Master's degree thesis

LOG953 Logistics

Modelling a cyclic maritime inventory routing problem

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Number of pages including this page: 91

Molde, 23.05.2022

Molde University College
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Acknowledgements

We would like to acknowledge our master thesis supervisor, professor Lars Magnus Hvattum and co-supervisor, Professor Sebastian Alberto Urrutia, at Molde University College for their support and guidance.

We also want to appreciate all those who have shown us the true way to live: respect for all religions, beliefs, customs, cultures, and races. And the desire for a world full of peace, love and smiles for all creatures.

Summary

Maritime transportation plays a vital role in the global trade and supply chain. Transportation cost is a significant factor affecting the final product price. On the other hand, continuity in goods flow is another measure that affects customer satisfaction and business success.

This thesis presents a mathematical model for cyclic inventory routing problem in maritime transportation with a cost reduction approach. The model contains the transport of a single product between producers and customers. During the planning horizon, both production and consumption rates remain unchanged. A heterogeneous fleet of vessels is assumed for the voyage; each ship has its unique operational character say, speed and capacity. Ports are divided into two groups of production and consumption ports, based on their sole capability. The required constraints are added to the model to make a cyclic approach.

Two different datasets are implemented to test the model's performance, run for the cyclic and base models, and analyze the outputs for each model. In the next step, the base and cyclic model is compared.

The results reveal that production ports are full or near their upper stock level, and consumption ports are empty or near their lower stock bound due to the end of horizon effect in the base model; hence, the base model cannot repeat the voyage for subsequent periods. But since, in the cyclic model, at the end of the voyage, all ports and ships are in the same state as when they start the trip, meaning that the stock level at production and consumption ports and vessels are equal at the first and last time of the voyage in the planning horizon. Moreover, ships return to their initial location at the end of the time horizon; the cyclic model can be repeated infinitely.

Contents

1.0 Introduction

Annual statistics show that marine transportation accounts for more than 80% of global commerce volume and 70% of global trade value (Review of Maritime Transport, 2020). Ocean transportation links to commerce at all times. This connection dates back thousands of years. On the other hand, ocean shipping established a separate commercial activity in the mid-nineteenth century. The steam engine provided a level of dependability and predictability that sailing boats could not match. However, the separation of shipping and commerce created a slew of new problems.

Logistics exist in various organizations, and it is reasonable to conclude that no firm can function without them. Even so, all companies recognize that improving company performance necessitates the addition of a distinct logistics part. Logistics has become more critical because of the business environment, such as globalization, changing client requirements, and technology advancements (Song & Panayides, 2015).

As the backbone of international commerce and logistics networks, maritime transportation requires to deliver not only transportation-related services but also other related and broader logistical services more efficiently and effectively. The significant advantage of maritime logistics has been maximizing the rate of operational efficiency and service effectiveness (Song & Panayides, 2015).

Two inland alternatives of seaborne trade are road and rail transportation, which differ in terms of transportation period, time, budget, and accessibility. Big sea vessels carry heavy items in huge containers easily, while in inland transportation, there are more barriers for size and weights of goods; they should be in a standard format fit for transport by cars on roads or by trains on railways. In addition, access to different parts of the world, from the North Pole to the South, with various climatic conditions, is provided by the seabed without particular infrastructure. Also, geopolitical issues such as war, security and friendly relations between countries have less impact on maritime transport. And compared to inland transportation and air freight, sea transportation is more affordable.

In the case of maritime transportation, many uncertainties belong to operational constraints which can affect the optimality, say weather condition, fuel consumption, vessel speed and ports functional status (time windows), delay raised from ships technical issues (repairing time).

Optimal voyage scheduling extracted from outstanding inventory management has many environmental advantages, say less fuel consumption and decreased CO2 emission and maritime threats.

Global supply chain management aims to involve and integrate the various parties in a business process from scratch (raw material within the production phase) until the top of the transportation phase when the ultimate product or service delivers to end-users. Such integration can cause ease within the flow of knowledge, optimal efficiency and increased profit. Inventory management and transportation are two crucial fractions of world supply chain management, which play a vital role within the continuity and survival of the worldwide supply chain in terms of customer satisfaction and price competitiveness.

Since in the globalization process, companies face actors in different regions of the world (close, far or remote), both routing and inventory management have crucial effects on the robustness of the supply chain. Supply chain management is divided into two principal subcategories, inventory management and transportation. And inventory routing problem (IRP) is referred to as joint optimization of the two main subcategories (Rahimi, 2019).

Historically, inventory routing problems (IRP) mainly refer to cost minimization problems raised from transportation, operation, and inventory handling costs (Zaitseva, 2017). In fact, when transportation costs and operational costs (such as inventory holding cost and setup price) are essential simultaneously in a supply chain network, it is a sample of the Inventory Routing Problem (IRP); IRP is based on the Vendor-Managed Inventory policy (Zhao, 2019). According to the VMI rule and to improve the supply chain performance, the supplier is responsible for controlling and managing its customer inventory level and its stock via delivery of a proper amount of goods at a suitable time when the supply chain will not interrupt. (Zhong & Aghezzaf, 2011).

An important class of IRP is the Cyclic Inventory Routing Problem (CIRP); when there are constant orders from the customer over an infinite planning horizon, the case calls CIRP. The meaning of a cyclic route is that when a vessel returns to its initial location, it should operate loading or unloading and instantaneously departs to repeat its delivery duty continually (Zenker, Emde, & Boysen, 2016). Discovering a long-term repeatable (cyclic) distribution schedule for particular products or services among a set of customers having stable demand in which the transportation costs and operational costs are minimized is the objective of CIRP (Raa & Aghezzaf, 2009).

The crucial issue in changing a typical inventory routing design to a cyclic model is the concept of the end of the horizon effect, which recommends zero inventory level for consumption ports at the end of the planning time horizon. The end of horizon effect prevents the final stock level from being equal to the initial stock level; thus, it is a barrier for the schedule to be repeated unlimitedly.

In recent years, quite a bit of research has been done on cyclic inventory routing problems (CIRP) and quite a bit of research on maritime inventory routing problems (MIRP); on the other hand, there aren't previous research on the combination we want to consider in this thesis, cyclic maritime inventory routing problem (CMIRP). The importance of this problem becomes manifest when looking at the real world; since, at the end of the planning horizon of maritime transportation, the service cycle should be repeated unlimited times.

1.1 Thesis's purpose

In this thesis, the cyclic maritime inventory problem (CMIRP) is studied to generate a mathematical model appropriate for modelling a cyclic schedule applicable for different scenarios such as the different number of production and consumption ports, various production and consumption rate, varying time period, homogeneous or heterogeneous fleet type and the changeable initial stock level. Indeed, the vendor undertakes the production, distribution of goods and controlling the inventory level of several customers always to remain in the desired interval in the assigned time horizons.

Furthermore, in order to evaluate the performance of the proposed mathematical model, we generated the AMPL codes based on the mathematical model. Then we perform a computational study on both randomly generated instances and real-life instances provided by (Hemmati, Hvattum, Fagerholt, & Norstad, 2014).

This thesis is organized as follows. A review of previous research connected to this thesis's subject and presented solution methods is done in chapter 2, the literature review. In chapter 3, the problem is described. In chapter 4, the mathematical formulation and relevant definitions are presented. In chapter 5, the computational evaluation of the model is implemented. And in chapter 6, results have discussed.

2.0 Literature review

The number of articles on optimization of maritime transportation (Maritime Economics & Logistics) experienced an ascending trend between 2001 and 2012. Most of their context addressed include shipping performance or management and shipping finance (Talley, 2013). Economic theory, management science, logistics science and business administration are the contexts that can help to develop such academic discipline.

2.1 Definition of logistics

The word logistics is extracted from the Greek name of 'Logistikos' and means skilled in calculating. In the Roman and Byzantine armies, Logista was responsible for various duties such as material selection, procurement, transportation and distribution and supplies. In the 17th century, in the French armies, the Loger was an official title for the person responsible for sectioning and transporting troops. The word logistics was defined as the practical art of moving armies by Baron de Jomini at the beginning of the 19th century. Also, in the last decade of the 19th century, admiral Alfred Mahan defined logistics as the support of armed forces by economic and industrial mobilization of a nation. Before the second world war, the word logistics was not as cherished as today (Russell, 2000).

In terms of technology, logistics divides into four concepts, namely, the utility of form (production according to the local demand and production potential or design of product or service), the utility of place (including transportation and warehouses/inventories management), the utility of time (refers to availability of materials when is requested) and the utility of information (to earn detailed knowledge about the customers, transportation method, production scheme, routes and price) (Gribkovskaia, 2020).

In a firm, the leading logistics tasks by considering the respective logistics operations and management and the supply chain contain the following sections: (R. Stock & M. Lambert, 2001):

- Customer service
- Traffic and transportation
- Warehousing and storage
- Plant and warehouse site selection

- Inventory management
- Order processing
- Logistics communications
- Procurement
- Materials handling
- Packaging
- Demand forecasting
- Parts and service support
- Salvage and scrap disposal (Reverse logistics)
- Return goods handling (Reverse logistics)

Stock and Lambert (2001) also defined the final objective function of logistics duties as minimizing the total operational and nonoperational costs for a final ordered product or service. Minimize total costs = minimize (transportation + warehousing + processing and information + lot quantity + inventory holding) costs

Toyota Production System (TPS), which is based on lean manufacturing principles in supply chain management, is implemented by many firms due to its achievements. Customer, sequential recovery, waste management which can lead to higher quality and integration of upstream and downstream sections in the value chain are the key points in the TPS (Liker & Morgan, 2006).

2.2 Vehicle routing problem

The vehicle routing problem (VRP) objective is to find the cheapest delivery routes to end-users or warehouses distributed in different close or remote geographical regions considering a set of respective constraints. Concerning the vital role of VRP in the economy and its methodological challenges, the VRP has been one of the popular subjects in operational research for more than a half-century (Toth & Vigo, Vehicle routing: Historical perspective and recent contributions, 2013).

Dantzig and Ramser (1959) found a primary matching-based heuristic model for solving the capacitated vehicle routing problems (CVRP). Dantzig and Ramser's heuristic was a breakthrough for upcoming developments models. Clarke and Wright (1964) introduced the most prominent model in terms of speed, simplicity and reasonably high accuracy.

Fisher and Jaikumar presented a heuristic model for solving the vehicle routing problem in 1981. The concept of the time period for the first time considered to VRP by research presented by Dror and Levy (1986), where they adjust a weekly VRP to the primary heuristic's models. In future studies, stochastic demand is included in the VRP by Dror and Ball (1987).

Christofides et al., in the year 1981, published two papers, namely Christofides, Mingozzi and Toth in Networks, in which presented dynamic programming with state-space relaxation, and Mathematical Programming, in which two mathematical formulations are making use of q-paths and k-shortest spanning trees were introduced. Three years later, in 1984, the primary cutting plane approach according to the solution of linear relaxation of an integer model for vehicle routing problem presented by Laporte et al. (1984), an approach for some of the current algorithms.

In the following developments, vehicle's flow or goods' flow variables are solved by branch-and-cut attached to the proposed mathematical programming. Fukasawa et al. (2006) and Baldacci et al. (2008) presented methodologies in which partitioning is applied to formulate the vehicle routing problem; moreover, the issue contains some inequalities. The vehicle routing problem has gained its modern heuristics after the emersion of metaheuristics since 1990. Metaheuristics that can execute both broad and deep search of a solution at the same time and also can solve multi-variant problems are the best. Furthermore, Pisinger and Ropke (2007) implemented an adaptive large neighborhood search, and Vidal et al. (2012) presented a hybrid genetic algorithm.

2.3 Periodic Vehicle routing problem

In a general context, in periodic vehicle routing problems (PVRP), customers should be visited one day or more according to a scheduled time horizon and through a set of feasible routes. For the first time, the vehicle routing problem was introduced by Dantzig and Ramser (1959). Then in 1974, the periodic vehicle routing problem context was presented by Beltrami and Bodin, including a formal definition and mathematical modelling for municipal waste collection (Archetti, Fernández, & Muñoz, The Flexible Periodic Vehicle Routing Problem, 2017).

In 2006 Mourgaya and Vanderbeck ran a survey in which different variants of periodic vehicle routing problems were classified. They classified different variants based on differences in objective functions, for instance, minimization of distance, travelling time, fuel consumption and transportation cost. It is possible to improve efficiency and achieve optimal service if consumers or customers are allowed to be visited more often than their minimum requested frequencies in a given time horizon cost.

The vehicle routing problem aims to design an optimal route in which a fleet of vehicles starts its route from depot to visit a set of customers based on their order demands (Vigo, Archetti, & Speranza, 2014).

2.4 Inventory routing problem

The subject of the inventory-routing problem (IRP) was studied comprehensively for the first time in 1983 by Bell et al. (1983). The context links vehicle routing, delivery plan and inventory management.

In the primary variants of IRP, researchers faced problems with just a single objective function of decreasing transportation cost. Also, the number of customer orders was stochastic, and it was vital to keep customers' inventory stock level. Although the number of primary research in transportation and distribution was eye-catching, it was challenging to combine both transportation problems and distribution problems due to the lack of proper computing tools and the disability of introduced algorithms to solve more significant complex combinatorial transportation and distribution problems. Three years after introducing the first heuristic model, to include the inventory costs and the shortage costs in a random order condition, their model was amended by Federgruen and Zipkin in 1984.

In order to make the issue more realistic other operational costs, say base set up fees, distribution costs and inventory holding costs added to the problem by Blumenfeld et al. (1985). At the same time, trade-offs analysing between transportation and inventory costs implemented by Burns et al. (1985) and short-term approaches reviewed by Dror, Ball, and Golden (1985). The clustering algorithm for the first time was proposed by Anily and Federgruen (1990) for the inventory routing problem.

Also, a comprehensive literature review of inventory routing problems was provided by Coelho, Cordeau and Laporte (2014). With respect to the structural variants and the customer demands, they classified IRP. Archetti et al. (2014) introduced o formulation for multi-vehicle IRP; that mathematical formulation aimed to increase profit by transportation and inventory holding costs. Also, in 2016, to avoid the drawback of zero inventory at the end of the finite time horizon, they presented a non-linear objective function in IRP then compared their results to those of classical IRP. An overview of the inventory routing problem in supply chain management in 2007 was presented by Moin and Salhi. They classified the articles based on the periods applied in their modelling, namely single period, multiperiod and infinite horizon models with deterministic and stochastic demand patterns.

Historically in a wide range of research in inventory routing problems (IRP) with pickup and delivery, the planning horizon is divided into many sub-periods (Bertazzi, Savelsbergh, & Grazia Sp, Inventory Routing, 2008) (C. Coelho, Cordeau, & Laporte, 2014). Furthermore, the inventory routing problem is portioned into two categories IRP with discretized time periods and IRP with continuous time periods. Since research indicates that the accuracy of the second category is higher than the first category, IRP with a continuous-time approach has been the subject of the majority of maritime transportation (Al-Khayyala & Hwang, 2007) (Christiansen, Decomposition of a Combined Inventory and Time Constrained Ship Routing Problem, 1999) (Christiansen, Fagerholt, Nygreen, & Ronen, Ship routing and scheduling in the new millennium, 2013) (Siswanto, Essam, & Sarker, 2011).

2.5 Maritime inventory routing problem

It is helpful to study the existing maritime inventory routing problems (these models are not cyclic, so they suffer from end-of-horizon effects). Maritime inventory routing problem (MIRP) is about transportation of single or multiple products from production ports to consumption ports and monitoring the inventory level of products in all ports constantly (Christiansen, Fagerholt, Nygreen, & Ronen, Maritime Transportation, 2007). An inventory routing problem with uncertain demands and sailing times was solved heuristically by Cheng and Duran (2004). Rakke et al. (2011) and Sherali and Al-Yakoob (2006) introduced penalty functions for deviating from customer contracts and inventory limits.

An overview of routing problems and combined inventory management was presented by Andersen et al. (2010). They described industrial aspects and a comprehensive literature review. An optimization model for the liquified natural gas by coordinating vessel routing in the oil and gas industry was presented by Fodstad et al. (2010). Moreover, a maritime inventory routing problem for liquified natural gas (LNG) was introduced by Grønhaug et al. (2010), who compared the features of the LNG inventory routing problem with other maritime IRPs.

A detailed description of deterministic single product maritime inventory routing problems in deep-sea, including inventory recording of each port, was presented by Papageorgiou et al. (2014).

The profit maximization in the problem of distributing a limited amount of products among consumers by a fleet of vehicles is studied by Chien, Balakrishnan and Wong (1989). Andersson, Christiansen and Fagerholt (2010) introduced a profit maximization model for scheduling a daily delivery problem based on a sophisticated Lagrangian relaxation algorithm for LNG supply chain, route planning and inventory management. Papageorgiou et al. (2014) comprehensively presented a mixed-integer linear program for profit maximization of deterministic single product maritime inventory routing problem case.

2.6 Cyclic inventory routing problem

The introduction of the cyclic inventory routing problem (CIRP) was done in 2006 by Aghezzaf et al. for the first time. Three years later, in 2009, Raa and Aghezzaf presented a heuristic model for CIRP. Inventory routing problem for an unlimited time horizon with the objective of minimizing the total transportation cost from a single warehouse to a group of consumers was investigated by Özener et al. in 2014. The subject of CIRP was developed via optimization of the size of the fleet by Raa in 2015. Minimizing the number of required vehicles was the centre stage. In 2018, multi-objective research was accomplished on green CIRP to minimize total cost and reduce emissions (Rau, Budiman, & Widyadana, 2018).

In 2019, Luca Bertazzi et al. designed a lean production system for a production base fed by many suppliers distributed in different zones. Standard inventory routing problem extracted from the outbound logistics problem refers to all feasible routes connecting a set of consumers who need particular products or services to a group of producers at an optimal cost. The optimal cost encompasses all transportation, inventory holding, setup, and other expenses. Unlike the traditional outbound IRP in the Luca Bertazzi et al. model, the objective is to collect the required components required for producing a unique good from different suppliers at minimum cost with respect to the lean principles of standard work and level production planning in a T-day time horizon which was repeated every T-day.

2.7 End of horizon effect

The vital issue occurring in the IRP is the end of horizon effect (Ben Ahmed, Okoronkwo, Okoronkwo, & Hvattum, 2021). In a typical optimal solution, the consumption and production ports' stock levels are low and high, respectively, at the end of time horizon. Moreover, there are some approaches suggested by Agostinho Agra et al. in 2017 to avoid end of horizon effect. Making tight inventory bounds for the end of the time horizon is the first alternative and increasing the amount of minimum delivery capacity is the second alternative. Also, in 2021 Agra et al. to complete their approaches, defined a fraction of F containing a discrete value between 0 and 1 to handle the stock level at the production and consumption nodes and then curb the end of horizon effect. When F gets the value of 1, it means the final stock is equal to the primary stock level in all nodes.

3.0 Problem description

This thesis is a kind of inventory routing problem in which conditions are created to form a cycle that works forever. This thesis examines how the inventory levels at the end of the time horizon can match the initial inventory levels for all ports and vessels and how vessels can cyclically repeat this operational planning for an unlimited time. Therefore, the centre stage of this thesis is the condition that the final stock level should be equal to the initial stock level for all ports and vessels. Also, ships should return to their initial position to continue the voyage plan for an infinite time.

The problem in this thesis considers the geographical area in which maritime transport of a single product comes about. The transportation is carried out by a heterogeneous fleet of vessels that differ in size, speed, amount of loads/unload goods rate per time unit, capacity, daily travelling cost, and operational cost. Travelling distances are contained into the problem. There are several ports, which are divided between consumption and production ports. None of the ports is able to have both consumption and production. Both production and consumption ports have fixed rates throughout the time horizon. Both ports of consumption and production have inventory, and each port has storage facilities with fixed upper and lower bounds. Depending on the quantity of storage and the number of items to be loaded or unloaded, each port may be visited many times by various vessels during the planning horizon. The objective function is to minimize shipping and operating costs by considering the cyclic condition. It is assumed that inventory holding cost is negligible.

