Master's degree thesis

LOG950 Logistics

Speed Optimization in a Maritime Inventory Routing Problem

Line Eide

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Preface

This thesis fulfills the requirements for the Master of Science in Logistics degree. It was written at Molde University College – Specialized University in Logistics. In addition, a part was written at Federal University of Minas Gerais during a four-week stay in Brazil.

The work was supervised by Professor of Quantitative Logistics of Molde University College (Norway) Lars Magnus Hvattum and Professor of Computer Science Department of Federal University of Minas Gerais (Brazil) Sebastián Alberto Urrutia.

The authors would like to thank Lars Magnus Hvattum and Sebastián Alberto Urrutia for the helpful feedback during the semester.

Summary

This thesis focuses on a maritime inventory routing problem (MIRP) that seeks to find optimal routes for seagoing vessels between ports. The problem in this paper considers both load and speed as important factors when calculating the fuel consumption and daily sailing costs of ships. The speed and load function of the fuel consumption is non-linear. The thesis describes how a linear approach can be used to find solutions to the problem. Four models are presented, one that finds optimal routes without considering load and speed, two models with fixed routes which optimizes the speed and load, and one model that finds optimal routes when both speed and load are considered. The models are tested on different instances and compared to each other. The solutions show that the speed and load does have an impact on the selection of routes in Maritime inventory routing problems (MIRP).

Contents

1. Introduction

Maritime transport is important when it comes to trade and development. According to the Review of Maritime Transport (2017), seaborn trade stood for over 80 % of global trade by volume and over 70 % by value. On specific routes, seaborn trade can compete with rail- and road transportation when it comes to accessibility, time, cost, speed, and other constraints. In other routes it is the single link between regions and continents.

As there has been a strong growth in world trade and development, the need for higher speeds in shipping grew. According to Psaraftis and Kontovas (2013), the increase in speed was made possible by technological advances in hull design, hydrodynamic performance of vessels, engine and propulsion efficiency, to mention a few. However, increasing fuel prices, depressed market conditions and the rising focus on environmental issues has shifted the attention over on the disadvantages of high speed. Due to the non-linear dependency between speed and fuel consumption, high vessel speed leads to an increase in the fuel consumption, and hence the total cost of cargo deliveries, and in air emissions (Psaraftis and Kontovas, 2013). As a result, the usage of "slow steaming" has increased. According to Psaraftis and Kontovas (2013), slow steaming is to go slower than the designed speed of a vessel and hence lead to a reduction in air emissions. Furthermore, speed reduction affects the total cost of transport, which shows that high speed is not always the best solution when it comes to optimization of delivery costs.

Further, maritime transport is characterized by high level of uncertainty. This can be uncertain connected to weather conditions, port congestions, mechanical problems at ports, among others. In cases of uncertain weather conditions, the vessels might need to reduce the speed, which will affect the sailing time. This might lead to deviations in the initial timetable. However, this deviation can be retrieved on later stretches by increasing the speed. In addition, companies have individual inventory policies. These policies may differ in capacity, type of product, storage time, and several other. As maritime transport is conducted on different distances, the uncertainty associated with the sailing time increases the importance of inventory routing. In the cases where the sailing time is affected, the inventory can be used as a buffer to prevent stockouts. Moreover, the speed of a vessel can be increased in the uncertain situations to make the policies more flexible.

The relationship between speed and fuel consumption is a topic which has been studied a lot. However, there is little research on the impacts which the load of the vessels may have on the fuel consumption, and hence the total transport costs. As there are different sized vessels, they also differ in weight according to the load they are transporting. It is therefore interesting to see if the load in combination with speed has a greater effect on the fuel consumption than speed alone.

In this thesis, Maritime Inventory Routing Problem (MIRP) with speed optimization is studied. A MIRP is used to find optimal delivery routes between producers and consumers and at the same time maintain a reasonable inventory level. The main objective of this thesis is to minimize the total costs, which includes transportation and operation costs for each vessel in the routes they conduct. To be able to achieve this, speed and load is introduced as key variables into the existing optimization tool.

This thesis considers four different models. The first model finds routes for each vessel, where each vessel is running at a fixed speed. In the second model, the routes created by the first model are taken as fixed and used along a speed choice between minimum and maximum for each vessel. The third model builds further on the second model and introduces a minimum and maximum level of load. The fourth model provides optimal routes, speeds, and loads for each vessel at once. One of the main assumptions in model 2, 3 and 4 is the non-linear dependency between fuel cost and speed. According to Andersson et al. (2015), the fuel consumption, and thus also the sailing cost, has a non-linear dependency of the speed of the vessel. This thesis examines how the changes introduced in the models influences the total costs.

The thesis is structured in the following way: chapter 2 represents literature review which is relevant to this research topic and includes different solution methods. Chapter 3 describes the problem itself. Chapter 4 includes the mathematical models as well as relevant definitions. Chapter 5 represent a computational evaluation of the models. Chapter 6 describes the conclusion.

2. Literature review

The problem considered in this thesis can be described as a maritime inventory routing problem, where the fuel consumption, and hence the costs, is minimized by optimizing the speed and load of the vessels. This chapter contains information about maritime inventory routing problems (MIRPs) and speed optimization problems. In addition, information about inventory routing problems (IRPs) is added to provide a broader theoretical understanding of MIRPs. Hence, articles which describe modeling approaches and various models for IRP and MIRP are relevant for this research. Articles which describe different strategies for speed optimization are also relevant. Some research papers have been used as a source of primary data, and other as a source of fuel consumption functions.

2.1 Inventory routing problem

According to Coelho et al. (2014), the inventory routing problem (IRP) integrates inventory management, vehicle routing and delivery scheduling decisions. The motivation for this integration has to do with the optimization of resources. One must know how much capacity both the vehicle and customer has available to be able to optimize the delivery schedules. In recent years, exact algorithms, and several powerful metaheuristics and matheuristic approached has been developed to solve these problems. Coelho et al. (2014) presents a survey within IRP literature, which is based on a new classification of the problem, where the comparison of the literature is based on the structure of the problem and the solution time.

When the IRP is solved, a business practice called vendor managed inventory replenishment (VMI) can be implemented. According to Kleywegt et al. (2002), VMI refers to a situation where a central decision maker (vendor) controls the replenishment of inventory at a number of locations. IRPs are hard to solve, especially with many customers involved, however, if they are able to solve the IRP they can manage to save costs both in inventory and transportation. Kleywegt et al. (2002) formulates the IRP as a Markov decision process and uses approximation methods to find good solutions.

Bertazzi and Speranza (2012) studied the IRPs, where they classify the characteristics and presents different models and policies for problems where the crucial decision is when to serve customers. This problem involves using capacitated vehicles and direct shipping, to ship products from a supplier to a customer, in terms of IRPs. Bertazzi and Speranza (2012) also presents an overview of some pioneering papers of IRPs, and literature that concerns direct shipping problems. In addition, they cite surveys that focuses on IRP decisions over time and space.

Zaitseva (2017) worked on introducing profit maximization in IRPs, with an insight into how the models formulated can increase possibilities and help companies to make better decisions in planning aspects. This problem involves using piecewise linear approximation.

As mentioned in Coelho et al. (2014), the IRP dates back over 30 years, till 1983 when Bell et al (1983) presented a study integrating inventory management, vehicle routing, and delivery scheduling decisions. The earlier papers which Coelho et al. (2014) mentioned have applied simple heuristics to simplified versions of the IRP and the papers are as follows: Blumenfeld et al. (1985) studied trade-offs between transportation, inventory, and production set-up costs to be able to determine optimal shipping strategies on freight networks. Burns et al. (1985) examined trade-offs between transportation and inventory, based on approximate routing costs. Dror et al. (1985) described an assignment heuristic over a short planning period. Dror and Levy (1986) examines an interchanging algorithm, where three heuristic route improvement schemes are used to improve a given solution to a vehicle routing problem (VRP). Anily and Federgruen (1990) studies clustering heuristics to determine feasible replenishment strategies minimizing routing- and inventory costs.

The recent paper which Coelho et al. (2014) mentioned are able to obtain high-quality solutions to difficult optimization problems, and they rely on the concept of metaheuristics. The papers are as follows: Ribeiro and Lourenço (2003) proposed a heuristic methodology based on the iterated local search to solve the multi-period inventory routing problem with stochastic and deterministic demand. Campbell and Savelsbergh (2004) introduced a greedy randomized adaptive search for large-scale real-life instances. Zhao et al. (2008) used variable neighborhood search to solve an integrated inventory and routing problem in a three-echelon logistic system. Boudia and Prins (2009) proposed a memetic algorithm to solve an NP-hard multi-period production-distribution problem to minimize the sum of costs associated with production setups, inventories, and distribution. Archetti et al. (2012) introduced a tabu search scheme to solve an IRP in discrete time where a supplier has to serve a set of customers over a multiperiod horizon. Coelho et al. (2012) proposed an adaptive large neighborhood search heuristic to solve the inventory routing problem with transshipment (IRPT).

In addition, Coelho et al. (2014) mentions the work of Chien et al. (1989) who proposed a construction and improvement heuristic, with a heterogenous fleet; Campbell et al. (1998) who proposed a two-phase heuristic based on a linear programming model; Bertazzi et al.

(2002) who proposed a fast-local search algorithm for the single-vehicle case in which an order-up-to level (OU) inventory policy is applied.

2.2 Maritime inventory routing problem

In this part of the thesis, solution methods for maritime inventory routing problems are presented. According to Song and Furman (2013) the IRP turns into a maritime inventory routing problem (MIRP) when the transportation is carried out by a seagoing vessel. They present flexible modeling and algorithmic framework as a solution method to this type of problems. Further, the paper includes a case study on a practical MIRP, which shows that the mentioned modeling and algorithmic framework is flexible and effective enough to be a choice for practical IRP.

Agra et al. (2015) write about a stochastic short sea shipping problem, where the port times and sailing times are considered as stochastic parameters. The company presented is responsible for both distribution between the isles and inventory management at the ports. Ship routing and scheduling is related with unstable weather conditions and unpredictable waiting times at ports. They use a two-stage stochastic programming model where the first stage includes decisions related to routing, loading and unloading, and the second stage consist of decisions related to scheduling and inventory.

Agra et al. (2016a) studied a single product MIRP where the production and consumption rates were constant over the planning horizon. The problem that is presented contains a heterogenous fleet of vessels and several production and consumption ports with limited storage capacity. As also mentioned in Agra et al. (2015), the weather conditions are uncertain, and this effects the sailing times. Hence, the travel time between the ports is assumed to be random and it follows a log-logistic distribution. The authors proposed a two-stage stochastic programming problem with recourse to be able to deal with the random sailing times. Further, they developed a MIP based local search heuristic to be able to solve the problem.

Agra et al. (2016b) considered a single product robust MIRP with constant production and consumption rates, a heterogenous fleet and multiple consumption and production ports with capacities regarding storage. The article introduces a robust model to a MIRP and presents a two-stage decomposition algorithm. Further, the authors introduce several improvement strategies for the decomposition procedure.

De et al. (2017) also studied a MIRP with demand at different ports during the planning horizon. The usage of slow steaming policy within ship routing is explored as a possibility.

To consider constraints like various scheduling and routing, loading/unloading and vessel capacity, the authors presented a mixed integer non-linear programming model. A non-linear equation is used to capture the sustainability aspects between fuel consumption and vessel speed. Further, they include several time window constraints to enhance the service level at each port, and in addition to this they have penalty costs associated with vessels arriving to early or finishes too late according to the time windows. To solve the problems in the paper they have used an effective search heuristic named Particle Swarm Optimization for Composite Particle (PSO-CP).

Andersson et al. (2010) writes about industrial aspects of combined inventory management and routing problems. They describe the current industrial practice and gives examples of when inventory management and routing can be combined. In addition, they present a classification and comprehensive literature survey of around 90 papers that focuses on IRP and MIRP.

2.3 Speed optimization problem

This thesis looks at previous articles on speed optimization problems to provide information on different strategies and how speed optimization works. Fuel consumption is a non-linear function of the vessel speed, which means that an increase in the speed will lead to an even higher increase in fuel consumption. One strategy to lower fuel consumption is to slow steam, however, if the demand is high enough this might lead to an increase in the fleet size.

Norstad et al. (2011) state that in traditional ship routing and scheduling problems, the speed of the vessels is fixed and that fuel consumption rate for each vessel is given. However, in real life the speed of a vessel will vary in different intervals and fuel consumption per time unit can be approximated by a cubic function of speed. Further they present the tramp ship routing and scheduling problem with speed optimization, where the speed of the vessels in different intervals will be characterized as a decision variable. To be able to solve this problem a multi-start local search heuristic is applied.

According to Andersson et al. (2015) it is common to use a sequential approach when planning shipping routes, where firstly each vessel sails with a given speed, and then later optimize the sailing speeds along the routes based on the execution of the routes. This article proposes a new modeling approach for integrating speed optimization in the planning of shipping routes and uses a rolling horizon heuristic to solve the combined problem. Further, the article considers a real deployment and routing problem in roll-on roll-off (RoRo)-shipping.

Wen et al. (2016) write about simultaneous optimization of routing and sailing speed in full-shipload tramp shipping. The article presents a heuristic that can find good solutions in short time. The problem consists of different cargos that needs to be transported from load ports directly to discharge ports. There is a heterogenous fleet of vessels, which have different speed ranges and load-dependent fuel consumption. The authors present solutions that determines which cargo to pick up, which route each vessel should follow, and the speed the vessels should have on each leg to maximize the profit. To find a solution, the paper presents a three-index formulation and a set packing formulation. Then, a branch-and-price algorithm is proposed, implemented and tested.

Psaraftis and Kontovas (2013) present a survey and a taxonomy of models in maritime transportation, where speed is one of the decision variables. The article discusses pros and cons of reducing the speed of vessels. These pros and cons are related to both costs and emissions. Different fuel consumption functions are described. In addition, the authors give examples on how the inventory costs can impact the speed. The taxonomy of the different models is based on predefined parameters, such as optimization criterion, and whether the model can find the optimal speed as a function of payload or not.

Psaraftis and Kontovas (2014) focus on clarifying issues in general speed optimization problems in maritime inventory routing, and then presents models that optimize the speed for different routing scenarios, for a single vessel. The article incorporates fundamental parameters and considerations, like fuel price, freight rate, inventory cost of the cargo and the dependency of fuel consumption on payload in the models. The authors also consider the difference between solutions that optimizes the economic performance, and the ones that optimize the environmental performance.

Bialystocki and Konovessis (2016) suggest an approach for constructing an accurate fuel consumption and speed curve. In the article, different factors that can affect the fuel consumption is presented and taken into consideration. An algorithm is introduced and is proven to be both simple and accurate when estimating the fuel consumption.

Evsikova (2017) worked on speed optimization in maritime inventory routing, with an insight into how the speed of the vessel may affect cost savings for different size of problems and emissions reduction for problems of a large size.

This thesis studies a MIRP with speed and load optimization, to see how the introduction of load as a decision variable influences the total sailing costs in comparison to only speed optimization and route planning with fixed speed and load. This thesis can be compared to the article by Wen et al. (2016), since both considers speed and load optimization, however, Wen et al. (2016) only considers full-shipload. To be able to optimize load, this thesis builds on a fuel consumption function presented in the article by Psaraftis and Kontovas (2014).

3. Problem description

This thesis describes how load and speed influences the cost function of a maritime inventory routing problem. Further, it considers a geographical region where maritime transportation of a single product takes place. The transportation is carried out by a heterogenous fleet of vessels, which differ in size, capacity, and cost. Travelling distances are included into the problem. There are several ports, which are divided between consumption and production facilities. None of the ports are able to have both consumption and production. The production facilities have fixed production rates, while the consumption facilities have fixed consumption rates.

Both the consumption and the production facilities have inventories and hence each port has storage facilities with fixed lower and upper limits. The production facilities are not allowed to exceed the maximum storage level, while the consumption ports are not allowed to have shortages. Further, each port can be visited multiple times by different vessels in the planning horizon depending on the size of storage and the number of products to be loaded or unloaded. Each vessel is given a starting location and executes its route in the best possible way. Throughout the routes, each vessel transports different loads between ports in accordance with the demand and can use different operating speeds during the execution of the routes.

