear serial dependence in the time series. The test null hypothesis is that the remaining residuals, after fitting the model, are independent and identically distributed. The BDS tests for nonlinearity and is one of the indications of chaos. The BDS test is defined as follows:

The time series has  $N$  observations, the first difference of the logarithms of data.

$$(x_i) = [x_1, x_2, x_3, \dots, x_N]$$

The correlation integral, is estimated by

$$C_{\varepsilon, m} = \frac{1}{N_m(N_m - 1)} \sum_{i \neq j} I_{i, j; \varepsilon}$$

$$\text{where, } I_{i, j; \varepsilon} = \begin{cases} 1 & \text{if } \|x_i^m - x_j^m\| \leq \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

The estimates above is used to state a test statistic:

$$BDS_{\varepsilon, m} = \frac{\sqrt{N}[C_{\varepsilon, m} - (C_{\varepsilon, 1})^m]}{\sqrt{V_{\varepsilon, m}}}$$



The test is a two-tailed test. The null hypothesis can be rejected if the statistics shown in Table 1 is greater or less than the critical values. The BDS test statistics of the spot price returns shows significant dependence in the data.

I apply the Augmented Dickey-Fuller test (ADF)(Dickey and Fuller 1979) and the Kwiatkowski, Phillips, Schmidt and Shin test (KPSS) (Kwiatkowski et al. 1992) to test for stationarity. The results of the spot price are presented in Table 1. The ADF shows a rejection of the null hypothesis at some level of confidence. This means that the time series are stationary, with no unit root present. The KPSS null hypothesis assumes the series stationary, which differs from the ADF test. The test shows; we cannot reject the null hypothesis. Both tests support stationary time series, which means that we can follow through with our analysis and be confident of our results.

Value at Risk(Rockafellar and Uryasev 2000) is used measure the level of risk (financial), within an investment portfolio over time. The VaR calculates the maximum loss expected on an investment, given a set period and a certain degree of confidence. The Conditional Value at Risk (Rockafellar and Uryasev 2000) measures the average loss in the tail of the distribution. Financial markets often have heavier tails than a normal distribution, this is also the case in the El-certificate market (kurtosis). The technique assesses the likelihood of a loss exceeding the Value at Risk. The Value at Risk and expected shortfall (CVaR) number report percentile shortfall numbers for long positions at less than 2.5% and 1%.

#### 4.2.2 One-Year Forward Statistics

**Table 2. Statistics for One-Year Forward Elcertificate Market 2005-2018**

	Mean (all)/	Median	Maximum /	Moment	Quantile	Quantile	Cramer-	Serial dependence		VaR
<b>Returns</b>	M (-drop)	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	von-Mises	Q(12)	Q <sup>2</sup> (12)	(1%; 2,5%)
	-0.21472	-0.10610	26.7190	15.63537	0.33916	3.6505	2.5525	61.120	315.54	-8.798%
	0.21129	3.30060	-26.4499	-0.29847	-0.05876	{0.1612}	{0.0000}	{0.0000}	{0.0000}	-6.307%
	BDS-Z-statistic ( $e = 1$ )		KPSS (Stationary)		Augmented	ARCH	RESET	CVaR		
	m=2	m=3	m=4	m=5	Intercept	I + Trend	DF-test	(12)	(12;6)	(1%; 2,5%)
	9.2063	11.1401	12.9148	14.5024	0.18546	0.04439	-11.7253	17.145	5.596287	-14.869%
	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.7390}	{0.2160}	{0.0000}	{0.0000}	{0.0000}	-9.971%

Table 2; Statistics for One-Year Forward Price El-certificate Market

Table 2 illustrates the characteristics of the One Year Forward price data set. The mean is negative, and the standard derivation is high (3.3). The minimum value is relatively low (-26.45), and the maximum value relatively high (26.72). The kurtosis is high and positive (15.63), indicating heavier tails than a normal distribution (more data in the tails). The skewness measures the symmetry in the dataset. In this case the skewness is -0.30 and indicates a symmetrical data set. Figure 7 illustrates the One-Day ahead conditional returns density for unconditional mean. The chart shows negative tails and a higher, slimmer distribution than the normal distribution. The higher tails are compatible with the data for the kurtosis.

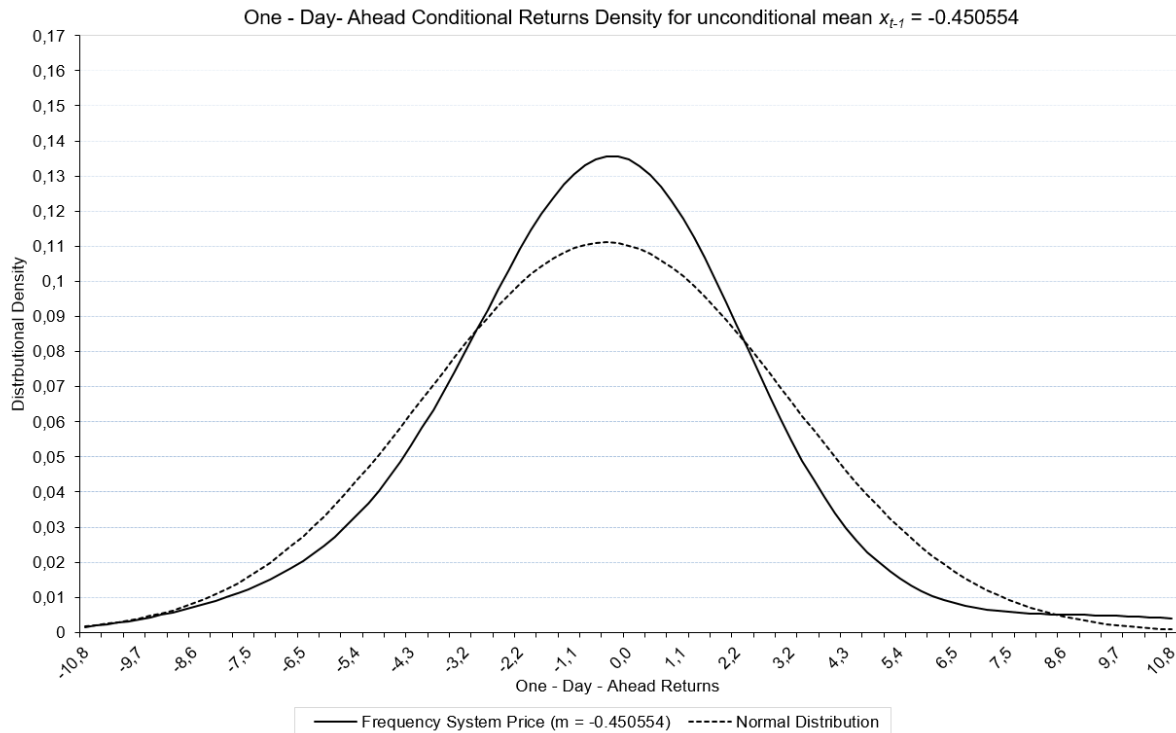


Figure 7; One-Day-Ahead Conditional returns density for unconditional mean

Testing for autocorrelation, The Ljung Box test ( $Q$ ) is used as described earlier in the Spot Price Statistics. The test use 12 lags ( $k$ ). In the spot price statistics, the  $Q$ -statistics confirms significant autocorrelation and dependency in our data. The BDS test statistics of the One Year Forward price returns show significant dependence in the data. Test for stationarity by applying Augmented Dickey-Fuller test (ADF) and Kwiatkowski, Phillips, Schmidt and Shin test(KPSS). The ADF shows a rejection of the null hypothesis at some level of confidence. This means that the time series are stationary, with no unit root present. The KPSS test shows; we cannot reject the null hypothesis. Both tests support stationary time series. The Value at Risk and expected shortfall (CVaR) number report percentile shortfall numbers for long positions at less than 2.5% and 1%.

## 5.0 The Empirical Analysis and Findings

In this section of the thesis the two sets of data, Spot price and One-Year Forward, will be analyzed and the findings will be presented. Firstly, we check the model for misspecification to assure we have the optimal semi-nonparametric model. Secondly, we analyze the impulse response output together with the bootstrap and the persistence. This is done for the Spot price and the One-Year forward respectively.

### 5.1 The Empirical Analysis and Findings, Spot Price

#### 5.1.1 The conditional density

The SNP model is based on an expansion of Hermite functions, for estimation of the conditional density. The model is fitted by following a path where the model becomes more richly parametrizes, at each level along the path. The expansion stops when the best model under the Schwarz criterion is obtained. Table 3 reports the maximum likelihood (ML) estimates of the parameters of the BIC-optimal SNP model.

**Table 3; Swedish/Norwegian Elcertificate Spot Price**

Statistical Model SNP-14,111,12, 000 -fit; semi-parametric-GARCH model

Mean Equation	SNP Coeff.	Mode	Standard error	t-statistics
<i>Hermite Polynoms</i>				
$\eta_1$	$a_0[1]$	-0,14650	0,05102	-2,87147
$\eta_2$	$a_0[2]$	-0,08655	0,0434	-1,99427
$\eta_3$	$a_0[3]$	0,05917	0,02842	2,08209
$\eta_4$	$a_0[4]$	0,04611	0,02741	1,68243
$\eta_5$	$a_0[5]$	-0,01902	0,03039	-0,62593
$\eta_6$	$a_0[6]$	-0,12338	0,02559	-4,82164
$\eta_7$	$a_0[7]$			
$\eta_8$	$a_0[8]$			
<i>Mean Correlation</i>				
$\eta_{13}$	$b_0[1]$	0,21443	0,06585	3,25647
$\eta_{14}$	$B(1,1)$	0,34733	0,04211	8,24833
$\eta_{15}$				
$\eta_{16}$				
<i>Variance Equation</i>				
$\eta_{27}$	$R_0[1]$	0,24681	0,05153	4,78985
$\eta_{28}$	$P[1,1]$	0,42318	0,13844	3,05686
$\eta_{29}$	$Q[1,1]$	0,82292	0,04803	17,13234
$\eta_{30}$	$V[1,1]$	-0,33552	0,18651	-1,79899
$\eta_{31}$	$W[1,1]$	0,52916	0,1056	5,01103
<hr/>				
Observations (incl. drops)	670		$s_n$	1,0854466
Log Likelihood	-738,1036656		$aic$	1,1045642
			$bic$	1,1477901
Largest eigenvalue of mean function companion matrix:				0,347332
Largest eigenvalue of variance function P&Q companion m:				0,856288

Table 3; BIC optimal SNP model estimates

The model selected under the Schwartz criterion is a semi-nonparametric GARCH with six Hermite polynomials ( $K_z$ ) for non-normal features of the series. The model is a GARCH (1,1) ( $L_g, L_r$ ) with one lag in VAR ( $L_u$ ). The mean correlation is significant implying dependence. The Variance coefficients are all significant ( $n_{27} - n_{31}$ ). Conditional heteroscedasticity ( $n_{27} - n_{29}$ ), asymmetry ( $n_{30}$ ) and level effects ( $n_{31}$ ) are present. The R0 coefficient for the volatility is high, 0.25, which suggests constant and long lasting conditional volatility for the market. The P coefficients shows the shock effect from the previous period and the Q coefficient reports the serial correlation (ARCH and GARCH). Coefficient V reports negative symmetry (-0.34), and W reports a high-level effect (0.53). The numbers suggest a high volatility process. The eigenvalue of the variance function is 0.856 and the eigenvalue of the mean function is 0.347. The Hermite function coefficients ( $n_1 - n_6$ ) are BIC preferred up to the sixth lag expansion. From this information we suggest a different distribution than the normal distribution.

Figure 8 shows the projected spot price conditional variance together with a moving average of 4 and 15 lags (weeks). From the figure we suggest that the volatility changes randomly, and the projected volatility tends to be relatively compact for  $m=4$  and  $m=15$ . Figure 9 reports the GAUSS-Hermite Quadrature Density Distribution. The distribution shows a higher density for positive price movements and suggests asymmetry. We also registrar a higher amount of observations on the negative (left) side of the distribution.

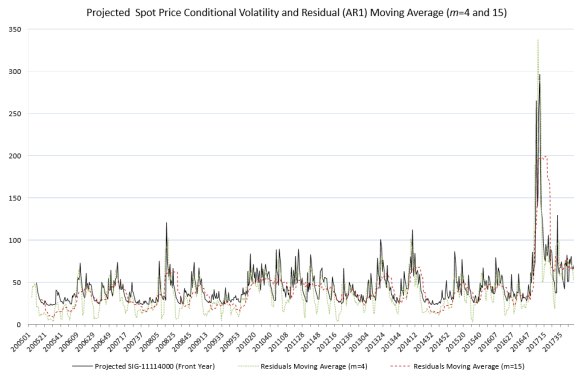


Figure 8; Projected Spot Price Conditional Volatility and Residual. Moving Average (m04 and 15)

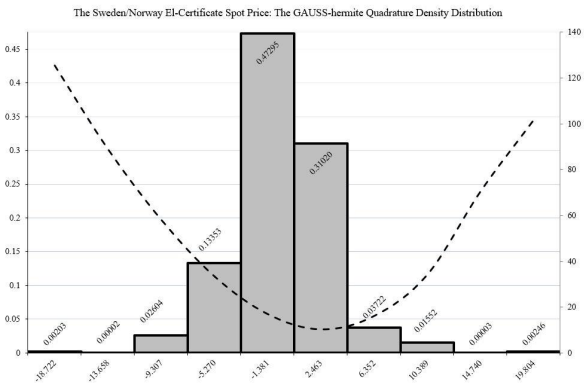


Figure 9; The GAUSS-hermite Quadrature Density Distribution

**Table 4 Residual Statistics for the Swedish/Norwegian Elcertificate market, Spot Price**

Residuals	Mean / Mode	Median / Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer- von-Mises	Serial dependence Q(12)	Q <sup>2</sup> (12)
	-0,00058	-0,01995	6,81126	5,09506	0,32470	2,95637	1,13089	7,1947	10.692
		1,00317	-4,34508	0,68121	-0,01080	{0,2281}	{0,0000}	{0,8440}	{0,5560}
BDS-statistic ( $\varepsilon=1$ )					ARCH	RESET	Joint	VaR	CVaR
	m=2	m=3	m=4	m=5	(12)	(12;6)	Bias	5%/ 1%	5%/ 1%
	(-0.524877)	(-0.697266)	(-0.384598)	0.246726	0.874236	1.522853		-1,5743 %	-2,1410 %
	{0,5997}	{0,4865}	{0,7005}	{0,8051}	{0,5733}	{0,1680}		-2,3008 %	-2,8384 %

*Table 4; Residual Statistics, Spot Price*

Table 4 reports residual statistics, test specification statistics. The residual statistics show data closer to the normal distribution. The mean is almost 0, and the standard derivation is close to one, which is compatible with a normal distribution. The Maximum, minimum values are 6.8 and -4.3, the kurtosis is 5.1 and the skewness is 0.7.

The Cramer-von-Mises test statistics suggest deviations from a standard normal distribution (1,13089). This means that the residuals have a small deviant from a standard normal distribution. This shall not have an impact on the result of the analysis. Furthermore, we calculate the Ljung and Box statistics (twelfth order) for the residuals and the squared residuals, the BDS-test (Brock et al. 1993) statistics, the 12<sup>th</sup> lag ARCH test statistics, the Ramsey reset test (Ramsey, 1969) statistics and the joint bias test (Engle and Ng, 1993). The serial dependence ( $Q$  and  $Q^2$ ) is non-significant, the ARCH and RESET test reports non-significant results. The BDS-test states no dependency in the data. From the results we can conclude with a well specified model, and all the systematic element from the earlier data set is gone. The residuals are stochastic. The semi-nonparametric GARCH model can be used for the impulse-response analysis with expectations of correct results.

The SNP projection gives possibilities for one-step-ahead densities  $f_k(y_t/x_{t-1}, \theta)$ , conditionals for the values for  $x_{t-1}$  (= unconditional mean). Figure 6 shows the volatility at the mean of the time series and displays small deviations from the plotted normal distribution. The Spot Price has a distribution higher and narrower than the normal distribution, in addition to heavier tails. Features like this are commonly seen in financial markets. Figure 10 model lags set explicitly, from -20% to +20%. Comparing the baseline profiles with the impulse profiles we find that the densities are wider after adding a shock. We read from the figure that the larger the shock gets; the more uncertainty lies in the spot price for the following day. We also see that a negative shock indicates greater amount of uncertainty, than positive

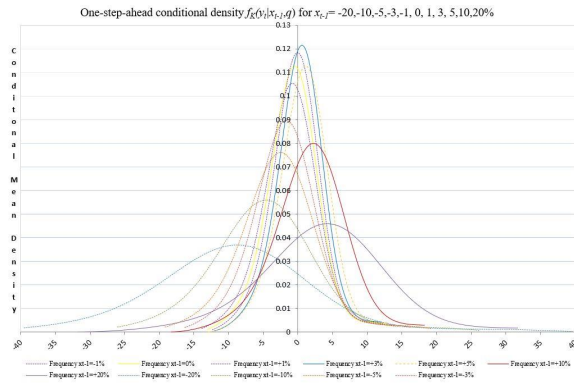


Figure 10; One-step-ahead conditional density for  $x = -20, \dots, +20$ .

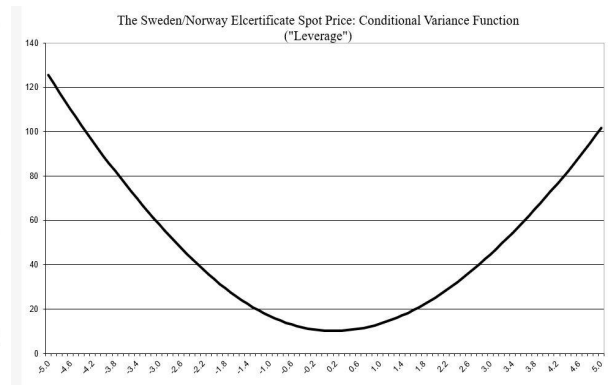


Figure 11; Conditional variance function

shocks, the following day. Figure 6, and 10 simply indicates that the uncertainty is higher when the price movements (daily volatility) are high. Figure 11 illustrates the conditional variance function. The graph indicates a higher response to negative shocks than to positive shocks. This indicates asymmetry in the data set.

### 5.1.2 The impulse-response analysis

The results from the impulse response analysis is presented using figures of the Mean impulse-response dynamics ( $\mu_j$ ) and the Variance Impulse-Response Dynamics ( $\sigma^2_j$ ) for the Swedish/Norwegian Elcertificate Spot Price Market. Figure 12 and 13 shows the dynamic impulse response of future mean return, volatility and variance to price shocks.

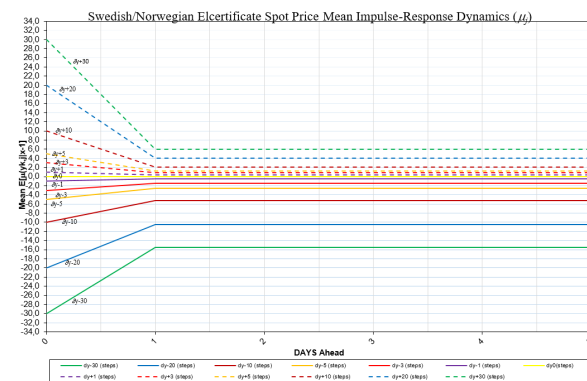


Figure 12; Mean Impulse-Response Dynamics

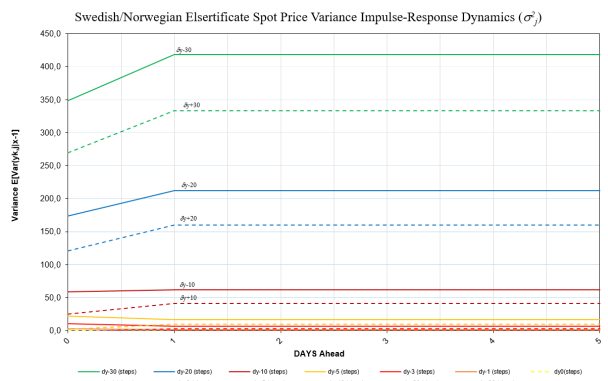


Figure 13; Variance impulse response Dynamics

Figure 12 displays the conditional mean profiles  $\left\{y^i_j - y^0_j\right\}_{j=1}^5$  for the impulses  $i = -30\%, \dots, +30\%$ . The percentage of the impulses are gathered from the maximum and minimum values of the data set (Table 1). This gives thirteen impulses and five steps

ahead  $j = 1, \dots, 5$ . The impulse response function measures the effect of shocks on future values of a time series, and for the conditional mean it shows the characteristics of mean reversion. Figure 12 show the baseline mean profile  $y^0_j$ , where the shock is 0 at day 0. The model is given several shocks between -30% and +30%. The shock ranging from negative 30% to positive 30% are plotted in the chart. The first thing noticeable on the chart is the response difference between negative and positive impulses. The negative impulses collect greater response than the positive impulses suggesting asymmetry. Figure 13 displays the conditional variance profiles  $\{\psi^i_j - \psi^0_j\}$  for the impulses  $i = -30\%, \dots, +30\%$ . This gives thirteen impulses and five steps ahead  $j = 1, \dots, 5$ . The figure shows the baseline variance profile  $\psi^0_j$  where the impulse at day 0 is 0. The model gives several impulses between -30% and +30%. The shock ranging from negative 30% to positive 30% are plotted in the chart. The chart indicates a small and symmetric variance response on impulses ranging from negative 10% to positive 10%. The impulses between positive/negative 10% and 30% shows an increasing variance response with increased impulses. Together with the increasing variance response we also detect a higher volatility for negative impulses than for the same positive impulse. The higher the impulse the greater the difference. The result of the impulse response analysis suggests negative asymmetry in the Swedish/Norwegian El-certificate Market. From figure 14 we detect the volatility differences between the positive and negative shock at the same level. From the plot we see that the volatility is higher for the negative impulses.