Each vessel can choose the starting location between ports and executes its route in the best possible way. The load onboard the vessel when leaving its initial position is considered variable. Each vessel transports different loads between ports following the demand throughout the routes. The speed at which ships sail for each voyage between ports can vary. Here the speed can be different even for a port-to-port route along the route for various reasons that may occur along the way. At the end of the program period, the ship may be in the middle of the sea and need extra time to return to its previous position. In this model, this extra time is considered a delay time. The initial inventory for production and consumption ports is considered variable. The sum of the production rates must be equal to the sum of the consumption rates because it would not be possible otherwise to have a cyclic plan. Production ports are not allowed to exceed their maximum storage capacity, and shortages are not permitted at consuming ports. Each ship must return to its starting position for this cycle to operate indefinitely. Also, each port must be serviced by vessels at least once. At the end of the timetable, the vessel's stock level should equal the initial inventory level. Also, at the end of the planning horizon, the port's inventory level should equal the initial inventory level. For further clarification of the problem, an example is given below, in which full explanations are presented.

In this example, there are three ports located Shanghai, Savannah, and Hamburg. The port of Shanghai is a production port, and the other two ports are consumers. The production rate in the port of Shanghai is 90 tons per hour, and the consumption rate in the ports of Savannah and Hamburg is 40 and 50 tons per hour, respectively. There are two identical Post Panamax Container ships with a capacity of 81,200 tons and a speed of 26 miles per hour. Vessels also have an operating rate of 2,000 tons per hour. The total duration of the planning horizon is two months.

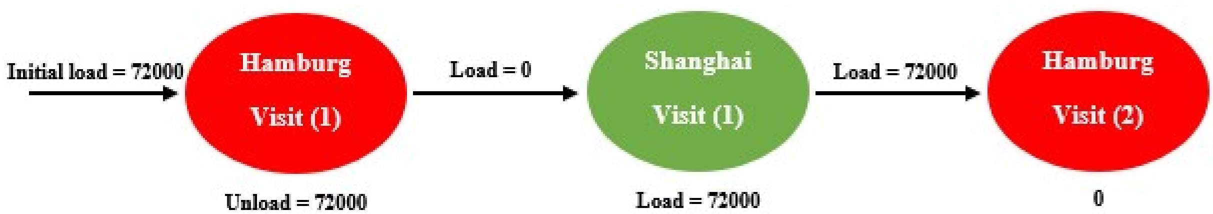

Figure 1: Rout planning for vessel 1, Source: Made by authors

Ship route 1 (V1):

Initial load = 72000, Origin: Hamburg port, Destination: Hamburg port

Hamburg (1) → Shanghai (1) → Hamburg (2)

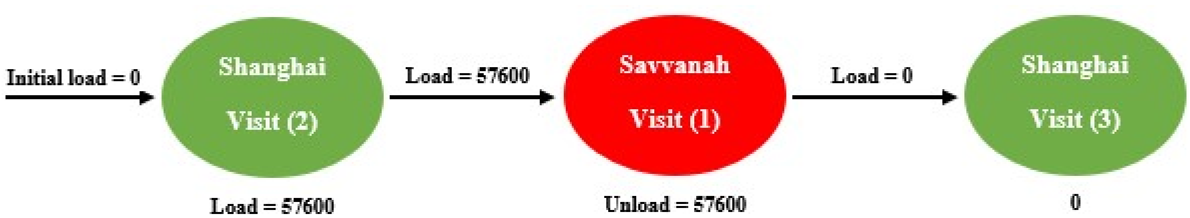

Figure 2: Rout planning for vessel 2, Source: Made by authors

Ship route 2 (V2):

Initial load = 0, Origin: Shanghai port, Destination: Shanghai port

Shanghai (2) → Savannah (1) → Shanghai (3)

From Figures 1 and 2, it is simple to detect how two vessels sail their paths. From Figure 3, When the two ships arrive at the desired ports, it is possible to see when they dock at the selected ports. The green port (Shanghai) is the production port, and the red ports (Savannah and Hamburg) are the consumption ports. The initial inventory stock level at Shanghai, Savannah and Hamburg ports are 56391.5, 49466.3 and 0 respectively.

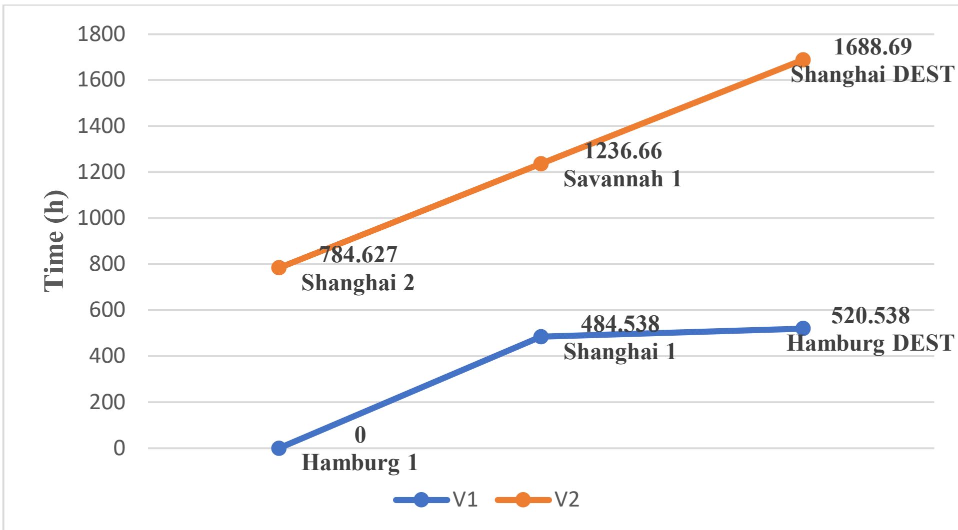

Figure 3: Vessels' location in time horizon, Source: Made by authors

The first vessel, which is blue, starts from the first visit to Hamburg port at the first visit with 72000 initial inventories; and unloads 72000 products at Hamburg port, then goes on a second visit to Shanghai port to load 72000 products. The time to reach Shanghai port is 484.538 hours after starting time horizon. After that, it sails to its destination, Hamburg port, the same port as the start point. The arrival time to destination for vessel one is 520.538.

The orange vessel's second ship starts from Shanghai port on the first visit with zero initial inventory level and loads 57600 products at Shanghai port, then goes on a first visit to Savannah port to unload 57600 products. The time to reach Savannah port is 1236.66 hours after starting time horizon. Then sails to Shanghai port, the destination port. The sail to the destination port includes 248.689 hours of delay time to return to its original condition and participate in the next cycle with the same conditions.

4.0 Mathematical modeling

Cyclic maritime inventory routing problem is modeled in the same approach and with the same notation as Agra et al. (2016a). In this section, two models are presented. Also, mathematical formulations are introduced. Model 1 is presented first, then model 2. Model 2 is a cyclic case of model 1. Both models use speed optimization. Andersson et al. (2015) presented a model in which speed optimization combined with shipping route planning to get better results than the previous sequential method.

4.1 Introduction to models

The distinctions between the two models are described in Table 1. As depicted in Table 1, the initial inventory of ports and vessels are parameters in model 1; however, both are variables in model 2. Also, the initial position in model 1 is assigned, but it is variable in model 2. Furthermore, model 2 has a constraint which vessels should return to their initial position at the end of the time planning horizon, but model 1 does not have this limitation.

Table 1: Differences between models

Source: Made by authors

Characteristics	**Model 1 (Base)**	**Model 2 (Cyclic)**
Initial ports inventory	parameter	variable
Initial stock of vessels	parameter	variable
Initial position of vessels	parameter	variable
Vessels return to their start position	No	Yes

In the models, V indicates a set of vessels, and N indicates a set of ports. Each vessel $v \in V$ has its starting point in model 1, which can be a point at ports, but it is variable in model 2. Each port can have several visits throughout the time horizon. Nodes in a network are denoted by a pair (i, m), where i is a port, and m is the visit number. If there are arcs of direct travel from node (i, m) to node (j, n), it is labeled as (i, m, j, n).

4.1.1 Model 1. Maritime Inventory Routing Problem

In this model, ports' initial inventory, the initial stock of vessels, and the initial position of vessels are parameters. Also, the amount of delay time that may occur for each ship is not considered. There are also no restrictions on returning the ship to its original location.

Notation:

Sets:

V : set of vessels

N : set of production and consumption ports

S^A : set of possible nodes (i,m)

S_v^A : set of nodes that can be visited by vessel v

S_v^X : set of all possible moves (i,m,j,n) of vessel v

S_v^S : set of breakpoints for the speed of vessel v, with $S_v^S = \{1,2,...,U\}$

Parameters:

T : number of time units in the planning horizon

H_i : minimum number of visits to port $i \in N$

M_i : maximum number of visits to port $i \in N$

D_i : consumption or demand at port $i \in N$ per unit of time

J_i : 1 if production facilities are located in port i, and -1 if consumption facilities are located in port $i, i \in N$

P_{iv} : port cost at port $i \in N$ for vessel $v \in V$

C_v : capacity of vessel $v \in V$

L_v : initial load onboard vessel $v \in V$

$\underline{S}_i$: lower bound on the inventory level at port $i \in N$

$\overline{S}_i$: upper bound on the inventory level at port $i \in N$

S_i^O : the initial stock level in port $i \in N$ at the beginning of the planning horizon

A_{im} : earliest time for starting visit m to port i, $(i,m) \in S^A$

B_{im} : latest time for starting visit m to port i, $(i,m) \in S^A$

K_i : minimum time between two consecutive visits to port $i \in N$

$\underline{Q}_i$: minimum load/unloaded quantity in port $i \in N$

U_{im} : latest time for finishing visit m to port i, $(i,m) \in S^A$

T_v^Q : time for unloading or loading each unit by vessel $v \in V$

T_{ijvs}^{PP} : time required by vessel $v \in V$ to sail from port $i \in N$ to port $j \in N$ with speed $s \in S_v^S$

T_{ijvs}^{OP} : time required by vessel $v \in V$ to sail from its origin to port $i \in N$ with speed $s \in S_v^S$

C_{ijvs}^{PP} : sailing cost from port $i \in N$ to port $j \in N$ by vessel $v \in V$ with speed $s \in S_v^S$

C_{ivs}^{OP} : sailing cost from its origin to port $i \in N$ by vessel $v \in V$ with speed $s \in S_v^S$

Variables:

x_{imjnv} : 1 if vessel $v \in V$ moves directly between nodes (i,m) and (j,n), 0 otherwise, $v \in V$,

$(i,m,j,n) \in S_v^X$

x_{imv}^O : 1 if vessel v departs from its initial position to node (i,m), 0 otherwise, $v \in V$, $(i,m) \in S_v^A$

z_{imv} : 1 if vessel v finishes its route at node (i,m), 0 otherwise, $v \in V$, $(i,m) \in S_v^A$

z_v^O : 1 if vessel v is not used in the planning horizon, 0 otherwise, $v \in V$

q_{imv} : the amount of product loaded or unloaded by vessel v at node (i,m), $v \in V$, $(i,m) \in S_v^A$

f_{imjnv} : the amount of product that vessel v transports from node (i,m) to node (j,n), $v \in V$, $(i,m,j,n) \in S_v^X$

f_{imv}^O : the amount of product that vessel v transports from the origin to node (i,m), $v \in V$, $(i,m) \in S_v^A$

f_{imv}^D : the amount of product that vessel v transports from node (i,m) to the destination, $v \in V$, $(i,m) \in S_v^A$

t_{im} : start time of visit number m to port i, $(i,m) \in S_v^A$

o_{imv}: 1 if vessel v operates in node (i,m), 0 otherwise, $v \in V$, $(i,m) \in S_v^A$

y_{im} : 1 if there is visit (i,m) to port i, 0 otherwise, $i \in N$, $(i,m) \in S^A$

s_{im} : stock level at ports at the start of visit m to port i, $(i,m) \in S^A$

g_{imjnvs} : auxiliary variable to determine the speed of vessel v when going from node (i,m) to (j,n), with s corresponding to a given choice of speed, $v \in V$, $s \in S_v^S$, $(i,m,j,n) \in S_v^X$

g_{imvs}^O : auxiliary variable to determine the speed of vessel v when going from its origin to (i,m), with s corresponding to a given choice of speed, $v \in V$, $s \in S_v^S$, $(i,m) \in S_v^A$

Mathematical model:

$$\min \sum_{v\in V}\sum_{(i,m,j,n)\in S_v^X}\sum_{s\in S_v^S} C_{ijvs}^{PP} g_{imjnvs} + \sum_{v\in V}\sum_{(i,m)\in S_v^A}\sum_{s\in S_v^S} C_{ijvs}^{OP} g_{imvs}^{O} + \sum_{v\in V}\sum_{(i,m)\in S_v^A} P_{iv} o_{imv} \qquad (1)$$

Expresses the minimization of the sum of traveling costs between ports depending on the chosen speed and operational costs in each port.

Subject to:

$$\sum_{(j,n)\in S^A} x_{jnv}^{O} + z_v^{O} = 1 \qquad v \in V \qquad (2)$$

Show that a vessel must either depart from the origin to a port or not be used at all.

$$o_{imv} - \sum_{(j,n):(j,n,i,m)\in S_v^X} x_{jnimv} - x_{imv}^{O} = 0 \qquad v \in V, (i,m) \in S_v^A \qquad (3)$$

Define that if a node is visited by vessel v, the vessel must either arrive at the node from the origin or from another node.

$$o_{imv} - \sum_{(j,n):(i,m,j,n)\in S_v^X} x_{imjnv} - z_{imv} = 0 \qquad v \in V, (i,m) \in S_v^A \qquad (4)$$

Ensure that if a vessel is at node i it must either leave to another node or end its route there.

$$\sum_{v\in V} o_{imv} = y_{im} \qquad (i,m) \in S^A \qquad (5)$$

Show that a vessel can only visit node (i,m) if there are at least m visits to port i.

$$y_{i(m-1)} - y_{im} \geq 0 \qquad (i,m) \in S^A : H_i + 1 \leq m \leq M_i \qquad (6)$$

Guarantee that if a port i is visited m times, then it also has been visited m-1 times.

$$y_{im} = 1 \qquad (i,m) \in S^A : m \in \{1, ..., H_i\} \qquad (7)$$

Defines the number of mandatory visits for port i.

$$\sum_{s\in S_v^S} g_{imjnvs} = x_{imjnv} \qquad v \in V, (i,m,j,n) \in S_v^X \qquad (8)$$

Enforce that speed of a vessel must be set for a travel from node (i,m) to node (j,n) if and only if that travel exists.

$$\sum_{s \in S_v^S} g_{imvs}^O = x_{imv}^O \qquad v \in V, (i,m) \in S_v^A \qquad (9)$$

Enforce that speed of a vessel must be set for a travel from the origin to node (i,m) if and only if that travel exists.

$$q_{imv} \leq \min\{C_v, \overline{S}_i\} o_{imv} \qquad v \in V, (i,m) \in S_v^A \qquad (10)$$

Ensure the quantity loaded/unloaded cannot exceed the vessel capacity nor the maximum port capacity.

$$\underline{Q}_i o_{imv} \leq q_{imv} \qquad v \in V, (i,m) \in S_v^A \qquad (11)$$

Show that if a vessel visits the port, then the amount loaded/unloaded should be at least equal to the minimum quantity.

$$f_{imv}^O = L_v x_{imv}^O \qquad v \in V, (i,m) \in S_v^A \qquad (12)$$

Determine flow from the initial position which is equal to initial load if ship travels from the initial position.

$$f_{jnv}^O + \sum_{(i,m):(i,m,j,n) \in S_v^X} f_{imjnv} + J_j q_{jnv} = \sum_{(i,m):(j,n,i,m) \in S_v^X} f_{jnimv} + f_{jnv}^D \qquad v \in V, (j,n) \in S_v^A \qquad (13)$$

It is flow conservation constraints which sum of incoming flow from origin, flow from a particular port and the amount of loaded/unloaded should be equal to outgoing flow from port plus outgoing flow to the destination.

$$f_{imjnv} \leq C_v x_{imjnv} \qquad v \in V, (i,m) \in S_v^A, \; (j,n) \in S_v^A \qquad (14)$$

Convey flow from a port to another port should not be more than the capacity of the vessel.

$$f_{jnv}^D \leq C_v z_{jnv} \qquad v \in V, (j,n) \in S_v^A \qquad (15)$$

Show flow to the destination is less or equal ship capacity if ship travels to the destination.

$$t_{im} - t_{i(m-1)} - \sum_{v \in V} T_v^Q q_{i(m-1)v} - K_i y_{im} \geq 0 \qquad (i,m) \in S^A : m > 1 \qquad (16)$$

Enforce the minimum time period between two consecutive visits of port i.

$$t_{im} + \sum_{v \in V} T_v^Q q_{imv} - t_{jn} + \sum_{v \in V, s \in S_v^S} \max\{U_{im} + T_{ijvs}^{PP} - A_{jn}, 0\} g_{imjnvs} \leq U_{im} - A_{jn} \qquad (i,m) \in S^A, \; (j,n) \in S^A \qquad (17)$$

Relate the start time associated with node (i, m) to the start time associated with node (j, n) when a vessel travels between ports i and j.

$$\sum_{v\in V}\sum_{s\in S_v^S} T_{ivs}^{OP} g_{imvs}^{O} \le t_{im} \qquad (i,m)\in S^A \qquad (18)$$

Show that the travel time for a vessel traveling from origin should not exceed the start time of the visit to the port.

$$t_{im} \ge A_{im} \qquad (i,m)\in S^A \qquad (19)$$

Express time windows for the start time of visits.

$$t_{im} \le B_{im} \qquad (i,m)\in S^A \qquad (20)$$

exhibit time windows for the end time of visits.

$$s_{i1} = S_i^O + J_i D_i t_{i1} \qquad i\in N \qquad (21)$$

Set the stock level at the start time of the first visit to a port.

$$s_{im} = s_{i(m-1)} - J_i \sum_{v\in V} q_{i(m-1)v} + J_i D_i (t_{im} - t_{i(m-1)})) \qquad (i,m)\in S^A : m>1 \qquad (22)$$

Show that the stock level at the start of the m[th] visit is set by to the stock level at the start of the previous visit, the load or unload operation in the previous visit and the time elapsed between the two visits.

$$s_{im} + \sum_{v\in V} q_{imv} - D_i \sum_{v\in V} T_v^Q q_{imv} \le \overline{S}_i \qquad (i,m)\in S^A : J_i = -1 \qquad (23)$$

Impose limitation of the stock level at the end time of a visit to a consumption port.

$$s_{im} - \sum_{v\in V} q_{imv} + D_i \sum_{v\in V} T_v^Q q_{imv} \ge \underline{S}_i \qquad (i,m)\in S^A : J_i = +1 \qquad (24)$$

Impose limitation of the stock level at the end time of a visit to a production port.

$$s_{iM_i} + \sum_{v\in V} q_{iM_i v} - D_i (T - t_{\mathrm{iM}_i}) \ge \underline{S}_i \qquad i\in N : J_i = -1 \qquad (25)$$

Show lower bound on the inventory level until the end of the time horizon for consumption ports.

$$s_{iM_i} - \sum_{v\in V} q_{iM_i v} + D_i (T - t_{\mathrm{iM}_i}) \le \overline{S}_i \qquad i\in N : J_i = +1 \qquad (26)$$

Exhibit upper bound on the inventory level until the end of the time horizon for production ports.

$$s_{im} \ge \underline{S}_i \qquad (i,m)\in S^A : J_i = -1 \qquad (27)$$

Illustrate lower bound on the stock level at the start of each visit for consumption ports.

$$s_{im} \leq \overline{S}_i \qquad (i,m) \in S^A : J_i = +1 \qquad (28)$$

Show upper bound on the stock level at the start of each visit for production ports.

