LOG950

MASTER'S DEGREE THESIS

Allocating Farmed Fish to Customer Orders Using Multi-Objective Optimization

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TOTAL NUMBER OF PAGES INCLUDING THIS PAGE: 123

May 25, 2021

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Preface

This thesis is written as a part of the degree in the Master of Science in Logistics programme at Molde University College.

The master's thesis is part of the project FishTraOpt. The project is led by Maritech Systems AS, with Lerøy Seafood AS, Møreforsking AS, and Molde University College as partners. The FishTraOpt project aims to provide decision support for automated and optimized planning for traders in aquaculture, and their main goal is to develop and test a prototype solver for the planning problems faced by traders of farmed fish. Based on this project, the case of the thesis revolves around Lerøy.

Firstly, we would like to personally thank our supervisors Professor Lars M. Hvattum, Professor Arild Hoff, and Dr. Johan Oppen, for feedback and good support. Special thanks go to Ragnar Nystøyl at Kontali Analyse for providing updated statistical reports on aquaculture, and Associate Professor Øystein Klakegg for access to valuable lectures in Marine Logistics. We would also like to give our thanks and appreciations to Jan P. Halvorsen and Alexander B. Johannessen from Lerøy Seafood AS for information, and Maritech Systems AS for providing data. Lastly, a thank goes out to our fellow students for moral support during the master's programme.

Knudseth thanks her colleagues for all support the last 2 years, and especially during the last 5 months - for keeping up with her nagging and reminding her to take breaks. She would also thank her family and friends for all support, and Foxen for making delicious pizza.

Molland thanks his family, friends, and colleagues for support. A modest thanks goes to himself for putting in the work.

What did the dry fish say to the other?
- Long time, no sea!

Even Molland | Sunniva H. Knudseth

Abstract

Aquaculture is an important industry in Norway and in need of technological development.

The background for this thesis is an operational planning problem regarding allocating supply to demand, with potentially conflicting objectives. The goal is to make the allocation process less time-consuming and more manageable by creating a decision support tool for the planners at Lerøy Seafood AS.

In this thesis a Mixed Integer Linear Multi-Objective Optimization model is developed for allocating supply to demand. As far as the authors know, there exists no model for this particular problem in this field of study.

The chosen solution method is the Augmented ε-constraint (AUGMECON) Method by Mavrotas [18] and is used to construct Pareto fronts for five different instances in two scenarios. The generated instances are fictional but inspired by the allocation process at Lerøy.

The results from the computational study show that the outcomes are depending on the scenarios. In the scenario with sufficient supply, the results show that it may not always be necessary to solve for both the chosen objectives, and there was no apparent conflict between the two. On the other hand, the results from the scenario with insufficient supply show that there exists a trade-off between the two objectives, and that including both is necessary.

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1. Introduction

Norway has natural advantages for production of farmed salmon, such as deep fjords, good sea current conditions, and oxygen-rich water with suitable temperatures. The Gulf Stream provides regular replacement of water, hence the mentioned temperature conditions [25].

The aquaculture industry in Norway got a breakthrough in the 1970's when seawater farming of salmon in sea cages became successful [25]. This has been the leading technology up until today since it is adaptive to the natural advantages along the coast of Norway.

During the last decades, the aquaculture industry has undergone significant restructuring. The number of production facilities has become fewer, but larger, due to consolidation. The number of players responsible for 80% of the total production in Norway has decreased from 55 to 20 during the last two decades [15].

Along with the decrease in players, the number of permits has increased, as seen in Figure 1.1. Thus, the production potential in the aquaculture industry has increased.

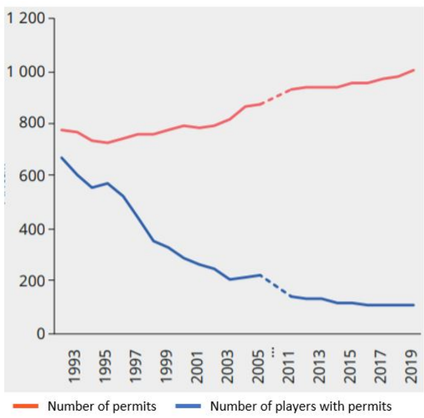


Figure 1.1: Food fish permits, 1992-2019. Directorate of Fisheries, in [25, p. 35].

In a global perspective, Norway produced 52% of the Atlantic Salmon in 2019. This is a slight decrease in a global scale from earlier years even though the production in Norway has increased. The reason for this decrease being that the total global production has increased more than the production in Norway, see Figure 1.2.

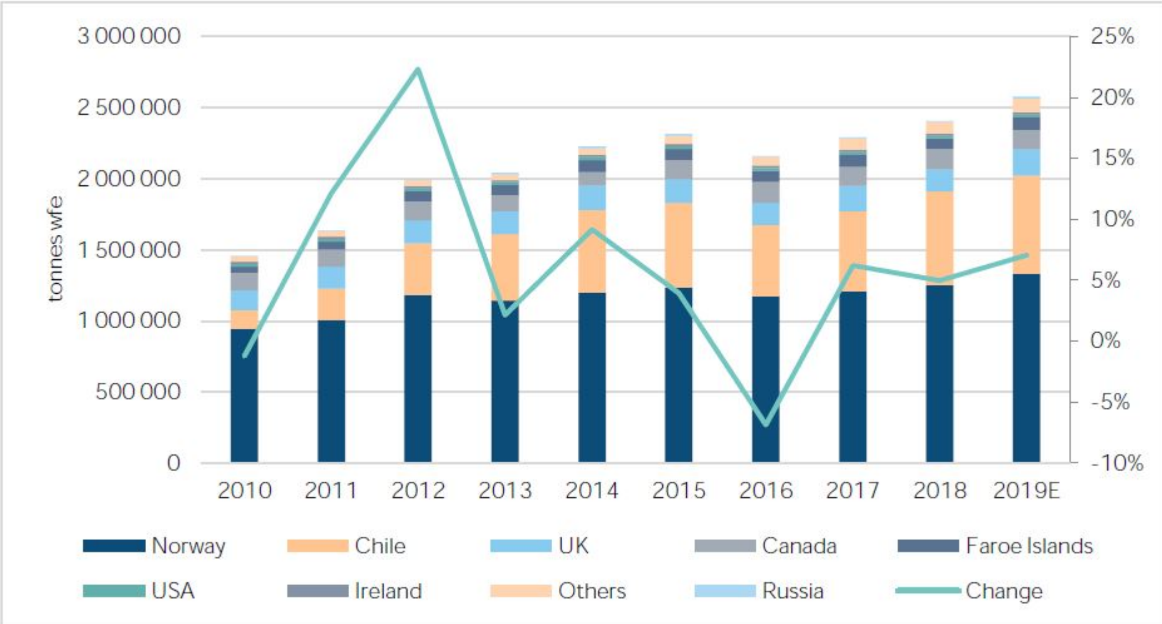


Figure 1.2: Global Harvest of Atlantic salmon in tonnes Whole Fish Equivalents, Change in % [15, p.8].

Lerøy Seafood Group is a world leading seafood company that dates to 1899. Their core business is production of salmon and trout, whitefish catch, processing, product development, marketing, and sales and distribution of seafood. They deliver an amount of seafood equivalent to five million meals every day, to over 80 different countries.

The main office is in Bergen, but Lerøy engage in fishing and aquaculture along the entire coast. They actively participate in all parts of the production of salmon and trout. This involves packing and processing at the factories, and distribution, in addition to fishing and aquaculture.

Seeing that the production in Norway is increasing, there will also be an increase in processing and distribution. The fish needs to be harvested, slaughtered, processed, packed, stored, and distributed to customers. The latter requires a process of allocating the fish to the right customers.

This process has its difficulties such as uncertainties in the supply regarding size, quality, and quantity of the fish. Allocation is decided by planners who rely on forecasts with this information, and uncertainties causes re-planning as soon as new information is available. Creating a plan can be time consuming, as it is done manually and only a handful of individuals have the right skill to carry out that plan. In addition, the allocation may not always be efficient, and decision making can be hard.
To improve the decision making, the planners need decision support systems. This will open for getting efficient solutions within reasonable time and eliminate a lot of manual work. Different solutions can be tested without being too time consuming. Time is a critical factor as the planning horizon is short, and the allocation plans should be created as fast as possible with satisfying results.

The scope of this thesis revolves around the process of planning, and allocating fish to customer orders, as seen from the perspective of the planners at Lerøy. This involves handling multiple objectives, that can be conflicting. This thesis aims to assist Lerøy by creating decision support for allocating supply to demand. In the perspective of Lerøy and their distribution department, it is of interest to get new methods for solving a challenge they frequently face. The research objective is to find an appropriate optimization method that produce efficient solutions that the decision maker can evaluate and choose from. To be able to find an appropriate optimization method, it is necessary to discover which approaches has been used before in similar contexts and incorporate that into a model. So far, there is no existing model that addresses the specific short-term allocation problem this thesis faces.

Furthermore, it is necessary to evaluate whether Multi-Objective optimization is appropriate regarding the decision making. Lastly, time is a critical factor so finding efficient solutions within an appropriate time frame is of importance.

Being able to create a tool based on the research from this thesis, can support the decision making for allocation problems. To be able to do so it is necessary to research similar problems within aquaculture, but also within agriculture due to its resemblance with aquaculture.

The aquaculture industry is of significant size and importance and does not appear to diminish anytime soon. For that reason alone, getting better decision support regarding allocation is essential.

The thesis is structured as follows. Chapter 2 introduces the aquaculture industry in Norway. Chapter 3 describes the problem background and presents the problem researched in this thesis. Relevant literature is presented in Chapter 4. Chapter 5 gives an insight to relevant methodology before the model is formulated in Chapter 6. Chapter 7 presents the computational study. Lastly, a conclusion in Chapter 8 and suggestions for future research in Chapter 9.

2. Aquaculture

This chapter provides background information about the aquaculture industry and its importance in Norway. Emphasizing the importance of the industry justifies the need of a good decision support tool for those involved. The focus is mainly on salmon and rainbow trout.

Section 2.1 gives an overview over the size of the industry. In Section 2.2 a brief overview of regulation, health, and certification is presented, before the characteristics of the value chain is presented in Section 2.3.

2.1 The Norwegian Aquaculture Industry

Norwegian aquaculture consists primarily of salmon and rainbow trout farming. Figure 2.1 gives an overall image of the development of the industry from 1986 to 2018.

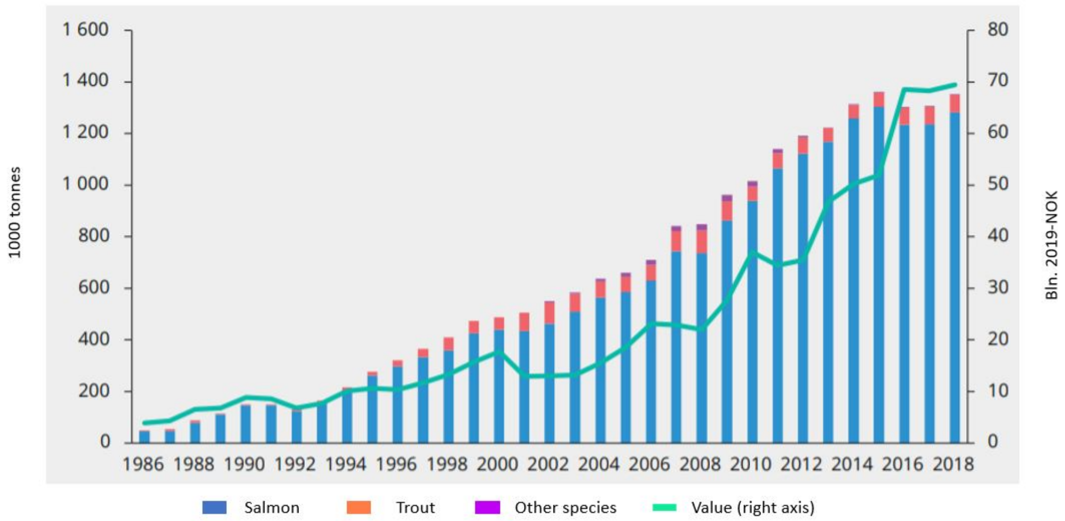


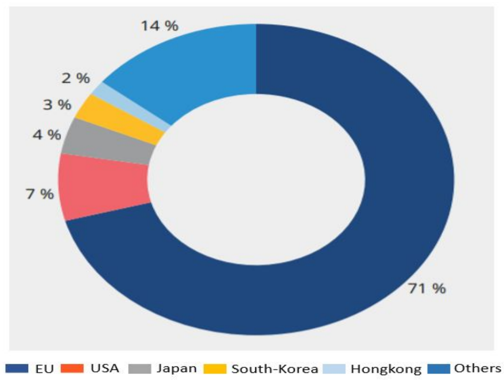
Figure 2.1: Sales of slaughtered food fish in Norway, from 1986-2018.
Amount in 1000 tonnes, value in billion 2019-NOK.
Statistics Norway, in [25, p. 30].

The aquaculture industry started off as an unimportant industry with several small players and has developed into one of the most essential export industries in Norway.

Compared to the second largest producer nation Chile, Norway has a relatively large number of players. In 2019 Norway had 20 players responsible for 80% of the total production, while Chile had 13. In the other salmon producing countries there are only 2 to 4 players responsible for the largest part of the production [15].

There has been a significant increase in export of salmon from the year 2000 and up to today, as seen in Figure 2.2a [25].
Norway exports 95% of the salmon, and exports seafood to over 140 countries.

(a) Export of salmon from 2000-2018
Statistics Norway & Norwegian Seafood Council, in [25, p.33].



(b) Export of salmon in 2018
Norwegian Seafood Council, in [25, p.33].

Figure 2.2: Overview of the salmon export.

Figure 2.2b shows the total distribution of the export in 2018, and 70% (measured in value) was exported to the EU. The largest markets are Poland, France, and Denmark. Poland and Denmark are markets for further processing, which means most of the fish going there is transported to other markets. Furthermore, 81% of the exported salmon in 2019 were fresh and whole, and approximately 10% were fresh fish fillets. The remainder consisted of frozen fillets and whole fish, and other value-added products [15].

The aquaculture industry is localized along the coast, and especially concentrated in some areas such as Hordaland, Sogn & Fjordane, Trøndelag, and Nordland.
The aquaculture industry is present in more than 160 municipalities and 10 county municipalities. In some of them it accounts for more than 10% of the employment, at the same time as it has a high value creation per man-year. The production of salmon and trout also generates employment in the processing industry both in Norway and abroad. The aquaculture industry employed around 8000 people in Norway in 2017.
This is an increase of 75% from the year 2000, and 20% from 2015, and just below 7500 of these are directly connected to production of salmon and trout. Indirectly, the industry contributes to over 40 000 man-years in Norway [13], including transport and finance.

Figure 2.3 shows how much the aquaculture industry contributes to value creation compared to other industries[1], both in development and in average.

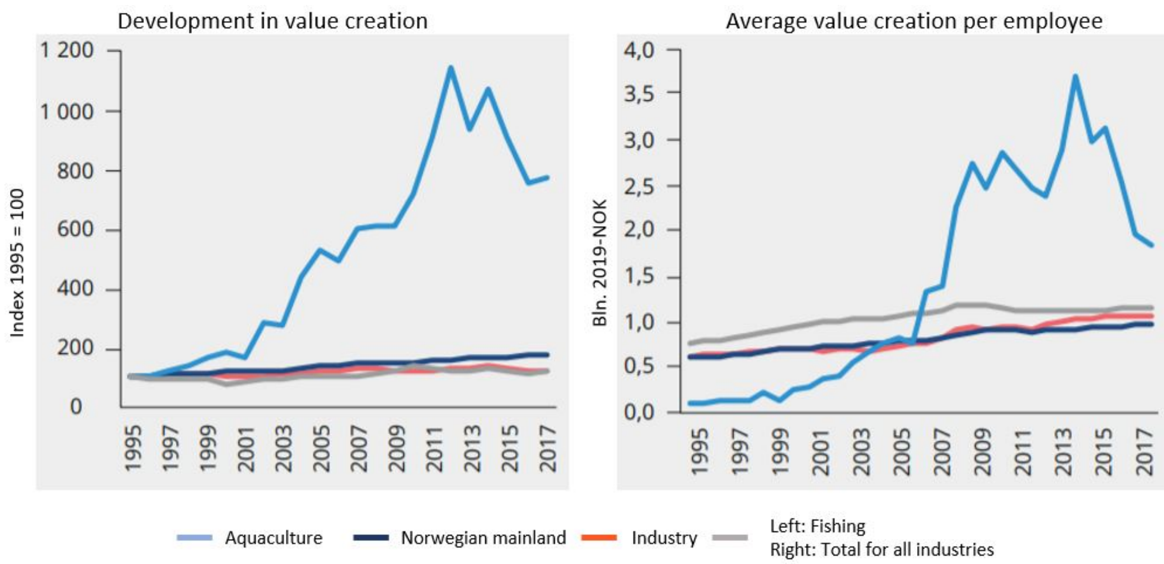


Figure 2.3: Value creation in aquaculture and other industries
Statistics Norway, in [25, p.32].

[1]Statistics Norway standard for grouping of industries. The expression "industry" contains food and beverages, textiles, paper/stationery, chemical and pharmaceutical product, metals, production of electronic and optical product

2.2 Regulations, Health, and Certifications

The aquaculture industry has its regulations regarding the number of licenses allowed per player, and how much standing biomass they can have. There are several permissions needed for farming of trout and salmon, but this is not of relevance for the thesis problem.

Health

Farmed fish lives close to each other inside a sea cage system. An outbreak of a disease spreads quickly between the fish, so preventing and dealing with an outbreak is of high importance. One of the big health issues is Salmon Lice. The lice make sores and damages to the fish, which opens for more infections [24]. The geographical area where aquaculture exists is divided into smaller areas, and if an area has an outbreak of disease there is a risk that all fish there may be affected.

Certification

Certain countries demand that farms have certain certifications before they import fish produced there. Certifications are used to prove that the fish holds a certain standard. Some certifications worth mentioning are Global G.A.P - a global recognized standard for production of farmed fish, and ASC Salmon Standard - documents responsible and sustainable aquaculture [14].

Maximum Allowed Biomass

Each farm has a permitted capacity of how much fish can be in the sea phase at the same time. This involves that the actual standing biomass cannot exceed the permission. Usually, it would be rational to always stay as close to the theoretical capacity as possible. When the fish is harvested, the farmers would want to set out new batches of fish, or let some fish stay for further growth to utilize the permitted capacity. However, the growth of the fish is highly seasonally dependent, and the growth increases with higher water temperatures, which would lead to an uneven pattern. Figure 2.4 shows the ratio between standing biomass and theoretical capacity from 2005-2019 in the Norwegian aquaculture industry overall. The gap between the blue area and the red line shows the proportion of the allocated capacity that has not been utilized.

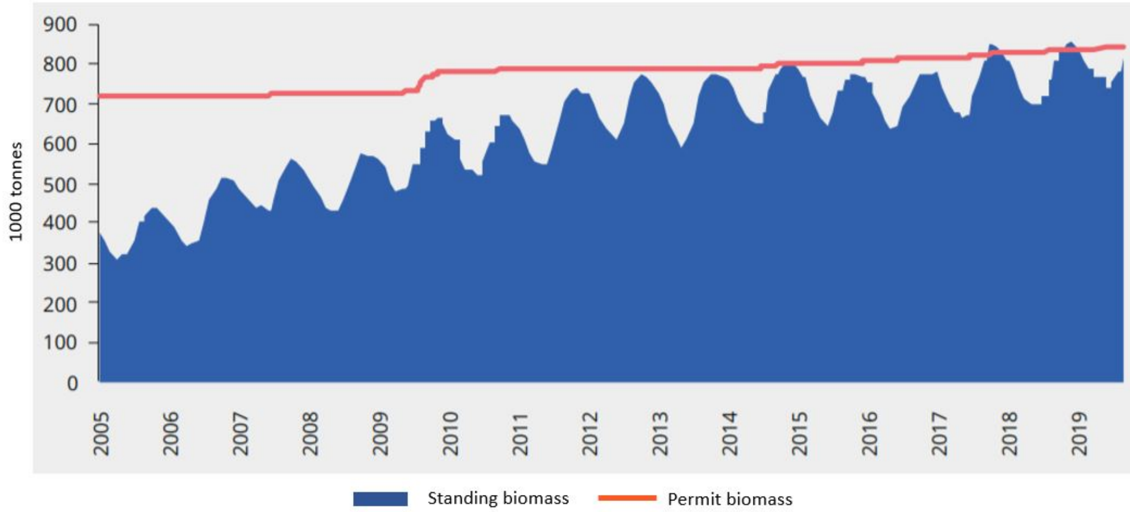


Figure 2.4: Development in theoretical capacity and standing biomass, 2005-2019, Directorate of Fisheries, in [25, p.41].

