INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

WILEY

INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

Intl. Trans. in Op. Res. 29 (2022) 2995–3030
DOI: 10.1111/itor.12998

Long-term effects of short planning horizons for inventory routing problems

Mohamed Ben Ahmed*, Onyemaechi Linda Okoronkwo, Edwin Chimezie Okoronkwo and Lars Magnus Hvattum

Faculty of Logistics, Molde University College, PO Box 2110, Molde NO-6402, Norway
E-mail: mohamed.ben.ahmed@himolde.no [Ahmed]

Received 11 August 2020; received in revised form 24 February 2021; accepted 13 May 2021

Abstract

This paper presents a detailed study concerning the importance of the planning horizon when solving inventory routing problems (IRPs). We evaluate the quality of decisions obtained by solving a finite-horizon IRP. We also discuss the relevance of explicitly considering profit maximization models rather than the traditional cost minimization variant. As a means to this end, we describe four classes of the IRP corresponding to different types of markets. Two of them lead to nonlinear models, which are linearized. Furthermore, we provide a deterministic simulator to evaluate the long-term effects arising from using planning horizons of varying lengths when solving the IRP. A computational study is performed on cases generated from benchmark data instances. The results confirm that the long-term performance of the IRP decisions is, in part, contingent on the length of the selected planning horizon. They also show that considering profit maximization instead of cost minimization leads to different decisions, generating considerably more revenue and profits, albeit not nearly as much as suggested by individual solutions to static IRPs with short planning horizons.

Keywords: profit maximization; path flow; linearization; end effect; simulation

1. Introduction

The inventory routing problem (IRP) integrates two fundamental logistic problems: inventory management and vehicle routing. Precisely, a supplier must decide *(i)* when to serve a given customer, *(ii)* how much to deliver to this customer when it is served, and *(iii)* how to combine customers into vehicle routes. Most often, the vehicles have limitations on the quantity of product they can transport, and the objective is to minimize the total transportation and inventory holding costs.

The IRP literature differentiates between instant, finite, and infinite planning horizons (Andersson et al., 2010). If at most one visit per customer is required, then it is deemed to be instant. The

*Corresponding author.

problem is termed finite if the length of the planning horizon $T < \infty$; otherwise, it is an infinite horizon problem. To reduce complexity, the IRP has often been modeled over finite planning horizons. However, this comes at the cost of generating end-of-horizon effects. A typical consequence is that customers' inventories are empty or near empty at the end of the last period. As goods have a relatively high cost of shipping and holding, myopic decisions involve delivering fewer goods to customers in the final planning periods. If additional planning periods are modeled, this shortage would be corrected by more intensives deliveries, and the end effects would be transferred to the new final period. The distortions occur because models fail to account for the future impact of decisions made in the final period; only the immediate consequences of these decisions are considered. When more planning periods are considered, the end effects become less significant, but it becomes more intricate to compute solutions of the IRP.

Given these concerns, we investigate *(i)* the potential long-term gain from explicitly maximizing profits rather than minimizing costs in an IRP setting, and *(ii)* the effect of varying the length of the planning horizon on the quality of the decisions obtained by solving a finite-horizon IRP model. First, we consider four different models for the IRP, which are adapted from Zaitseva et al. (2018). All of the models described hereafter account for limits on the number stops a vehicle can make within a route. These characteristics can be used to represent real-life applications of the IRP, such as perishable goods delivery problems and time-critical delivery problems. The models are:

1. **A basic cost minimization model.**
2. **A profit maximization model with lost sales.** By modifying the basic cost minimization model to allow for lost sales, a profit maximization variant can be obtained. Here, the company may scale back the volume of its deliveries, in case it is more profitable to accept penalties for lost sales than to service the full demand at a high transportation cost.
3. **A profit maximization model under monopoly.** This model examines the case of a price-setting company. The company can adjust its prices, whereupon the customer demands follow accordingly.
4. **A profit maximization model with variable production costs.** The last model extends the profit maximization with lost sales variant by incorporating variable production costs. Production costs can increase or decrease following the company's production volume.

The profit maximization models are generalizations of the cost minimization model. Therefore, when considering a particular instance and a fixed planning horizon, the profit obtained by profit maximization models are necessarily at least as good as the profit obtained by the optimal decisions from the cost minimization model. Zaitseva et al. (2018) showed that the increase in profits could be substantial. However, just as end-of-horizon effects can influence the value of decisions in a given model, they can also distort the comparison between two different models. To properly assess the actual difference between cost minimization models and profit maximization models and the actual impact of varying the length of the planning horizon used when solving IRPs, we propose using simulation. In this simulation, decisions about scheduling, routing, and potentially pricing and production rates are determined by solving a sequence of IRPs. Although simulation is an accepted tool for evaluating such policies in the face of uncertainty, this work shows that policies based on solving problems with finite planning horizons exhibit similar behavior even when no stochasticity is present. In summary, this paper makes the following contributions:

- We use a deterministic simulator to calculate the long-term effects of making decisions by solving IRPs with short planning horizons.
- We present path flow formulations of four distinct classes of the IRP, including profit maximization variants with nonlinear expressions that are linearized.
- We perform an extensive computational study using benchmark test instances. The experiment is designed to examine the impact of three factors: the length of the planning horizon, the maximum route length allowed, and the market structure.

The remainder of this paper is organized as follows. Section 2 contains a review of relevant literature. In Section 3, we formally present the problem at hand and provide a detailed description of the proposed models. Next, in Section 4 we describe the deterministic simulator. This is followed by the results of extensive computational experiments in Section 5. Concluding remarks follow in Section 6.

2. Literature review

For recent surveys of research on IRPs, see the work by Andersson et al. (2010) and Coelho et al. (2014b). In the following, we focus on contributions related to the planning horizon of IRPs (Section 2.1) and on profit maximization in IRPs (Section 2.2).

2.1. IRPs with long planning horizons

A substantial body of IRP literature is concerned with the deficiency of short planning horizons. Several notable contributions ventured to solve IRP for long planning horizons. Dror et al. (1985) proposed a way to take into consideration what happens after the short-term planning period. The authors computed the stock-out probability, the average cost to deliver, and the anticipated cost of a stock-out for each customer and each day in the planning horizon. By dint of these metrics, they determined the optimal replenishment day, which yields a minimum expected total cost. If this day falls within the short-term planning period, the customer will be visited. Otherwise, an anticipated delivery is scheduled, and a future benefit is included within the objective. These computed values reflect the long-term effects of short-term decisions.

Bard et al. (1998) formulated an extension of the ideas by Dror et al. (1985) to address an IRP with satellite facilities. A satellite facility is a location apart from the depot where vehicles can be loaded. They proposed a rolling horizon framework, in which shipment schedules are determined over a two-week moving period. Only customers scheduled for the first week are routed. The two-week planning horizon is then shifted by a week, and the process is repeated. The routing decisions are fixed by employing three heuristics (randomized Clarke-Wright, GRASP, and modified sweep). An analysis similar to Dror et al. (1985) was performed to determine an optimal replenishment day for each customer. A handful of contributions adapted and enhanced these solution strategies and attempted to mitigate the effect of short-term decisions, including Anily and Federgruen (1990), Trudeau and Dror (1992), and Jaillet et al. (2002).

Archetti et al. (2016) presented a new variant of the IRP in which the ratio between the total costs and the total delivered quantity is minimized. This ratio represents the unitary distribution cost, and the problem was labeled the IRP with a logistic ratio. The authors' motivation for using this objective function is to avoid low inventory levels at the end of the planning horizon and capture the IRP's long-term impact. Using a branch-price-and-cut based-algorithm, the authors could optimally solve instances with up to five vehicles and 15 customers over three periods. Recently, Archetti et al. (2019) proposed an exact iterative algorithm to solve larger instances with more customers and a longer planning horizon.