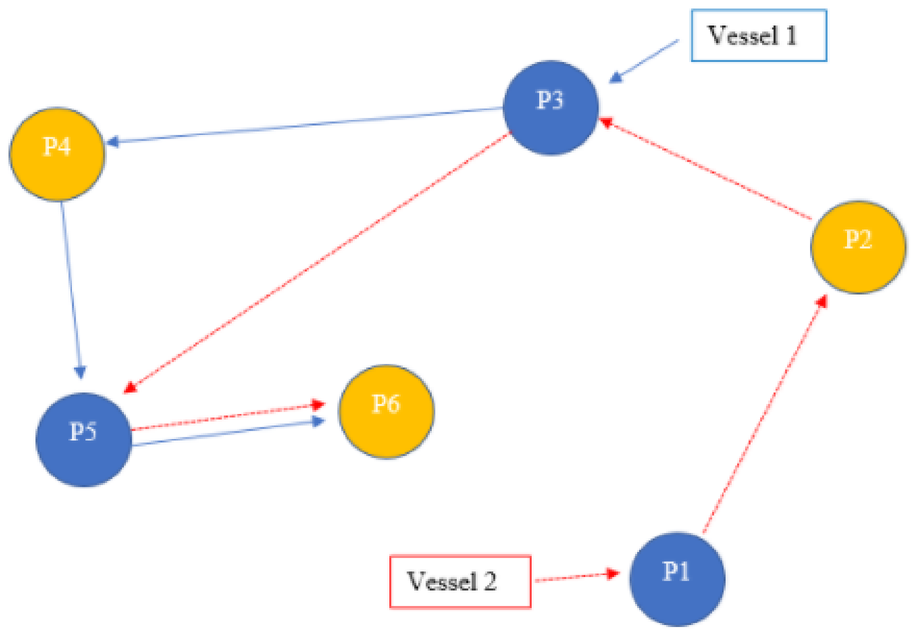

Figure 1: Description of Maritime Inventory Routing Problem. Source: Made by the authors

Figure 1 shows how two vessels travel their routes. The blue ports are production ports, and the orange ports are consumption ports. The first vessel, which is blue, goes to port 3 to fill up before it sails to port 4. Here it unloads before sailing to port 5 to fill up again. From port 5 it sails to port 6 to unload and ends its journey. The solid blue line shows this route. Another vessel, which is red, goes to port 1 to fill up before it sails to port 2 to unload. From here it sails to port 3 to partially fill up and continues to port 5. Here it also gets partially filled up, before it sails to port 6 to unload and ends its journey. The red dashed line shows this route.

This thesis focuses on the minimization of the sum of traveling costs between ports depending on the chosen speed and load, and operational costs in each port. Each port is located with different distances between each other, different demand rates and individual operational costs. The vessels can operate with different speeds and loads. In addition, the vessels have different sailing costs which will be reviewed in the next chapter.

The vessels have different properties. They differ in size to better meet the various demands. In this thesis, the size of the vessels is categorized by deadweight tonnage (DWT), in other words how much the vessels can transport. In addition, the vessels operate with different speeds which differs in knots. The operating speed of the vessels depends on the size of the vessel, where the larger vessels operate with higher speeds. Furthermore, the vessels have different load rates, which also is dependent on the size of the vessels.

The main objective is to minimize the total routing costs, which includes transportation and operating costs for each vessel in the routes they conduct. According to Andersson et al. (2015), the fuel consumption, and thus also the sailing cost, has a non-linear dependency of the speed of the vessel. They approximated the non-linear fuel consumption function by using three speed alternatives and combines these to find linear overestimations of the consumption. Figure 2 shows this overestimation. There is a certain minimum speed that the vessel must perform due to safety reasons, and to run at a minimum cost. In addition, there is a maximum speed that can be achieved when there are perfect weather conditions. This is a theoretical upper limit, and the highest speed alternative should be slightly lower.

Bialystocki and Konovessis (2016) present a regression formula with two constant values that shows the non-linear relationship between speed and fuel consumption. The function of the fuel consumption is equal to $0.1727\mu^2 - 0.217\mu$, where μ is the speed of the vessel. This function is illustrated in figure 3. This function is independent of load; thus, the load is a given constant. As figure 2 shows, the fuel consumption curve is convex. Therefore, the linearization of the curve will be an overestimation of the fuel consumption.

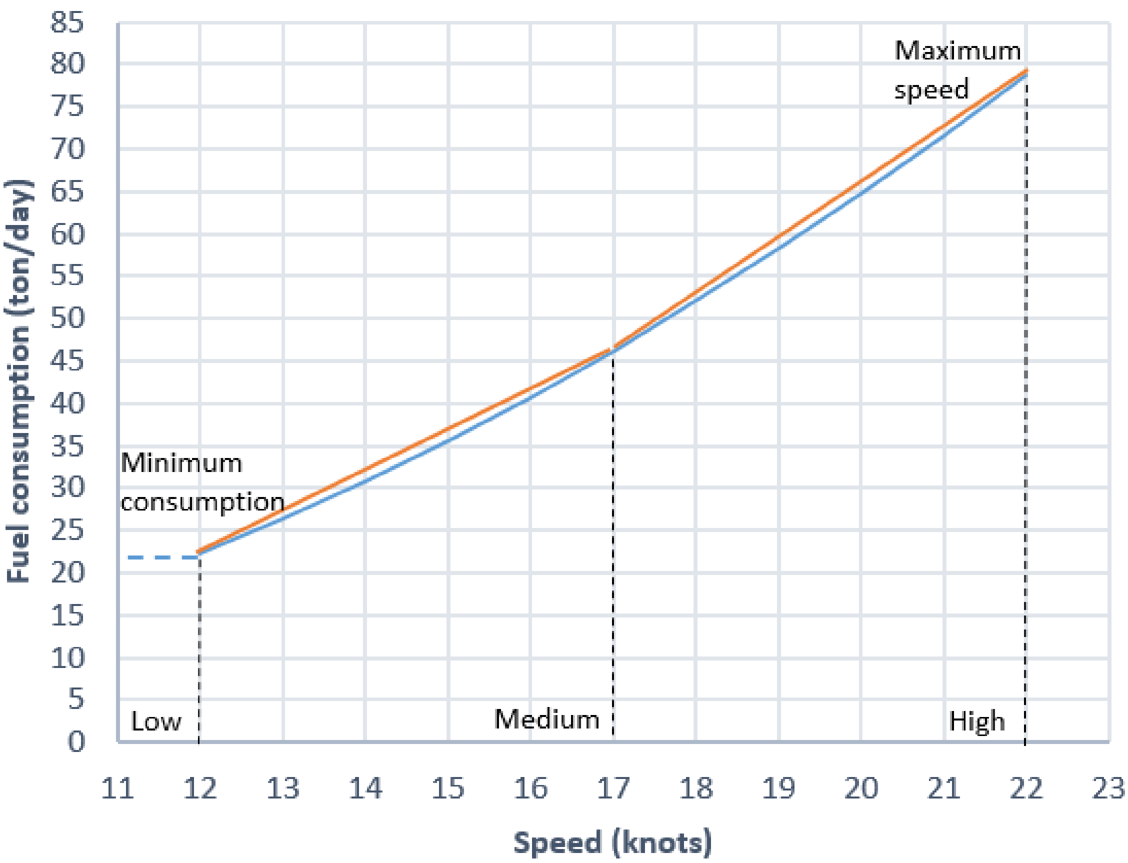

Figure 2: The non-linear relationship between speed and fuel consumption. Source: Made by the authors based on Bialystocki and Konovessies (2016), and Andersson et al. (2015)).

According to Andersson et al. (2015), there is an additional overestimation when it comes to the non-linear dependency between time and speed. Hence, there is also an overestimation when it comes to the travel time. The approach is illustrated in Figure 3. This graph is built on the same speed curve as for Figure 2. As we see in Figure 3, a selected speed from the speed options will result in a higher travel time than the actual time being used. Higher speed reduces traveling time and *vice versa*.

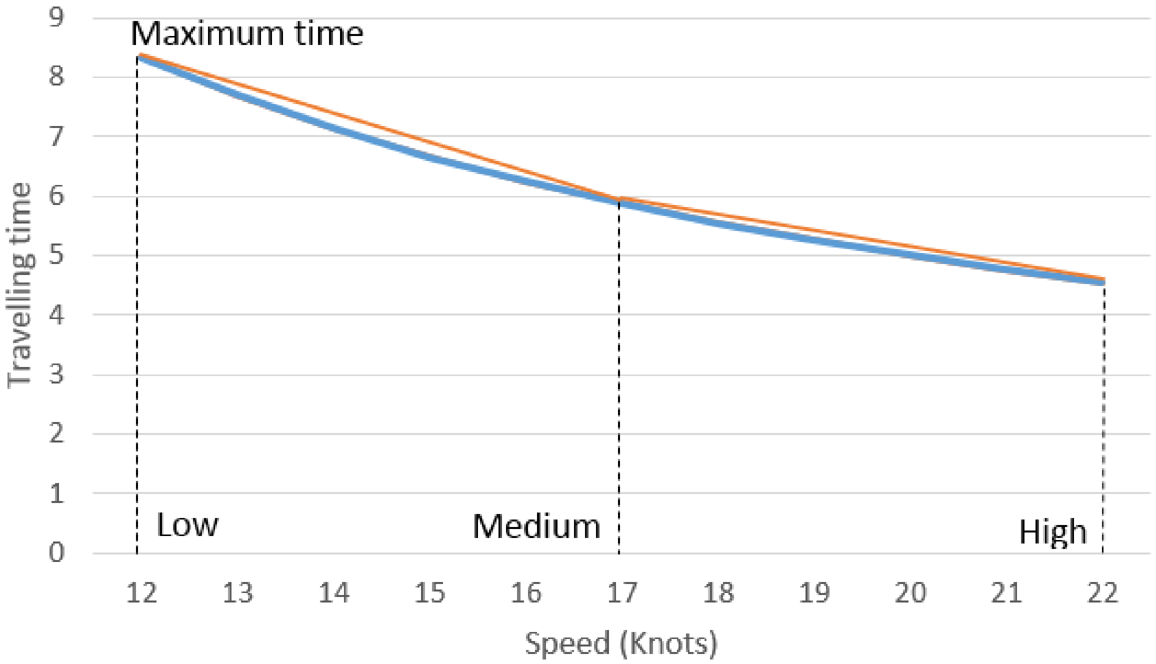

Figure 3: The non-linear function of travel time as a function of speed. Source: Made by the authors based on Bialystocki and Konovessies (2016), and Andersson, Fagerholt, and Hobbesland (2015)).

According to Psaraftis and Kontovas (2014), the fuel consumption has a non-linear dependency on both the speed and the load of a vessel. The daily fuel consumption is equal to $FC = k\mu^3(l + A)^{2/3}$, where k is a given constant, μ is the speed of the vessel, l is the payload and A is the "lightship weight", or the weight of the vessel when it has no load except fuel. The daily sailing costs from this fuel consumption formula is illustrated in the graph in Figure 4, for a single vessel with five different load levels. The part of the formula that is related to speed generates a convex function, illustrated by the green lines in Figure 4. The part of the formula that are dependent on load, generates a concave function, which is illustrated by the orange lines in the figure.

Linear approximation of a convex function will give an overestimate of the costs, while linear approximation of a concave function will give an underestimate of the costs.

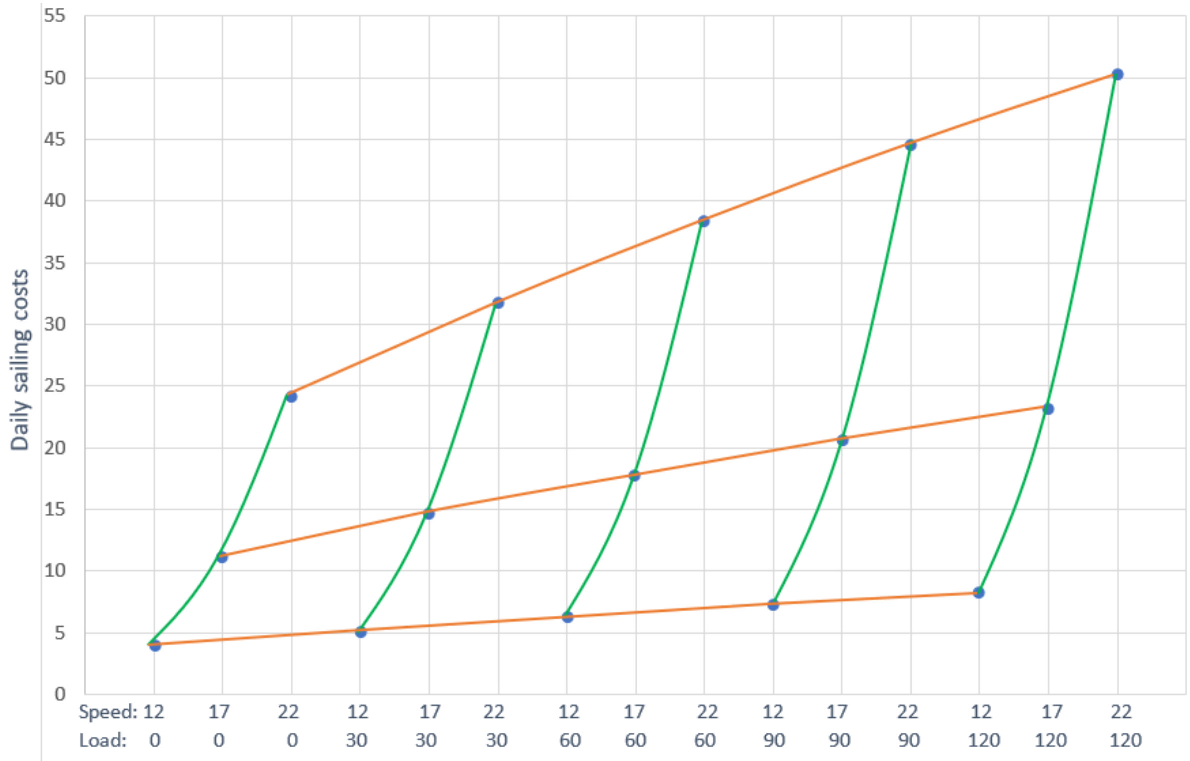

Figure 4: The relationship between speed, load, and daily sailing costs of a vessel. Source: Made by the authors based on Psaraftis and Kontovas (2014).

4. Mathematical models

The maritime inventory routing problem is modeled in a similar way as in Evsikova (2017), and with the same notations. Model 1 is presented first. Model 2 is presented second. Model 3 is presented third, and model 4 is presented last.

4.1 Introduction to models

In this section the thesis focuses on how, for each model, different assumptions on load and speed are handled. Mathematical formulations are introduced and considered in the first model. The maritime inventory routing problem presented is created in the same way as in Evsikova (2017) and uses the same notations. Speed optimization is introduced in a similar way as in Andersson et al. (2015).

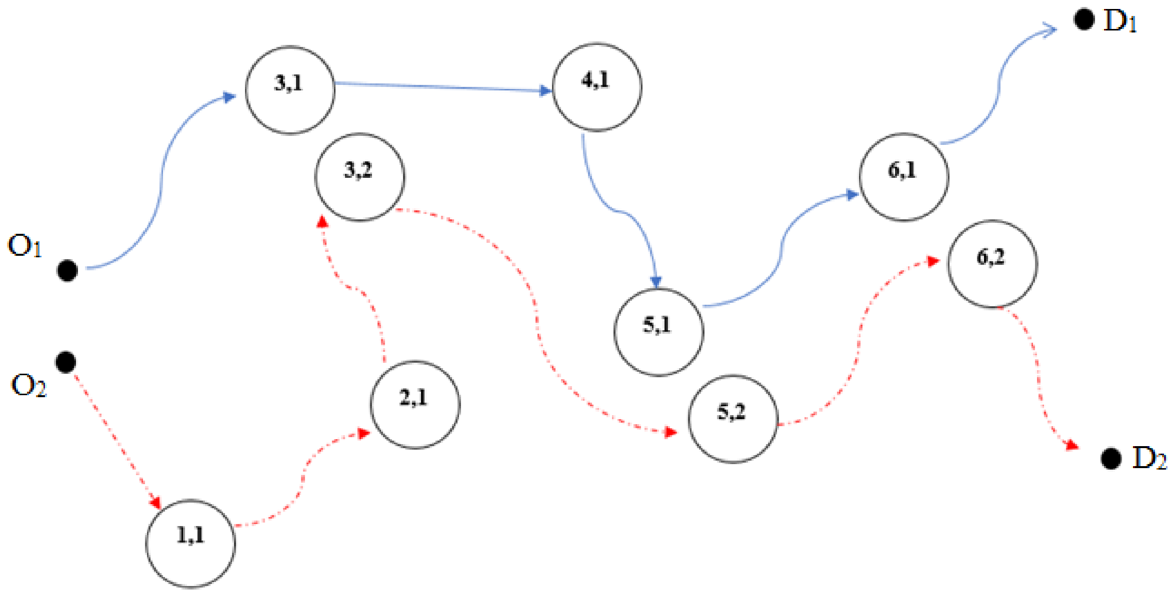

Figure 5: Example of ship routes. Source: Made by the authors based on Agra et al. (2015).

In the models, V indicates a set of vessels and N indicates a set of ports. Each vessel $v \in V$ has its starting point, which can be a point at sea. Each port can have multiple visits during the time horizon. Nodes in a network is indicated by a pair *(i, m)*, where *i* is a port and *m* is the visit number. If there are arcs of direct movement from node *(i, m)* to node *(j, n)*, it is denoted as (*i, m, j, n)*.

Figure 5 shows the origins and destinations of two different vessels, and how they move from port to port, as also seen in Figure 1. Vessel 1 starts from origin O_1 and goes to port 3 for the first visit, then sails to port 4 for the first visit, continues to port 5 for the first visit, followed by port 6 for the first visit, and ends up in destination D_1, as the route is completed. The blue line shows this. Vessel 2 starts from origin O_2 and sails to port 1 for the first visit, continues to port 2 for the first visit, continues to port 3 for the second visit, sails further to port 5 for the second visit, followed by port 6 for the second visit, and ends up in destination D_2, as the route is completed. The red dashed line shows this.

The first model optimizes routes and has speed of the vessels as a fixed parameter. The speed is set to the maximum speed of each vessel. Each of the vessels has a specific daily sailing cost. The daily sailing costs of each ship in model 1 is a parameter and is calculated using the formula of Psaraftis and Kontovas (2014). As this formula considers load, the costs are calculated as if the ships sail with a load level equal to the total capacity.