The rest of the constraints enforce the binary or non-negative nature of the variables.

$$x_{imjnv} \in \{0,1\} \qquad v \in V, (i,m,j,n) \in S_v^X \qquad (29)$$

$$x_{imv}^O \in \{0,1\} \qquad v \in V, (i,m) \in S_v^A \qquad (30)$$

$$o_{imv} \in \{0,1\} \qquad v \in V, (i,m) \in S_v^A \qquad (31)$$

$$z_{imv} \in \{0,1\} \qquad v \in V, (i,m) \in S_v^A \qquad (32)$$

$$z_v^O \in \{0,1\} \qquad v \in V \qquad (33)$$

$$y_{im} \in \{0,1\} \qquad (i,m) \in S^A \qquad (34)$$

$$q_{imv} \geq 0 \qquad v \in V, (i,m) \in S_v^A \qquad (35)$$

$$f_{imjnv} \geq 0 \qquad v \in V, (i,m,j,n) \in S_v^X \qquad (36)$$

$$f_{imv}^O \geq 0 \qquad v \in V, (i,m) \in S_v^A \qquad (37)$$

$$f_{imv}^D \geq 0 \qquad v \in V, (i,m) \in S_v^A \qquad (38)$$

$$0 \leq g_{imvs}^O \leq 1 \qquad v \in V, (i,m) \in S_v^A, s \in S_v^s \qquad (39)$$

$$0 \leq g_{imjnvs} \leq 1 \qquad v \in V, (i,m,j,n) \in S_v^X, s \in S_v^s \qquad (40)$$

$$s_{im} \geq 0 \qquad (i,m) \in S^A \qquad (41)$$

$$t_{im} \geq 0 \qquad (i,m) \in S^A \qquad (42)$$

4.1.2 Model 2. Cyclic Maritime Inventory Routing Problem

In this model, an attempt has been made to consider the real-world conditions to obtain a more practical model in the industry. This model can also be beneficial because at the end of the time horizon, everything, including the inventory of the ships, the ports and the place where the vessels finish their journey, are equal to their initial values so that this cycle can work indefinitely. According to this model, ports' initial inventory, the initial stock of vessels, and the initial position of vessels are variables. There are also propound restrictions on returning the ship to its original location. Appendix 3 contains the cyclic model's AMPL code (.mod file).

Notation:

Sets:

V : set of vessels

N : set of production and consumption ports

S^A : set of possible nodes (i,m)

S_v^A : set of nodes that can be visited by vessel v

S_v^X : set of all possible moves (i,m,j,n) of vessel v

S_v^S : set of breakpoints for the speed of vessel v, with $S_v^S = \{1,2,...,U\}$

Parameters:

T : number of time units in the planning horizon

H_i : minimum number of visits to port $i \in N$

M_i : maximum number of visits to port $i \in N$

D_i : consumption or demand at port $i \in N$ per unit of time

J_i : 1 if production facilities are located in port i, and -1 if consumption facilities are located in port $i, i \in N$

P_{iv} : port cost at port $i \in N$ for vessel $v \in V$

C_v : capacity of vessel $v \in V$

$\underline{S}_i$: lower bound on the inventory level at port $i \in N$

$\overline{S}_i$: upper bound on the inventory level at port $i \in N$

A_{im} : earliest time for starting visit m to port i, $(i,m) \in S^A$

B_{im} : latest time for starting visit m to port i, $(i,m) \in S^A$

K_i : minimum time between two consecutive visits to port $i \in N$

$\underline{Q}_i$: minimum load/unloaded quantity in port $i \in N$

U_{im} : latest time for finishing visit m to port i, $(i,m) \in S^A$

T_v^Q : time for unloading or loading each unit by vessel $v \in V$

T_{ijvs}^{PP} : time required by vessel $v \in V$ to sail from port $i \in N$ to port $j \in N$ with speed $s \in S_v^S$

C_{ijvs}^{PP} : sailing cost from port $i \in N$ to port $j \in N$ by vessel $v \in V$ with speed $s \in S_v^S$

Variables:

w_{iv} : 1 if vessel v starts from port i, 0 otherwise

x_{imjnv} : 1 if vessel $v \in V$ moves directly between nodes (i,m) and (j,n), 0 otherwise, $v \in V$,

$(i,m,j,n) \in S_v^X$

x_{ijnv}^O : 1 if vessel v departs from its origin at port i to node (j,n), 0 otherwise, $v \in V$, $i \in N$,

$(j,n) \in S_v^A$

x_{jniv}^D : 1 if vessel v goes from node (j,n) and ending its route at port i, 0 otherwise, $v \in V$,

$i \in N, (j,n) \in S_v^A$

l_v: initial load onboard vessel $v \in V$

q_{imv}: the amount of product loaded or unloaded by vessel v at node (i,m), $v \in V$, $(i,m) \in S_v^A$

f_{imjnv}: the amount of product that vessel v transports from node (i,m) to node (j,n), $v \in V$, $(i,m,j,n) \in S_v^X$

f_{ijnv}^O: the amount of product that vessel v transports from its origin at port i to node (j,n), $v \in V$, $i \in N, (j,n) \in S_v^A$

f_{jniv}^D: the amount of product that vessel v transports from node (j,n) to its destination at port i, $v \in V$, $(i,m) \in S_v^A$

t_{im}: start time of visit number m to port i, $(i,m) \in S_v^A$

t_v^S: The amount of delay that may occur for each vessel $v \in V$

o_{imv}: 1 if vessel v operates in node (i,m), 0 otherwise, $v \in V$, $(i,m) \in S_v^A$

y_{im}: 1 if there is visit (i,m) to port i, 0 otherwise, $i \in N$, $(i,m) \in S^A$

s_{im}: stock level at ports at the start of visit m to port i, $(i,m) \in S^A$

s_i^O: the initial stock level in port $i \in N$ at the beginning of the planning horizon

g_{imjnvs}: auxiliary variable to determine the speed of vessel v when going from node (i,m) to (j,n), with s corresponding to a given choice of speed, $v \in V$, $s \in S_v^S$, $(i,m,j,n) \in S_v^X$

g_{ijnvs}^O: auxiliary variable to determine the speed of vessel v when going from its origin at port i to node (j,n), with s corresponding to a given choice of speed, $v \in V$, $s \in S_v^S$, $i \in N$,

$(j,n) \in S_v^A$

g_{jnivs}^{D} : auxiliary variable to determine the speed of vessel v when going from node (j,n) to destination at port i, with s corresponding to a given choice of speed, $v \in V$, $s \in S_v^S$, $i \in N$, $(j,n) \in S_v^A$

Mathematical model:

$$\min \sum_{v \in V} \sum_{(i,m,j,n) \in S_v^X} \sum_{s \in S_v^S} C_{ijvs}^{PP} g_{imjnvs} + \sum_{v \in V} \sum_{(j,n) \in S_v^A} \sum_{s \in S_v^S} \sum_{i \in N} C_{ijvs}^{PP} g_{ijnvs}^{O} + \tag{1}$$

$$\sum_{v \in V} \sum_{(j,n) \in S_v^A} \sum_{s \in S_v^S} \sum_{i \in N} C_{jivs}^{PP} g_{jnivs}^{D} + \sum_{v \in V} \sum_{(i,m) \in S_v^A} P_{iv} o_{imv}$$

Expresses the minimization of the sum of traveling costs between ports depending on the chosen speed and operational costs in each port.

$$\sum_{i \in N} w_{iv} \le 1 \qquad v \in V \tag{2}$$

Show that each vessel must have a maximum of one start position.

$$\sum_{(j,n) \in S^A} x_{jniv}^{D} = w_{iv} \qquad v \in V, i \in N \tag{3}$$

Illustrate each vessel should return to its start position.

$$\sum_{(j,n) \in S^A} x_{ijnv}^{O} = w_{iv} \qquad v \in V, i \in N \tag{4}$$

Illustrate each vessel should start from initial position.

$$o_{imv} - \sum_{(j,n):(i,m,j,n) \in S_v^X} x_{imjnv} - \sum_{j \in N} x_{imjv}^{D} = 0 \qquad v \in V, (i,m) \in S_v^A \tag{5}$$

Ensure that if a vessel is at node i it must either leave to another node or end its route there.

$$o_{imv} - \sum_{(j,n):(j,n,i,m) \in S_v^X} x_{jnimv} - \sum_{j \in N} x_{jimv}^{O} = 0 \qquad v \in V, (i,m) \in S_v^A \tag{6}$$

Define that if a node is visited by vessel v, the vessel must either arrive at the node from the origin or from another node.

$$\sum_{v \in V} o_{imv} = y_{im} \qquad (i,m) \in S^A \qquad (7)$$

show that a vessel can only visit node (i,m) if there are at least m visits to port i.

$$y_{i(m-1)} - y_{im} \geq 0 \qquad (i,m) \in S^A : H_i + 1 \leq m \leq M_i \qquad (8)$$

Guarantee that if a port i is visited m times, then it also has been visited m-1 times.

$$y_{im} = 1 \qquad (i,m) \in S^A : m \in \{1, ..., H_i\} \qquad (9)$$

Defines the number of mandatory visits for port i.

$$\sum_{s \in S_v^S} g_{imjnvs} = x_{imjnv} \qquad v \in V, (i,m,j,n) \in S_v^X \qquad (10)$$

Enforce that speed of a vessel must be set for a travel from node (i,m) to node (j,n) if and only if that travel exists.

$$\sum_{s \in S_v^S} g_{ijnvs}^O = x_{ijnv}^O \qquad v \in V, (j,n) \in S_v^A, i \in N \qquad (11)$$

Enforce that speed of a vessel must be set for a travel from the origin to node (j,n) if and only if that travel exists.

$$\sum_{s \in S_v^S} g_{jnivs}^D = x_{jniv}^D \qquad v \in V, (j,n) \in S_v^A, i \in N \qquad (12)$$

Enforce that speed of a vessel must be set for a travel from node (j,n) to the destination if and only if that travel exists.

$$q_{imv} \leq \min\{C_v, \overline{S}_i\} o_{imv} \qquad v \in V, (i,m) \in S_v^A \qquad (13)$$

Ensure the quantity loaded/unloaded cannot exceed the vessel capacity nor the maximum port capacity.

$$\underline{Q}_i o_{imv} \leq q_{imv} \qquad v \in V, (i,m) \in S_v^A \qquad (14)$$

Show that if a vessel visits the port, then the amount loaded/unloaded should be at least equal to the minimum quantity.

$$f_{ijnv}^O \geq l_v - C_v(1 - x_{ijnv}^O) \qquad v \in V, (j,n) \in S_v^A, i \in N \qquad (15)$$

Determine flow from the initial position which is equal to initial load if ship travels from the initial position.

$$f_{ijnv}^{O} \le l_v + C_v(1 - x_{ijnv}^{O}) \qquad v \in V, (j,n) \in S_v^A, \; i \in N \qquad (16)$$

Determine flow from the initial position which is equal to initial load if ship travels from the initial position.

$$\sum_{i \in N} f_{ijnv}^{O} + \sum_{(i,m):(i,m,j,n) \in S_v^X} f_{imjnv} + J_j q_{jnv} = \sum_{(i,m):(j,n,i,m) \in S_v^X} f_{jnimv} + \sum_{i \in N} f_{jniv}^{D} \qquad v \in V, (j,n) \in S_v^A \qquad (17)$$

It is flow conservation constraints which sum of incoming flow from origin, flow from a particular port and the amount of loaded/unloaded should be equal to outgoing flow from port plus outgoing flow to the destination.

$$f_{ijnv}^{O} \le C_v x_{ijnv}^{O} \qquad v \in V, (j,n) \in S_v^A, \; i \in N \qquad (18)$$

Determine flow from the initial position should be less or equal to the capacity of vessel; Also, may not move any products.

$$f_{imjnv} \le C_v x_{imjnv} \qquad v \in V, (i,m) \in S_v^A, \; (j,n) \in S_v^A \qquad (19)$$

Convey flow from a port to another port should not be more than the capacity of the vessel.

$$f_{jniv}^{D} \le C_v x_{jniv}^{D} \qquad v \in V, (j,n) \in S_v^A, \; i \in N \qquad (20)$$

Show flow to the destination is less or equal ship capacity if ship travels to the destination.

$$l_v = \sum_{(i,m) \in S^A} \sum_{j \in N} f_{imjv}^{D} \qquad v \in V \qquad (21)$$

Show vessels load at the end of planning horizon should be equal to the initial inventory level.

$$t_{im} - t_{i(m-1)} - \sum_{v \in V} T_v^Q q_{i(m-1)v} - K_i y_{im} \ge 0 \qquad (i,m) \in S^A : m > 1 \qquad (22)$$

Enforce the minimum time period between two consecutive visits of port i.

$$t_{im} + \sum_{v \in V} T_v^Q q_{imv} - t_{jn} + \sum_{v \in V, s \in S_v^S} \max\{U_{im} + T_{ijvs}^{PP} - A_{jn}, 0\} g_{imjnvs} \le U_{im} - A_{jn} \qquad (i,m) \in S^A, \; (j,n) \in S^A \qquad (23)$$

Relate the start time associated with node (i, m) to the start time associated with node (j, n) when a vessel travels between ports i and j.

$$t_v^S + \sum_{s \in S_v^S} \sum_{j \in N} T_{jivs}^{PP} g_{jimvs}^O \leq t_{im} + T(1 - o_{imv}) \qquad (i,m) \in S^A, \; v \in V \qquad (24)$$

Show that the travel time for a vessel traveling from origin plus delay time should not exceed the start time of the visit to the port.

$$t_{jn} + T_v^Q q_{jnv} + \sum_{i \in N} \sum_{s \in S_v^S} T_{jivs}^{PP} g_{jnivs}^D \leq T + t_v^S + T(1 - o_{jnv}) \qquad v \in V, (j,n) \in S^A \qquad (25)$$

Express time at node (j,n) plus travel time for a vessel traveling from node (j,n) to destination at node i plus operation time should not exceed the Time horizon plus delay time.

$$t_{im} \geq A_{im} \qquad (i,m) \in S^A \qquad (26)$$

Express time windows for the start time of visits.

$$t_{im} \leq B_{im} \qquad (i,m) \in S^A \qquad (27)$$

Exhibit time windows for the end time of visits.

$$s_{i1} = s_i^O + J_i D_i t_{i1} \qquad i \in N \qquad (28)$$

Set the stock level at the start time of the first visit to a port.

$$s_{im} = s_{i(m-1)} - J_i \sum_{v \in V} q_{i(m-1)v} + J_i D_i (t_{im} - t_{i(m-1)})) \qquad (i,m) \in S^A : m > 1 \qquad (29)$$

Show that the stock level at the start of the m^{th} visit is set by to the stock level at the start of the previous visit, the load or unload operation in the previous visit and the time elapsed between the two visits.

$$s_{im} + \sum_{v \in V} q_{imv} - D_i \sum_{v \in V} T_v^Q q_{imv} \leq \overline{S}_i \qquad (i,m) \in S^A : J_i = -1 \qquad (30)$$

Impose limitation of the stock level at the end time of a visit to a consumption port.

$$s_{im} - \sum_{v \in V} q_{imv} + D_i \sum_{v \in V} T_v^Q q_{imv} \geq \underline{S}_i \qquad (i,m) \in S^A : J_i = +1 \qquad (31)$$

Impose limitation of the stock level at the end time of a visit to a production port.

$$s_{iM_i} + \sum_{v \in V} q_{iM_i v} - D_i (T - t_{\mathrm{iM}_i}) \geq \underline{S}_i \qquad i \in N : J_i = -1 \qquad (32)$$

Show lower bound on the inventory level until the end of the time horizon for consumption ports.

$$s_{iM_i} - \sum_{v \in V} q_{iM_i v} + D_i (T - t_{\mathrm{iM}_i}) \le \overline{S}_i \qquad i \in N : J_i = +1 \qquad (33)$$

Exhibit upper bound on the inventory level until the end of the time horizon for production ports.

$$s_{im} \ge \underline{S}_i \qquad (i,m) \in S^A : J_i = -1 \qquad (34)$$

Illustrate lower bound on the stock level at the start of each visit for consumption ports.

$$s_{im} \le \overline{S}_i \qquad (i,m) \in S^A : J_i = +1 \qquad (35)$$

Show upper bound on the stock level at the start of each visit for production ports.

$$\sum_{v \in V} \sum_{m \in 1..M_i} q_{imv} = TD_i \qquad i \in N \qquad (36)$$

Determine inventory level at the end of planning horizon should be equal to the initial inventory level for all ports.

The rest of the constraints enforce the binary or non-negative nature of the variables.

$$x_{imjnv} \in \{0,1\} \qquad \mathrm{v} \in V, \ (i,m,j,n) \in S_v^X \qquad (37)$$

$$x_{ijnv}^O \in \{0,1\} \qquad v \in V, (j,n) \in S_v^A, \ i \in N \qquad (38)$$

$$x_{jniv}^D \in \{0,1\} \qquad v \in V, (j,n) \in S_v^A, \ i \in N \qquad (39)$$

$$w_{iv} \in \{0,1\} \qquad \mathrm{v} \in V, i \in N \qquad (40)$$

$$o_{imv} \in \{0,1\} \qquad \mathrm{v} \in V, (i,m) \in S_v^A \qquad (41)$$

$$y_{im} \in \{0,1\} \qquad (i,m) \in S^A \qquad (42)$$

$$q_{imv} \ge 0 \qquad \mathrm{v} \in V, (i,m) \in S_v^A \qquad (43)$$

$$f_{ijnv}^O \ge 0 \qquad v \in V, (j,n) \in S_v^A, \ i \in N \qquad (44)$$

$$f_{imjnv} \geq 0 \qquad v \in V, \ (i,m,j,n) \in S_v^X \tag{45}$$

$$f_{jniv}^{D} \geq 0 \qquad v \in V, (j,n) \in S_v^A, \ i \in N \tag{46}$$

$$0 \leq g_{imvs}^{O} \leq 1 \qquad v \in V, (i,m) \in S_v^A, \ s \in S_v^s \tag{47}$$

$$0 \leq g_{imjnvs} \leq 1 \qquad v \in V, \ (i,m,j,n) \in S_v^X, \ s \in S_v^s \tag{48}$$

$$l_v \geq 0 \qquad v \in V \tag{49}$$

$$s_i^O \geq 0 \qquad i \in N \tag{50}$$

$$s_{im} \geq 0 \qquad (i,m) \in S^A \tag{51}$$

$$t_{im} \geq 0 \qquad (i,m) \in S^A \tag{52}$$

$$t_v^S \geq 0 \qquad v \in V \tag{53}$$

5.0 Computational study

This section examines the test instances and presents the computational results and interpretation. The instances implemented to test the proposed model have been collected from two different data sources. And the computational tests were run on a computer with a 3.20 GHz Intel i7-8700 CPU, 16GB of RAM, and under the Microsoft Windows 10 Enterprise 64-bit version. AMPL language is used to code the models, and they are run in CPLEX 20.1.0.0.

5.1 Test instances explanation of data group one

The first group of instances used to test the model in the computational study are based on the six artificial data of A, B, C, D, F and G taken from Agra et al. (2016a). It should be mentioned that the original database includes seven instances. As regards instance E and instance D are identical, the deviation is in the inventory level; in the computational study, we just have used one of them, instance D.

Table 2 reveals the detailed data of each instance in terms of the number of ports, number of vessels and length of the planning horizon. Each instance is divided into four subcategories based on the planning time horizon, 30 days or 60 days, and based on the problem, base, or cyclic model. So, the instances' index includes:

- original representation in the resource.
- the number of ports.
- the number of vessels.
- the length of the planning horizon (day).
- problem model (base model/cyclic model).

There is an extra alphabet character in the last position of the instance index in the evaluation texts in which B stands for the base model and C stands for the cyclic model. And the route schematic of each instance in the base and cyclic models are shown in Appendix 1.

Table 2: Test instances for computational study

Source: Made by authors

Instance index	Number of ports	Number of vessels	Time horizon (day)
A-4-1-30	4	1	30
A-4-1-60	4	1	60
B-3-2-30	3	2	30
B-3-2-60	3	2	60
C-4-2-30	4	2	30
C-4-2-60	4	2	60
D-5-2-30	5	2	30
D-5-2-60	5	2	60
F-4-3-30	4	3	30
F-4-3-60	4	3	60
G-6-5-30	6	5	30
G-6-5-60	6	5	60

Moreover, each vessel used in the instances introduced by its operational characteristics in terms of:

- Vessel capacity (ton).
- Initial load level (ton).
- Operational load rate (ton/day).
- Possible sailing speeds (knots/hour).
- Operational cost at each port (1000 USD).
- Daily sailing costs related to each speed alternative (1000 USD/day).