As seen in the figure, the standing biomass fluctuates throughout the year. Furthermore, the capacity has been better utilized from 2012 than before. The permit capacity has increased by a little over 20% since 2005, and the production has doubled. This is partly because of a more efficient operation.

2.3 Value Chain

The value chain for farming of salmon and trout includes brood-stock (mature fish used for breeding purposes), production of juvenile fish, production of food fish, slaughter and processing, and sales and export, see Figure 2.5. Other essential input factors are related to breeding, genetics, vaccines, feed, well-boats, and other equipment.

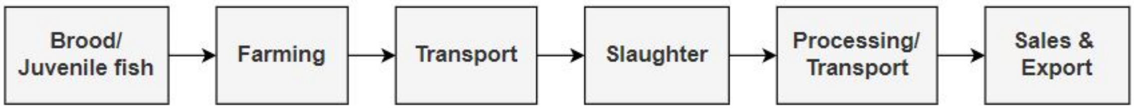


Figure 2.5: Value chain in aquaculture.

The production can roughly be divided into two parts - freshwater and saltwater production. Smolt is the stage in the growth of salmon or trout where it is transferred from freshwater to saltwater. It takes 8 to 18 months to breed smolt sized 100 grams. Today there are investments on production of larger smolts (250-500 grams), which are more resistant against diseases and parasites. It also yields a shorter sea phase which again can be positive for the health and welfare of the fish. This again leads to reduced costs. However, production of larger smolt can be cost driving as the land-based production is energy consuming. When the smolt is large enough, it is transferred from freshwater tanks to sea cages and reaches the sea phase where it grows until it reaches the proper weight for slaughtering. It can take up to 12-18 months to reach the weight of 3-6 kg, but that relies on factors such as temperature and feeding. When the fish is fully grown, they are harvested and transported from the cages to the slaughter sites with well-boats, also called Live-fish-carriers. When the fish is harvested, the breeder would want to either set out new fish or let some of the fish stay to utilize the Maximum Allowed Biomass.

After slaughtering, the fish is packed and sent to customers. The transportation can either be by truck, train, plane, or ship. This part of the process is more thoroughly described in Chapter 3 in the perspective of Lerøy Seafood Group.

2.3.1 Sorting

The fish can in some cases also be sorted at harvest by the well-boats. There are several reasons for doing this.

This also helps utilize the Maximum Allowed Biomass. A cage containing fish of different sizes can also be sorted into several cages for each size. It makes it easier to harvest only the fish that has reached the proper size for slaughter, leaving the smaller sized fish behind for further growth.

Sometimes, the fish at the top can dominate and get most of the feed and thus grow better. This impacts the non-dominant fish further down. Harvesting just the dominant fish allows non-dominant fish get another chance to grow before being harvested. Sorting can also help the slaughter process, due to the machinery needing adjustment between the sizes of fish. This is not that widespread in Norway yet, but is practiced more in Scotland [10].

2.3.2 Costs

Near half of the production cost is related to feed, and in 2016 85% of that was the cost of the ingredients [25]. The production costs differ from region to region, due to different growth rates and different extents of disease problems. Along with the cost of feeding, there are also costs related to treatments of illnesses and lice during the sea phase. The costs are both for the treatments and the loss as a result of the treatment. The economic impact of the loss depends on when in the production phase it happens, and loss late in the phase creates a higher impact. Another factor is the escaping fish. It has costs connected to the loss for capturing the escaped fish and maintenance and repairs. It also has an environmental cost because it can affect the genetics of the wild salmon and creates challenges on spawning grounds.

3. Problem Background

The thesis focuses on the process of planning, and allocating fish to customer orders, from the perspective of the planners. This process is referred to as the allocation process throughout the thesis. In this chapter, the details of this process are going to be described. The purpose of this is to get insight into how planning and allocating is done at Lerøy, and the different challenges the planners face. This insight is used when stating the specific problem that is going to be solved. The problem background is based on allocating farmed fish of species salmon and trout.

In Section 3.1 the attributes of the fish are described. Section 3.2 explains the physical flow and is followed by Section 3.3 which details the necessary information for the plan. The elements of the planning horizon are described in Section 3.4. The steps of the allocation process are explained in Section 3.5. Sections 3.6 and 3.7 explains transportation and costs related to the allocation process. The problem description with limitations and assumptions is presented in Section 3.8.

3.1 Fish

In the allocation process the fish is described by three attributes, namely species, size class, and quality. Figure 3.1 presents a simplified illustration of the three attributes and how they are related.

Figure 3.1: Simplified illustration of the relationship between species, quality, and size class.

In this example the species is salmon. Salmon is divided into two qualities: ordinary and superior. Both qualities are further divided into three different size classes.

In reality, salmon and trout are available in three different qualities and up to nine different size classes.

3.2 Physical Flows

The main entities in the supply chain are fish farms, plants, and distribution centers. The physical flow of this supply chain is illustrated in Figure 3.2, including delivery point.

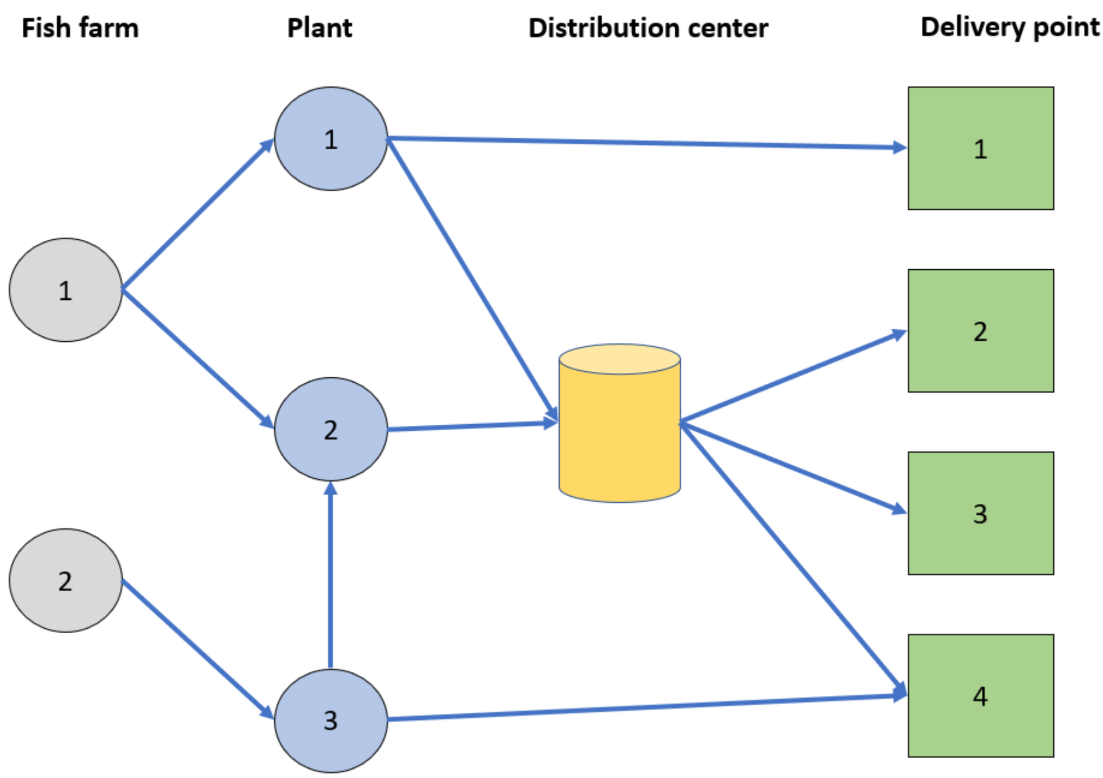


Figure 3.2: Simplified illustration of the physical flow.

Fish Farm

Lerøy has fish farms located along the entire coast of Norway. The purpose of the fish farms in the supply chain is to breed the fish. At the time of harvest, the fish is loaded onto a well-boat and transported from farm to the plants.

Plant

In the supply chain, the plant is the first arrival point for the fish. Depending on the time of year, there are around three to six plants open. The plant consists of a slaughterhouse, a packery, and possibly a processing facility located in the same building. When the well-boat arrives at the plant, the fish is unloaded and delivered to the slaughterhouse. In the slaughterhouse, the fish is first stunned and then gutted. When the process of slaughter is done, the fish is either forwarded to the internal processing facility or transported out from the plant. If the next step is the internal processing facility, the fish is sent directly from the slaughterhouse to the facility, and further processed into fillets or other fish products. If not, the fish must first be packed into boxes at the packery. The boxes can weigh 5, 10 or 20 kg when fully packed, depending on the size of the box. After packing, the boxes are ready for storage or further transport. The storage capacity at plant is limited, and boxes are only stored there for a short period of time. If the boxes are transported out from plant, the next destination can be another plant, directly to delivery point, to a distribution center, or another intermediary site.

Distribution Centers

The distribution center is used as an intermediary hub and has the features of consolidation and storage. All the incoming supply to the distribution center comes from the plants. There is distance between the plant and the distribution centers, and the lead time is usually one or two days. If necessary, it is possible to consolidate supply from multiple plants before further transport. Having significantly larger inventory capacity than the plants, allows the distribution center to hold more inventory. If preferred, it is possible to store the boxes at the distribution center instead of the plant. Long term storage is allowed, and the boxes can be stored up until the expiration date of the fish.

Delivery Point

After all the transport is done, the requested boxes of fish arrives at its destination. The delivery point is specified in the order.

3.3 Information Flow

The planners require specific information to create the plan for allocation.

3.3.1 Farmed Fish

Fish farms can acquire different certifications and have different health statuses. All fish coming from the same farm, have the same certifications and health statuses as that farm. Diseases affecting farmed salmon and trout can be highly contagious. Since the density of the fish in the cages are high, an outbreak can spread quickly. If a farm has had an outbreak of a given disease recently, the health status for fish bred there states that. This information is related to all the fish, even if the fish did not get the actual disease. Fish that get a disease, are either treated or discarded.

When the fish has grown to an acceptable weight and size, they are harvest ready. The planners have minimal influence regarding when the fish are harvested. The fish are harvested on a specific day and sent directly to the plant for further processing. The planners know the day and time the fish arrive at plant.

3.3.2 Forecast

The planners have access to a continuously updated forecast. This is one of the key sources of information used when creating the plan for allocation. The purpose of the forecast is to get an estimation of the available amount of fish. The species of the incoming fish is known, but the forecast holds more specific data regarding size classes, quality, and amount. The estimates are used as basis for the allocation of fish to orders, so inaccuracies have an impact on the plan.

The forecast contains information about:

- total amount of fish
- amount of fish in each size class
- amount of fish with a given quality

When the fish is slaughtered, it is also weighed, and its quality is checked. The planners will at that point start to receive more accurate information about the available amount of supply. This information is used to adjust the plan for allocation to fit the available supply.

3.3.3 Sales Department

The sales department works closely with the planners. They handle all the orders, customer relationships, contracts, and price. The prioritization of the orders is decided by the planners and sales department in cooperation.

3.3.4 Customer Orders

The customer order is the source of information describing the customer's demand. The main elements in an order are details about requested species, size class, quality, number of boxes, certification, health status, and delivery date. In the following paragraphs, each of these elements are described.

Species, Size Class, and Quality

The orders contain details regarding the requested number of boxes for a given combination of species, size class, and quality. An order can allow for flexibility in the number of boxes delivered for the given combination. The flexibility is given as upper and lower bounds on how many boxes that must be delivered. If it possible to deliver within the bounds, the order is considered fulfilled. Another type of flexibility is that the order specifies a certain species but allows delivery of multiple different size classes and qualities. E.g., the species salmon of the quality superior is requested, but it is allowed to deliver from size class 2-3 kg and 3-4 kg.

Day of Delivery

The orders specify a date the customers want the demand to arrive at the delivery point. From the plant to the delivery point, there is a need for transportation which takes an amount of time. This lead time must be factored in when deciding which date the supply must be sent from plant. Day of delivery is calculated based on requested date minus lead time. When the planners create their plan, they only use day of delivery for deciding when the supply should be sent from plant. An order can allow for some flexibility regarding the requested delivery date.

Certification and Health Status

Fish is mostly used in the food industry, so having the proper certifications and health statuses are important. Orders can specify that the fish must have one or more certifications. Customers can also reserve themselves from getting fish with specific health statuses or that is bred in a specific area. Federal governments in certain countries have their own requirements regarding the health and certification of the fish. This means that if the fish is to enter those countries, it needs to have the correct specifications.

External and Internal Orders

An important distinction made in this thesis is between external and internal orders. The term internal order is referring to orders coming from the internal processing facility at the plant. External orders have delivery points outside of the plant, which requires transportation.

3.4 Planning Horizon

When enough information is available, the planning can start. The initial planning starts on Wednesday the week prior to the planning horizon. The planning horizon is one week, starting on Monday and ending on Friday. This is illustrated in Figure 3.3.

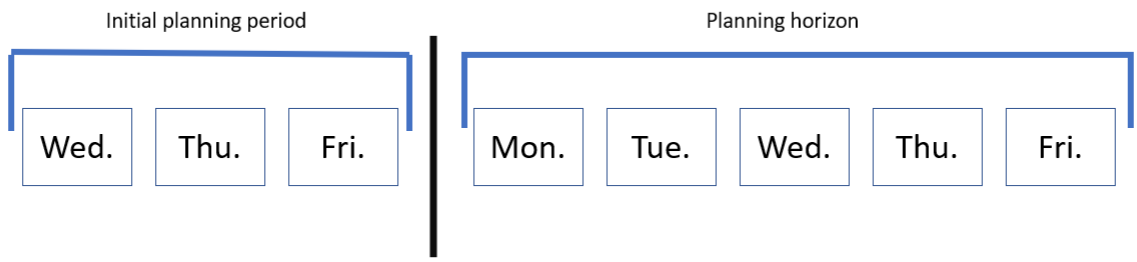


Figure 3.3: The initial planning period and the planning horizon.

Orders are placed by the customers on Wednesday-Friday, and the first initial plan should be available by Thursday. This plan is then re-planned regularly as soon as new or updated information that impacts the plan is available. The planning horizon decreases for each planning day that passes, as illustrated in Figure 3.4.

On Friday until mid-day the period from Monday to Friday is planned. After mid-day on Friday the plan for Monday is sent to the packery coordinator. Monday is then locked, and the packery coordinator implements and deals with situations that might arise for all deliveries that occur on Monday. The planners can now only re-plan for the remaining horizon, which is from Tuesday until Friday. At mid-day on Monday the plan for Tuesday is sent to the packery coordinator. Tuesday is then locked, and the packery coordinator implements and deals with situations that might arise for all deliveries that occur on Tuesday. The planners can now only re-plan for the remaining planning horizon, which is from Wednesday until Friday. This cycle is repeated until there are no more days left in the week.

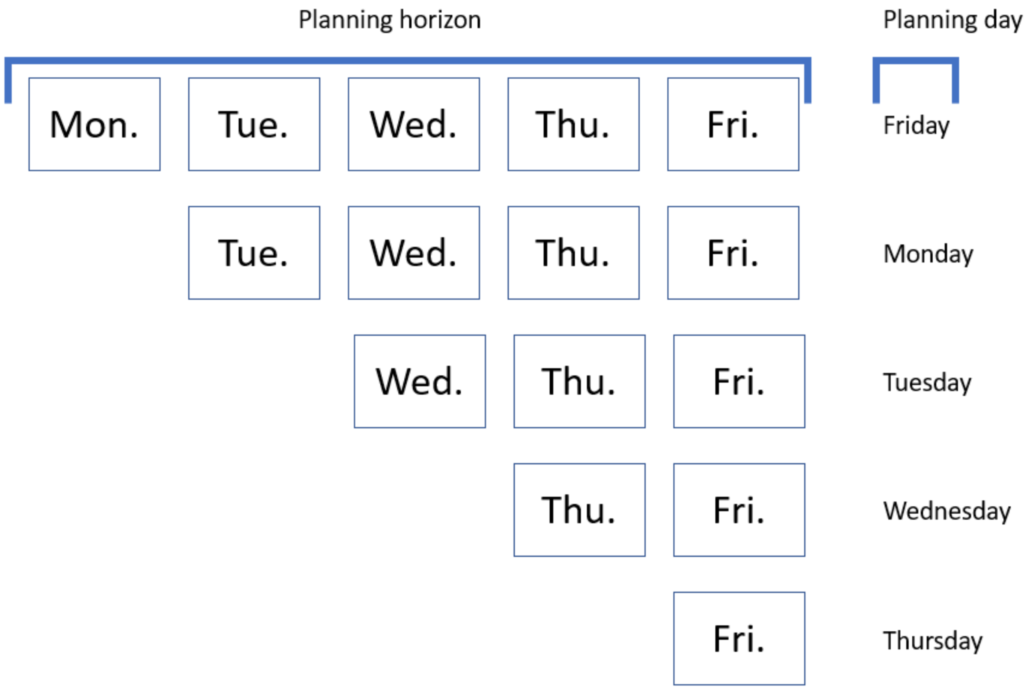


Figure 3.4: The remaining planning horizon for each planning day of the week.

3.5 Allocation Process

The allocation process has the goal of allocating supply to demand in the best possible way. The planners go through five defined steps when creating the plan:

1. Receive orders
2. Check orders
3. Prioritize orders
4. Check if it is possible to fulfill the order
5. Deliver or cancel the order

Receive and Check

The planners receive the order details from the sales department, and check what is requested.

Priority

The orders are given their priorities, which are decided by the planners and the sales department in cooperation. Main aspects that affect the priority of an order are contracts, customer relationship, and price. Prioritization influences the ranking of the order. Highly prioritized orders get fish allocated to them first, while lower prioritized orders get the remaining available fish. Another important factor for the priority ranking of an order, is deviation. Sometimes a customer must deal with deviations from their exact demand. It is a goal for the planners to deliver with as little deviation as possible to customers over a longer period of time. Multiple customers shop repeatedly, which opens for possibility of leveling out the deviation. If a customer has significant deviation from their demand this week, their next order is set to a higher priority.

Is it Possible to Fulfill the Order?

Now, the planners have enough information about the demand for each order, and its prioritization. The available supply is either known exactly or estimated. This is enough information to start the allocation of supply to demand. How the order is fulfilled is decided in this step.

Accept or Cancel

If it is possible to fulfill the order, it can be accepted. If not, the order is cancelled.

Aftermath

When all the orders that can be fulfilled are dealt with, there might be unsold fish left. The sales department tries to find even more orders, so all the fish can be sold. If this is impossible, the fish is stored until the next planning horizon, usually at a distribution center.

3.6 Transportation

When the fish has been allocated to orders, it is necessary to transport it out from the plant. The planners decide which vehicle takes which order, and the logistics department is responsible for booking enough vehicles. The main vehicle used for transport out from plant is a truck with a loading capacity of 891 boxes weighing 20 kg. Achieving a high fill-rate for all vehicles leaving the plant is a priority, as it decreases the transport costs. The planners therefore attempt to load as much as possible into each vehicle. After the fish is allocated to vehicles, the planners are done with their responsibilities regarding transport, and the logistics department handles the rest of the transportation. The destination for the vehicles sent from plant can either be the final delivery point requested in the order, or an intermediary hub. At the intermediary hub, the boxes are either stored, consolidated, or cross-docked for the next leg of the journey.

3.7 Costs

The entire value chain for farmed fish has many elements creating costs. Examples are farming, processing, transportation, and storage. When it comes to the perspective of the planners, the costs are related to transportation and storage of the boxes. The transportation costs are connected to the vehicles used to transport boxes out from plant. Increasing the fill-rate on each vehicle allows for using fewer vehicles and thus lower the costs. Storing unsold boxes requires handling, space, and insurance, which all creates more costs. Whether the boxes are stored at the plant or at the distribution center, affect the storage cost per box.