In the second model, the routes which are given by model 1 are used as input parameters. In addition, the model includes a set of speeds and speed variables. The input data includes 3 different speeds for each vessel, which is the same speed intervals as presented in Evsikova (2017). Evsikova (2017) defined two of three speeds according to corresponding ranges for vessels with general cargo and different capacities. The lowest speed was calculated according to the slow steaming policy.

In the third model, two speed- and load-variables are introduced. The first one is g^0_{imvls}, which is an auxiliary variable used to determine the speed and load on the route between the origin and the port; the second is $g_{imjnvls}$, which is an auxiliary variable used to determine the speed and load along the route between ports. These variables can be equal to zero, one or a fractional value, as presented in figure 6. If it is zero, the given speed and load is not chosen, but if it is one it is chosen. It will be a fractional value if a middle speed and load is chosen.

In the fourth and last model, routes, speed and load are optimized at the same time by combining model 1 and model 3.

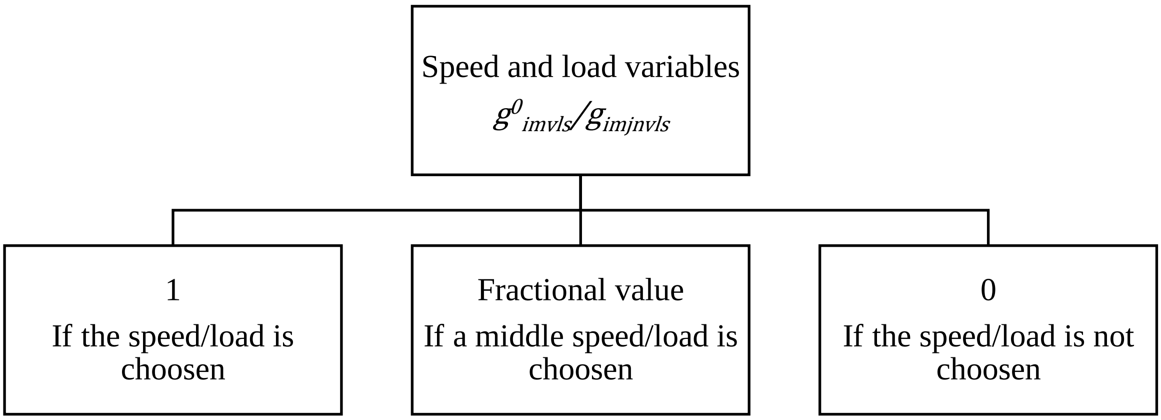

Figure 6: Explanation of the speed and load variables. Source: Made by the authors

4.2 Model 1. Maritime inventory routing problem with fixed speed

The first model is deterministic and finds routes for each vessel between ports by focusing on minimizing the sum of traveling costs between ports and operational costs in each port. The model assumes that each vessel is running at a fixed speed.

Notation:

Set:

V : set of vessels

N : set of production and consumption ports

S^A : set of possible nodes *(i, m)*

S_v^A : set of nodes that can be visited by vessel *v*

S_v^X : set of all movements *(i, m, j, n)* of vessel *v*

Variables:

x_{imjnv} : 1 if vessel $v \in V$ moves directly between nodes (i, m) and (j, n),

0 otherwise, $\quad v \in V, \quad (i, m, j, n) \in S_v^X$

x_{imv}^0 : 1 if vessel *v* departs from its initial position to node (i, m),

0 otherwise, $\quad v \in V, \quad (i, m) \in S_v^A$

z_{imv} : 1 if vessel *v* finishes its route at node (i, m),

0 otherwise, $\quad v \in V, \quad (i, m) \in S_v^A$

z_v^0 : 1 if origin of vessel *v* is the same as destination,

0 otherwise, $\quad v \in V$

q_{imv} : the amount of product loaded/unloaded from vessel *v* at node (i, m),

$v \in V, \quad (i, m) \in S_v^A$

f_{imjnv} : the amount of product that vessel *v* transports from node (i, m) to node (j, n),

$v \in V, \quad (i, m, j, n) \in S_v^X$

f_{imv}^0 : the amount of product that vessel *v* transports from the origin to node (i, m),

$v \in V, \quad (i, m) \in S_v^A$

f_{imv}^d : the amount of product that vessel *v* transports from the origin to destination node (i, m) $\ v \in V, \quad (i, m) \in S_v^A$

t_{im} : starting time of the m^{th} visit to port i,

$(i, m) \in S_v^A$

o_{imv} : 1 if vessel v operates in port (i, m),

0 otherwise, $v \in V$, $(i, m) \in S_v^A$

y_{im} : 1 if there is a visit (i, m) to port i,

0 otherwise, $i \in N$, $(i, m) \in S^A$

s_{im} : stock levels at ports at the start of the visit m, to port i,

$(i, m) \in S^A$

Parameters:

T : number of time units in the planning horizon

H_i : minimum number of visits to port $i \in N$

M_i : maximum number of visits to port $i \in N$

D_i : consumption or demand at port $i \in N$

J_i : 1 if production facilities are located in port i, -1 if consumption facilities are located in port i, $i \in N$

P_{iv} : port cost at port $i \in N$ for vessel $v \in V$

C_v : capacity of vessel $v \in V$

L_v : initial load onboard vessel $v \in V$ when leaving port i

$\underline{S_i}$: lower bound on the inventory level at port $i \in N$

$\overline{S}_i$: upper bound on the inventory level at port $i \in N$

S_i^0 : the initial stock level in port $i \in N$ at the beginning of the planning horizon

A_{im} : earliest time for starting visit m to port i, $(i, m) \in S^A$

B_{im} : latest time for starting visit m to port i, $(i, m) \in S^A$

K_i : minimum time between two consecutive visits to port $i \in N$

$\underline{Q_i}$: minimum load/unload quantity in port $i \in N$

U_{im} : latest time for finishing visit m to port i, $(i, m) \in S^A$

T_v^Q : time for unloading/loading each unit by vessel $v \in V$

C_{ijv}^{PP} : sailing cost from port $i \in N$ to port $j \in N$ with vessel $v \in V$

C_{iv}^{OP} : sailing cost from origin to port $i \in N$ by vessel $v \in V$

T_{ijv}^{PP} : time required by vessel $v \in V$ to sail from port $i \in N$ to port $j \in N$

T_{iv}^{OP} : time required by vessel $v \in V$ to sail from its origin to port $i \in N$

Formulation:

$$\min \sum_{v\in V} \sum_{(i,m,j,n)\in S_v^X} C_{ijv}^{PP} x_{imjnv} + \sum_{v\in V} \sum_{(i,m)\in S_v^A} C_{iv}^{OP} x_{imv}^0 + \sum_{v\in V} \sum_{(i,m)\in S_v^A} P_{iv} o_{imv} \qquad (1)$$

subject to

$$\sum_{(j,n)\in S_v^A} x_{jnv}^0 + z_v^o = 1 \qquad \forall\ v \in V \qquad (2)$$

$$o_{imv} - \sum_{(j,n,i,m)\in S_v^x} x_{jnimv} - x_{imv}^o = 0 \qquad \forall\ v \in V,\ \ (i,m) \in S_v^A \qquad (3)$$

$$o_{imv} - \sum_{(i,m,j,n)\in S_v^x} x_{imjnv} - z_{imv} = 0 \qquad \forall\ v \in V,\ \ (i,m) \in S_v^A \qquad (4)$$

$$\sum_{v\in V} o_{imv} = y_{im} \qquad \forall\ (i,m) \in S^A \qquad (5)$$

$$y_{i(m-1)} - y_{im} \geq 0 \qquad \forall\ (i,m) \in S^A : H_i + 1 \leq m \leq M_i \qquad (6)$$

$$y_{im} = 1 \qquad \forall\ (i,m) \in S^A :\ m \in \{1 \dots H_i\} \qquad (7)$$

$$q_{imv} \leq \min\{C_v, \overline{S}_i\} o_{imv} \qquad \forall\ v \in V,\ \ (i,m) \in S_v^A \qquad (8)$$

$$\underline{Q}_i o_{imv} \leq q_{imv} \qquad \forall\ v \in V,\ \ (i,m) \in S_v^A \qquad (9)$$

$$f_{imv}^o = L_v x_{imv}^o \qquad \forall\ v \in V,\ \ (i,m) \in S_v^A \qquad (10)$$

$$f_{jnv}^o + \sum_{(i,m,j,n)\in S_v^X} f_{imjnv} + J_i q_{jnv} = \sum_{(j,n,i,m)\in S_v^X} f_{jnimv} + f_{jnv}^d \qquad \forall\ v \in V,\ \ (j,n) \in S_v^A \qquad (11)$$

$$f_{imjnv} \leq C_v x_{imjnv} \qquad \forall\ v \in V,\ \ (i,m),(j,n) \in S_v^A \qquad (12)$$

$$f_{jnv}^d \leq C_v z_{jnv} \qquad \forall\ v \in V,\ \ (j,n) \in S_v^A \qquad (13)$$

$$t_{im} + \sum_{v\in V} T_v^Q q_{imv} - t_{jn} + \sum_{v\in V} \max\{U_{im} + T_{ijv}^{PP} - A_{jn}, 0\}\ x_{imjnv} \leq U_{im} - A_{jn} \qquad (i,m),(j,n) \in S^A \qquad (14)$$

$$t_{im} - t_{i(m-1)} - \sum_{v\in V} T_v^Q q_{i(m-1)v} - K_i y_{im} \geq 0 \qquad \forall\ (i,m) \in S_v^A : m > 1 \qquad (15)$$

$$\sum_{v\in V} T_{iv}^{OP} x_{imv}^o \leq t_{im} \qquad \forall\ (i,m) \in S^A \qquad (16)$$

$$t_{im} \geq A_{im} \qquad \forall\ (i,m) \in S^A \qquad (17)$$

$$t_{im} \leq B_{im} \qquad \forall\ (i,m) \in S^A \qquad (18)$$

$$s_{i1} = S_i^o + J_i D_i t_{i1} \qquad \forall\ i \in N \qquad (19)$$

$$s_{im} = s_{i(m-1)} - J_i \sum_{v\in V} q_{i(m-1)v} + J_i D_i (t_{im} - t_{i(m-1)}) \qquad \forall\ (i,m) \in S^A : m > 1 \qquad (20)$$

$$s_{im} + \sum_{v\in V} q_{imv} - D_i \sum_{v\in V} T_v^Q q_{imv} \leq \overline{S}_i \qquad \forall\ (i,m) \in S^A : J_i = -1 \qquad (21)$$

$s_{im} - \sum_{v \in V} q_{imv} + D_i \sum_{v \in V} T_v^Q q_{imv} \geq \underline{S}_i$	$\forall\ (i,m) \in S^A : J_i = 1$	(22)
$s_{iM_i} + \sum_{v \in V} q_{iM_i v} - D_i(T - t_{iM_i)}) \geq \underline{S}_i$	$\forall\ i \in N : J_i = -1$	(23)
$s_{iM_i} - \sum_{v \in V} q_{iM_i v} + D_i(T - t_{iM_i)}) \leq \overline{S}_i$	$\forall\ i \in N : J_i = 1$	(24)
$s_{im} \geq \underline{S}_i$	$\forall\ (i,m) \in S^A : J_i = -1$	(25)
$s_{im} \leq \overline{S}_i$	$\forall\ (i,m) \in S^A : J_i = 1$	(26)
$x_{imjnv} \in \{0,1\}$	$\forall\ v \in V,\ \ (i,m,j,n) \in S_v^X$	(27)
$x_{imv}^o \in \{0,1\}$	$\forall\ v \in V,\ \ (i,m) \in S_v^A$	(28)
$o_{imv} \in \{0,1\}$	$\forall\ v \in V,\ \ (i,m) \in S_v^A$	(29)
$z_{imv} \in \{0,1\}$	$\forall\ v \in V,\ \ (i,m) \in S_v^A$	(30)
$z_v^o \in \{0,1\}$	$\forall\ v \in V$	(31)
$y_{im} \in \{0,1\}$	$(i,m) \in S^A$	(32)
$q_{imv} \geq 0$	$\forall\ v \in V,\ \ (i,m) \in S_v^A$	(33)
$f_{imjnv} \geq 0$	$\forall\ v \in V,\ \ (i,m,j,n) \in S_v^X$	(34)
$f_{imv}^o \geq 0$	$\forall\ v \in V,\ \ (i,m) \in S_v^A$	(35)
$f_{imv}^d \geq 0$	$\forall\ v \in V,\ \ (i,m) \in S_v^A$	(36)
$s_{im} \geq 0$	$\forall\ (i,m) \in S^A$	(37)
$t_{im} \geq 0$	$\forall\ (i,m) \in S^A$	(38)

The objective function (1) expresses the minimization of the sum of traveling costs between ports and the operational costs at each port. Constraints (2) – (7) is routing constraints. Constraint (2) shows that a vessel must either depart from the origin to a port or from the origin to the destination. Constraint (3) defines that if the port is visited, the vessel must either depart from the origin to the node, or from another node to the node. Constraint (4) ensures that if a vessel is at node *i*¸ it must either leave or end the route. Constraint (5) shows that a vessel can only visit node *(i, m)* if Y_{im} is equal to one. Constraint (6) guarantees that if a port *i* is visited m^{th} times, then it must also have been visited *m-1* times. Constraint (7) defines the mandatory visits for port *i*.

Constraints (8) – (9) are loading and unloading constraints. Constraint (8) ensures the quantity loaded/unloaded cannot exceed the lowest amount of vessel capacity or port capacity. Constraint (9) shows that if a vessel visits the port, then the amount loaded/unloaded should be more or equal to the minimum quantity.

Constraints (10) – (13) are arc flow constraints. Constraint (10) defines that if a vessel travels from the initial position, then the transported amount is equal to the initial load of the vessel. Constraint (11) guarantees that the amount of incoming product flow and the amount loaded/unloaded must be equal to the outgoing product flow. Constraint (12) shows that the product flow from port to port should be less or equal to the capacity of the vessel. Constraint

(13) ensures that the product flow to the destination is less or equal to the capacity of the vessel.

Constraints (14) – (18) are time constraints. Constraint (14) relates the start time associated with node *(i, m)* to the start time associated with node *(j, n)* when a vessel travels between ports. Constraint (15) impose a minimum interval between two consecutive visits at port *i*. Constraint (16) shows that the travel time for a vessel traveling from origin should not exceed the start time of the visit to the port. Constraints (17) – (18) defines time windows for the start and end time of the visits.

Constraints (19) – (26) are inventory constraints. Constraint (19) ensure the stock level at the start time of the first visit to a port. Constraint (20) shows that the stock level at the start of the m^{th} visit is connected to the stock level at the start of the previous visit.

Constraints (21) – (22) guarantees that the inventory is within the limit at the end of the visit. Constraints (23) – (24) defines upper- and lower bound on the inventory level at time T for production and consumption ports. Constraints (25) – (26) ensures that the stock level is within their limits at the start of each visit.

Constraints (27) – (32) states that the variables are binary. Constraints (33) – (38) ensures that variables are nonnegative.

4.3 Model with fuel consumption as a non-linear function of speed and load

Speed and load has a non-linear influence on the costs. This section shows the objective function and constraints of the non-linear model. In this thesis, the non-linear problem is solved by using linear approximation. Therefore, this model can be considered as an extra model, and is not used in any computational studies in this thesis.

The non-linear additions to model 1 is as follows:

Variables:

μ_{imjnv} : The speed of vessel v between nodes (i,m) and (j,n) $\quad v \in V, \ (i,m,j,n) \in S_v^X$

μ_{imv}^0 : The speed of vessel v from origin (i,m) $\quad v \in V, \ (i,m,j,n) \in S_v^X$

l_{imjnv} : The load of vessel v between nodes (i,m) and (j,n) $\quad v \in V, \ (i,m,j,n) \in S_v^X$

l_{imv}^0 : The load of vessel v from origin (i,m) $\quad v \in V, \ (i,m,j,n) \in S_v^X$

Parameters:

E_{ij} : Distance from port $i \in N$ to port $j \in N$

E_{iv}^{0} : Distance from port $i \in N$ to port $j \in N$

$\overline{\mu}_v$: Maximum speed of vessel $v \in V$

$\underline{\mu}_v$: Minimum speed of vessel $v \in V$

$\overline{l}_v$: Maximum load of vessel $v \in V$

$\underline{l}_v$: Minimum load of vessel $v \in V$

Z : Fuel cost per ton

Formulation:

$$\min \sum_{v \in V} \sum_{(i,m,j,n) \in S_v^X} x_{imjnv}\, h_v\big(\mu_{imjnv}, l_{imjnv}\big) \frac{E_{ij}}{24\mu_{imjnv}} Z + \sum_{v \in V} \sum_{(i,m) \in S_v^A} x_{imv}^0\, h_v(\mu_{imv}^0, l_{imv}^0) \frac{E_{iv}^0}{24\mu_{imv}^0} Z + \sum_{v \in V} \sum_{(i,m) \in S_v^A} P_{iv} o_{imv} \qquad (1)$$

$$t_{im} + \sum_{v \in V} T_v^Q q_{imv} - t_{jn} + \sum_{v \in V} \max\left\{U_{im} + \frac{E_{ij}}{\mu_{imjnv}} - A_{jn}, 0\right\} x_{imjnv} \le U_{im} - A_{jn} \qquad (i,m),(j,n) \in S^A \qquad (14)$$

$$\sum_{v \in V} \frac{E_{iv}^0}{\mu_{imv}^0} x_{imv}^o \le t_{im} \qquad \forall\, (i,m) \in S^A \qquad (16)$$

$$\underline{\mu}_v \le \mu_{imjnv} \le \overline{\mu}_v \qquad \forall\, v \in V,\ (i,m,j,n) \in S_v^X$$

$$\underline{\mu}_v \le s_{imv}^0 \le \overline{\mu}_v \qquad \forall\, v \in V,\ (i,m,j,n) \in S_v^X$$

$$\underline{l}_v \le l_{imjnv} \le \overline{l}_v \qquad \forall\, v \in V,\ (i,m,j,n) \in S_v^X$$

$$\underline{l}_v \le l_{imv}^0 \le \overline{l}_v \qquad \forall\, v \in V,\ (i,m,j,n) \in S_v^X$$

In the objective function (1), $h_v(\mu, l)$ is the daily fuel consumption function of the speed and load from Psaraftis and Kontovas (2014), where $h_v(\mu, l) = k_v \mu^3 (l + A_v)^{2/3}$. The last four constraints decide lower and upper bounds for the speed and load. The non-linear model can be simplified by linear approximation. According to Williams (1999), a function is separable if it is a function of a single variable, and non-separable if it is a function of more than one variable. In the non-linear model, both the objective function and constraint (14) and (16) are non-separable.