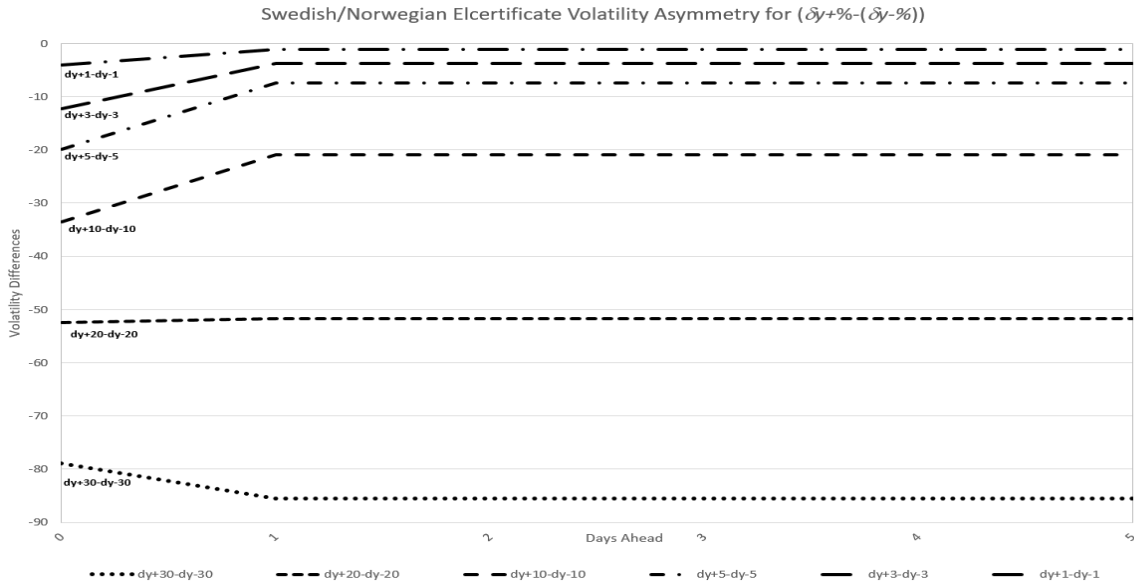


Figure 14; Volatility Asymmetry



### 5.1.3 Implementing bootstrapping.

Figure 15, 16 and 17 reports 95% sub-norm bands of the mean made using the bootstrapping theory presented in the theory section of the paper. The figure illustrates the mean set at 10% and negative 10%. First, we look at the illustration of the negative 10% Spot Price impulse, figure 15. The 95% sub-norm band is located between -4.67 and -5.35, with an expectation of -5. Figure 16, illustrating the positive 10% Spot Price impulse, shows the 95% sub-norm band ranges between 2.9 and 1.8 with an expectation of 2.35. The sub-norm bands for both negative and positive 10% does not include zeroes. The band responses for the negative 10% are greater than the ones for the positive 10%. Figure 17 illustrates the sub norm band for the difference of the positive and negative impulse. The band is located between -2.2 and -3.1 with an expectation of -2.7. In addition, we see a greater confidence interval for the +10% than the negative 10%. This indicates a greater amount of certainty that the -10% will fall the next day. The +10% will rise, but with a lower amount of certainty. None of the intervals include 0, which shows asymmetry. Negative return has a larger ability to last, than positive returns. Observed negative return, gives an anticipation of more negative returns the following day.

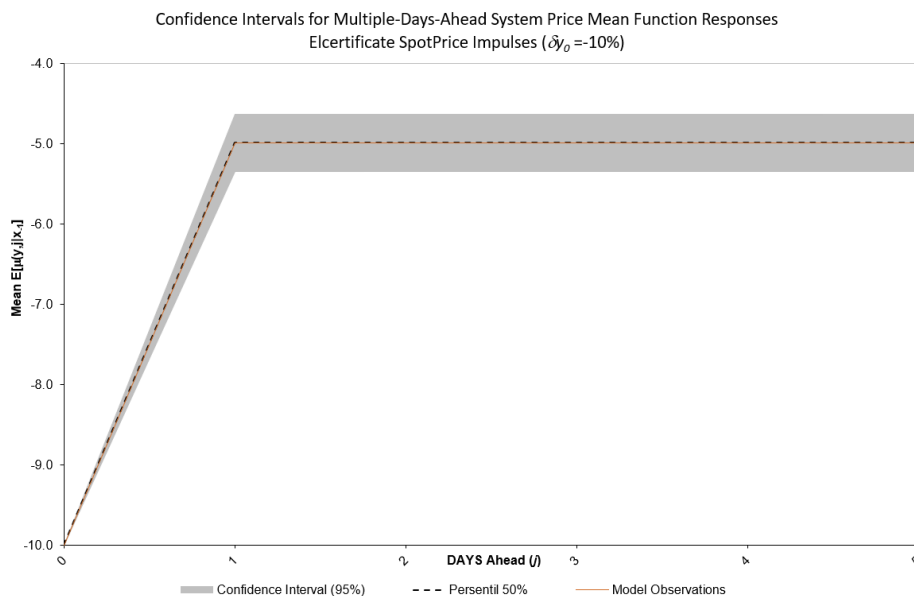


Figure 15; Confidence intervals for Multiple-Days-Ahead, mean ( $\delta y_0 = -10\%$ )

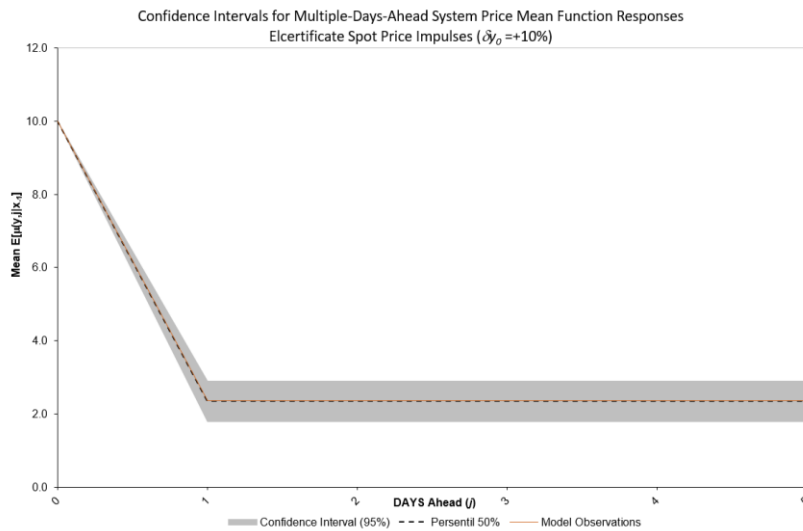


Figure 16; Confidence intervals for Multiple-Days-Ahead, mean ( $\delta y_0 = +10\%$ )

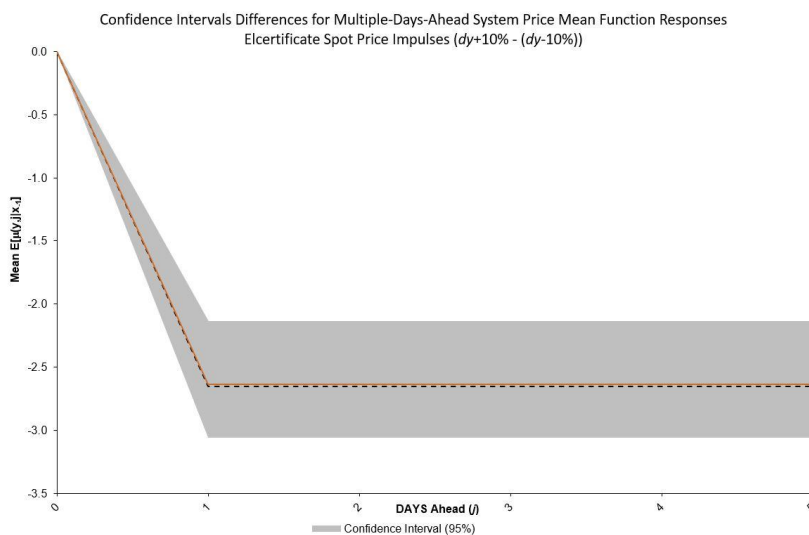


Figure 17; Confidence intervals for Multiple-Days-Ahead, mean ( $dy + 10\% - (dy - 10\%)$ )

Figure 18. 19 and 20 reports 95% sub-norm bands of the volatility. The figure illustrates 95% confidence intervals for the negative and positive 10% impulses. Figure 18 illustrates the 95% confidence interval for the negative 10% and figure 19 for the positive 10%. The sub norm band response for the negative 10% impulses are multi step ahead lifted higher than the positive 10% impulses. The band for day 0 is wider for the positive impulses than for the negative, naturally. The confidence intervals illustrated in both figure 18 and 19 indicates a significant increase in volatility for both negative and positive impulse. Figure 20 illustrates the sub norm band (95%) for the difference between the two previously discussed impulses. This response sub-norm band does not include zero for positive step ahead, it is possible to reject, at 5% statistical significant, the null hypothesis of symmetry. The band for day zero includes 0 and indicates a rejection of asymmetry at 5% statistical significance.

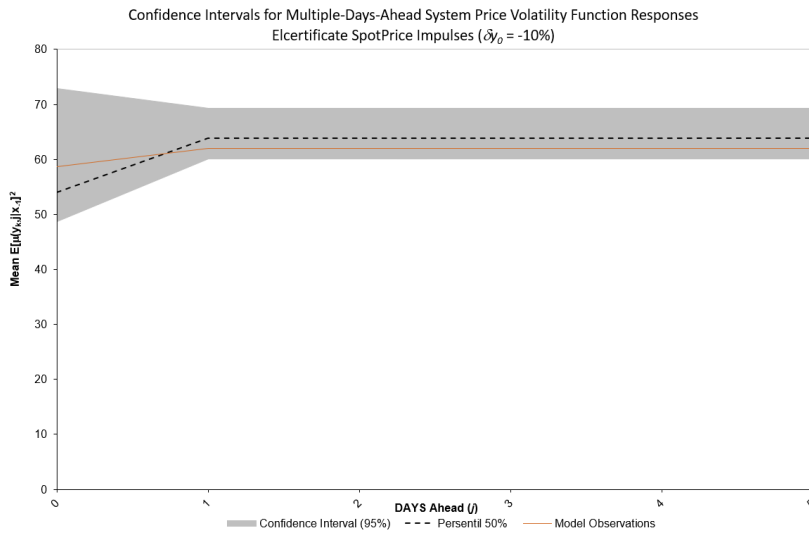


Figure 18; Confidence intervals for Multiple-Days-Ahead, volatility ( $\delta y_0 = -10\%$ )

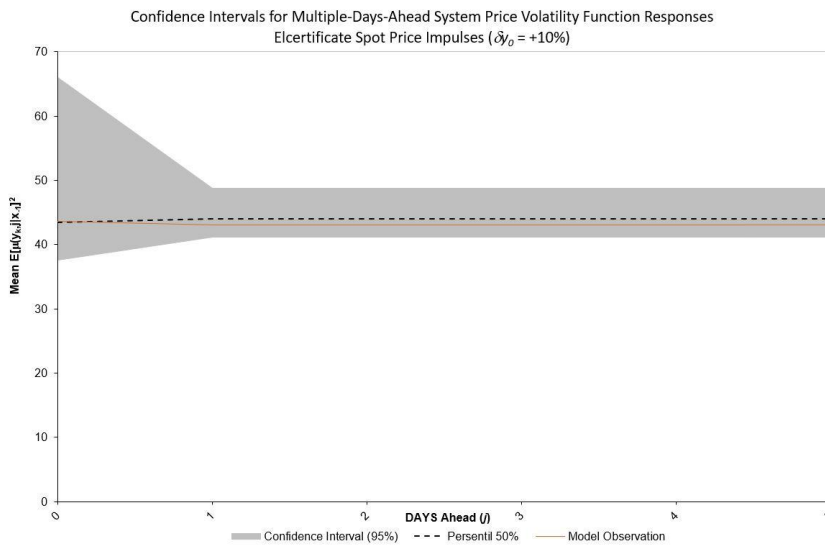


Figure 19; Confidence intervals for Multiple-Days-Ahead, volatility ( $\delta y_0 = +10\%$ )

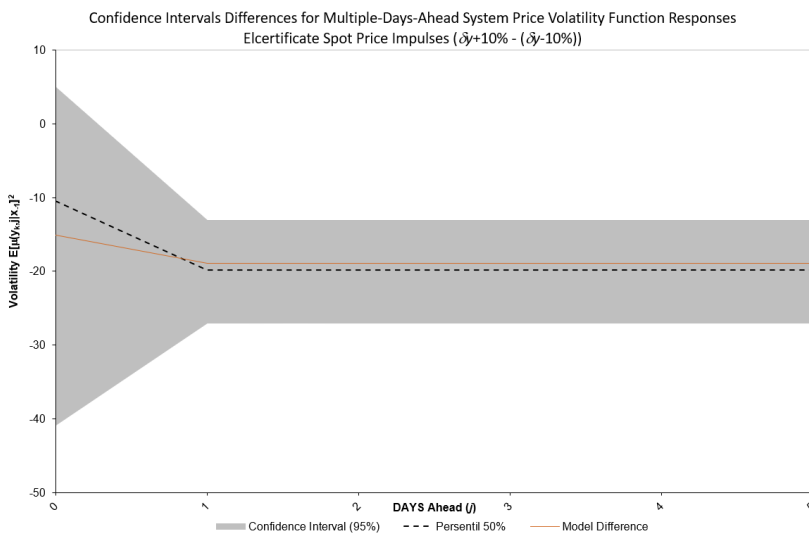


Figure 20; Confidence intervals for Multiple-Days-Ahead, volatility ( $dy + 10\% - (dy - 10\%)$ )

#### 5.1.4 Persistence

Figure 21 reports profile bundles for the volatility. If the thickness of the bundle decreases to zero fast, it tells us that the process reverts to its mean. If this is not the case, and the bundle keeps its width, the process is persistent. The half-life of the Spot Price time series is reported to be 13 days, and there for a relatively slow persistence after a shock (short memory). But; the standard derivation is very high, more than twice the average half-life, which shows that the value of persistence is highly uncertain.

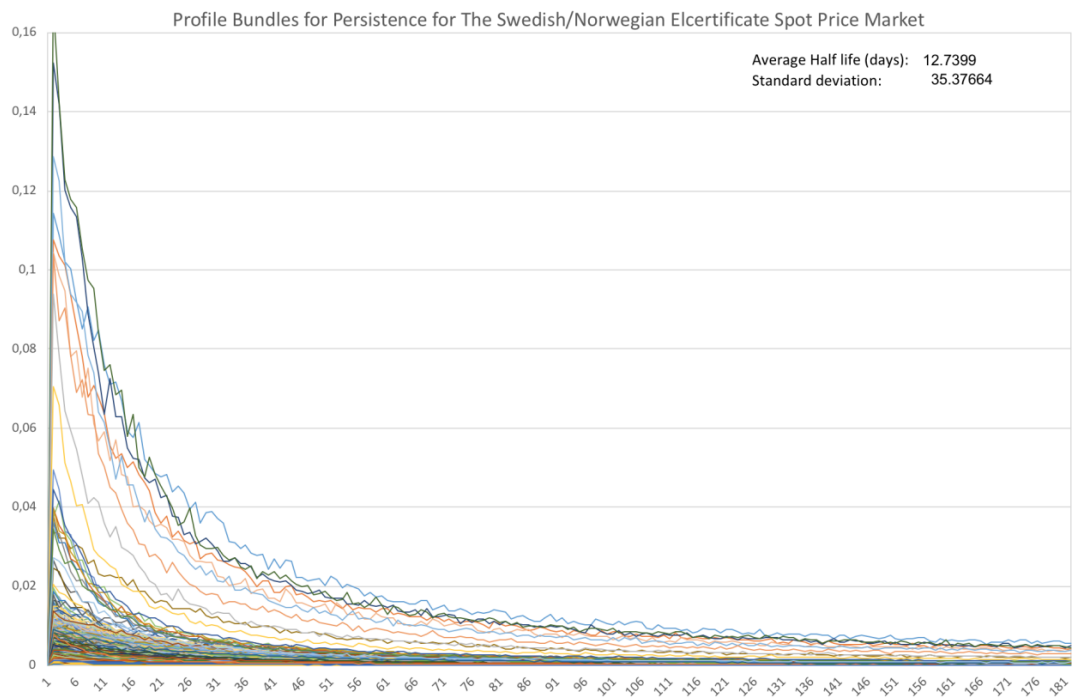


Figure 21; Profile bundles for Persistence, Spot Price

## 5.2 The Empirical Analysis and Findings, One-Year Forward March

### 5.2.1 The conditional density

The SNP model is based on an expansion of Hermite functions, for estimation of the conditional density. The model is fitted by following a path where the model becomes more richly parametrizes, at each level along the path. The expansion stops when the best model under the Schwarz criterion is obtained. Table 5 reports the maximum likelihood (ML) estimates of the parameters of the BIC-optimal SNP model.

**Table 5. Swedish/Norwegian Elcertificate One-Year Forward**

Statistical Model SNP-14,111,12, 000 -fit; semi-parametric-GARCH model

Mean Equation		Standard		
Var	SNP Coeff.	Mode	error	t-statistics
<i>Hermite Polynoms</i>				
$\eta_1$	a0[1]	-0,15006	0,0484	-3,10038
$\eta_2$	a0[2]	-0,07964	0,04105	-1,93989
$\eta_3$	a0[3]	0,09053	0,02949	3,06939
$\eta_4$	a0[4]	0,03983	0,02706	1,47186
$\eta_5$	a0[5]	-0,01112	0,02968	-0,37464
$\eta_6$	a0[6]	-0,11648	0,02428	-4,79716
$\eta_7$	a0[7]			
$\eta_8$	a0[8]			
<i>Mean Correlation</i>				
$\eta_{13}$	b0[1]	0,23266	0,05728	4,06208
$\eta_{14}$	B(1,1)	0,30792	0,04280	7,19516
$\eta_{15}$				
$\eta_{16}$				
$\eta_{17}$				
<i>Variance Equation</i>				
$\eta_{27}$	R0[1]	0,27771	0,0555	5,00369
$\eta_{28}$	P[1,1]	0,45863	0,15419	2,97448
$\eta_{29}$	Q[1,1]	0,79037	0,05651	13,9876
$\eta_{30}$	V[1,1]	-0,37584	0,16663	-2,2556
$\eta_{31}$	W[1,1]	0,42248	0,12974	3,25627
<hr/>				
<i>Observations (incl. drops)</i>		670	$s_n$	1,0858296
<i>Log Likelihood</i>		-738,3640987	<i>aic</i>	1,1049472
			<i>bic</i>	1,1481731
Largest eigenvalue of mean function companion matrix:				0,307921
Largest eigenvalue of variance function P&Q companion m:				0,835024

Table 5; BIC Optimal SNP model estimates

The model selected under the Schwarz criterion is a semi-nonparametric GARCH with six Hermite polynomials ( $K_z$ ) for non-normal features of the series. The model is a GARCH (1,1) ( $L_g, L_r$ ) with one lag in VAR ( $L_u$ ). The mean correlation is significant implying dependence. The Variance coefficients are all significant ( $n_{27} - n_{31}$ ). Conditional heteroscedasticity ( $n_{27} - n_{29}$ ), asymmetry ( $n_{30}$ ) and level effects ( $n_{31}$ ) are present. The R0 coefficient for the

volatility is high, 0.28, which suggests constant and long lasting conditional volatility for the market. The P coefficients shows the shock effect from the previous period and the  $\phi$  coefficient reports the serial correlation (ARCH and GARCH). Coefficient V reports negative symmetry (-0.37), and W reports a high-level effect (0.53). The numbers suggest a high volatility process. The eigenvalue of the variance function is 0.835 and the eigenvalue of the mean function is 0.307. The Hermite function coefficients ( $n_1 - n_6$ ) are BIC preferred up to the sixth lag expansion. From this information we suggest a different distribution than the normal distribution. Figure 22 shows the projected spot price conditional variance together with a moving average of 4 and 15 lags (weeks). From the figure we suggest that the volatility changes randomly, and the projected volatility tends to be relatively compact for  $m=4$  and  $m=15$ . Figure 23 reports the GAUSS-Hermite Quadrature Density Distribution. The distribution shows a higher density for positive price movements and suggests asymmetry. We also registrar a higher amount of observations on the negative (left) side of the distribution.

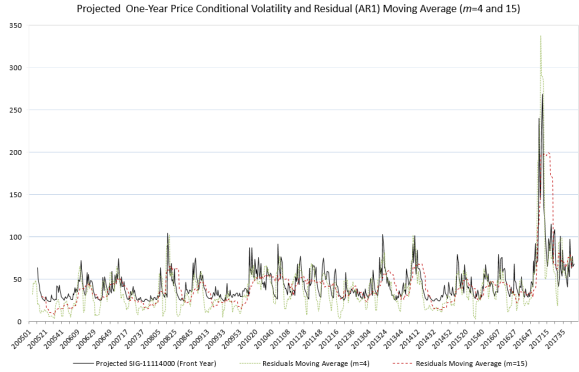


Figure 22; Projected One-Year Price Conditional Volatility and Residual. Moving Average (m04 and 15)

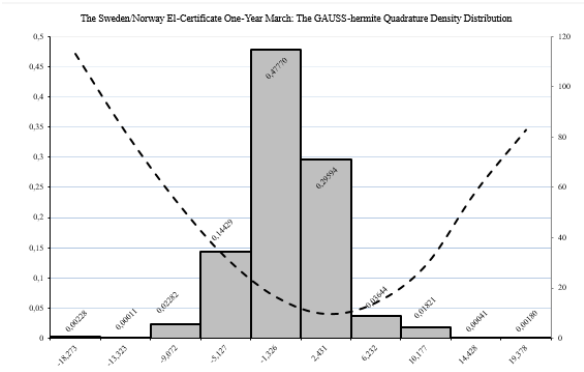


Figure 23; The GAUSS-hermite Quadrature Density Distribution

Table 6 reports residual statistics, test specification statistics. The residual statistics show data closer to the normal distribution. The mean is almost 0, and the standard derivation is close to one, which is compatible with a normal distribution. The Maximum, minimum values are 6.3 and -4.96, the kurtosis is 4.1 (normal distribution 3) and the skewness is 0.38.

**Table 6 Residual Statistics for the Swedish/Norwegian Elcertificate market, One Year Forward**

Residuals	Mean / Mode	Median / Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer- von-Mises	Serial dependence Q(12)	$Q^2(12)$
	-0,00699	-0,02745	6,33737	4,06937	0,21522	1,38922	0,87874	10,8530	9,3357
		1,00096	-4,96242	0,38351	0,02934	{0,4993}	{0,0000}	{0,5420}	{0,6740}
BDS-statistic ( $\varepsilon=1$ )					ARCH	RESET	Joint	VaR	CVaR
	m=2	m=3	m=4	m=5	(12)	(12;6)	Bias	5%/ 1%	5%/ 1%
	-0,97937	-0,40852	0,14031	0,65305	0,787644	1,158792		-1,6136 %	-2,1320 %
	{0,3274}	{0,6829}	{0,8884}	{0,5137}	{0,6636}	{0,3267}		-2,4372 %	-3,2012 %

Table 6; Residual Statistics

The Cramer-von-Mises test statistics suggest deviations from a standard normal distribution (0,87874). This means that the residuals have a small deviant from a standard normal distribution. This shall not have an impact on the result of the analysis. Furthermore, we calculate the Ljung and Box statistics (twelfth order) for the residuals and the squared residuals, the BDS-test (Brock et al. 1993) statistics, the 12<sup>th</sup> lag ARCH test statistics, the Ramsey reset test (Ramsey, 1969) statistics and the joint bias test (Engle and Ng, 1993). The serial dependence ( $Q$  and  $Q^2$ ) is non-significant, the ARCH and RESET test reports non-significant results. The BDS-test states no dependency in the data. From the results we can conclude with a well specified model, and all the systematic element from the earlier data set is gone. The residuals are stochastic. The semi-nonparametric GARCH model can be used for the impulse-response analysis with expectations of correct results.