3.8 Problem Description

The purpose of this section is to present the thesis problem, including assumptions and limitations. The aim of the thesis is to assist Lerøy by creating decision support for allocating supply to demand. The decision support should be able to aid in solving challenges they frequently face.

Determining how to allocate supply to demand is influenced by multiple factors. The planners have limited control over incoming supply. This creates different scenarios dependent on how well the supply matches the demand. If there is insufficient supply, they are forced to decide which orders to fulfill. When there is a surplus of supply, challenges regarding where to store the boxes arises. The decision about finding an appropriate location to store boxes, is affected by storage costs, available inventory capacity, and shelf life. Another layer of complexity added into the process is the element of transportation. Decisions regarding transport are affected by costs, routing, and the goal of achieving a high fill-rate. These are all challenges that the planners face frequently, and that they want to solve in a satisfactory manner. Seeing the magnitude of different goals the planners have, using an approach that can handle multiple goals seems to be appropriate.

Two goals are chosen to be the focus of this thesis problem. The first goal is to deliver as much as possible of the available supply to orders. This is going to lower the total number of unsold boxes left at the end of the planning horizon. Lowering the number of unsold boxes left, can potentially lower storage costs. The second goal is to fulfill as many highly prioritized orders as possible. The priority of an order is, as previously stated, based upon contracts, customer relationship, and price. Satisfying this goal is going to please important customers and can increase profits. On the other end, the goal overlooks challenges related to unsold boxes, inventory, and fill-rate on vehicles.

The interesting connection between the two goals are that they can be conflicting and non-conflicting, based upon the balance between supply, demand, and the prioritization of the orders. To elaborate around this connection a simplified example with four scenarios is used. The overall setting of the scenarios is that there are two hundred boxes of supply available and two orders. The supply meets all the requirements of both orders.

Scenario 1
Order one and two demand one hundred boxes and are equally prioritized. In this scenario both orders can be fulfilled without any conflict between the goals.

Scenario 2
Order one and two demand one hundred boxes, but order two are prioritized higher than order one. In this scenario there is enough supply to deliver to both orders, so there is no conflict between the two goals.

Scenario 3
Order one have a demand of two hundred boxes, and order two have a demand of one hundred boxes. Order one is prioritized higher than order two. Only order one is fulfilled, but there is no conflict between the two goals.

Scenario 4
Order one have a demand of two hundred boxes, while order two demands one hundred boxes. Order two is prioritized higher than order one. In this scenario there is a conflict between the two goals. There is not enough supply to fulfill both orders, so one must be chosen over the other. This choice depends on which goal is the most important.

Finding a way to efficiently solve these goals can assist the planners in creating the allocation plan for a given time period.

3.8.1 Assumptions and Limitations

The scope of the realistic allocation process is quite extensive. Assumptions and limitations have been done to narrow the scope down.

Boxes

A box is used as the measurement of stating how much fish there is. The box is defined as a standardized unit containing fish, and it is assumed that all boxes are of equal weight and size. The assumption of equality is made because there exists boxes of different weights and sizes, but not all plants deliver all different sizes.

Plant

A plant has both incoming and outgoing flows. The only source of incoming flow to a plant is supply from the fish farms. Outgoing flow is defined as either sending fish directly to delivery point, to distribution center, or to the internal processing facility. The processes of slaughter and packing are omitted. It is assumed that the slaughter and packing processes happen quickly enough for the fish to arrive and departure on the same day. The plant has the feature of storage if there exists inventory capacity at the plant. The possibility of having initial inventory is included, so inventory from the previous planning horizon can be used. All surplus supply that cannot be stored at the plants, is sent to the distribution center.

Distribution Center

The distribution center has both incoming and outgoing flow. All the incoming supply to the distribution center comes from the plants. Outgoing flow consists of sending boxes to the delivery point specified in the order. The distribution center has inventory capacity, which allows for storage. Initial inventory is also included for the distribution center, so it is possible to use supply that arrived in the previous planning horizon.

Planning Horizon

The problem scope is set to focus on both the plant and distribution center. The plant is open from Monday until Friday, and the distribution center is open from Monday to Sunday. The boxes of fish are only counted as available supply for a brief period in the allocation process. The period is from the box is ready for distribution at plant until it is delivered. After delivery, the boxes are outside of the scope. In real life there is a connection between consecutive planning periods, which opens for the possibility of planning ahead. This aspect of the planning is omitted. Each planning horizon is independent of each other, except for what is left in inventory from the previous planning horizon.

Methods of Delivery

The problem has three different methods of delivery:

1. From plant directly to delivery point
2. From plant, through distribution center, to delivery point
3. From plant to internal processing facility

Diseases and Certifications

The location of the farms where the fish have been bred determines the health status and certifications related to the fish. If a farm has had an outbreak of disease, this information affects all fish coming from that farm. Problems related to medical treatment of fish at an individual level, is omitted. It is assumed that all fish with a disease are discarded and are not a part of the supply.
If a farm has any certifications, then all fish coming from that farm possesses the certification. It is assumed that all fish can get all the same diseases and certifications.

Order

The order specifies a certain number of boxes for a given combination of species, size class, and quality. To allow for flexibility in the number of boxes delivered, an order can state upper and lower bounds on the demand. Delivering inside of the given bounds, while meeting all other requirements is regarded as a fulfilled order.

When it comes to substituting, some limitations are set. If a specific combination of species and quality is requested, it cannot be substituted with another combination of species and quality. On the other hand, it is possible to substitute one size class with another size class within the same combination of species and quality.

If an order specifies requirements regarding disease and certification, the requirement applies to the entire order. It is assumed that it is not allowed to request one amount of fish with one type of certification and health status, and another amount with a different certification and health status in the same order.

Transportation
The problem does not focus on the role the transportation has in the allocation process. Decisions regarding number of vehicles used, routing, and achieving a high fill-rate are for that reason omitted. This is done to limit the size of the problem.

Costs
Costs have an extensive role throughout the entire supply chain. The allocation process has certain aspects directly affected by costs, but the specific decisions regarding which orders have supply allocated to them is only affected to a limited degree. Storage costs are implicitly implemented into the goal of delivering as much as possible of the available supply. Pursuing this goal can result in a low number of unsold boxes, which in return can lower the total storage cost. Outside of this, costs are omitted. Having also left out the element of transport, a major source of cost in the allocation process is eliminated. Not explicitly including costs, allows for studying how the balance between supply and demand, and the orders priority affect the relationship between the two goals.

4. Relevant Literature

This chapter contains an overview of relevant literature about optimization and planning problems regarding supply chains involving perishable products. The specific areas are chosen to get an overview over what has been done before, and what this thesis can contribute with.

It is assumed that the reader is familiar with the concepts of supply chains, value chains, forecasts, operations research, mathematical modelling and programming. Perishable supply chains regarding agriculture are included due to similarities with aquaculture when it comes to uncertainties in harvest and supply. For the same reason, supply chains containing more stable production and supplies are omitted.

Section 4.1 gives an overview over the characteristics of perishable supply chains, mainly containing fish. In Section 4.2, different issues regarding planning in agricultural supply chains are presented before different previous solution approaches are presented in Section 4.3.

4.1 Supply Chains

Livestock, food, or other perishable products increases the complexity of a supply chain. The complexity factors widely mentioned are the limited shelf-life [12, 3], the fact that the value of the product decreases whilst moving downstream [22], and the variability of price and demand [3].

Regarding the distribution of perishable products, the main objective is attending to the freshness of the products [22]. This again affects the flexibility of the network directly. The value of the products reduces over time from the point of production or harvest, so minimizing shipping time or maximizing quality of the products in delivery time should be considered.

Jensen et al. [12] mentions several characteristics in supply chains in the fish industry. In general, different species of fish belong to different chains from the time of catch to consumption, though there may be substitutions of species which leads to an inter-dependency between the different chains.

Aquaculture can occasionally be a parallel source to wild catch but can also result in independent supply chains from farm to fork. This parallel source can be compared with farming of other animals for production of food.

In the upstream end of the supply chain, the fishermen and catching of fish takes place, along with breeding of species in fish farming [12]. The fresh and processed sales are in the downstream end. Between these, there are several agents that handle and process the fish and the products.

Abedi and Zhu [2] divides supply chains involving fish or other livestock into two parts; the warm chain and the cold chain, see Figure 4.1. The warm chain contains only one type of product, namely live fish and other livestock. The cold chain on the other hand, contains the product after harvest and processing e.g., frozen and packaged. The thesis problem takes part in the cold chain, and thus omits any literature only concentrated on the warm chain.

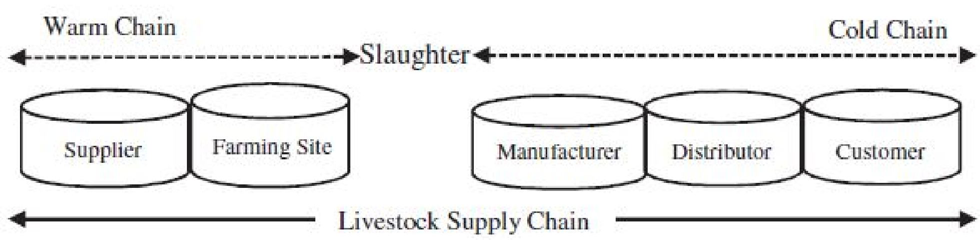


Figure 4.1: Elements of warm and cold chain [2].

The profit of fish farmers may be influenced by factors including quantity of spawns purchased, time to harvest, and the customer demand [2]. The profit is maximized when the supply perfectly matches the demand at the end of a particular period. Another factor is the time the fish is harvested. Longer culturing time gives higher feeding costs, at the same time as the value of the fish increases. However, the customer demand may change throughout the year, and thus impacts the decision making.

4.2 Planning in Supply Chains

The planning process in supply chains is divide into 3 levels: strategic, tactical, and operational. Strategic is less detailed and has a long time span, while operational planning is high detailed and spans over hours, days, and weeks. Tactical planning is in between. The problem in this thesis is on the operational level, and the literature is chosen accordingly. Even though demand management is under strategic planning, it is included for the sake of context.

4.2.1 Demand Management

Croxton et al. [8] describes demand management as the process that balances the requirements of the customers (demand) with the capabilities of the supply chain (supply, capacity). The process includes the forecasting of the demand and synchronizing it with production, procurement, and distribution. A company can become more proactive to expected demand, and more reactive to unexpected demand with a good demand management process. An important part of demand management is to reduce demand variability and improve flexibility. The reduced variability helps reduce costs and the increased flexibility helps to respond quickly to events, both external and internal. The overall goal of demand management is to meet the customer's demands effective and efficiently.

Variability can be seen as the enemy of planning, and Croxton et al. addresses two things that can be done to reduce the negative impacts of variability. One can either reduce the variability or increase the flexibility to handle it. Reducing the variability is outside the scope of the thesis, as it occurs in the upstream end. This leaves increasing the flexibility. Flexibility can be costly and should not be used as a temporary solution for problems that can be avoided. By gaining flexibility, one can better manage the variabilities. It can also influence factors such as reliability, quality, and costs of the process. First one needs to identify the level of flexibility needed, and make sure it is consistent with the needs of the supply chain. After identifying the flexibility needs, one should find ways to achieve it. This can be identifying bottlenecks and restrictions.

4.2.2 Planning Issues in Agricultural Supply Chains

Agriculture faces issues such as when to harvest and how often, to satisfy the requirements from the customers. These requirements include colour and ripeness of the harvest, for instance tomatoes. Ahumada and Villalobos [4] addresses the trade-off between balancing value loss due to perishability and the costs of preventing that loss.

There is a difference in planning on tactical and operational level when it comes to harvesting and distributing perishable crops. The growers have better estimates of the yields and the market conditions at the time of operational planning. Regardless, there can be factors influencing the decisions such as market fluctuations; expected yield and maturity of the crops - which are dependent on weather conditions - and the behavior of the crops after harvest. Figure 4.2 illustrates these factors and how they affect the planning.

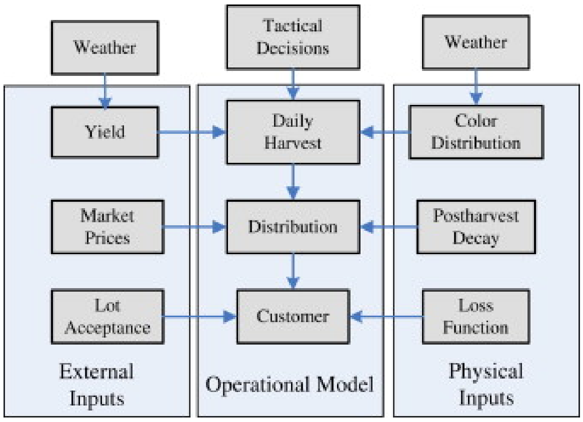


Figure 4.2: Factors affecting operational planning [4].

A way to deal with these short-term planning issues is having a planning model that includes the post-harvest behavior of the crops. In addition to this, the planning model should also include weather effects, transportation time, post-harvest decay, labor, and delivery costs. Simultaneously, the decision variables should include transportation mode and harvest policy so that the crops reach the right customer at the right quality, with a proper remaining shelf-life, and with the appropriate routing through the supply chain.

In the supply chain, crops can be contracted or sold in open or spot markets. Customers in the open market usually wish to pick up the products at the warehouses, whereas contracted customers usually have delivery included. There are many different agreements in between the two mentioned, illustrated in Figure 4.3.

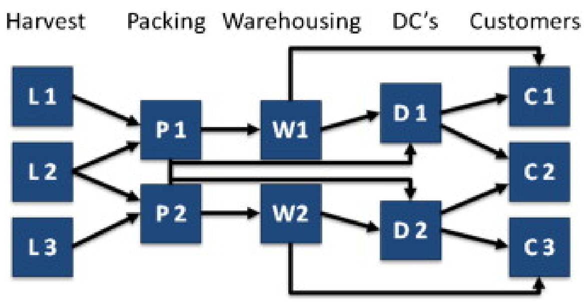


Figure 4.3: Supply chain of fresh horticultural crops [4].

Customer Priority and Requirements

Customers can be grouped into A, B, and C customers based on their accumulated revenue [2], where class A are the first 80%, class B the next 15%, and class C the last 5%. Distribution based on that classification will help companies to meet more profitable demand with higher priority. Although the priority of customers can be handled in different manners, such as seeing the revenue contribution over time instead of in one single purchase, the 80-15-5 concept can be a suitable starting point.

The planning model should also consider the requirements for the crops at their final destinations. Some customers such as restaurants prefer mature green tomatoes that are harvested before maturity and are ripen right before shipping. Retailers usually prefer vine ripe tomatoes that are harvested at breaker stage - the stage where the tomato has a definite red, yellow, or pink colour on a maximum of 10% of the surface [16].
The limited shelf life of the crops prevents lengthy storage, which forces producers to supply demand with the current production, despite the yield distribution. This is similar to the case in this thesis where the plants have limited storage, and other factors that forces harvest and slaughter. This forced harvest may yield different sizes and qualities than expected, and the allocation may be affected accordingly.

4.3 Solution Methods Used

Ahumada and Villalobos [4] presents a Mixed Integer Programming model that decides which products to harvest, how often in times per week, and on which day. They also consider restrictions on time and labor, and how the harvest decisions affect the quality of the products. However, these decisions may be easier in agriculture due to the maturity of the crops being more visual than size and quality of fish in aquaculture. The advantage of the model provided, is that it can be solved with commercially available software, even with realistic instances of operational planning problem. Even though the model only has one objective, it is suitable for further development due to many of the basic requirements in short term planning of fresh produce is addressed.

Amorim et al. [5] presents a Multi-Objective Mixed Integer Programming model which integrates production and distribution. The focus of the model is to minimize total cost and maximize mean remaining shelf-life. Their logistic setting is multi-product, multi-plant, multi-distribution center, and what differs the most from the scope of this thesis - multi-period. The model is developed for two product types: one with fixed shelf-life, and one with loose shelf-life. These models always consider the decreasing value of the products until they perish. A Pareto front is also constructed, but the method used is not particularly mentioned.

Abedi and Zhu [2] has an optimization model that maximize the profit of a trout fish supply chain. The output of this plan determines the purchase quantity (of trout spawns), harvest plan, and distribution plan. The distribution plan also involves customer prioritization based on quantity in demand, in order to efficiently find a way to deliver fish. Their model is formulated as a Mixed Integer Linear Programming problem, with the main contribution being to simultaneously consider factors in both the warm and the cold chain. As a remark for future work, they mention that so far (2016), fish farming companies have not taken much advantage of distribution planning.

Musavi and Bozorgi-Amiri [22] compares features of earlier related papers with their own work, such as objectives, perishability, limitations on vehicles, and scheduling. They focus on cost, delivery time, and CO_2-emission. They chose Metaheuristics (more specifically a non-dominated sorting genetic algorithm-II - NSGA-II) due to the multi-objective hub scheduling problem needing an effective solution to provide a proper Pareto frontier. Hub location problems are proved to be NP-hard, and their model was of a dimension so that solving it with exact approaches would be too time consuming even for small instances. They also compared the results from NSGA-II with those of the ε-constraint method - the latter being an exact solution approach. They also mention that this approach is widely used as decision making in multi-objective models. The improved version of the ε-constraint method, AUGMECON, is used to evaluate their proposed metaheuristic method. As a result of this evaluation, they found that the metaheuristic could achieve suitable Pareto solutions, and is also gave more Pareto points. More Pareto points means more options for the decision maker. Regarding solution time, the proposed NSGA-II algorithm yields good solutions with small gaps in a much more appropriate time compared to ε-constraint method, which they claim more appropriate for small sized problems.

There has also been done research regarding optimizing production in context with feeding, when to buy smolt, and when to harvest [2]. Similar research has also been done in the agriculture industry [4] regarding when to harvest to get the right ripeness.

Previous, and similar, problems have cost as a common factor in the problem solving. This is of course not beneficial for the thesis problem, where the details of cost and revenue are omitted. Many also seem to integrate production and distribution. In aquaculture it would mean that factors such as growth rate, feeding, temperature, diseases, and weather would affect the harvest and thus affect the allocation of supply to demand. In addition, there are no existing models for the short-term allocation problem presented in this thesis and its field of study. They are either multi-period or integrated production and distribution.

5. Methodology

The previous chapter gave an overview of what approaches has been done to solve similar problems in comparable supply chain settings. This chapter describes some of the optimization methods that were considered appropriate for the thesis.

Section 5.1 gives a brief introduction to the assignment problem and an overview over Multi-Criteria decision making. Section 5.2 describes different constraint methods. Lastly, Section 5.3 gives a justification to the method chosen, and why other methods were rejected.

5.1 Assignment Problem and Multiple Objectives

Assignment problems are usually considered as problems of minimizing cost or time. However, real-world assignment problems do not necessarily have a single criterion as the two mentioned [21]. When a problem has multiple goals, it is more appropriate to use a multi-criteria approach, as it allows the decision situation to be more accurately captured in an optimization model. One of the most commonly used multi-criteria optimization techniques is goal programming. It requires the decision-maker to specify the weighting of the objectives that are considered. Although the goal programming approach offers a certain flexibility, it has major limitations regarding considering single weights for each objective function [21].

5.1.1 Assignment Problem

Assignment problems are related to combinatorial optimization, which is a sub-field of mathematical optimization. The assignment problem is relevant in dealing with problems related to production, scheduling and distribution [29]. The concept of assignment problems is that there are m resources/people and n demands/jobs. The resource i has a cost c_{ij} of being assigned to demand j. The goal is to assign the resource to the demand based on a certain objective such as minimizing time or cost. In order to do so, the problem would be formulated as

$$\text{Optimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{5.1}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, ..., m \tag{5.2}$$

$$\sum_{i=1}^{m} x_{ij} = 1, \quad j = 1, ..., n \tag{5.3}$$

where

$$x_{ij} = \begin{cases} 1 & \text{if resource } i \text{ is assigned to demand } j, \\ 0 & \text{otherwise} \end{cases}$$

Constraints (5.2) enforce that each resource m need to be assigned to one and only one demand n, and constraints (5.3) enforce that every demand n needs to be assigned to only one resource m. This implies that $n = m$.