Table 3 exhibits the vessels' operational speeds, which vary between 13.5 and 21 knots. Again, it is noteworthy that there are two variants of the base model (B) and cyclic model (C) for each instance; for example, instance A-4-1-3 is divided into two subcategories of A-4-1-3-B and A-4-1-3-C.

As is depicted from the Table 3, different vessel types with their unique functional character are used in instances; for example, in the instance A-4-1-30-B, one vessel is used, but in the instance G-6-5-60-C, five ships are hired. The operational rates for vessels' speed extracted from the Evsikova's study (Evsikova, 2017).

Table 3: Speed alternatives for vessels in the instances

Source: Made by authors

Instance index	**Speed alternatives (knots)**		
	13.5-15-19	**14.4-16-20**	**16.2-18-21**
A-4-1-30			Vessel 1
A-4-1-60			Vessel 1
B-3-2-30	Vessel 1&2		
B-3-2-60	Vessel 1&2		
C-4-2-30	Vessel 1&2		
C-4-2-60	Vessel 1&2		
D-5-2-30	Vessel 1&2		
D-5-2-60	Vessel 1&2		
F-4-3-30	Vessel 1&2&3		
F-4-3-60	Vessel 1&2&3		
G-6-5-30	Vessel 1	Vessels 2&4	Vessels 3&5
G-6-5-60	Vessel 1	Vessels 2&4	Vessels 3&5

The maximum operational capacity of each ship reveals in Table 4. The vessels' capacity varies between 100 (1000 DWT), small and 160 (1000 DWT), large. DWT stands for the deadweight tonnage and defines the total capacity of a vessel for contents, say, cargo, passenger, and food.

Table 4: Operational capacity for vessels in the instances

Source: Made by authors

Instance index	**Vessels' capacity (1000 DWT)**					
	100	**120**	**130**	**140**	**150**	**160**
A-4-1-30-B						Vessel 1
A-4-1-30-C						Vessel 1
A-4-1-60-B						Vessel 1
A-4-1-60-C						Vessel 1
B-3-2-30-B	Vessel 1		Vessel 2			
B-3-2-30-C	Vessel 1		Vessel 2			
B-3-2-60-B	Vessel 1		Vessel 2			
B-3-2-60-C	Vessel 1		Vessel 2			
C-4-2-30-B			Vessel 2			Vessel 1
C-4-2-30-C			Vessel 2			Vessel 1
C-4-2-60-B			Vessel 2			Vessel 1
C-4-2-60-C			Vessel 2			Vessel 1
D-5-2-30-B	Vessel 2				Vessel 1	
D-5-2-30-C	Vessel 2				Vessel 1	
D-5-2-60-B	Vessel 2				Vessel 1	
D-5-2-60-C	Vessel 2				Vessel 1	
F-4-3-30-B			Vessel 2		Vessel 3	Vessel 1
F-4-3-30-C			Vessel 2		Vessel 3	Vessel 1
F-4-3-60-B			Vessel 2		Vessel 3	Vessel 1
F-4-3-60-C			Vessel 2		Vessel 3	Vessel 1
G-6-5-30-B			Vessel 2	Vessels 4	Vessels 3&5	Vessel 1
G-6-5-30-C			Vessel 2	Vessels 4	Vessels 3&5	Vessel 1
G-6-5-60-B			Vessel 2	Vessels 4	Vessels 3&5	Vessel 1
G-6-5-60-C			Vessel 2	Vessels 4	Vessels 3&5	Vessel 1

5.2 Assessment of computational results of the first data group

5.2.1 Computational time

This section provides comparisons of computational time for two models. Table 5 demonstrates the computational time in second for two models on the different instances. The base model always shows lower computational time compared to the cyclic model. This is due to the problem's additional constraints; the cyclic model has some restrictions about the stock level of the ports and the vessels at the end of the time planning horizon. Moreover, another restriction imposes the condition in which the vessels must come back to the initial location at the end of the route.

As can be seen in the Table 5, as the number of ports, the number of vessels and the time of the planning horizon increases, the problem's difficulty rises, so the computation time increases. The real computational time is the same for instances C, D, F, and G in a 60-day planning horizon in the cyclic model because we have defined a time limit of two hours for solving the problem. If we do not consider the time limit for solving the problem, we expect that the instance G at the 60-day planning horizon will need the most time to solve the problem.

Table 5: Real solving time of instances

Source: Made by authors

Instance index	Real solving time (s) (Base)	Real solving time (s) (Cyclic)
A-4-1-30	0.13	0.56
A-4-1-60	0.45	0.48
B-3-2-30	0.36	2132.95
B-3-2-60	-	0
C-4-2-30	0.34	595.92
C-4-2-60	1.58	7200
D-5-2-30	1.09	7200
D-5-2-60	23.84	7200
F-4-3-30	3.63	1597.56

F-4-3-60	2.91	7200
G-6-5-30	81.23	7200
G-6-5-60	-	7200

5.2.2 Cost comparison between the two models

This section compares the total cost between the two models, which is the sum of the travelling cost and operating cost. It is better to mention that the operating cost is only related to the operation in the ports, and its cost does not depend on the number of goods delivered or picked up at the ports. The costs are based on 1000 US Dollars. Obviously, the cost comparison should be formed based on the optimal solution for economic analysis. However, in this section, the total costs are extracted from the best solution in a two-hour predefined solving time.

Table 6 shows the total cost, the cost deviation, and the percentage of cost deviation for all instances. The cost deviation refers to the differences in cost for the cyclic model compared to the base model due to more conditions in its model, such as equality of port inventory at the beginning and end of the schedule and the return of ships to their initial position.

Table 6: Total cost comparison of the instances

Source: Made by authors

Instance index	Total cost (1000 USD) (Base)	Total cost (1000 USD) (Cyclic)	Cost deviation (1000 USD)	Cost deviation (%)
A-4-1-30	74.87	230.60	155.73	67.54
A-4-1-60	184.93	366.56	181.63	49.55
B-3-2-30	158.53	342.15	183.62	53.66
B-3-2-60	-	integer infeasible	-	-
C-4-2-30	75.06	347.11	272.05	78.37
C-4-2-60	299.45	521.50	222.04	42.57
D-5-2-30	214.60	555.56	340.96	61.37
D-5-2-60	458.33	783.91	325.58	41.53
F-4-3-30	156.88	315.75	158.88	50.32

F-4-3-60	323.15	514.80	191.64	37.22
G-6-5-30	411.60	605.23	193.63	31.99
G-6-5-60	-	no integer solution	-	-

Table 6 also reveals that instance B-3-2-60-C is integer infeasible because the solver cannot solve instance G-6-5-60-C within the time limit. Chapter 4 described that since the values of initial stock at ports, initial ships' load and locations are parameters in the base model, we need input for those parameters from the solved cyclic model. Therefore, since the cyclic model in both B-3-2-60 and G-6-5-60 remains unsolved, there is no input for such parameters in the basic model, so the basic models cannot be solved.

Reasons that caused instance B-3-2-60-C to become integer infeasible, and its solution approach is discussed in detail in section 5.2.3. And concerning instance G-6-5-60-C, the reason for getting no integer solution is that the problem assumptions and inputs cannot meet all constraints to get the answer; thus, changing the parameters is inevitable.

Table 7 exhibits the average total cost for the two models and demonstrates the average total cost deviation, the percentage of average total cost deviation, and the maximum and minimum cost deviation. As shown in Table 7, the average total cost deviation percentage is 48.56%, the highest cost deviation is 78.37%, and the lowest cost deviation is 31.99%, which is very high. It should be spent about 30% more than the base model, even with the least cost deviation.

Table 7: Average total cost for the base and the cyclic models

Source: Made by authors

Model	**Average total cost (1000 USD)**	**Average total cost deviation (1000 USD)**	**Average total cost deviation (%)**	**Max cost deviation (%)**	**Min cost deviation (%)**
Base Model	235.74	222.57	48.56	78.37	31.99
Cyclic Model	458.32				

At first glance, it seems that the base model encompasses the higher economic justification. But this is just one dimension of the issue and is meaningful when a single period of goods distribution is scheduled. There is another story when companies are to have a program that is always run sequentially because some companies, such as Oil companies and petrochemical companies, cannot stop their production. In section 5.2.3, we did the tests that prove how non-cyclic solutions leave the company in an untenable position.

Also, Table 8 illustrates the average total cost in 30- and 60-days planning horizon for both the base and the cyclic models. As exhibited in Table 8, If we implement our base model for 60 days, instead of performing the model twice for 30 days, it will save approximately 14.97% in cost. But the point to consider in the base model is that we cannot apply the model twice.

Table 8: Comparing the average total cost between 30 and 60 days for the base and the cyclic models

Source: Made by authors

Model	**Average total cost 30 days (1000 USD)**	**Average total cost 60 days (1000 USD)**
Base Model	181.92	316.46
Cyclic Model	399.40	546.69

In the cyclic model, a 30-day timetable can be repeated twice, and it can be compared with a 60-day model because all conditions are the same. We will save approximately 46.11 % of the cost if we run our cyclic model for a single period of 60 days instead of repeating the 30 days model two times. Moreover, Table 9 compares the percentage of total cost deviation in 30 and 60 days.

Table 9: Comparing the average cost deviation between 30 and 60 days for the base and the cyclic models

Source: Made by authors

Average cost deviation 30 days (%)	**Average cost deviation 60 days (%)**
57.20	42.72

The above table shows that the average cost deviation in a 30-day schedule is more than a 60-day planning horizon. In case of stability in demand and the need for continuity, there is only one option on the table in companies, the cyclic model. Changing the base model to the cyclic model in a 60-day planning horizon compared to a 30-day timetable imposes lower transportation costs.

5.2.3 Feasibility test of the base model for repeatable schedules

The following experiment examined the base model's performance for repeatable schedules. First, the instances solved for the base model, then the last stock inventory level at ports and vessels and the ships' locations recorded. Then, we solved the base model again, but this time starting from the state that is left at the end of the planning period when solving the base model for the first time. Table 10 shows the tests' results.

Table 10: Comparison of the first and second routes for the base model

Source: Made by authors

Instance index	First route total cost (1000 USD)	Second route total cost (1000 USD)	Added cost (1000 USD)	Cost Increase (%)
A-4-1-30	74.87	integer infeasible	-	-
A-4-1-60	184.93	integer infeasible	-	-
B-3-2-30	158.53	integer infeasible	-	-
B-3-2-60	-	-	-	-
C-4-2-30	75.06	integer infeasible	-	-
C-4-2-60	299.45	integer infeasible		
D-5-2-30	214.60	integer infeasible	-	-
D-5-2-60	458.33	integer infeasible	-	-
F-4-3-30	156.88	integer infeasible	-	-
F-4-3-60	323.15	integer infeasible	-	-
G-6-5-30	411.60	integer infeasible	-	-
G-6-5-60	-	-	-	-

Results in Table 10 don't present a situation where a feasible solution can be found, and none of the instances can continue their cycle for the second round because of the end of the horizon effect, and vessels' last locations which are mostly different from the position where they started the voyage. The end of horizon effect in the base model makes the inventory level of almost all consumption ports and ships zero or near to minimum inventory level at the end of the time horizon. Also, it makes the inventory level in the production ports full or near the upper bound at the end of the first program.

5.2.4 Evaluation of the role of maximum visit number in the solution

As depicted in Table 6 in section 5.2.2, the AMPL software output for the cyclic model of the instance B-3-2-60 was integer infeasible, meaning that the solver could not find any integer values for problem variables that could meet all constraints.

As explained in chapter 4, the values of the initial stock of ports, the initial load of vessels and the initial position of vessels are defined as a parameter in the base model and as a variable in the cyclic model. Therefore, the outputs from the cyclic model are used as inputs for the base model. And since the problem remained unsolved in the cyclic model of the instance B-3-2-60, there is no input to solve the base model for this instance. Checking the data file reveals that the problem has arisen from the upper bound in the number of visits.

The maximum number of visits for each port was assigned 4; while this number can satisfy the model for a planning horizon of 30 days, it is not enough for a 60-day planning horizon to fulfil the demands. Thus, we start to increase the interval for visit numbers, increasing one by one, until finding a feasible integer solution. Furthermore, in order to evaluate the role of maximum visit numbers on the solution, the problem is solved for the maximum visit numbers of 5, 6, 7 and 8 for each port. Table 11 illustrates the result after changing the maximum visit number.

Table 11: Test outputs for the cyclic model of instance B-3-2-60-C

Source: Made by authors

Maximum visit number	Real Solving time (Base)	Real Solving time (Cyclic)	Total Cost (Base)	Total Cost (Cyclic)	Gap (Base)	Gap (Cyclic)
4	-	0	-	integer infeasible	-	-
5	12.72	7200	345.31	632.14	0.00	0.21
6	8.58	7200	356.76	568.11	0.00	0.14
7	5.86	7200	294.21	568.11	0.00	0.14
8	6.63	7200	346.37	568.11	0.00	0.14

The first solution achieved for maximum visit number of 5, containing the total costs of 345.31 and 632.14 for the base model and cyclic model, respectively. The outputs' appraisal exhibits that while the total cost for the cyclic model remained constant for the maximum visit numbers of 6 and higher, that amount follows a fluctuating trend for the base model. Figure 4 shows the changes in the total cost for two models.

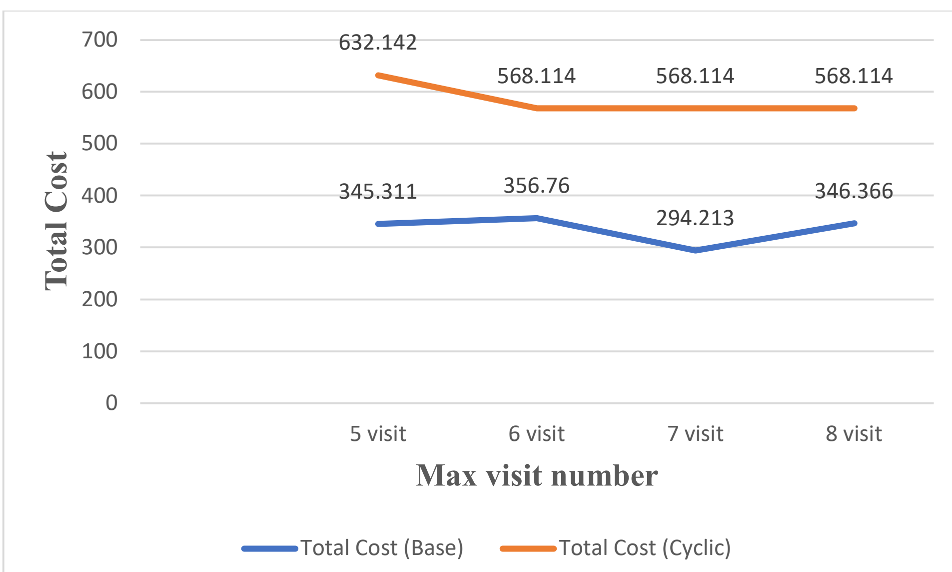

Figure 4: Changes in the total cost for two models, Source: Made by authors

Considering the total cost for both base and cyclic models and concerning the amount of the gap, the best output could be extracted when the value of the maximum visit number for all ports is equal to 7. Then values of the total cost, 294.21 and 568.11, and the amount of gap, 0 and 0.14, are for the base model and cyclic model, respectively.

5.2.5 Structural analysis of the solution

As shown in Table 12 in the cyclic model, the total distance traveled by ships in the 60-day program is less than two times the 30-day program. Therefore, the cost of a 60-day program is less than running a 30-day program twice. Another point that can be mentioned in the analysis of the results is the number of visits made by the vessels. As can be seen in the results, in all instances, in the cyclic model, the number of visits made in 60 days is less than two times the 30-day program. As mentioned in Section 5.2.2, the total cost in the 60-day cyclic model is less than the total cost in the twice 30-day model; this is due to the lees travelled distance proposed by the 60-day cyclic.

As was expected, the total number of visits and the total travelled distance in the cyclic model is higher than the base model in all instances because of the defined additional constraints in the cyclic model to make it repeatable for unlimited time.

Table 12: Comparison of total travelled distance and total visit numbers between two models

Source: Made by authors

Instance index	Base model		Cyclic model	
	Total travelled distance (nautical miles)	Total visit numbers	Total travelled distance (nautical miles)	Total visits numbers
A-4-1-30	2332	3	8161	4
A-4-1-60	6218	5	12825	7
B-3-2-30	7774	4	16325	7
B-3-2-60	-	-	-	-
C-4-2-30	2332	4	12825	7
C-4-2-60	8161	7	20986	12
D-5-2-30	7773	6	19432	8

D-5-2-60	19043	10	33034	14
F-4-3-30	6218	4	12825	7
F-4-3-60	12436	10	20986	12
G-6-5-30	17491	8	26431	10
G-6-5-60	-	-	-	-

5.2.6 Evaluation of vessels' speed in different instances

This section investigates the speeds of ships in routes for both the base model and the cyclic model within the 30-day planning horizon. Table 13 exhibits the speed alternatives used by vessels during the voyage in different instances. In addition, numbers assigned to each speed alternative in each cell express the number of routes travelled with that speed by all vessels. The calculated speed average during the voyage for each instance in each model is shown in the respective column. And the last column in the tables reveals the last day on which both voyage and loading or unloading operations are made, the end of the delivery mission.

As is perceived from Table 13, except for instance A, in which the vessels' average speed in the base model is negligibility more than the cyclic model, 0.22 higher, in other instances, the ships' average speed in the cyclic model is higher than the base model or equal to the base model, instance F. The vessels use a higher speed during the voyage in the cyclic model because, as proved in section 5.2.5, in the cyclic model, ships should make more trips and visit than in the base model in the same time horizon. Therefore, increasing the speed for fulfilling the task is inevitable.

As regards higher speed imposing higher voyage cost, to minimize the total cost solver considers the minimum speed for the vessels during the trip; Table 13 can certify such a condition. Moreover, the last column illustrates that when ships can finish their voyage sooner than the last day on the planning horizon, they mostly travel at the minimum speed. But when voyage time last until the last days on the planning horizon, vessels increase their speed to finish their task. For example, in the instance D-5-2-30-B, ships just in the 0.90 of the routes move with of 19 knots, but in the instance D-5-2-30-C, the number of ways in which vessels' speed was 19 knots increased to 3.27 because the delivery task in the first and second instances conducted by 14.80 days and 28.97 days respectively.

Table 13: Speed alternatives for vessels in the instances

Source: Made by authors

Instance index	**Speed alternatives (knots)**						**Average speed (knots)**	**Last Operation Time (day)**
	13.50	**14.40**	**15.00**	**16.20**	**18.00**	**19.00**		
A-4-1-30-B	-	-	-	1.75	0.25	-	16.42	7.19
A-4-1-30-C	-	-	-	4.00	0.00	-	16.20	25.49
B-3-2-30-B	2.00	-	0.72	-	-	0.28	14.37	19.55
B-3-2-30-C	3.00	-	3.00	-	-	1.00	14.93	30.00
C-4-2-30-B	2.00	-	0.00	-	-	0.00	13.50	12.53
C-4-2-30-C	2.00	-	4.64	-	-	0.36	14.77	30.00
D-5-2-30-B	3.00	-	0.10	-	-	0.90	14.40	14.80
D-5-2-30-C	4.00	-	0.73	-	-	3.27	15.88	28.97
F-4-3-30-B	4.00	-	-	-	-	-	13.50	15.99
F-4-3-30-C	7.00	-	-	-	-	-	13.50	30.00
G-6-5-30-B	2.00	2.00	-	3.00	-	-	14.91	26.76
G-6-5-30-C	-	4.00	2.56	2		0.44	15.19	28.44

5.2.7 Computational Gap

The computational gap in cost for both the base model and the cyclic model is evaluated in this section. The formulation of the computational gap in the CPLEX solver defines as the deviation between the upper bound and the lower bound divided by the lower bounds ((UB-LB)/LB) in the specified solving time. Table 14 exhibits the computational gap for all instances.

