



Master's degree thesis

LOG950 Logistics

An exact approach for collecting manures from local farms by the planned biogas plant Hustadvika Biokraft AS

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This master thesis represents my final academic work as a master student at Molde University College – Specialized University in Logistics. From November 2018 to June 2020, the research has been carried out in order to obtain a MSc in Logistics.

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Abstract

This thesis considers the real case vehicle routing problem (VRP) of Hustadvika Biokraft AS. They are dependent of the logistics of collecting the manure from the local farms and delivery of the decomposing residue to the local farms to be efficient. The thesis focuses on the logistics part concerning collecting manure from the local farms, which can be defined as an inventory routing problem (IRP).

Through the research work we have studied the problem and collected data from Hustadvika Biokraft AS, followed by development of a mathematical model that is based on a basic version of IRP presented by Coelho, Cordeau, and Laporte (2014). The basic version was rewritten from a delivery problem to a pickup problem. We then tested and analysed the model, which is a simplified version of the real case problem at this point. Great adaptations are needed in order for the model to be useable for a biogas plant such as Hustadvika Biokraft AS. Though the aim of this thesis is meant to be only a starting point and lay a foundation where the end goal is to create a heuristic for finding the best routing plan.

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1.0 Introduction

In this section the thesis topic is presented, the aim of the thesis and research questions are described and presented, the motivation and background for thesis topic are described, and an overview of the structure of the thesis is given.

1.1 Motivation and background for the thesis

Norway has a goal of becoming carbon neutral by 2030 (Miljøverndepartementet 2012) and has therefore among others created a strategy focusing on bioenergy in order to reach this goal. The government's ambition is to have 30% of the manures in Norway to go through a biogas reactor by 2020 (Landbruks- og matdepartementet 2009). So far, there are about 40 biogas plants in Norway and few of them are using manures as feedstock (Norges Bondelag 2011). Considering the bioenergy strategy of the government of Norway and the increase of demand for biogas, we can expect there will be an increase of biogas plants in Norway in the short future.

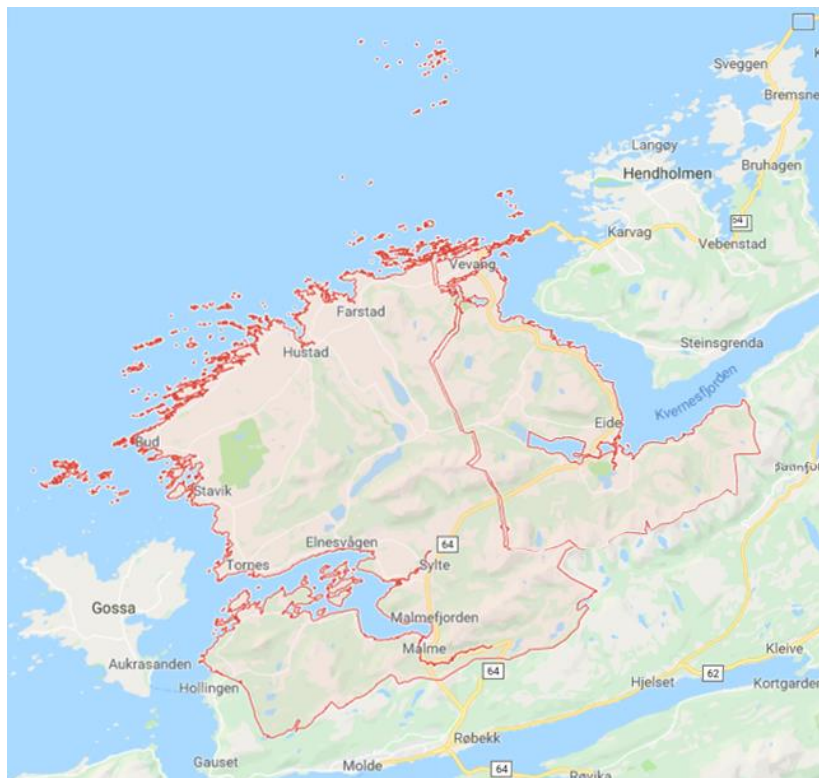


Figure 1 Map of Hustadvika municipality

The two municipalities Fræna and Eide were fused together in January 2020 and became Hustadvika municipality (see Figure 1). They are now the largest municipality in the western part of Norway (in the three counties Rogaland, Vestland, Møre & Romsdal and

Trøndelag) between Stavanger and Trondheim when it comes to amount of cattle, as you can see in Figure 2 (Statistisk Sentralbyrå 2017, Norwegian Government 2018).

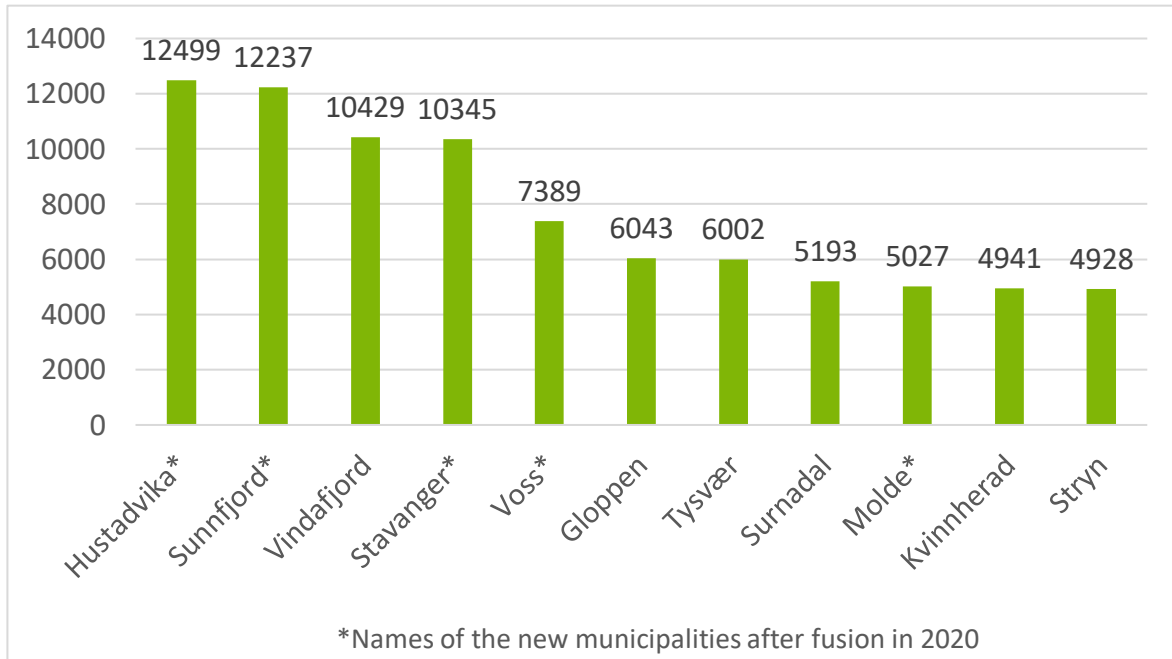


Figure 2 Top 10 municipalities between Stavanger and Trondheim in amount of cattle in 2017

It has therefore good potential as a location for a biogas plant based on manure as raw material. Hustadvika Biokraft AS is planning to do just that at Harøysundet. In addition to using manure, they are also planning to use fish slurry from a planned local fish farm. They are also planning to use wood waste from RIR (Romsdal Interkommunale Renovasjonsselskap), to heat the biogas reactor. The outputs will be biogas, excess heat and decomposing residue to be used as fertilizer. It takes 20 days in average from the arrival of the manure and fish slurry at the biogas plant, until the process of producing biogas is finished (see Figure 3).