We encounter a more advanced treatment of long-term decisions in the cyclic IRP. It is a long-term decision problem where the distribution policy and vehicle routes, once computed for a baseline period, will remain the same in the following periods. Raa and Aghezzaf (2008) brought forth a cyclic planning approach, in which a vehicle travels in every period of the cyclic planning horizon the same route to serve customers. Replenishment frequencies that minimize total costs are heuristically derived. With these patterns at hand, a steady-state distribution plan is constructed, which can be repeatedly reproduced no matter how long the planning horizon is. Later, Lefever et al. (2016) reformulated the problem as a convex optimization problem and proposed a modified branch-and-bound procedure for its solution. Raa and Aghezzaf (2009) extended the former approaches by integrating a fleet sizing problem. Vansteenwegen and Mateo (2014) examined a particular case of the cyclic IRP where a single vehicle is at disposal and employed an iterated local search to solve it. Chitsaz et al. (2016) studied the multiple vehicles variant of the problem and solved it using a decomposition algorithm. Ekici et al. (2015), Zenker et al. (2016), Raa and Dullaert (2017), and Dai et al. (2020) provide further examples for solving the IRP over a perpetual time horizon. Cyclic IRP plans can mitigate the end-of-horizon effects as they allow for equal ending inventory levels at each planning period. However, a cyclic IRP is not necessarily appropriate when some parameters of the problem, such as customer demands, vary over time.

A completely different solution strategy for tackling long-term IRPs is concerned with using rolling horizon heuristics (RHH). This approach operates by splitting up the planning horizon into shorter and more manageable periods. Rakke et al. (2011) used a rolling horizon matheuritics when studying a maritime IRP. A later effort to incorporate rolling horizon matheuristics was performed by Agra et al. (2014). The authors developed a mathematical programming formulation for the problem and a heuristic algorithm, which uses a rolling horizon decomposition, local branching, and a feasibility pump procedure. The relevance of RHH is high for maritime IRPs as these naturally involve long planning horizons. However, a handful of studies have used RHH when solving basic IRPs (Coelho et al., 2014a).

Many of the contributions documented above tend to reduce the long-term IRP into a more basic form. In doing so, they neglect the long-term impact of such decisions, and are in principle, still dealing with short planning horizons.

2.2. IRPs with profit maximization

A key characteristic of the class of IRPs with profit maximization is that the set of customers to be served and the demands to be satisfied are not necessarily known in advance. Therefore, a decision-maker must identify the customers to serve and determine their fill rates to maximize the profit. Revenue is usually associated with each customer to quantify their attractiveness. In other settings, a penalty cost for lost sales is attributed to each customer.

Several variations of this problem have been addressed in the literature. They respond to challenges faced by a logistic provider. In what follows, we provide a classification perspective that may help identify the relevance of IRPs with profit maximization models.

- **Lost sales.** For a supplier with limited resources (e.g., production, fleet), it may be impossible to meet customer demands. Only a proportion of demands can be fulfilled, while the remaining part is treated as lost sales and not backlogged. These shortages are discouraged by a penalty term specific to each customer. The objective is to choose a distribution policy that maximizes the profits (revenues minus costs). Chien et al. (1989) proposed a day-to-day version of the problem where inventory holding costs are null. The solution approach breaks down the problem into two subproblems: an inventory allocation subproblem and a combined customer assignment and vehicle routing subproblem, both of which were solved to optimality. Numerous contributions have drawn inspiration from the seminal work of Chien et al. (1989) and successfully incorporated lost sales within profit maximization models, for example, Goyal and Gunasekaran (1995), Al-Ameri et al. (2008), Kleywegt et al. (2002), Kleywegt et al. (2004), and Liu and Lee (2011).
- **Customer selection.** In some applications, the supplier must select a subset of customers from those available in the market. For each selected customer, the supplier gets a revenue, while no penalty is incurred from those not selected. The objective is to maximize the sum of revenues minus operations costs. These variants are often encountered in the IRP's maritime applications, where a shipping company has a mix of contracted cargoes (spot) and optional cargoes. Hemmati et al. (2015) described a profit maximization IRP that arises in tramp shipping. In that problem, revenue is collected from mandatory cargoes as well as optional cargoes. A two-phase heuristic was proposed to compute routes and schedules for the shipping company. Akin models with profit maximization are found in Grønhaug et al. (2010), Stålhane et al. (2014), Papageorgiou et al. (2014a), and Andersson et al. (2015).
- **Maximum collection.** The counterpart of delivery to customers is the collection from customers, where goods are picked-up from selective points to the depot based on profitability. The objective is to select pick-up points and quantities that maximize total profits. Montagné et al. (2019) introduced an application of this problem for waste collection. The developed approach relies on linear programming to build delivery schedules, while service routes are computed heuristically.
- **Variable production rates.** Some IRPs involve decisions regarding the production rate at the supplier. Solutions to these planning problems specify how much to produce, how much to sell, and how to combine deliveries within vehicle routes to maximize profits. The problem is not to be confounded with the production routing problem (PRP), which incorporates as well as lot-sizing decisions. Papageorgiou et al. (2014b) described a maritime variant of this problem, where a single product can be loaded from a set of production ports and discharged in specified consumption ports. An arc-flow based formulation was proposed to model the problem, while a branch-and-cut algorithm was developed to solve it. Similar problems were also considered by Grønhaug and Christiansen (2009) and Papageorgiou et al. (2014b, 2015).

3. Mathematical formulations for multiple classes of the IRP

In the following, we describe mathematical formulations for four classes of the IRP. The considered models are adapted from Zaitseva et al. (2018) and correspond to specific market structures within the IRP.

Although the models may seem incompatible, they can all be used for the same application, which involves a price-setting company whose unit production cost varies with the production rate, and that can choose not to meet the available demand at the cost of a penalty and lost sales. If prices and production quantities are decided *a priori*, and it is assumed that all demands must be met, decisions can be made using a cost minimization model. If only prices and production quantities are decided *a priori*, a profit maximization model with lost sales can be used to make decisions. If only deciding the production rates *a priori*, a model for profit maximization under monopoly can be used. Finally, if only deciding prices *a priori*, a model for profit maximization with variable production costs can be used. These four models are now described in turn. For either model, its performance can be evaluated under the assumption that the decisions can be converted into a final profit over the long term.

3.1. Problem definition, terminology, and notation

The inventory routing problem considered in this paper is formally defined as follows. Let $G = (V, A)$ be an undirected graph where $V = \{0, 1, \ldots, n\}$ is the vertex set, and $A = \{(i, j) : i, j \in V, i \leq j\}$ is the arc set, all defined over a finite and discrete time horizon H. Vertex 0 represents the depot, whereas the set $V' = \{1, \ldots, n\}$ corresponds to n customers. Let $T = \{1, \ldots, H\}$ be a set of time periods. A fleet of m identical vehicles of capacity Q are stationed at the depot. At each time period $t \in T$, a quantity r_{0t} is produced by the supplier and r_{it} units are demanded by customer $i \in V'$. Production costs are represented by the unit cost function $f(r_{0t})$. A starting inventory level I_i^0 and a maximum inventory level U_i are given for each $i \in V$. The inventory levels cannot be negative. That is, stock-out is not allowed. A holding cost is charged both at the supplier and at the customers. The unit inventory cost at $i \in V$ is denoted by h_i.

The problem is to determine, at each time period $t \in T$ and for each customer $i \in V'$, a shipped quantity, and a traveled route, providing that:

1. Each customer $i \in V'$ is visited at most once.
2. Each vehicle performs at most one tour, delivering at most Q units, visiting at most L customers, and starting and ending at the depot 0.
2. Inventory levels at both the supplier and customers are nonnegative and must not exceed a maximum holding capacity.