Table 14: Computational gap for all instances

Source: Made by authors

Instance index	**Gap (Base)**	**Gap (Cyclic)**
A-4-1-30	0.00	0.00
A-4-1-60	0.00	0.00
B-3-2-30	0.00	0.00
B-3-2-60	-	-

C-4-2-30	0.00	0.00
C-4-2-60	0.00	0.00
D-5-2-30	0.00	0.30
D-5-2-60	0.00	0.20
F-4-3-30	0.00	0.00
F-4-3-60	0.00	0.09
G-6-5-30	0.00	0.46
G-6-5-60	-	-

Moreover, Table 15 reveals a comparison of the computational gap between instances. Results show for the base model, the average gap in the 30-day planning horizon is equal to the 60-day planning horizon because all instances were solved by the two hours defined solving time, so the amount of gap was zero. The cyclic model's outputs exhibit that the average gap for a 30 days schedule is more than a 60-days timetable because the maximum gap, which is seen in the G-6-5-30, significantly affects the average gap.

Furthermore, the average gap for the cyclic model was higher than the base model for both 30 days and 60 days planning horizons. The main reason for such a result is that the optimal result is achieved by the defined solving time for all instances in the base model. But in the cyclic model, for some instances, the calculated output is the best feedback in the defined solution interval, which is far from the optimal solution. Increasing the predefined solution time in the programming software could improve the outputs by decreasing the gap.

Table 15: Comparison of computational gap

Source: Made by authors

Gap	**Planning horizon**			
	30 days		**60 days**	
	Base Model	**Cyclic Model**	**Base Model**	**Cyclic Model**
Average gap	0.00	0.12	0.00	0.07
Maximum gap	0.00	0.46	0.00	0.20
Minimum gap	0.00	0.00	0.00	0.09

5.3 Test instances explanation of data group two

The cases utilized to test the model in this section comprised five seaports of Savannah, Hamburg, Shanghai, Santos and San Francisco are based on the actual dataset (Hemmati, Hvattum, Fagerholt, & Norstad, 2014). It should be stated that the dataset is real in terms of distances from port to port, the capacity of vessels, and the speed of the ships. The ports are located on the different continents of Asia, Europe, and North and South America, as is shown in Figure 5. Moreover, Table 16 illustrates the actual distances between the seaports.

Figure 5: The location of ports, Source: Made by authors

Table 16: Distances (miles) between the seaports

Source: Made by authors

	Hamburg	**Santos**	**Shanghai**	**Savannah**	**San Francisco**
Hamburg	0	5880	11662	4200	9128
Santos	5880	0	11312	5054	8456
Shanghai	11662	11312	0	11004	5586
Savannah	4200	5054	11004	0	5488
San Francisco	9128	8456	5586	5488	0

All instances comprised of homogeneous vessel type of Post Panamax Container. The vessel has the operational character of 81200-ton capacity, and the sailing speed varies between 23 and 28 miles per hour. In this research, the travelling speed of ships in all instances is assumed to be 26 miles per hour. Furthermore, all instances have only been evaluated for the cyclic model. In the next section the impacts of modifying the number of production ports, consumption ports, and vessels on outputs for the cyclic model will be investigated.

5.4 Assessment of computational results of the second data group

5.4.1 Impacts of modifying the number of production ports, consumption ports, and vessels on outputs

Table 17 provides the detailed data of each instance in terms of the number of ports, production ports, consumption ports, and vessels. As a result, the instances' index includes:

- representation of real data.
- the number of ports.
- the number of production ports.
- the number of consumption ports.
- the number of vessels.

Table 17: Test instances for computational study

Source: Made by authors

Instance index	Number of ports	Number of production ports	Number of consumption ports	Number of vessels
R-5-1-4-5	5	1	4	5
R-5-1-4-4	5	1	4	4
R-5-1-4-3	5	1	4	3
R-5-2-3-5	5	2	3	5
R-5-2-3-4	5	2	3	4
R-5-2-3-3	5	2	3	3
R-5-3-2-5	5	3	2	5

R-5-3-2-4	5	3	2	4
R-5-3-2-3	5	3	2	3

Table 18 illustrates the test outputs. In all tested instances, the total number of ports is five, and the difference between the samples is in the number of production ports, consumption ports and ships. As shown in Table 18, in instances R-5-1-4-5, R-5-1-4-4 and R-5-1-4-3, when the number of production ports is one, and the number of consumption ports is 4, the total cost of the voyage for 5, 4 and 3 ships are equal. But the gap, for instance R-5-1-4-3 which has the minimum number of vessels in a limited run time of 7200 seconds, is less than other instances. Therefore, implementing three ships will be a better option when there is one production port and four consumption ports in the assigned solving time. Appendix 2 exhibits the schematic of routes for instances R-5-1-4-5, R-5-1-4-4 and R-5-1-4-3.

Table 18: Total cost and real solving time for instances

Source: Made by authors

Instance index	**Total Cost (USD)**	**Real Solving time (s)**	**Gap**
R-5-1-4-5	19327200	7200	0.30
R-5-1-4-4	19327200	7200	0.25
R-5-1-4-3	19327200	7200	0.09
R-5-2-3-5	15540700	7200	0.38
R-5-2-3-4	15540700	7200	0.28
R-5-2-3-3	15540700	7200	0.09
R-5-3-2-5	12405000	7200	0.35
R-5-3-2-4	12405000	7200	0.22
R-5-3-2-3	12405000	7200	0.06

Outputs show a similar trend for the rest six instances, meaning that although increasing the number of vessels from 3 to 5 does not affect the total cost, it increases the gap. Expressing that when there is a feasible solution for different scenarios of a different number of vessels in a problem, the total cost of the best solution in the specified solving time is independent of the number of ships, and it is equal for all scenarios. Of course, this point should not be forgotten that adding each ship will also impose additional costs that are not considered in this model.

Therefore, the solver feedback exhibits that the best results in the assigned solving time are accessible when the number of vessels is minimum, instance R-5-1-4-3, R-5-2-3-3 and R-5-3-2-3, due to the minimum gap. Moreover, comparing the mentioned three instances reveals that increasing the number of production ports from one in the instance R-5-1-4-3 to three in instance R-5-3-2-3 will significantly decrease the total cost. The main reason is that increasing the number of suppliers and decreasing the number of consumers and orders can reduce the complexity of the problem to meet customer satisfaction constraints. And the total cost is not independent of the number of production and consumption ports.

6.0 Conclusion

Maritime transportation is one of the vital arteries of global trade and economy. Shipping costs play a decisive role in the price of the final product. Thus, to benefit from the merits of the competitive price, companies seek approaches that result in cost reduction in transportation and distribution sectors in maritime transportation. In addition to effective price, continuity and non-delay in the supply of goods are essential criteria of the global supply chain because disruption in the supply of commodities causes enormous and irreparable damage to some companies and industries. Thus, proposing an approach which guarantees the repeatable cycle of material flow between suppliers and consumers in a competitive cost in maritime logistics could warmly be welcomed by players.

The previous studies on the maritime inventory routing problem are affected by the end of horizon effect, which is a barrier to repeating the voyage for an unlimited time in the assigned planning horizons. Therefore, they cannot be considered a cyclic maritime inventory routing problem. This research aims to present a cyclic approach to the maritime inventory routing problem.

This thesis examines two models, both of which aim to reduce the cost of transportation and operating ships in ports. In the first model, the base model, the initial inventory level of vessels and ports are parameters; also, the initial position of the ships is specified. The base model does not contain restrictions on returning ships to their initial position. In addition, it does not comprise constraints imposing equal inventory levels for the first and last state in ports and vessels. The second model, the cyclic model, is a complementary approach to the base model by adding such not considered constraints to the model to let the planned horizon have an absolute continuity.

The analysis of outputs in the computational section reveals that since the cyclic model encompasses more constraints to be continuable, it comprises more complexity than the base model; therefore, the complex cyclic model needs more time to be solved by solvers. But the key point here is that companies do one-time planning for an unlimited repeatable voyage in the cyclic model. However, in the base model, the players need to discover a new feasible plan before each trip; if it exists, it increases the operational cost and wastes plenty of time.

Furthermore, section 5.2.3 proves that starting the second voyage in the base model is impossible exactly after ending the first voyage due to infeasibility.

The cost evaluation exhibits that since vessels must return to their initial location in the cyclic model, they need more trips. It is also required to balance the inventory level at ports and ships to equal the initial state. Thus, the cyclic model contains a higher travelling cost in a period than the base model. But when customers have constant orders over a repeatable time horizon, cyclic demand, the base model is impractical.

6.1 Research barriers

Accessibility to a professional computing system and time constraints were to main barriers on the path. As mentioned in chapter 5, complex instances containing a higher number of ports and vehicles, such as G-6-5-60-C, needed more solution time to get accurate feedback; the solution time varies between hours and days. Since this research was limited to a maximum of 5 months, the solution time was reduced to two hours for complex cases. Such a barrier could affect the computational solutions and a broader canvas on the computational evaluations by increasing the gaps and errors.

6.2 Suggestions for future developments on subject

As mentioned in the first chapter, the research on the cyclic inventory routing problem (CIRP) in maritime is scant; therefore, a wide range of approaches can be added into context to turn a raw theme into a mature one. In the following, you will read some of the authors' proposals for upcoming research in a similar scop.

Since uncertainties, say weather conditions, technical issues, and maritime traffic in ports and unique places such as straits play a crucial role in marine transport, adding the measures related to uncertainties to the objective function is highly recommended to make the problem more realistic.

Continuity and on-time delivery are two essential factors in the supply chain management system which affect customer satisfaction and business survival. Hence, almost in most commercial contracts, the penalty costs resulting from the delay in delivery are included. Adding the penalty cost into the objective function could help find routes containing minimum delay time.

The vessels' load on the routes could affect the speed and fuel consumption; on a broader picture, it could affect the amount of the sailing days and sailing cost. So, the load dependent speed optimization approach is another suggestion that can be considered.

All instances studied in this research include ports in which a single loading or unloading operation is assumed. In the actual cases, it is possible to handle both loading and unloading operations simultaneously. Although it makes the problem more complex, our suggestion is to consider a multifunctional operational capability, loading and unloading, into the problem.

And the last suggestion refers to the mentioned research barriers in section 6.1.1. saying it is essential to implement a professional computing system and extend the assigned solution time to get more precise feedback from the AMPL or other auxiliary software to boost the accuracy of evaluations.

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Appendix 1

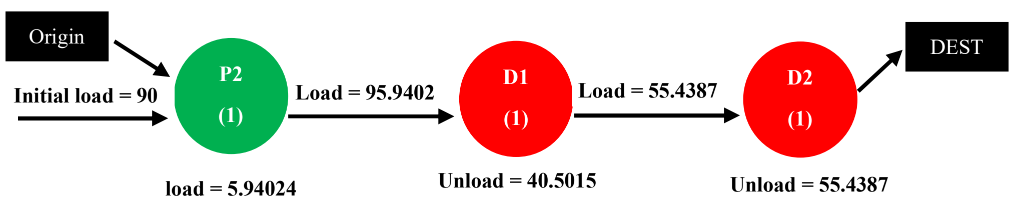
This section contains the voyage schematic of instances for the base and the cyclic models in data group one. The consumption and production ports are shown in red and green, respectively. Each port is depicted with its own name and visit number written in parentheses. The number under each port indicates how much the vessel loaded or unloaded during each visit. The vessel load between two ports is indicated by the number above each leg. Also, the sample name, total trip cost, and computational time are shown in the box before each sample's trip route map.

Model 1: A-4-1-30-B

Total Cost= 74.8741

Computational Time= 0.13

Ship route 1 (V1):

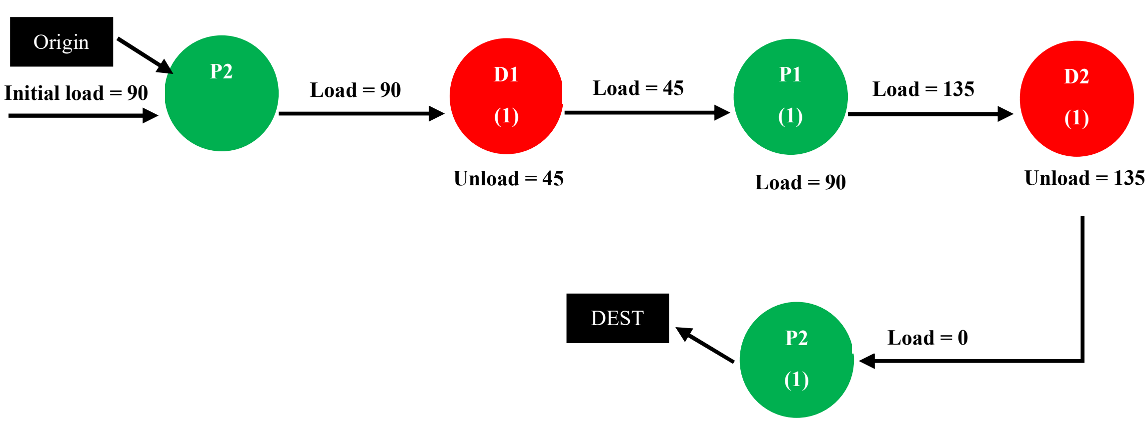


Model 1: A-4-1-30-C

Total Cost= 230.602

Computational Time= 0.56

Ship route 1 (V1):

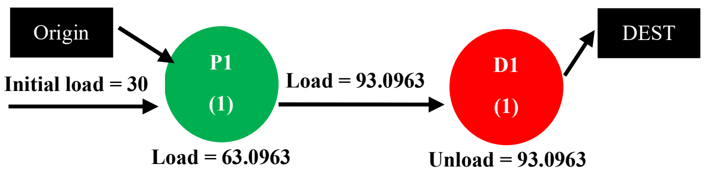


Model 1: B-3-2-30-B

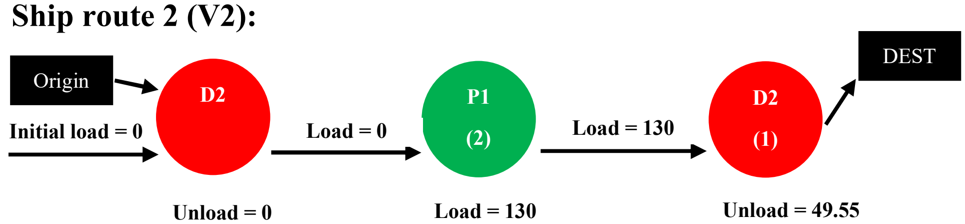
Total Cost= 158.53

Computational Time= 0.36

Ship route 1 (V1):



Ship route 2 (V2):

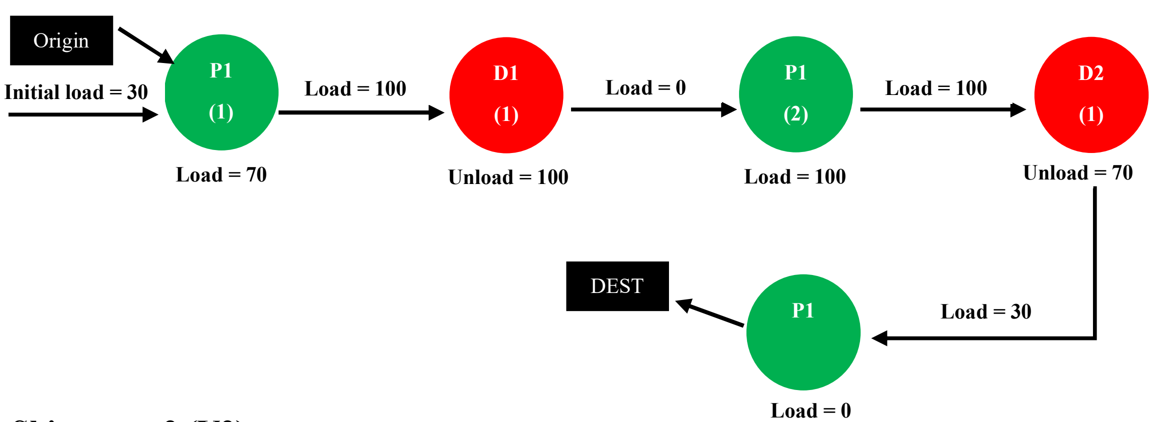


Model 1: B-3-2-30-C

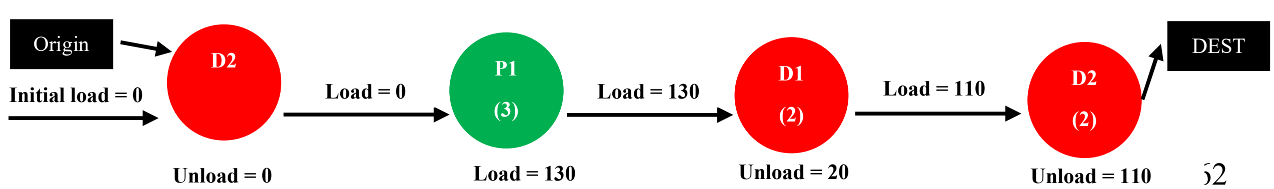
Total Cost= 342.153

Computational Time= 2132.95

Ship route 1 (V1):



Ship route 2 (V2):

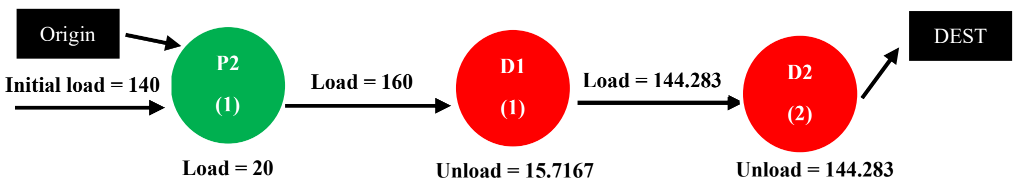


Model 1: C-4-2-30-B

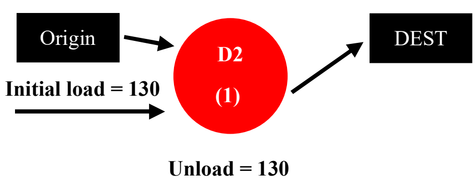
Total Cost= 75.0591

Computational Time= 0.34

Ship route 1 (V1):



Ship route 2 (V2):

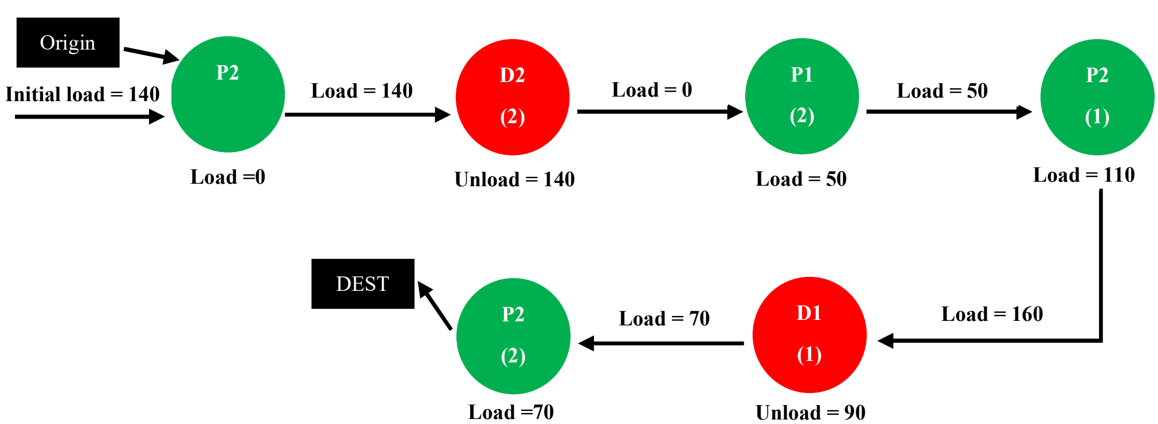


Model 1: C-4-2-30-C

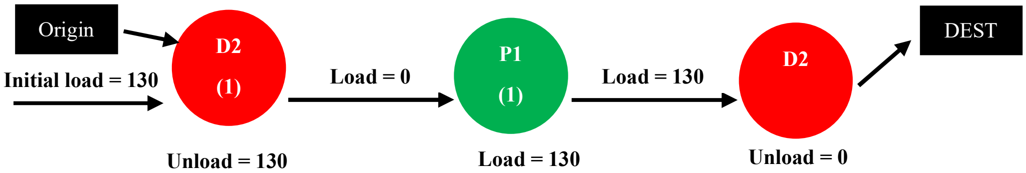
Total Cost= 347.106

Computational Time= 595.92

Ship route 1 (V1):