4.4 Model 2. Maritime inventory routing problem with fixed routes and fuel consumption as a function of speed

This part examines the second model, which is a special variant of model 1. In the second model, the routes created by the first model are taken as fixed and used along a speed choice between minimum and maximum for each vessel. As the speed is affecting the fuel consumption in a non-linear way, the daily sailing costs will become non-linear. To linearize the model, each vessel is given a set of speed choices with associated costs, and optimization tools will be able to provide which speed each vessel should operate with. Because of this linearization, the solution of the model will be an overestimate of the costs. The research paper of Andersson et al. (2015) was used to provide a modelling approach for speed optimization.

Notation:

Set:

S_v^S – set of speeds which can be used by vessel v

Variables:

g_{imjnvs} – auxiliary variables to determine the speed of a vessel when going from *(i, m)* to *(j, n)*, with *s* corresponding to a given choice of speed.

$v \in V \qquad s \in S_v^S \qquad (i,m,j,n) \in S_v^X$

g_{imvs}^0 – auxiliary variables to determine the speed of a vessel when going from origin to node *(i, m)*, with *s* corresponding to a given choice of speed.

$v \in V \qquad s \in S_v^S \qquad (i,m) \in S_v^A$

Parameters

x_{imjnv} : 1 if vessel $v \in V$ moves directly between nodes (i,m) and (j,n),

0 otherwise, $v \in V, \quad (i,m,j,n) \in S_v^X$

x_{imv}^0 : 1 if vessel *v* departs from its initial position to node (i,m),

0 otherwise, $v \in V, \quad (i,m) \in S_v^A$

z_{imv} : 1 if vessel v finishes its route at node (i, m),

0 otherwise, $v \in V, \quad (i, m) \in S_v^A$

z_v^0 : 1 if origin of vessel v is the same as destination,

0 otherwise, $v \in V$

o_{imv} : 1 if vessel v operates in port (i, m),

0 otherwise, $v \in V, \quad (i, m) \in S_v^A$

y_{im} : 1 if there is a visit (i, m) to port i,

0 otherwise, $i \in N, \quad (i, m) \in S^A$

C_{ijvs}^{PP} : sailing cost from port $i \in N$ to port $j \in N$ with vessel $v \in V$ with speed $s \in S_v^S$

C_{ivs}^{OP} : sailing cost from origin to port $i \in N$ by vessel $v \in V$ with speed $s \in S_v^S$

T_{ijvs}^{PP} : time required by vessel $v \in V$ to sail from port $i \in N$ to port $j \in N$ with speed $s \in S_v^S$

T_{ivs}^{OP} : time required by vessel $v \in V$ to sail from its origin to port $i \in N$ with speed $s \in S_v^S$

Formulation:

$$min \sum_{v \in V} \sum_{(i,m,j,n) \in S_v^X} \sum_{s \in S_v^S} C_{ijvs}^{PP} g_{imjnvs} + \sum_{v \in V} \sum_{(i,m) \in S_v^A} \sum_{s \in S_v^S} C_{ivs}^{OP} g_{imvs}^{o} + \sum_{v \in V} \sum_{(i,m) \in S_v^A} P_{iv} o_{imv} \tag{1}$$

$\sum_{s \in S_v^S} g_{imjnvs} = x_{imjnv}$	$\forall\ v \in V, \ (i,m,j,n) \in S_v^x$	(39)
$\sum_{s \in S_v^S} g_{imvs}^{o} = x_{imn}^{o}$	$\forall\ v \in V, \ (i,m) \in S_v^A$	(40)
$0 \leq g_{imjnvs} \leq 1$	$\forall\ v \in V, \ (i,m,j,n) \in S_v^x, \ s \in S_v^S$	(41)
$0 \leq g_{imvs}^{o} \leq 1$	$\forall\ v \in V, \ (i,m) \in S_v^A, \ s \in S_v^S$	(42)

Changes:

$$t_{im} + \sum_{v \in V} T_v^Q q_{imv} - t_{jn} + \sum_{v \in V} \sum_{s \in S_v^S} max\{U_{im} + T_{ijvs}^{PP} - A_{jn}, 0\} g_{imjnvs} \leq U_{im} - A_{jn} \qquad \forall\ (i,m),(j,n) \in S^A \tag{43}$$

$$\sum_{v \in V} \sum_{s \in S_v^S} T_{ivs}^{OP} \, g_{imvs}^{o} \leq t_{im} \qquad \forall \ (i,m) \in S^A \tag{44}$$

The first model generates routes which are used in model 2 to be able to provide the given speed for each vessel. Hence, the routing constraints (2) – (7) is removed since the routes are now fixed in model 2 as input variables. These input variables are as follows: x_{imjnv}, x_{imv}^0, z_{imv}, z_v^0, o_{imv}, y_{im}.

Model 2 includes all sets from model 1 and a new set of speeds. The objective function (1) is now considering the speed for each vessel and expresses the minimization of the sum of traveling costs between ports depending on the chosen speed and operational costs in each port. Further, constraints (43) and (44) are extensions of time constraints (14) and (16) respectively and is now also considering the speed of each vessel.

Loading and unloading constraints (8) – (9), arc flow constraints (10) – (13), time constraints (15), (17) – (18), inventory constraints (19) – (26), and nonnegative constraints on variables (33) – (38) stays the same way as in model 1.

In addition, model 2 has some new constraints. Constraints (39) – (40) are speed constraints. Constraint (39) shows that the speed travelled from one port to anther port can only exist if a vessel travels this arc. Constraint (40) states that the speed travelled from origin to a port can only exist if a vessel travels this arc. Constraints (41) – (42) ensures that variables are between zero and one.

4.5 Model 3. Maritime inventory routing problem with fixed routes and fuel consumption as a function of speed and load

This part considers the third model, which builds further on model 2. The third model builds further on the second model and introduces a minimum and maximum level of load. As in the second model, the routes provided from the first model is fixed parameters. As mentioned, the model is looking at cost minimization, and with the choice between minimum and maximum load and speed, the optimization tools are able to find the optimal speed and load for each vessel. The load of the vessels has a non-linear effect on the daily sailing costs, hence, the model will be non-linear. Therefore, we will have to approximate linear results by using breakpoints of loads as sets. The fuel consumption has a concave dependency of the load. Since it is a minimization problem, it is necessary to use special ordered set of type 2

(SOS2) constraints to ensure that two adjacent breakpoints of the load are chosen in the solution.

Notation:

Set:

$S_v^L = \{1, 2, \ldots, R\}$

Variables:

$g_{imjnvls}$ **:** auxiliary variable to determine the speed and load of vessel *v* when going from *(i, m)* to *(j, n)*, with *s* corresponding to a given choice of speed and *l* of a level of load, $v \in V$ $s \in S_v^S$ $l \in S_v^L$ $(i, m, j, n) \in S_v^X$

g_{imvls}^0 : auxiliary variable to determine the speed and load of vessel *v* when going from origin to *(i, m)*, with *s* corresponding to a given choice of speed and *l* of a level of load, $v \in V$ $s \in S_v^S$ $l \in S_v^L$ $(i, m) \in S_v^A$

p_{imjnvl} : 1 if the interval between two adjacent breakpoints on route (i, m, j, n), is chosen, 0 otherwise, $v \in V$ $l \in S_v^{L-\{R\}}$ $(i, m, j, n) \in S_v^X$

p_{imvl}^0 : 1 if the interval between two adjacent breakpoints from origin to node (i, m) is chosen, 0 otherwise, $v \in V$ $1 \in S_v^{L-\{R\}}$ $(i, m) \in S_v^A$

Parameters:

L_{vl} : possible levels of load $l \in L$ that can be transported on vessel $v \in V$

C_{ijvls}^{PP} : sailing cost from port $i \in N$ to port $j \in N$ with vessel $v \in V$ with load $l \in S_v^L$ and with speed $s \in S_v^S$

C_{ivls}^{OP} : sailing cost from origin to port $i \in N$ by vessel $v \in V$ with load $l \in S_v^L$ and with speed $s \in S_v^S$

Formulation:

$$min \sum_{v \in V} \sum_{(i,m,j,n) \in S_v^X} \sum_{l \in S_v^L} \sum_{s \in S_v^S} C_{ijvls}^{PP} g_{imjnvls} + \sum_{v \in V} \sum_{(i,m) \in S_v^A} \sum_{l \in S_v^L} \sum_{s \in S_v^S} C_{ivls}^{OP} g_{imvls}^{o} + \sum_{v \in V} \sum_{(i,m) \in S_v^A} P_{iv} o_{imv} \quad (1)$$

$\sum_{l\in S_v^L}\sum_{s\in S_v^S} L_{vl}\, g_{imjnvls} = f_{imjnv}$	$\forall\, v\in V,\ \ (i,m,j,n)\in S_v^x$	(45)
$\sum_{l\in S_v^L}\sum_{s\in S_v^S} L_{vl}\, g_{imvls}^0 = f_{imv}^0$	$\forall\ v\in V,\ \ (i,m)\in S_v^A$	(46)
$\sum_{l\in S_v^{L-\{R\}}} p_{imjnvl} = x_{imjnv}$	$\forall\, v\in V,\ \ (i,m,j,n)\in S_v^x$	(47)
$\sum_{s\in S_v^S} g_{imjnv1s} \le p_{imjnv1}$	$\forall\ v\in V,\ \ (i,m,j,n)\in S_v^x$	(48)
$\sum_{s\in S_v^S} g_{imv1s}^0 \le p_{imv1}^0$	$\forall\ v\in V,\ \ \ (i,m)\in S_v^A$	(49)
$\sum_{s\in S_v^S} g_{imjnvls} \le p_{imjnv(l-1)} + p_{imjnvl}$	$\forall\ v\in V,\ \ (i,m,j,n)\in S_v^x$ $l\in S_v^{L-\{1,R\}}$	(50)
$\sum_{s\in S_v^S} g_{imvls}^0 \le p_{imv(l-1)}^0 + p_{imvl}^0$	$\forall\ v\in V,\ \ (i,m)\in S_v^A$ $l\in S_v^{L-\{1,R\}}$	(51)
$\sum_{s\in S_v^S} g_{imjnvRs} \le p_{imjnv(R-1)}$	$\forall\ v\in V,\ \ (i,m,j,n)\in S_v^x$	(52)
$\sum_{s\in S_v^S} g_{imvRs}^0 \le p_{imv(R-1)}^0$	$\forall\ v\in V,\ \ (i,m)\in S_v^A,$	(53)
$p_{imjnvl}\in\{0,1\}$	$\forall\ v\in V,\ \ (i,m,j,n)\in S_v^x$	(54)
$p_{imvl}^0\in\{0,1\}$	$\forall\ v\in V,\ \ (i,m)\in S_v^A$	(55)

Changes:

$t_{im} + \sum_{v\in V} T_v^Q\, q_{imv} - t_{jn} + \sum_{v\in V}\sum_{l\in S_v^L}\sum_{s\in S_v^S} max\left\{U_{im} + T_{ijvs}^{PP} - A_{jn}, 0\right\} g_{imjnvls} \le U_{im} - A_{jn}$	$\forall\ (i,m),(j,n)\in S^A$	(56)
$\sum_{v\in V}\sum_{l\in S_v^L}\sum_{s\in S_v^S} T_{ivs}^{OP}\, g_{imvls}^o \le t_{im}$	$\forall\ (i,m)\in S^A$	(57)
$\sum_{l\in S_v^L}\sum_{s\in S_v^S} g_{imjnvls} = x_{imjnv}$	$\forall\, v\in V,\ \ (i,m,j,n)\in S_v^x$	(58)
$\sum_{l\in S_v^L}\sum_{s\in S_v^S} g_{imvls}^o = x_{imn}^o$	$\forall\ v\in V,\ \ (i,m)\in S_v^A$	(59)
$0\le g_{imjnvls}\le 1$	$\forall\ v\in V,\ \ (i,m,j,n)\in S_v^x,$ $s\in S_v^S,\ \ l\in S_v^L$	(60)
$0\le g_{imvls}^o\le 1$	$\forall\ v\in V,\ \ (i,m)\in S_v^A,$ $s\in S_v^S,\ \ l\in S_v^L$	(61)

Model 3 includes all sets from model 2. The objective function (1) is now considering both the speed and the load for each vessel and expresses the minimization of the sum of traveling costs between ports depending on the chosen speed and load, and operational costs in each port. Further, constraints (56) and (57) are extensions of time constraints (43) and (44). Constraints (58) and (59) are extensions of speed constraints (39) and (40). Constraints (60) and (61) are extensions of constraints (41) and (42), which indicate that the variables must be between zero and one. The extensions of constraints (56) – (61) includes the load of each vessel in addition to the speed.

Loading and unloading constraints (8) – (9), arc flow constraints (10) – (13), time constraints (15), (17) – (18), inventory constraints (19) – (26), and nonnegative constraints on variables (33) – (38) stays the same way as in model 1.

In addition, model 3 has some new constraints. Constraints (45) – (46) are load constraints. Constraint (45) shows that the speed travelled from one port to another by a vessel when transporting the load must be equal to the product flow. Constraint (46) states that the speed travelled from origin to the port by a vessel when transporting the load must be equal to the product flow from origin to the port. Constraints (47) – (53) are breakpoint constraints, known as special ordered set of type 2 (SOS2). Constraint (47) ensures that if a vessel travels this arc then an adjacent breakpoint must be chosen. Constraint (48) shows that the value of the speed and the load used by a vessel on route *(i, m, j, n)* with load *L1* can only be more than 0 if the first interval is chosen. Constraint (49) shows the same as constraint (48), but from origin to port (*i, m)*. Constraint (50) ensures that the value of the speed and the load used by a vessel on route *(i, m, j, n)* can only be more than 0 if one of the intervals connected to the breakpoint is chosen. Constraint (51) ensures the same as constraint (50), but from origin to port *(i, m)*. Constraint (52) guarantees that the value of the speed and the load used by vessel on route *(i, m, j, n)* can only be more than 0 if the last interval is chosen. Constraint (53) guarantees the same as constraint (52), but from origin to *(i, m)*. Constraints (54) – (55) states that the variables are binary.

4.6 Model 4. Maritime inventory routing problem with fuel consumption as a function of speed and load

This part considers the final model, which is a combination of the first- and third model. The fourth model provides optimal routes, speeds, and loads for each vessel at once.

Model 4 includes all sets from model 1, and the ones presented in model 2 and model 3. The objective function (1) stays the same as presented in model 3. Further, model 4 uses routing constraints (2) – (7), loading and unloading constraints (8) – (9), arc flow constraints

(10) – (13), time constraints (15), (17) – (18), inventory constraints (19) – (26), binary constraints on variables (27) – (33), and nonnegative constraints on variables (33) – (38) as presented in model 1.

In addition, it uses time constraints (56) and (57), speed constraints (58) – (59), fraction between zero and one constraints on variables (60) – (61), load constraints (45) – (46), SOS2 or breakpoint intervals constraints (47) – (53), and binary constraints on variables (54) – (55) as presented in model 3.

5. Computational study

This chapter describes data instances and presents computational results and analysis. The computational tests were run via an external connection to a computer with 2.30 GHz Intel Xeon CPU E5-2698 v3 processor and 32 GB of RAM under Microsoft Windows Server 2008 R2 Enterprise 64-bit version. The models are coded in AMPL language and run in CPLEX 12.7.1.0.

5.1 Test instance description

The computational study in this thesis is based on data from seven main instances, A, B, C, D, E, F and G. These instances and the corresponding data are taken from Agra et al. (2016a). The seven instances vary in the number of ports and ships. Each of the instances are divided into three sub-instances, with differences in the number of days in the planning horizon and initial stock and demand rates at the ports. To separate the instances from each other, each instance has a specific name based on the characteristics of the data. The name of each instance consists of the instance letter, the number of ports, the number of ships, number of days in the planning horizon, and an index number. The index number identifies the sub-instances. Table 1 shows the main characteristics of the instances.

Table 1: Test instances.
Source: Made by the authors.