The SNP projection gives possibilities for one-step-ahead densities  $f_k(y_t/x_{t-1}, \theta)$ , conditionals for the values for  $x_{t-1}$  (= unconditional mean). Figure 7 shows the volatility at the mean of the time series and displays small deviations from the plotted normal distribution. The Spot Price has a distribution higher and narrower than the normal distribution, in addition to heavier tails. Features like this are commonly seen in financial markets. Figure 24 models' lags set explicitly, from -20% to +20%. Comparing the baseline profiles with the impulse profiles we find that the densities are wider after adding a shock. We read from the figure that the larger the shock gets; the more uncertainty lies in the spot price for the following day. We also see that a negative shock indicates greater amount of uncertainty the following day. Figure 7, and 24 shows that the uncertainty is higher when the price movements (daily volatility) are high. Figure 25 illustrates the conditional variance function. The graph indicates a higher response to negative shocks than to positive and suggest asymmetry.

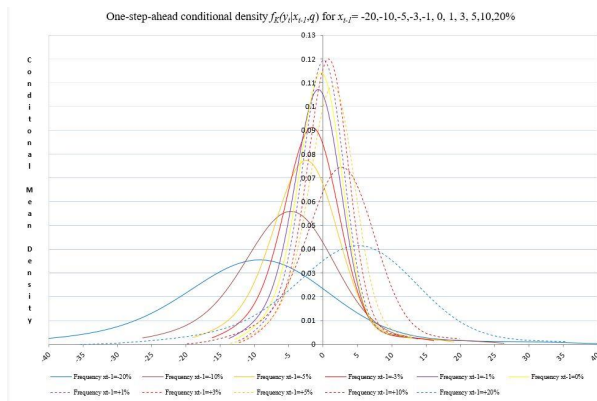


Figure 24; One-step-ahead conditional density for  $x = -20, \dots, +20$ .

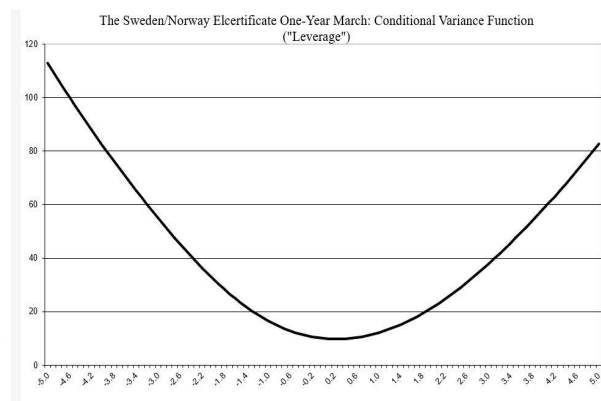


Figure 25; Conditional variance function

### 5.2.2 The impulse-response analysis

The results from the impulse response analysis is presented using figures of the Mean impulse-response dynamics ( $\mu_j$ ) and the Variance Impulse-Response Dynamics ( $\sigma_j^2$ ) for the Swedish/Norwegian Elcertificate One Year Forward Market. Figure 26 and 27 shows the dynamic impulse response of future mean return, volatility and variance to price shocks.

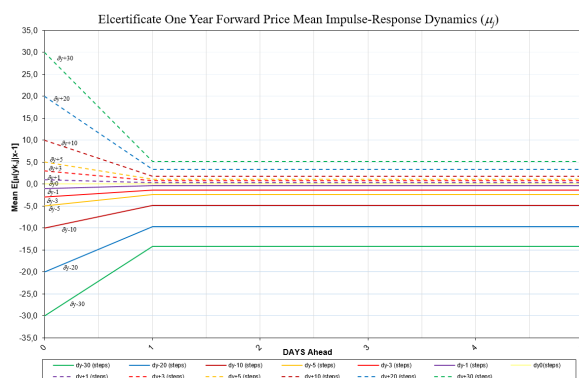


Figure 26; Mean impulse-response Dynamics

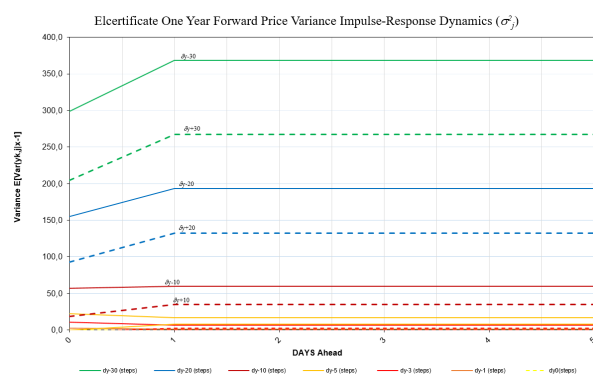


Figure 27; Variance impulse-response Dynamics

Figure 26 displays the conditional mean profiles  $\{y_j^i - y_j^0\}_{j=1}^5$  for the impulses  $i = -30\%, \dots, +30\%$ . The percentage of the impulses are gathered from the maximum and minimum values of the data set (see Table 2). The impulse response function measures the effect of shocks on future values of a time series, and for the conditional mean it shows the characteristics of mean reversion. The figure shows the baseline mean profile  $y_j^0$ , where the shock is 0 at day 0. The model is gives several shocks between  $-30\%$  and  $+30\%$ . The shock ranging from negative 30% to positive 30% are plotted in the chart. The first thing noticeable



on the chart is the response difference between negative and positive impulses. The negative impulses collect greater response than the positive impulses suggesting asymmetry.

Figure 27 displays the conditional variance profiles  $\{\psi^i_j - \psi^0_j\}$  for the impulses  $i = -30\%, \dots, +30\%$ . The figure shows the baseline variance profile  $\psi^0_j$  where the impulse at day 0 is 0. The model is given several impulses between -30% and +30%. The shock ranging from negative 30% to positive 30% are plotted in the chart. The chart indicates a small and symmetric variance response on impulses ranging from negative 10% to positive 10%. The impulses between positive/negative 10% and 30% shows an increasing variance response with increased impulses. Together with the increasing variance response we also detect a higher volatility for negative impulses than for the same positive impulse. The higher the impulse the greater the difference. The result of the impulse response analysis suggests negative asymmetry in the Swedish/Norwegian El-certificate Market. From figure 28 we detect the volatility differences between the positive and negative shock at the same level. From the plot we see that the volatility is higher for the negative impulses.

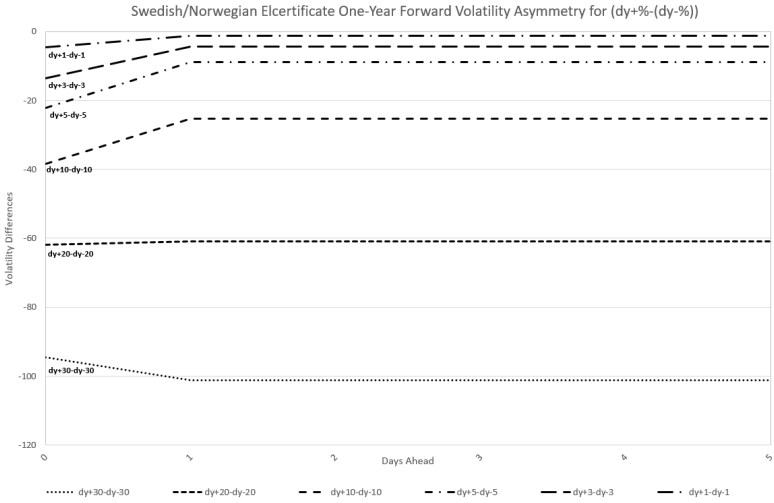


Figure 28; Mean impulse-response Dynamics

5.2.3 Implementing bootstrapping.

Figure 29, 30 and 31 reports 95% sub-norm bands of the mean made using the bootstrapping theory presented in the theory section of the paper. The figure illustrates the mean set at 10% and negative 10%. First, we look at the illustration of the negative 10% Spot Price impulse, figure 29. The 95% sub-norm band is located between -4.35 and -4.98, with an expectation of -4.67. Figure 30, illustrating the positive 10% Spot Price impulse, shows the 95% sub-norm band ranges between 2.5 and 1.5 with an expectation of 2. The sub-norm bands for both negative and positive 10% does not include zeroes. The band responses for the negative 10% are greater than the ones for the positive 10%. Figure 31 illustrates the sub norm band for the difference of the positive and negative impulse. The band is located between -2.3 and -3.2 with an expectation of -2.75. In addition, we see a greater confidence interval for the +10% than the negative 10%. This indicates a greater amount of certainty that the -10% will fall the next day. The +10% will rise, but with a lower amount of certainty. None of the intervals include 0, which shows asymmetry. Negative return has a larger ability to last, than positive returns. Observed negative return, gives an anticipation of more negative returns the following day.

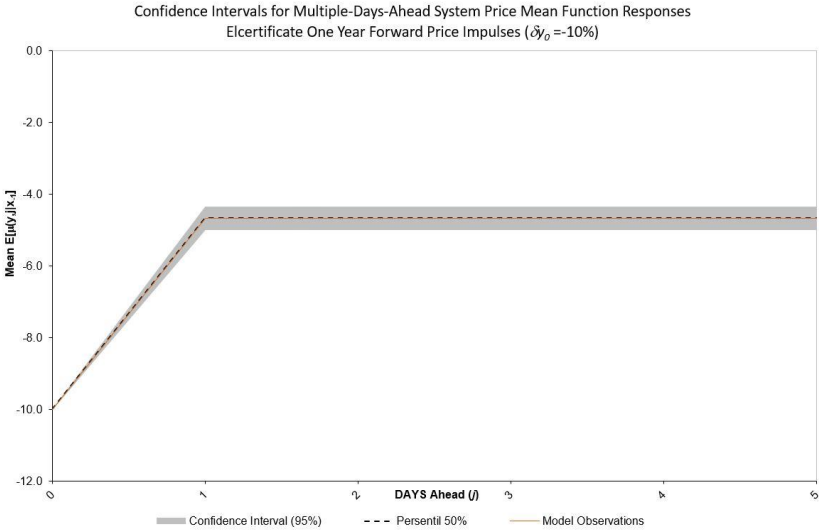


Figure 29; Confidence intervals for Multiple-Days-Ahead, mean ( $\delta y_0 = -10\%$ )

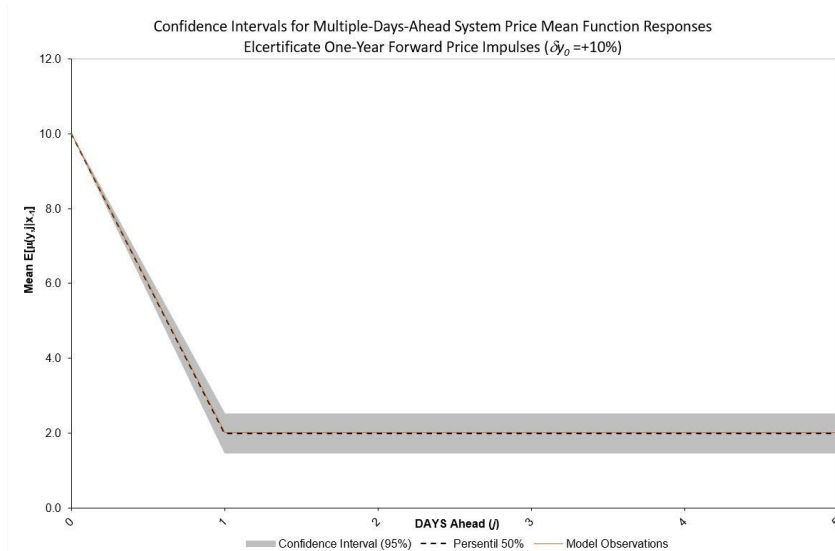


Figure 30; Confidence intervals for Multiple-Days-Ahead, mean ( $\delta y_0 = +10\%$ )

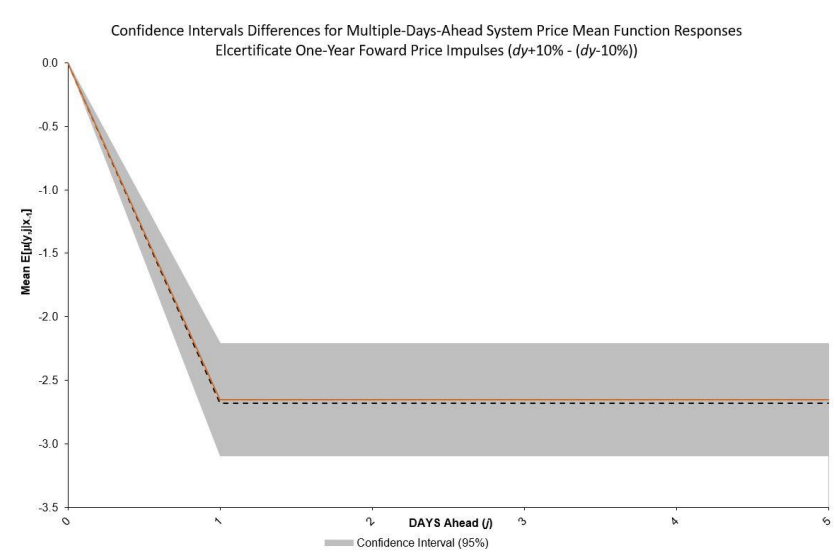


Figure 31; Confidence intervals for Multiple-Days-Ahead, mean ( $dy + 10\% - (dy - 10\%)$ )

Figure 32, 33 and 34 reports 95% sub-norm bands of the volatility. The figure illustrates 95% confidence intervals for the negative and positive 10% impulses. Figure 32 illustrates the 95% confidence interval for the negative 10% and figure 31 for the positive 10%. The sub norm band response for the negative 10% impulses are multi step ahead lifted higher than the positive 10% impulses. The band for day 0 is wider for the positive impulses than for the negative, naturally. The confidence intervals illustrated in both figure 32 and 33 indicates a significant increase in volatility for both negative and positive impulse. Figure 34 illustrates the sub norm band (95%) for the difference between the two previously discussed impulses. This response sub-norm band does not include zero for positive step ahead, it is possible to reject, at 5% statistical significant, the null hypothesis of symmetry. The band for day zero includes 0 and indicates a rejection of asymmetry at 5% statistical significance.

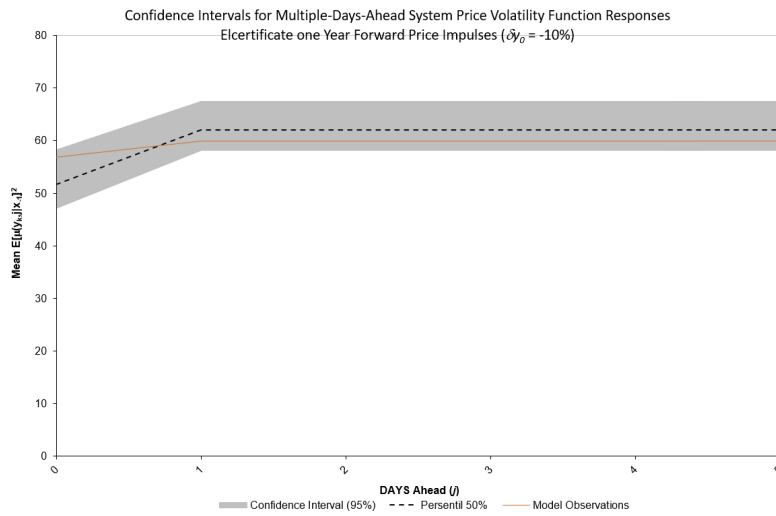


Figure 32; Confidence intervals for Multiple-Days-Ahead, volatility ( $\delta y_0 = -10\%$ )

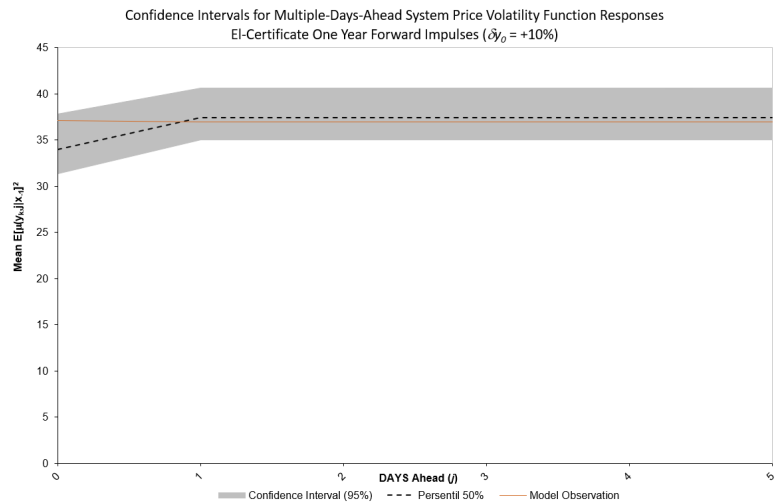


Figure 33; Confidence intervals for Multiple-Days-Ahead, volatility ( $\delta y_0 = +10\%$ )

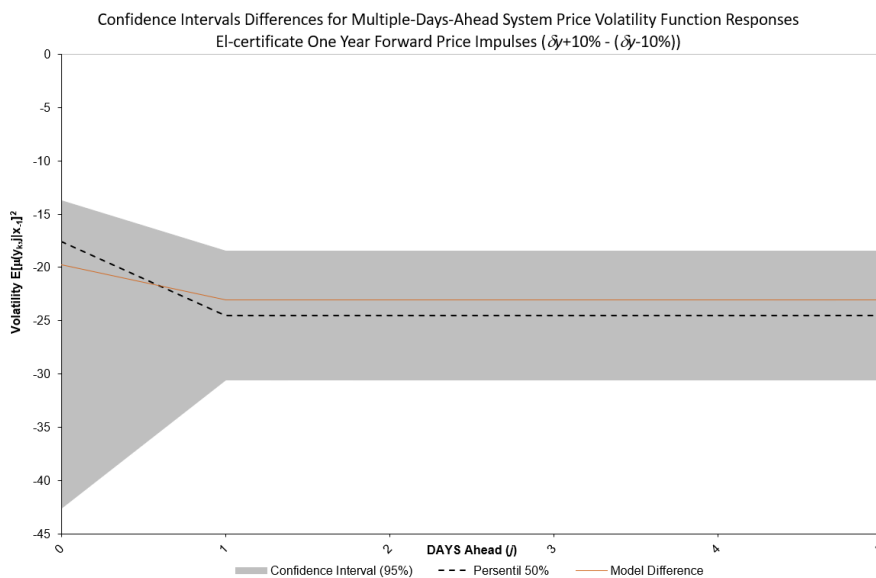


Figure 34; Confidence intervals for Multiple-Days-Ahead, volatility ( $dy + 10\% - (dy - 10\%)$ )

## 5.2.4 Persistence

Figure 35 reports profile bundles for the volatility. If the thickness of the bundle decreases to zero fast, it tells us that the process reverts to its mean. If this is not the case, and the bundle keeps its width, the process is persistent. The half-life of the Spot Price time series is reported to be 13 days, and there for a relatively slow persistence after a shock (short memory). But; the standard derivation is very high, almost three times the average half-life, which shows that the value of persistence is highly uncertain.

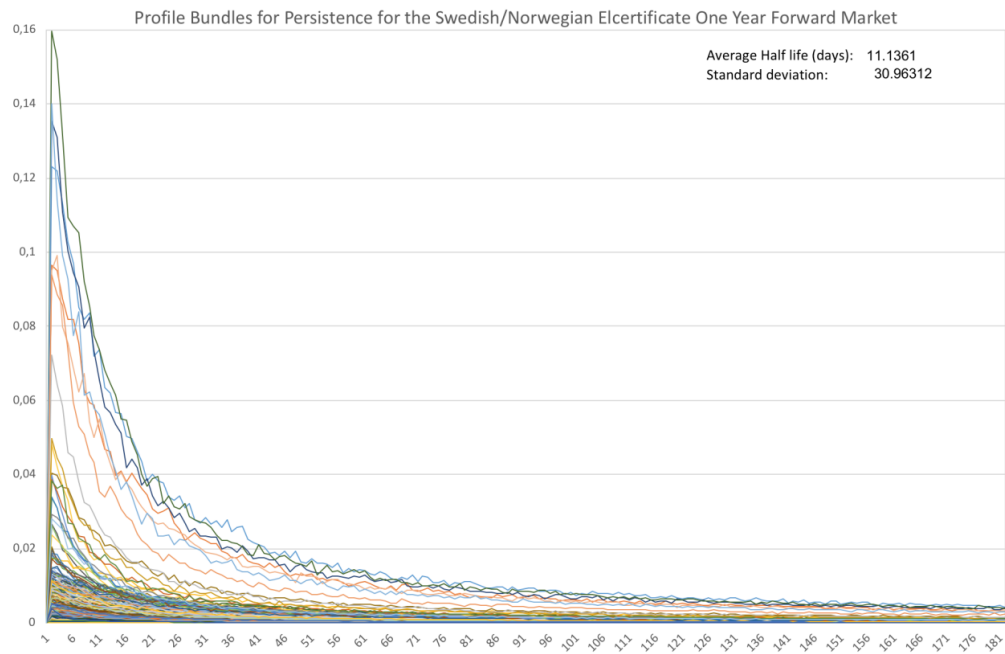


Figure 35; Profile bundles for Persistence, One-Year Forward Price

## 6.0 Conclusion

### 6.1 Summary of results

In this thesis I have analyzed the mean and volatility responses from probable price impulses that will hit the EI-certificate market occasionally. The thesis has studied the conditional densities, performed an impulse-response analysis which also includes bootstrapping for confidence intervals and volatility persistence.

My results show two well-established SNP specifications which report residuals suggesting non-significant statistical model misspecifications. The semi-nonparametric GARCH model is therefore well specified for executing the impulse-response analysis. The impulse-response analysis reveal, for the mean, impulse responses symmetric around the mean with a positive (negative) expected return response from positive (negative) price movement impulses. The negative price changes are followed by a greater expected return the following days. This suggest asymmetry suggesting market is more sensitive to negative impulses and the negative price changes lasts. It is a large skewness in returns after shocks. The volatility response to the impulses are quite different for small impulses and large impulses. The volatility shows visibly larger responses to large price changes, than for small price changes, it also displays larger impact with negative impulses.