Ship route 2 (V2):

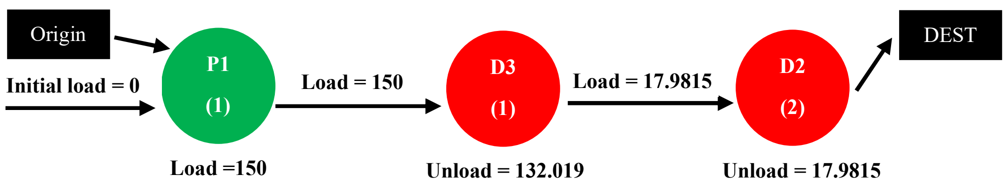


Model 1: D-5-2-30-B

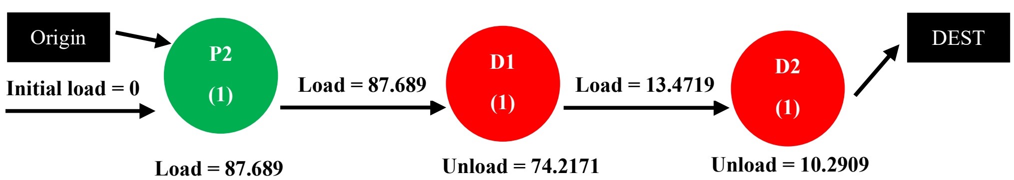
Total Cost= 214.604

Computational Time= 1.09

Ship route 1 (V1):



Ship route 2 (V2):

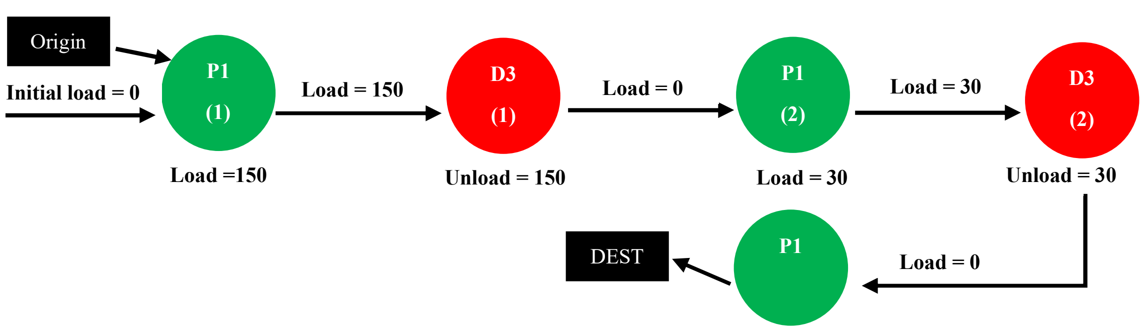


Model 1: D-5-2-30-C

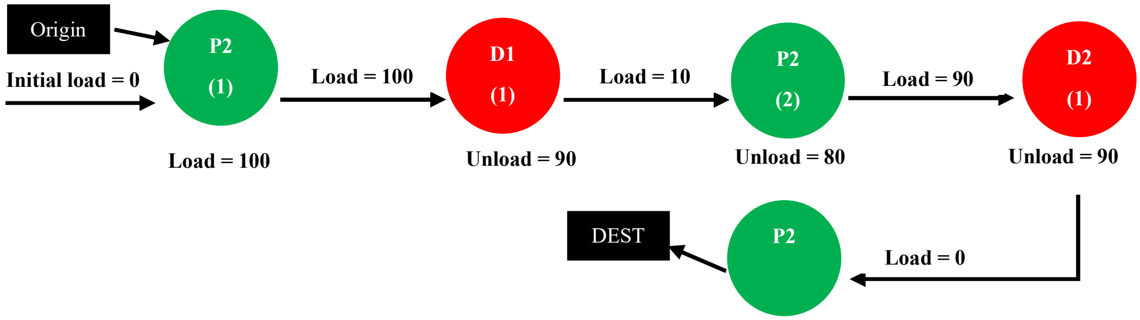
Total Cost= 555.563

Computational Time= 7202.11

Ship route 1 (V1):



Ship route 2 (V2):



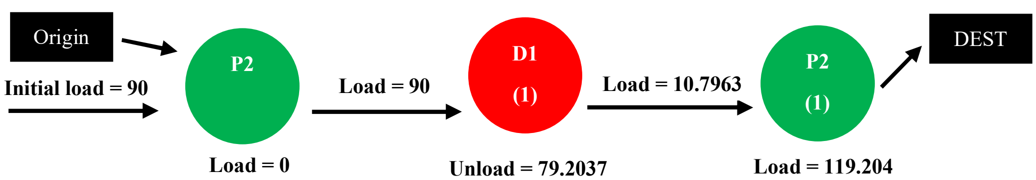
Model 1: F-4-3-30-B

Total Cost= 156.877

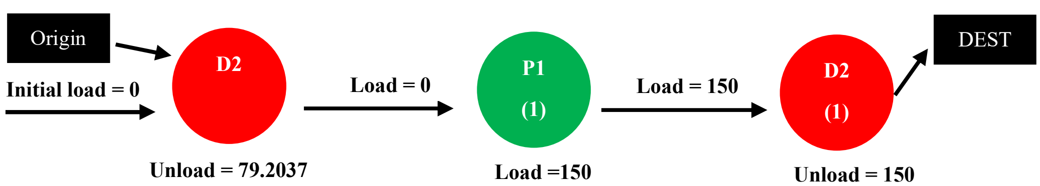
Computational Time= 3.63

Ship route 1 (V1): Not used

Ship route 2 (V2):



Ship route 3 (V3):

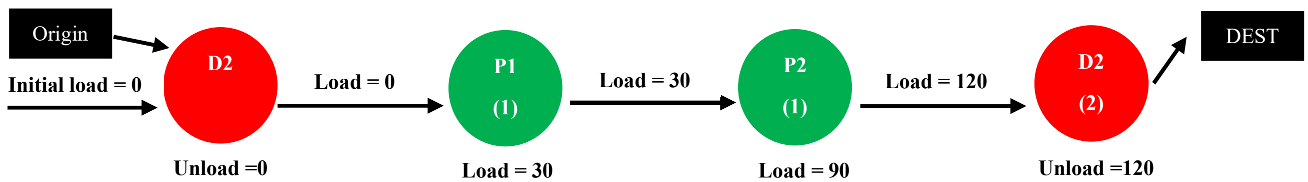


Model 1: F-4-3-30-C

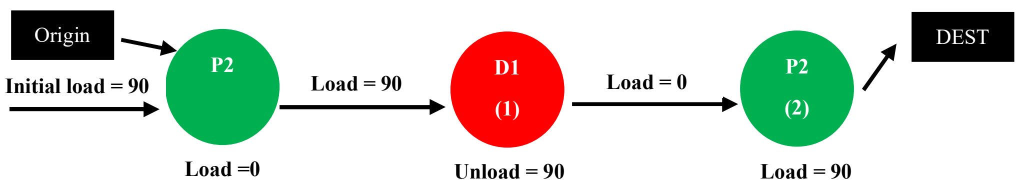
Total Cost= 315.754

Computational Time= 1597.56

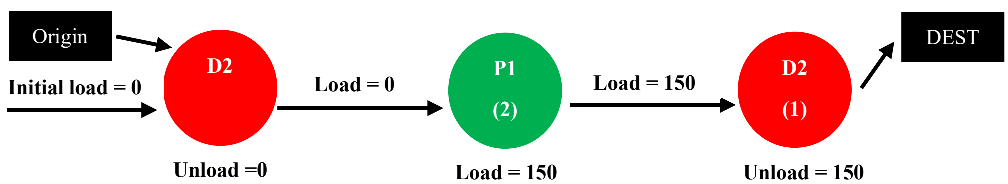
Ship route 1 (V1):



Ship route 2 (V2):



Ship route 3 (V3):

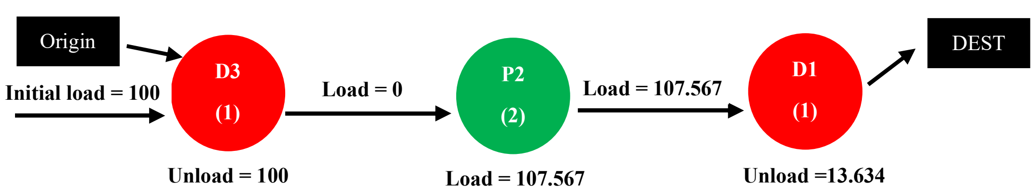


Model 1: G-6-5-30-B

Total Cost= 411.598

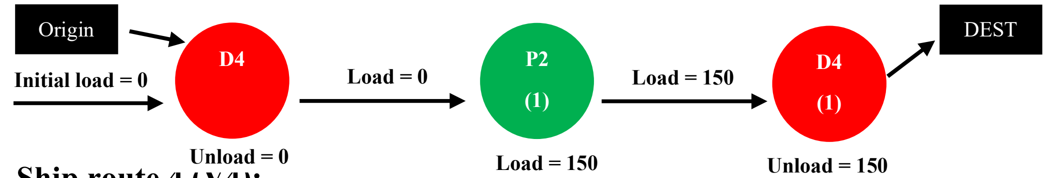
Computational Time= 81.23

Ship route 1 (V1):

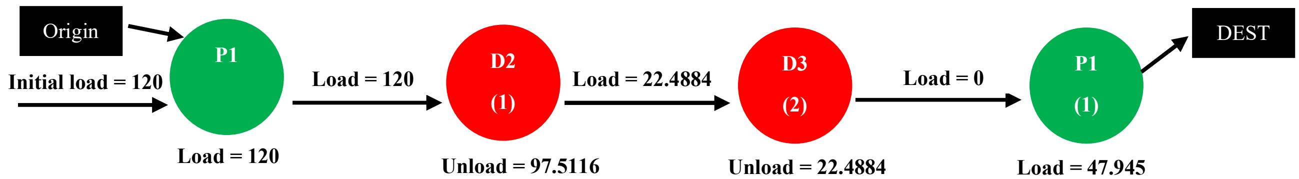


Ship route 2 (V2): Not used

Ship route 3 (V3):



Ship route 4 (V4):

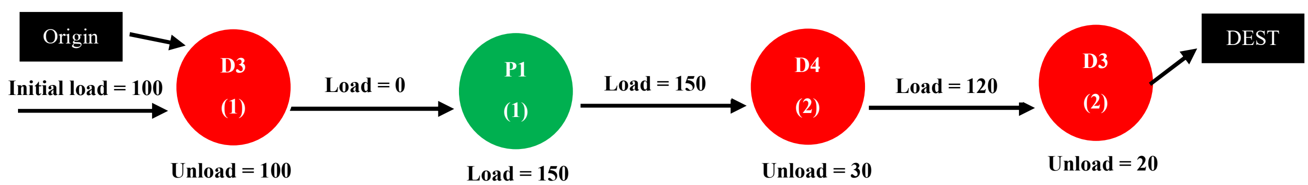


Model 1: G-6-5-30-C

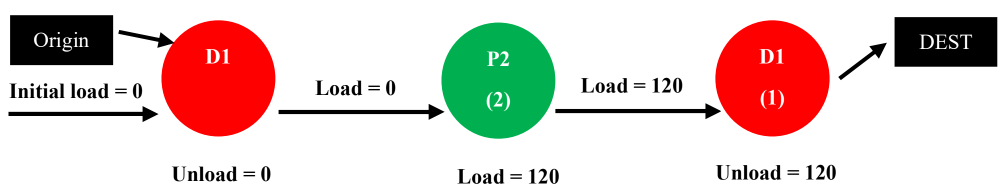
Total Cost= 605.231

Computational Time= 7207.66

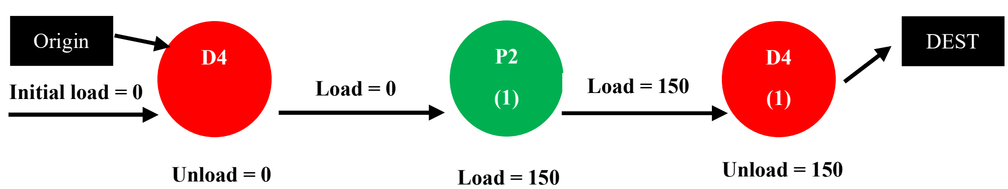
Ship route 1 (V1):



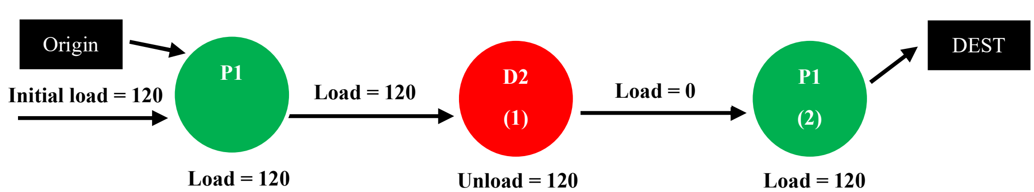
Ship route 2 (V2):



Ship route 3 (V3):



Ship route 4 (V4):



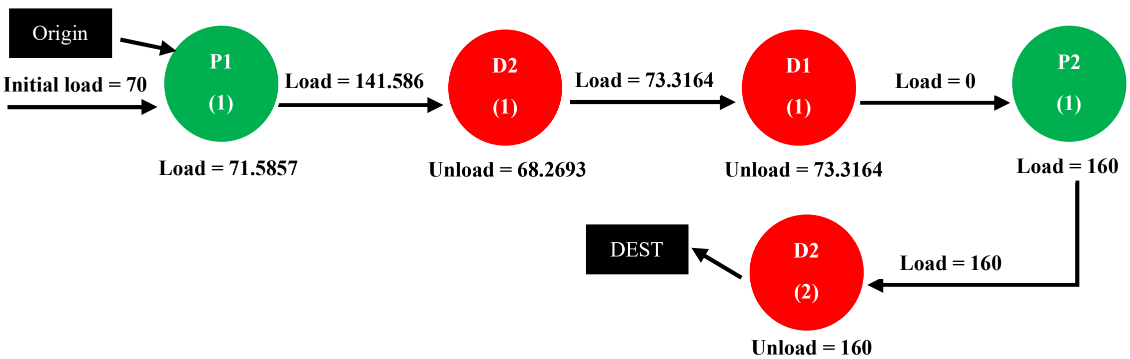
Ship route 5 (V5): Not used

Model 1: A-4-1-60-B

Total Cost= 184.928

Computational Time= 0.45

Ship route 1 (V1):

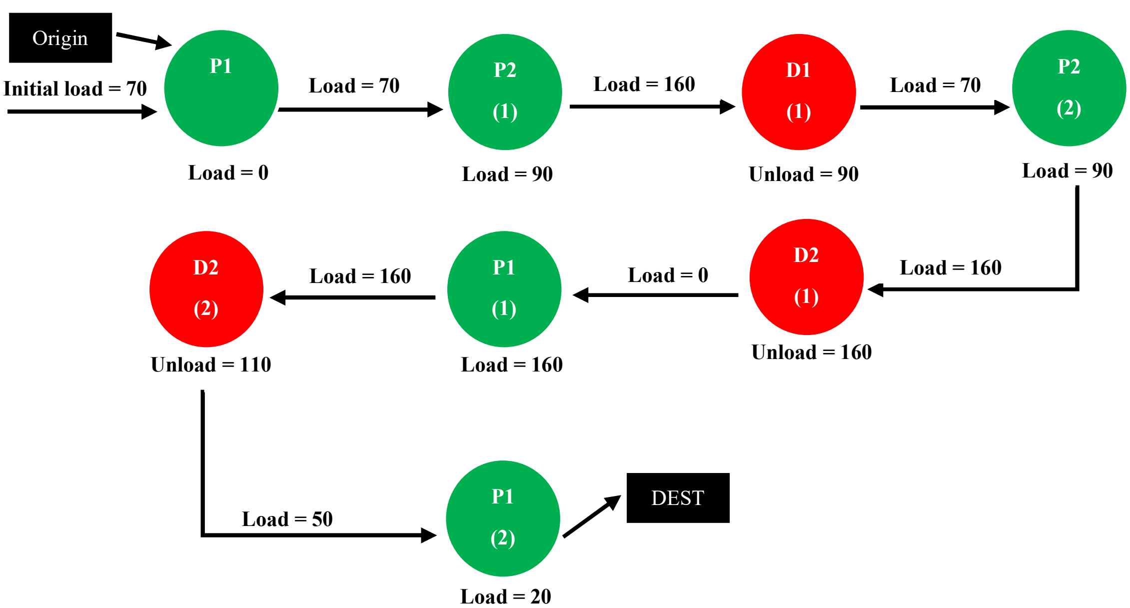


Model 1: A-4-1-60-C

Total Cost= 366.561

Computational Time= 0.48

Ship route 1 (V1):



Model 1: B-3-2-60-B

Total Cost= -

Computational Time= -

Model 1: B-3-2-60-C

Total Cost= integer infeasible

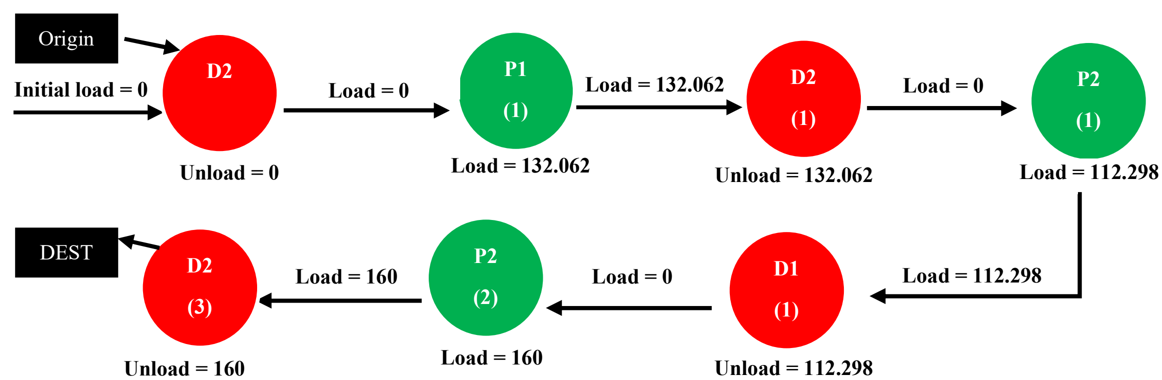
Computational Time= 0

Model 1: C-4-2-60-B

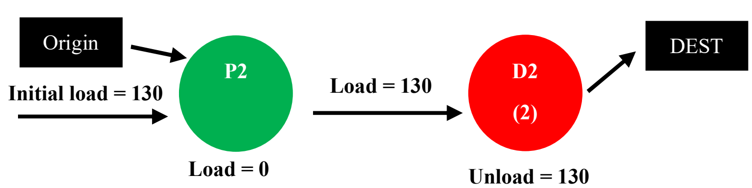
Total Cost= 299.454

Computational Time= 1.58

Ship route 1 (V1):



Ship route 2 (V2):