Name of the Instance	Main instance	Number of ports	Number of vessels	Number of days in the planning horizon
A-4-1-30-1	A	4	1	30
A-4-1-60-1	A	4	1	60

A-4-1-60-2	A	4	1	60
B-3-2-30-1	B	3	2	30
B-3-2-60-1	B	3	2	60
B-3-2-60-2	B	3	2	60
C-4-2-30-1	C	4	2	30
C-4-2-60-1	C	4	2	60
C-4-2-60-2	C	4	2	60
D-5-2-30-1	D	5	2	30
D-5-2-60-1	D	5	2	60
D-5-2-60-2	D	5	2	60
E-5-2-30-1	E	5	2	30
E-5-2-60-1	E	5	2	60
E-5-2-60-2	E	5	2	60
F-4-3-30-1	F	4	3	30
F-4-3-60-1	F	4	3	60
F-4-3-60-2	F	4	3	60
G-6-5-30-1	G	6	5	30
G-6-5-60-1	G	6	5	60
G-6-5-60-2	G	6	5	60

Each vessel has operational characteristics which varies between the instances. These characteristics are the capacity of the vessel, the initial load of the vessel, possible speeds of the vessels and the daily sailing costs for each speed and load. The ships have different operational speeds, which differs from 13.5 to 21 knots. In the computational study, the ships have a set of speeds with three options for each vessel. The speed options are based on the speed ranges in Evsikova (2017). The speeds of the vessels in each instance are shown in Table 2.

Table 2: Speed ranges of the vessels in the instances.
Source: Made by the authors.

Instance	**Speed ranges, knots**		
	13.5-15-19	**14.4-16-20**	**16.2-18-21**
A-4-1-30-1			Vessel 1

A-4-1-60-1			Vessel 1
A-4-1-60-2			Vessel 1
B-3-2-30-1	Vessel 1	Vessel 2	
B-3-2-60-1	Vessel 1	Vessel 2	
B-3-2-60-2	Vessel 1	Vessel 2	
C-4-2-30-1	Vessel 1	Vessel 2	
C-4-2-60-1	Vessel 1	Vessel 2	
C-4-2-60-2	Vessel 1	Vessel 2	
D-5-2-30-1	Vessel 1		Vessel 2
D-5-2-60-1	Vessel 1		Vessel 2
D-5-2-60-2	Vessel 1		Vessel 2
E-5-2-30-1	Vessel 1		Vessel 2
E-5-2-60-1	Vessel 1		Vessel 2
E-5-2-60-2	Vessel 1		Vessel 2
F-4-3-30-1	Vessel 1	Vessel 2	Vessel 3
F-4-3-60-1	Vessel 1	Vessel 2	Vessel 3
F-4-3-60-2	Vessel 1	Vessel 2	Vessel 3
G-6-5-30-1	Vessel 1	Vessel 2 Vessel 4	Vessel 3 Vessel 5
G-6-5-60-1	Vessel 1	Vessel 2 Vessel 4	Vessel 3 Vessel 5
G-6-5-60-2	Vessel 1	Vessel 2 Vessel 4	Vessel 3 Vessel 5

Table 3 shows the maximum capacity of the vessels in each instance. The smallest ships in the instances has a total capacity of 100 000 DWT, while the ships with the largest capacity has a DWT of 160 000. In the computational study, the ships have a set of loads, with three options for each vessel. The largest load option equals to the capacity of the vessel, and the smallest load is 0 and represents an empty vessel. In the middle, the load level equals to exactly half of the ship capacity.

Table 3: Capacities of the vessels in the instances.

Source: Made by the authors

Instance	**Capacities, DWT**					
	100	**120**	**130**	**140**	**150**	**160**
A-4-1-30-1						Vessel 1
A-4-1-60-1						Vessel 1
A-4-1-60-2						Vessel 1
B-3-2-30-1	Vessel 1		Vessel 2			
B-3-2-60-1	Vessel 1		Vessel 2			
B-3-2-60-2	Vessel 1		Vessel 2			
C-4-2-30-1	Vessel 1		Vessel 2			
C-4-2-60-1	Vessel 1		Vessel 2			
C-4-2-60-2	Vessel 1		Vessel 2			
D-5-2-30-1	Vessel 1				Vessel 2	
D-5-2-60-1	Vessel 1				Vessel 2	
D-5-2-60-2	Vessel 1				Vessel 2	
E-5-2-30-1	Vessel 1				Vessel 2	
E-5-2-60-1	Vessel 1				Vessel 2	
E-5-2-60-2	Vessel 1				Vessel 2	
F-4-3-30-1		Vessel 1	Vessel 2		Vessel 3	
F-4-3-60-1		Vessel 1	Vessel 2		Vessel 3	
F-4-3-60-2		Vessel 1	Vessel 2		Vessel 3	
G-6-5-30-1		Vessel 1	Vessel 2	Vessel 4	Vessel 3 Vessel 5	
G-6-5-60-1		Vessel 1	Vessel 2	Vessel 4	Vessel 3 Vessel 5	
G-6-5-60-2		Vessel 1	Vessel 2	Vessel 4	Vessel 3 Vessel 5	

5.2 Assessment of computational results

5.2.1 Computational time

In this part, the computational time for all four models are compared, and is shown in seconds. As seen in Table 4, the computational time is shown for all the models on the different instances. Model 1 use fixed load and speed to provide the best routes for the problem. As Table 4 shows, the computational time is increasing as the problem is introduced to higher numbers of ports and vessels.

For model 2 and model 3, the computational time is under 1 second, with the exception of instance G-6-5-60-1 for model 2, which uses just under 3 seconds. The reason model 2 and model 3 is so fast, is because it uses the predetermined routes from model 1 to optimize the speed and load used on the different routes, respectively, which reduces the amount of work for the model.

Model 4 provides optimal routes, speed, and load all at once. When comparing model 1 and model 4, the computational time has increased, especially from instance C-4-2-30-1 and out. The reason for this is because more variables are introduced in model 4 as compared to model 1 and hence the problem is growing in size.

Table 4: Computational time in seconds for every model on the different instances.
Source: Made by the authors.

Instance	Model 1	Model 2	Model 3	Model 4
A-4-1-30-1	0.1	0.1	0.1	0.3
A-4-1-60-1	1.1	0.2	0.1	2.1
A-4-1-60-2	1.0	0.3	0.1	2.7
B-3-2-30-1	0.9	0.1	0.2	59.8
B-3-2-60-1	4.4	0.2	0.3	42.4
B-3-2-60-2	4.5	0.2	0.3	13.8
C-4-2-30-1	5.8	0.2	0.1	123.2
C-4-2-60-1	8.5	0.2	0.1	183.3
C-4-2-60-2	18.6	0.2	0.1	2459.0
D-5-2-30-1	5.8	0.1	0.2	6011.5
D-5-2-60-1	81.5	0.1	0.3	1422.5
D-5-2-60-2	13.4	0.1	0.1	20.8

E-5-2-30-1	23.1	0.1	0.1	38682.9
E-5-2-60-1	77.3	0.1	0.1	5312.7
E-5-2-60-2	59.4	0.2	0.1	4132.5
F-4-3-30-1	151.9	0.1	0.3	2111.6
F-4-3-60-1	16.0	0.2	0.3	211.0
F-4-3-60-2	21.8	0.2	0.2	1474.1
G-6-5-30-1	464.2	0.2	0.1	54009.1
G-6-5-60-1	398.5	2.8	0.1	50084.5
G-6-5-60-2	979.1	0.3	0.1	36005.0

Instance G-6-5-30-1 has the highest computational time, with 54009 seconds. As the number of vessels and the length of the planning horizon increase, the models use more time to be able to provide the best solution among all possible alternatives. Instance G-6-5-30-1 uses roughly 54000 seconds while instance G-6-5-60-2 uses roughly 36000 seconds, and the reason for this is because there is a time limit in the models, to be able to give solutions within reasonable time. Instance G-6-5-30-1 ran for 55 hours before the personal computer which was used had troubles and ended the program. Instance G-6-5-60-2 ran for 40 hours, before the authors ended it. This time usage justifies the need for a time limit in the models. In addition, there is a great need for storage capacity on the computer. As the problem increases in size, the size of the node file in Cplex is also increasing. The node file is a temporarily file which stores all the data which the model needs to find the best possible solution on every available possibility, and at most this reached over 150 gigabytes.

5.2.2 Sailing costs and savings

In this part, a comparison of sailing costs and savings are presented. The sailing costs are in 1000 US Dollars. Table 5 shows the average sailing cost of all instances, the percentage savings in average, and the maximum- and minimum savings for the different models. The average sailing costs are calculated by adding the sailing cost for each instance and dividing it by the number of instances. The sailing costs are the actual costs calculated from the distances, visits, speeds and loads found by the models, using the following formula, $FC = k\mu^3(l + A)^{2/3}$, which is provided from Psaraftis and Kontovas (2014) and was discussed in chapter 3. The percentage savings shows how much the different models are able to save in comparison to model 1. For more detailed information on each instance, please see Appendix 1.

Model 4-2 is the same as model 4 but has the same cost for each load level. Thus, the route optimization will only be based on speed and not consider load. Model 3-2 is the same as model 3 but uses the predetermined routes from model 4-2. Model 2-2 is not included in this comparison since is gives the same sailing costs as model 4-2. The reason for this, is because both models optimizes speed on the same routes.

Table 5: Cost comparison of the models.
Source: Made by the authors

		Initial solution	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)	**Average total cost**	*Model 1*	*Model 2*	*Model 3*
		1212.762	853.904	630.238
	Average savings	0.0 %	29.59 %	48.03 %
	Max savings	0.0 %	44.75 %	62.90 %
	Min savings	0.0 %	6.58 %	29.20 %
Route-planning with speed (without load)	**Average total cost**		*Model 4-2*	*Model 3-2*
			740.416	546.717
	Average savings		38.95 %	54.92 %
	Max savings		62.23 %	69.18 %
	Min savings		6.86 %	41.97 %
Route-planning with speed and load	**Average total cost**			*Model 4*
				530.537
	Average savings			56.25 %
	Max savings			69.18 %
	Min savings			48.91 %

Table 5 shows that every model is able to save costs in comparison to model 1. Model 2 is the model that gives the lowest savings and has an average saving of 29.59%, closely followed by model 4-2, which gives an average saving of 38.95 %. This is because these models only consider speed optimization and has a fixed load. Model 3 and model 3-2 optimizes both speed and load and gives a better result than model 2 and model 4-2. This shows that by introducing load as a decision variable it is possible to decrease the sailing cost, and hence increase the savings in comparison to model 1. In total, it is model 4 which is able to provide the highest savings. By optimizing routes, speed, and load all at once it is possible to reduce the average costs with 56.25% in comparison to route-planning with fixed speed and load.

The minimum saving is 6.58 % and is obtained in model 2, while the maximum saving is 69.18% and is obtained by both model 3-2 and model 4. More details on which instances these minimum and maximum savings occurs can be seen in Appendix 1.

As mention, Table 5 shows the average sailing cost for each model, to be able to compare the savings provided by the different models. To show how the different models perform on the given instances, as shown in Appendix 1, Figure 7 gives a graphical comparison of the sailing costs.

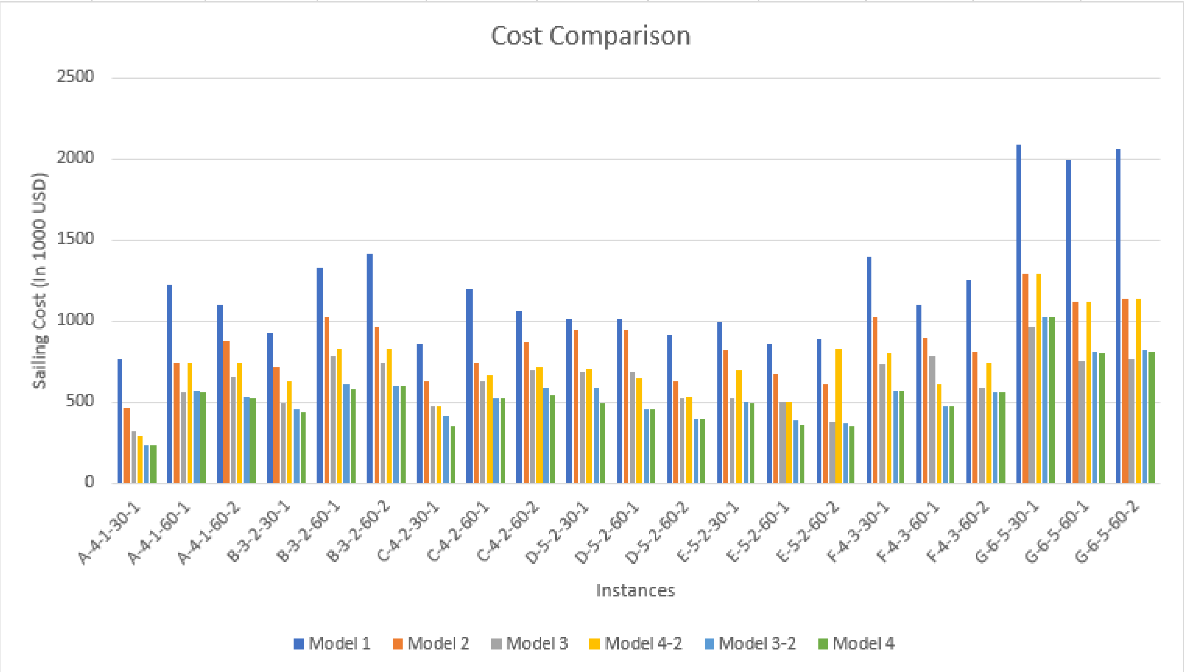

Figure 7: Cost comparison of the different instances. Source: Made by the authors.

It is clear to see that model 1 gives the highest sailing costs in every instance, especially in the three last instances. However, in instance D-5-2-30-1 and D-5-2-60-1 the sailing costs between model 1 and model 2 is not that different, which indicates that the optimal speed found in model 2 is close to the ones which were fixed in model 1. It is model 4 which gives the best solution in every instance. In some of the instances model 3-2 can give the same sailing cost as model 4, which indicates that the routes given by 4-2 is equal to the ones found in model 4.

5.2.3 Speed of vessel

In this part, the average speed in knots of the vessels is compared. The comparison looks at the different models, and all the speeds used in the instances in total. To get more details on how the average speed is in the different instances, please see Appendix 3.

Table 6 shows the average speed from the instances, average reduction in percentage, and maximum- and minimum speed. Each vessel uses only one speed between two ports but can use different speeds for the different visits on the routes. The average speed for each instance is calculated by adding together the speeds used by the models on the given instances and divided by the number of visits. In Table 6, all the average speeds from each of the instances are summarized and divided by the number of instances. Thus, the table shows the average speed from the instances for each model.

Table 6: Average speed of vessels.
Source: Made by the authors.

	Model 1	**Model 2**	**Model 3**	**Model 4-2**	**Model 2-2**	**Model 3-2**	**Model 4**
Average speed	19.83	16.05	16.02	14.75	14.87	14.77	14.80
Average reduction of speed (in %)	0.00	19.06 %	19.21 %	25.62 %	25.01 %	25.52 %	25.37 %
Maximum speed	21	18.82	18.62	16.26	16.26	16.31	16.55
Minimum speed	19.20	14.34	14.34	13.90	13.89	13.97	13.86

Model 1 has the highest average speed as shown in Table 6. This is because the vessels operate at max speed to get the best possible route solutions. Model 2 uses the predetermined routes from model 1 to optimize the speed, hence the average speed is lower in comparison to model 1. Model 3 also uses the predetermined routes and optimizes both the speed and the load of the vessels. As shown in Table 6, the average speed in model 3 is fairly similar to the average speeds provided by model 2. Model 4, which optimize the routes, speed, and load at once gives lower average speed compared to model 1, 2 and 3.

Model 4-2 is the same as model 4, but the cost for each load level is the same, hence it does not consider load when it finds the optimal routes and speeds for each vessel. Model 4-2 gives the lowest average speeds in comparison to model 1. Further, model 2-2 and model 3-2 is the same as model 2 and model 3, respectively, but uses the predetermined routes from model 4-2. The average speeds in model 2-2 and model 3-2 are similar to each other, but when compared to model 2 and model 3 they provide lower average speeds.

5.2.4 Load of vessel

The load of the vessels on each trip is the same as the flow between the ports. This flow must meet the demand requirements of the ports. As the demand and consumption rates differs between the instances, comparing the loads between the instances is useless. Thus, when looking at the load levels, the best comparison would be to look at the differences in the solutions provided from the different models.

In this part, the average load of the vessels from each model is compared. This comparison only looks at the different instances in model 3, model 3-2, and model 4, since these models considers load as a decision variable. The average load on each trip is calculated by summarizing the loads used by the models and dividing it by the total number of visits. A full table of average load of trips for each instance is given in Appendix 3. Table 7 presents the average of the average load levels of each instance provided by the different models.

Table 7: Average load of vessels in 1000 tons.
Source: Made by the authors.

	Model 3	**Model 3-2**	**Model 4**
Average load on a trip	52.80	55.61	51.36

When comparing the three models, Table 7 shows that model 3-2 has the highest average load level, while model 4 has the lowest average load level. As model 3 and 3-2 optimizes load on predefined routes, the flexibility is lower when fulfilling the requirement of demand. In model 4, routes and load (and speed) is optimized at the same time. Thus, adjustments can be made such that the vessels can carry less load and still meet the demand in the ports.