For modeling purposes, we use a path-flow formulation where $K = \{1, 2, \ldots, k\}$ is the set of routes that satisfy the problem requirements. These routes are enumerated *a priori* and fed into the formulation. We first generate tours by representing them as random combinations $(X_0, \ldots, X_k)$ of $(0, \ldots, n)$, where each tour contains at most L customers. Then, a travelling salesman problem (TSP) is solved for each tour to obtain the optimal sequencing of nodes. A symmetric cost matrix c_{ij}

is defined on A. The cost of a route $k \in K$, denoted by c_k, is computed using the arc costs c_{ij}. Strictly speaking, given a route k that successively visits vertices $i_0, i_1, \ldots, i_p,\ 0 < p \leq L$, its associated cost is given by

$$c_k = \sum_{j=0}^{p-1} c_{i_j i_{j+1}}.$$

Let a_{ik} be a binary coefficient equal to 1 if and only if customer $i \in V'$ belongs to route k. Define y_{kt}, a binary variable equal to 1 if and only if a route k is used in the optimal solution in period $t \in T$, and 0 otherwise. Continuous variables x_{ikt} represent the quantity of product delivered to each customer $i \in V'$ at time $t \in T$ by means of route $k \in K$. The inventory levels at the end of period t at the supplier and customers are described by I_{0t} and I_{it}, respectively.

3.2. Cost minimization

In this formulation, transportation and inventory holding costs are minimized. We deliberately omit production costs as they are assumed to be fixed. Accordingly, the cost minimization variant can be stated as follows:

$$\text{Minimize} \sum_{k \in K} \sum_{t \in T} c_k y_{kt} + \sum_{i \in V} \sum_{t \in T} h_i I_{it}, \tag{1.1}$$

subject to

$$\sum_{k \in K} y_{kt} \leq m, \quad t \in T, \tag{1.2}$$

$$x_{ikt} \leq Q a_{ik} y_{kt}, \quad i \in V', k \in K, t \in T, \tag{1.3}$$

$$x_{ikt} \leq U_i - I_{i,t-1}, \quad i \in V', t \in T, k \in K, \tag{1.4}$$

$$\sum_{i \in V} x_{ikt} \leq Q y_{kt}, \quad t \in T, k \in K, \tag{1.5}$$

$$I_{it} = I_{i,t-1} + x_{ikt} - r_{it}, \quad i \in V', t \in T, k \in K, \tag{1.6}$$

$$I_{0,t} = I_{0,t-1} + r_{0t} - \sum_{i \in V'} x_{ikt}, \quad t \in T, k \in K, \tag{1.7}$$

$$I_{i0} = I_i^0, \quad i \in V, \tag{1.8}$$

$$0 \leq I_{0,t-1} + r_{0t} \leq U_0, \quad t \in T, \tag{1.9}$$

$$I_{it} \geq 0, \quad i \in V, t \in T, \tag{1.10}$$

$$(x) \quad \text{positive}, \tag{1.11}$$

$$(y) \quad \text{binary}. \tag{1.12}$$

The objective function (1.1) is to minimize the sum of all costs incurred by transportation, inventory holding at customers, and inventory holding at the supplier. Constraints (1.2) restrict the maximum number of routes in a solution to the number of available vehicles while constraints (1.3) ensure that a customer is visited at most once during each period, $t \in T$. Constraints (1.4) establish that if a customer is visited, the quantity delivered is such that the maximum inventory level is not exceeded. The vehicle capacity constraints are provided by constraints (1.5). Furthermore, inventory balance for customer nodes, defined at the end of each period, is ensured in constraints (1.6). Inventory levels at the supplier are represented by (1.7). The initial conditions for the inventory levels are given by constraints (1.8), and the inventory bounds are enforced by constraints (1.9) and (1.10). Finally, constraints (1.11) and (1.12) set the domains for the decision variables. Given integer values of x satisfying constraints (1.6) and (1.7), constraints (1.6) and (1.7) necessarily induce the $I-$variables to be integer-valued.

3.3. *Profit maximization with lost sales*

The seller can earn a sales revenue of p_0 per unit of product consumed by customer $i \in V'$ in period $t \in T$, which is the unit price for that customer. Let b_i a penalty for each unit of unsatisfied demand, while $f(r_{0,t})$ is a function that approximates the unit production cost. We also introduce a new decision variable w_{it} that represents the proportional amount of product consumed by customer $i \in V'$ at period $t \in T$. Accordingly, the profit maximization IRP with lost sales becomes

$$\text{Maximize} \sum_{i\in V'}\sum_{t\in T} p_0 w_{it} - \sum_{k\in K}\sum_{t\in T} c_k y_{kt} - \sum_{i\in V}\sum_{t\in T} h_i I_{it} - \sum_{i\in V'}\sum_{t\in T} b_i(r_{it} - w_{it}) - \sum_{t\in T} f(r_{0,t}) r_{0,t}, \tag{2.1}$$

subject to

$$(1.2)\text{–}(1.5), \quad (1.7)\text{–}(1.10),$$

$$I_{it} = I_{i,t-1} + \sum_{k\in K} x_{ikt} - w_{it}, \quad i \in V', t \in T, \tag{2.2}$$

$$0 \le w_{it} \le r_{it}, \quad i \in V', t \in T, \tag{2.3}$$

$$(x, w) \quad \text{positive}, \tag{2.4}$$

$$(y) \quad \text{binary}. \tag{2.5}$$

The objective function (2.1) maximizes the earned net profit, which is equal to the total revenues minus transportation, inventory holding, and production costs as well as a penalty term that is induced by unsatisfied demands. Constraints (2.2) are inventory conservation constraints at customer $i \in V$, linking the inventory of period t with that of period $t-1$, plus any deliveries minus the consumption w_{it}. Constraints (2.3) guarantee for each customer $i \in V$ that the consumed quantity of products in period t does not exceed the preset demand r_{it}. The routing constraints and inventory conservation constraints at supplier are identical to (1.2)–(1.5) and (1.7)–(1.10). Finally, constraints (2.4) and (2.5) define the integrality conditions of variables.

3.4. Profit maximization under monopoly

We introduce the pricing variable p_i, representing the revenue per unit consumed at customer $i \in V'$. There is a maximum price p_{max}, and the demand, $r_{it} = g(p_i)$, is a decreasing function of the price.

Proposition 1. *A demand curve in a monopolistic market can be a linear downward-sloping curve. To sell more, the seller must decrease the price; when reducing the sales, the prices can increase (Krugman and Wells, 2015, Chapter 13). Accordingly, it can be approximated by a function of the type* $g(p_i) = u_1 p_i + u_2, \quad i \in V$*, where* $u_1, u_2 \in \mathbb{R}$*, and* u_2 *have strictly negative values.*

Using these definitions, we can formulate the monopolistic variant of the IRP as follows.

$$\text{Maximize} \sum_{i \in V'} \sum_{t \in T} p_i w_{it} - \sum_{k \in K} \sum_{t \in T} c_k y_{kt} - \sum_{i \in V} \sum_{t \in T} h_i I_{it} - \sum_{i \in V'} \sum_{t \in T} b_i (g(p_i) - w_{it}) - \sum_{t \in T} f(r_{0,t}) r_{0,t}, \tag{3.1}$$

subject to

$$(1.2)-(1.5), \quad (1.7)-(1.10), \quad (2.2),$$
$$0 \le w_{it} \le g(p_i), \quad i \in V', t \in T, \tag{3.2}$$

$$0 \le p_i \le p_{max}, \quad i \in V', \tag{3.3}$$

$$(x, w, p) \quad \text{positive}, \tag{3.4}$$

$$(y) \quad \text{binary}. \tag{3.5}$$

The objective function (3.1) is to maximize the total earned profits. Constraints (3.2) ensure that the consumed amount of product by each customer $i \in V'$ does not exceed the demand, $r_{it} = g(p_i)$. Constraints (3.3) ensure the restriction on the maximum allowed price, while (3.4) and (3.5) define the integrality conditions for the variables. The remaining set of constraints are reproduced from the previous model.

Remark 1. The objective function (3.1) contains a nonlinear term $\sum_{i \in V'} \sum_{t \in T} p_i w_{it}$, which is the product of two variables (w) and (p). We emphasize that this term is nonseparable.

Proposition 2. *We can derive an equivalent mixed-integer linear programming model for the IRP under monopoly by applying an approximating piecewise linear function, upon the transformation of the nonseparable function.*

The linearization process, as well as the resulting linear model, are described in Appendix A.