The logistics of collecting the manure from the farms and delivery of the decomposing residue to be used as fertilizer to the farms are in itself vast, and we will therefore be focusing on this side of the transportation. The farms that will be involved in the pickup and delivery, are to be located 2.6-48.8 km from the biogas plant and are producing 150 000 – 200 000 tons manures in total per year (500-7000 tons per farm). This is equivalent to 10-20 semi-trailers per day in average. With such a high number it is crucial for the company to have

good logistical solutions, including a vehicle routing plan, that works in a cost efficient and environmental way.



Figure 3 Inputs and outputs of Hustadvika Biokraft AS

There are some aspects that complicates what could have been a basic vehicle routing problem (VRP) with backhauls.

Decomposing residue vs. manure

The first challenge is there will be more decomposing residue to deliver to the farms than the amount of manure to pick up, since there will also be residue from the fish slurry. This means that there will be more vehicles going out delivering than coming back with pickups. Since the manure and decomposing residue should not mix, the vehicles need to deliver all the decomposing residue first before starting to pick up manure on their routes.

Types of farms/storage facilities

Another challenge is that there are two types of farms when it comes to storage facilities for manure and decomposing residue. The first type of farms has several storage facilities for the manure. At the farm itself is a manure tank located in the same building as the cattle, in addition to 1-3 manure/slurry stores located a bit away from the farm. With these farms Hustadvika Biokraft AS plans to fill up the manure/slurry stores with decomposing residue while emptying the manure tanks for manure, through the whole year. Of the 84 farms that will be delivering manure and receiving decomposing residue, 36 of these have several storage facilities. The storage capacity of the manure/slurry stores have a capacity from 500 to 6000 tons.

The second type of farms has only one storage facility, which is a tank/cellar located in the same building as the cattle. Since these farms have only one storage facility and the

manure and residue cannot be mixed, Hustadvika Biokraft AS will have to prioritize to empty as much as possible the tanks/cellars before delivering the decomposing residue that is needed, not too long before the farmers are to fertilize their fields. Of the 84 farms that will be delivering manure and receiving decomposing residue, 48 of these have only one storage facility.

As part of solving these issues, Hustadvika Biokraft AS is planning to have a storage facility for the residue at the biogas plant. In addition, there is a possibility for the farms to collaborate on storage facilities between neighboring farms where residue is stored at one farm while manure is stored at the other.

Inaccessible roads

Another challenge is roads that might not be accessible all year round. At some of the farms the storage facilities are located a certain distance away from the main farmhouse, which can be reached by using a separate road. This road is often blocked by snow during the winter, meaning Hustadvika Biokraft AS's vehicles cannot access these storage facilities in periods with snow. In addition, there are some roads that has restrictions for heavy vehicles during periods where the ground frost is thawing, usually in the spring. The manures at these farms should therefore be picked up before the periods of inaccessibility strikes and be a priority for delivery of decomposing residue around April, so it is ready for when the farmers spread the fertilizer onto their fields in the end of April/beginning of May.

The annual cycle of demand

The farms' demand of the decomposing residue through the year is also a challenge. The annual cycle of the farms when it comes to fertilizing the fields and harvesting, is shown in Figure 4. As we can see, there are three periods where the farmers are fertilizing their fields: spring, summer and autumn. The first fertilization period takes place as soon as the snow has disappeared and the ground frost has thawed, while the second fertilization period takes place as soon as the first harvest is finished. The third fertilization period takes place after the second harvest is finished and is used to get rid of any leftover manure (or in future decomposing residue), so the farms will have enough space in their storage facilities over the winter. Not all the farms have had any need of fertilizing the fields after the second harvest period in the past. But since there will be more tons decomposing residue received at the farms than tons manure going out, we can expect that most of the farms will need to have a third fertilization period. In Figure 4 the third fertilization period is longer than the two first periods. Although most farmers fertilize their fields as soon as the second harvest

is finished, usually by September 1st, they can fertilize the fields until October 31st by law, unless the ground frost has set.

Hustadvika Biokraft AS must therefore make sure all the farms have all the decomposing residue they will need during the fertilizing periods, before they start. For the first fertilization period, each farm will need approximately 60% of its total storage capacity of decomposing residue delivered beforehand. For the second fertilization period, each farm will need approximately 40% of its total storage capacity of decomposing residue delivered beforehand. As mentioned previously, the third fertilization period is used for getting rid of any leftover manure (or in future decomposing residue).

What exact time of the year the fertilization and harvesting periods take place depends very much on the weather, and it varies therefore from year to year.



Figure 4 Annual cycle of fertilization and harvest at the farms

Daily deliveries and pickups

Hustadvika Biokraft AS is planning to have two vehicles with four drivers to do the deliveries and pick-ups at the farms Mondays to Fridays from 7am until 9pm-10pm and possibly Saturdays from 9am until 4pm. This means that there might be more manure to pick

up and decomposing residue to deliver at the beginning of the week than the end of the week, since it has piled up over the weekend. It also means that the amount of manure picked up from Monday to Saturday must cover the demand of manure at the factory for Sunday as well.

This VRP will be a combination of VRP with inventory, backhauls and multiple vehicles. In other words, it can be categorized as a rich VRP, which is a big research field.

1.2 Research questions

Due to the complexity of the problem, the thesis aims to lay a foundation where the end goal is to create a heuristic for finding the best routing plan for collecting manure and delivering decomposing residue at local farms by Hustadvika Biokraft AS. The main research question will be:

- **What is the shortest route measured in time?**

With the following sub-questions:

- **When should each farm be serviced?**
- **How often should each farm be serviced?**
- **How much manure should be picked up for each visit at each farm?**
- How much decomposing residue should be delivered for each visit at each farm?
- Will two vehicles be able to serve all the routes?

In this thesis we will study and try to formulate the problem based on the pickup of manure from the farms, the relevant research questions for the problem are highlighted above. The main research task will be to analyse the problem and create a mathematical model, and then to use the model and the data received from Hustadvika Biokraft AS in a solver in order to analyse the model.

1.3 Structure of the thesis

The remainder of the thesis is structured as followed: In section 2 we present relevant literature for the research. The approach chosen in order to answer the research questions are described in section 3, as well as describing the case problem and developing the mathematical model. This is closely followed by testing of the model and an analysis of the model in section 4. In the end the conclusion and recommendations for further work come in section 5.

2.0 Literature Review

The vehicle routing problem (VRP) is a common field within optimization. The problem often includes product(s) to be delivered from one place (a depot) to multiple places (customers) by a vehicle, where the goal is to e.g. minimize total costs or maximize profit. There are many different types of vehicle routing problems (Toth and Vigo 2002a), such as:

- The inventory routing problem (IRP): the inventory management and routing problem is combined (Andersson et al. 2010).
- The VRP with backhauling: there is both delivery and pickup of products (Koç and Laporte 2018).
- VRPs with multiple vehicles: instead of using only one vehicle for the delivery, multiple vehicles can be used.
- The VRP with time windows: routing problems where the delivery must take place within certain time windows at the customers (Koç and Laporte 2018).
- VRPs with multiple objectives: routing problems with multiple objectives such as minimizing routes and minimizing costs (Koç and Laporte 2018).