The bootstrapped confidence intervals show asymmetry, with larger confidence intervals for the positive impulses (mean), and a larger volatility for negative impulses than positive (negative difference). Again, showing asymmetry. This indicates a nervous market when it comes to negative price movements. Given the microstructure of the market it seems like the people in position of certificates seems to create a sale pressure when there are negative price movements, this seen in the increased volatility when the price movement is negative. When the price drops, the producers wish to sell their certificates because of an anticipation of greater reductions. The volatility does not show a similar will to sell when the price movements are positive (lower volatility). The persistence of impulse on volatility is 13 days for Spot Price and 11 days for One-Year Forward, but both has a standard derivation of over 30. This indicates a short time for mean reversion with large differences.

It is possible to trade derivatives bilateral with skm.se. (Kraftmekling 2018) There is today no active market for the derivatives. If there in the future is such a market, the volatility index (see 6.2 Further research) will be an important aspect to price the derivatives.

## 6.2 Further research

The dataset is for weekly prices, because of the lack of daily prices in the market. The possibility to obtain daily prices should be further examined. A similar analysis for daily prices would be interesting

An interesting expansion, given optimal SNP model, would be to establish a volatility index for the el-certificate model.

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## Appendix

The appendix contains all the tables and figures of the thesis.

**Table 1 Statistics for Spot Price Elcertificate Market 2005-2018**

**Panel A: Spot Price Series**

	Mean (all)/	Median	Maximum /	Moment	Quantile	Quantile	Cramer-	Serial dependence		VaR
<b>Returns</b>	M (-drop)	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	von-Mises	Q(12)	Q <sup>2</sup> (12)	(1%; 2,5%)
	-0.16549	-0.09988	27.4355	17.76170	0.48125	6.8609	2.5439	62.0	291.64	-7.586%
	█ -0.16189	3.29699	-27.1800	-0.31074	-0.05136	{0.0324}	{0.0000}	{0.0000}	{0.0000}	-14.596%
	BDS-Z-statistic ( $e = 1$ )				KPSS (Stationary)		Augmented	ARCH	RESET	CVaR
	m=2	m=3	m=4	m=5	Intercept	I + Trend	DF-test	█ (12)	(12;6)	(1%; 2,5%)
	9.5262	10.6936	12.0988	13.6935	0.20068	0.04632	-20.1549	18.65	16.02314	-14.596%
	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.7390}	{0.2160}	{3.9716}	{0.0000}	{0.0000}	-9.613%

Table 1; Statistics for Spot Price El-certificate Market.

Table 1 reports the characteristics of the Spot price data set. The mean is negative, and the standard derivation is quite high (3.3). The minimum value is relatively low (-27.18), and the maximum value relatively high (27.43). The kurtosis is high and positive (17.76), indicating heavier tails than a normal distribution (more data in the tails), see figure 6. The skewness measures the symmetry in the dataset. In this case the skewness is -0.31 and indicates a symmetrical data set with a bias to the left of the distribution.

**Table 2. Statistics for One-Year Forward Elcertificate Market 2005-2018**

	Mean (all)/	Median	Maximum /	Moment	Quantile	Quantile	Cramer-	Serial dependence		VaR
<b>Returns</b>	M (-drop)	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	von-Mises	Q(12)	Q <sup>2</sup> (12)	(1%; 2,5%)
	-0.21472	-0.10610	26.7190	15.63537	0.33916	3.6505	2.5525	61.120	315.54	-8.798%
	█ -0.21129	3.30060	-26.4499	-0.29847	-0.05876	{0.1612}	{0.0000}	{0.0000}	{0.0000}	-6.307%
	BDS-Z-statistic (e = 1)				KPSS (Stationary)		Augmented	ARCH	RESET	CVaR
	m=2	m=3	m=4	m=5	Intercept	I + Trend	DF-test	█ (12)	(12;6)	(1%; 2,5%)
	9.2063	11.1401	12.9148	14.5024	0.18546	0.04439	-11.7253	17.145	5.596287	-14.869%
	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.7390}	{0.2160}	{0.0000}	{0.0000}	{0.0000}	-9.971%

Table 2; Statistics for One-Year Forward Price El-certificate Market

Table 2 illustrates the characteristics of the One Year Forward price data set. The mean is negative, and the standard derivation is high (3.3). The minimum value is relatively low (-26.45), and the maximum value relatively high (26.72). The kurtosis is high and positive (15.63), indicating heavier tails than a normal distribution (more data in the tails). The skewness measures the symmetry in the dataset. In this case the skewness is -0.30 and indicates a symmetrical data set.

**Table 3; Swedish/Norwegian Elcertificate Spot Price**

Statistical Model SNP-14,111,12, 000 -fit; semi-parametric-GARCH model

Mean Equation		Standard		
Var	SNP Coeff.	Mode	error	t-statistics
<i>Hermite Polynoms</i>				
$\eta_1$	a <sub>0</sub> [1]	-0,14650	0,05102	-2,87147
$\eta_2$	a <sub>0</sub> [2]	-0,08655	0,0434	-1,99427
$\eta_3$	a <sub>0</sub> [3]	0,05917	0,02842	2,08209
$\eta_4$	a <sub>0</sub> [4]	0,04611	0,02741	1,68243
$\eta_5$	a <sub>0</sub> [5]	-0,01902	0,03039	-0,62593
$\eta_6$	a <sub>0</sub> [6]	-0,12338	0,02559	-4,82164
$\eta_7$	a <sub>0</sub> [7]			
$\eta_8$	a <sub>0</sub> [8]			
<i>Mean Correlation</i>				
$\eta_{13}$	b <sub>0</sub> [1]	0,21443	0,06585	3,25647
$\eta_{14}$	B(1,1)	0,34733	0,04211	8,24833
$\eta_{15}$				
$\eta_{16}$				
<i>Variance Equation</i>				
$\eta_{27}$	R <sub>0</sub> [1]	0,24681	0,05153	4,78985
$\eta_{28}$	P[1,1]	0,42318	0,13844	3,05686
$\eta_{29}$	Q[1,1]	0,82292	0,04803	17,13234
$\eta_{30}$	V[1,1]	-0,33552	0,18651	-1,79899
$\eta_{31}$	W[1,1]	0,52916	0,1056	5,01103
<hr/>				
<i>Observations (incl. drops)</i>		670	$s_n$	1,0854466
<i>Log Likelihood</i>		-738,1036656	<i>aic</i>	1,1045642
			<i>bic</i>	1,1477901
Largest eigenvalue of mean function companion matrix:				0,347332
Largest eigenvalue of variance function P&Q companion m:				0,856288

Table 3; BIC optimal SNP model estimates

**Table 4 Residual Statistics for the Swedish/Norwegian Elcertificate market, Spot Price**

Residuals	Mean /	Median /	Maximum /	Moment	Quantile	Quantile	Cramer-	Serial dependence	
	Mode	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	von-Mises	Q(12)	Q <sup>2</sup> (12)
	-0,00058	-0,01995	6,81126	5,09506	0,32470	2,95637	1,13089	7,1947	10.692
		1,00317	-4,34508	0,68121	-0,01080	{0,2281}	{0,0000}	{0,8440}	{0,5560}
	BDS-statistic ( $\varepsilon=1$ )				ARCH	RESET	Joint	VaR	CVaR
	m=2	m=3	m=4	m=5	(12)	(12;6)	Bias	5%/ 1%	5%/ 1%
	(-0.524877)	(-0.697266)	(-0.384598)	0.246726	0.874236	1.522853		-1,5743 %	-2,1410 %
	{0,5997}	{0,4865}	{0,7005}	{0,8051}	{0,5733}	{0,1680}		-2,3008 %	-2,8384 %

*Table 4; Residual Statistics, Spot Price*

*Table 4 reports residual statistics, test specification statistics. The residual statistics show data closer to the normal distribution. The mean is almost 0, and the standard derivation is close to one, which is compatible with a normal distribution. The Maximum, minimum values are 6.8 and -4.3, the kurtosis is 5.1 and the skewness is 0.7. The Cramer-von-Mises test for normal distribution suggest a deviation. The rest of the tests are nonsignificant.*

**Table 5. Swedish/Norwegian Elcertificate One-Year Forward**

Statistical Model SNP-14,111,12, 000 -fit; semi-parametric-GARCH model

Mean Equation		Standard		
Var	SNP Coeff.	Mode	error	t-statistics
<i>Hermite Polynoms</i>				
$\eta_1$	$a_0[1]$	-0,15006	0,0484	-3,10038
$\eta_2$	$a_0[2]$	-0,07964	0,04105	-1,93989
$\eta_3$	$a_0[3]$	0,09053	0,02949	3,06939
$\eta_4$	$a_0[4]$	0,03983	0,02706	1,47186
$\eta_5$	$a_0[5]$	-0,01112	0,02968	-0,37464
$\eta_6$	$a_0[6]$	-0,11648	0,02428	-4,79716
$\eta_7$	$a_0[7]$			
$\eta_8$	$a_0[8]$			
<i>Mean Correlation</i>				
$\eta_{13}$	$b_0[1]$	0,23266	0,05728	4,06208
$\eta_{14}$	$B(1,1)$	0,30792	0,04280	7,19516
$\eta_{15}$				
$\eta_{16}$				
$\eta_{17}$				
<i>Variance Equation</i>				
$\eta_{27}$	$R_0[1]$	0,27771	0,0555	5,00369
$\eta_{28}$	$P[1,1]$	0,45863	0,15419	2,97448
$\eta_{29}$	$Q[1,1]$	0,79037	0,05651	13,9876
$\eta_{30}$	$V[1,1]$	-0,37584	0,16663	-2,2556
$\eta_{31}$	$W[1,1]$	0,42248	0,12974	3,25627
<hr/>				
<i>Observations (incl. drops)</i>		670	$s_n$	1,0858296
<i>Log Likelihood</i>		-738,3640987	<i>aic</i>	1,1049472
			<i>bic</i>	1,1481731
<b>Largest eigenvalue of mean function companion matrix:</b>				0,307921
<b>Largest eigenvalue of variance function P&amp;Q companion m:</b>				0,835024

Table 5; BIC optimal SNP model estimates

**Table 6 Residual Statistics for the Swedish/Norwegian Elcertificate market, One Year Forward**

	Mean / Mode	Median / Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer- von-Mises	Serial dependence Q(12)	Q <sup>2</sup> (12)
Residuals	-0,00699	-0,02745	6,33737	4,06937	0,21522	1,38922	0,87874	10,8530	9,3357
		1,00096	-4,96242	0,38351	0,02934	{0,4993}	{0,0000}	{0,5420}	{0,6740}
BDS-statistic ( $\varepsilon=1$ )					ARCH	RESET	Joint	VaR	CVaR
	m=2	m=3	m=4	m=5	(12)	(12;6)	Bias	5%/ 1%	5%/ 1%
	-0,97937	-0,40852	0,14031	0,65305	0,787644	1,158792		-1,6136 %	-2,1320 %
	{0,3274}	{0,6829}	{0,8884}	{0,5137}	{0,6636}	{0,3267}		-2,4372 %	-3,2012 %

*Table 6; Residual Statistics, One-Year Forward Price*

Table 6 reports residual statistics, test specification statistics. The residual statistics show data closer to the normal distribution. The mean is almost 0, and the standard derivation is close to one, which is compatible with a normal distribution. The Maximum, minimum values are 6.3 and -4.96, the kurtosis is 4.1 and the skewness is 0.4. The Cramer-von-Mises test for normal distribution suggest a deviation. The rest of the tests are nonsignificant.

## Elcertificates Spot Prices 2005 - 2018



Figure 1; Spot Prices, 2005-2018

The figure illustrates the Spot Price for the El-certificates from 2005-2018. We detect a negative trend from 2008 until today.



## Elcertificates One-Year Forward March Prices 2005 - 2018



Figure 2; One-Year Forward Prices, 2005-2018

The figure illustrates the One Year Forward price for the El-certificates from 2005-2018. We detect a negative trend from 2008 until today.

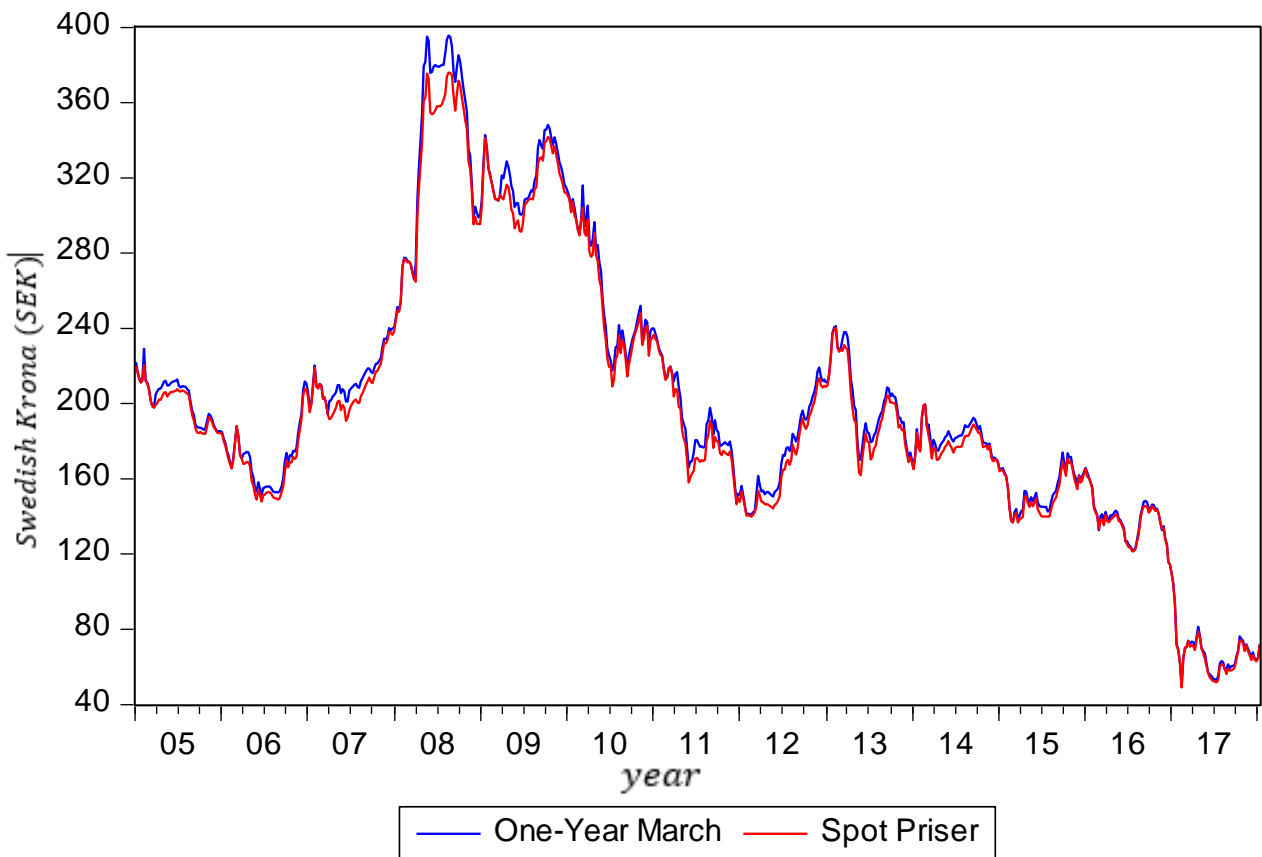


Figure 3; Spot and One-Year Prices, 2005-2018

The price movements of the Spot Price and the One-Year Forward Price. We detect a very similar movement pattern.

## Elcertificates Spot Price Movements

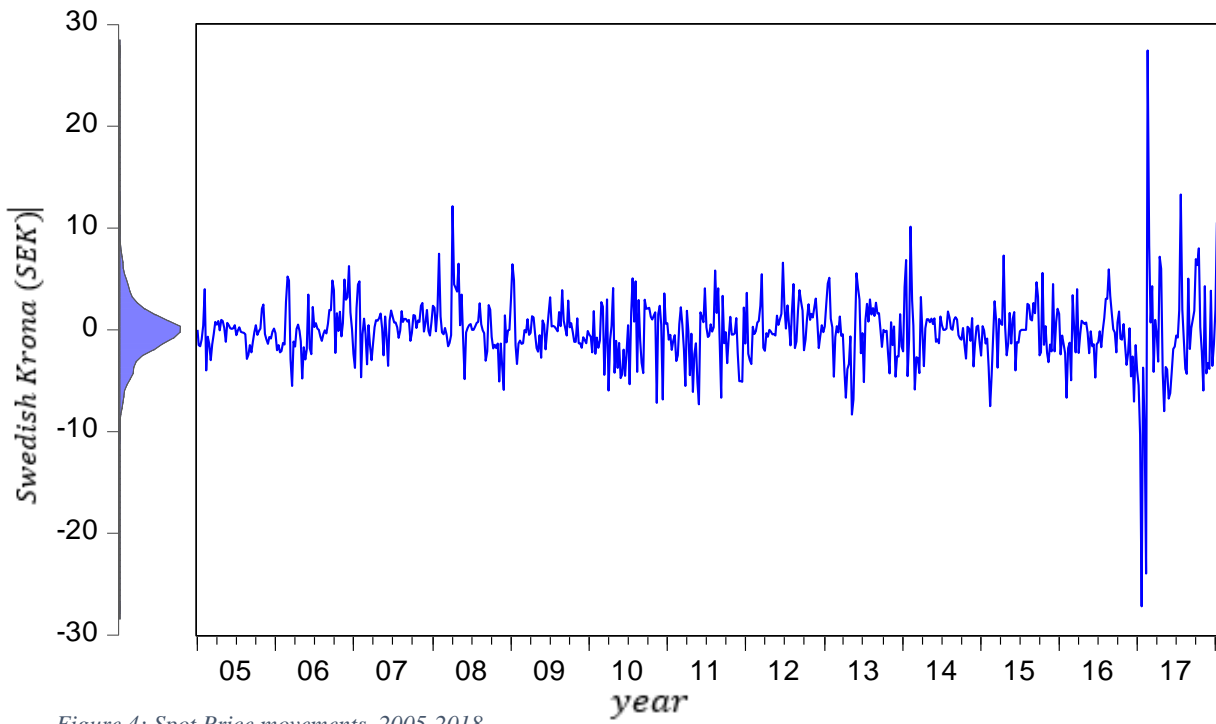


Figure 4; Spot Price movements, 2005-2018

The logarithmic returns for the El-certificate Spot price, together with a kernel distribution on the left.

## Elcertificates One-Year Forward March Price Movements

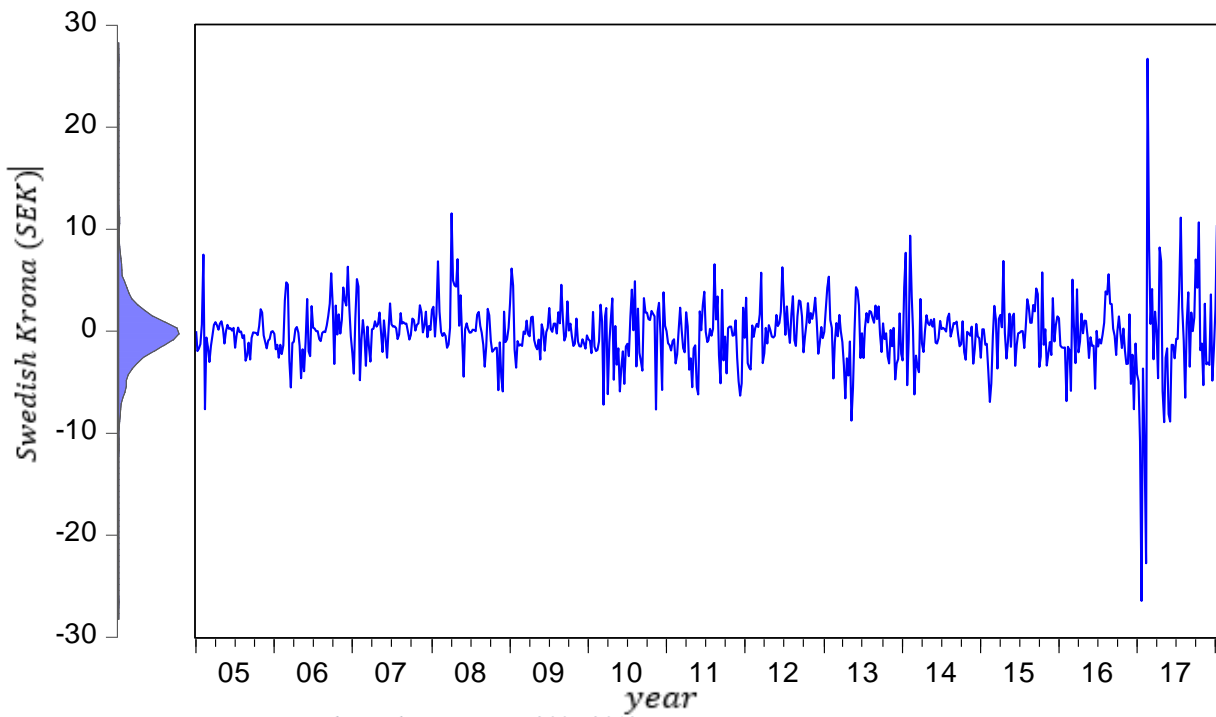


Figure 5; One-Year Forward March movements, 2005-2018

The logarithmic returns for the El-certificate One-Year Forward price, together with a kernel distribution on the left.