In every assignment problem, there is a matrix $[c_{ij}]$, where c_{ij} as mentioned is the cost of assigning resource i to demand j, called the *assignment matrix* [9].

$$\begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{pmatrix}$$

An assignment problem can be solved as a regular Linear Programming model, but the models can become very large [29]. Assigning 100 people to 100 tasks would result in a 100 by 100 matrix, or 10 000 variables. When solving, constraints must be fulfilled under certain conditions. These constraints are so called hard constraints, which means they must meet any condition, and satisfying the conditions could yield a feasible solution. Other constraints are soft constraints, which are seen as needed, but not crucial. These constraints can be put in the objective function, and when they are adhered, they do not affect the feasibility of the solution. However, to get a solution with high quality the constraints must be fulfilled as much as possible [9]. The assignment problem has many techniques that can be used to model it. Exact methods such as linear programming, integer programming, dynamic programming [9], and the Quadratic Assignment problem [1] are some of them. Other methods such as Heuristics and metaheuristics produces good, but not optimal, solutions. These methods are often used when the problems are too large for exact methods. Heuristics find a good solution faster, but exact methods find the optimal solution. When the problem becomes complex, the duration of solving with exact methods becomes more complicated than by solving with heuristic methods [9]. Deciding which method to use becomes a trade-off between time consumed, and the quality of the solution.

5.1.2 Multi-Criteria Decision-Making

It was previously stated that the research problem is inside the operation research field, to be more precise in the sub-discipline of Multi-Criteria Decision-Making. Multi-Criteria Decision-Making refers to making decisions in the presence of multiple, usually conflicting, objectives [27]. The field of research focuses on both qualitative and quantitative research problems. The focus in this thesis is on the quantitative side. This sub-discipline has had extensive amounts of research done over many years, indicating that the field is highly attractive for research [17]. Mardan et al. [17] showed that Multi-Criteria Decision-Making has been applied in a wide range of fields such as operations research, supply chain, production, and risk management.

5.1.3 Dominance and Pareto Optimality

In the case of two conflicting objectives, one to minimize the impact of production and the other to maximize profit, increasing the profit results in increasing the impact on the environment. Figure 5.1 shows a potential trade-off between the profit and the impact. Point A has the same profit as point B and the same amount of toxic waste as point C. This is clearly undesirable, since point B gives the same profit with less impact and point C gives the same impact with even higher profit. Points B and C *dominates* point A, along with all points on the curve between B and C. If an alternative is dominated, it means that there are other alternatives that provides better values for at least one of the objectives without worsening the other. A decision maker would prefer non-dominated alternatives [26].

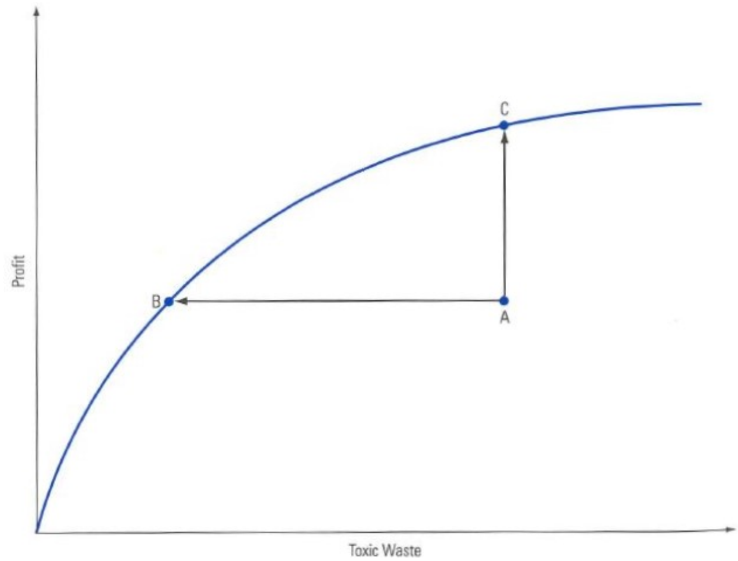


Figure 5.1: Trade-offs between objectives and dominant decision solution alternatives [26].

In Multi-Objective Mathematical Programming there exists multiple objective functions, and usually there is no solution that optimizes all objective functions simultaneously. In these cases, the decision makers would look for the most preferred solution instead of the optimal solution. In Multi-Objective Mathematical Programming, the optimality concept is replaced with Pareto optimality or efficiency. Pareto optimal solutions are solutions where one objective function cannot be improved without worsening at least one of the other.

5.1.4 Finding the Efficient Solution

Mavrotas [18] defines a solution x as efficient if there are no other feasible solutions x' such as $f_i(x') \geq f_i(x)$ for every $i = 1, .., p$ with at least one strict inequality.
Hwang and Masud [11] define three steps to Multi Objective Decision Making:

1. At which stage the information is needed
2. What type of information
3. Major classes of methods

The first step is again divided into 4, and says something about where the decision maker comes in:

1. No preference information
2. A priori
3. Interactive
4. A posteriori (Generation method)

In the A Priori method, the decision maker is involved before the solution process. However, they may not always know their preferences beforehand. In the interactive method, there are phases of dialogue with the decision maker alternating with phases of calculation. Here the decision maker drives the search towards a preferred solution. The downside to this method is that the decision maker never sees the whole picture, or the Pareto set, so their most preferred solution may not be the most preferred after all. In the A Posteriori method, the efficient solutions are generated, and then presented to the decision maker who then selects among them. This method was less popular earlier due to computational effort [11] but has significant advantages such with solutions being at hand when the decision maker is involved, and no potential solution is left undiscovered [18]. There have also been improvements since the seventies regarding computers and computational software, which makes the A Posteriori more popular [19]. Depending on where the decision maker is involved in the process, and the type of information, one can find the appropriate method for solving the problem. E.g., in A Priori the Lexicographic Method and Goal Programming are recommended, and in the A Posteriori the ε-constraint Method is recommended.

5.2 Constraint Methods

The ε-constraint method is one of the recommended methods for A Posteriori. This method generates non-dominated solutions and is one of the two most popular methods for generating the Pareto front along with the Weighting Method [20].

This section describes how to calculate a payoff-table and presents the ε-constraint method along with two improved versions; AUGMECON developed by Mavrotas [18], and AUGMECON2 developed my Mavrotas and Florios [19, 20].

5.2.1 Lexicographic Method

The lexicographic method requires the objectives to be ranked by importance [11]. The preferred solution is obtained by maximizing the objectives starting with the most important and proceed with the other objectives in their order of importance.
After solving the first objective and obtaining $f_1 = z_1^*$, the second objective is optimized by adding $f_1 = z_1^*$ as a constraint.
After solving the second objective and obtaining $f_2 = z_2^*$, the third objective is optimized by adding both $f_1 = z_1^*$ and $f_2 = z_2^*$ as constraints, and so on. The solutions gained are sensitive to the ranking of the objectives, so when two objectives are almost equally important one should exercise caution in applying this method.
On the other hand, Waltz's lexicographic method [28] may reduce the sensitivity of the decision maker's priority. The first objective is optimized, then the second objective is optimized subject to the first objective being kept within a certain percentage of its optimum. The third objective is then optimized subject to the two first objectives staying within a certain percentage, and so on.

5.2.2 Calculating the Payoff Table

A payoff table presents the values for all objectives in a problem, after optimizing each objective subject to the constraints. There are several ways of calculating the payoff table. Chowdhury and Tan [7] describes an easy approach. Each n objective functions $(Z_1, .., Z_n)$ are optimized individually, giving minimum and maximum values for all.
Minimize or maximize:

$$Z_i = \sum_{j=1}^{m} X_j c_j \tag{5.4}$$

An example is given in Table 5.1, with 4 objectives. When minimizing objective function 1, Z_1 refers to the value of that objective, Z_2 refers to the value of objective 2, and so on. When maximizing objective function 2, Z_1 takes a higher value, but so does Z_2 as it is the one being optimized.

Table 5.1: Example payoff table

	Z_1	Z_2	Z_3	Z_4
Min obj. 1	2	2	60	100
Max obj. 2	5	9	50	75
Max obj. 3	4	9	70	50
Max obj. 4	5	7	50	100
Min. value	2	2	45	50
Max. value	5	9	70	100

5.2.3 ε-constraint

The method is applied by optimizing one of the n objectives, while the other $n-1$ objectives are used as constraints with a value ε on the right-hand side. This ε value is chosen from the payoff table which presents the minimum and maximum values the corresponding objective function can take. The payoff-table is calculated by using the lexicographic method.

$$
\begin{aligned}
& \max f_1(x) \\
\text{subject to} \quad & \\
& f_2(x) \geq \varepsilon_2 \\
& f_3(x) \geq \varepsilon_3 \\
& ... \\
& f_p(x) \geq \varepsilon_p \\
& x \in S
\end{aligned} \tag{5.5}
$$

where x is the decision variable, S is the feasible region, and $\varepsilon_2, .., \varepsilon_p$ are parameters for the right-hand side for the specific iteration drawn from the grid points of the $p-1$ objective functions.

However, it is pointed out by Mavrotas [18] that the optimal solution of the problem is only guaranteed to be efficient if all $p-1$ objective function constraints are binding. It is further proposed to transform the inequalities to equalities by adding slack or surplus variables.

These variables are also used in the objective function to force production of efficient solutions:

$$
\begin{aligned}
&\max\big(f_1(x) + \lambda(S_2 + S_3 + ... + S_p)\big) \\
\text{subject to}\quad & \\
&f_2(x) - S_2 = \varepsilon_2 \\
&f_3(x) - S_3 = \varepsilon_3 \\
&... \\
&f_p(x) - S_p = \varepsilon_p \\
&x \in S \text{ and } S_k \in R^+
\end{aligned}
\tag{5.6}
$$

where $\lambda \in \left[10^{-6}, 10^{-3}\right]$

The range of the k^{th} objective function is divided into q_k equal intervals using $q_k - 1$ intermediate evenly spaced grid points. This gives $q_k + 1$ points that are used to vary the right-hand side of the k^{th} objective function.

The ε-constraint method has an advantageous feature, namely that it is possible to control how dense the efficient set becomes by assigning proper values to q_k. The more grid points used, the denser the efficient set becomes. However, this results in longer computation time.

5.2.4 AUGMECON and AUGMECON2

AUGMECON, or Augmented ε-constraint, enhances the original ε-constraint method [18]. It also addresses some weak points of the ε-constraint method, namely the guarantee of Pareto optimality of the obtained solutions in the payoff table and the generation process, and the increased solution time for problems with more than two objective functions.

The AUGMECON2 improvement exploits the information from the slack variables in every iteration and reduces computational time as many redundant iterations are avoided [19]. AUGMECON2 can be used to produce approximations of the Pareto set effectively. The density of the approximation can be controlled by controlling the number of grid points. However, the major advantage is that in Multi-Objective Integer Programming the method can be adjusted to produce a complete, exact Pareto set under two conditions:

- Objective function coefficients must be integer
- The nadir points of the Pareto set must be known

The nadir points are constructed by the worst objective values in the Pareto optimal front, which is a difficult task compared to the ideal points which are found by optimizing the objectives. This becomes more difficult as the number of objective functions increases [6]. However, for problems with only two objective functions it is guaranteed that the payoff-table provides the nadir points [19].

In the original AUGMECON method [18] the objective function in (5.6) is modified to:

$$\max \left(f_1(x) + \lambda \left(\frac{S_2}{r_2} + \frac{S_3}{r_3} + \ldots + \frac{S_p}{r_p} \right) \right) \tag{5.7}$$

The new parameter r_k is the range of the respective objective function found in the payoff table.

In AUGMECON2 [20] the objective function (5.7) is slightly modified:

$$\max\left(f_1(x) + \lambda\left(\frac{S_2}{r_2} + 10^{-1}\frac{S_3}{r_3} + \ldots + 10^{-(p-2)}\frac{S_p}{r_p}\right)\right) \tag{5.8}$$

This is in order to perform a lexicographic-like optimization on the rest of the objective functions if there are any alternative optima. With this formulation, the solver finds the optimum for f_1, and then it tries to optimize f_2 and so on. With the AUGMECON formulation, the sequence for optimizing f_2 to f_p is not of importance, while in AUGMECON2 the sequential optimization of the constrained objective functions is forced.

For each objective function 2..p the objective function range r_k is calculated. The range of the k^{th} objective function is then divided into q_k equal intervals using $q_k - 1$ intermediate evenly spaced grid points. This gives a total of $q_k + 1$ grid points that is used to vary the right-hand side (ε_k) of the k^{th} objective function. The total number of runs becomes: $(q_2 + 1)(q_3 + 1)...(q_p + 1)$.
The discretization step for this objective function: $step_k = \frac{r_k}{q_k}$
The right-hand side of the corresponding constraint in the t^{th} iteration for the k^{th} objective function becomes: $\varepsilon_{kt} = fmin_k + t(step_k)$

$fmin_k$ is the minimum value obtained from the payoff table and t is the counter for the k^{th} objective function. The surplus variable that corresponds to the innermost objective function is checked in each iteration. The bypass coefficient is then calculated:

$$b = int\left(\frac{S_k}{step_k}\right)$$

When the surplus variable S_k is larger than $step_k$, the next iteration gives the same solution with the surplus variable as difference with value $S_k - step_k$. This is what makes the iteration redundant and can be bypassed as no new Pareto optimal solution is generated. So, the bypass coefficient b indicates how many consecutive iterations that can be bypassed. By using this information, the process is highly accelerated as redundant iterations are avoided.

5.3 Solution Method Decision

Constructing a Pareto front would help visualize the solutions and make it easier for the decision maker to choose between the available solutions.

After reviewing different methods, the Augmented ε-constraint method was chosen as the most fitting. The most emphasized reason for the choice is the guarantee of Pareto optimality of obtained solutions. As claimed by Musavi and Bozorgi-Amiri [22], the Augmented ε-constrain method is more appropriate for small sized problem. The reason for still choosing this method is that their proposed metaheuristic method NSGA-II only gives good solutions. On the other hand, NSGA-II yields these solutions in a more appropriate time. Getting efficient solutions was decided more important than getting good solutions even if it would be more time-consuming.

AUGMECON2 was rejected since the problem was narrowed down to two objective functions. With only two objective functions, the lexicographic order of the non-primary objective functions would not be useful. This is because the factor $10^{-(p-2)}$ is only applied if there are three or more objective functions.

6. Model Formulation

This chapter describes the mathematical model of the allocation process. The model is a mixed integer linear multi-objective model. Section 6.1 describes the scope of the model. Section 6.2 describes the notation before objective functions and constraint are presented in Sections 6.3 and 6.4. Section 6.5 explains how AUGMECON is implemented into the model.

6.1 Model Scope

The intention behind the model design is to capture the fundamental essence of the allocation process. Simplifications, assumptions, and exclusions of elements have been done to create a mathematical description of the process. The key elements incorporated into the model are farms, plants, distribution center, supply, demand, and attributes related to the fish. These elements are decided to be the most fundamental parts of the allocation process. The mathematical model is designed for use in the period before the planning horizon starts, and not meant for re-planning inside the horizon. The model is built with the intent of supporting the decision makers at Lerøy in the allocation process. Solving a problem instance described with the model, results in a plan for how to allocate supply to orders for a given planning horizon. The plan is for decision support, and the result can be used in aiding the planners handling the decisions made when they are creating a detailed plan for a planning horizon. The final decisions are made by the planners themselves.

6.2 Formal Definitions

6.2.1 Sets

Farm

The fish are bred at given farms. All the farms used in a problem instance is an element of the set $\mathcal{L}$.

Plant

The plant is modeled as a hub, with both incoming and outgoing flow. After the fish is harvested from a farm, it is sent to a plant for slaughter and packing. Opposite to the short-term storage of fish at plant in real-life, the model has no limits on how long a box of fish can be stored. All plants in a problem instance are elements of the set $\mathcal{A}$.

Fish

Fish is described using the attributes species, size class, and quality. In the model it is possible to add as many different species, size classes, and qualities as needed. All combinations of species, size class, and qualities are allowed. All species in a problem instance are elements of set $\mathcal{S}$. All size classes in a problem instance are elements of set $\mathcal{Z}$. All qualities in a problem instance are elements of set $\mathcal{Q}$.

Orders

The orders are divided into two distinct categories, namely external and internal orders. External orders have a delivery point outside of the plant, while internal orders are related to processing facilities located in the same building as the plant. All external orders in a problem instance are elements of the set $\mathcal{O}^{\mathcal{EX}}$, and all internal orders in a problem instance are elements of the set $\mathcal{O}^{\mathcal{IN}}$.

Diseases

Outbreaks of disease can occur at all available farms. All diseases included into a problem instance are elements of the set $\mathcal{E}$.

Certifications

The farms can get certified if certain criteria are met. All available certifications a farm can attain in each problem instance, are elements of the set $\mathcal{H}$.

6.2.2 Frequently Used Superscripts

A, DC and IN are three superscripts that are used repeatedly in the notation for the model. The superscript A is associated with plants, DC is associated with distribution center, and IN is associated with internal processing facilities. When a symbol uses one of these as a superscript, the symbol is related to what that superscript represents.

6.2.3 Planning Horizon

The model is designed to fit a seven-day schedule, but this is designed to be flexible in the model. The length of the horizon is based around the number of days a plant is open, plus the longest lead time from plant to distribution center. The plant is open from day 1 until day T, and all plants are open the same number of days. A requirement is set that all supply sent from a plant at day T, must have arrived at the distribution center before the planning horizon ends. The reason for this constraint is to avoid that supply sent from plant at day T arrives in the next planning horizon. The lead time from plant a to distribution center is found in parameter R_a. The lead time is assumed to be minimum one day. The parameter F represent the longest lead time from plant to distribution center and is defined as $F = \max_{a \in \mathcal{A}}\{R_a\}$. The distribution center is by this logic open from day 1 up to and including day $T + F$. I.e., the length of the planning horizon is equal to the number of days the distribution center is open.

6.2.4 Methods of Delivery

The model has incorporated three different methods of delivery. An order can only be served by one of these methods. External orders have two different methods of delivery, while the internal orders have one.

Direct Delivery
Direct delivery is defined as sending boxes directly from one plant to the requested delivery point without the need for consolidating from other plants. For direct delivery to be allowed, one plant alone must be able to deliver the entire demand specified in the order. It is not allowed to deliver directly to one order in cooperation with the distribution center or other plants. The variables $x^{A}_{l,a,s,z,q,t,o}$ represent the number of boxes containing fish of species s, size class z, and quality q harvested at farm l, sent directly from plant a to order o, on day t. The binary variables $y^{A}_{a,t,o}$ are 1 if order o is delivered directly from plant a on day t, 0 otherwise.

Delivery Through Distribution Center
Delivery through distribution center is defined as delivering from the distribution center. If delivery is done from the distribution center to an order, it is not allowed to have direct delivery from a plant to the same order. The variables $x^{DC}_{l,a,s,z,q,t,o}$ represent the number of boxes containing fish of species s, size class z, and quality q, harvested at farm l, sent from plant a through the distribution center to order o, on day t. The binary variables $y^{DC}_{t,o}$ are 1 if order o is delivered from distribution center on day t, 0 otherwise.

In-House Delivery
If an order comes from the processing facility located in the same building as the plant, the order is defined as internal. Internal orders have only one method of delivery, which is in-house delivery. It is not allowed to serve an internal processing facility at plant a from the distribution center or other plants. The variables $x^{IN}_{l,a,s,z,q,t,o}$ represent the number of boxes containing fish of species s, size class z, and quality q, harvested at farm l, delivered to the internal processing facility at plant a to order o, on day t. The binary variables $y^{IN}_{t,o}$ are 1 if order o is delivered to on day t, 0 otherwise. These types of orders are related to only one plant.

6.2.5 Prioritization of Orders

The priority of an order is decided in cooperation between the sales department and the planners. Priority is implemented into the model as a value associated with each order. Each order has a prioritization coefficient which is used to determine its importance. If the value of the prioritization coefficient of an order is relatively higher than others, the order is of higher priority. If multiple orders receive the same coefficient value, none of the orders are more or less prioritized. The parameters K_o hold a value indicating the importance of order o.