3.5. Profit maximization with variable productions costs

We introduce the variable $R_{0,t}$ representing the production rate in time period $t \in T$, and the total production cost $f(R_{0,t})$ as a function of the production rate.

Proposition 3. *The average total cost is equal to the sum of the average fixed cost and average variable cost. It has a* U*-shape because these two components vary in opposite directions when production increases (see Krugman and Wells, 2015, Chapter 11). Thus, the average total cost function can be approximated by a function of the type* $f(R_{0,t}) = u'_1 R_{0,t} + \frac{u'_3}{R_{0,t}} + u'_2, \quad t \in T$, *where* u'_1, u'_2, *and* $u'_3 \in \mathbb{R}^+ \setminus \{0\}$.

The profit maximization variant with variable production costs describes as follows:

$$\text{Maximize} \sum_{i \in V'} \sum_{t \in T} p_0 w_{it} - \sum_{k \in K} \sum_{t \in T} c_k y_{kt} - \sum_{i \in V} \sum_{t \in T} h_i I_{it} - \sum_{i \in V'} \sum_{t \in T} b_i (r_{it} - w_{it}) - \sum_{t \in T} f(R_{0,t}) R_{0,t}, \tag{4.1}$$

subject to

$$(1.2)\text{–}(1.6), \quad (1.8), \quad (2.2)\text{–}(2.3),$$

$$I_{0,t} = I_{0,t-1} + R_{0,t} - \sum_{i \in V'} x_{ikt}, \quad t \in T, k \in K, \tag{4.2}$$

$$0 \le I_{0,t-1} + R_{0,t} \le U_0, \quad t \in T, \tag{4.3}$$

$$0 \le R_{0,t} \le r_{max}, \quad t \in T, \tag{4.4}$$

$$(x, w, R) \quad \text{positive}, \tag{4.5}$$

$$(y) \quad \text{binary}. \tag{4.6}$$

The objective function (4.1) maximizes the total revenue minus transportation, inventory holding, production costs, and the penalty term for lost sales. Constraints (4.2) define the inventory at the supplier, while Constraints (4.3) prohibit stock-outs at the supplier. Constraints (4.4) require that the total produced amount does not exceed the production capacity. Constraints (4.5) and (4.6) represent the logical restrictions. The remaining constraints are identical to those described in the monopoly variant of the IRP.

Remark 2. The objective function (4.1) includes a quadratic term $\sum_{t \in T} f(R_{0,t}) R_{0,t}$ that is equivalent after factorization to $\sum_{t \in T} u'_1 R_{0,t}^2 + u'_2 R_{0,t} + u'_3$.

Proposition 4. *We can derive an equivalent linear model representation of the IRP with variable production costs by applying an approximating piecewise linear function.*

The linearization process and the linear counterpart of this problem are described in Appendix B.

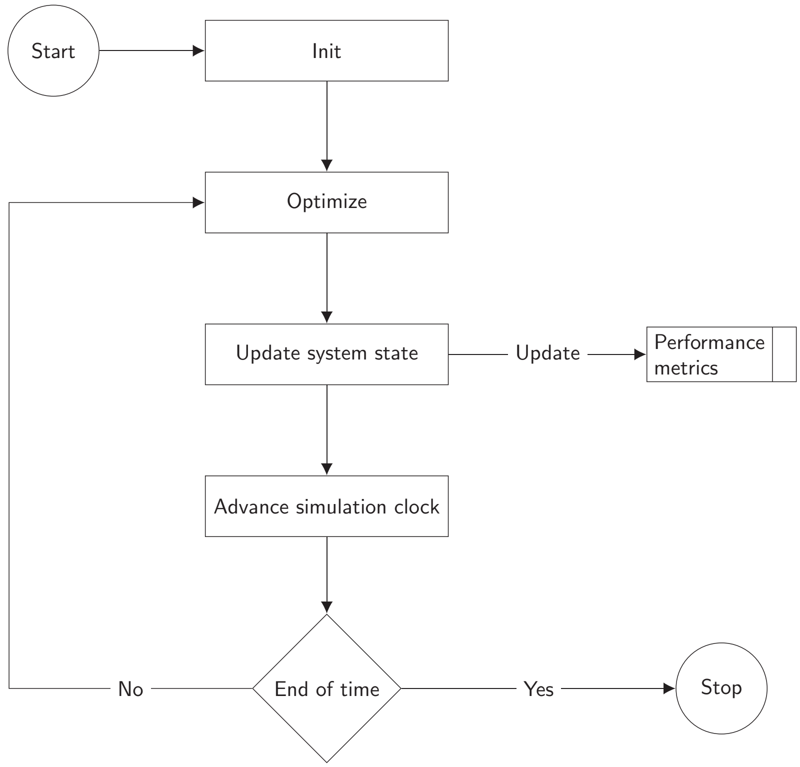


Fig. 1. Generic framework of the deterministic simulation.

4. Deterministic simulator for the IRP

In this section, we address the issue of evaluating IRP decisions made using the optimization models over a longer time horizon, so that the consequences of end-of-horizon effects can be measured. For this purpose, we implement a deterministic simulator that embeds decisions made from the optimization model, which is updated with fixed time increments. Fagerholt et al. (2010) formerly investigated the idea of combining simulation and optimization similarly in the context of strategic maritime planning. The simulator developed in this paper is built on three phases: an optimization stage, an update procedure, and a time advance mechanism. Figure 1 illustrates the general steps of the simulation framework.

The procedure operates by initializing the system state, as well as the simulation clock. The system state contains all available information about customer demands, starting inventory levels, and pricing (i.e., input parameters of the mathematical model). The optimization stage relies on solving an IRP model whose planning horizon H and start date t are specified by the simulation clock. Whenever the optimization model is invoked, distribution, production, and pricing decisions, over H periods are determined, out of which the decisions for the first Δt periods are implemented. The system state is updated afterward to account for these changes, and performance metrics, such as incurred costs and profits, are recorded. Next, the simulation clock is advanced with a fixed time

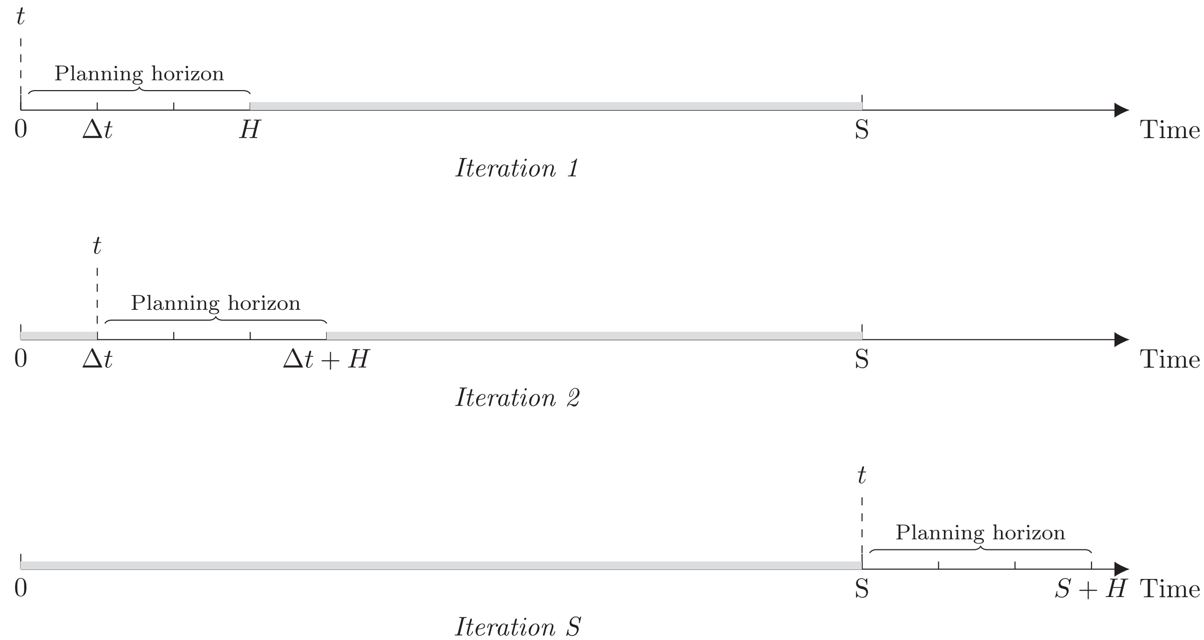


Fig. 2. Illustration of the time advance mechanism.

increment ($\Delta t \leq H$), the current time (t) is updated, and the procedure is reiterated. The simulation continues until reaching the end of the simulation horizon at S. Figure 2 describes the time advance mechanism between iterations. At each time instant (t), information about customers' demand is known for the next H periods.