5.2.5 Structural analysis of the solution

The structure of the solution can be analyzed by looking at the route decisions. In model 1, the routes are decided without considering speed and load. In model 4-2, the routes are decided by considering speed, and not load. In model 4, both load and speed have an impact of the costs, and hence the routes are decided based on both two factors. Thus, the solutions from model 4-2 and model 4 can be compared to the solutions from model 1 to see if speed and load had an impact on the routes that are chosen.

Table 8 shows that the routes in model 4-2 changed from the routes in model 1 in 15 of 21 instances. This means that the speed of the vessels has an impact of the structure in 71.4% of the solutions. When comparing the routes of model 4 to the routes of model 1, we see that the structure changes in 18 out of the 21 instances. Thus, speed and load together have an impact of the structure in 85.7% of the cases. When looking at the differences in the structure between model 4-2 and model 4, we see that the model that considers load in addition to speed changes the optimal routes from the solution of the model that only considers speed in 10 out of the 21 instances. This means that load alone has an impact of the route decision in 47.6% of the instances.

Table 8: Comparison of the routes generated from the models.
Source: Made by the authors.

Instance	**Model 1**		**Model 4-2**		**Model 4**	
	Distance travelled in nautical miles	*Number of visits*	*Distance travelled in nautical miles*	*Number of visits*	*Distance travelled in nautical miles*	*Number of visits*
A-4-1-30-1	6 996	5	4 275	4	4 275	4
A-4-1-60-1	11 271	8	11 271	8	11 271	8
A-4-1-60-2	10 105	7	11 271	8	11 271	8
B-3-2-30-1	13 605	6	15 159	8	14 382	7

B-3-2-60-1	20 212	10	20 212	10	21 767	11
B-3-2-60-2	19 824	9	21 767	11	21 767	11
C-4-2-30-1	12 438	7	12 049	7	13 604	8
C-4-2-60-1	16 325	9	17 102	9	17 102	9
C-4-2-60-2	16 323	10	19 433	12	17 489	10
D-5-2-30-1	12 825	8	12 825	8	13 601	9
D-5-2-60-1	15 157	10	17 489	11	17 489	11
D-5-2-60-2	12 048	8	13 991	9	13 991	9
E-5-2-30-1	12 047	7	13 990	8	13 990	8
E-5-2-60-1	10 881	7	10 881	8	11 270	8
E-5-2-60-1	13 213	9	12 047	9	14 379	10
F-4-3-30-1	17 491	10	17 491	10	17 491	10
F-4-3-60-1	12 438	9	14 769	10	14 769	10
F-4-3-60-2	17 489	11	18 655	11	17 878	11
G-6-5-30-1	22 931	14	22 931	14	25 654	14
G-6-5-60-1	24 486	14	24 486	14	24 486	14
G-6-5-60-2	26 043	13	25 939	13	26 819	13

5.2.6 Computational error

In the computational test of the models, three breakpoints are used in the sets for both load and speed. If a speed and load that are equal to the breakpoints are chosen, the approximated cost will be the same as the real cost. If a load that is equal to one of the breakpoint is chosen, but the chosen speed lies between two breakpoints, the approximated cost will be higher than the real cost. In the opposite way, if the chosen speed is on one of the breakpoints, but the chosen load is between two points, the approximated cost will be lower than the real cost. Because of this, the deviation from the real cost and the approximated cost from the models can vary between negative and positive in the different instances. As there are only three breakpoints in use, the deviation from the real costs can be high. The more breakpoints that are used, the more accurate will the approximated costs be.

This part shows the computational errors of the model. The computational error shows how much the approximated cost from the models deviates from the actual costs that are calculated from the formula of Psaraftis and Kontovas (2014) based on the variable values generated from the models. Expanded tables that shows all computational errors of all the instances are presented in Appendix 4, 5 and 6. Table 9 presents the average deviation based on all the deviations of the instances. The table also shows how high the highest deviation in the instances is and how low the lowest deviation is.

Table 9: Computational errors of the models
Source: Made by the authors.

	Model 1	**Model 2**	**Model 3**	**Model 4-2**	**Model 2-2**	**Model 3-2**	**Model 4**
Average deviation of all instances	0.0098%	0.236%	1.760%	0.236%	0.236%	0.959%	0.732%
Maximum deviation of all instances	0.047%	3.784%	8.230%	4.023%	4.023%	5.106%	3.276%
Minimum deviation of all instances	0.0002%	0.007%	0.045%	0.003%	0.003%	0.148%	0.0007%

As Table 9 shows, the deviation between real costs and approximated costs from the models are relatively low for all the models.

Model 1 has an average deviation of 0.0098%, model 2 an average deviation of 0.236% and the average deviation of model 3 is 1.76%. The average deviations are calculated regardless of the deviations being negative or positive. As model 1 considers both speed and load as constants, the cost function is linear, and hence, the real costs and the approximated costs are close to each other. In model 2, there are linear approximation of the speed. Thus, the average deviation from this model is higher than model 1. Model 3 has a higher average deviation than model 1 and 2 because model 3 is approximating both the speed and the load.

When looking at model 4-2 where speed and routes are optimized, and load is not considered as each load level is given the same cost, the average deviation is 0.236%. The average deviation of model 2-2 and 3-2 where the fixed routes are taken from the routes generated in model 4-2, is 0.0236% and 0.959%. The average deviation of model 4-2 and 2-2 generates the exact same solutions. This was expected as model 4-2 finds optimal routes and optimizes the speed, and model 2-2 uses the routes from model 4-2 and optimizes the speed.

As the table shows, the average deviations are highest for the models that considers load as a decision variable; model 3, model 3-2 and model 4. The objective function of the models is minimizing costs, and the parts of the function that considers load is concave. Because of this, the optimization tool that uses the approximately linearized models will try to choose a load that generates the lowest costs as possible but is large enough to meet the demands at the ports. Thus, a load level that lies between two breakpoints has a high chance of being chosen. When a load level between two breakpoints is chosen, the approximated costs from the model will be lower than the real costs as the real costs are calculated by using a non-linear function. An example of the this can be drawn from model 3, where one of the deviations from the real cost is 6.589%. This deviation is from instance C-4-2-30-1, where the real cost is $632.271 and the approximated cost from the model is $590.612. In the solution of model 3 for this instance, almost all the load levels of the ships lie between two breakpoints. This leads to a high deviation between the approximated costs from the model and the real costs calculated by the non-linear formula.

The average deviation of model 4 in the Table 9 is 0.723%. This average deviation is relatively small, when taking into consideration that the model considers both load and speed when optimizing the routes. Therefore, model 4 will generate an answer that is close to the actual costs in many cases and can be useful in real-life situations.

6. Conclusion

Maritime transport plays a huge role in the global trade business. To gain competitive advantages, costs reductions can be an important factor for the companies that operates within maritime transportation. To minimize the costs, models for MIRP with speed optimization can be used to find the routes and speeds that will generate the lowest costs. Like the speed, the vessels load level do also have an impact of the fuel consumption, and hence the daily sailing costs. This impact is non-linear. In this thesis, a linear approach is used to find solutions to the MIRP with speed and load optimization.

The thesis presents four different MIRP models, all with the objective of minimizing total costs. The first one is a deterministic model with speed as a fixed parameter, where load does not affect the costs. The second and third model are special variants of model 1, where the routes are fixed, and the speed and load of the ships are added as sets. In the fourth model, routes are generated, and the speed and load are optimized.

The computational tests conducted on the different data instances shows that speed optimization alone can generate high savings regarding the total costs. When the load levels impact on the fuel consumption function are added, the cost savings becomes even higher. The tests also show that the solutions from the linear approximation approach in the models has a relatively small deviation from the actual costs of the non-linear problem. Thus, the models can be applied in real data and can be used to help real companies make better decisions.

6.1 Limitations of the study

Although this thesis was able to reach its aim, there are still some limitations. First of all, the computational time for instance G-6-5-30-1, G-6-5-60-1 and G-6-5-60-2 in model 4 is very high. As mentioned, the longest time used on an instance without a time limit was over 55 hours before it was cancelled either by storage capacity or by the authors. Even though there were a solution to instance G-6-5-30-1 after 15 hours, there might be a better solution available. Secondly, because of restricted time on this thesis, the authors were not able to improve the model to decrease the computational time used. Finally, in the computational tests of the thesis, only three breakpoints are used for both speed and load. When using so few breakpoint, some computational errors occurs, especially in the models that considers load.

6.2 Suggestions for future research

The limitations of this thesis show the need for further research to be able to decrease the time usage of the final model. The authors tried to introduce dummy ports into the models as a measure to reduce the number of possible alternatives for the models to run, hence reduce the time usage, but were unable to get it to work because of the time restriction on the thesis. If the limitation with time usage is solved, there might be found better solutions to the instances than those presented in this thesis. Furthermore, if it is solved, it makes it possible to expand the size of the problem without increasing the required time usage too much.

Another suggestion for further research is to increase the number of breakpoints for load and speed and analyze how this influence the computational errors and the computational time.

Reference list

Agra A., Christiansen M., Delgado A., Hvattum L.M. (2015). A maritime inventory routing problem with stochastic sailing and port times. Computers & Operations Research. 61:18–30.
doi: http://doi.org/10.1016/j.cor.2015.01.008

Agra A., Christiansen M., Hvattum L.M., Rodrigues F., (2016a). A MIP Based Local Search Heuristic for a Stochastic Maritime Inventory Routing Problem. A. Paias et al. (Eds.): Computational Logistics. ICCL 2016, LNCS 9855, pp. 18–34
doi: http://doi.org/10.1007/978-3-319-44896-1_2

Agra A., Christiansen M., Hvattum L.M., Rodrigues F. 2016b. Robust optimization for a maritime inventory routing problem. Transportation Science.

Andersson H., Fagerholt K., Hobbesland K. (2015). Integrated maritime fleet deployment and speed optimization: Case study from RoRo shipping. Computers & Operations Research. 55:233–240.

Andersson H., Hoff A., Christiansen M., Hasle G., Løkketangen A. (2010). Industrial aspects and literature survey: Combined inventory management and routing. Computers & Operations Research. 37: 1515-1536.

Anily S., Federgruen A. (1990). One warehouse multiple retailer systems with vehicle routing costs. Management Sci. 36(1):92–114.

Archetti C., Bertazzi L., Hertz A., Speranza M.G. (2012). A hybrid heuristic for an inventory routing problem. INFORMS J. Comput. 24(1):101–116.

Bell W.J., Dalberto L.M., Fisher M.L., Greenfield A.J., Jaikumar R., Kedia P., Mack R.G., Prutzman P.J. (1983). Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer. Interfaces 13(6):4–23.

Bertazzi L., Paletta G., Speranza M. G. (2002). Deterministic order-up-to level policies in an inventory routing problem. Transportation Sci. 36(1):119–132.

Bertazzi L., Speranza M.G. (2012). Inventory routing problems: an introduction.
EURO J Transp Logist. 1:307-326.
doi: 10.1007/s13676-012-0016-7.

Bialystocki N., Konovessis D. 2016. On the estimation of ship's fuel consumption and speed curve: A statistical approach. Journal of Ocean Engineering and Science
1: 157-166.

Blumenfeld D.E., Burns L.D., Diltz J.D., Daganzo C.F. (1985). Analyzing trade-offs between transportation, inventory and production costs on freight networks. Transportation Res. Part B: Methodological 19(5):361–380.

Boudia M., Prins C. (2009). A memetic algorithm with dynamic population management for an integrated production-distribution problem. Eur. J. Oper. Res. 195(3):703–715.

Burns L.D., Hall R.W., Blumenfeld D.E., Daganzo C.F. (1985). Distribution strategies that minimize transportation and inventory costs. Oper. Res. 33(3):469–490.

Campbell A.M., Clarke L., Kleywegt A.J., Savelsbergh M.W.P. (1998). The inventory routing problem. Crainic TG, Laporte G, eds. Fleet Management and Logistics (Springer, Boston), 95–113.

Campbell A.M., Savelsbergh M.W.P. (2004). A decomposition approach for the inventory-routing problem. Transportation Sci. 38(4):488–502.

Chien T.W., Balakrishnan A., Wong R.T. (1989). An integrated inventory allocation and vehicle routing problem. Transportation Sci. 23(2):67–76.

Coelho L.C., Cordeau J-F., Laporte G. (2012). The inventory routing problem with transshipment. Comput. Oper. Res. 39(11): 2537–2548.

Coelho L.C., Cordeau J-F., Laporte G. (2014) Thirty Years of Inventory Routing. Transportation Science 48(1):1-19. https://doi.org/10.1287/trsc.2013.0472

De A., Kumar S.K., Gunasekaran A., Tiwari M.K., (2017). Sustainable maritime inventory routing problem with time window constraints. Engineering Applications of Artificial Intelligence. 61:77–95.

Dror M., Ball M.O., Golden B.L. (1985). A computational comparison of algorithms for the inventory routing problem. Ann. Oper. Res. 4:3–23.

Dror M., Levy L. (1986). A vehicle routing improvement algorithm comparison of a “greedy” and a matching implementation for inventory routing. Comput. Oper. Res. 13(1):33–45.

Evsikova, N. 2017. *Speed optimization in maritime inventory routing."* Masteroppgave, Høgskolen i Molde, Vitenskapelig høgskole i logistikk. http://hdl.handle.net/11250/2465249

Kleywegt A.J., Nori V.S, Savelsbergh M.W.P (2002). The Stochastic Inventory Routing Problem with Direct Deliveries. Transportation Science 36(1):94-118. https://doi.org/10.1287/trsc.36.1.94.574

Norstad I., Fagerholt K., Laporte G. (2011). Tramp ship routing and scheduling with speed optimization. Transportation Research Part C: Emerging Technologies. 19:853-865

Psaraftis H.N., Kontovas C.A. (2013). Speed models for energy-efficient maritime transportation: A taxonomy and survey. Transportation Research Part C: Emerging Technologies. 26:331-351.
doi: http://dx.doi.org/10.1016/j.trc.2012.09.012.

Psaraftis H.N., Kontovas C.A. (2014). Ship speed optimization: Concepts, models and combined speed-routing phases. Transportation Research Part C: Emerging Technologies. 44:52-69.
doi: http://dx.doi.org/10.1016/j.trc.2014.03.001.

Ribeiro R., Lourenço H.R. (2003). Inventory-routing model for a multi-period problem with stochastic and deterministic demand. Technical report 275, Department of Economics and Business, Universitat Pompeu Fabra, Barcelona.

Song J-H., Furman K.C. 2013. "A maritime inventory routing problem: practical approach". *Computers & operations research* Vol 40, no 3 657-665 https://doi.org/10.1016/j.cor.2010.10.031

United Nations Conference on Trade and Development. (2017). Review of Maritime Transport.

Wen M., Ropke S., Petersen H.L., Larsen R., Madsen O.B.G., 2016. "Full-shipload tramp ship routing and scheduling with variable speeds." *Computers & Operations Research 70 1-8. https://doi.org/10.1016/j.cor.2015.10.002*

Williams, H.Paul. 1999. Model Building in Mathematical Programming, 4th Edition. United Kingdom: John Wiley & Sons.

Zaitseva A. 2017. *"Introducing profit maximization in inventory routing problems."* Masteroppgave, Høgskolen i Molde, Vitenskapelig høgskole i logistikk. http://hdl.handle.net/11250/2465754

Zhao Q-H., Chen S., Zang C-X. (2008). Model and algorithm for inventory/routing decisions in a three-echelon logistics system. Eur. J. Oper. Res. 19(3):623–635.

Appendix 1

This part includes the tables which show the total costs and savings for every model on the different instances. Each instance has its own table, where the total costs, based on the different models and savings of the models are compared to the solution for model 1. The percentage change shown for each model is calculated according to model 1. These savings are presented in table 10 till 30.

Table 10: Cost comparison of instance A-4-1-30-1.
Source: Made by the authors

Instance A-4-1-30-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	764.062	465.789	321.276
	Savings	0.0%	39.04 %	57.95 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		288.584	235.462
	Savings		62.23 %	69.18 %
Route-planning with speed and load				*Model 4*
	Total cost			235.462
	Savings			69.18 %

Table 11: Cost comparison of instance A-4-1-60-1. Source: Made by the authors.

Instance A-4-1-60-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1228.210	747.673	563.759
	Savings	0.0%	39.12 %	54.10 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		747.673	570.874
	Savings		39.12 %	53.52 %
Route-planning with speed and load				*Model 4*
	Total cost			563.653
	Savings			54.11 %

Table 12: Cost comparison of instance A-4-1-60-2. Source: Made by the authors.

Instance A-4-1-60-2		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1100.433	881.027	657.089
	Savings	0.0%	19.94 %	40.29 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		747.673	535.067
	Savings		32.06 %	51.38 %
Route-planning with speed and load				*Model 4*
	Total cost			522.919
	Savings			52.48 %

Table 13: Cost comparison of instance B-3-2-30-1.
Source: Made by the authors

Instance B-3-2-30-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	924.795	711.474	492.654
	Savings	0.0%	23.07 %	46.73 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		626.720	454.101
	Savings		32.23 %	50.90 %
Route-planning with speed and load				*Model 4*
	Total cost			436.245
	Savings			52.83 %

Table 14: Cost comparison of instance B-3-2-60-1.
Source: Made by the authors.