6.2.6 Other Assumptions and Limitations

Boxes

The need to differentiate between the size and weight of boxes is avoided, since it is assumed that all boxes are of equal weight and size. This simplifies the calculation of boxes in circulation. Fractional number of boxes are allowed. This is because the number of boxes is so high, that it is assumed to not have any significant effect on the result.

Distribution Center

The model is designed to only have one distribution center. The reason for this is to create only one point of consolidation.

Inventory

The model only allows to store boxes at the plant or the distribution center. The distribution center has inventory capacity, which allows for storage. Initial inventory is included in the model, so it is possible to use supply that arrived in the previous planning horizon. The model has no limits on how long a box of fish can be stored. It is assumed that the inventory capacity at the distribution center is infinite. This assumption is needed for all instances to be feasible.

Table 6.1: Parameters

Parameters	
T	Number of days a plant is open
F	Longest lead time from a plant to the distribution center.
$P_{l,a,s,z,q,t}$	Supply coming from farm l to plant a of species s, size class z, and quality q, on day t
$D^{TOTAL}_{s,q,o}$	Total demand of boxes of fish of species s and quality q, in order o
$D^{MAX}_{s,z,q,o}$	Maximum demand of boxes of fish of species s, size class z, and quality q, in order o
$D^{MIN}_{s,z,q,o}$	Minimum demand of boxes of fish of species s, size class z, and quality q, in order o
K_o	A value indicating the importance of order o
G_o	Value in the interval from 0 to and including 1
$\sigma^{A}_{t,o}$	1 if order o does not allow delivery on day t, 0 otherwise
$\sigma^{DC}_{t,o}$	1 if order o does not allow delivery on day t, 0 otherwise
$\sigma^{IN}_{t,o}$	1 if order o does not allow delivery on day t, 0 otherwise
$I^{A}_{l,a,s,z,q}$	Initial number of boxes of fish, of species s, size class z, and quality q, from farm l, in inventory at plant a
$I^{DC}_{l,a,s,z,q}$	Initial number of boxes of fish, of species s, size class z, and quality q coming from farm l through plant a, in inventory at distribution center
C^{A}_{a}	Inventory capacity at plant a
R_a	Lead time from plant a to distribution center
$\delta^{A}_{a,o}$	1 if plant a can deliver to order o, 0 otherwise
U^{O}_{oe}	1 if order o does not accept fish from a farm with disease e, 0 otherwise
U^{L}_{le}	1 if fish comes from a farm l without disease e, 0 otherwise
π^{O}_{oh}	1 if order o requires fish with certificate h, 0 otherwise
π^{L}_{lh}	1 if fish from farm l have certificate h, 0 otherwise
J^{A}	Value for delivering directly
J^{DC}	Value for delivering through distribution center
M	Large value

Table 6.2: Variables

Variables	
$x^{A}_{l,a,s,z,q,t,o}$	Number of boxes of fish, of species s, size class z, and quality q from farm l, sent directly from plant a to order o on day t
$x^{DC}_{l,a,s,z,q,t,o}$	Number of boxes of fish, of species s, size class z, and quality q from farm l, sent from plant a through the distribution center to order o on day t
$x^{IN}_{l,a,s,z,q,t,o}$	Number of boxes of fish, of species s, size class z, and quality q from farm l, sent from plant a to internal order o on day t
$y^{A}_{a,t,o}$	1 if delivery to order o on day t directly from plant a is done, 0 otherwise
$y^{DC}_{t,o}$	1 if delivery to order o on day t from distribution center is done, 0 otherwise
$y^{IN}_{t,o}$	1 if delivery to internal order o on day t from plant is done, 0 otherwise
$\mu^{A}_{l,a,s,z,q,t}$	Number of boxes of fish, of species s, size class z, and quality q from farm l, stored at plant a on day t
$\mu^{DC}_{l,a,s,z,q,t}$	Number of boxes of fish, of species s, size class z, and quality q from farm l, sent from plant a, stored at the distribution center on day t
$b_{l,a,s,z,q,t}$	Number of boxes of fish, of species s, size class z, and quality q from farm l, sent from plant a to distribution center on day t

6.3 Objective Functions

The model is formulated with two objective functions. These objective functions are based upon the two goals defined in Chapter 3.

Maximize the Total Number of Boxes Delivered to all Orders

The first goal is to deliver as much as possible of the available supply to orders. This goal has been formulated as maximizing the total number of boxes delivered to all orders.

$$\max W_1 = \sum_{l\in\mathcal{L}} \sum_{a\in\mathcal{A}} \sum_{s\in\mathcal{S}} \sum_{z\in\mathcal{Z}} \sum_{q\in\mathcal{Q}} \left(\sum_{t=1}^{T} \sum_{o\in\mathcal{O}^{\mathcal{EX}}} J^A x^A_{l,a,s,z,q,t,o} + \sum_{t=1}^{T+F} \sum_{o\in\mathcal{O}^{\mathcal{EX}}} J^{DC} x^{DC}_{l,a,s,z,q,t,o} + \sum_{t=1}^{T} \sum_{o\in\mathcal{O}^{\mathcal{IN}}} x^{IN}_{l,a,s,z,q,t,o} \right) \tag{6.1}$$

It is possible for the planners to pre-define which delivery method that is preferred for external orders. The weights for direct delivery J^A and delivery through distribution center J^{DC} are multiplied with their respective variables. A relatively higher weight on one of the two parameters indicates which of the delivery methods that are preferred. The value on the weights must be a positive number equal to or less than 1. The reason for not allowing a higher weight, is because it would create the effect of preferring to deliver boxes to external orders over delivery to internal orders.

Maximize the Total Value of Prioritized Orders Fulfilled

The second goal is to fulfill as many highly prioritized orders as possible. Having implemented the priority as a value associated with each order, fulfilling orders is going to generate values. This goal has therefore been formulated as maximizing the total value of prioritized orders fulfilled.

$$\max W_2 = \sum_{o\in\mathcal{O}^{\mathcal{EX}}} K_o \left(\sum_{a\in\mathcal{A}} \sum_{t=1}^{T} y^A_{a,t,o} + \sum_{t=1}^{T+F} y^{DC}_{t,o} \right) + \sum_{o\in\mathcal{O}^{\mathcal{IN}}} K_o \sum_{t=1}^{T} y^{IN}_{t,o} \tag{6.2}$$

When the decision variable multiplied with the prioritization coefficient for an order is 1, then the order is fulfilled, and the value of the coefficient is obtained. The purpose of maximizing the function is to fulfill the most important orders first.

6.4 Constraints

Initial Inventory

Constraint set (6.3) ensures that the initial inventory at plant is set.

$$\mu^{A}_{l,a,s,z,q,t} = I^{A}_{l,a,s,z,q}, \quad l \in \mathcal{L},\ a \in \mathcal{A},\ s \in \mathcal{S},\ z \in \mathcal{Z},\ q \in \mathcal{Q},\ t = 0 \tag{6.3}$$

Constraint set (6.4) ensures that the initial inventory at the distribution center is set.

$$\mu^{DC}_{l,a,s,z,q,t} = I^{DC}_{l,a,s,z,q}, \quad l \in \mathcal{L},\ a \in \mathcal{A},\ s \in \mathcal{S},\ z \in \mathcal{Z},\ q \in \mathcal{Q},\ t = 0 \tag{6.4}$$

Inventory Capacity Constraints at Plant

Constraint set (6.5) ensures that the value of the variables $\mu^{A}_{l,a,s,z,q,t}$ is less than or equal to the inventory capacity at plant a.

$$\sum_{l \in \mathcal{L}} \sum_{s \in \mathcal{S}} \sum_{z \in \mathcal{Z}} \sum_{q \in \mathcal{Q}} \mu^{A}_{l,a,s,z,q,t} \leq C^{A}_{a}, \qquad a \in \mathcal{A},\ t \in \{0, \ldots, T\} \tag{6.5}$$

The distribution center has no inventory capacity constraint, due to the assumption that the capacity is infinite.

Balance Constraint at Plant

Constraint set (6.6) balances the inventory, and incoming and outgoing flow for plant a. Available supply in each period t is incoming supply $P_{l,a,s,z,q,t}$ and inventory from previous periods $\mu^{A}_{l,a,s,z,q,t-1}$. If there is no available supply for a given combination of farm, plan, species, size class, and quality, all variables with that combination are set to zero. Variables $x^{A}_{l,a,s,z,q,t,o}$ and $x^{IN}_{l,a,s,z,q,t,o}$ state the number of boxes delivered using either direct or in-house delivery. Variables $b_{l,a,s,z,q,t}$ represent the number of boxes sent from plant a to distribution center. Sending boxes from plant to distribution center is done to ensure that there are enough boxes available for further delivery, or to store the boxes at the distribution center instead of the plant. All boxes left on day t at plant a is stored to the next period.

$$P_{l,a,s,z,q,t} + \mu^{A}_{l,a,s,z,q,t-1} - \sum_{o \in \mathcal{O}^{\mathcal{EX}}} x^{A}_{l,a,s,z,q,t,o} - \sum_{o \in \mathcal{O}^{\mathcal{IN}}} x^{IN}_{l,a,s,z,q,t,o} - b_{l,a,s,z,q,t} = \mu^{A}_{l,a,s,z,q,t},$$
$$l \in \mathcal{L},\ a \in \mathcal{A},\ s \in \mathcal{S},\ z \in \mathcal{Z},\ q \in \mathcal{Q},\ t \in \{1, \dots, T\} \quad (6.6)$$

Balance Constraint at Distribution Center

Three constraint sets are used to ensure the correct balance of inventory, incoming and outgoing flow at the distribution center. All incoming flow to the distribution center comes from the plants. All outgoing flow is the number of boxes delivered from distribution center to orders. The lead time from plant a to distribution center is given in parameter R_a. It takes R_a number of days from plant a until the supply arrives at the distribution center. The distribution center is open from day 1, but incoming supply can only arrive R_a number of days after departure from plant a. This affects the incoming flow at the distribution center. Constraint set (6.7) ensures that there is balance in inventory and outgoing flow in the periods before new supply can arrive from plant a.

$$\mu^{DC}_{l,a,s,z,q,t-1} - \sum_{o \in \mathcal{O}^{\mathcal{EX}}} x^{DC}_{l,a,s,z,q,t,o} = \mu^{DC}_{l,a,s,z,q,t},$$
$$l \in \mathcal{L},\ a \in \mathcal{A},\ s \in \mathcal{S},\ z \in \mathcal{Z},\ q \in \mathcal{Q},\ t \in \{1, \dots, R_a\} \quad (6.7)$$

The earliest day that supply sent from plant a can arrive at the distribution center, is on day $1 + R_a$. Constraint set (6.8) ensures balance in inventory, incoming and outgoing flow at distribution center from period $1 + R_a$ until $T + R_a$. The variables $b_{l,a,s,z,q,t-R_a}$ represent the number of boxes sent on day $t - R_a$ from plant a, that arrive on day t at the distribution center.

$$b_{l,a,s,z,q,t-R_a} + \mu^{DC}_{l,a,s,z,q,t-1} - \sum_{o \in \mathcal{O}^{\mathcal{EX}}} x^{DC}_{l,a,s,z,q,t,o} = \mu^{DC}_{l,a,s,z,q,t},$$
$$l \in \mathcal{L},\ a \in \mathcal{A},\ s \in \mathcal{S},\ z \in \mathcal{Z},\ q \in \mathcal{Q},\ t \in \{1 + R_a, \dots, T + R_a\} \quad (6.8)$$

The distribution center is open in the period from 1 until $T+F$, but the constraint set (6.8) is only valid from $1+R_a$ until $T+R_a$. If R_a is less than F for plant a, then this causes a problem. The variables $\mu^{DC}_{l,a,s,z,q,t-1}$, $x^{DC}_{l,a,s,z,q,t,o}$ and $\mu^{DC}_{l,a,s,z,q,t}$ related to plant a are unbounded in the periods after $T+R_a$. Constraint set (6.9) is used to deal with the problem and ensures that from $T+R_a+1$ up to and including $T+F$, all variables related to plants with lead times shorter than F are bounded.

$$\mu^{DC}_{l,a,s,z,q,t-1} - \sum_{o\in\mathcal{O}^{\mathcal{EX}}} x^{DC}_{l,a,s,z,q,t,o} = \mu^{DC}_{l,a,s,z,q,t},$$
$$l\in\mathcal{L},\ a\in\mathcal{A},\ s\in\mathcal{S},\ z\in\mathcal{Z},\ q\in\mathcal{Q},\ t\in\{T+R_a+1,\dots,T+F\} \quad (6.9)$$

The variables $b_{l,a,s,z,q,t-R_a}$ are omitted from constraint set (6.9). An example is used to describe why the variables are omitted. If the lead time from plant a is 1 day and it is now day $T+2$, then variables $b_{l,a,s,z,q,t-R_a}$ are related to the number of boxes that departed from plant a on day $T+1$. The plant is only open from day 1 until day T, so no boxes depart from plant a after day T. It is therefore redundant to have variables related to the number of boxes sent on day $T+1$. This is the reason for not including $b_{l,a,s,z,q,t-R_a}$ in the constraint set.

Delivery

Constraint set (6.10) ensures that if an external order is accepted, only one of the two available delivery methods for external orders are used. If direct delivery from a plant is used, then only one plant can deliver to the order. It is only allowed to deliver one time to an order throughout the entire planning horizon, regardless of the chosen delivery method.

$$\sum_{a\in\mathcal{A}}\sum_{t=1}^{T} y^{A}_{a,t,o} + \sum_{t=1}^{T+F} y^{DC}_{t,o} \le 1, \quad o\in\mathcal{O}^{\mathcal{EX}} \quad (6.10)$$

Constraint set (6.11) ensures that if an internal order is accepted, it is only delivered to once throughout the entire planning horizon.

$$\sum_{t=1}^{T} y^{IN}_{t,o} \le 1, \quad o\in\mathcal{O}^{\mathcal{IN}} \quad (6.11)$$

Day of Delivery

The day of delivery is the day the supply must be sent, so it can arrive on the day specified in the order. Constraint set (6.12) ensures that it is only possible to deliver directly from plant to an external order on the allowed day of delivery.

$$\sum_{l\in\mathcal{L}}\sum_{a\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{z\in\mathcal{Z}}\sum_{q\in\mathcal{Q}} x^{A}_{l,a,s,z,q,t,o} \leq M\left(1-\sigma^{A}_{t,o}\right), \quad t\in\{1,\ldots,T\},\ o\in\mathcal{O}^{\mathcal{EX}} \tag{6.12}$$

Constraint set (6.13) ensures that it is only possible to deliver from distribution center to an external order on the allowed day of delivery.

$$\sum_{l\in\mathcal{L}}\sum_{a\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{z\in\mathcal{Z}}\sum_{q\in\mathcal{Q}} x^{DC}_{l,a,s,z,q,t,o} \leq M\left(1-\sigma^{DC}_{t,o}\right), \quad t\in\{1,\ldots,T+F\},\ o\in\mathcal{O}^{\mathcal{EX}} \tag{6.13}$$

Constraint set (6.14) ensures that it is only possible to deliver to an internal order on the allowed day of delivery.

$$\sum_{l\in\mathcal{L}}\sum_{a\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{z\in\mathcal{Z}}\sum_{q\in\mathcal{Q}} x^{IN}_{l,a,s,z,q,t,o} \leq M\left(1-\sigma^{IN}_{t,o}\right), \quad t\in\{1,\ldots,T\},\ o\in\mathcal{O}^{\mathcal{IN}} \tag{6.14}$$

Demand - Upper Bound

The model allows for the order to have flexibility in number of boxes delivered and the possibility of substituting size classes used in delivery.

It is allowed for flexibility in the number of boxes specified in the order. Parameter $D^{TOTAL}_{s,q,o}$ represents the demand for a specific combination of species s and quality q, for order o. The next three constraint sets are related to their own method of delivery and ensures the upper bound on this demand.

Constraint set (6.15) ensures that the number of boxes delivered directly from plant a is less than or equal to the maximum number of boxes specified in order o for a given combination of species s and quality q.

$$\sum_{l \in \mathcal{L}} \sum_{z \in \mathcal{Z}} x^{A}_{l,a,s,z,q,t,o} \leq D^{TOTAL}_{s,q,o} y^{A}_{a,t,o}, \ a \in \mathcal{A}, \ s \in \mathcal{S}, \ q \in \mathcal{Q}, \ t \in \{1, \ldots, T\}, \ o \in \mathcal{O}^{\mathcal{EX}} \quad (6.15)$$

Constraint set (6.16) ensures that the number of boxes delivered from distribution center is less than or equal to the maximum number of boxes specified in order o for a given combination of species s and quality q.

$$\sum_{l \in \mathcal{L}} \sum_{a \in \mathcal{A}} \sum_{z \in \mathcal{Z}} x^{DC}_{l,a,s,z,q,t,o} \leq D^{TOTAL}_{s,q,o} y^{DC}_{t,o}, \ s \in \mathcal{S}, \ q \in \mathcal{Q}, \ t \in \{1, \ldots, T+F\}, \ o \in \mathcal{O}^{\mathcal{EX}} \quad (6.16)$$

Constraint set (6.17) ensures that the number of boxes delivered in-house is less than or equal to the maximum number of boxes specified in order o for a given combination of species s and quality q.

$$\sum_{l \in \mathcal{L}} \sum_{a \in \mathcal{A}} \sum_{z \in \mathcal{Z}} x^{IN}_{l,a,s,z,q,t,o} \leq D^{TOTAL}_{s,q,o} y^{IN}_{t,o}, \ s \in \mathcal{S}, \ q \in \mathcal{Q}, \ t \in \{1, \ldots, T\}, \ o \in \mathcal{O}^{\mathcal{IN}} \quad (6.17)$$

Demand - Lower Bound

The parameter G_o is a value in the interval from 0 to and including 1. If parameter $D^{TOTAL}_{s,q,o}$ is multiplied with G_o, the lower bound on demand for a specific combination of species s and quality q, for order o is found.

Constraint set (6.18) ensures that the number of boxes delivered directly from plant a is greater than or equal to the minimum number of boxes specified in order o for a given combination of species s and quality q.

$$\sum_{l \in \mathcal{L}} \sum_{z \in \mathcal{Z}} x^{A}_{l,a,s,z,q,t,o} \geq D^{TOTAL}_{s,q,o} G_o y^{A}_{a,t,o}, \qquad a \in \mathcal{A},\ s \in \mathcal{S},\ q \in \mathcal{Q},\ t \in \{1, \dots, T\},\ o \in \mathcal{O}^{\mathcal{EX}} \tag{6.18}$$

Constraint set (6.19) ensures that the number of boxes delivered from distribution center is greater than or equal to the minimum number of boxes specified in order o for a given combination of species s and quality q.

$$\sum_{l \in \mathcal{L}} \sum_{a \in \mathcal{A}} \sum_{z \in \mathcal{Z}} x^{DC}_{l,a,s,z,q,t,o} \geq D^{TOTAL}_{s,q,o} G_o y^{DC}_{t,o}, \qquad s \in \mathcal{S},\ q \in \mathcal{Q},\ t \in \{1, \dots, T+F\},\ o \in \mathcal{O}^{\mathcal{EX}} \tag{6.19}$$

Constraint set (6.20) ensures that the number of boxes delivered in-house is greater than or equal to the minimum number of boxes specified in order o for a given combination of species s and quality q.

$$\sum_{l \in \mathcal{L}} \sum_{a \in \mathcal{A}} \sum_{z \in \mathcal{Z}} x^{IN}_{l,a,s,z,q,t,o} \geq D^{TOTAL}_{s,q,o} G_o y^{IN}_{t,o},\ s \in \mathcal{S},\ q \in \mathcal{Q},\ t \in \{1, \dots, T\},\ o \in \mathcal{O}^{\mathcal{IN}} \tag{6.20}$$

Demand - Upper Bound - Size Class

The parameter $D^{MAX}_{s,z,q,o}$ represents the upper bound on a given size class z that can be delivered with a combination of species s and quality q, for order o. The parameter must be set to less than or equal to $D^{TOTAL}_{s,q,o}$. If $D^{MAX}_{s,z,q,o}$ is set to zero, it is not allowed to deliver anything of that size class z for that combination of species s and quality q for order o.