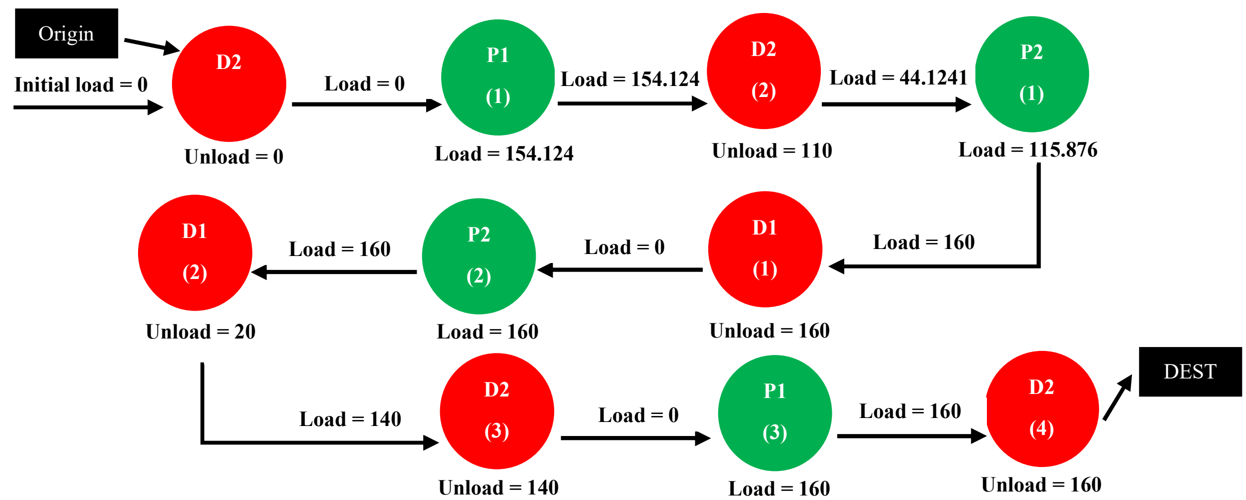


Model 1: C-4-2-60-C

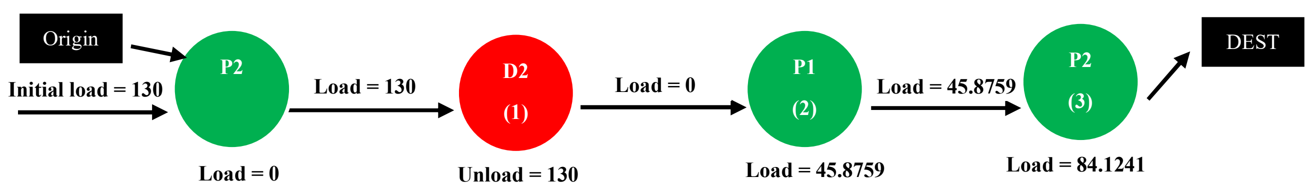
Total Cost= 521.49

Computational Time= 7201.09

Ship route 1 (V1):



Ship route 2 (V2):

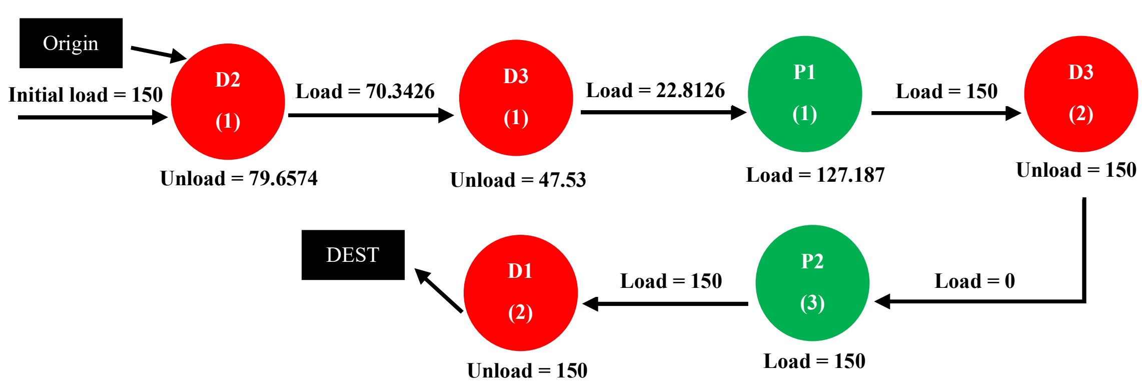


Model 1: D-5-2-60-B

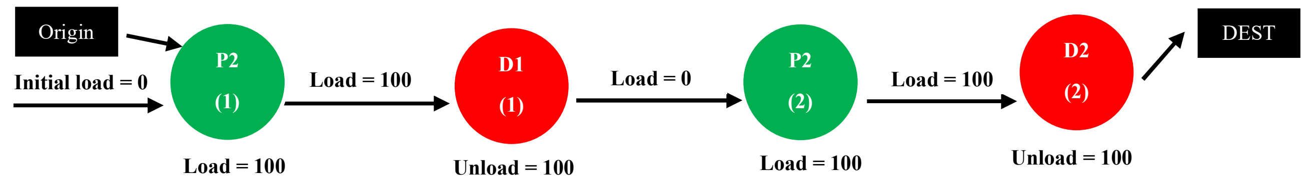
Total Cost= 458.331

Computational Time= 23.84

Ship route 1 (V1):



Ship route 2 (V2):

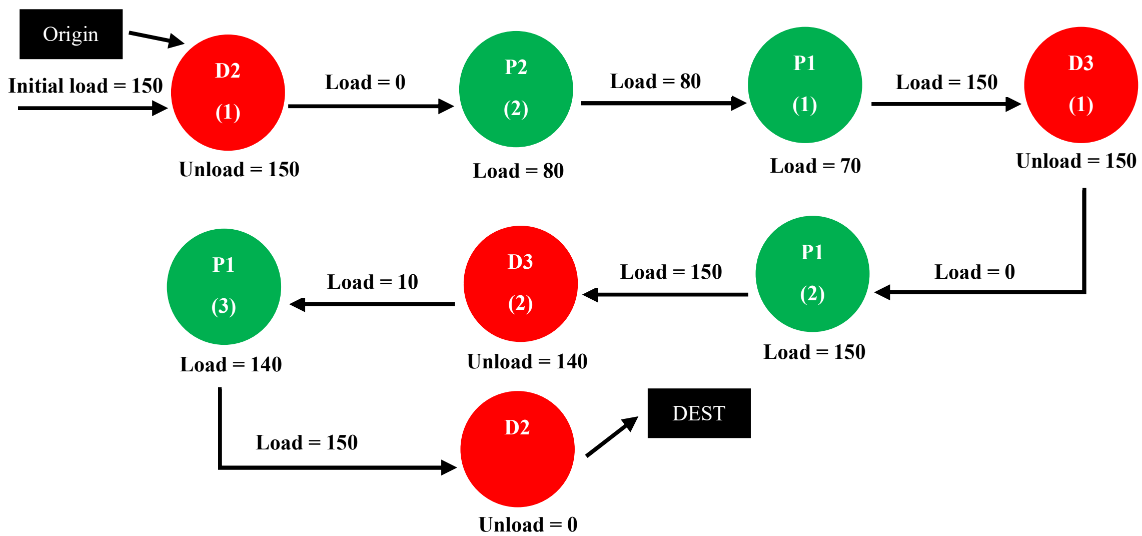


Model 1: D-5-2-60-C

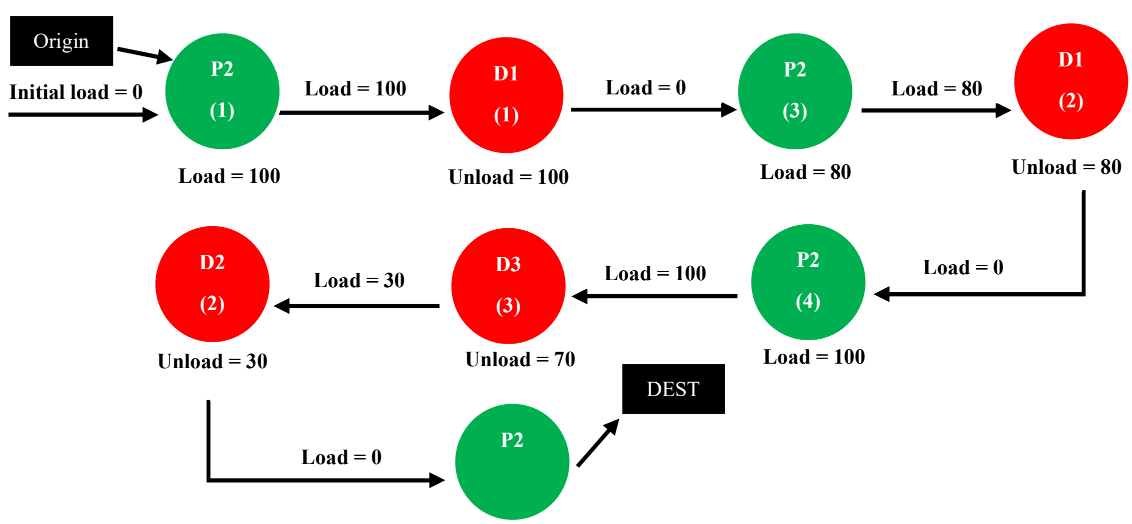
Total Cost= 783.91

Computational Time= 7201.88

Ship route 1 (V1):



Ship route 2 (V2):

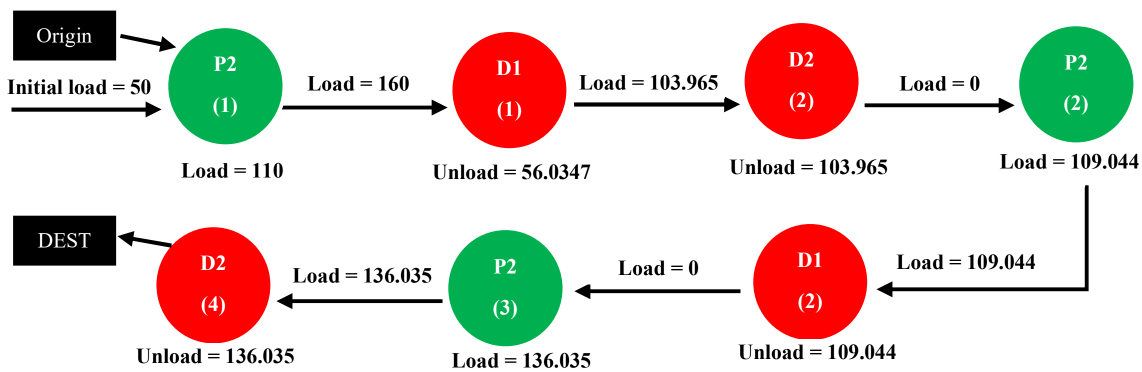


Model 1: F-4-3-60-B

Total Cost= 323.154

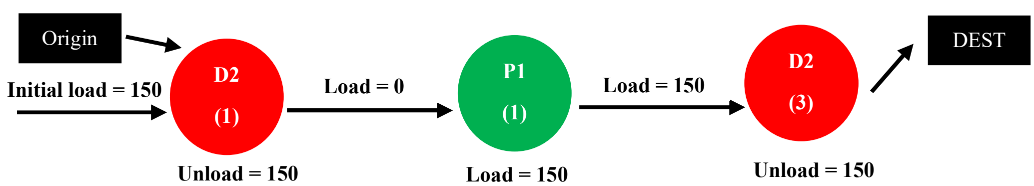
Computational Time= 2.91

Ship route 1 (V1):



Ship route 2 (V2): Not used

Ship route 3 (V3):

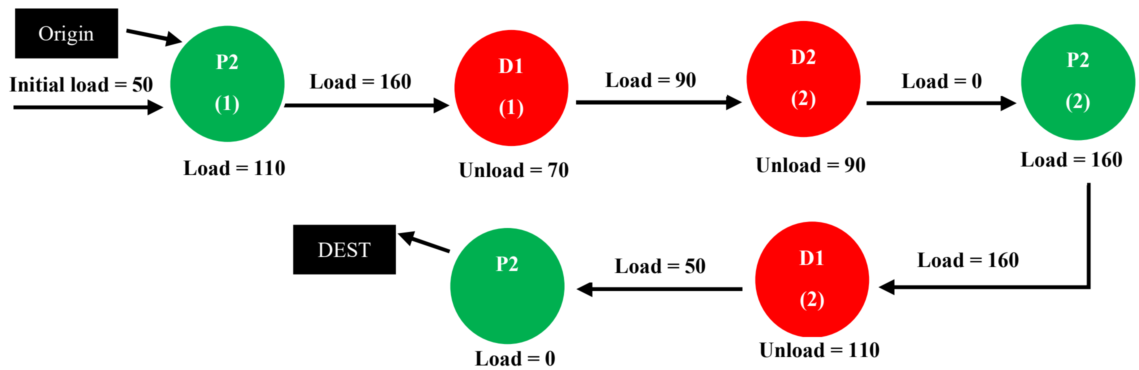


Model 1: F-4-3-60-C

Total Cost= 514.79

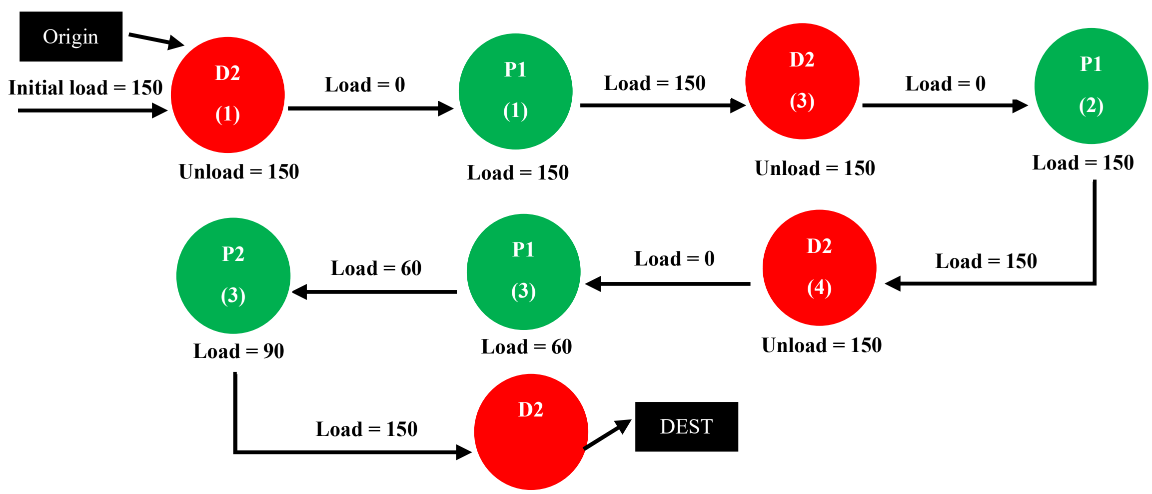
Computational Time= 7202.27

Ship route 1 (V1):



Ship route 2 (V2): Not used

Ship route 3 (V3):



Appendix 2

This section contains the schematic of routes in the cyclic model for three instances in the second data group. The sample name and total trip cost are shown in the box before each sample's trip route map.

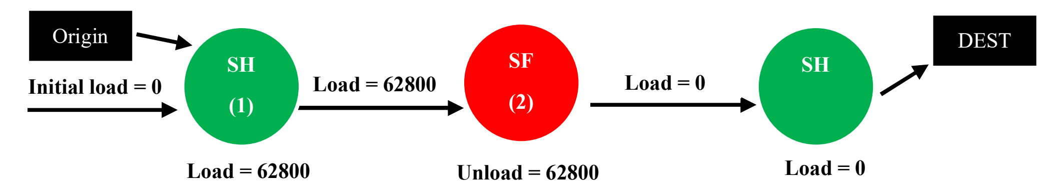
Real instances name:

Port name	Shanghai	Savannah	Hamburg	Santos	San Francisco
Abbreviation	SH	SV	HH	SA	SF

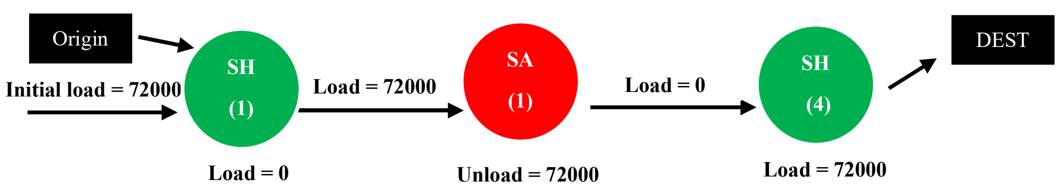
Model 1: R-5-1-4-5

Total Cost= 19327200

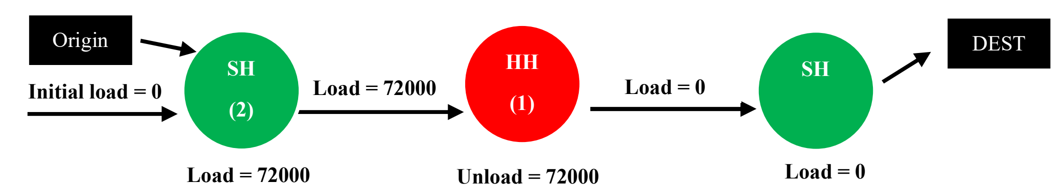
Ship route 1 (V1):



Ship route 2 (V2):

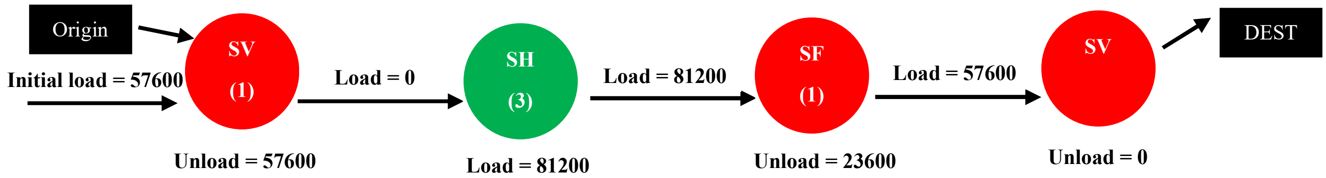


Ship route 3 (V3):



Ship route 4 (V4): Not used

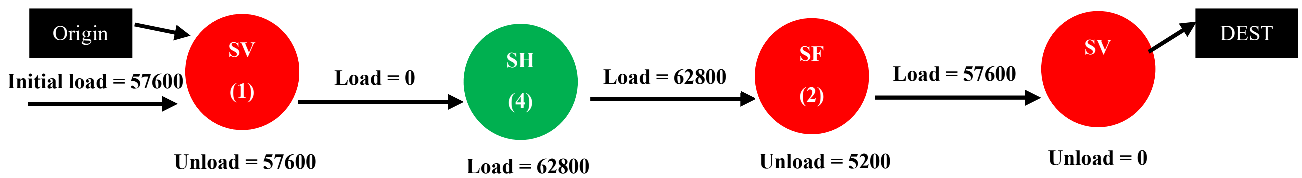
Ship route 5 (V5):



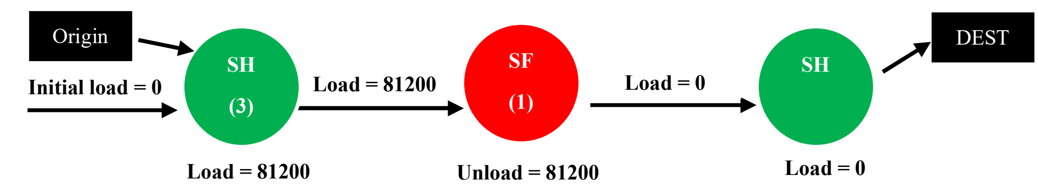
Model 1: R-5-1-4-4

Total Cost= 19327200

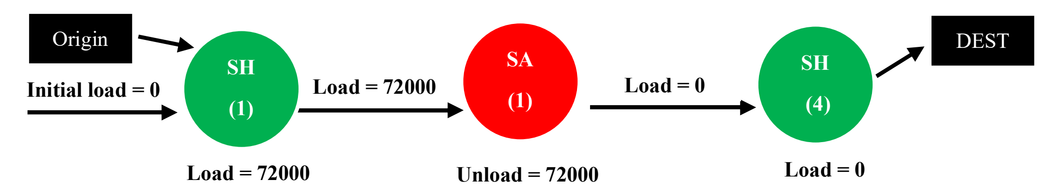
Ship route 1 (V1):