Instance B-3-2-60-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1328.108	1025.809	778.106
	Savings	0.0%	22.76 %	41.41 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		829.951	605.246
	Savings		37.51 %	54.43 %
Route-planning with speed and load				*Model 4*
	Total cost			582.698
	Savings			56.13 %

Table 15: Cost comparison of instance B-3-2-60-2.
Source: Made by the authors.

Instance B-3-2-60-2		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1420.735	965.891	747.291
	Savings	0.0%	32.01 %	47.40 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		828.132	601.460
	Savings		41.71 %	57.67 %
Route-planning with speed and load				*Model 4*
	Total cost			601.460
	Savings			57.67 %

Table 16: Cost comparison of instance C-4-2-30-1
Source: Made by the authors.

Instance C-4-2-30-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	859.577	624.595	474.532
	Savings	0.0%	27.34 %	44.79 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		474.993	412.129
	Savings		44.74 %	52.05 %
Route-planning with speed and load				*Model 4*
	Total cost			353.070
	Savings			58.93 %

Table 17: Cost comparison of instance C-4-2-60-1.
Source: Made by the authors.

Instance C-4-2-60-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1197.127	739.500	632.271
	Savings	0.0%	38.23 %	47.18 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		668.456	522.524
	Savings		44.16 %	56.35 %
Route-planning with speed and load				*Model 4*
	Total cost			522.524
	Savings			56.35 %

Table 18: Cost comparison of instance C-4-2-60-2.
Source: Made by the authors.

Instance C-4-2-60-2		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1064.085	869.244	691.914
	Savings	0.0%	18.31 %	34.98 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		716.572	558.501
	Savings		32.66 %	47.51 %
Route-planning with speed and load				*Model 4*
	Total cost			543.621
	Savings			48.91 %

Table 19: Cost comparison of instance D-5-2-30-1.
Source: Made by the authors

Instance D-5-2-30-1		Initial solution	Optimized speed (constant load)	Optimized speed and load
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1008.713	942.303	681.309
	Savings	0.0%	6.58 %	32.46 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		705.398	585.309
	Savings		30.07%	41.97%
Route-planning with speed and load				*Model 4*
	Total cost			491.124
	Savings			51.31 %

Table 20: Cost comparison of instance D-5-2-60-1.
Source: Made by the authors

Instance D-5-2-60-1		Initial solution	Optimized speed (constant load)	Optimized speed and load
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1008.713	942.303	681.309
	Savings	0.0%	6.58 %	32.46 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		645.979	451.071
	Savings		35.96 %	55.28 %
Route-planning with speed and load				*Model 4*
	Total cost			451.092
	Savings			55.28 %

Table 21: Cost comparison of instance D-5-2-60-2.
Made by the authors

Instance D-5-2-60-2		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	914.987	626.414	517.930
	Savings	0.0%	31.54 %	43.39 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		536.334	401.965
	Savings		41.38 %	56.07 %
Route-planning with speed and load				*Model 4*
	Total cost			401.117
	Savings			56.16 %

Table 22: Cost comparison of instance E-5-2-30-1.
Source: Made by the authors.

Instance E-5-2-30-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	990.605	818.016	526.315
	Savings	0.0%	17.42 %	46.87 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		695.921	505.936
	Savings		29.75 %	48.93 %
Route-planning with speed and load				*Model 4*
	Total cost			489.002
	Savings			50.64 %

Table 23: Cost comparison of instance E-5-2-60-1.
Source: Made by the authors.

Instance E-5-2-60-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	859.576	674.176	505.721
	Savings	0.0%	21.57 %	41.17 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		500.028	388.869
	Savings		41.83 %	54.76 %
Route-planning with speed and load				*Model 4*
	Total cost			356.716
	Savings			58.50%

Table 24: Cost comparison of instance E-5-2-60-2.
Source: Made by the authors.

Instance E-5-2-60-2		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	889.768	611.248	379.119
	Savings	0.0%	31.30 %	57.39 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		828.720	366.047
	Savings		6.86 %	58.86 %
Route-planning with speed and load				*Model 4*
	Total cost			351.768
	Savings			60.47 %

Table 25: Cost comparison of instance F-4-3-30-1.
Source: Made by the authors.

Instance F-4-3-30-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1397.923	1026.465	733.711
	Savings	0.0%	26.57 %	47.51 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		803.792	565.972
	Savings		42.50 %	59.51 %
Route-planning with speed and load				*Model 4*
	Total cost			565.972
	Savings			59.51 %

Table 26: Cost comparison of instance F-4-3-60-1.
Source: Made by the authors.

Instance F-4-3-60-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1098.796	900.146	777.959
	Savings	0.0%	18.08 %	29.20 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		613.379	475.096
	Savings		44.18 %	56.76 %
Route-planning with speed and load				*Model 4*
	Total cost			475.096
	Savings			56.76 %

Table 27: Cost comparison of instance F-4-3-60-2.
Source: Made by the authors.

Instance F-4-3-60-2		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1256.122	811.145	593.243
	Savings	0.0%	35.42 %	52.77 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		743.435	561.271
	Savings		40.82 %	55.32 %
Route-planning with speed and load				*Model 4*
	Total cost			558.793
	Savings			55.51 %

Table 28: Cost comparison of instance G-6-5-30-1.
Source: Made by the authors.

Instance G-6-5-30-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	2093.876	1292.800	960.407
	Savings	0.0%	38.25%	54.13 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		1292.799	1024.376
	Savings		38.26 %	51.08 %
Route-planning with speed and load				*Model 4*
	Total cost			1021.698
	Savings			51.21 %

Table 29: Cost comparison of instance G-6-5-60-1.
Source: Made by the authors.

Instance G-6-5-60-1		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	1997.086	1115.115	753.117
	Savings	0.0%	44.16 %	62.29 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		1115.114	808.557
	Savings		44.16 %	59.51 %
Route-planning with speed and load				*Model 4*
	Total cost			804.005
	Savings			59.74 %

Table 30: Cost comparison of instance G-6-5-60-2.
Source: Made by the authors.

Instance G-6-5-60-2		**Initial solution**	**Optimized speed (constant load)**	**Optimized speed and load**
Route-planning (without speed and load)		*Model 1*	*Model 2*	*Model 3*
	Total cost	2064.710	1140.853	765.959
	Savings	0.0%	44.75%	62.90 %
Route-planning with speed (without load)			*Model 4-2*	*Model 3-2*
	Total cost		1139.077	821.233
	Savings		44.83 %	60.23 %
Route-planning with speed and load				*Model 4*
	Total cost			813.233
	Savings			60.61 %

Appendix 2

Table 31 presents the average speeds in knots provided by the different models on the given instances. The average speed is calculated by adding the speed used on the different routes together and dividing it by the number of port visits.

Table 31: Average speed by the vessels.
Source: Made by the authors.

Instance	**Model 1**	**Model 2**	**Model 3**	**Model 4-2**	**Model 2-2**	**Model 3-2**	**Model 4**
A-4-1-30-1	21.00	16.20	16.20	16.20	16.20	16.20	16.20
A-4-1-60-1	21.00	16.20	16.20	16.20	16.20	16.20	16.20
A-4-1-60-2	21.00	18.82	18.62	16.20	16.20	16.20	16.20
B-3-2-30-1	19.33	16.34	16.38	14.78	14.78	14.72	14.73
B-3-2-60-1	19.20	16.39	16.53	14.45	14.45	14.49	14.15
B-3-2-60-2	19.44	15.47	15.54	13.95	13.95	14.01	14.01
C-4-2-30-1	19.43	15.68	16.13	14.01	14.01	14.01	13.86
C-4-2-60-1	19.44	14.90	15.03	14.01	14.01	14.01	14.01
C-4-2-60-2	19.20	16.17	16.00	13.90	13.89	13.97	14.53
D-5-2-30-1	19.75	16.31	16.31	16.26	16.26	16.31	16.55
D-5-2-60-1	19.40	17.69	17.91	14.16	14.16	14.16	14.16
D-5-2-60-2	19.75	15.46	15.46	14.38	14.38	14.43	14.42
E-5-2-30-1	20.14	17.37	16.45	14.85	15.62	15.19	14.96
E-5-2-60-1	19.86	15.99	15.99	14.85	15.30	14.85	14.51
E-5-2-60-2	19.44	15.80	15.05	14.40	15.60	14.40	15.13
F-4-3-30-1	19.80	16.24	16.30	14.58	14.58	14.58	14.58
F-4-3-60-1	20.11	16.62	16.62	14.22	14.22	14.22	14.22
F-4-3-60-2	19.36	14.99	15.04	14.11	14.11	14.11	14.25

G-6-5-30-1	20.21	15.48	15.54	15.48	15.48	15.34	15.40
G-6-5-60-1	19.71	14.34	14.34	14.34	14.34	14.34	14.34
G-6-5-60-2	19.85	14.67	14.70	14.47	14.47	14.47	14.47

The lowest average speed is 13.86 knots and is given in model 4 on instance C-4-2-30-1. Further, the highest speed is 21 knots and is observed in model 1 on instances A-4-1-30-1, A-4-1-60-1 and A-4-1-60-2. On the different instances, it is models 4-2, 2-2, and 3-2 which gives the lowest average speeds. Model 4-2 has the lowest speed on 15 instances, while model 2-2 and model 3-2 has the lowest speed on 14 and 13 instances, respectively. This shows that on some of the instances, these models give the same average speed. Model 4 has the lowest speed on 13 of the instances.

Further, table 31 shows that the instances with 60 days planning horizon is able to reduce the average speed when compared to the same instances with 30 days planning horizon. Hence, longer planning horizons gives an opportunity to reduce the speed along the different routes.

Appendix 3

Table 32 presents the average load on each trip between ports provided by the models that considers load. The average load is calculated by summing the loads from each instance and dividing the total load by the number of visits.

Table 32: Average load of vessels in the different instances in tons
Source: Made by the authors.

Instance	Model 3	Model 3-2	Model 4
A-4-1-30-1	50.0	64.5	64.5
A-4-1-60-1	69.6	63.0	69.6
A-4-1-60-2	58.9	49.3	51.5
B-3-2-30-1	52.8	59.4	53.6
B-3-2-60-1	49.9	52.8	47.7
B-3-2-60-2	53.5	55.1	55.1
C-4-2-30-1	33.6	59.6	40.3
C-4-2-60-1	60.2	57.7	57.7
C-4-2-60-2	50.3	55.3	52.4
D-5-2-30-1	58.6	58.6	38.1
D-5-2-60-1	42.7	37.6	37.6
D-5-2-60-2	55.1	44.4	41.4
E-5-2-30-1	49.4	47.2	47.2
E-5-2-60-1	29.0	62.5	33.0
E-5-2-60-2	30.8	37.3	32.6
F-4-3-30-1	60.6	56.8	56.8
F-4-3-60-1	69.7	58.1	58.1
F-4-3-60-2	55.6	55.6	55.6
G-6-5-30-1	63.1	73.5	66.8
G-6-5-60-1	61.9	62.6	61.9
G-6-5-60-1	53.5	57.0	57.0

The lowest average load on a trip is 29.0 tons. This load is provided by model 3 for instance E-5-2-60-1. The highest average load on a trip is 73.5 tons and is provided by model 3-2 for instance G-6-5-30-1.

When comparing model 3 and model 4, Table 32 shows that model 3 has the highest average load in 9 instances, while model 4 has the highest average load in 8 instances. In instances A-4-1-60-1, F-4-3-60-2 and G-6-5-60-1 the average loads are identical for model 3 and model 4.

Comparisons of model 3 and model 3-2 shows that model 3-2 gives the highest average loads on 11 of the instances, while model 3 has the highest average on 8 of the instances. Further, there are two instances with identical loads, which is D-5-2-30-1 and F-4-3-60-2.

The last comparison is of model 3-2 and model 4. Table 32 shows that model 3-2 is the best model to provide the highest average loads, as it does so on 9 of the instances, while model 4 only does so on 2 instances. However, on 8 of the instances, they have identical loads, which evens out the difference a bit.

Appendix 4

This part includes the computation errors which occurs in the different instances in model 1, 2 and 3. Table 33 shows approximated cost, the real cost, and the deviation.

Table 33: Computational errors in model 1, model 2 and model 3.
Source: Made by the authors.

Instances	Model 1			Model 2			Model 3		
	Approximated Cost	Real cost	Deviation	Approximated Cost	Real cost	Deviation	Approximated Cost	Real cost	Deviation
A-4-1-30-1	764.201	764.062	+ 0.018 %	465.910	465.789	+ 0.026 %	320.049	321.276	- 0.380 %
A-4-1-60-1	1228.430	1228.210	+ 0.018 %	747.867	747.673	+ 0.026 %	561.324	563.759	- 0.432 %
A-4-1-60-2	1100.630	1100.433	+ 0.018 %	881.227	881.027	+ 0.023 %	652.592	657.089	- 0.689%
B-3-2-30-1	924.793	924.795	-0.0002%	715.506	711.474	+0.567%	491.667	492.654	-0.2003%
B-3-2-60-1	1328.130	1328.108	+ 0.0012%	1027.460	1025.809	+ 0.161%	775.187	778.106	- 0.375%
B-3-2-60-2	1420.695	1420.735	- 0.0028%	967.385	965.891	+ 0.155%	746.955	747.291	- 0.045%

C-4-2-30-1	C-4-2-60-1	C-4-2-60-2	D-5-2-30-1	D-5-2-60-1	D-5-2-60-2	E-5-2-30-1	E-5-2-60-1	E-5-2-60-2
859.676	1197.29	1064.59	1008.770	1008.770	915.029	990.641	859.611	889.82
859.577	1197.127	1064.085	1008.713	1008.713	914.987	990.605	859.576	889.768
+ 0.012 %	+ 0.014 %	+ 0.047 %	+ 0.0057%	+ 0.0057%	+ 0.0046%	+ 0.004 %	+ 0.004 %	+ 0.006 %
624.781	739.905	872.697	943.326	943.326	627.516	819.632	674.805	611.29
624.595	739.500	869.24415	942.303	942.303	626.414	818.016	674.176	611.248
+ 0.030 %	+ 0.055 %	+ 0.396 %	+ 0.109%	+ 0.109%	+ 0.176%	+ 0.197 %	+ 0.093%	+ 0.007 %
470.669	590.612	689.589	668.289	668.289	515.752	522.996	520.834	377.076
474.532	632.271	691.914	681.309	681.309	517.930	526.315	505.721	379.119
- 0.814 %	- 6.589 %	- 0.360 %	- 1.911%	- 1.911%	- 0.405%	- 0.631 %	+ 2.988 %	- 0.539 %

F-4-3-30-1	F-4-3-60-1	F-4-3-60-2	G-6-5-30-1	G-6-5-60-1	G-6-5-60-2
1398.000	1098.810	1256.23	2094.01	1997.32	2064.96
1397.923	1098.796	1256.122	2093.876	1997.086	2064.710
+ 0.0055%	+ 0.0013%	+ 0.0086%	+ 0.006 %	+ 0.012 %	+ 0.012 %
1027.354	900.601	812.559	1291.43	1112.44	1184.02
1026.465	900.146	811.145	1292.800	1115.115	1140.853
+ 0.087%	+ 0.051%	+ 0.174%	- 0.106 %	- 0.240 %	+ 3.784 %
734.806	776.035	590.308	989.7	802.222	828.998
733.771	777.959	593.243	960.407	753.117	765.959
+ 0.141%	- 0.247%	- 0.495%	+ 3.050 %	+ 6.520 %	+ 8.230 %

Appendix 5

This part includes the computation errors which occurs in the different instances in model 4-2, 2-2 and 3-2. Table 34 shows approximated cost, the real cost, and the deviation.

Table 34: Computational errors in model 4-2, model 2-2 and model 3-2.
Source: Made by the authors.