Constraint set (6.21) ensures that the number of boxes of size class z delivered directly from plant a is less than or equal to the maximum number of boxes specified in order o for a given combination of species s, size class z, and quality q.

$$\sum_{l \in \mathcal{L}} x^{A}_{l,a,s,z,q,t,o} \leq D^{MAX}_{s,z,q,o} y^{A}_{a,t,o}, \qquad a \in \mathcal{A},\ s \in \mathcal{S},\ z \in \mathcal{Z},\ q \in \mathcal{Q},\ t \in \{1, \ldots, T\},\ o \in \mathcal{O}^{\mathcal{EX}} \tag{6.21}$$

Constraint set (6.22) ensures that the number of boxes of size class z delivered from distribution center is less than or equal to the maximum number of boxes specified in order o for a given combination of species s, size class z, and quality q.

$$\sum_{l \in \mathcal{L}} \sum_{a \in \mathcal{A}} x^{DC}_{l,a,s,z,q,t,o} \leq D^{MAX}_{s,z,q,o} y^{DC}_{t,o}, \qquad s \in \mathcal{S},\ z \in \mathcal{Z},\ q \in \mathcal{Q},\ t \in \{1, \ldots, T+F\},\ o \in \mathcal{O}^{\mathcal{EX}} \tag{6.22}$$

Constraint set (6.23) ensures that the number of boxes of size class z delivered in-house is less than or equal to the maximum number of boxes specified in order o for a given combination of species s, size class z, and quality q.

$$\sum_{l \in \mathcal{L}} \sum_{a \in \mathcal{A}} x^{IN}_{l,a,s,z,q,t,o} \leq D^{MAX}_{s,z,q,o} y^{IN}_{t,o}, \qquad s \in \mathcal{S},\ z \in \mathcal{Z},\ q \in \mathcal{Q},\ t \in \{1, \ldots, T\},\ o \in \mathcal{O}^{\mathcal{IN}} \tag{6.23}$$

Demand - Lower Bound - Size Class

The parameter $D^{MIN}_{s,z,q,o}$ represents the lower bound of boxes of a given size class s that must be delivered with the combination of species s and quality q, for order o. The value on parameter $D^{MIN}_{s,z,q,o}$ must be less than or equal to $D^{MAX}_{s,z,q,o}$.

Constraint set (6.24) ensures that the number of boxes of size class z delivered directly from plant a is greater than or equal to the minimum number of boxes specified in order o for a given combination of species s, size class z, and quality q.

$$\sum_{l\in\mathcal{L}} x^{A}_{l,a,s,z,q,t,o} \geq D^{MIN}_{s,z,q,o} y^{A}_{a,t,o}, \qquad a \in \mathcal{A},\ s \in \mathcal{S},\ z \in \mathcal{Z},\ q \in \mathcal{Q},\ t \in \{1,\dots,T\},\ o \in \mathcal{O}^{\mathcal{EX}} \tag{6.24}$$

Constraint set (6.25) ensures that the number of boxes of size class z delivered from distribution center is greater than or equal to the minimum number of boxes specified in order o for a given combination of species s, size class z, and quality q.

$$\sum_{l\in\mathcal{L}} \sum_{a\in\mathcal{A}} x^{DC}_{l,a,s,z,q,t,o} \geq D^{MIN}_{s,z,q,o} y^{DC}_{t,o}, \qquad s \in \mathcal{S},\ z \in \mathcal{Z},\ q \in \mathcal{Q},\ t \in \{1,\dots,T+F\},\ o \in \mathcal{O}^{\mathcal{EX}} \tag{6.25}$$

Constraint set (6.26) ensures that the number of boxes of size class z delivered in-house is greater than or equal to the minimum number of boxes specified in order o for a given combination of species s, size class z, and quality q.

$$\sum_{l\in\mathcal{L}} \sum_{a\in\mathcal{A}} x^{IN}_{l,a,s,z,q,t,o} \geq D^{MIN}_{s,z,q,o} y^{IN}_{t,o}, \qquad s \in \mathcal{S},\ z \in \mathcal{Z},\ q \in \mathcal{Q},\ t \in \{1,\dots,T\},\ o \in \mathcal{O}^{\mathcal{IN}} \tag{6.26}$$

Disease

If external order o is to receive fish grown at farm l, then constraint set (6.27) ensures that the health status regarding disease e is as specified in the order.

$$\sum_{a\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{z\in\mathcal{Z}}\sum_{q\in\mathcal{Q}}\left(\sum_{t=1}^{T} x^{A}_{l,a,s,z,q,t,o} + \sum_{t=1}^{T+F} x^{DC}_{l,a,s,z,q,t,o}\right) \leq M\big((1-U^{O}_{o,e}) + U^{L}_{l,e}\big),$$
$$l\in\mathcal{L},\ o\in\mathcal{O}^{\mathcal{EX}},\ e\in\mathcal{E} \quad (6.27)$$

If internal order o is to receive fish bred at farm l, then constraint set (6.28) ensures that the health status regarding disease e is as specified in the order.

$$\sum_{a\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{z\in\mathcal{Z}}\sum_{q\in\mathcal{Q}}\left(\sum_{t=1}^{T} x^{IN}_{l,a,s,z,q,t,o}\right) \leq M\big((1-U^{O}_{o,e}) + U^{L}_{l,e}\big),$$
$$l\in\mathcal{L},\ o\in\mathcal{O}^{\mathcal{IN}},\ e\in\mathcal{E} \quad (6.28)$$

Certifications

If external order o is to receive fish bred at farm l, then constraint set (6.29) ensures that the fish meets the requirements regarding certification h as specified in the order.

$$\sum_{a\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{z\in\mathcal{Z}}\sum_{q\in\mathcal{Q}}\left(\sum_{t=1}^{T} x^{A}_{l,a,s,z,q,t,o} + \sum_{t=1}^{T+F} x^{DC}_{l,a,s,z,q,t,o}\right) \leq M\big((1-\pi^{O}_{o,h}) + \pi^{L}_{l,h}\big),$$
$$l\in\mathcal{L},\ o\in\mathcal{O}^{\mathcal{EX}},\ h\in\mathcal{H} \quad (6.29)$$

If internal order o is to receive fish bred at farm l, then constraint set (6.30) ensures that the fish meets the requirements regarding certification h as specified in the order.

$$\sum_{a\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{z\in\mathcal{Z}}\sum_{q\in\mathcal{Q}}\left(\sum_{t=1}^{T} x^{IN}_{l,a,s,z,q,t,o}\right) \leq M\big((1-\pi^{O}_{o,h}) + \pi^{L}_{l,h}\big),$$
$$l\in\mathcal{L},\ o\in\mathcal{O}^{\mathcal{IN}},\ h\in\mathcal{H} \quad (6.30)$$

Internal Processing Facility

An internal order is only related to one plant. Constraint set (6.31) is used to ensure that only the plant related to order o is allowed to deliver. If $\delta^A_{a,o}$ is 1, then order o is related to plant a, 0 otherwise.

$$\sum_{l \in \mathcal{L}} \sum_{s \in \mathcal{S}} \sum_{z \in \mathcal{Z}} \sum_{q \in \mathcal{Q}} \sum_{t=1}^{T} x^{IN}_{l,a,s,z,q,t,o} \leq M\delta^A_{a,o}, \; a \in \mathcal{A}, \; o \in \mathcal{O}^{\mathcal{IN}} \tag{6.31}$$

Domain of Variables

All the variables in the model are non-negative.

$$x^A_{l,a,s,z,q,t,o} \geq 0 \qquad l \in \mathcal{L}, \; a \in \mathcal{A}, \; s \in \mathcal{S}, \; z \in \mathcal{Z}, \; q \in \mathcal{Q}, \; t \in \{1, \ldots, T\}, \; o \in \mathcal{O}^{\mathcal{EX}} \tag{6.32}$$

$$x^{DC}_{l,a,s,z,q,t,o} \geq 0 \qquad l \in \mathcal{L}, \; a \in \mathcal{A}, \; s \in \mathcal{S}, \; z \in \mathcal{Z}, \; q \in \mathcal{Q}, \; t \in \{1, \ldots, T+F\} \; o \in \mathcal{O}^{\mathcal{EX}} \tag{6.33}$$

$$x^{IN}_{l,a,s,z,q,t,o} \geq 0 \qquad l \in \mathcal{L}, \; a \in \mathcal{A}, \; s \in \mathcal{S}, \; z \in \mathcal{Z}, \; q \in \mathcal{Q}, \; t \in \{1, \ldots, T\}, \; o \in \mathcal{O}^{\mathcal{IN}} \tag{6.34}$$

$$y^A_{a,t,o} \in \{0, 1\} \qquad a \in \mathcal{A}, \; t \in \{1, \ldots, T\}, \; o \in \mathcal{O}^{\mathcal{EX}} \tag{6.35}$$

$$y^{DC}_{t,o} \in \{0, 1\} \qquad t \in \{1, \ldots, T+F\}, \; o \in \mathcal{O}^{\mathcal{EX}} \tag{6.36}$$

$$y^{IN}_{t,o} \in \{0, 1\} \qquad t \in \{1, \ldots, T\}, \; o \in \mathcal{O}^{\mathcal{IN}} \tag{6.37}$$

$$\mu^A_{l,a,s,z,q,t} \geq 0 \qquad l \in \mathcal{L}, \; a \in \mathcal{A}, \; s \in \mathcal{S}, \; z \in \mathcal{Z}, \; q \in \mathcal{Q}, \; t \in \{0, \ldots, T\} \tag{6.38}$$

$$\mu^{DC}_{l,a,s,z,q,t} \geq 0 \qquad l \in \mathcal{L}, \; a \in \mathcal{A}, \; s \in \mathcal{S}, \; z \in \mathcal{Z}, \; q \in \mathcal{Q}, \; t \in \{0, \ldots, T+F\} \tag{6.39}$$

$$b_{l,a,s,z,q,t} \geq 0 \qquad l \in \mathcal{L}, \; a \in \mathcal{A}, \; s \in \mathcal{S}, \; z \in \mathcal{Z}, \; q \in \mathcal{Q}, \; t \in \{1, \ldots, T\} \tag{6.40}$$

6.5 Augmented ε-constraint Method

The model is created to use multiple objectives. Augmented ε-constraint method is used to solve problem instance with both objectives.

The objective function maximizes total number of boxes delivered is the primary objective.

$$max \sum_{l\in\mathcal{L}}\sum_{a\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{z\in\mathcal{Z}}\sum_{q\in\mathcal{Q}}\left(\sum_{t=1}^{T}\sum_{o\in\mathcal{O}^{\mathcal{EX}}} J^{A}x^{A}_{l,a,s,z,q,t,o} + \sum_{t=1}^{T+F}\sum_{o\in\mathcal{O}^{\mathcal{EX}}} J^{DC}x^{DC}_{l,a,s,z,q,t,o} + \sum_{t=1}^{T}\sum_{o\in\mathcal{O}^{\mathcal{IN}}} x^{IN}_{l,a,s,z,q,t,o}\right) + \lambda\left(\frac{S_2}{r_2}\right) \tag{6.41}$$

$\lambda \in \left[10^{-6}, 10^{-3}\right]$

The parameter r_k is the range of the respective objective function found in the payoff table, and S_k is the slack/surplus variables.

The objective function maximizing total value of prioritized orders is transformed into a constraint.

$$\sum_{o\in\mathcal{O}^{\mathcal{EX}}} K_o\left(\sum_{a\in\mathcal{A}}\sum_{t=1}^{T} y^{A}_{a,t,o} + \sum_{t=1}^{T+F} y^{DC}_{t,o}\right) + \sum_{o\in\mathcal{O}^{\mathcal{IN}}} K_o\sum_{t=1}^{T} y^{IN}_{t,o} - S_2 = \varepsilon_2 \tag{6.42}$$

Where ε_k is calculated as:

$$\varepsilon_k = f_k^{min} + t\left(\frac{r_k}{q_k}\right)$$

where f_k^{min} is the minimum obtained from payoff table, t is the counter for the specific objective function, and q_k is the number of intervals for the k^{th} objective function. The surplus variables that correspond to the innermost objective function is checked in each iteration.

7. Computational Study

This chapter presents the computational study performed. The purpose of the study is to get a better understanding of the relationship between the two objective functions. The study can assist in understanding whether there is a need for using multi-objective optimization. If a precise solution can be found within a reasonable time limit is also examined. The study uses a generated dataset, and not data provided by Lerøy. This is done with the intent of not using sensitive data.

Section 7.1 describes the implementation of the model. Section 7.2 describes the generated test case. In Section 7.3, an issue arising when solving problem instances described with the model regarding the problem size is discussed. The analysis of the test results is in Section 7.4.

7.1 Implementation of the Model

The model is written in AMPL and solved using the CPLEX solver, version 20.1.0.0 64-bit. The specifications of the computer used are presented in Table 7.1. The model is attached in Appendix A.

Table 7.1: Specifications

Processor	3.7 GHz Intel Core i5-9600K
Number of processors	1
Cores per processor	6
Memory (RAM)	16 GB
Operating system	Windows 10 Home

7.2 Test Case

In the test case, the orders with low demand have a high priority, and orders with high demand have a low priority. The purpose of using this example is to present a case with potential conflict between the two objective functions. The test instances are presented in Subsection 7.2.1. The fixed data for all instances are described in Subsection 7.2.2, and in Subsection 7.2.3 the different test scenarios are explained.

7.2.1 Test Instances

Five different test instances were created. The differences between these are the number of farms, plants, species, size classes, qualities, certifications, diseases, and orders. The values for each of these elements are presented in Table 7.2.

Table 7.2: Test instances

Instance	Farm	Plant	Species	Size class	Quality	Diseases	Certifications	Orders
1	2	2	2	3	2	2	2	50
2	3	3	2	4	3	3	3	100
3	4	4	2	4	4	4	4	150
4	5	5	2	6	4	4	4	200
5	6	6	3	6	4	5	5	250

7.2.2 Fixed Data

The dataset used is generated and fictional. However, elements such as the number of plants, species, size classes, qualities, certifications and diseases are based on reality. Number of orders, demand, supply, and prioritization are fictitious. Having realistic values on these elements are not decisive for this test.

Each order specifies the demand for a specific combination of fish attributes, and number of boxes. To create a conflict, the orders are divided in groups of ten orders requesting the same combination, but different number of boxes. The order requesting the highest number of boxes for the combination gets the lowest priority, while the order requesting the lowest number of boxes gets the highest priority.

The prioritization coefficients has a value from a discrete interval of integer numbers from 1 to 10, where 10 is the highest. The relationship between priority and number of boxes has a strong negative correlation. The demand for one combination in an order is set to be a random value between 100 and 1000 boxes. The reason for this interval is to eliminate the problem related to having orders demanding extreme outlier values, either excessively small or large. The total demand for the given combination is the sum of demand from all orders requesting the combination. The reason for having only ten orders in conflict over one combination of fish is because it is a sufficient number of orders for it to be useful to use computational power to get an exact solution.

The values for required day of delivery are set so it is possible to deliver to all orders on any day throughout the planning horizon. The requirements about diseases and certifications are set so that all orders accept all fish, independent of health status and certifications. The reason for setting these values is to remove the issue of an order not being fulfilled, because the supply cannot meet the requirements. This layer of decision making is not necessary for the example.

The only source of incoming supply is set to be the farms. Initial inventory is set to zero for all plants and the distribution center. The inventory capacity is set to zero for all plants, so any supply that needs to be stored must be sent to the distribution center.

All orders allow a deviation of 10 % from the requested number of boxes for a given combination. This relaxation is done to allow for flexibility in demand. All orders are set to be external orders. The weight for the delivery methods for external orders are set so direct delivery is weighted higher than delivery through distribution center.

The parameter T is set to 5. The lead time from plant to the distribution center is set to be either 1 or 2 days. The longest lead time F will be 2.

7.2.3 Scenario

One source of conflict is the balance between supply and demand for a given combination of fish. To test different balances of supply and demand, the five instances presented is used in two different scenarios. In the first scenario all five instances have more supply than demand, and in the second scenario all five instances have less supply than demand. For the purpose of comparison, all data except the available supply is similar for the same instance in the two scenarios. Having the data fixed allows for a valid comparison of the solutions.

7.3 Problem Size

The problem size for each instance is presented in Table 7.3.

Table 7.3: Problem size

Instance	Variables	Constraints
1	30562	28934
2	265504	138142
3	935378	337942
4	2909200	744230
5	7834500	1608570

One of the main issues when solving problem instances described by the model, is the generation of many redundant variables and constraints. Many combinations of fish attributes coming from a specific farm through a given plant have no supply. The balance constraint sets for plant and distribution center sets these variables to zero due to the fact that there is no supply for them. Constraint sets for day of delivery, diseases, and certificates are generated, even though all orders allow delivery on all days and have no requirements about diseases and certifications in this case. These constraint sets are designed using the big M approach, so many non-binding constraints with a large right-hand side value are created. The results from solving the problem instances are not affected by the number of superfluous variables and constraints, but the solution time can potentially be increased.

7.4 Test Results

This section presents the analysis of the results for each problem instance found by using the proposed solution method. The payoff tables created for all instances in both scenarios are presented in Tables 7.4 and 7.5.

If the solution found in an instance is the same regardless of the order in which the objectives were solved, it is possible to state that there exists only one Pareto optimal solution for that given instance. In the scenario with more supply than demand, the solutions within each instance were the same for both objectives regardless of the order in which they were solved. Both objectives have different values to maximize, but their overall intention is to serve all orders. Having more supply than demand enables the possibility of fulfilling all orders and causes no conflict between the objectives.

Compared to the first scenario, the opposite is observed for the scenario with less supply than demand. The solutions are affected by the order in which the objectives are solved. The improvement of one objective value seems to worsen the other objective value, which is indicative for the existence of two Pareto optimal solutions. The effect of having less available supply than demand, prompts the need to decide which order is the most important to fulfill. The two objectives decide the importance of an order differently. One objective considers the number of boxes specified in an order to be the indicator of importance, while the other objective focuses on the value of the priority coefficient. In this case, the number of boxes demanded, and the prioritization coefficient values are negatively correlated. The evaluation of the importance of an order, is in this case most likely not coinciding for both objective functions. It is expected to observe different Pareto optimal solutions when the order in which the objectives are solved is different.

Table 7.4: More supply than demand

Instance	Max	Value		CPU time
1	W_1	W_1	27706	0.09375
		W_2	275	0.125
	W_2	W_1	27006	0.078125
		W_2	275	0.109375
2	W_1	W_1	56665	0.109375
		W_2	550	0.3125
	W_2	W_1	56665	0.078125
		W_2	550	0.015625
3	W_1	W_1	82728	0.140625
		W_2	825	0.359375
	W_2	W_1	82728	0.140625
		W_2	825	0.046875
4	W_1	W_1	110210	0.21875
		W_2	1100	0.625
	W_2	W_1	110210	0.28125
		W_2	1100	0.125
5	W_1	W_1	141377	0.203125
		W_2	1375	0.90625
	W_2	W_1	141377	0.28125
		W_2	1375	0.09375

Table 7.5: Less supply than demand

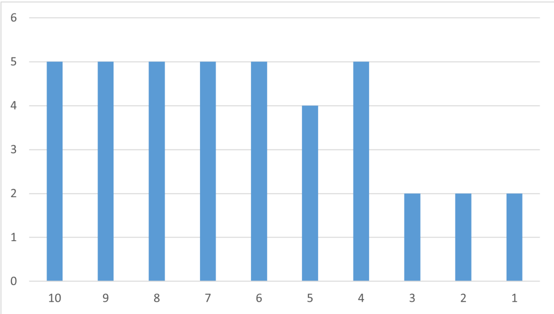
Instance	Max	Value		CPU time
1	W_1	W_1	19398	0.078125
		W_2	252	2.85938
	W_2	W_1	18756	267.688
		W_2	260	0.21875
2	W_1	W_1	39673	1.17188
		W_2	506	3.40625
	W_2	W_1	38301	31.8594
		W_2	520	0.125
3	W_1	W_1	57919	0.40625
		W_2	763	6.09375
	W_2	W_1	56186	1551.25
		W_2	782	0.171875
4	W_1	W_1	77160	0.765625
		W_2	1021	11.25
	W_2	W_1	74823	8344.38
		W_2	1044	0.78125
5	W_1	W_1	98978	0.59375
		W_2	1275	6.95312
	W_2	W_1	96385	1727.89
		W_2	1304	0.4375

The CPU time is included for all instances. The time is relatively low for all instances in the scenario with more supply than demand. However, when solving instances with less supply than demand the solution time increases. This is especially evident when maximizing total number of boxes delivered to all orders, with the optimal total value of prioritized orders fulfilled used as a constraint.