We could go on with the list. In addition, there are also several variants of the VRP variants listed above. A real case problem is often a combination of several VRP variants and makes the problem quite complex. The VRP in this case is as mentioned earlier a combination of inventory, backhauling and multiple vehicles.

2.1 The inventory routing problem (IRP)

Problems combining vehicle routing and inventory management has been most commonly named as the inventory routing problems (Campbell, Clarke, and Savelsbergh 2002). The objective is usually to minimize the total cost while making sure no customer stocks out at any time (Bard et al. 1998, Campbell, Clarke, and Savelsbergh 2002).

According to Campbell, Clarke, and Savelsbergh (2002), there are three decisions to be made while solving an IRP:

1. Decide when to serve each customer.
2. Decide how much one should deliver to a customer when served.
3. Decide which delivery routes to use.

An IRP is often a long-term problem, though to make routing plans for a long-term horizon would make the problem too big, challenging and unreliable. Therefore, it is

necessary to reduce the planning horizon from long-term to short-term. Bard et al. (1998) proposed a step by step approach based upon the idea of rolling horizon:

1. Identify customers that are to be visited during the planning horizon, say two weeks. If the next visit of a customer falls within the planning horizon, the customer is selected.
2. Make an adjustment to balance daily demand.
3. Route the customers that are scheduled for the first week of the planning horizon.
4. Then extend the planning horizon and repeat the process.

In their paper, Bard et al. (1998) also developed and tested three VRP-based heuristics modified to account for satellite facilities: randomized version of the Clarke-Wright algorithm, GRASP and a revised sweep algorithm. They found that the Clarke-Wright algorithm outperformed the others based on distance traveled and total incremental costs.

Both Andersson et al. (2010) and (Coelho, Cordeau, and Laporte 2014) have done a literature survey of the IRP up through the years. In the paper *Thirty Years of Inventory Routing* by Coelho, Cordeau, and Laporte (2014) a basic version of the IRP using an exact algorithm is presented. It was first developed by Archetti et al. (2007) and later extended by Coelho and Laporte (2013) and Adulyasak, Cordeau, and Jans (2013). This algorithm can be used to formulate and solve IRP where multiple vehicles deliver a product from the depot to multiple customers in order to cover their demand, in other words one-to-many.

2.2 The vehicle routing problem with backhauls (VRPB)

The vehicle routing problem with backhauls are also known as the linehaul-backhaul problem. The customers in a VRPB are split into two subsets; linehaul customers and backhaul customers. The linehaul customers need deliveries of outbound product, while backhaul customers have inbound products to be picked up.

Toth and Vigo (2002b) described the VRPB as follows:

- Each vehicle performs one route.
- Each route starts and finishes at the depot.
- On each route the backhaul customers, if any, are visited after all linehaul customers.
- For each route the total load associated with linehaul and backhaul customers does not exceed, separately, the vehicle capacity.
- The total distance traveled by the vehicles is to be minimized.

Koç and Laporte (2018) added two more points to Toth and Vigo's description:

- Each customer is visited by exactly one vehicle.
- Each route must contain at least one linehaul customer. Routes with only backhaul customers are not permitted.

In the survey of VRPB by Parragh, Doerner, and Hartl (2008), the authors split the VRPB into four classes. The customers in the first and second classes are either linehaul or backhaul customers, never both. In the first class, all the linehaul customers are visited before any backhaul customer, while in the second class any sequence of the linehauls and backhauls is permitted. In the third and fourth classes however, the customers are both linehaul and backhaul customers. The customers in the third class that needs both delivery and pickup, can be visited twice. In the fourth class these customers must be visited exactly once.

The VRPB was introduced by Deif and Bodin back in 1984, and in 1997 the first exact solution method for VRPB was developed by Toth and Vigo. Koç and Laporte (2018) performed an extensive review of the existing literature on VRPBs up until 2017 and found that most of the research has focused on several extensions of the VRPB, such as multiple depots, multiple trips, two-dimensional loading and time windows, though there still exist numerous research opportunities within the different variants of VRPB. Some of the research papers mentioned by Koç and Laporte (2018) that focuses on different variants of the VRPB are as follows:

- Liu and Chung (2008) described a hybrid heuristic method that combines variable neighborhood search and tabu search to solve the VRPB with inventory management (VRPBI).
- García-Nájera, Bullinaria, and Gutiérrez-Andrade (2015) proposed a new heuristic for finding multi-objective solutions for VRPBs called similarity-based selection multi-objective evolutionary algorithm.
- Wassan et al. (2017) introduced a new VRP variant called the Multiple Trip Vehicle Routing Problem with Backhauls (MT-VRPB), where the vehicles may perform multiple trips within the planning horizon.

3.0 Methodology and Model

In this section we will describe the approach that was chosen in order to answer the research questions of the thesis. We will then describe and develop a mathematical model for the IRP of collecting manure from farms to Hustadvika Biokraft AS

3.1 Methodology

There are two methods of research, qualitative and quantitative. Qualitative research is an in-depth study approach on how and why things are as they are, while quantitative research is a study approach that quantifies the problem. This thesis will be using a quantitative method.

First of all, the problem had to be analysed in order to define which parameters are needed. In collaboration with Hustadvika Biokraft AS, we received the relevant information for the problem. While analyzing the information received, we were able to define the parameters and decide which assumptions to make. Simultaneously while receiving the information, the data was collected from the company.

As soon as the information was analysed, the development of the model could start. Mathematical model was chosen as the exact method, which will be a good foundation for the future development. The model were based on a basic version of an IRP described by (Coelho, Cordeau, and Laporte 2014) and adapted to our case. The model was then tested with a limited selection of farms, followed by an analysis of the model versus the real case problem, giving an insight to what should be done further development of the model in order to make it closer to the real case problem.

3.2 Model description and assumptions

Sets

Vertices are the factory and all the farms. Note that the manure/slurry stores that are located a bit away from the farm buildings, we have together with Hustadvika Biokraft AS agreed to assume that the manure/slurry stores are located at the same place as the manure tank at the farm and are reachable all year through, in order to simplify the problem.

There are in total 84 relevant farms to be included in delivering manure, though the list might be shortened by the time Hustadvika Biokraft AS is up and running. Due to the problem being NP hard, only a small selection of farms is included in the testing of the model. Since Hustadvika Biokraft AS will prioritize farms that has multiple storage

facilities, and then in the following order availability of the storage location, distance from the factory and amount of manure, the four farms chosen

- are among those farms with multiple storage facilities.
- are located from the factory such that they represent farms located a short, medium and long distance away, and the average distance is representative for all the farms.
- produces such that they represent farms with a low, medium and high production rate of manure, and the average amount of manure produced is representative for all the farms.

The farms chosen are as follows:

Farm nr. (nr. in AMPL)	Distance from factory in km (hours)	Production per year (daily production)	Min. inventory holding capacity (slurry store)
Farm 7 (1)	2.8 (0.07)	5333 (14.6)	3555.3 (3000)
Farm 21 (2)	18.4 (0.46)	1483 (4.1)	988.7 (1000)
Farm 50 (3)	24.5 (0.61)	3250 (8.9)	2166.7 (1400)
Farm 81 (4)	29.5 (0.74)	7000 (19.2)	4666.7 (6000)

Table 1 Farms chosen for testing the model

The arcs are the roads between the vertices that Hustadvika Biokraft AS must travel in order to pick up the manure.