	Instances	**A-4-1-30-1**	**A-4-1-60-1**	**A-4-1-60-2**	**B-3-2-30-1**	**B-3-2-60-1**	**B-3-2-60-2**
Model 4-2	Approximated Cost	288.657	747.867	747.867	627.512	831.597	828.165
	Real cost	288.584	747.673	747.673	626.720	829.951	828.132
	Deviation	+ 0.025 %	+ 0.026 %	+ 0.026 %	+ 0.126%	+ 0.198%	+ 0.004%
Model 2-2	Approximated Cost	288.657	747.867	747.867	627.512	831.597	828.165
	Real cost	288.584	747.673	747.673	626.720	829.951	828.132
	Deviation	+ 0.025 %	+ 0.026 %	+ 0.026 %	+ 0.126%	+ 0.198%	+ 0.004%
Model 3-2	Approximated Cost	234.286	568.437	532.144	453.314	600.878	598.857
	Real cost	235.462	570.874	535.067	454.101	605.246	601.460
	Deviation	- 0.500 %	- 0.427 %	- 0.546 %	- 0.173%	- 0.722%	- 0.433%

C-4-2-30-1	**C-4-2-60-1**	**C-4-2-60-2**	**D-5-2-30-1**	**D-5-2-60-1**	**D-5-2-60-2**	**E-5-2-30-1**	**E-5-2-60-1**	**E-5-2-60-2**
475.111	668.656	716.893	706.893	646.132	536.506	695.939	500.049	528.748
474.993	668.456	716.572	705.398	645.979	536.334	695.921	500.028	528.720
+ 0.025 %	+ 0.030 %	+ 0.045 %	+ 0.212%	+ 0.024%	+ 0.032%	+ 0.003 %	+ 0.004 %	+ 0.005 %
475.111	668.656	716.893	706.893	646.132	536.506	695.939	500.049	528.748
474.993	668.456	716.572	705.398	645.979	536.334	695.921	500.028	528.720
+ 0.025 %	+ 0.030 %	+ 0.045 %	+ 0.212%	+ 0.024%	+ 0.032%	+ 0.003 %	+ 0.004 %	+ 0.005 %
411.257	521.388	557.199	583.470	447.854	397.403	480.104	372.433	364.644
412.129	522.524	558.501	585.309	451.071	401.965	505.936	388.869	366.047
- 0.212 %	- 0.218	- 0.233 %	- 0.314%	- 0.713%	- 1.135%	- 5.106 %	- 4.227 %	- 0.383 %

F-4-3-30-1	**F-4-3-60-1**	**F-4-3-60-2**	**G-6-5-30-1**	**G-6-5-60-1**	**G-6-5-60-2**
803.696	613.339	743.489	1294.21	1115.23	1184.9
803.792	613.379	743.435	1292.799	1115.114	1139.077
- 0.0119%	- 0.006%	+ 0.007%	+ 0.109 %	+ 0.010 %	+ 4.023 %
803.696	613.339	743.489	1294.21	1115.23	1184.9
803.792	613.379	743.435	1292.799	1115.114	1139.077
- 0.0119%	- 0.006%	+ 0.007%	+ 0.109 %	+ 0.010 %	+ 4.023 %
564.848	474.395	559.496	989.7	802.222	818.65
565.972	475.096	561.271	1024.376	808.557	821.233
- 0.199%	- 0.148%	- 0.316%	- 3.385 %	- 0.783	- 0.315 %

Appendix 6

This part includes the computation errors which occurs in the different instances in model 4. Table 35 shows approximated cost, the real cost, and the deviation.

Table 35: Computational errors in model 4
Source: Made by the authors.

Instance	**Model 4**		
	Approximated cost	*Real cost*	*Deviation From real cost*
A-4-1-30-1	234.286	235.462	- 0.502 %
A-4-1-60-1	561.324	563.653	- 0.415 %
A-4-1-60-2	520.502	522.919	- 0.464 %
B-3-2-30-1	436.242	436.245	- 0.0007%
B-3-2-60-1	578.299	582.698	- 0.755%
B-3-2-60-2	598.857	601.460	- 0.433%
C-4-2-30-1	351.329	353.070	- 0.496 %
C-4-2-60-1	521.388	522.524	- 0.218 %
C-4-2-60-2	544.668	543.621	+ 0.192 %
D-5-2-30-1	484.077	491.124	- 1.435%
D-5-2-60-1	447.854	451.092	- 0.718%
D-5-2-60-2	396.663	401.117	- 1.110%
E-5-2-30-1	480.104	489.002	- 1.819 %
E-5-2-60-1	351.199	356.716	- 1.547 %
E-5-2-60-2	349.872	351.768	- 0.539 %
F-4-3-30-1	564.848	565.972	- 0.196%
F-4-3-60-1	474.395	475.096	- 0.148%
F-4-3-60-2	557.589	558.793	- 0.216%
G-6-5-30-1	989.281	1021.698	- 3.276 %
G-6-5-60-1	802.222	804.00468	- 0.222 %
G-6-5-60-2	818.65	813.23302	+ 0.662 %

Appendix 7

This part includes the AMPL codes from the .mod-file of model 4. The .mod contains sets, parameters, variables, objective function, and constraints. Further, the variables and the constraints are further divided into types, to get a better overview of the structure. To be able to run the .mod-file, it is required to have files with .dat and .run extensions. The solution is transferred into a file with .sol extension.

```
##########SETS##########
set PORTS;
set SHIPS;
set SPEEDS;
set LOADS ordered;

##########PARAMETERS##########
param T>=0;
param Vmin {i in PORTS} >=0;
param Vmax {i in PORTS} >=0;
param PortCost {i in PORTS, v in SHIPS} >=0;
param DemandRate {i in PORTS} >=0;
param J {i in PORTS};
param ShipCap {v in SHIPS} >=0;
param InitialLoad {v in SHIPS} >=0;
param LoadRate {v in SHIPS} >=0;
param UpperStock {i in PORTS} >=0;
param LowerStock {i in PORTS} >=0;
param InitialStock {i in PORTS} >=0;
param Distance {i in PORTS, j in PORTS} >=0;
param Speed {v in SHIPS, s in SPEEDS} >=0;
param load {v in SHIPS, l in LOADS} >= 0;
param DailySailCost {v in SHIPS, l in LOADS, s in SPEEDS} >=0;
param Evisit {i in PORTS, m in 1..Vmax[i]}:=0;
param Qmin {i in PORTS} >=0;
param TB {i in PORTS}>=0;
```

```
param Lvisit {i in PORTS, m in 1..Vmax[i]}:=T;
param Ltime {i in PORTS, m in 1..Vmax[i]}:=min(T, T+Lvisit[i,m]*UpperStock[i]);
param TL {v in SHIPS}:=1/LoadRate[v];
param DistOrig {i in PORTS, v in SHIPS} >=0;
param TravelTime {i in PORTS, j in PORTS, v in SHIPS, s in SPEEDS}:=
Distance[i,j]/(24*Speed[v,s]);
param TravelCost {i in PORTS, j in PORTS, v in SHIPS, l in LOADS, s in SPEEDS}:=
DailySailCost[v,l,s]*TravelTime[i,j,v,s];
param TravelTimeOrig {i in PORTS, v in SHIPS, s in
SPEEDS}:=DistOrig[i,v]/(24*Speed[v,s]);
param TravelCostOrig {i in PORTS, v in SHIPS,l in LOADS, s in SPEEDS}:=
DailySailCost[v,l,s] * TravelTimeOrig[i,v,s];

##########VARIABLES##########

###ROUTING VARIABLES###
var X {i in PORTS, m in 1..Vmax[i], j in PORTS, n in 1..Vmax[j], v in SHIPS: i<>j}
binary;
var Xo {i in PORTS, m in 1..Vmax[i], v in SHIPS} binary;
var Z {i in PORTS, m in 1..Vmax[i], v in SHIPS} binary;
var Zo {v in SHIPS} binary;
var O {i in PORTS, m in 1..Vmax[i], v in SHIPS} binary;
var Y {i in PORTS, m in 1..Vmax[i]} binary;

###SPEED AND LOAD VARIABLES###
var G {i in PORTS, m in 1..Vmax[i], j in PORTS, n in 1..Vmax[j], v in SHIPS, l in
LOADS,
s in SPEEDS: i<>j} >=0, <=1;
var Go {i in PORTS, m in 1..Vmax[i], v in SHIPS, l in LOADS, s in SPEEDS} >=0, <=1;
var P {i in PORTS, m in 1..Vmax[i], j in PORTS, n in 1..Vmax[j],v in SHIPS, R in 1..card
(LOADS)-1} binary;
var Po {i in PORTS,m in 1..Vmax[i],v in SHIPS,R in 1..card(LOADS)-1} binary;
```

```
###FLOW VARIABLES###
var Q {i in PORTS, m in 1..Vmax[i], v in SHIPS} >=0;
var F {i in PORTS, m in 1..Vmax[i], j in PORTS, n in 1..Vmax[j], v in SHIPS: i<>j} >=0;
var Fo {i in PORTS, m in 1..Vmax[i], v in SHIPS} >=0;
var Fd {i in PORTS, m in 1..Vmax[i], v in SHIPS} >=0;

###TIME VARIABLES###
var t {i in PORTS, m in 1..Vmax[i]} >=0, <=T;

###STOCK VARIABLES###
var S {i in PORTS, m in 1..Vmax[i]}>=0;


##########OBJECTIVEFUNCTION##########
minimize Total_Cost:
sum {i in PORTS, m in 1..Vmax[i], j in PORTS, n in 1..Vmax[j], v in SHIPS, l in LOADS,
s in SPEEDS: i<>j} TravelCost[i,j,v,l,s] * G[i,m,j,n,v,l,s] +
sum {v in SHIPS, i in PORTS, m in 1..Vmax[i], l in LOADS, s in SPEEDS}
TravelCostOrig[i,v,l,s] * Go[i,m,v,l,s] +
sum {v in SHIPS, i in PORTS, m in 1..Vmax[i]} PortCost[i,v] * O[i,m,v];
##########CONSTRAINTS##########

###BREAKPOINTS INTERVAL CONSTRAINTS###
subject to FIRST{i in PORTS,m in 1..Vmax[i],j in PORTS,n in 1..Vmax[j],v in SHIPS}:
sum {R in 1..card(LOADS)-1} P[i,m,j,n,v,R] <= 1 ;

subject to SECOND{i in PORTS,m in 1..Vmax[i],j in PORTS,n in 1..Vmax[j],v in SHIPS:
i<>j}: sum {s in SPEEDS}G[i,m,j,n,v,1,s] <= P[i,m,j,n,v,1];

subject to SECOND2{i in PORTS,m in 1..Vmax[i],j in PORTS,n in 1..Vmax[j],v in SHIPS:
i<>j}: sum {s in SPEEDS}Go[i,m,v,1,s] <= Po[i,m,v,1];
```

```
subject to THIRD{i in PORTS, m in 1..Vmax[i], j in PORTS,n in 1..Vmax[j],v in SHIPS,
R in 2..card(LOADS)-1: i<>j}:
sum {s in SPEEDS}G[i,m,j,n,v,R,s] <= P[i,m,j,n,v,R-1] + P[i,m,j,n,v,R] ;

subject to THIRD2{i in PORTS, m in 1..Vmax[i], j in PORTS, n in 1..Vmax[j], v in SHIPS,
R in 2..card(LOADS)-1: i<>j}:
sum {s in SPEEDS}Go[i,m,v,R,s] <= Po[i,m,v,R-1] + Po[i,m,v,R] ;

subject to FOURTH {i in PORTS, m in 1..Vmax[i], j in PORTS, n in 1..Vmax[j], v in
SHIPS, R in 1..card(LOADS)-1: i<>j}:
sum {s in SPEEDS}G[i,m,j,n,v,card(LOADS),s] <= P[i,m,j,n,v,card(LOADS)-1];

subject to FOURTH2 {i in PORTS, m in 1..Vmax[i], j in PORTS,n in 1..Vmax[j], v in
SHIPS, R in 1..card(LOADS)-1: i<>j}:
sum {s in SPEEDS}Go[i,m,v,card(LOADS),s] <= Po[i,m,v,card(LOADS)-1];

###ROUTING CONSTRAINTS###
subject to FLOW1 {v in SHIPS}:
sum {j in PORTS, n in 1..Vmax[j]} Xo[j,n,v] + Zo[v]=1;

subject to FLOW2 {v in SHIPS, i in PORTS, m in 1..Vmax[i]}:
O[i,m,v] - sum {j in PORTS, n in 1..Vmax[j]: j<>i} X[j,n,i,m,v] - Xo[i,m,v]=0;

subject to FLOW3 {v in SHIPS, i in PORTS, m in 1..Vmax[i]}:
O[i,m,v] - sum {j in PORTS, n in 1..Vmax[j]: i<>j} X[i,m,j,n,v] - Z[i,m,v]=0;

subject to SHIP_VISIT {i in PORTS, m in 1..Vmax[i]}: sum {v in SHIPS}
O[i,m,v] = Y[i,m];

subject to PORT_VISIT {i in PORTS, m in 2..Vmax[i]}:
Y[i,m-1] - Y[i,m] >= 0;

subject to MANDATORY_VISITS {i in PORTS, m in 1..Vmin[i]}:
Y[i,m] = 1;
```

```
###SPEED and LOAD CONSTRAINTS###
subject to SPEED_CHOICE1 {i in PORTS, m in 1..Vmax[i], j in PORTS, n in 1..Vmax[j],
v in SHIPS: i<>j}: sum {l in LOADS, s in SPEEDS} G[i,m,j,n,v,l,s] = X[i,m,j,n,v];

subject to SPEED_CHOICE2 {i in PORTS, m in 1..Vmax[i], v in SHIPS}:
sum {l in LOADS, s in SPEEDS} Go[i,m,v,l,s] = Xo[i,m,v];

###LOADING AND UNLOADING CONSTRAINTS###
subject to CONSTRAINT2 {i in PORTS, m in 1..Vmax[i], v in SHIPS}:
Q[i,m,v] <= min(ShipCap[v], UpperStock[i]) * O[i,m,v];

subject to CONSTRAINT3 {i in PORTS, m in 1..Vmax[i], v in SHIPS}:
Qmin[i] * O[i,m,v] <= Q[i,m,v];

subject to CONSTRAINT4 {v in SHIPS, i in PORTS, m in 1..Vmax[i]}:
Fo[i,m,v] = InitialLoad[v] * Xo[i,m,v];

subject to LOADCONSTRAINT {i in PORTS, m in 1..Vmax[i], j in PORTS, n in
1..Vmax[j], v in SHIPS: i<>j}:
sum {l in LOADS, s in SPEEDS} load[v,l] * G[i,m,j,n,v,l,s] = F[i,m,j,n,v];
subject to LOADCONSTRAINT2 {i in PORTS, m in 1..Vmax[i], v in SHIPS}:
sum {l in LOADS, s in SPEEDS} load[v,l] * Go[i,m,v,l,s] = Fo[i,m,v];

###ARC - FLOW MODEL###
subject to CONSTRAINT5 {v in SHIPS, j in PORTS, n in 1..Vmax[j]}:
Fo[j,n,v] + sum {i in PORTS, m in 1..Vmax[i]: i<>j}
F[i,m,j,n,v] + J[j] * Q[j,n,v] =sum {i in PORTS, m in 1..Vmax[i]: j<>i} F[j,n,i,m,v] +
Fd[j,n,v];

subject to CONSTRAINT6 {i in PORTS, j in PORTS, m in 1..Vmax[i], n in 1..Vmax[j], v
in SHIPS: j<>i}: # and J[i]=1}:
F[i,m,j,n,v] <= ShipCap[v] * X[i,m,j,n,v];
```

```
subject to CONSTRAINT7 {j in PORTS, n in 1..Vmax[j], v in SHIPS}:
Fd[j,n,v] <= ShipCap[v] * Z[j,n,v];

###TIME CONSTRAINTS###
subject to START_TIME {i in PORTS, j in PORTS, m in 1..Vmax[i], n in 1..Vmax[j]:
i<>j}: t[i,m] + sum {v in SHIPS} TL[v] * Q[i,m,v] - t[j,n] +
sum {v in SHIPS, s in SPEEDS, l in LOADS} max(Ltime[i,m] +
TravelTime[i,j,v,s] -Evisit[j,n],0) * G[i,m,j,n,v,l,s] <= Ltime[i,m]-Evisit[j,n];

subject to MIN_INTERVAL {i in PORTS, m in 1..Vmax[i]:m>1}:
t[i,m] - t[i,m-1] - sum {v in SHIPS} TL[v] * Q[i,m-1,v] - TB[i] * Y[i,m] >=0;

subject to CONSTRAINT {i in PORTS, m in 1..Vmax[i]}:
sum {v in SHIPS, s in SPEEDS, l in LOADS} TravelTimeOrig[i,v,s] * Go[i,m,v,l,s] <=
t[i,m];

subject to TIME_WINDOW1 {i in PORTS, m in 1..Vmax[i]}:
t[i,m] >= Evisit[i,m];

subject to TIME_WINDOW2 {i in PORTS, m in 1..Vmax[i]}:
t[i,m] <= Lvisit[i,m];

###INVENTORY CONSTRAINRS###
subject to STOCK_START {i in PORTS}:
S[i,1] = InitialStock[i] + J[i] * DemandRate[i] * t[i,1];

subject to RELATE_STOCK {i in PORTS, m in 1..Vmax[i]:m>1}:
S[i,m] = S[i,m-1] - J[i] * sum{v in SHIPS} Q[i,m-1,v] + J[i] * DemandRate[i] * (t[i,m] -
t[i,m-1]);

subject to STOCK_LIMIT1 {i in PORTS, m in 1..Vmax[i]:J[i]=-1}:
S[i,m] + sum{v in SHIPS} Q[i,m,v] - DemandRate[i] * sum{v in SHIPS} TL[v] * Q[i,m,v]
<= UpperStock[i];
```

```
subject to STOCK_LIMIT2 {i in PORTS, m in 1..Vmax[i]:J[i]=1}:
S[i,m] - sum{v in SHIPS} Q[i,m,v] + DemandRate [i] * sum{v in SHIPS} TL[v] * Q[i,m,v]
>= LowerStock[i];

subject to LBOUND {i in PORTS:J[i]=-1}:
S[i, Vmax[i]] + sum{v in SHIPS} Q[i, Vmax[i],v] - DemandRate[i] * (T-t[i, Vmax[i]])
>= LowerStock[i];

subject to UBOUND {i in PORTS:J[i]=1}:
S[i, Vmax[i]] - sum{v in SHIPS} Q[i, Vmax[i],v] + DemandRate[i] * (T-t[i, Vmax[i]])
<= UpperStock[i];

subject to LIMIT1 {i in PORTS, m in 1..Vmax[i]: J[i]=-1}:
S[i,m] >= LowerStock[i];

subject to LIMIT2 {i in PORTS, m in 1..Vmax[i]: J[i]=1}:
S[i,m] <= UpperStock[i];
```