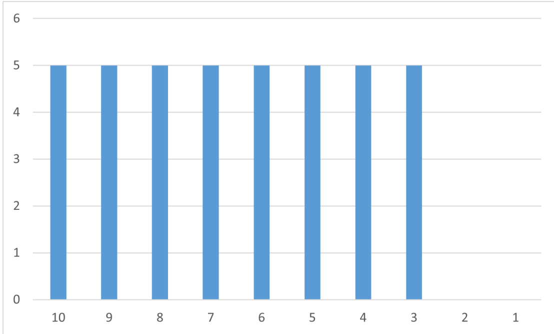
Having observed that each of the instances in the scenario with less supply than demand have at least two Pareto optimal solutions, further analysis will be done for this scenario. The result for each instance in the scenario with more supply than demand is that they have only one Pareto optimal solution. This reduces the necessity for further analysis.

7.4.1 Evaluation of Order Fulfillment

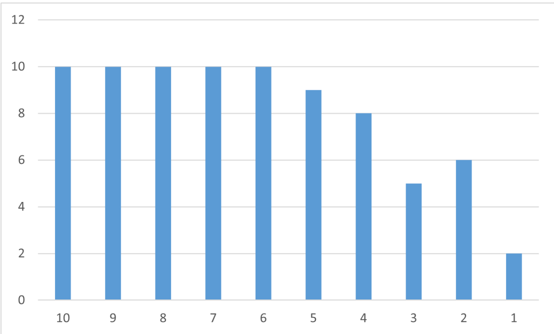
As previously stated, the two objectives evaluate the importance of an order differently. Ten orders specify demand for the same combination of fish attributes. Each of these orders have a prioritization coefficient in the interval from one to ten. Barplots for each instance are created, using the prioritization coefficient value on the x-axis. On the y-axis, is the number of orders fulfilled that have the same coefficient value. The barplots are presented in Figures 7.1 and 7.2. If the plot caption states W_1, the objective of maximizing the number of boxes delivered is solved first. If W_2 is written, then the objective of maximizing total value of prioritized orders fulfilled is solved first.



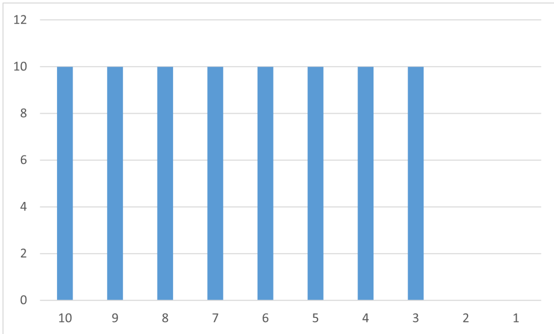
(a) W_1 instance 1.



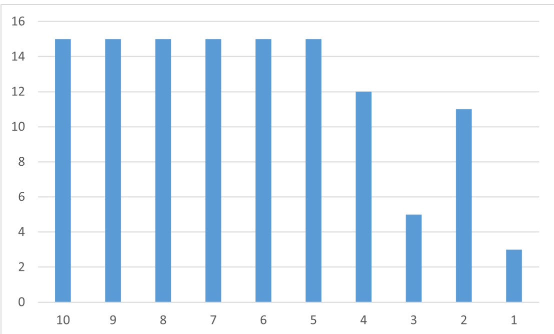
(b) W_2 instance 1.



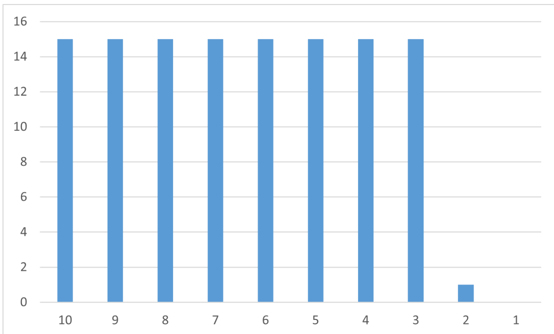
(c) W_1 instance 2.



(d) W_2 instance 2.

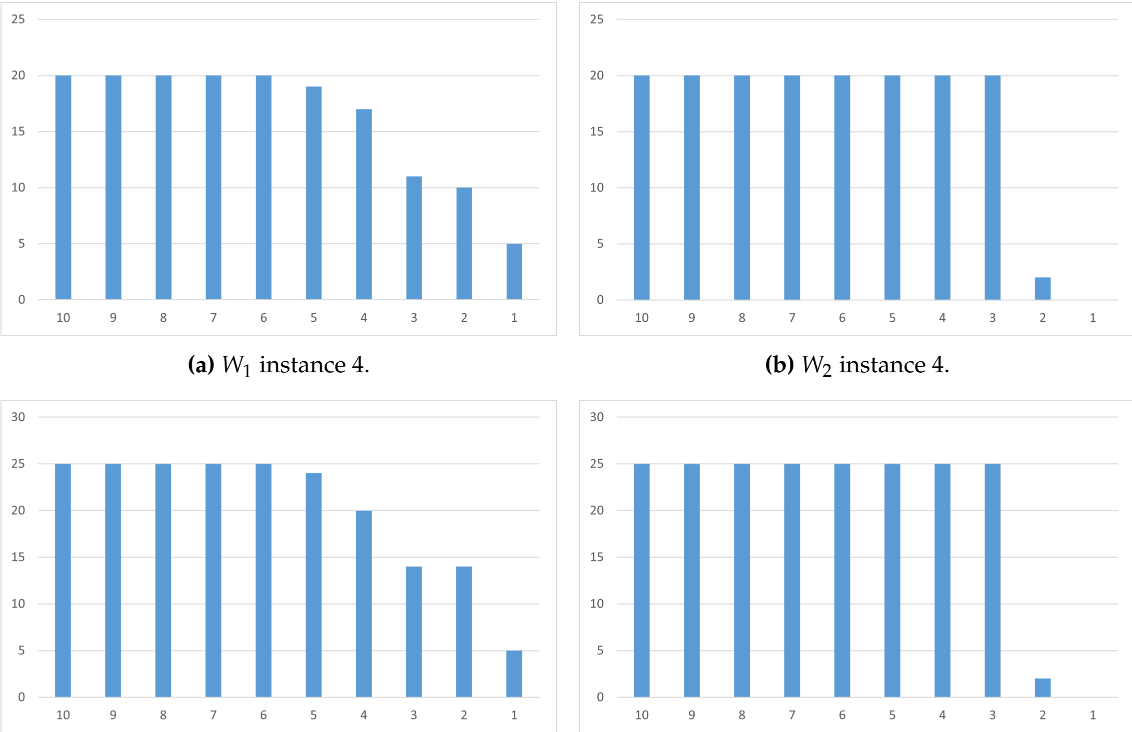


(e) W_1 instance 3.



(f) W_2 instance 3.

Figure 7.1: Barplots instances 1-3.



(a) W_1 instance 4.

(b) W_2 instance 4.

(c) W_1 instance 5.

(d) W_2 instance 5.

Figure 7.2: Barplots instances 4-5.

The interesting observation made for each instance, is that independent of the order in which the objectives were solved, the same number of orders were fulfilled. This can be seen in column 7 in all tables in Appendix B. The only difference was which of the orders were fulfilled. The barplots visualize the change in distribution of orders fulfilled over their given prioritization coefficient. In general, when maximizing number of boxes delivered first, more low prioritized orders are fulfilled compared to the opposite ordering. This is due to the fact that orders with a low prioritization coefficient request the highest number of boxes for a given combination of fish attributes. Also observed is the pattern that highly prioritized orders for a given combination is always fulfilled, independent of the order in which the objectives are solved. This is owing to the fact that higher prioritized orders are requesting a relatively low number of boxes compared to the available supply. This makes it possible to fulfill many orders with a high priority.

7.4.2 Pareto Front

Observed from the payoff tables (Tables 7.5 and 7.4), the improvement of one objective value seemed to worsen the other objective value when there was less supply than demand. To further investigate this, Pareto fronts were created for all the instances in that scenario.

In a realistic planning scenario, developing a plan for allocation must be done within a certain time frame. A time limit is set to 700 seconds, which reflects a reasonable time to use for finding one solution. The need for re-solving occurs frequently in the realistic planning process, due to updated information that affects the allocation. Setting the time limit too high is problematic, since the planners would spend significant amounts of time waiting for solutions.

The optimality gap is for each instance set quite low. This is done to ensure that a solution with an optimality gap lower than the requirement, is a close approximation of the actual Pareto optimal solution. The optimality gap and number of grid points for each instance can be seen in table 7.6.

Table 7.6: Optimality gap and grid points

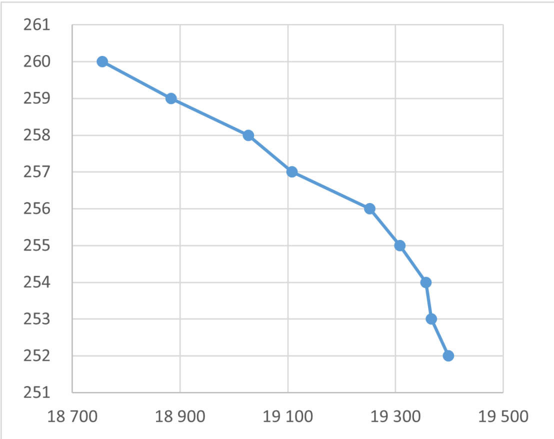
Instance	Optimality Gap %	Grid points
1	0.01	9
2	0.01	15
3	0.01	20
4	0.01	24
5	0.05	30

Having instances of increasing problem sizes, opens for observing if any iteration is unable to meet the required optimality gap before the time limit is reached. Table 7.7 describes a brief summary of the number of unique solutions for each instance, max and min of optimality gaps in percentage and number of times the time limit was reached. The entire result can be found in appendix B.

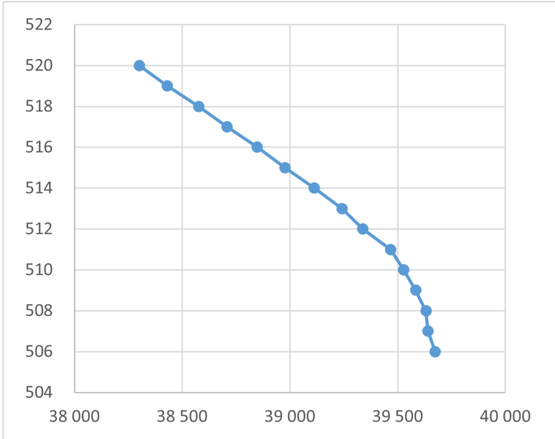
Table 7.7: Summary of the instance solutions

Instance	Unique Solutions	Max Gap %	Min Gap %	Number of times the time limit was reached
1	9	0.01	<0.001	0
2	15	0.01	0	0
3	20	0.017	0	1
4	24	1.265	<0.001	18
5	30	1.161	0.033	18

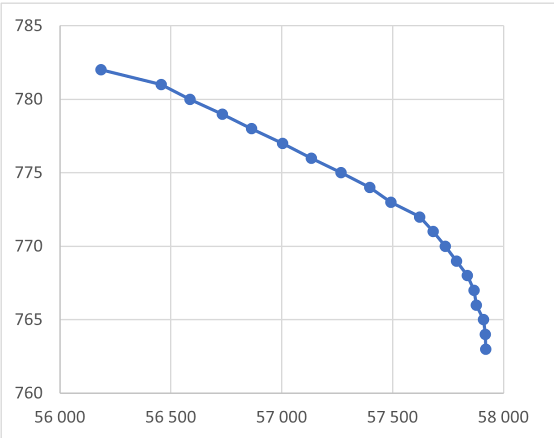
When creating Pareto fronts for instance 1 and 2, all iterations found Pareto optimal solutions with an optimality gap of 0.01% or lower within the time limit. As the problem size increased, fewer iterations could find a solution satisfying the required optimality gap within the time limit. This is especially apparent for instance 4 and 5, which both had 18 iterations with solutions that did not meet the required optimality gap within the time limit. At most, two of the solutions had an optimality gap of over 1%. This causes an issue regarding the certainty of having actually found a Pareto optimal solution, since the gap away from optimality is so large. A trade-off relation seems to exist between finding a solution within a reasonable time limit and a low optimality gap. The Pareto front for each instance is illustrated in figure 7.3.



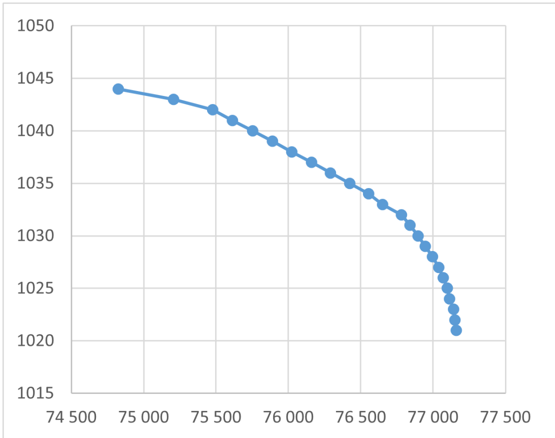
(a) Instance 1



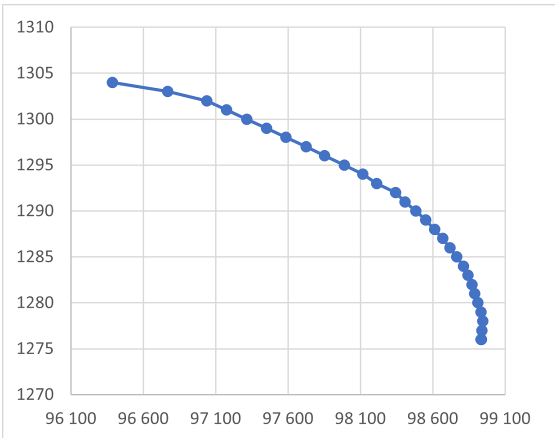
(b) Instance 2



(c) Instance 3



(d) Instance 4



(e) Instance 5

Figure 7.3: Pareto fronts for all instances

The gradual increase in problem size for each instance, allows for observing the evolution of the Pareto fronts pattern. When the total value for prioritized orders increases, the total number of boxes delivered decreases. This displays the negative correlation between the two objective functions for this scenario. Following the increase in total number of boxes delivered, is the increased density of Pareto optimal solutions. This behaviour is especially apparent in instance three, four and five (Figures 7.3c, 7.3d and 7.3e). The approximate Pareto fronts provide evidence that there can be many Pareto optimal points. It is not possible to be absolutely certain since the Pareto fronts are only approximations.

8. Conclusion

The scope of this thesis revolved around the allocation process, as seen from the perspective of the planners at Lerøy.

In this thesis the allocation of farmed fish to customer orders has been described and modelled.

The aim of the thesis was to provide decision support in the allocation process for the planners at Lerøy. The problem background gave insight into the allocation process at Lerøy, and the different challenges they faced. The scope of the allocation process was narrowed down, and assumptions and limitations were described. Multiple goals were identified that the planners had to find solutions to, some of which were conflicting.

The problem was formulated as a multi-objective assignment model with the objectives chosen being:

- Maximize the total number of boxes delivered to all orders
- Maximize the total value of prioritized orders fulfilled

These two objective functions were chosen since they represent a realistic allocation dilemma.

There was not found any previous multi-objective assignment model that had solved a similar allocation problem handling the same goals.

Regarding the research objective of finding an appropriate optimization method, there was a trade-off between computational time, and quality of the solutions, where quality was considered more important. The method chosen for the allocation problem was the Augmented ε-constraint method (AUGMECON).

Solving a problem instance described with the model provides a plan for allocation of fish to customer orders for a given horizon. This plan can be used to assist the planners in determining how the supply should be allocated to the orders.

A computational study with five instances in two different scenarios was performed. The study demonstrated an example where orders requesting a high number of boxes were set to have a low priority, while orders requesting a low number of boxes had a high priority. The results showed that in the scenario with more supply than demand, no conflict was observed between the objectives. This shows that in cases where there is more supply than demand, it might not be necessary to solve for both objective functions. When there was less supply than demand a conflict occurred, which resulted in different Pareto optimal solutions depending on which objective was solved first. In these cases, using multi-objective optimization is appropriate.
The CPU time also increased significantly when there was less supply than demand, indicating that it can be more time consuming to find an optimal solution in those scenarios. Pareto fronts were created for all the instances in that scenario. A time limit reflecting a reasonable time for how long it can take to find a solution, and the required optimality gap was set. The results showed that as the problem size grew, fewer solutions with an optimality gap satisfying the requirement was found before the time limit was reached. Natural questions that arise here are whether to increase the allowed solution time or the required optimality gap. Finding an answer to these questions requires further research.

The overall results from the computational study are not conclusive since it does not cover enough scenarios. However, when solving a problem where the number of boxes demanded and the prioritization of an order is negatively correlated in a situation where multiple orders are demanding the same fish, the results found here can be indicative of the expected solution.

9. Future Research

The allocation process has more objectives than the two incorporated into the model. A natural extension of the model would be to expand it by including more objective functions. Transportation was completely omitted from the problem scope, so including it in future research would be reasonable. The planners have a role in deciding number of vehicles used and what is going on which vehicles. Maximizing the fill-rate for the vehicles would be a fitting objective function related to this decision. Adding more objective functions would potentially increase the solution time. Applying a more efficient solution method than AUGMECON can be useful. AUGMECON2 and AUGMECON-R [23] are both improvements of AUGMECON and are proven to have a quicker solution time.

In the realistic planning process re-planning occur during the week. The model has not included the possibility for re-planning inside of the planning horizon. Improving the model to efficiently re-planning while inside of a planning period, would make it more useful in aiding the planners.

In the computational study it was evident that the problem size increased significantly when the problem instance got larger. Especially pointed out was the generation of variables and constraints that were unnecessary. A considerable improvement of the model would be to create the possibility of eliminating redundant variables and constraints.