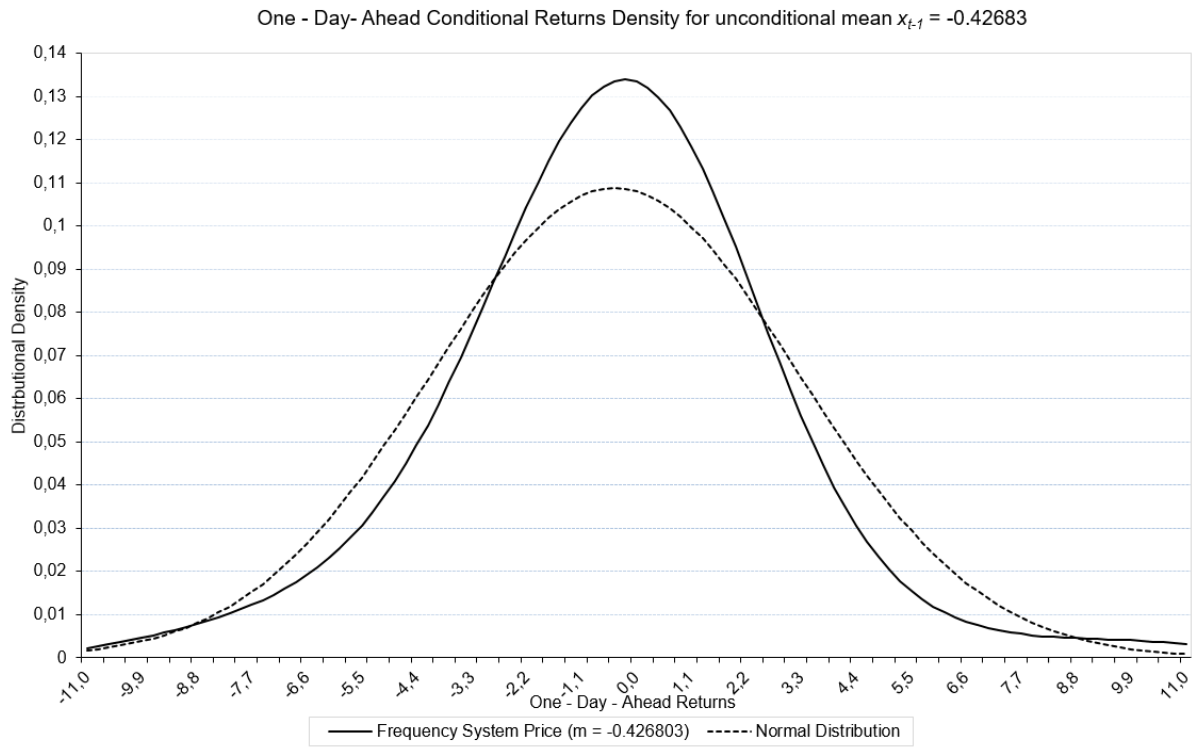


Figure 6; One-Day-Ahead Conditional returns density for unconditional mean, Spot

The chart shows negative tails and a higher, slimmer distribution than the normal distribution. The higher tails are compatible with the data for the kurtosis.

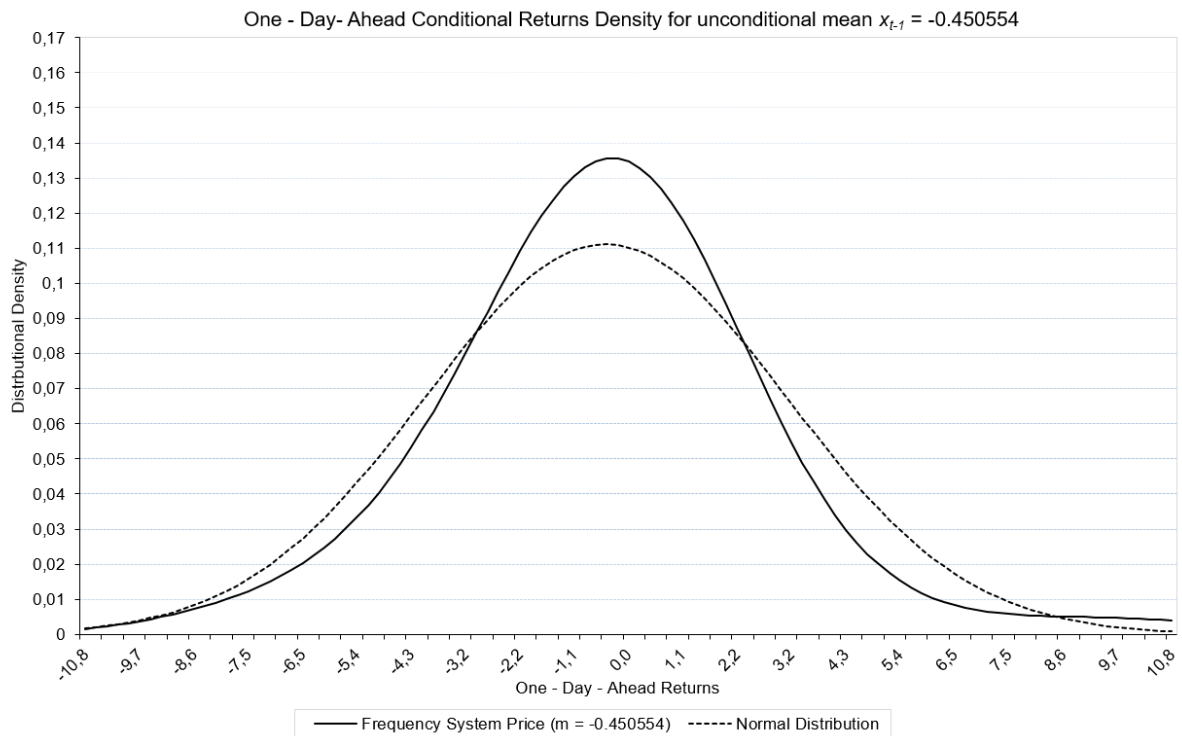


Figure 7; One-Day-Ahead Conditional returns density for unconditional mean, One-Year Forward

The chart shows negative tails and a higher, slimmer distribution than the normal distribution. The higher tails are compatible with the data for the kurtosis.

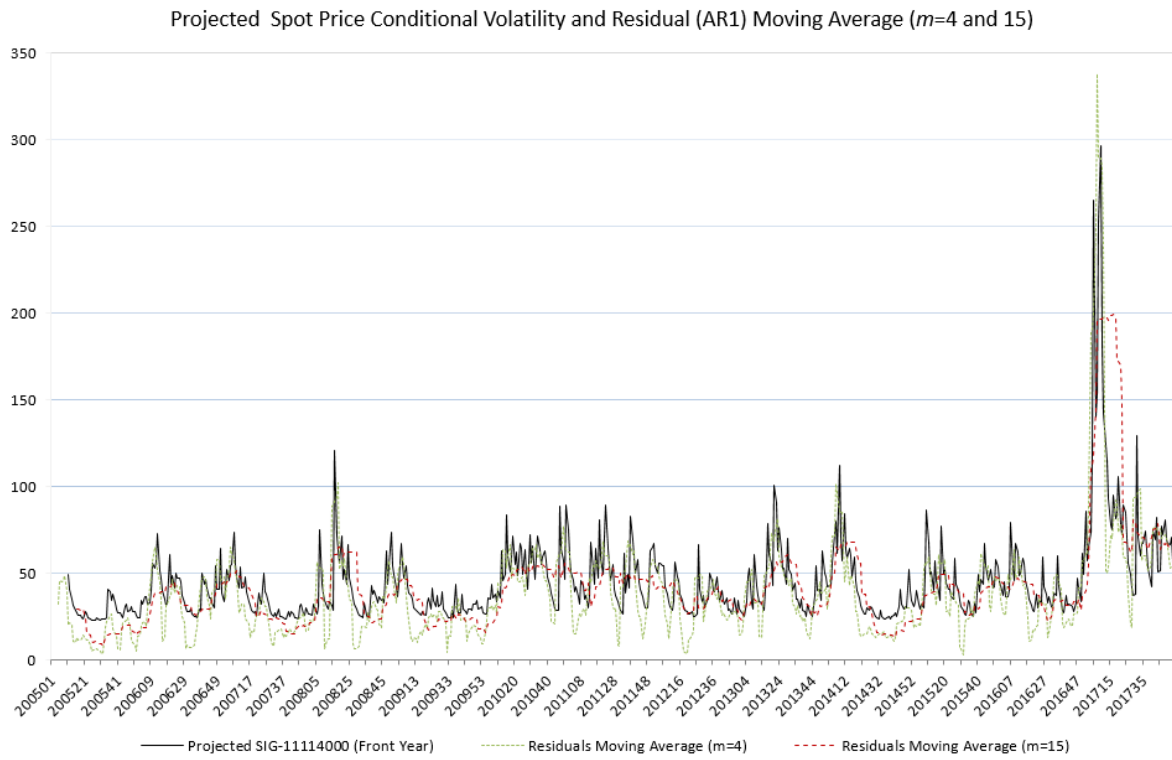


Figure 8; Projected Spot Price Conditional Volatility and Residual. Moving Average ( $m=4$  and  $15$ )

The Sweden/Norway EI-Certificate Spot Price: The GAUSS-hermite Quadrature Density Distribution

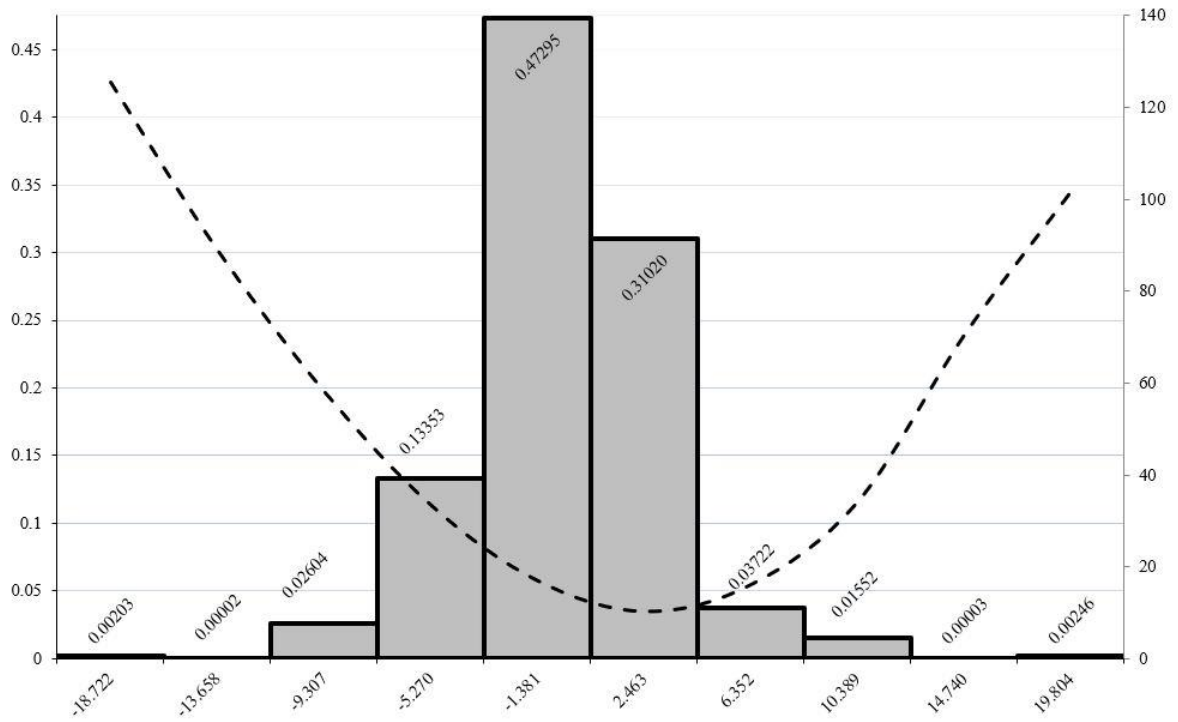


Figure 9; The GAUSS-Hermite Quadrature Density.

Shows the density of the price movements. We see a large quantity of the price movements is in the low negative side.



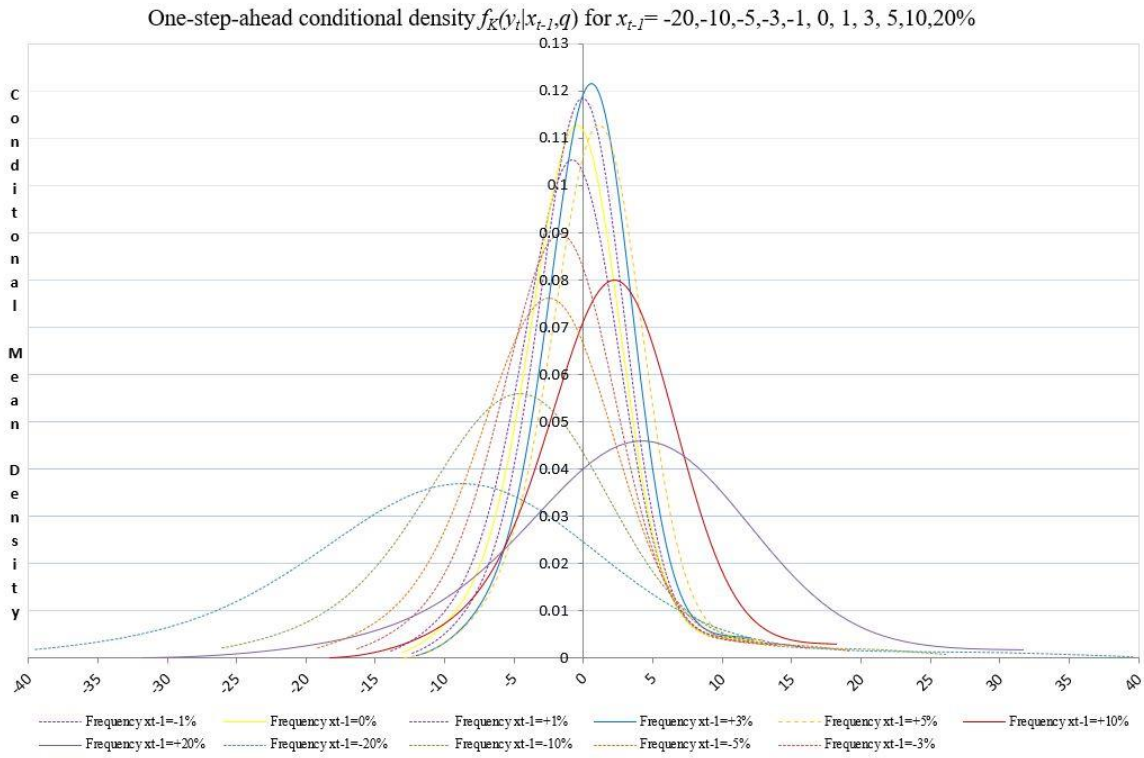


Figure 10; One-step-ahead conditional density;

for  $x = -20, \dots, +20$ . The densities are widest after large shocks. This reads that the larger the impulse gets; the more uncertainty lies in the price for the following day.

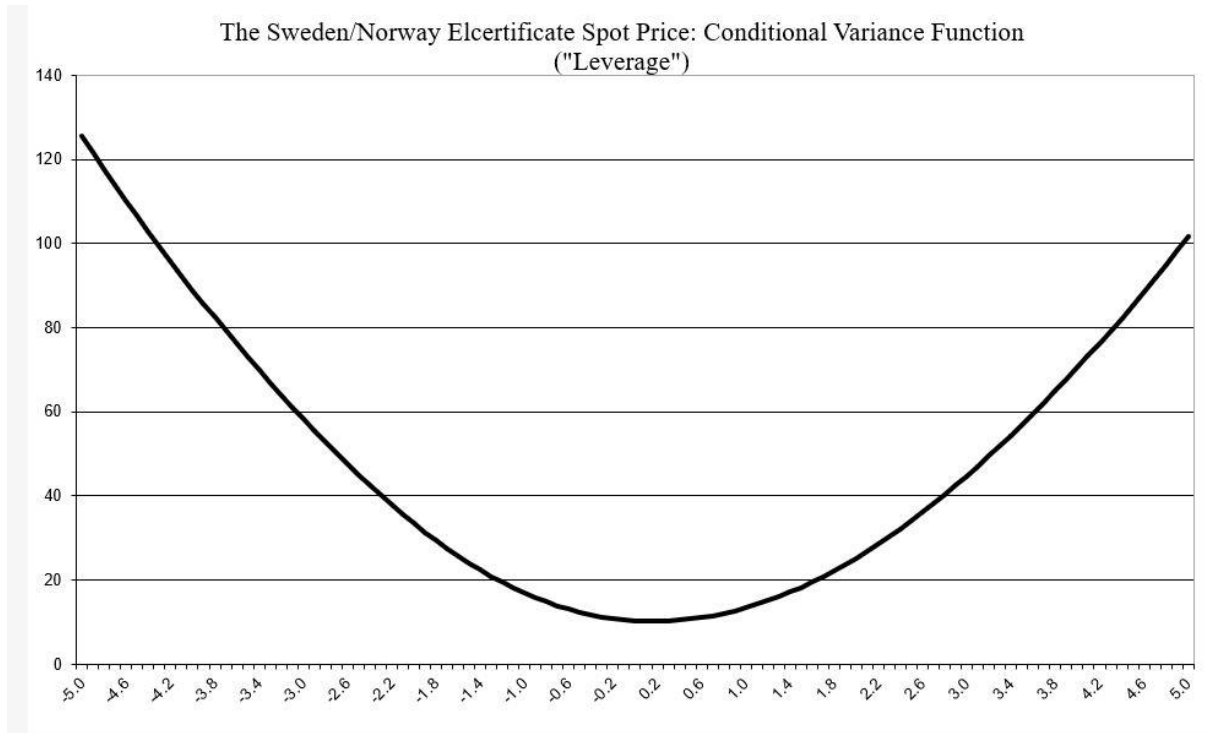


Figure 11; Conditional variance function.

This supports the suggestion of asymmetry, showing higher response to negative impulses.

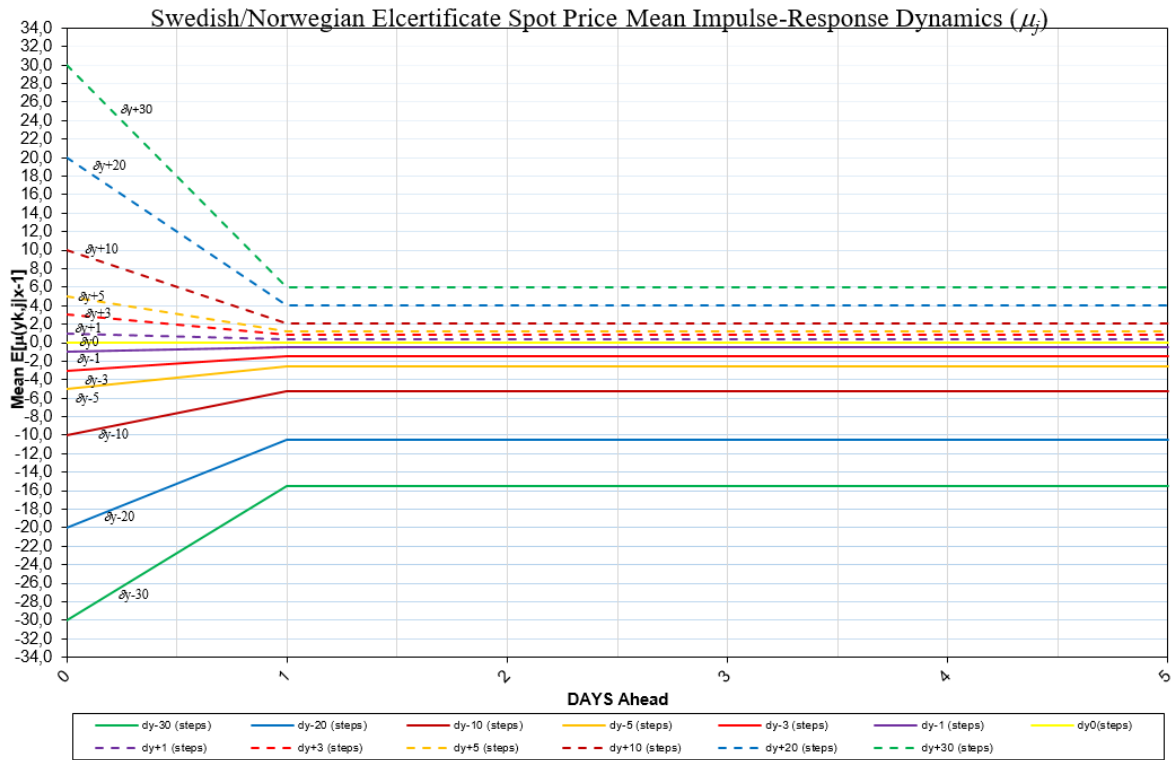


Figure 12; Mean Impulse-Response Dynamics.

The impulses revert to the mean, and the positive (negative) impulses generates a positive(negative) expected return. The negative impulses are followed by a greater negative expected return the following day.

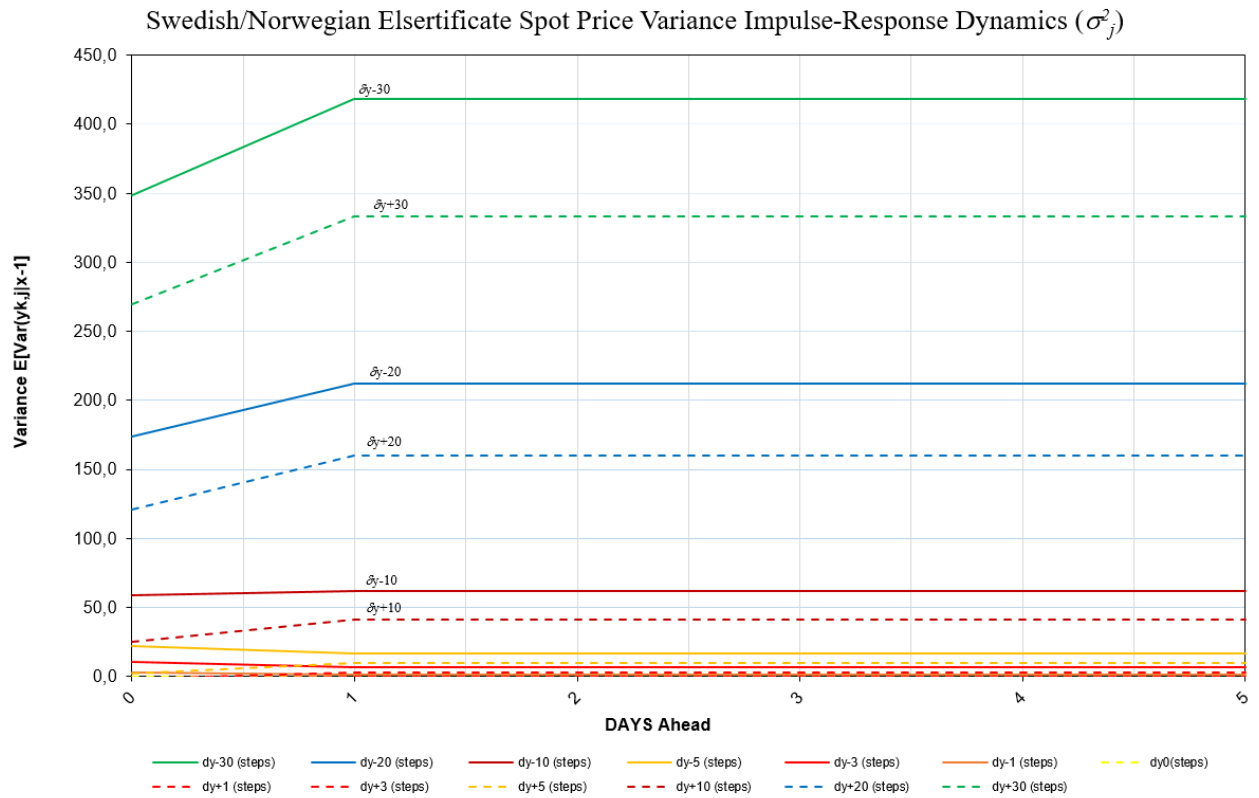


Figure 13; Variance impulse response Dynamics.