5. Computational study

In this section, we present computational experiments that were carried out on data instances of the IRP adopted from Archetti et al. (2007). All runs were made on a PC equipped with an Intel Core i5-6300U CPU running at i7-8700, with 16GB of RAM. The mathematical models described in Section 3 and the simulation framework described in Section 4 were coded in AMPL language and compiled with CPLEX Optimization Studio 12.8 using its default setting.

5.1. Description of problem instances

The data set comprises nine generic instances, hereafter referred to as Ins_n_L, where n corresponds to the number of customers, $n \in \{5, 10, 15\}$, and L represents the maximum route length, $L \in \{2, 3, 4\}$. A fleet of $m = 3$ identical vehicles is used in each instance. Data related to the supplier, customers, and vehicles' capacities are kept as in Archetti et al. (2007). More specifically, the production quantity r_{it} consumed by a customer $i \in V'$ at time $t \in T$ is constant over time, and represented by an integer value in the interval [10,100]. Inventory holding costs are selected in the interval [0.1,0.5]. For each of the nine instances, the planning horizon is varied in the range $H = 1, \ldots, 8$.

Parameter values for the four different models are set to correspond to the cost minimization case with standard parameter values from the benchmark instances. Hence, the production function f is given by $f(r) = 5 \times 10^{-4}r + \frac{3}{r} + 2$, where the production rate r can be either stationary or variable. The maximum production rate r_{max} was set equal to the sum of customers' demands. Similarly, the demand function g is defined by $g(p) = -2.5p + 113$, where the price limit p_{max} is equal to 41. Given fixed demand values, the corresponding price can be calculated by the inverse function g^{-1}. The penalty term for unsatisfied demanded b_i is a proportion of the price and is set to $0.2p_i$. In the particular case of monopoly, penalty values were fixed *a priori* and were given values equal to those obtained in the other models.

Five grid points are used for the piece-wise linear transformation of the objective function in the monopoly case. Likewise, five grid points are selected to transform the nonlinear terms in the IRP model with variable production costs. Further empirical studies can be carried out to determine the optimal number of grid points; however, this is beyond this paper's scope. The simulation length (S) was set to 30 days, and jumps of $\Delta t = 1$ day in time are used to reproduce a real planning situation and provide maximum flexibility for re-planning. It implies that for each simulation, 30 calculations of IRP decisions are conducted. A time limit of 30 minutes is enforced in each step of the simulation, providing an overall time limit of approximately 15 hours per simulation run.

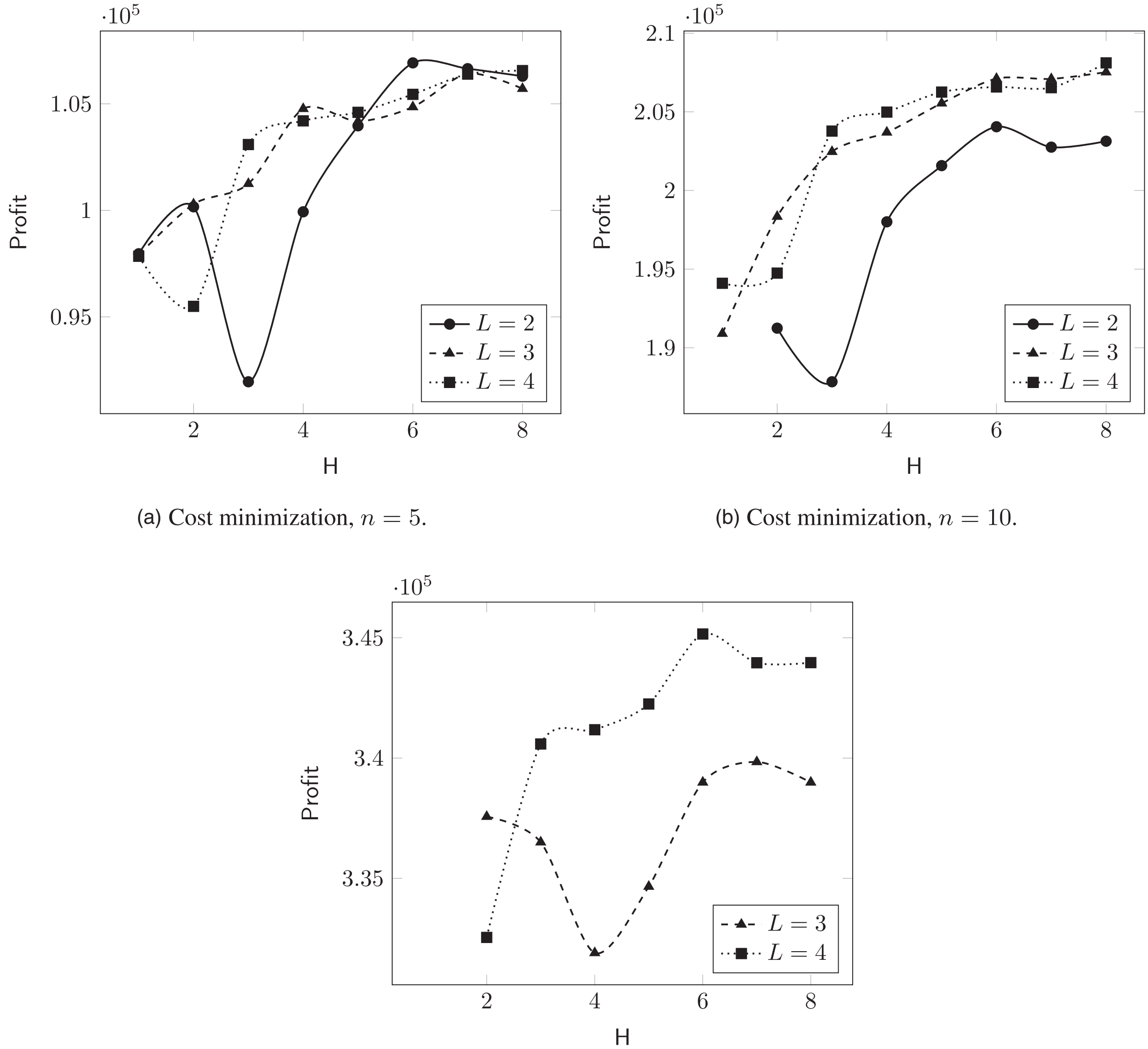
5.2. Performance of the cost minimization model

In this section, we discuss several experiments to test the long-term quality of policies produced by a cost minimization variant of the IRP. We examine the effect of varying the length of the planning horizon on the performance of IRP decisions. Figure 3 presents a summary of the results grouped by the number of customers. The horizontal axis represents the planning horizon, and the vertical axis represents recorded profits. A significant performance difference appears between one-period and eight-period tests. We see also that the profits obtained are, in general, increasing with the length of the planning horizon. Thus, there is evidence that a potential gain can be achieved by selecting longer planning horizons.

When planning horizons are short and the maximum route lengths low, the end of horizon effects can lead to simulations where the encountered IRPs do not have any feasible solutions. This happens when $L = 2$ and $n = 15$, as well as for $L = 2$ and $n = 10$ with $H = 1$, and these results are omitted in Fig. 3. For more detailed results, the reader can refer to Tables C.1–C.3 in Appendix C.