Vehicles are the number of vehicles that will be used for picking up manure at the farms. There will be two vehicles available, and in this model, they will be limited to drive only one route per day each in the exact model.

The set of time periods has been decided to be 6 days for now while testing the model, since the drivers of the vehicles will be collecting manure Monday to Saturday.

Parameters

The cost of driving the arcs will be measured in time (hours). Hustadvika Biokraft AS assumes that the average speed will be approximately 40 km/h for the vehicles when collecting manure. The distance in hours is therefore found by dividing the distance in km on the average speed 40 km/h. It might be worth mentioning that the data of distance in km is for the shortest route, though it might not be the fastest route in reality due to speed limits, the quality of the road, etc.

The inventory holding cost is the cost of storing manure at the vertices and is set to 0 at this point, though it is included in model in case that changes.

The vehicle capacity will be 30 tons and is the same amount for all vehicles. If this changes, it is easy enough to change in the model by adding an index $k \in K$ to the parameter.

The inventory holding capacity is the amount of manure each vertex can store. We have received the data for the capacity of the manure/slurry stores of the farms, but do not have the information of the total storage capacity at each farm at this point. We have therefore calculated this amount for the farms by multiplying the annual production at each farm with $\frac{2}{3}$, since the farms are required by law to have a minimum storage capacity for the manure of 8 months production of manure (Forskrift om organisk gjødsel 2003, §20). This means in the data we use for testing the model, some of the farms' inventory holding capacity is less than what it is in real life. Some farms have a higher inventory holding capacity on their manure/slurry stores than the minimum requirement. For those we will use the capacity of the manure/slurry stores in testing of the model.

We calculate the inventory holding capacity for the factory by dividing its demand when it is properly established and fully up and running (200 000 tons per year), on 365 (days per year) in order to find its daily capacity.

The initially inventory level at the vertices depends on when Hustadvika Biokraft AS will start picking up manure, and even then it might be challenging to say for certain the amount of manure in storage at each farm. When testing the model, we therefore set the initial inventory to 0 for all vertices.

In order to find the amount of manure produced at each farm per day we divide the annual production rate on 365 (days per year). But since there will be no pickups on Sundays, we double the amount produced on Mondays (day 1 in model).

The annual demand of manure at the factory will be 150 000 tons in the start. In order to find the daily demand, we divide this on 365 (days per year). But since there will be no pickups from the farms (meaning no delivery of manure at the factory) on Sundays, and the production at the factory will still be running on Sundays, we double the demand on Saturdays. We have chosen to keep the index for vertices on the demand-parameter, though it could easily be removed since there is no demand for manure at the farms.

Variables

In our model we would like to find the following:

1. Which routes should the vehicles drive in order to minimize the driving time?
2. When and how often will the farms be visited?

3. How much manure should be *picked up* for each visit at each farm?

Additionally, we would like to find what the *inventory level* will be at each vertex at the end of each day.

3.3 Model development

The base for the mathematical model developed in this section, is a basic version of the IRP presented in the paper by Coelho, Cordeau, and Laporte (2014). As mentioned in 2.1 under Literature Review, this is an exact algorithm that can be used to solve an IRP with delivery from one-to-many. The algorithm therefore must be rewritten to fit our problem of pickup from many-to-one.

The first change is the constraints defining the inventory level at the depot and customers (constraints (2) and (4) in Coelho, Cordeau, and Laporte (2014)). In our problem the production of manure takes place at the farms (customers) whilst the demand of manure is at the factory (depot). In addition, the delivery is from the farms to the factory. It is also worth mentioning that the parameter for amount produced has been added an extra index, since the production does not take place at only one vertex. Constraint (3) is excess since it is covered by constraint (5) and is therefore unnecessary to include in our model. Constraint (7) ensures the delivery at a customer does not exceed what there's room for. In our model it is changed to ensure that the vehicles cannot pickup more than what is in stock at the farm. Constraint (8) is not included in our model since it is not applicable for a pickup-problem such as ours. Constraint (9) is kept the same though now it means there is no pickup at a farm without it being visited. The rest of the model by Coelho, Cordeau, and Laporte (2014) are kept the same and included in our model. When it comes to the objective function (1), it has been kept unchanged, even though the inventory holding costs are set to 0.

Below, the notations for sets, parameters and variables are presented followed by the mathematical formulation to the IRP.

Sets:

$V = \{0, \dots, n\}$	– vertex set where $0 = \text{factory}$, $V' = \text{farms}$
$A = \{(i, j) : i, j \in V, i < j\}$	– arc set
$K = \{1, \dots, k\}$	– vehicle set
$T = \{1, \dots, t\}$	– time period set

Parameters

c_{ij} = time used to drive from vertex i to vertex j (arc (i, j))	$(i, j) \in A$
h_i = inventory holding cost at vertex i	$i \in V$
q = vehicle capacity	
g_i = inventory holding capacity at vertex i	$i \in V$
l_i = initial inventory level at vertex i	$i \in V$
r_i^t = manure produced at farm i on day t (daily production)	$i \in V', t \in T$
d_i^t = demand for manure at vertex i at day t	$i \in V, t \in T$

Variables

I_i^t = inventory level at vertex i at the end of day t	$i \in V, t \in T$
P_i^{kt} = amount of manure picked up at vertex i by vehicle k on day t	$i \in V', k \in K, t \in T$
$X_{i0}^{kt} = \{0, 1, 2\}$ = how many times vehicle k travels the arc between farm i and factory on day t	$i \in V', k \in K, t \in T$
$X_{ij}^{kt} = \{0, 1\}$ = how many times vehicle k travels the arc between farms i and j on day t	$i, j \in V', k \in K, t \in T$
$Y_i^{kt} = \begin{cases} 1 & \text{if vertex } i \text{ is visited by vehicle } k \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$	$i \in V, k \in K, t \in T$

Formulation:

$$(1) \quad \text{Min} \sum_{i \in V} \sum_{t \in T} h_i I_i^t + \sum_{(ij) \in A} \sum_{k \in K} \sum_{t \in T} c_{ij} X_{ij}^{kt}$$

Subject to:

$$(2) \quad I_0^t = I_0^{t-1} + \sum_{i \in V'} \sum_{k \in K} P_i^{kt} - d_0^t \quad \forall t \in T$$

$$(3) \quad I_i^t = I_i^{t-1} + r_i^t - \sum_{k \in K} P_i^{kt} \quad \forall i \in V', t \in T$$

$$(4) \quad I_i^0 = l_i \quad \forall i \in V$$

- (5) $I_i^t \leq g_i$ $\forall i \in V, t \in T$
- (6) $\sum_{k \in K} P_i^{kt} \leq I_i^{t-1} + r_i^t$ $\forall i \in V', t \in T$
- (7) $P_i^{kt} \leq g_i Y_i^{kt}$ $\forall i \in V', k \in K, t \in T$
- (8) $\sum_{i \in V'} P_i^{kt} \leq q Y_0^{kt}$ $\forall k \in K, t \in T$
- (9) $\sum_{j \in V, i < j} X_{ij}^{kt} + \sum_{j \in V, i > j} X_{ji}^{kt} = 2Y_i^{kt}$ $\forall i \in V, k \in K, t \in T$
- (10) $\sum_{i \in S} \sum_{j \in S, i < j} X_{ij}^{kt} \leq \sum_{i \in S} Y_i^{kt} - Y_m^{kt}$ $\forall S \subseteq V', k \in K, t \in T, m \in S$
- (11) $I_i^t \geq 0$ $\forall i \in V, t \in T$
- (12) $P_i^{kt} \geq 0$ $\forall i \in V', k \in K, t \in T$
- (13) $X_{i0}^{kt} \in \{0, 1, 2\}$ $\forall i \in V', k \in K, t \in T$
- (14) $X_{ij}^{kt} \in \{0, 1\}$ $\forall (i, j) \in V', k \in K, t \in T$
- (15) $Y_i^{kt} \in \{0, 1\}$ $\forall i \in V, k \in K, t \in T$