The negative impulses are one straight line and the positive is dotted. The negative impulses generates a higher response than the positive impulses. Espesially for the impulses from +/- 10% to +/- 30%.

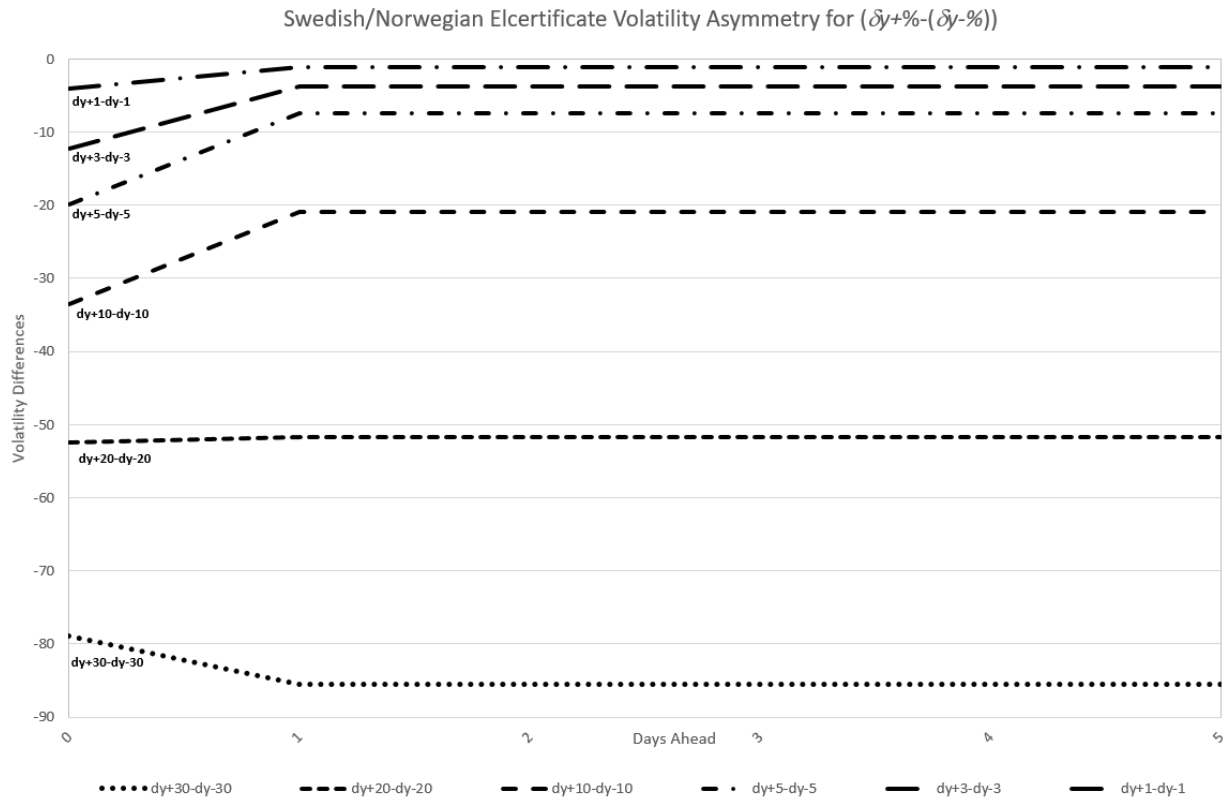


Figure 14; Volatility Asymmetry

The difference between the positive and negative impulse responses on volatility. The negative impulses clearly generate larger responses than the negative.

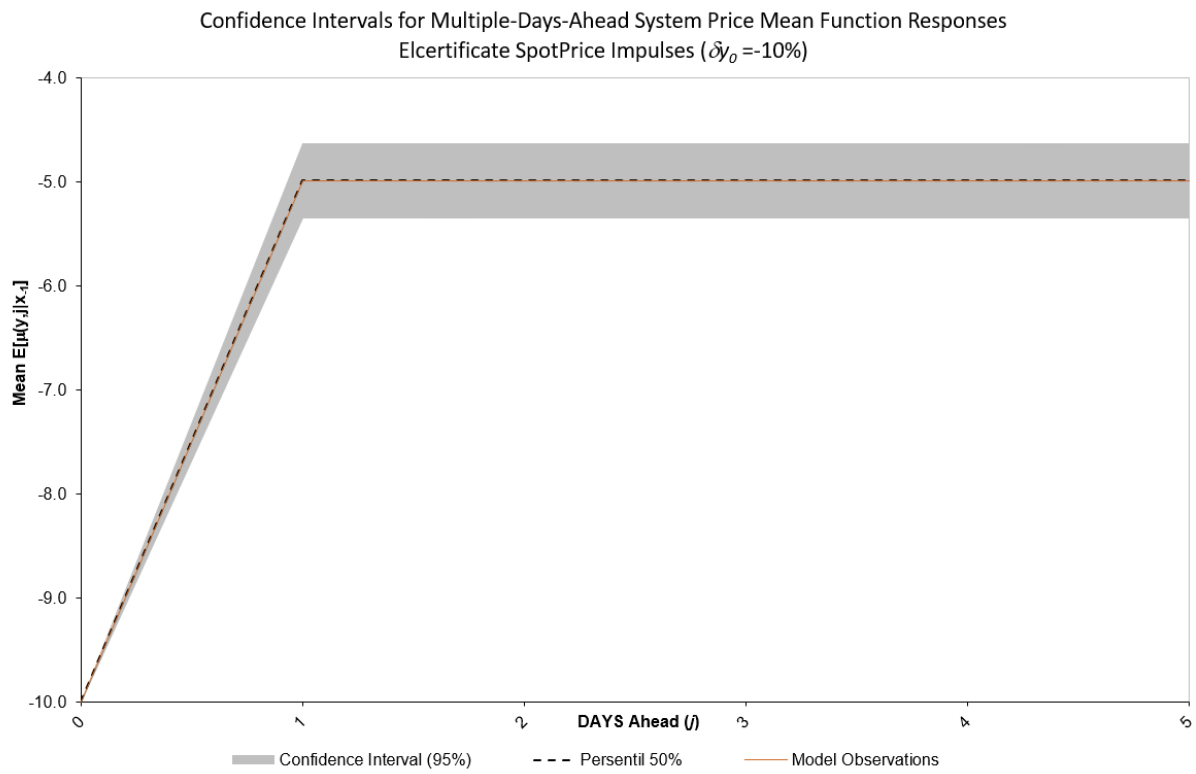


Figure 15; Confidence intervals for Multiple-Days-Ahead;  
mean ( $\delta y_0 = -10\%$ )

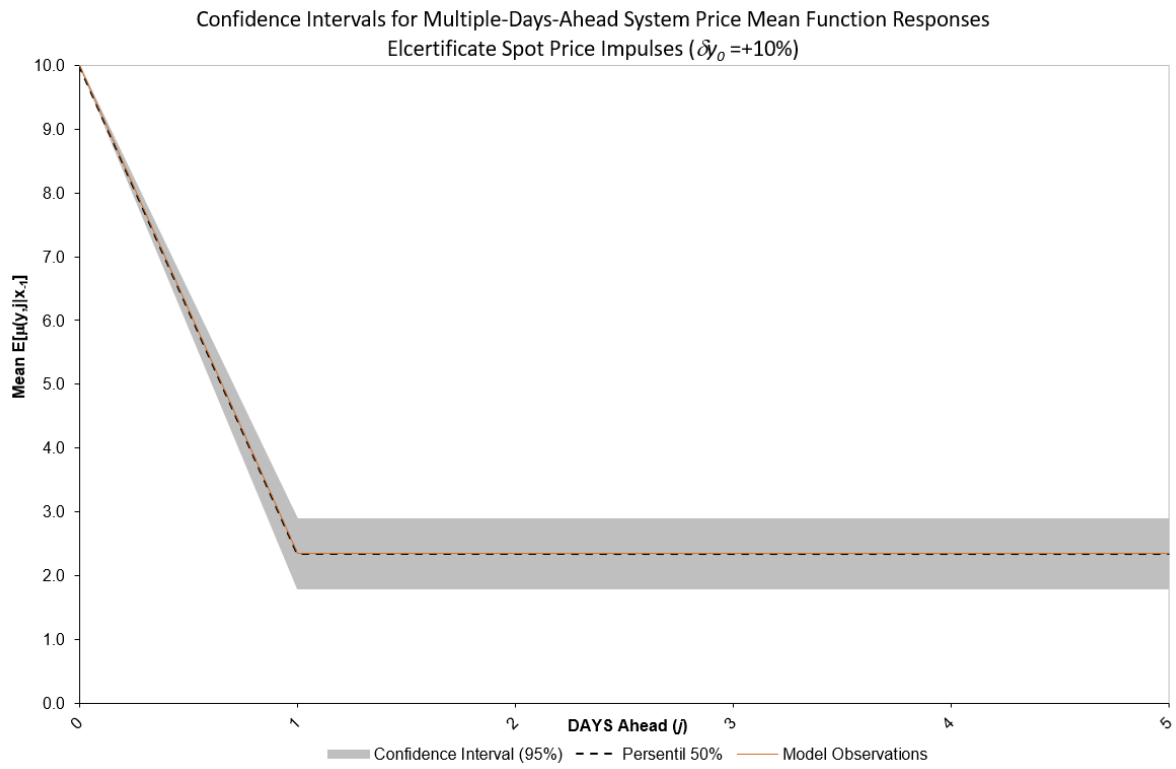


Figure 16; Confidence intervals for Multiple-Days-Ahead;

mean ( $\delta y_0 = +10\%$ )

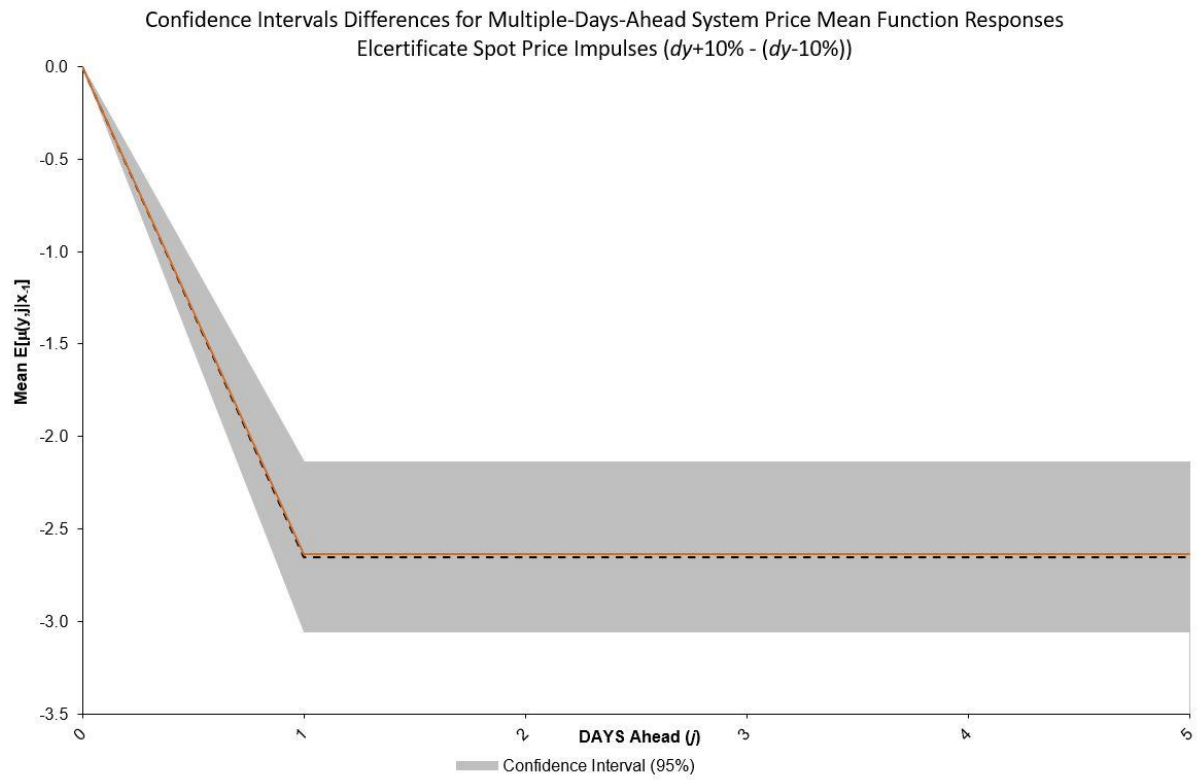


Figure 17; Confidence intervals for Multiple-Days-Ahead;

$mean(dy + 10\% - (dy - 10\%))$



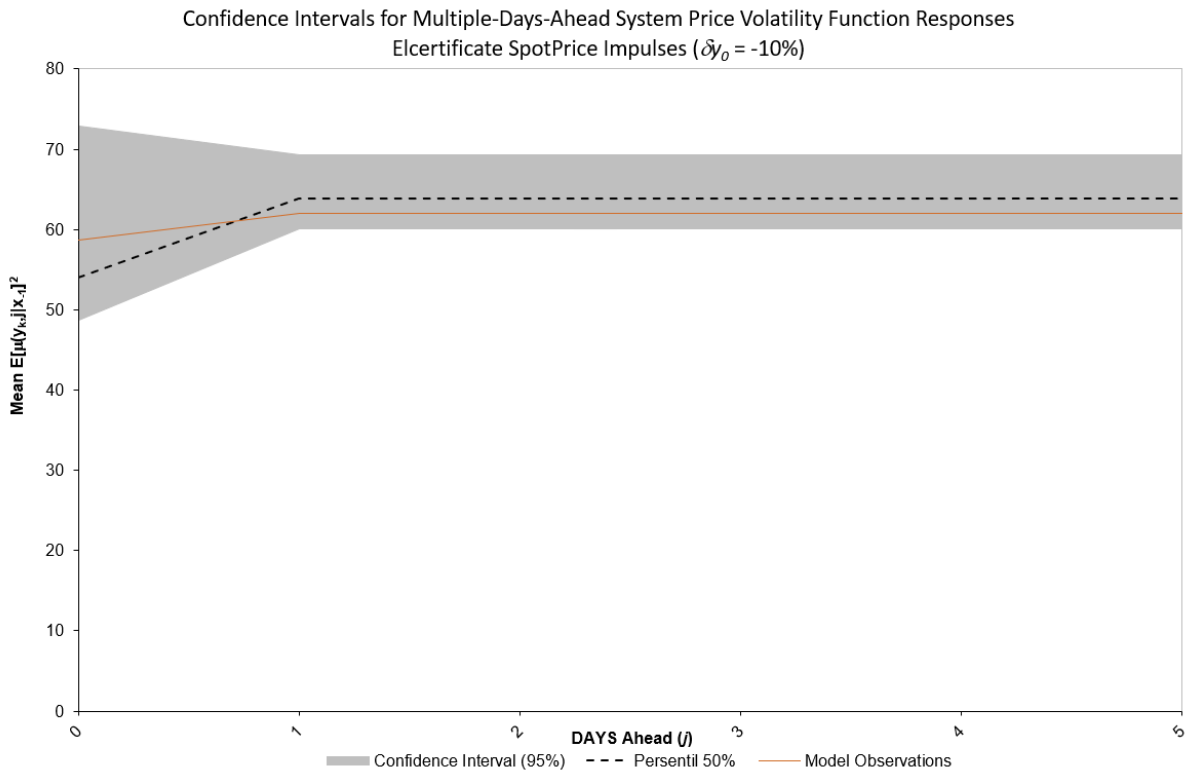


Figure 18; Confidence intervals for Multiple-Days-Ahead;

volatility ( $\delta y_0 = -10\%$ )

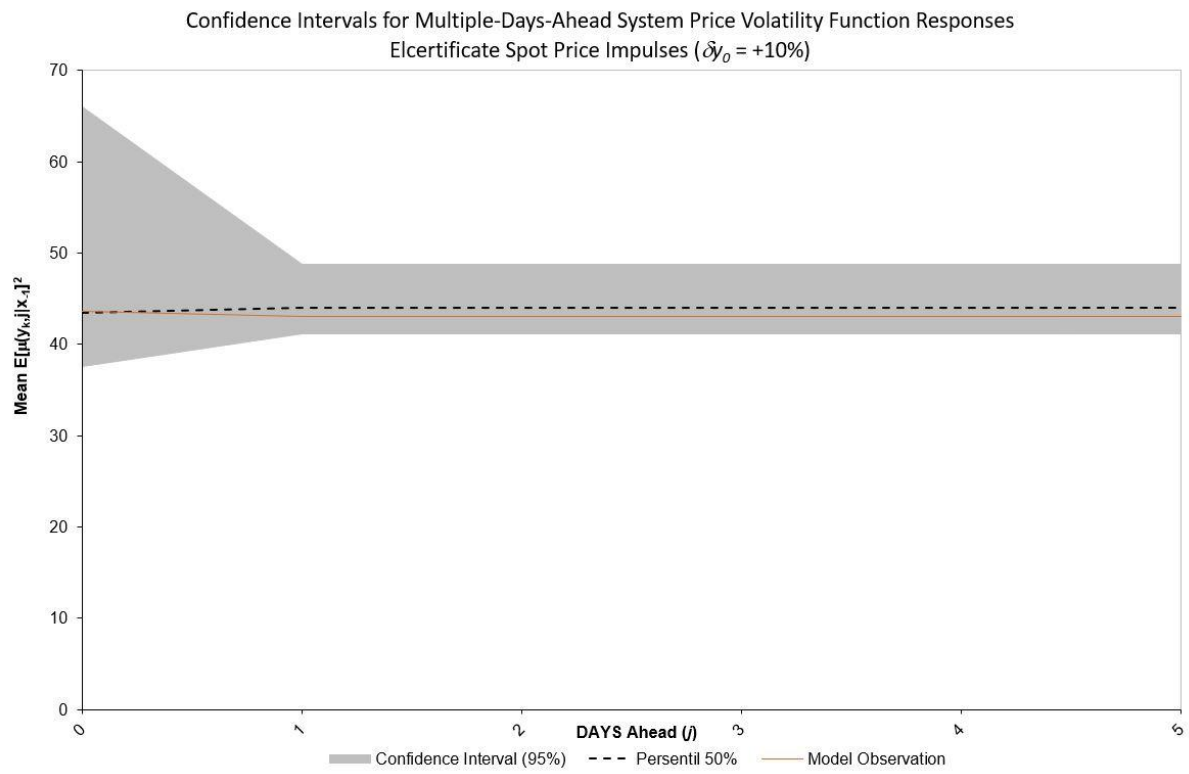


Figure 19; Confidence intervals for Multiple-Days-Ahead;  
volatility ( $\delta y_0 = +10\%$ )

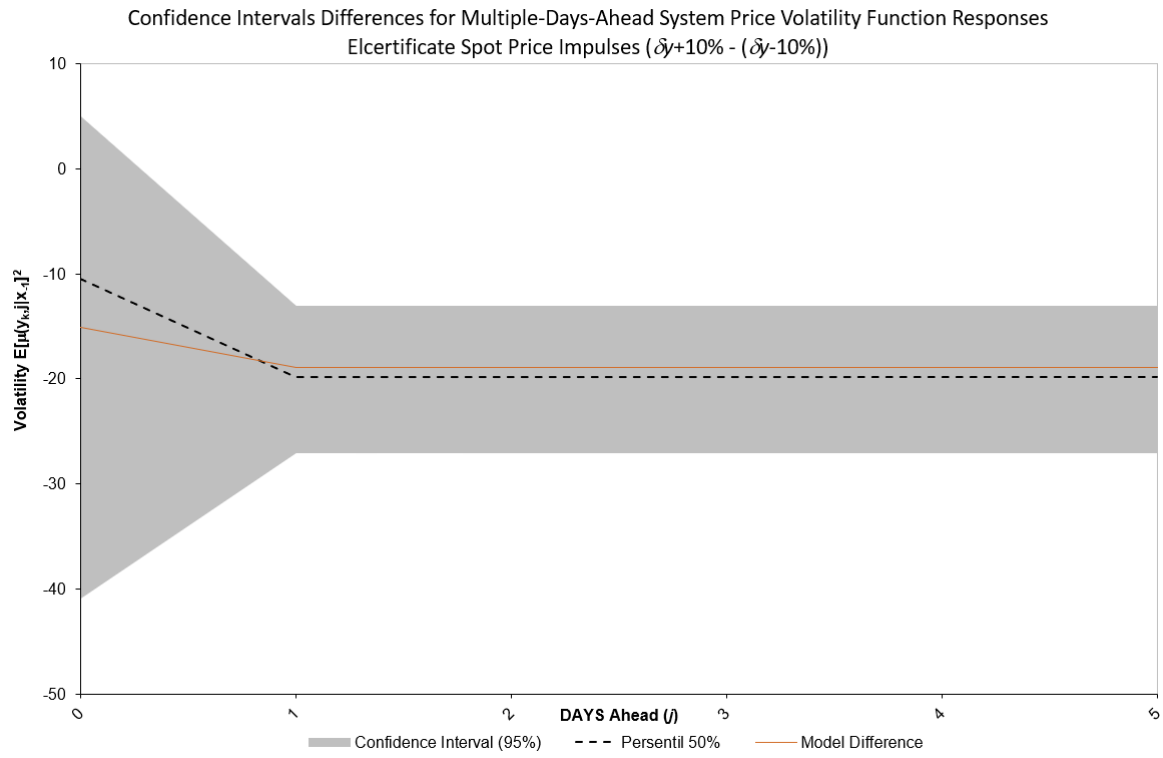


Figure 20; Confidence intervals for Multiple-Days-Ahead;  
volatility ( $dy + 10\% - (dy - 10\%)$ )

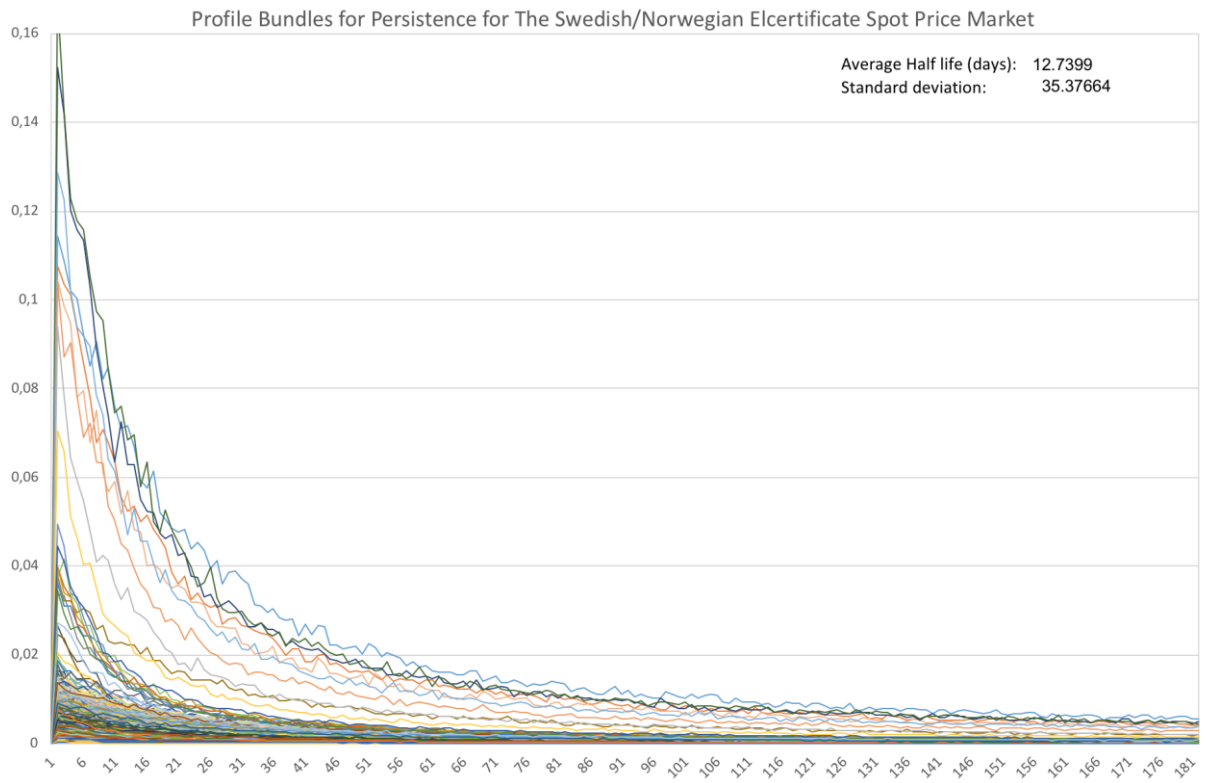


Figure 21; Profile bundles for Persistence, Spot Price.