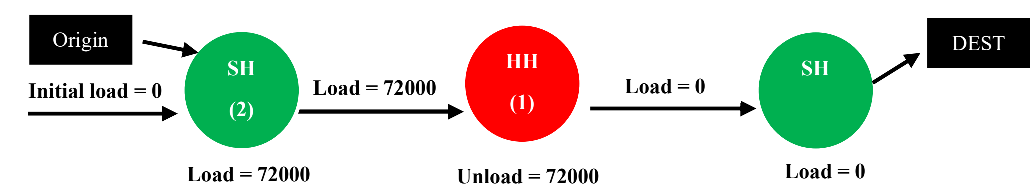
Ship route 2 (V2):



Ship route 3 (V3):



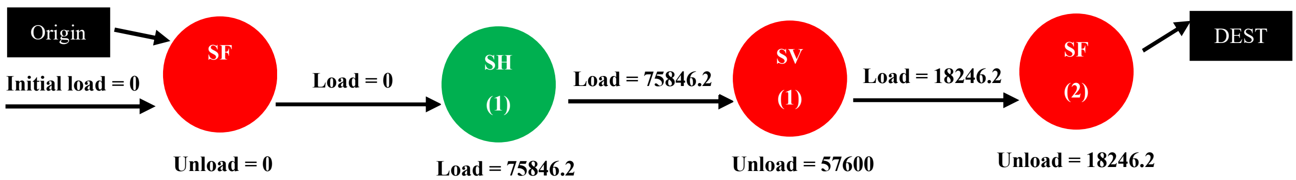
Ship route 4 (V4):



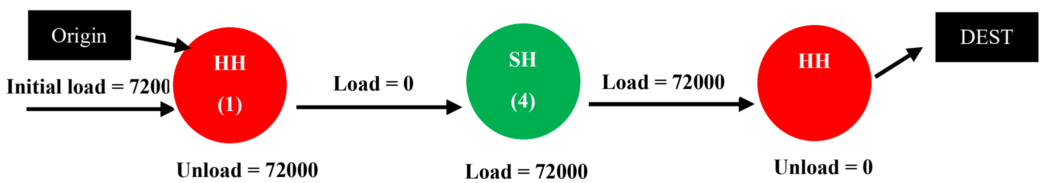
Model 1: R-5-1-4-3

Total Cost= 19327200

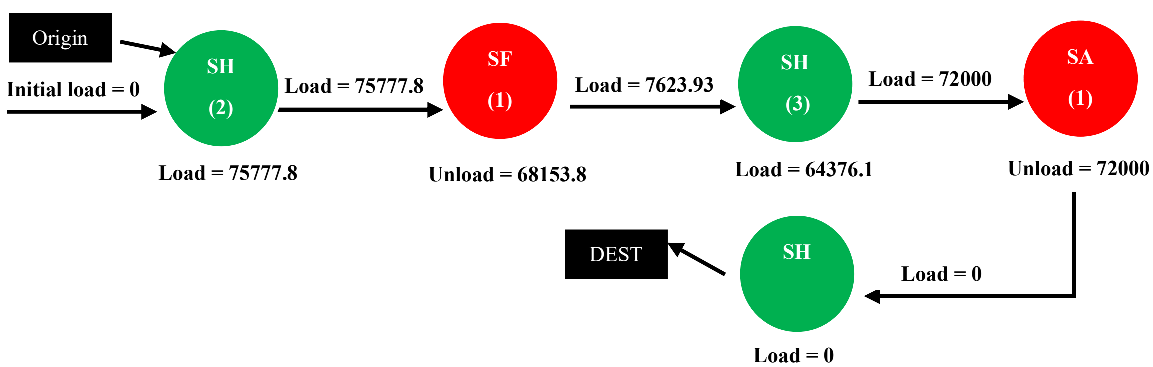
Ship route 1 (V1):



Ship route 2 (V2):



Ship route 3 (V3):



Appendix 3

This section contains the cyclic model's AMPL code (.mod file). The notation is based on the mathematical model presented by Agra et al. (2016a). Some changes and modifications described in sections problem description and mathematical model have been incorporated into this model.

```
#SETS
set N;       #set of production and consumption ports.
set V;       #set of ships.
set S;       #set of speeds.
#PARAMETERS
param T>=0;         #time planning horizon
param J {i in N};   #J(i) = 1 if i is a loading port and -1 if i is an unloading
                    #port
param D {i in N} >=0;     #consumption or demand at port i per unit of time
param US {i in N} >=0;    #the upper bound on the inventory level at port i in
                         # time period t
param LS {i in N} >=0;    #the lower bound on the inventory level at port i in
                         # time period t
param H {i in N} >=0;     #minimum number of visits at port i
param M {i in N} >=0;     #maximum number of visits at port i
param Q {i in N} >=0;     #minimum load/unload quantity at port i.
param K {i in N}>=0;      #minimum time between two consecutive visits at port i.
param LR {v in V} >=0;    #the fixed amount ship v loads/ discharges per time
                         #period
param TQ {v in V}:=1/LR[v];      #time for unload/load each unit.
param SP {v in V, e in S} >=0;   #speed s of ship v
param C {v in V} >=0;            #capacity of ship v
param DPP {v in V, e in S} >=0;  #daily traveling cost
param A {i in N, m in 1..M[i]}:=0;     #earliest time for starting vist m
                                       # to port i
param B {i in N, m in 1..M[i]}:=T;     #latest time for starting visit m
                                       #at port i.
param U {i in N, m in 1..M[i]}:=min(T, T+B[i,m]*US[i]);
                                       #latest time for finishing visit
param DI {i in N, j in N} >=0;         #port to port distance matrix
param P {i in N, v in V} >=0;          #port cost at port i for vessel v
param TPP {i in N, j in N, v in V, e in S}:=DI[i,j]/(24*SP[v,e]);
                                       #time (number of units) required by ship v
                                       #to sail from port i to port j (days)
param CPP {i in N, j in N, v in V, e in S}:=DPP[v,e]*TPP[i,j,v,e];
                                      #sailing cost from port i port j with ship v
###ROUTING VARIABLES
var w{i in N, v in V} binary;          #1 if vessel v starts from port i, 0
                                       #otherwise
var x {i in N, m in 1..M[i], j in N, n in 1..M[j], v in V: i<>j} binary;
                                       #1 if vessel v departs from node (i,m) to
                                       #node (j,n), 0 otherwise
var xo {i in N,j in N, n in 1..M[j], v in V} binary;
```

```
                                      #1 if vessel v departs from its initial
                                      #position to node (i,m), 0 otherwise
var xd {j in N, n in 1..M[j], i in N,v in V} binary;
                                      #1 if vessel v end its routes at
                                      #node (i,m); 0 otherwise
var o {i in N, m in 1..M[i], v in V} binary;
                                      #1 if vessel v operates in port
                                      # (i,m), 0 otherwise
var y {i in N, m in 1..M[i]} binary;   #1 if there is a visit (i,m); 0 otherwise

###FLOW VARIABLES
var q {i in N, m in 1..M[i], v in V} >=0;
                                      #the quantity loaded/unloaded at
                                      # node (i,m) by vehicle v
var f {i in N, m in 1..M[i], j in N, n in 1..M[j], v in V: i<>j} >=0;
                                      #flow associated to arcs X (from port to
                                      #port)
var fo {i in N, j in N, n in 1..M[j], v in V} >=0;
                                      #flow associated to arcs Xo (from initial
                                      #position to port)
var fd {j in N, n in 1..M[j], i in N,v in V} >=0;
                                      #associated to arcs Xd (to destination)

###TIME VARIABLES
var t {i in N, m in 1..M[i]} >=0, <=T; #start time at node (i,m)
var ts{v in V} >=0;                    #The amount of delay that may occur
                                       # for each vessel v

###STOCK VARIABLES
var so {i in N} >=0;                   #the stock level in port i at the
                                       # beginning of the planning horizon
var s {i in N, m in 1..M[i]}>=0;       #stock level at node (i,m)
var l {v in V} >=0;                    #initial load onboard ship v when
                                       # leaving the starting station.
var g {i in N, m in 1..M[i],j in N, n in 1..M[j], v in V,e in S: i<>j}>=0,<=1 ;
                                       #auxiliary variable to determine the
                                       # speed of vessel v when going from
                                       # node(i,m) to node(j,n), with s
                                       # corresponding to a given choice of speed
var go {i in N, j in N, n in 1..M[j], v in V,e in S}>=0,<=1 ;
                                      #auxiliary variable to determine the
                                      # speed of vessel v when going from
                                      # its origin at port i to node(j,n),
                                      # with s corresponding to a given
                                      # choice of speed
var gd {j in N, n in 1..M[j], i in N,v in V,e in S}>=0,<=1 ;
                                       #auxiliary variable to determine the
                                       #speed of vessel v when going from
                                       # node (j,n)to destination at port i,
                                       # with s corresponding to a given
                                       # choice of speed
###OBJECTIVEFUNCTION
minimize Total_Cost:
```

```
sum {i in N, m in 1..M[i], j in N, n in 1..M[j], v in V, e in S: i<>j}
CPP[i,j,v,e]* g[i,m,j,n,v,e] +
sum {i in N, j in N, n in 1..M[j], v in V, e in S} CPP[i,j,v,e]* go[i,j,n,v,e]+
sum {j in N, n in 1..M[j], i in N,v in V, e in S} CPP[i,j,v,e]* gd[j,n,i,v,e]+
sum {i in N, m in 1..M[i], v in V} P[i,v]*o[i,m,v];
                                       #(1)Expresses the minimization of
                                       # the sum of traveling costs between
                                       # ports depending on the chosen speed
                                       # and operational costs in each port.

###ROUTING CONSTRAINTS
subject to FLOW1 {v in V}:
sum {i in N} w[i,v]<=1;                #(2)Show that each vessel must have
                                       # a maximum of one start position.

subject to FLOW2 {i in N, v in V}:
sum {j in N, n in 1..M[j]} xd[j,n,i,v]=w[i,v];
                                       #(3)Illustrate each vessel should
                                       # return to its start position.

subject to FLOW3 {i in N,v in V}:
sum {j in N, n in 1..M[j]} xo[i,j,n,v]=w[i,v];
                                       #(4)Illustrate each vessel should
                                       # start from initial position.

subject to FLOW4 {v in V, i in N, m in 1..M[i]}:
o[i,m,v] - sum {j in N, n in 1..M[j]: i<>j} x[i,m,j,n,v] - sum {j in N}
xd[i,m,j,v]=0;                         #(5)Ensure that if a vessel is at
                                       # node i it must either leave to
                                       # another node or end its route there.

subject to FLOW5 {v in V, i in N, m in 1..M[i]}:
o[i,m,v] - sum {j in N, n in 1..M[j]: j<>i} x[j,n,i,m,v] - sum {j in N}
xo[j,i,m,v]=0;                         #(6)Define that if a node is visited
                                       # by vessel v, the vessel must either
                                       # arrive at the node from the origin
                                       # or from another node.

subject to SHIP_VISIT {i in N, m in 1..M[i]}:
sum {v in V} o[i,m,v] = y[i,m];        #(7)show that a vessel can only
                                      # visit node (i,m) if there are at least m
                                      #visits to port i.

subject to PORT_VISIT {i in N,m in 2..M[i]:H[i]+1<=m<=M[i]}:
y[i,m-1] - y[i,m] >= 0;                #(8)Guarantee that if a port i is
                                      # visited m times, then it also has been
                                      #visited m-1 times.

subject to MANDATORY_VISITS {i in N, m in 1..H[i]}:
y[i,m] = 1;                            #(9)Defines the number of mandatory
                                       # visits for port i.

subject to SPEED_ARC {v in V, i in N, m in 1..M[i], j in N, n in 1..M[j]: j<>i}:
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sum {e in S} g[i,m,j,n,v,e] = x[i,m,j,n,v];
                                        #(10)Enforce that speed of a vessel
                                        # must be set for a travel from node (i,m)
                                        #to node (j,n) if and only if that travel
                                        #exists.

subject to SPEED_ARC2 {v in V, i in N, j in N, n in 1..M[j]}:
sum {e in S} go[i,j,n,v,e] = xo[i,j,n,v];
                                        #(11)Enforce that speed of a vessel
                                        # must be set for a travel from the origin
                                        #to node (j,n) if and only if that travel
                                        #exists.

subject to SPEED_ARC3 {v in V, i in N, j in N, n in 1..M[j]}:
sum {e in S} gd[j,n,i,v,e] = xd[j,n,i,v];
                                        #(12)Enforce that speed of a vessel
                                        # must be set for a travel from node (j,n)
                                        #to the destination if and only if that
                                        # travel exists.

###LOADING AND UNLOADING CONSTRAINTS
subject to CONSTRAINT1 {i in N, m in 1..M[i], v in V}:
q[i,m,v] <= min(C[v], US[i]) * o[i,m,v];
                                        #(13)Ensure the quantity loaded/ unloaded
                                        # cannot exceed the vessel capacity nor the
                                        #maximum port capacity.

subject to CONSTRAINT2 {i in N, m in 1..M[i], v in V}:
Q[i] * o[i,m,v] <= q[i,m,v];            #(14)Show that if a vessel visits
                                         # the port, then the amount loaded/unloaded
                                        # should be at least equal to the minimum
                                        #quantity.

subject to CONSTRAINT3 {v in V, j in N, n in 1..M[j],i in N}:
fo[i,j,n,v]>= l[v]-C[v]*(1-xo[i,j,n,v]);
                                        #(15)Determine flow from the initial
                                        # position which is equal to initial load if
                                        #ship travels from the initial position.

subject to CONSTRAINT4 {v in V, j in N, n in 1..M[j],i in N}:
fo[i,j,n,v]<= l[v]+C[v]*(1-xo[i,j,n,v]);
                                        #(16)Determine flow from the initial
                                        # position which is equal to initial load if
                                        #ship travels from the initial position.

###ARC - FLOW MODEL
subject to CONSTRAINT5 {v in V, j in N, n in 1..M[j]}:
sum {i in N}fo[i,j,n,v] + sum {i in N, m in 1..M[i]: i<>j} f[i,m,j,n,v] + J[j] *
q[j,n,v] =
sum {i in N, m in 1..M[i]: j<>i} f[j,n,i,m,v] + sum {i in N}fd[j,n,i,v];
                                         #(17)It is flow conservation constraints
                                         # which sum of incoming flow from origin,
                                        #flow from a particular port and the amount
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                                        #of loaded/unloaded should be equal
                                        # to outgoing flow from port plus
                                        # outgoing flow to the destination.

subject to CONSTRAINT6 {i in N, j in N, n in 1..M[j], v in V}:
fo[i,j,n,v] <= C[v] * xo[i,j,n,v];      #(18)Determine flow from the initial
                                        # position should be less or equal to the
                                        #capacity of vessel; Also, may not move any
                                        #products.

subject to CONSTRAINT7 {i in N, j in N, m in 1..M[i], n in 1..M[j], v in V: j<>i}:
f[i,m,j,n,v] <= C[v] * x[i,m,j,n,v];    #(19)Convey flow from a port to
                                        # another port should not be more than the
                                        #capacity of the vessel.

subject to CONSTRAINT8 {i in N, j in N, n in 1..M[j], v in V}:
fd[j,n,i,v] <= C[v] * xd[j,n,i,v];      #(20)Show flow to the destination is
                                        # less or equal ship capacity if ship
                                        #travels to the destination.

subject to V_End_LOAD {v in V}:
l[v] = sum {i in N,m in 1..M[i],j in N} fd[i,m,j,v];
                                        #(21)Show vessels load at the end of
                                        # planning horizon should be equal to
                                        # the initial inventory level.

###TIME CONSTRAINTS
subject to MIN_INTERVAL {i in N, m in 1..M[i]:m>1}:
t[i,m] - t[i,m-1] - sum {v in V} TQ[v] * q[i,m-1,v] - K[i] * y[i,m] >=0;
                                        #(22)Enforce the minimum time period
                                        # between two consecutive visits of
                                        #port i.

subject to START_TIME {i in N, j in N, m in 1..M[i], n in 1..M[j]: i<>j}:
t[i,m] + sum {v in V} TQ[v] * q[i,m,v] - t[j,n] +
sum {v in V,e in S} max(U[i,m] + TPP[i,j,v,e] - A[j,n],0) * g[i,m,j,n,v,e]<=
U[i,m]- A[j,n];                         #(23)Relate the start time
                                        # associated with node (i, m) to the
                                        start time associated with node (j,
                                        #n) when a vessel travels between
                                        # ports i and j.

subject to START_TIME2 {i in N, m in 1..M[i],v in V}:
ts[v] +sum {j in N,e in S} TPP[j,i,v,e] * go[j,i,m,v,e] <= t[i,m]+T*(1-o[i,m,v]);
                                        #(24)Show that the travel time for a
                                        #vessel traveling from origin plus
                                        #delay time should not exceed the
                                        #start time of the visit to the port.

subject to START_TIME3 {j in N, n in 1..M[j],v in V}:
t[j,n] + TQ[v]*q[j,n,v] +sum {i in N,e in S} TPP[j,i,v,e] * gd[j,n,i,v,e] <= T
+ts[v]+T*(1-o[j,n,v]);                  #(25)Express time at node (j,n) plus
                                        #travel time for a vessel traveling
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                                      #from node (j,n) to destination at
                                      #node i should not exceed the Time
                                      #horizon plus delay time.

subject to TIME_WINDOW1 {i in N, m in 1..M[i]}:
t[i,m] >= A[i,m];                      #(26)Express time windows for the
                                      #start time of visits.

subject to TIME_WINDOW2 {i in N, m in 1..M[i]}:
t[i,m] <= B[i,m];                      #(27)Exhibit time windows for the
                                      #end time of visits.

###INVENTORY CONSTRAINRS
subject to CONS_STOCK_START {i in N}:
s[i,1] = so[i] + (J[i]*D[i] * t[i,1]);;
                                      #(28)Set the stock level at the
                                      #start time of the first visit to a port.

subject to RELATE_STOCK {i in N,m in 1..M[i]:m>1}:
s[i,m] = s[i,m-1] - sum{v in V} J[i]*q[i,m-1,v] + J[i]*D[i] * (t[i,m] - t[i,m-1]);
                                      #(29)Show that the stock level at
                                      #the start of the mth visit is set by
                                      #to the stock level at the start of
                                      #the previous visit, the load or
                                      #unload operation in the previous
                                      #visit and the time elapsed between
                                      #the two visits.

subject to CONS_STOCK_LIMIT1 {i in N, m in 1..M[i]:J[i]=-1}:
s[i,m] + sum{v in V} q[i,m,v] - sum{v in V}D[i] * TQ[v]*q[i,m,v] <= US[i];
                                      #(30)Impose limitation of the stock
                                      #level at the end time of a visit to
                                      #a consumption port.

subject to 1_PROD_STOCK_LIMIT1 {i in N, m in 1..M[i]:J[i]=+1}:
s[i,m] - sum{v in V} q[i,m,v] + sum{v in V}D[i] * TQ[v]*q[i,m,v] >= LS[i];
                                      #(31)Impose limitation of the stock
                                      #level at the end time of a visit to
                                      #a production port.

subject to CONS_LBOUND {i in N:J[i]=-1}:
s[i, M[i]] + sum{v in V} q[i, M[i],v] - (D[i] * (T - t[i, M[i]])) >= LS[i];
                                      #(32)Show lower bound on the
                                      #inventory level until the end of the
                                      #time horizon for consumption ports.

subject to 1_PROD_LBOUND {i in N:J[i]=+1}:
s[i, M[i]] - sum{v in V} q[i, M[i],v] + (D[i] * (T - t[i, M[i]])) <= US[i] ;
                                      #(33)Exhibit upper bound on the
                                      #inventory level until the end of the
                                      #time horizon for production ports.

subject to LIMIT1 {i in N, m in 1..M[i]: J[i]=-1}:
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s[i,m] >= LS[i];                       #(34)Illustrate lower bound on the
                                      #stock level at the start of each visit for
                                      #consumption ports.

subject to LIMIT2 {i in N, m in 1..M[i]: J[i]=1}:
s[i,m] <= US[i];                       #(35)Show upper bound on the stock
                                      #level at the start of each visit for
                                      #production ports.

subject to End_Inventory {i in N}:
sum{v in V,m in 1..M[i]} q[i, m ,v]=T*D[i];
                                      #(36)Determine inventory level at
                                      #the end of planning horizon should
                                      #be equal to the initial inventory
                                      #level for all ports.
```