Description:

The objective function (1) minimizes the total costs measured in time (hours), here including inventory holding costs and travel costs. Constraints (2)–(3) define the inventory at the factory and the farms respectively, while constraints (4) define the initial inventory of the factory and the farms. Constraints (5) ensure that the inventory level does not exceed the capacity at each farm and factory. As Coelho, Cordeau, and Laporte (2014) points out in their paper, “... these constraints assume that the inventory at the end of the period cannot exceed the maximum available holding capacity, which means that during the period, before all demand has happened, the inventory capacity could be temporarily exceeded. This is a usual assumption in IRP models”. Constraints (6) make sure that a vehicle cannot pick up more manure than what is in stock at each farm each day, while constraints (7) ensure that there is no pick up at a farm by a vehicle one day without the same farm being visited by that vehicle on that exact day. Constraints (8) ensure that the total vehicle load on a vehicle one day does not exceed the vehicle capacity, as well as making sure the same vehicle visits the factory that day. Constraints (9) ensure a vehicle arriving at a farm also leaves that farm (opposite for the factory), while constraints (10) eliminate subtours. Constraints (11)–(15) specify the variables as non-negative, integer and/or binary.

4.0 Testing and Analysing the Model

AMPL, an algebraic modelling language used to describe and solve mathematical problems, was used for solving the mathematical model. CPLEX version 12.2 was used as the solver. Appendix A shows the four AMPL-files of testing the model.

Since computing the subtour constraint (11) in AMPL is challenging and we are only testing the model for a small selection of farms, we do not include that constraint in solving the model. Instead we will add constraints for eliminating exact subtours that occur while testing the model. As you can see in the model-file in Appendix A, there was only need of one exact subtour constraint.

The model was solvable, and an optimal solution was found. In Figure 5 and Figure 6 on the next page, you can see the test results illustrated. The total cost became 10.664 hours. We see by the results that some farms are visited multiple times a day, which might not be convenient for the farmers nor the drivers, who might end up picking up manure at a farm at the same time and therefore having to wait until the other is finished. This is easily solved by adding a constraint ensuring a farm will be visited maximum once a day. The constraint can be formulated this way: $\sum_{k \in K} Y_i^{kt} \leq 1 \quad \forall i \in V', t \in T$.

In our model when setting the data of the inventory holding capacity, we made the assumption that all the storage capacity at each farm is available for the manure, though in reality most of it at farms with multiple storage facilities will be for the decomposing residue. This means that the actual inventory holding capacity at the farms are much lower than the data we used.