The mean reversion time is low for the volatility, but has a high standard derivation.

Projected One-Year Price Conditional Volatility and Residual (AR1) Moving Average ( $m=4$  and  $15$ )

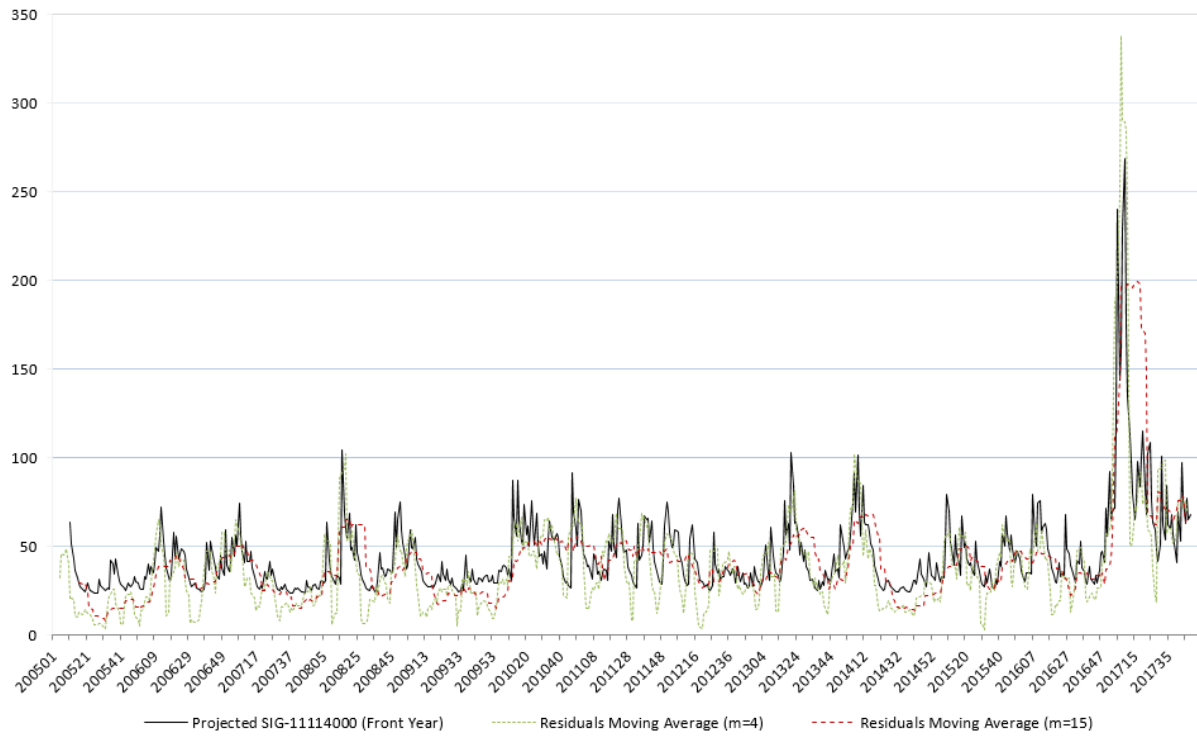


Figure 22; Projected One-Year Price Conditional Volatility and Residual. Moving Average ( $m=4$  and  $15$ )

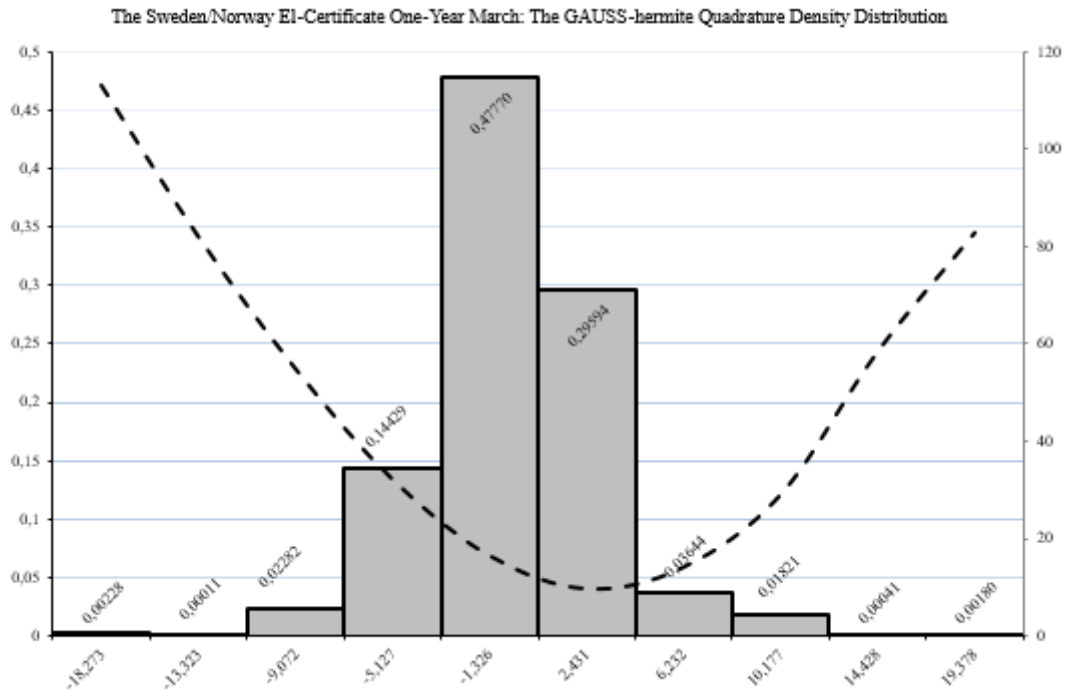


Figure 23; The GAUSS-hermite Quadrature Density.

Shows the density of the price movements. We see a large quantity of the price movements is in the low negative side.

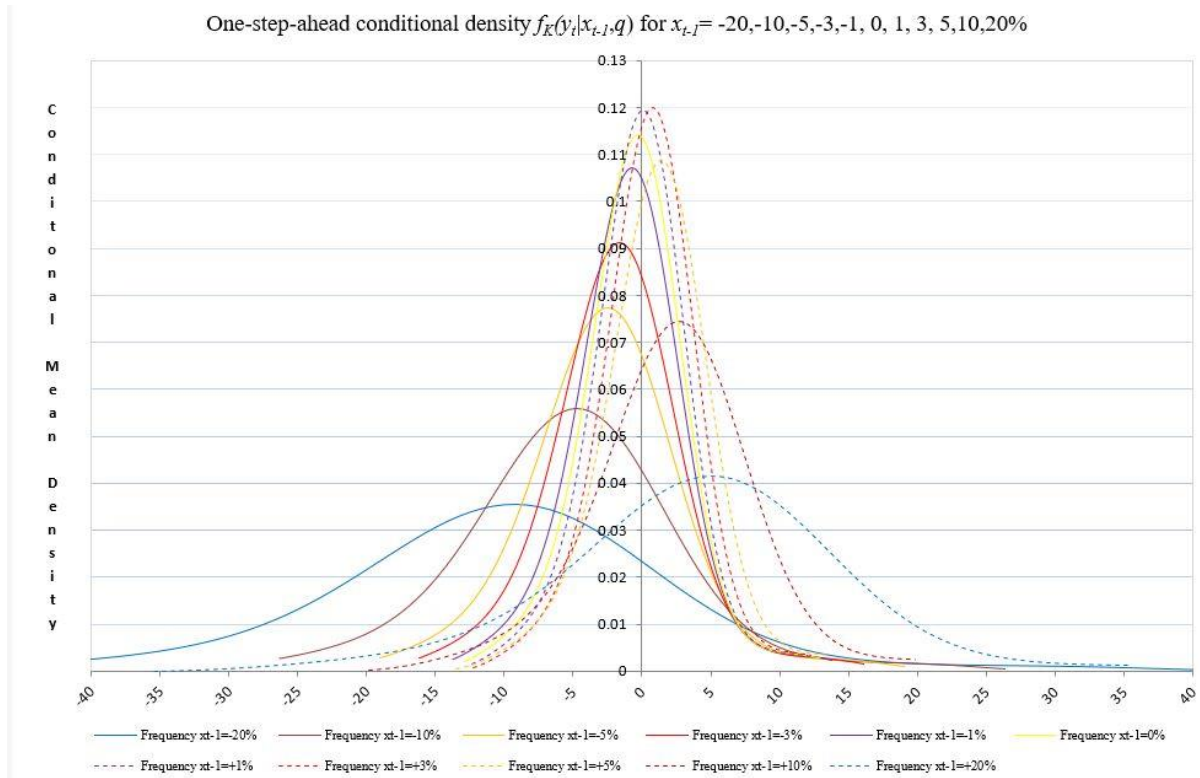


Figure 24; One-step-ahead conditional density for;

$x = -20, \dots, +20$ . The densities are widest after large shocks. This reads that the larger the impulse gets; the more uncertainty lies in the price for the following day.

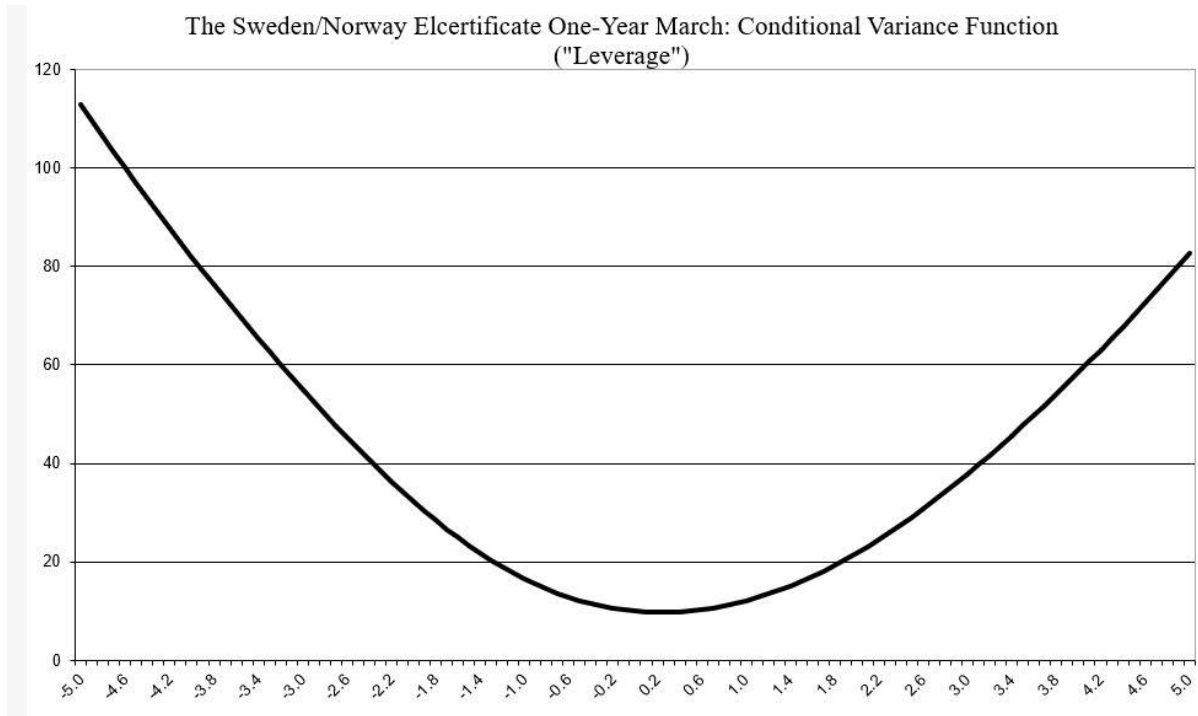


Figure 25; Conditional variance function.

This supports the suggestion of asymmetry, showing higher response to negative impulses.



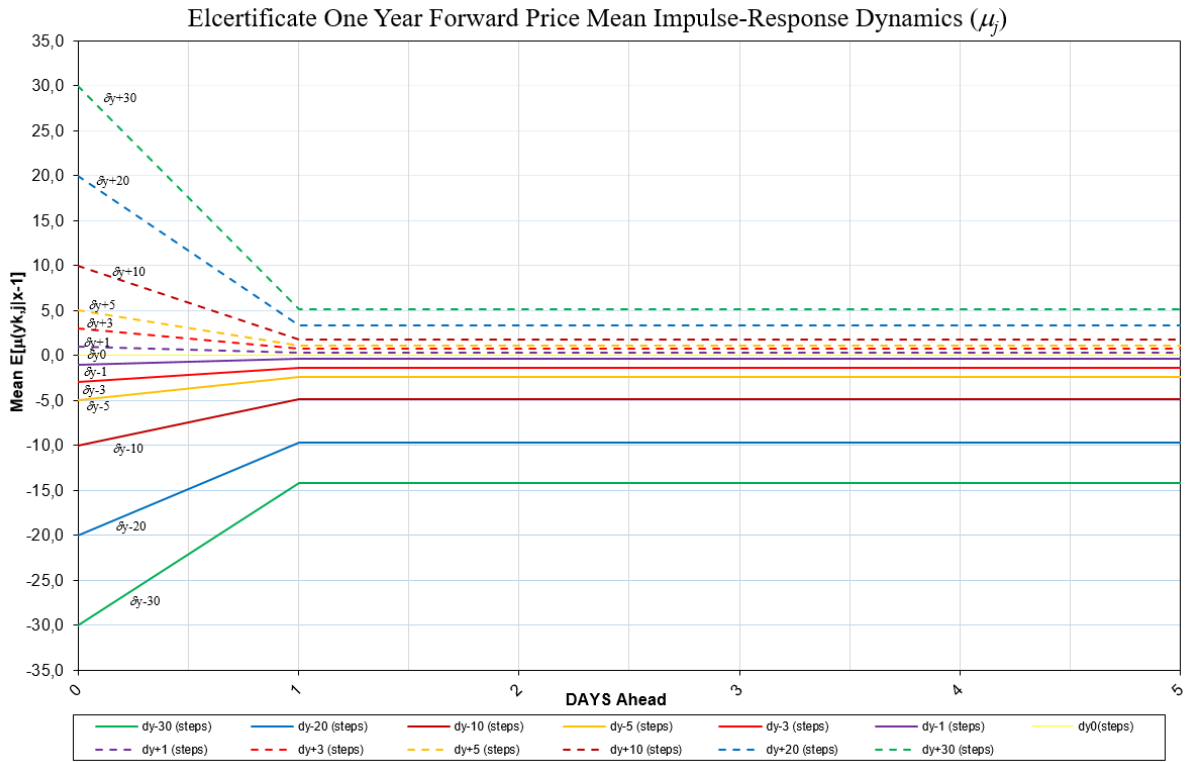


Figure 26; Mean impulse-response Dynamics.

The impulses revert to the mean, and the positive (negative) impulses generates a positive(negative) expected return. The negative impulses are followed by a greater negative expected return the following day.

Elcertificate One Year Forward Price Variance Impulse-Response Dynamics ( $\sigma_j^2$ )

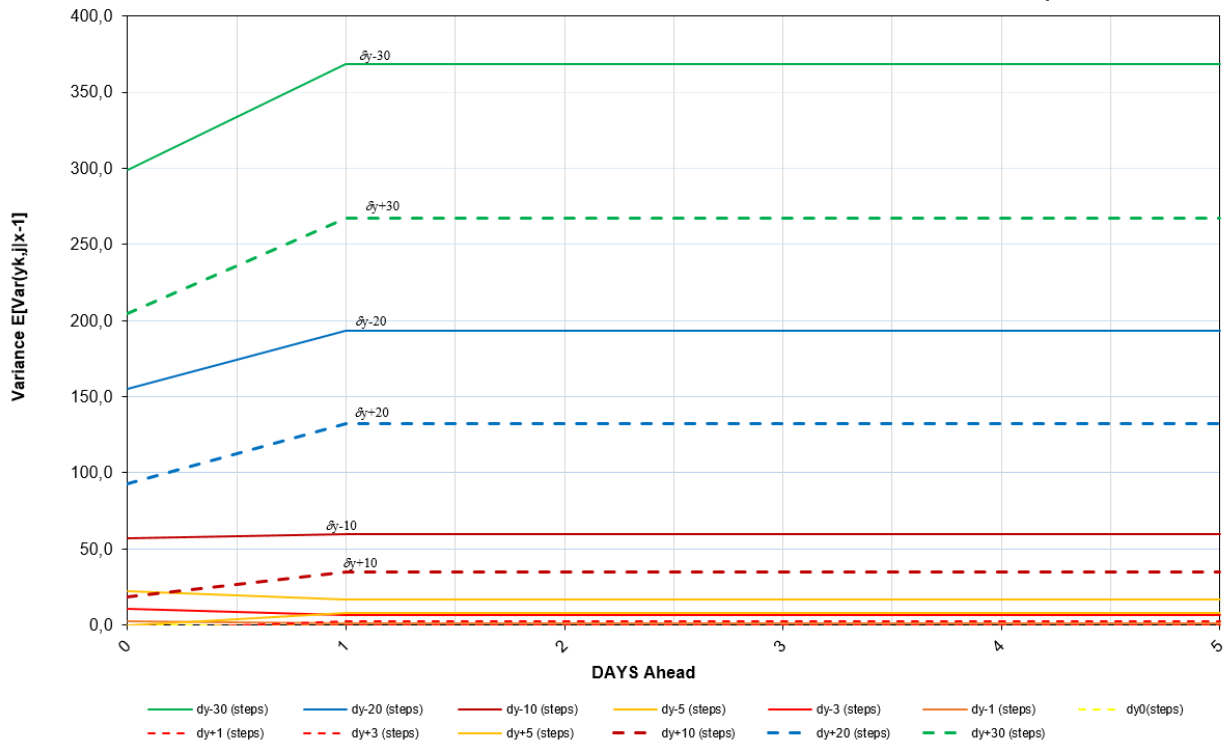


Figure 27; Variance impulse-response Dynamics.

The negative impulses are one straight line and the positive is dotted. The negative impulses generates a higher response than the positive impulses. Especially for the impulses from +/- 10% to +/- 30%.

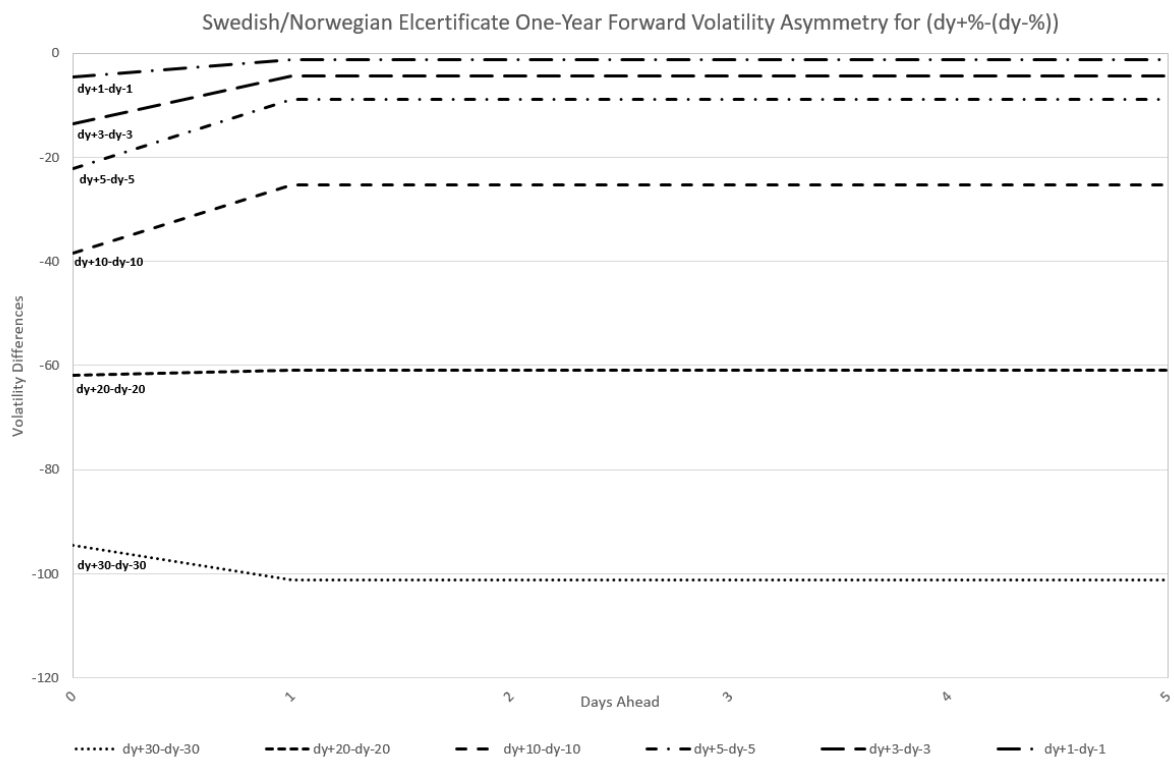


Figure 28; Volatility Asymmetry;

The difference between the positive and negative impulse responses on volatility. The negative impulses clearly generate larger responses than the positive.