5.3. Performance of the profit maximization model with lost sales

In Fig. 4, we present the results of tests conducted for the profit maximization case with lost sales. Again, higher profits are obtained for tests with longer planning horizons. In all tests, we were able to derive feasible decisions for the IRP with profit maximization. In contrast to the cost



(a) Cost minimization, $n = 5$.

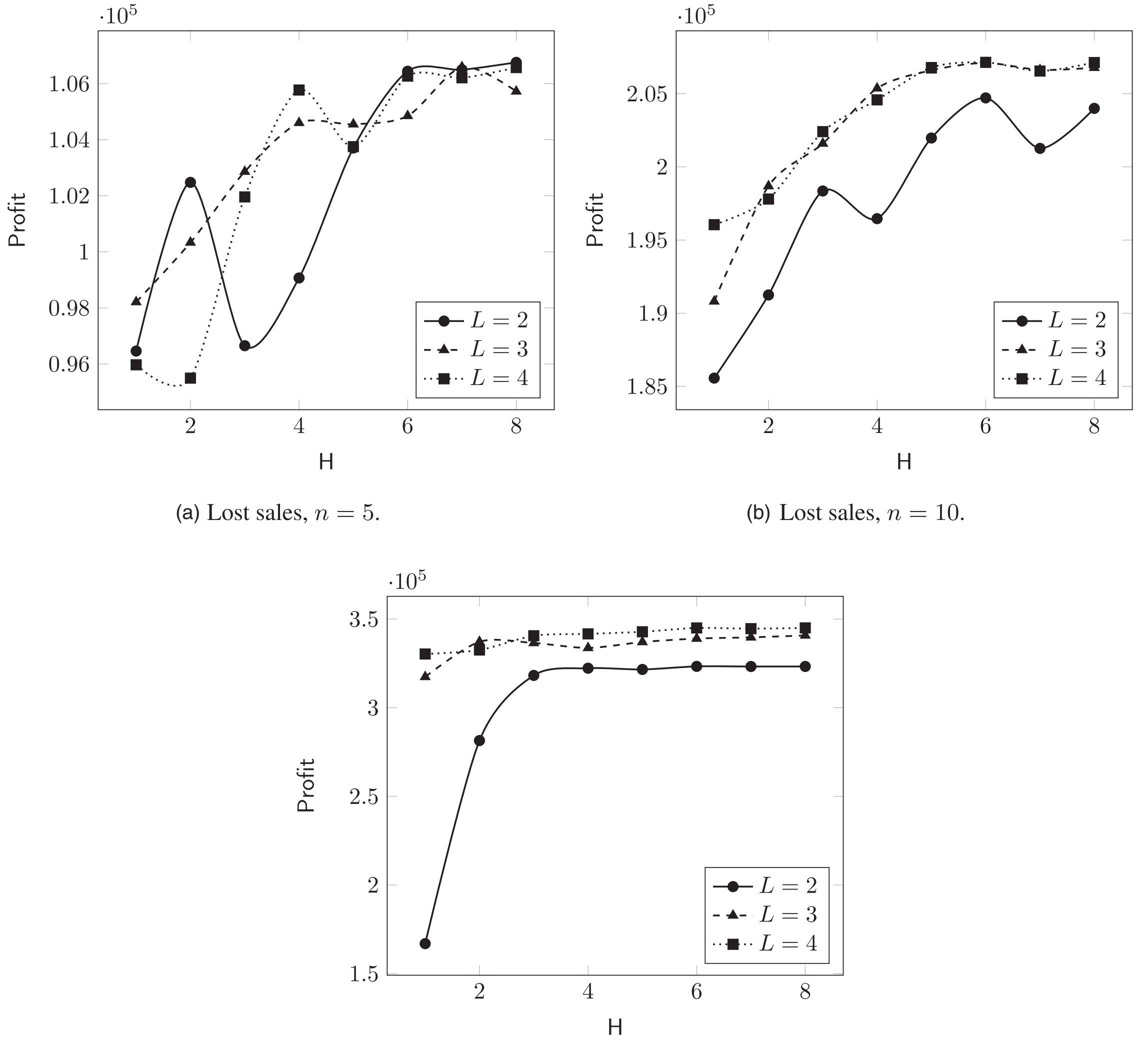
(b) Cost minimization, $n = 10$.

(c) Cost minimization, $n = 15$.

Fig. 3. Effect of varying the length of the planning horizon for the cost minimization case.

minimization variant, allowing lost sales induces more flexibility in the IRP solutions, thereby avoiding stock-outs at customers at the cost of added complexity.

Detailed experimental results are listed in Tables C.4–C.6 in Appendix C. They show high inventory costs at the supplier, low transportation costs, and low inventory holding costs at customers for reduced planning horizons. These cost patterns indicate near-empty inventories at customers; meanwhile, inventories are full at the supplier. Also, high costs of lost sales are recorded for short planning horizons; however, these costs decrease when the IRP is solved with longer planning



(a) Lost sales, $n = 5$.

(b) Lost sales, $n = 10$.

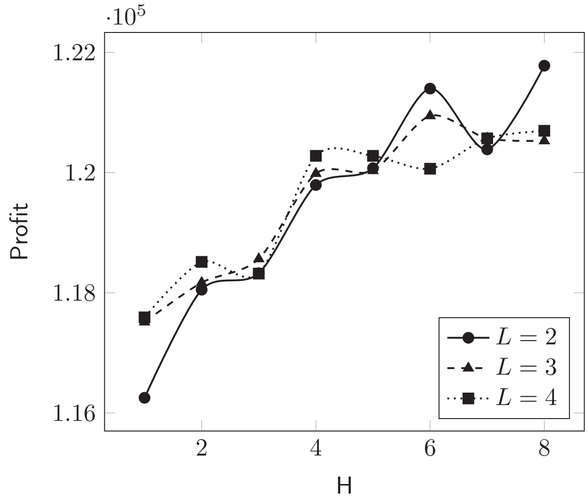
(c) Lost sales, $n = 15$.

Fig. 4. Effect of varying the length of the planning horizon for the case of profit maximization with lost sales.

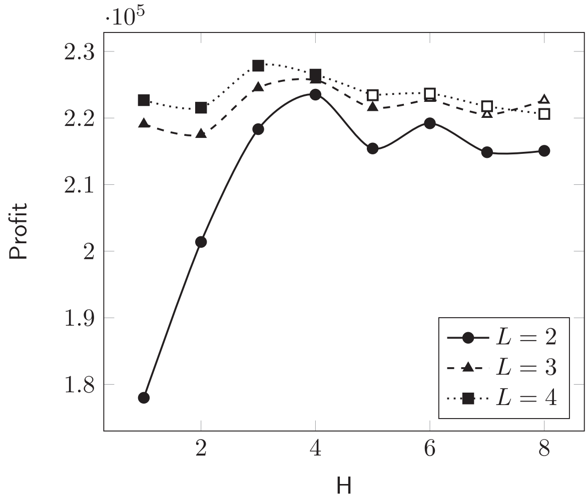
horizons. A natural explanation is that short term decision involves delivering less to customers. In the short run, it is more profitable to pay the penalty for unsatisfied customers rather than mobilizing resources.

5.4. *Performance of the profit maximization model under monopoly*

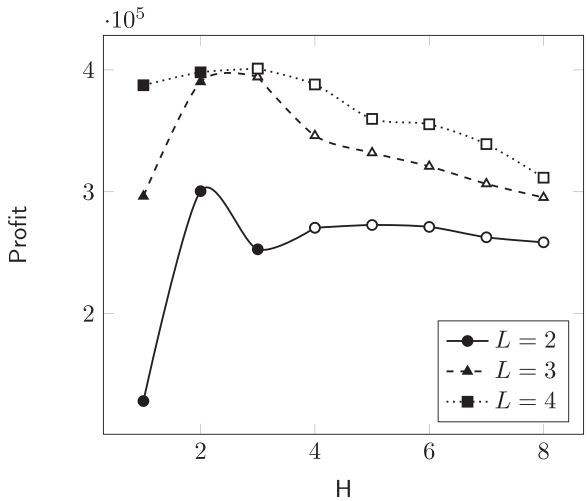
Figure 5 summarizes the results of experiments conducted for the case of a price-setting company. The corresponding mathematical model is harder to solve, and in 50% of the simulations, optimality



(a) Monopoly, $n = 5$.