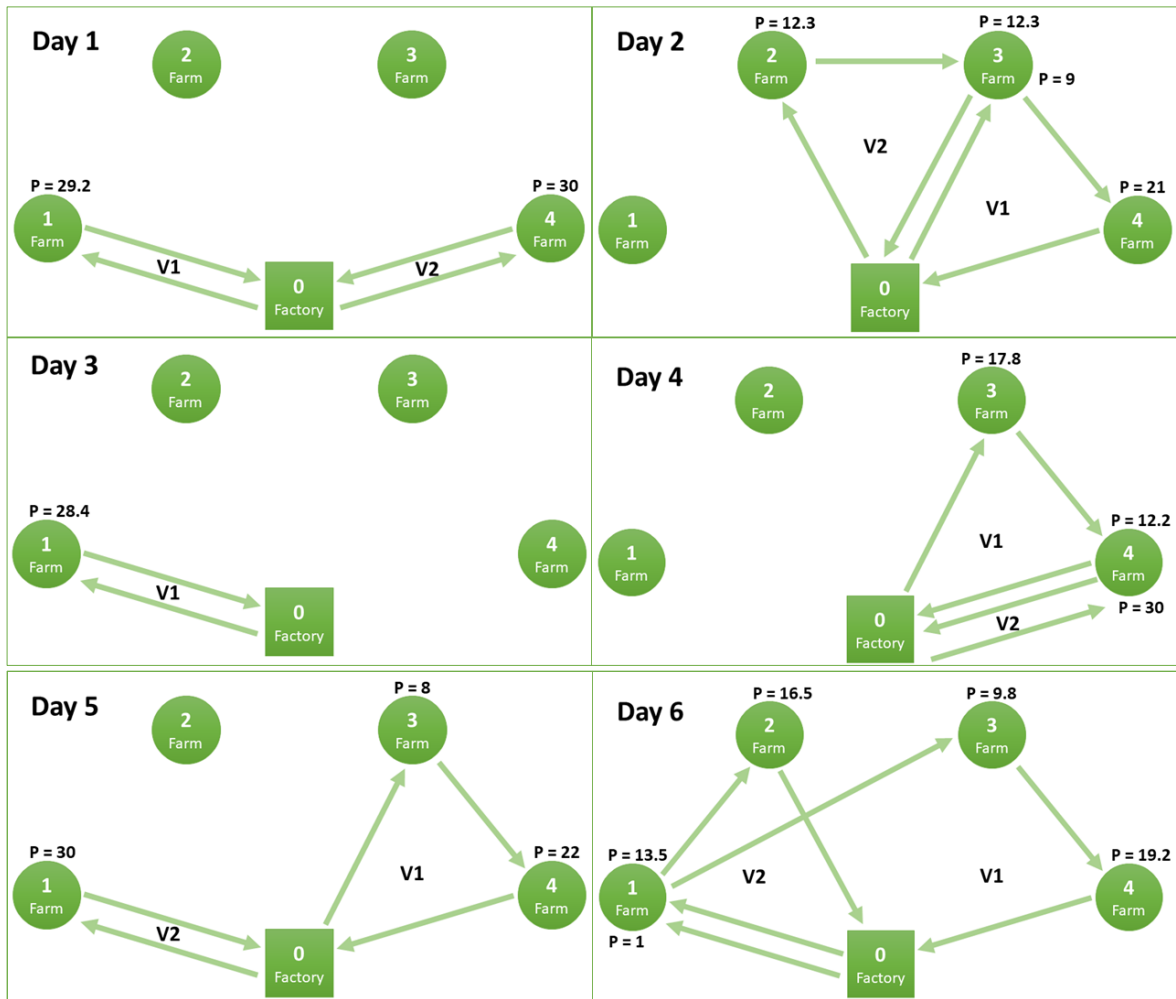


Figure 5 Test results - Vehicle routes and amount of manure picked up each day

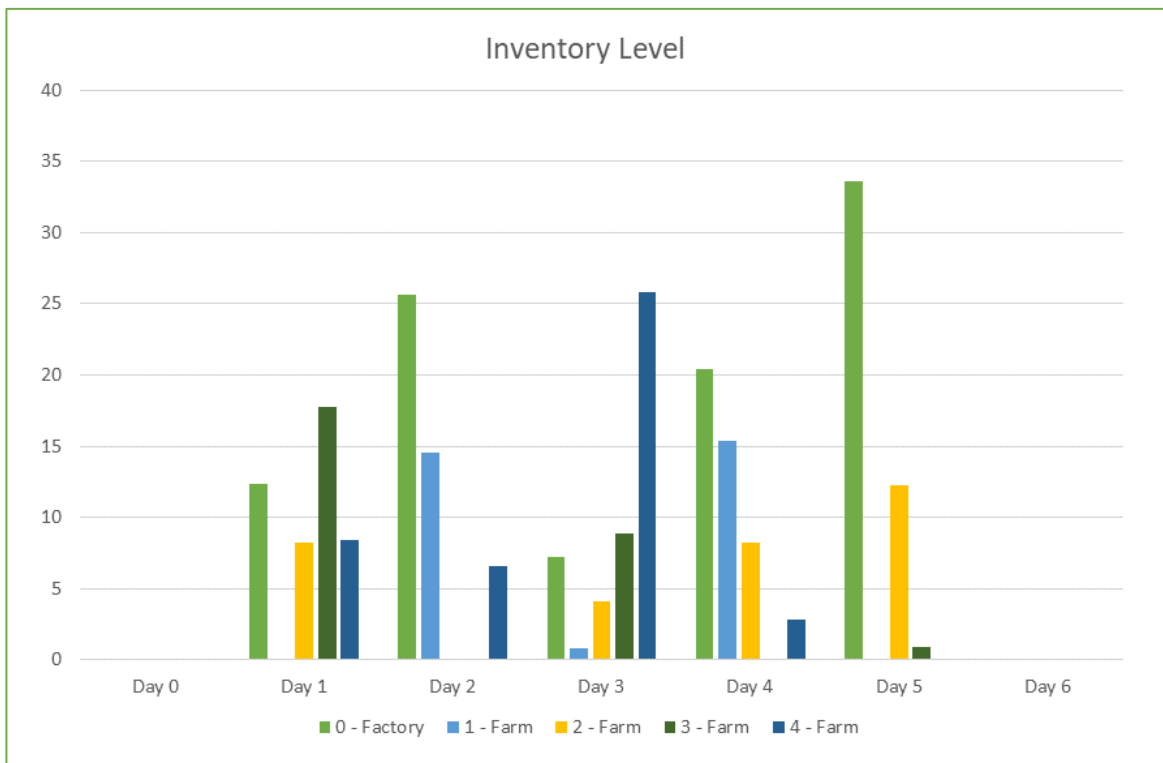


Figure 6 Test results - Inventory level at each vertex each day

Had all farms been included though, then the solver most likely would have run out of memory before finding an optimal solution. We could have run it of course just to be sure, though that is unnecessary since the problem is NP hard and the goal is only to test the model to see if it works as it should. As it does, the model can now be used as a foundation for developing a model that is closer to the real case problem, where such as delivering decomposing residue to the farms, loading and unloading time, and several routes per vehicle is included.

Additional time costs

Starting with the objective function: time it takes to load and unload the manure at the factory and farms should be included. Hustadvika Biokraft AS has estimated this to take about 10 minutes each time. The following should therefore be added to the objective function: $\sum_{i \in V} \sum_{k \in K} \sum_{t \in T} Y_i^{kt} * 10$. Additionally, a similar part should be added for the loading and unloading when delivering decomposing residue to the farms.

There might also be possible costs for the farms (or factory) for stirring the manure stored at the farms, before pickup takes place. This cost can also be measured in time and will probably depend on how much manure is stored at that time and how long since last pickup took place.

Backhauling

At this point the model takes only the pickup of manure from farms into consideration, though Hustadvika Biokraft AS is planning to use the vehicles also for delivering decomposing residue to the farms. This means that the model needs to be turned from an IRP with pickups, to an IRPB (Inventory Routing Problem with Backhauling). We will then need several new parameters and variables that will apply for the decomposing residue in the same way as the demand, production and inventory holding capacity parameters, the inventory level and the pickup variables do for the manure. Some of the constraints in the model by Coelho, Cordeau, and Laporte (2014) can be inserted just as they are, for instance the inventory level constraints, since the decomposing residue part is a delivery VRP. Other constraints that are already included in our model, such as the vehicle load constraints, must be changed to include the delivery of decomposing residue as well.

Multiple routes and shifts

In our model we have assumed that the two vehicles can only perform one route a day. In the real case they can of course do multiple routes in one day, and there are also two shifts a day Monday to Friday. So, on this point the model needs to be modified. There are several

ways of doing so. One way can be to add another set to the model, a set of possible routes, so that there will be another index on such as the X-variable. Additionally, one can add two vehicles to the already two vehicles in the vehicle set, instead of having an own set for shifts. Another way of solving this is to

1. switch the vehicle set out with a possible route set,
2. solve the model and find the optimal routes driven each day,
3. and then as a second step in solving the model divide the optimal routes for one day on the four vehicle shifts in such a way that they are less than the 6.75 hours a driver is working.

Longer time period

In our model we set the planning horizon very short and is not realistic at all. It might be more ideally to plan at least for 2-3 weeks ahead. The idea of a rolling horizon suggested by Bard et al. (1998) might be highly relevant to use in this problem.

Further challenges

There are still several challenges to take into consideration. The one that might be the biggest challenge of them all and cannot be overlooked, is delivery of decomposing residue to farms with only one storage facility. One solution might be to deliver decomposing residue only in the last weeks or months before the fertilization periods, which means that the factory will not be able to pick up nor use the manure produced those weeks/months and during the fertilization period, since the manures will be mixed with the decomposing residue. It will also most likely require all of the delivery capacity of the vehicles during those weeks/months, so that the farms with manure/slurry stores should have received all the decomposing residue they need for the fertilizing period beforehand.

Another challenge that might be worth considering is that the farmers might prefer some predictability of which days and time of day they can expect pickup of manure and delivery of decomposing residue at their farms, especially if the storage facilities are located close to their homes and the pickup or delivery will take place late evening.

We have already mentioned that this VRP is NP hard, and NP hard problems are not solvable by using exact methods. Though the mathematical models can be used as a framework for developing a heuristic or metaheuristic for solving the problem, which can come as close as be to finding the best solution. Therefore, the next step after improving the mathematical model and making closer to the real case problem, will be to develop a heuristic or metaheuristic in order to find the best solution one can find.

5.0 Conclusion and Further Research

The main purpose of the thesis was to lay a foundation for solving the real case problem of Hustadvika Biokraft AS, by creating a mathematical model for the part of the problem revolving the pickup of manure from local farms.

In this thesis we analysed the problem and formulated a mathematical model based on the pickup of manure from the farms. We then tested the model by using the solver CPLEX for a small selection of farms. From the results we could see that some farms were visited multiple times a day, which might not be quite ideal. Also, the data for the inventory holding capacity that was used should be a great deal lower and is a direct result of the model not taking into account the whole problem. The formulated model focuses only on a part of the real case problem and therefore there is still a lot to be done in order to achieve a realistic solution to the problem. For further research one can use the mathematical model developed in this thesis as a foundation, and adapt it by:

- Adding additional time costs, such as the loading and unloading time.
- Changing the model from an IRP to an IRPB.
- Adding a set of possible routes.
- Solving the model with a longer time period, by using a rolling horizon.
- Developing a heuristic/metaheuristic based on the formulated model, and use it to solve the problem.