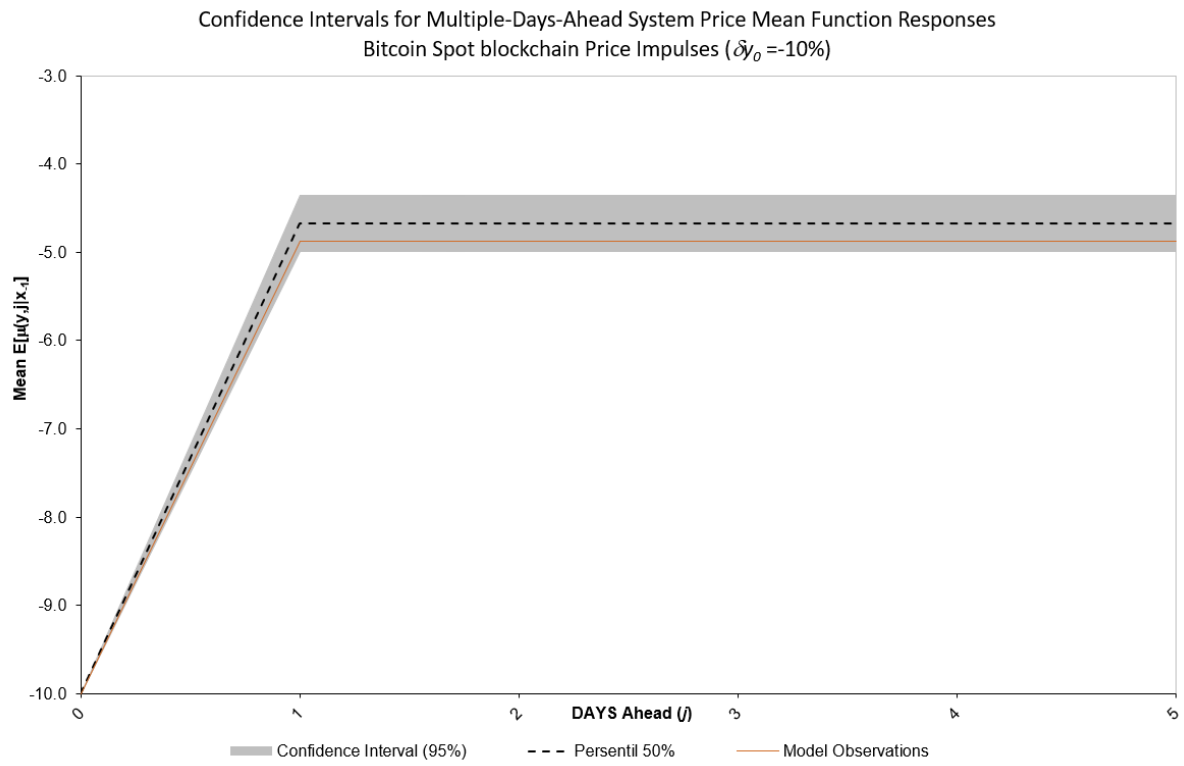


Figure 29; Confidence intervals for Multiple-Days-Ahead;

mean ( $\delta y_0 = -10\%$ )

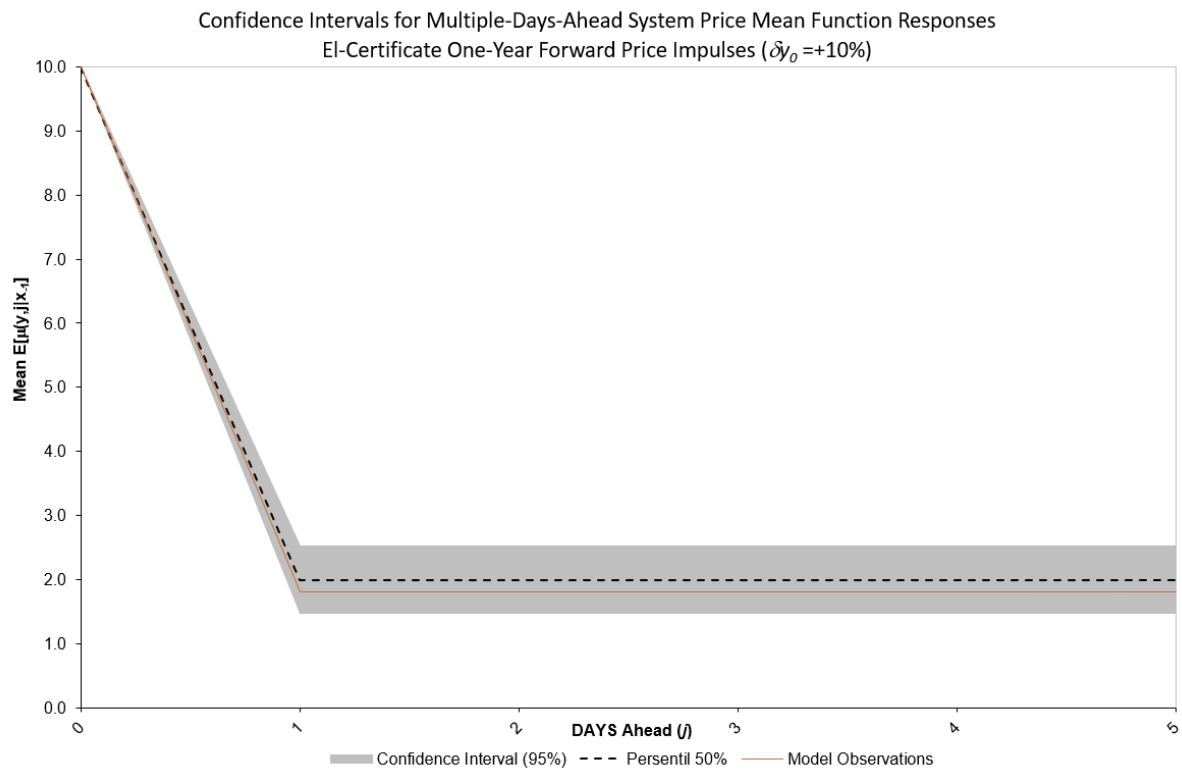


Figure 30; Confidence intervals for Multiple-Days-Ahead;

mean ( $\delta y_0 = +10\%$ )

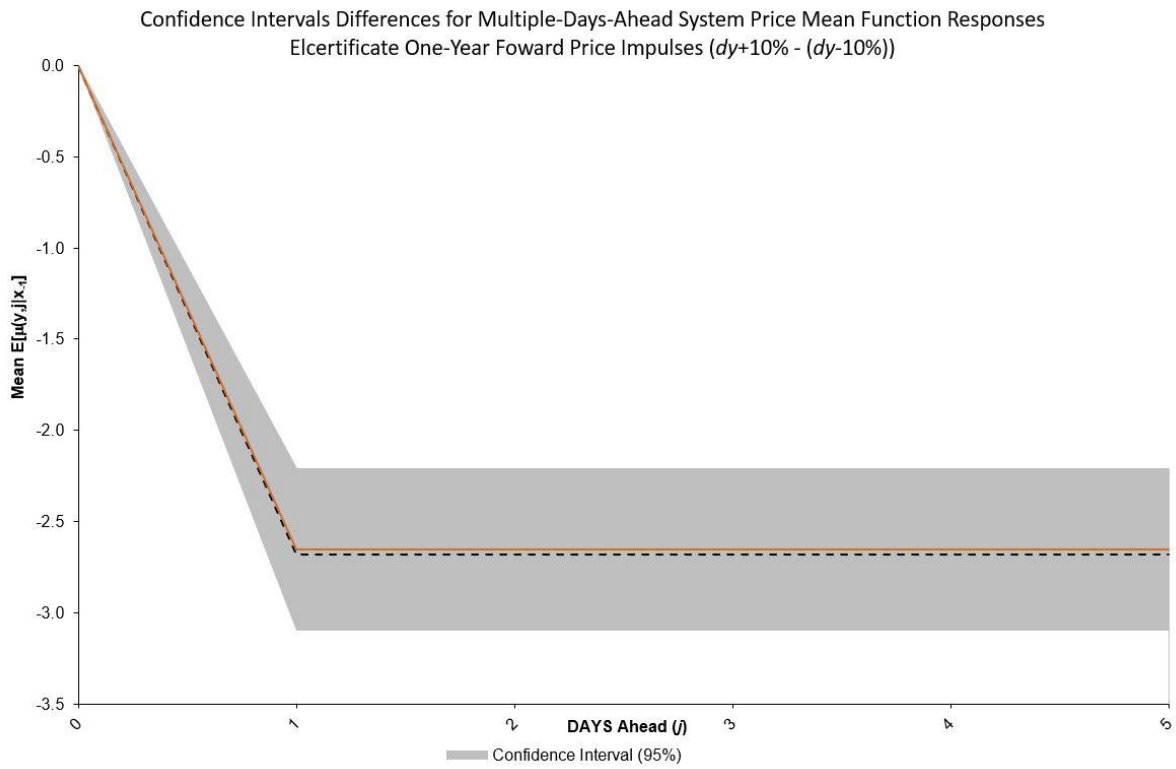


Figure 31; Confidence intervals for Multiple-Days-Ahead;

$mean(dy + 10\% - (dy - 10\%))$

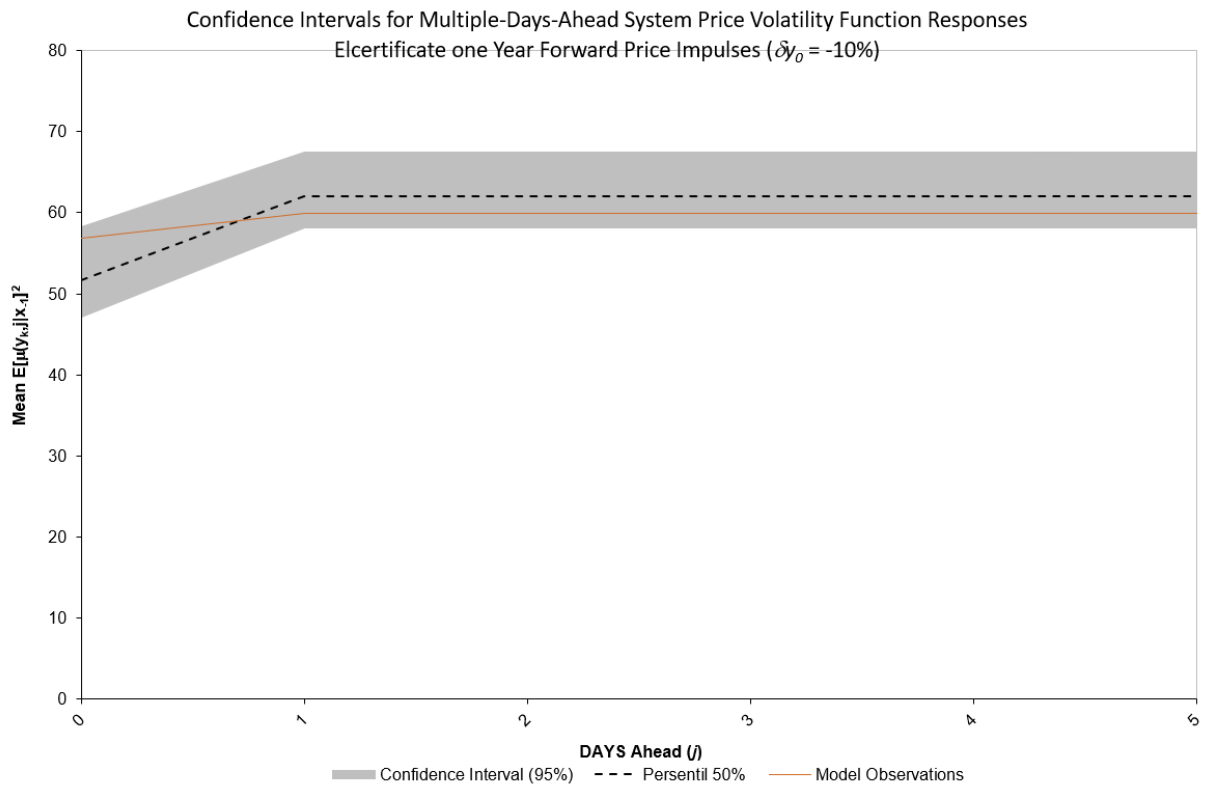


Figure 32; Confidence intervals for Multiple-Days-Ahead;

volatility ( $\delta y_0 = -10\%$ )

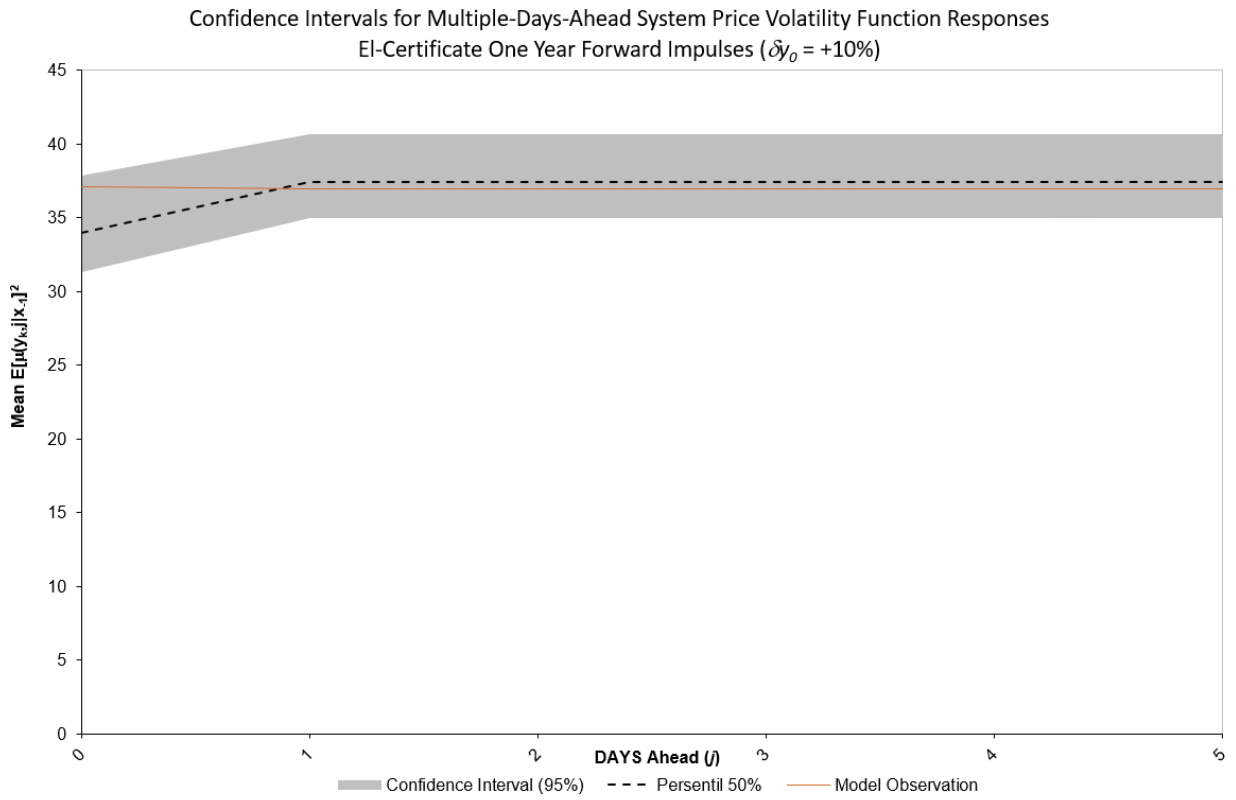


Figure 33; Confidence intervals for Multiple-Days-Ahead;

volatility ( $\delta y_0 = +10\%$ )



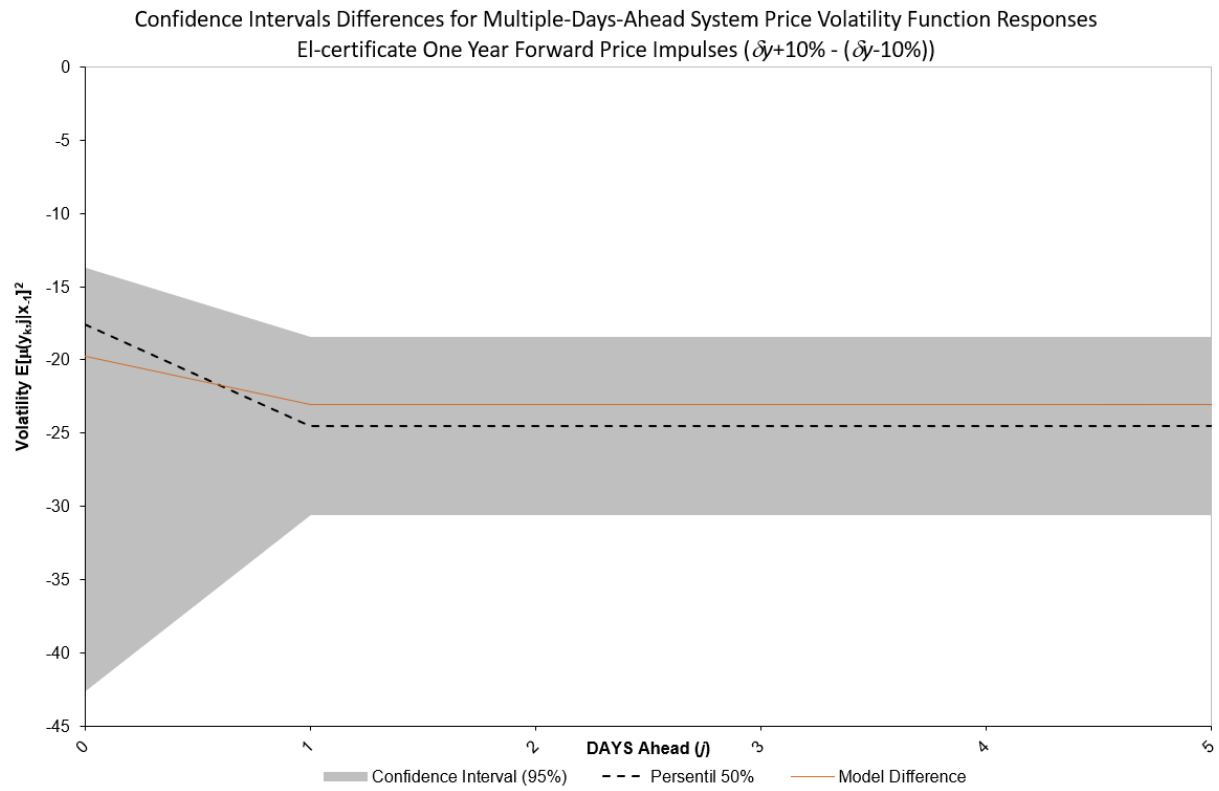


Figure 34; Confidence intervals for Multiple-Days-Ahead;

volatility ( $dy + 10\% - (dy - 10\%)$ )

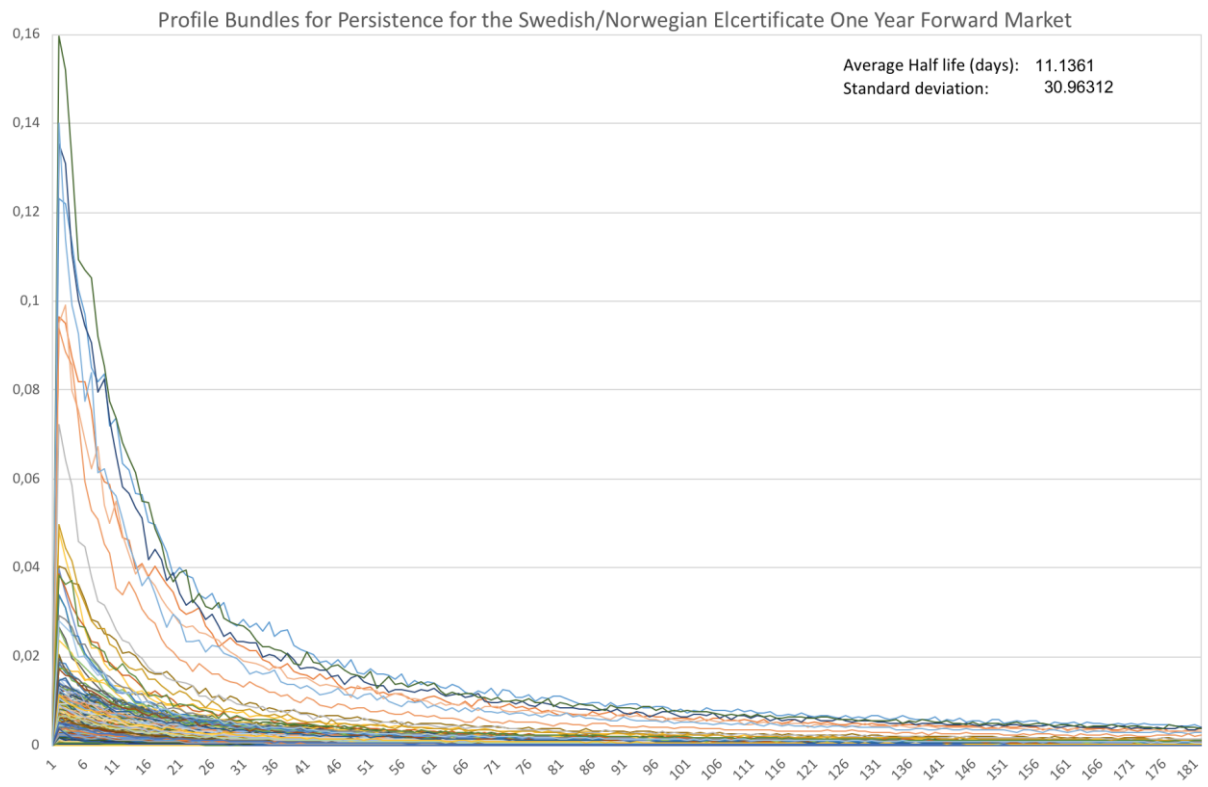


Figure 35; Profile bundles for Persistence, One-Year Forward Price.

The mean reversion time is short for the volatility but has a high standard derivation.