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A. AMPL Model

```
# Sets
param m := 2;                               # Longest lead time
param T >= 1;                               # Number of days the plant is open
set PROD;                                   # Farms
set PACK;                                   # Plants
set ART;                                    # Species of the fish
set SIZE;                                   # Size of the fish
set QUAL;                                   # Quality of the fish
set iORDERS;                                # Internal orders
set eORDERS;                                # External orders
set ORDERS = {iORDERS union eORDERS};       # All orders
set CERT;                                   # Certifications
set DISE;                                   # Diseases

# Parameters
param supply{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL, 1..T} >= 0; # Supply

# The total demand for a given combination of species and quality in an order:
param demand{ART,QUAL,ORDERS} >= 0;
# The upper bound on demand for given combination of species, size class
# and quality in an order:
param demUpper{ART,SIZE,QUAL,ORDERS} >= 0;
 # The lower bound on demand for given combination of species, size class
 # and quality in an order:
param demLower{ART,SIZE,QUAL,ORDERS} >= 0;

param orderValue{ORDERS} >= 0, <= 500;      # Prioritization coefficient
# The amount it is allowed to reduce the total quantity delivered
# and still be allowed to deliver:
param demDev{ORDERS} >= 0;
```

```
param delDayDir{1..T, eORDERS} binary;      # Day of delivery - Direct delivery
param delDayInt{1..T, iORDERS} binary;      # Day of delivery - In-house delivery
param delDayCen{1..T+m, eORDERS} binary;    # Day of delivery - from distribution center


param prodCert{p in PROD, c in CERT} binary;    # If the farm has certifications
param demCert{o in ORDERS, c in CERT} binary;   # If the demand require certification
param prodDise{p in PROD, c in DISE} binary;    # If the farm has had outbreaks of disease
param demDise{o in ORDERS, c in DISE} binary;   # If the demand has any requirement
                                                # regarding disease
param PTO{k in PACK, o in iORDERS} binary;      # If a plant can delivery to the order
                                                # from an internal processing facility

# Initial inventory at plant:
param init{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL} >= 0;
# Initial inventory at the distribution center
param initCenter{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL} >= 0;

param invPackCap{PACK} >= 0;            # Inventory capacity at plant

param deliverDirect >= 0;               # Value for delivering directly
param deliverThrough >= 0;              # Value for delivering though distribution center
param M >= 0;                           # Big value


# Variables
# Direct delivery - Number of boxes:
var X{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL, 1..T, o in eORDERS}  >= 0;

# In-house delivery - Number of boxes:
var R{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL, 1..T, o in iORDERS}  >= 0;

# Delivery from distribution center - Number of boxes:
var W{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL, 1..T+m, eORDERS}  >= 0;

# Inventory at plant - Number of boxes:
var I{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL, 0..T}  >= 0;
```

```
# Inventory at distribution center - Number of boxes:
var Io{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL, 0..T+m}  >= 0;

# Number of boxes sent from plant to distribution center:
var B{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL,1..T}  >= 0;


var Y{PACK,1..T, eORDERS} binary;           # Direct delivery
var Yr{1..T, iORDERS} binary;               # In-house delivery
var Yw{1..T+m, eORDERS} binary;             # Delivery from distribution center


# Objective function
#maximize TotalDelivery:
        sum{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL}
            (sum{t in 1..T, o in eORDERS} deliverDirect*X[p,k,a,s,q,t,o]
                + sum{t in 1..T+m, o in eORDERS} deliverThrough*W[p,k,a,s,q,t,o]
                + sum{t in 1..T, o in iORDERS} R[p,k,a,s,q,t,o]);

# Objective Function
#maximize PrioDelivery:
        sum{k in PACK,t in 1..T, o in eORDERS} orderValue[o]*Y[k,t,o]
                + sum{t in 1..T+m, o in eORDERS} orderValue[o]*Yw[t,o]
                + sum{t in 1..T, o in iORDERS} orderValue[o]*Yr[t,o];


# Constraints

# Inventory
subject to initPackeryCon{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL}:
        I[p,k,a,s,q,0] = init[p,k,a,s,q];

subject to initCenterCon{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL}:
        Io[p,k,a,s,q,0] = initCenter[p,k,a,s,q];

subject to InventoryPakeryCap{k in PACK, t in 0..T}:
        sum{p in PROD, a in ART, s in SIZE, q in QUAL} I[p,k,a,s,q,t]  <= invPackCap[k];
```

```
# Inventory balance at plant
subject to InvBalancePlant
{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL, t in 1..T}:
        supply[p,k,a,s,q,t] + I[p,k,a,s,q,t-1] - sum{o in eORDERS} X[p,k,a,s,q,t,o]
        - sum{o in iORDERS} R[p,k,a,s,q,t,o] - B[p,k,a,s,q,t] = I[p,k,a,s,q,t];



# Inventory balance at distribution center
subject to InvBalanceDC_1
{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL, t in 1..dayFor[k]}:
        Io[p,k,a,s,q,t-1] - sum{o in eORDERS} W[p,k,a,s,q,t,o] = Io[p,k,a,s,q,t];

subject to InvBalanceDC_2
{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL, t in 1+dayFor[k]..T+dayFor[k]}:
        B[p,k,a,s,q,t-dayFor[k]] + Io[p,k,a,s,q,t-1] - sum{o in eORDERS} W[p,k,a,s,q,t,o]
        = Io[p,k,a,s,q,t];

subject to InvBalanceDC_3
{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL, t in T+1+dayFor[k]..T+m}:
        Io[p,k,a,s,q,t-1] - sum{o in eORDERS} W[p,k,a,s,q,t,o] = Io[p,k,a,s,q,t];

# Direct delivery
subject to TotalDemandDirect
{k in PACK, t in 1..T, a in ART, q in QUAL, o in eORDERS}:
        sum{p in PROD, s in SIZE} X[p,k,a,s,q,t,o] <= demand[a,q,o]*Y[k,t,o];

subject to TotalDemandReducedDirect
{k in PACK, t in 1..T, a in ART, q in QUAL, o in eORDERS}:
        sum{p in PROD, s in SIZE}X[p,k,a,s,q,t,o]  >= demand[a,q,o]*demDev[o]*Y[k,t,o];

subject to SizeDemandUpperDirect
{k in PACK, t in 1..T, a in ART, s in SIZE, q in QUAL, o in eORDERS}:
        sum{p in PROD} X[p,k,a,s,q,t,o]  <= demUpper[a,s,q,o]*Y[k,t,o];

subject to SizeDemandLowerDirect
{k in PACK,  t in 1..T, a in ART, s in SIZE, q in QUAL, o in eORDERS}:
        sum{p in PROD} X[p,k,a,s,q,t,o] >= demLower[a,s,q,o]*Y[k,t,o];
```

```
# Delivery through distribution center
subject to TotalDemandThrough{t in 1..T+m, a in ART, q in QUAL, o in eORDERS}:
        sum{p in PROD, k in PACK, s in SIZE} W[p,k,a,s,q,t,o] <= demand[a,q,o]*Yw[t,o];

subject to TotalDemandReducedThrough{t in 1..T+m,a in ART, q in QUAL, o in eORDERS}:
        sum{p in PROD, k in PACK, s in SIZE} W[p,k,a,s,q,t,o]
        >= demand[a,q,o]*demDev[o]*Yw[t,o];

subject to SizeDemandUpperThrough
{t in 1..T+m, a in ART, s in SIZE, q in QUAL, o in eORDERS}:
        sum{p in PROD, k in PACK} W[p,k,a,s,q,t,o] <= demUpper[a,s,q,o]*Yw[t,o];

subject to SizeDemandLowerThrough
{t in 1..T+m, a in ART, s in SIZE, q in QUAL, o in eORDERS}:
        sum{p in PROD, k in PACK} W[p,k,a,s,q,t,o] >= demLower[a,s,q,o]*Yw[t,o];


# In-house delivery
subject to TotalDemandInternal{t in 1..T, a in ART, q in QUAL, o in iORDERS}:
        sum{p in PROD, k in PACK, s in SIZE} R[p,k,a,s,q,t,o] <= demand[a,q,o]*Yr[t,o];

subject to TotalDemandReducedInternal{t in 1..T, a in ART, q in QUAL, o in iORDERS}:
        sum{p in PROD, k in PACK, s in SIZE} R[p,k,a,s,q,t,o]
        >= demand[a,q,o]*demDev[o]*Yr[t,o];

subject to SizeDemandUpperInternal
{t in 1..T, a in ART, s in SIZE, q in QUAL, o in iORDERS}:
        sum{p in PROD, k in PACK} R[p,k,a,s,q,t,o] <= demUpper[a,s,q,o]*Yr[t,o];

subject to SizeDemandLowerInternal
{t in 1..T, a in ART, s in SIZE, q in QUAL, o in iORDERS}:
        sum{p in PROD, k in PACK} R[p,k,a,s,q,t,o] >= demLower[a,s,q,o]*Yr[t,o];
```

```
# Whole delivery
subject to WholeDeliveriesExternal{o in eORDERS}:
        sum{k in PACK, t in 1..T} Y[k,t,o] + sum{t in 1..T+m} Yw[t,o] <= 1;

subject to WholeDeliveriesInternal{o in iORDERS}:
        sum{t in 1..T} Yr[t,o] <= 1;


# Day of delivery
subject to DayOfDeliveryDirect{t in 1..T, o in eORDERS}:
        sum{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL} X[p,k,a,s,q,t,o]
        <= M*(1-delDayDir[t,o]);

subject to DayOfDeliveryCenter{t in 1..T+m, o in eORDERS}:
        sum{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL} W[p,k,a,s,q,t,o]
        <= M*(1-delDayCen[t,o]);

subject to DayOfDeliveryIntern{t in 1..T, o in iORDERS}:
        sum{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL} R[p,k,a,s,q,t,o]
        <= M*(1-delDayInt[t,o]);


# Certifications
subject to CertificationCon{p in PROD, o in eORDERS, c in CERT}:
        sum{k in PACK, a in ART, s in SIZE, q in QUAL, t in 1..T} X[p,k,a,s,q,t,o]
        + sum{k in PACK, a in ART, s in SIZE, q in QUAL, t in 1..T+m} W[p,k,a,s,q,t,o]
        <= M*((1-demCert[o,c])+prodCert[p,c]);

subject to CertificationConIntern{p in PROD, o in iORDERS, c in CERT}:
        sum{k in PACK, a in ART, s in SIZE, q in QUAL, t in 1..T} R[p,k,a,s,q,t,o]
        <= M*((1-demCert[o,c])+prodCert[p,c]);
```

```
# Disease
subject to DiseaseCon{p in PROD, o in eORDERS, c in DISE}:
        sum{k in PACK, a in ART, s in SIZE, q in QUAL, t in 1..T} X[p,k,a,s,q,t,o]
        + sum{k in PACK, a in ART, s in SIZE, q in QUAL, t in 1..T+m} W[p,k,a,s,q,t,o]
        <= M*((1-demDise[o,c])+prodDise[p,c]);

subject to DiseaseConIntern{p in PROD, o in iORDERS, c in DISE}:
        sum{k in PACK, a in ART, s in SIZE, q in QUAL, t in 1..T} R[p,k,a,s,q,t,o]
        <= M*((1-demDise[o,c])+prodDise[p,c]);

# Plant to order
subject to PlantToOrderInternal{k in PACK, o in iORDERS}:
        sum{p in PROD, a in ART, s in SIZE, q in QUAL, t in 1..T} R[p,k,a,s,q,t,o]
        <= M*PTO[k,o];



# Lexicographic

param OptLow1;
param OptLow2;

#Constraint
subject to TotalDeliveryCon:
        sum{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL}
        (sum{t in 1..T, o in eORDERS} deliverDirect*X[p,k,a,s,q,t,o]
        + sum{t in 1..T+m, o in eORDERS} deliverThrough*W[p,k,a,s,q,t,o]
        + sum{t in 1..T, o in iORDERS} R[p,k,a,s,q,t,o]) >= OptLow1;

subject to PrioDeliveryCon:
        (sum{k in PACK,t in 1..T, o in eORDERS} orderValue[o]*Y[k,t,o]
        + sum{t in 1..T+m, o in eORDERS} orderValue[o]*Yw[t,o]
        + sum{t in 1..T, o in iORDERS} orderValue[o]*Yr[t,o]) >= OptLow2;
```

```
# AUGMECON

param r1 >= 0;
param r2 >= 0;

param e2 >= 0;

var s2 integer >= 0;
param qDiv >= 0;

# g is an iterator in a for loop.
# The loop goes from 0..qDiv.
# g in 0..qDiv

e2 := lower2 + g*(r2/qDiv);


maximize TotalDeliveryAUGMECON:
        sum{p in PROD, k in PACK, a in ART, s in SIZE, q in QUAL}
        (sum{t in 1..T, o in eORDERS} deliverDirect*X[p,k,a,s,q,t,o]
                + sum{t in 1..T+m, o in eORDERS} deliverThrough*W[p,k,a,s,q,t,o]
                + sum{t in 1..T, o in iORDERS} R[p,k,a,s,q,t,o]) + 0.0001 * (s2/r2);

subject to PrioDeliveryEpsilon:
        (sum{k in PACK,t in 1..T, o in eORDERS} orderValue[o]*Y[k,t,o]
        + sum{t in 1..T+m, o in eORDERS} orderValue[o]*Yw[t,o]
        + sum{t in 1..T, o in iORDERS} orderValue[o]*Yr[t,o]) - s2 = e2;
```

B. Generated Pareto Solutions

The values for the variables in columns 4 through 9 are sums of the specific set of variables.

Table B.1: Values for the Pareto front, instance 1

Solution #	W_1	W_2	$x^A_{l,a,s,z,q,t,o}$	$x^{DC}_{l,a,s,z,q,t,o}$	$x^{IN}_{l,a,s,z,q,t,o}$	$Y^A_{a,t,o}$	$Y^{DC}_{t,o}$	$Y^{IN}_{t,o}$	Gap
1	19 398	252	19 398	0	0	40	0	0	<0.001%
2	19 366	253	19 366	0	0	40	0	0	0.009%
3	19 356	254	19 356	0	0	40	0	0	0.005%
4	19 308	255	19 308	0	0	40	0	0	<0.001%
5	19 252	256	19 252	0	0	40	0	0	0.006%
6	19 108	257	19 108	0	0	40	0	0	0.009%
7	19 027	258	19 027	0	0	40	0	0	0.009%
8	18 883	259	18 883	0	0	40	0	0	0.01%
9	18 756	260	18 756	0	0	40	0	0	0.009%

Table B.2: Values for the Pareto front, instance 2

Solution #	W_1	W_2	$x^{A}_{l,a,s,z,q,t,o}$	$x^{DC}_{l,a,s,z,q,t,o}$	$x^{IN}_{l,a,s,z,q,t,o}$	$Y^{A}_{a,t,o}$	$Y^{DC}_{t,o}$	$Y^{IN}_{t,o}$	Gap
1	39 673	506	39 673	0	0	80	0	0	0%
2	39 641	507	39 641	0	0	80	0	0	0.009%
3	39 631	508	39 631	0	0	80	0	0	0.01%
4	39 583	509	39 583	0	0	80	0	0	0.01%
5	39 528	510	39 528	0	0	80	0	0	0.01%
6	39 467	511	39 467	0	0	80	0	0	0.0091%
7	39 337	512	39 337	0	0	80	0	0	0.0096%
8	39 242	513	39 242	0	0	80	0	0	0.0091%
9	39 112	514	39 112	0	0	80	0	0	0.01%
10	38 978	515	38 978	0	0	80	0	0	0.0097%
11	38 848	516	38 848	0	0	80	0	0	0.0098%
12	38 707	517	38 707	0	0	80	0	0	0.0095%
13	38 577	518	38 577	0	0	80	0	0	0.01%
14	38 431	519	38 431	0	0	80	0	0	0.01%
15	38 301	520	38 301	0	0	80	0	0	0.0096%

Table B.3: Values for the Pareto front, instance 3

Solution #	W_1	W_2	$x^A_{l,a,s,z,q,t,o}$	$x^{DC}_{l,a,s,z,q,t,o}$	$x^{IN}_{l,a,s,z,q,t,o}$	$Y^A_{a,t,o}$	$Y^{DC}_{t,o}$	$Y^{IN}_{t,o}$	Gap
1	57 919	763	57 919	0	0	121	0	0	0%
2	57 918	764	57 918	0	0	121	0	0	0.0017%
3	57 909	765	57 909	0	0	121	0	0	0.017%
4	57 878	766	57 878	0	0	121	0	0	0.009%
5	57 867	767	57 867	0	0	121	0	0	0.0047%
6	57 836	768	57 836	0	0	121	0	0	0.0017%
7	57 788	769	57 788	0	0	121	0	0	0.0099%
8	57 737	770	57 737	0	0	121	0	0	0.0035%
9	57 683	771	57 683	0	0	121	0	0	0.01%
10	57 622	772	57 622	0	0	121	0	0	0.01%
11	57 492	773	57 492	0	0	121	0	0	0.01%
12	57 398	774	57 398	0	0	121	0	0	0.0083%
13	57 268	775	57 268	0	0	121	0	0	0.01%
14	57 134	776	57 134	0	0	121	0	0	0.01%
15	57 004	777	57 004	0	0	121	0	0	0.01%
16	56 863	778	56 863	0	0	121	0	0	0.01%
17	56 733	779	56 733	0	0	121	0	0	0.01%
18	56 587	780	56 587	0	0	121	0	0	0.01%
19	56 457	781	56 457	0	0	121	0	0	0.01%
20	56 186	782	56 186	0	0	121	0	0	0.01%

Table B.4: Values for the Pareto front, instance 4

Solution #	W_1	W_2	$x^A_{l,a,s,z,q,t,o}$	$x^{DC}_{l,a,s,z,q,t,o}$	$x^{IN}_{l,a,s,z,q,t,o}$	$Y^A_{a,t,o}$	$Y^{DC}_{t,o}$	$Y^{IN}_{t,o}$	Gap
1	77 160	1021	77 160	0	0	162	0	0	<0.001%
2	77 151	1022	77 151	0	0	162	0	0	0.012%
3	77 140	1023	77 140	0	0	162	0	0	0.026%
4	77 112	1024	77 112	0	0	162	0	0	0.062%
5	77 098	1025	77 098	0	0	162	0	0	0.08%
6	77 070	1026	77 070	0	0	162	0	0	0.01%
7	77 039	1027	77 039	0	0	162	0	0	0.16%
8	76 994	1028	76 994	0	0	162	0	0	0.01%
9	76 946	1029	76 946	0	0	162	0	0	0.01%
10	76 896	1030	76 896	0	0	162	0	0	0.01%
11	76 841	1031	76 841	0	0	162	0	0	0.34%
12	76 780	1032	76 780	0	0	162	0	0	0.185%
13	76 650	1033	76 650	0	0	162	0	0	0.099%
14	76 555	1034	76 555	0	0	162	0	0	0.157%
15	76 425	1035	76 425	0	0	162	0	0	0.962%
16	76 291	1036	76 291	0	0	162	0	0	0.75%
17	76 161	1037	76 161	0	0	162	0	0	0.6%
18	76 024	1038	76 024	0	0	162	0	0	0.466%
19	75 890	1039	75 890	0	0	162	0	0	0.65%
20	75 753	1040	75 753	0	0	162	0	0	0.698%
21	75 614	1041	75 614	0	0	162	0	0	0.793%
22	75 477	1042	75 477	0	0	162	0	0	0.657%
23	75 206	1043	75 206	0	0	162	0	0	0.01%
24	74 823	1044	74 823	0	0	162	0	0	1.265%

Table B.5: Values for the Pareto front, instance 5

Solution #	W_1	W_2	$x^{A}_{l,a,s,z,q,t,o}$	$x^{DC}_{l,a,s,z,q,t,o}$	$x^{IN}_{l,a,s,z,q,t,o}$	$Y^{A}_{a,t,o}$	$Y^{DC}_{t,o}$	$Y^{IN}_{t,o}$	Gap
1	98 937	1276	98 937	0	0	202	0	0	0.041%
2	98 933	1276	98 933	0	0	202	0	0	0.045%
3	98 938	1277	98 938	0	0	202	0	0	0.04%
4	98 945	1278	98 945	0	0	202	0	0	0.033%
5	98 932	1279	98 932	0	0	202	0	0	0.046%
6	98 911	1280	98 911	0	0	202	0	0	0.068%
7	98 890	1281	98 890	0	0	202	0	0	0.089%
8	98 869	1282	98 869	0	0	202	0	0	0.049%
9	98 841	1283	98 841	0	0	202	0	0	0.049%
10	98 810	1284	98 810	0	0	202	0	0	0.049%
11	98 765	1285	98 765	0	0	202	0	0	0.049%
12	98 717	1286	98 717	0	0	202	0	0	0.05%
13	98 667	1287	98 667	0	0	202	0	0	0.074%
14	98 611	1288	98 611	0	0	202	0	0	0.049%
15	98 551	1289	98 551	0	0	202	0	0	0.11%
16	98 483	1290	98 483	0	0	202	0	0	0.05%
17	98 409	1291	98 409	0	0	202	0	0	0.26%
18	98 341	1292	98 341	0	0	202	0	0	0.10%
19	98 211	1293	98 211	0	0	202	0	0	0.22%
20	98 117	1294	98 117	0	0	202	0	0	0.42%
21	97 987	1295	97 987	0	0	202	0	0	0.21%
22	97 853	1296	97 853	0	0	202	0	0	0.25%
23	97 723	1297	97 723	0	0	202	0	0	0.506%
24	97 586	1298	97 586	0	0	202	0	0	0.756%
25	97 452	1299	97 452	0	0	202	0	0	0.415%
26	97 315	1300	97 315	0	0	202	0	0	0.88%
27	97 176	1301	97 176	0	0	202	0	0	0.44%
28	97 039	1302	97 039	0	0	202	0	0	0.82%
29	96 768	1303	96 768	0	0	202	0	0	0.946%
30	96 385	1304	96 385	0	0	202	0	0	1.161%