(b) Monopoly, $n = 10$.



(c) Monopoly, $n = 15$.

Fig. 5. Effect of varying the length of the planning horizon for the monopoly case.

cannot be proven within the 30-minute time limit for all the individual IRP instances encountered. Furthermore, the gap to the dual bound can be up to 62% on average for the hardest instances. Instances that yield an average gap higher than 1% are highlighted by using a white fill color in Fig. 5. These large optimality gaps affect the slope of the profit function, which behaves differently from the other models. Better results are sometimes recorded for shorter planning horizons, where better solutions can be found for the instances encountered. This effect is most apparent for the instances with $n = 15$ customers.

For a price-setting company, the customer demands can be decreased by increasing the unit price. This does happen in the test cases examined here, with the consequence that inventories at the supplier become almost full. Besides, the cost of lost sales can be higher when using the monopoly model. This occurs when the solution implies a price and a corresponding demand such that the demand cannot be completely satisfied. For instance, the cost of lost sales is higher when the planning horizon or the route length is short. In this case, the inventory levels at the supplier may become very high. The results show that substantial savings can be achieved when the planning horizon is longer. The savings are mainly due to a reduction in transportation costs. A more limited reduction is owed to a decrease in costs at the supplier. The detailed cost results are shown in Tables C.7–C.9 in Appendix C.

5.5. *Performance of the profit maximization model with variable production costs*

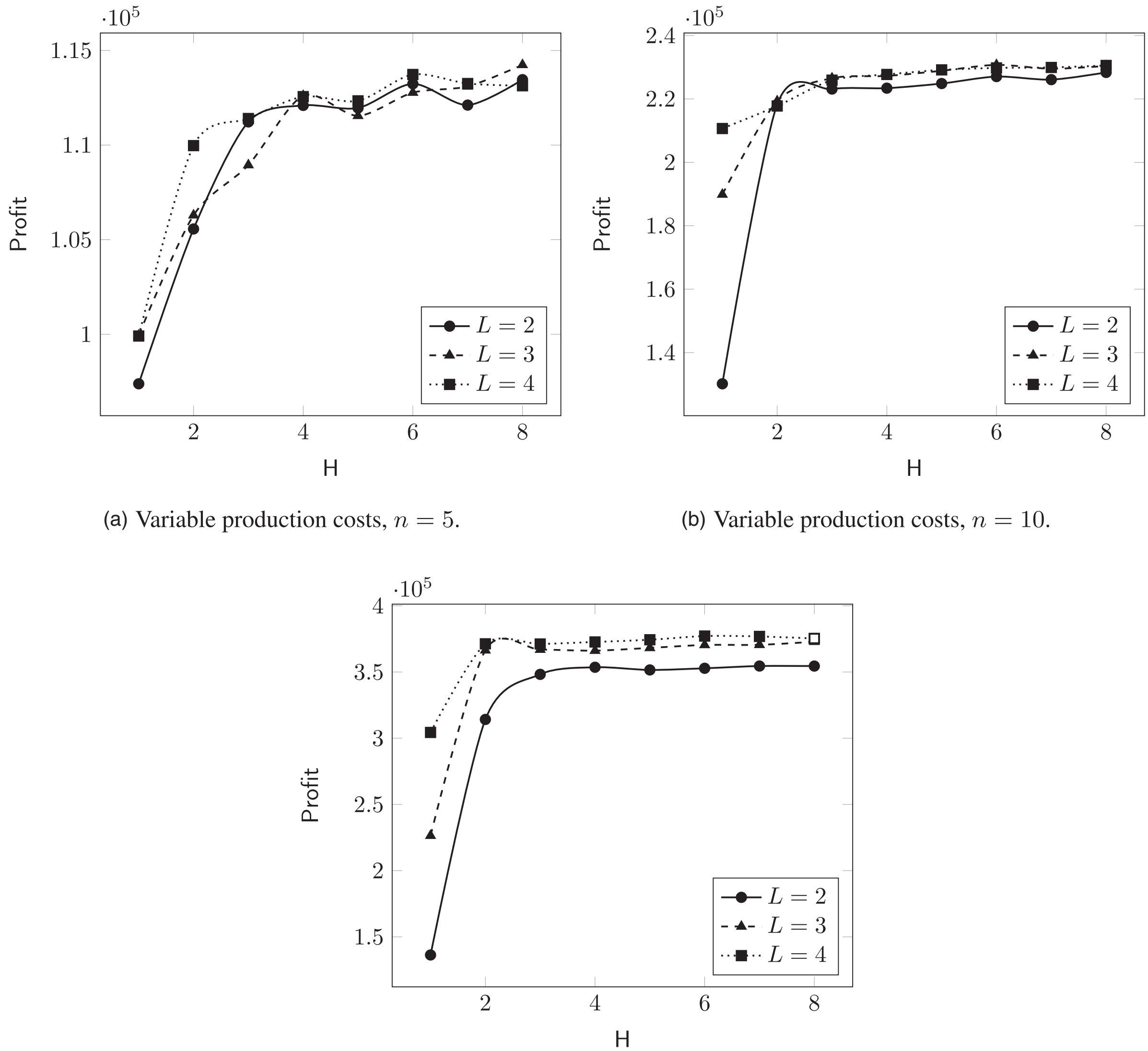
For the final IRP-variant, with variable production costs, almost all IRPs encountered were solved to optimality, except in 11 of the simulations. For these simulations, the maximum recorded optimality gap was 1.56%. The single simulation with an average optimality gap above 1% is highlighted in Fig. 6 using a white fill color. It appears that a stable performance is obtained once the length of the planning horizon reaches three periods.

Low inventory costs are incurred at the supplier. As the production rates can be adjusted, the model forces the company to produce no more than necessary. In contrast, inventory costs at customers, respectively, inventory levels, are high. If the planning horizon is longer, the optimal decisions involve increasing the production rate in early periods, leading to a lower unit production cost and, eventually, further savings. A longer horizon offers more possibilities for the coordination of production and transportation. Detailed results are presented in Tables C.10–C.12 in Appendix C.

5.6. *Comparing the different classes of the IRP*

Given the results provided in the previous section, we assess the long-term gain of using profit maximization in the IRP, rather than deciding prices and production rates *a priori* while not allowing lost sales. We make the following comparisons:

- *Cost minimization versus profit maximization with lost sales.* The profit maximization model with lost sales performs better than or equal to the cost minimization variant in only 47 out of 72 simulations. Allowing lost sales leaves room for flexibility in planning and induces substantial savings in transportation and inventory holding costs. These differences are considerable when route lengths are short, in which case allowing lost sales results in superior profits. When the route length is very restrictive, it is more important to select the customers to be served wisely. However, when the planning horizon is longer, the differences in quality between the two policies tend to be smaller. For $H = 6$, the cost minimization model leads to 0.45% higher profits than the lost sales model. From a purely theoretical perspective, the model with lost sales should perform at least as well as the cost minimization model: the only difference is increased flexibility through the



(a) Variable production costs, $n = 5$.

(b) Variable production costs, $n = 10$.

(c) Variable production costs, $n = 15$.

Fig. 6. Effect of varying the length of the planning horizon for the variable production costs case.

possibility of losing sales. This strongly suggests that the lost sales model's performance depends on having an even longer planning horizon than tested in this research.