The real case problem that has been studied in this thesis has some similarities to the problem and to a certain degree takes place in the same geographic area as the paper by Pasha, Hoff, and Løkketangen (2014). They developed a method using a clustering technique and a tabu search for solving their case problem, which can possibly be used as a starting point for creating a heuristic/metaheuristic for solving the real case problem of collecting manure and delivering decomposing residue to local farms by Hustadvika Biokraft AS.

References

- Adulyasak, Yossiri, Jean-François Cordeau, and Raf Jans. 2013. "Formulations and branch-and-cut algorithms for multivehicle production and inventory routing problems." *INFORMS Journal on Computing* ePub ahead of print June 14, <https://doi.org/10.1287/ijoc.2013.0550>.
- Andersson, Henrik, Arild Hoff, Marielle Christiansen, Geir Hasle, and Arne Løkketangen. 2010. "Industrial aspects and literature survey: Combined inventory management." *Computers & Operations Research* 37 (9):1515-1536.
- Archetti, Claudia, Luca Bertazzi, Gilbert Laporte, and Maria Grazia Speranza. 2007. "A branch-and-cut algorithm for a vendor-managed inventory-routing problem." *Transportation Science* 41 (3):382-391.
- Bard, Jonathan F., Liu Huang, Patrick Jaillet, and Moshe Dror. 1998. "A decomposition approach to the inventory routing problem with satellite facilities." *Transportation Science* 32 (2):90-203.
- Campbell, Ann M., Lloyd W. Clarke, and Martin W.P. Savelsbergh. 2002. "Inventory routing in practice." In *The vehicle routing problem*, edited by Paolo Toth and Daniele Vigo, 309-330. Philadelphia, PA: SIAM - Society for Industrial and Applied Mathematics.
- Coelho, Leandro C., Jean-François Cordeau, and Gilbert Laporte. 2014. "Thirty years of inventory routing." *Transportation Science* 48 (1):1-19.
- Coelho, Leandro C., and Gilbert Laporte. 2013. "The exact solution of several classes of inventory-routing problems." *Computers & Operations Research* 40 (2):558-565.
- Deif, I., and L.D. Bodin. 1984. "Extension of the clarke and wright algorithm for solving the vehicle routing problem with backhauling." In *Proceedings of the babson conference on software uses in transportation and logistics management*, edited by A. Kidder, 75-96. Babson Park, MA.
- Forskrift om organisk gjødsel. 2003. Forskrift om gjødselvarer mv. av organisk opphav. Fastsatt ved kgl.res. 4.juli 2003. <https://lovdata.no/dokument/SF/forskrift/2003-07-04-951>.
- García-Nájera, Abel, John A. Bullinaria, and Miguel A. Gutiérrez-Andrade. 2015. "An evolutionary approach for multi-objective vehicle routing problems with backhauls." *Computers & Industrial Engineering* 81:90-108.

- Koç, Çağrı, and Gilbert Laporte. 2018. "Vehicle routing with backhauls: Review and research." *Computers & Operations Research* 91:79-91.
- Landbruks- og matdepartementet. 2009. *Klimautfordringene - landbruket en del av løsningen*. Mld. St. 39 (2008-2009). Oslo: Landbruks- og matdepartementet.
- Liu, Shu-Chu, and Chich-Hung Chung. 2008. "A heuristic method for the vehicle routing problem with backhauls and inventory." *Journal of Intelligent Manufacturing* 20 (1):29-42.
- Miljøverndepartementet. 2012. *Norsk klimapolitikk*. Mld. St. 21 (2011-2012). Oslo: Miljøverndepartementet.
- Norges Bondelag. 2011. *Fakta om Biogass. Norsk kulturlandskap - det nye gassfeltet*. Oslo: Norges Bondelag.
- Norwegian Government. 2018. "Nye kommuner og navn." accessed 24.11.2018. <https://www.regjeringen.no/no/tema/kommuner-og-regioner/kommunereform/Hvorfor-kommunereform/nye-kommuner/id2470015/>.
- Parragh, Sophie N., Karl F. Doerner, and Richard F. Hartl. 2008. "A survey on pickup and delivery problems. Part I: Transportation between customers and depot." *Journal für Betriebswirtschaft* 58 (1):21-51.
- Pasha, Urooj, Arild Hoff, and Arne Løkketangen. 2014. "A hybrid approach for milk collection using trucks and trailers." *Annals of Management Science* 3 (1):85-108.
- Statistisk Sentralbyrå. 2017. "Husdyr etter region, husdyrslag, statistikkvariabel og år." accessed 24.11.2018. <http://www.ssb.no/statbank/sq/10014526/>.
- Toth, Paolo, and Daniele Vigo. 1997. "An exact algorithm for the vehicle routing problem with backhauls." *Transportation Science* 31 (4):372-385.
- Toth, Paolo, and Daniele Vigo. 2002a. *The vehicle routing problem*. Philadelphia, PA: SIAM - Society for Industrial and Applied Mathematics.
- Toth, Paolo, and Daniele Vigo. 2002b. "VRP with Backhauls." In *The vehicle routing problem*, edited by Paolo Toth and Daniele Vigo, 195-224. Philadelphia, PA: SIAM - Society for Industrial and Applied Mathematics.
- Wassan, Naveed, Niaz Wassan, Gábor Nagy, and Saïd Salhi. 2017. "The multiple trip vehicle routing problem with backhauls: Formulation and a two-level variable neighbourhood search." *Computers & Operations Research* 78:454-467.

Appendix A: AMPL-files

Testmodel_IRP.mod

```
param DAYS >= 0;           # number of days in the planning period
set VEHICLES;              # set of vehicles available
set VERTEX;                # set of vertices, including depot and all farms
set FARMS within VERTEX;  # subset of vertex, only farms

param drivetime {i in VERTEX, j in VERTEX: i<>j} >= 0;
                                # travel time between vertex i and vertex j in hours
param invcost {i in VERTEX} >= 0;           # inventory holding cost at vertex i
param vehiclecap >= 0;                     # vehicle capacity
param invcap {i in VERTEX} >= 0;           # inventory holding capacity at vertex i
param prodman {i in FARMS, t in 1..DAYS} >= 0; # manure produced at vertex i at day t
param demand {i in VERTEX, t in 1..DAYS} >= 0; # demand for manure at the depot at day t
param inv0 {i in VERTEX} >= 0;             # the initial inventory level at vertex i

# Variables:
var Level {i in VERTEX, t in 0..DAYS} >= 0;
                                # inventory level at the depot at the end of day t
var Pickedup {i in FARMS, k in VEHICLES, t in 1..DAYS} >= 0;
                                # amount of manure picked up from farm i by vehicle k at day t
var Used {i in VERTEX, j in VERTEX, k in VEHICLES, t in 1..DAYS: i<j} integer >=0 <= 2;
                                # how many times arc (i,j) is used by vehicle k at day t
var Visited {i in VERTEX, k in VEHICLES, t in 1..DAYS} binary;
                                # 1 if vertex i is visited by vehicle k at day t

# Objective Function:
minimize Total_Cost: sum {i in VERTEX, t in 1..DAYS} invcost[i] * Level[i,t]
                + sum {i in VERTEX, j in VERTEX, k in VEHICLES, t in 1..DAYS: i<j}
                drivetime[i,j] * Used[i,j,k,t];
# Minimizing all costs, including the inventory holding cost and the travel cost, measured in hours.

# Constraints:
subject to Used_farms {i in FARMS, j in FARMS, k in VEHICLES, t in 1..DAYS: i<j}:
Used[i,j,k,t] <= 1;
# Arcs between two farms cannot be used more than once

subject to Inv_level_depot {t in 1..DAYS}:
Level[0,t] = Level[0,t-1] + sum {i in FARMS, k in VEHICLES} Pickedup[i,k,t] - demand[0,t];
# The inventory level at the end of day t at the depot

subject to Inv_level_farms {i in FARMS, t in 1..DAYS}:
Level[i,t] = Level[i,t-1] + prodman[i,t] - sum {k in VEHICLES} Pickedup[i,k,t];
# The inventory level at the end of day t at farm i

subject to Inv_lev_start {i in VERTEX}:
Level[i,0] = inv0[i];
# The initial inventory level at vertex i

subject to Inv_capacity_dep {i in VERTEX, t in 1..DAYS}:
Level[i,t] <= invcap[i];
# The inventory level at vertex i can never be more than the capacity at any time
```

```

subject to Pickup_max {i in FARMS, t in 1..DAYS}:
sum {k in VEHICLES} Pickedup[i,k,t] <= Level[i,t-1] + prodman[i,t];
# Amount picked up at farm i by all vehicles at day t cannot be more than what's in stock

subject to Pickup_visit {i in FARMS, k in VEHICLES, t in 1..DAYS}:
Pickedup[i,k,t] <= invcap[i] * Visited[i,k,t];
# There cannot be any pickup at farm i by vehicle k on day t without being visited

subject to Vehicle_load {k in VEHICLES, t in 1..DAYS}:
sum {i in FARMS} Pickedup[i,k,t] <= vehiclecap * Visited[0,k,t];
# Makes sure the total vehicle load on vehicle k on day t doesn't exceed the vehicle capacity and
that vehicle k cannot pick up any manure at any farms at day t without visiting the depot the same
day

subject to Vehicle_balance {i in VERTEX, k in VEHICLES, t in 1..DAYS}:
sum {j in VERTEX: i<j} Used[i,j,k,t] + sum {j in VERTEX: i>j} Used[j,i,k,t] = 2 * Visited[i,k,t];
# Vehicle k arriving at vertex i at day t, must also leave (opposite for depot)

subject to Subtour_one {k in VEHICLES, t in 1..DAYS}:
Used[2,3,k,t] + Used[2,4,k,t] + Used[3,4,k,t] <= 2;
# Eliminates the subtour 2-3-4-2

```

Testmodel_IRP.dat

```
set VERTEX := 0 1 2 3 4;
set FARMS := 1 2 3 4;
set VEHICLES := V1 V2;
param DAYS := 6;
```

```
param drivetime: 0 1 2 3 4 :=
  0 . 0.07 0.46 0.52 0.66
  1 0.07 . 0.48 0.49 0.76
  2 0.46 0.48 . 0.31 0.43
  3 0.61 0.49 0.31 . 0.14
  4 0.74 0.76 0.43 0.14 .;
```

```
param invcost :=
  0 0
  1 0
  2 0
  3 0
  4 0;
```

```
param invcap :=
  0 548
  1 3555.3
  2 1000
  3 2166.6
  4 6000;
```

```
param inv0 :=
  0 0
  1 0
  2 0
  3 0
  4 0;
```

```
param vehiclecap:= 30;
```

```
param prodman: 1 2 3 4 5 6:=
  1 29.2 14.6 14.6 14.6 14.6 14.6
  2 8.2 4.1 4.1 4.1 4.1 4.1
  3 17.8 8.9 8.9 8.9 8.9 8.9
  4 38.4 19.2 19.2 19.2 19.2 19.2;
```

```
param demand: 1 2 3 4 5 6:=
  0 46.8 46.8 46.8 46.8 46.8 93.5
  1 0 0 0 0 0 0
  2 0 0 0 0 0 0
  3 0 0 0 0 0 0
  4 0 0 0 0 0 0;
```

Testmodel_IRP.run

```
model Testmodel_IRP.mod;
data Testmodel_IRP.dat;
option solver cplex;
option omit_zero_rows 1;
solve;
display Total_Cost > Testmodel_IRP.sol;
display Used, Pickedup, Level, Visited > Testmodel_IRP.sol;
exit;
```


Testmodel_IRP.sol

Total_Cost = **10.664**

Used

[0,*,V1,*] (tr)	[1,*,V1,*] (tr)	[2,*,V1,*] (tr)	[3,*,V1,*] (tr)
: 1 2 3 4 :=	: 2 3 4 :=	: 3 4 :=	: 4 :=
1 2 0 0 0	1 0 0 0	1 0 0	1 0
2 0 0 1 1	2 0 0 0	2 0 0	2 1
3 2 0 0 0	3 0 0 0	3 0 0	3 0
4 0 0 1 1	4 0 0 0	4 0 0	4 1
5 0 0 1 1	5 0 0 0	5 0 0	5 1
6 1 0 0 1	6 0 1 0	6 0 0	6 1

[0,*,V2,*] (tr)	[1,*,V2,*] (tr)	[2,*,V2,*] (tr)	[3,*,V2,*] (tr)
: 1 2 3 4 :=	: 2 3 4 :=	: 3 4 :=	: 4 :=
1 0 0 0 2	1 0 0 0	1 0 0	1 0
2 0 1 1 0	2 0 0 0	2 1 0	2 0
3 0 0 0 0	3 0 0 0	3 0 0	3 0
4 0 0 0 2	4 0 0 0	4 0 0	4 0
5 2 0 0 0	5 0 0 0	5 0 0	5 0
6 1 1 0 0	6 1 0 0	6 0 0	6 0;

Visited

[*,V1,*] (tr)	[*,V2,*] (tr)
: 0 1 2 3 4 :=	: 0 1 2 3 4 :=
1 1 1 0 0 0	1 1 0 0 0 1
2 1 0 0 1 1	2 1 0 1 1 0
3 1 1 0 0 0	3 0 0 0 0 0
4 1 0 0 1 1	4 1 0 0 0 1
5 1 0 0 1 1	5 1 1 0 0 0
6 1 1 0 1 1	6 1 1 1 0 0

Pickedup

[*,V1,*] (tr)	[*,V2,*] (tr)
: 1 2 3 4 :=	: 1 2 3 4 :=
1 29.2 0 0 0	1 0 0 0 30
2 0 0 9 21	2 0 12.3 17.7 0
3 28.4 0 0 0	3 0 0 0 0
4 0 0 17.8 12.2	4 0 0 0 30
5 0 0 8 22	5 30 0 0 0
6 1 0 9.8 19.2	6 13.5 16.4 0 0
	:

Level

[*,*] (tr)
: 0 1 2 3 4 :=
0 0 0 0 0 0
1 12.4 0 8.2 17.8 8.4
2 25.6 14.6 0 0 6.6
3 7.2 0.8 4.1 8.9 25.8
4 20.4 15.4 8.2 0 2.8
5 33.6 0 12.3 0.9 0
6 0 0.1 0 0 0;