- *Cost minimization versus monopoly.* When compared to the cost minimization variant, the monopoly variant records the overall best profit values. It outperforms the former in 91.6% of the simulations, while the average improvement in profit is 12.9%. The difference in quality reaches its peak with medium length planning horizons (generally $H = 3$, or $H = 4$). However, the monopoly model is too complex to allow optimal solutions to be found within the allotted

Table 1
Performance comparison of profit maximization models with regard to a baseline cost minimization test case with $H = 3$

n	L	Baseline profit	(%) improvement with lost sales	(%) improvement with monopoly	(%) improvement with variable production costs
5	2	91,955	5.1	28.7	21.0
5	3	101,250	1.6	17.1	7.6
5	4	103,090	−1.1	14.8	8.1
10	2	187,839	5.6	16.2	18.8
10	3	202,463	−0.4	10.9	11.9
10	4	203,776	−0.7	11.8	10.9
15	2	*	*	*	*
15	3	336,502	0.0	17.2	9.0
15	4	340,586	0.0	17.9	9.0

*This setting led to infeasible IRP instances.

running time. This means that for test cases with longer planning horizons and more customers, the potential improvement from allowing prices to be optimized is likely underestimated.

- *Cost minimization versus variable production costs.* The results demonstrate that the variable production costs policy consistently outperforms the cost minimization policy. Deciding about the production rate leads to lower unit production costs and lower inventory holding costs. Furthermore, the differences in quality appear to increase with the length of the planning horizon. Planning over longer planning horizons leads to better coordination of production and distribution. However, the variable production cost model produces solutions with almost empty inventories at the supplier, exemplifying an end-of-horizon effect that increases the risk of making bad decisions unless the planning horizon is sufficiently long.

Another way to compare the performance of the different IRP policies is to compute their relative gap. Table 1 shows the performance ratio $(P^x - P^c)/P^c$, where P^x refers to the profit recorded by a profit maximization model, and P^c represent the profit obtained by the cost minimization model. All of them are computed for a baseline test case with $H = 3$. The table shows that substantial savings can be achieved by adopting different profit maximization policies for the IRP. We also observe that increasing the planning horizon has a more pronounced effect for instances with small number of customers than in instances with large number of customers.

Our findings can be juxtaposed with those obtained by Zaitseva et al. (2018), where the authors conducted two experiments using a single planning instant to evaluate the quality of the different profit maximization policies. The two instances solved varied in terms of the length of the planning horizon, and the number of customers. Their results indicated that adopting a profit maximization policy for a price-setting company generates up to 283% in increased profits for a 5-customer instance, and up to 96% in increased profits for a 10-customer instance. Based on our simulations, these results are highly exaggerated, with more appropriate estimates being less than 15% and 8% profit increases, respectively. The discrepancy is due to the myopic nature of the IRP models: They merely consider the immediate gain, rather than anticipate the future effects taken into account through the simulations.

Table 2
Summary of cost minimization results for eight tests with $n = 5$ and no limit on the maximum route length

H	CPU(s)	%Gap	TC	VC	IHSU	IHC	TP
1	3.1	0.0	50,295	30,743	6950	373	97,281
2	4.0	0.0	52,081	32,678	5657	1518	95,495
3	6.1	0.0	44,486	25,089	5614	1554	103,090
4	11.5	0.0	41,523	22,098	5875	1321	106,053
5	17.3	0.0	42,974	23,535	5825	1386	104,602
6	32.7	0.0	42,130	22,711	5846	1344	105,446
7	57.2	0.0	41,659	22,220	5713	1497.67	105,917
8	73.4	0.0	41,002	21,556	5933	1284	106,574

Table 3
Summary of profit maximization with lost sales results for eight tests with $n = 5$ and no limit on the maximum route length

H	CPU(s)	%Gap	TC	VC	IHSU	IHC	Lost sales	TP
1	4.2	0.0	50,516	30,620	7122	355	191	96,107
2	8.5	0.0	52,081	32,678	5657	1518	0	95,495
3	26.9	0.0	44,591	24,912	5694	1564	192	102,025
4	55.3	0.0	40,929	21,498	5849	1352	0	106,647
5	192.4	0.0	43,233	23,566	5978	1339	122	103,731
6	328.3	0.0	41,272	21,849	5873	1321	0	106,304
7	535.8	0.0	41,168	21,717	5728.2	1494	0	106,408
8	1243.1	0.0	41,001	21,556	5937	1279	0	106,575

Table 4
Summary of profit maximization under monopoly results for eight tests with $n = 5$ and no limit on the maximum route length

H	CPU(s)	%Gap	TC	VC	IHSU	IHC	Lost sales	TP
1	4.8	0.00	54,568	39,724	2582	33	0	117,634
2	14.8	0.00	52,751	37,676	2171	513	162	118,738
3	53.7	0.00	51,889	37,243	1602	645	170	118,982
4	235.5	0.00	49,567	34,775	1518	828	217	120,456
5	1733.7	0.00	49,804	35,002	1521	944	109	120,545
6	8503.2	0.00	48,080	32,900	1582	1059	310	121,289
7	42,790.3	0.32	49,206	34,075	1541	1140	222	120,552
8	54,286.4	0.99	49,541	34,345	1640	1139	187	120,286

5.7. Investigating the impact of route limit constraints

In the results so far, it can be seen that increasing the maximum route length from three to four visits has a relatively small effect on the final profits. In this section, we examine how IRP decisions change when the route length limitations are lifted completely. We run new tests for instances with five customers with a full enumeration of routes. Tables 2–5 present results of tests conducted for the unconstrained versions of our IRP models.

Table 5
Summary of profit maximization with variable production costs results for eight tests with $n = 5$ and no limit on the maximum route length

H	CPU(s)	%Gap	TC	VC	IHSU	IHC	PC	Lost sales	TP
1	2.8	0.0	47,218	36,248	364	48	10,468	90	99,909
2	3.8	0.0	35,992	24,605	248	423	10,393	323	109,971
3	7.609	0.0	36,116	24,795	223	511	10,576	12	111,403
4	9.7	0.0	35,016	23,442	284	657	10,633	0	112,560
5	19.5	0.0	35,216	23,599	336	648	10,630	4	112,338
6	56.8	0.0	33,825	22,222	278	693	10,632	0	113,751
7	108	0.0	34,049	22,253	321	805	10,615	55	113,252
8	294	0.0	34,442	22,565	324	848	10,705	0	113,134

Observe that 15 out of the 32 unconstrained tests record higher profits than tests where the maximum number of stops is enforced. An increase in the route size results in lower transportation costs and an increase in profits in consequence. We also observe that the gain earned by using a maximum route size is relatively small in terms of the total profits. The average increase in profit is 0.09% for the basic cost minimization model, 0.15% for the profit maximization model with lost sales, 0.23% for the profit maximization model with monopoly, and 0.00% for the variable production costs model.

Interestingly, for 17 tests out of 32, it seems beneficial to enforce route limit constraint. The explanation may be simple. When only a few customers are assigned to a vehicle, they can be served with higher quantities. In the longer run, this translates into fewer visits, and subsequently, lower transportation costs.

6. Concluding remarks

The contribution of this paper is twofold. First, we investigated the consequences of explicitly maximizing profits rather than minimizing costs in an IRP setting. Four IRP models were implemented to take into account different market conditions: classic cost minimization, profit maximization with lost sales, profit maximization under monopoly, and profit maximization with variable production costs. The two latter variants include nonlinear expressions, which are linearized using a piece-wise linear transformation. Second, we examined the effect of varying the planning horizon's length on the quality of the decisions derived by solving an IRP model. In this regard, we developed a deterministic simulator that calculates the long-term effects of making IRP decisions over short planning horizons. In doing so, we diverge from the typical literature on the IRP, which tends to focus on increasing the problem size in terms of the number of nodes, while ignoring the consequences of considering an increased planning horizon.

Extensive computational experiments show that the length of the planning horizon can significantly affect decision-making performance. By considering planning horizons for up to eight periods, we showed that end-of- horizon effects could be mitigated. However, in some cases, it is unclear how many periods are required to achieve stable performance. The obtained results also demonstrate the viability of considering profit maximization within an IRP setting, particularly when the company can adjust its production volumes.

Several research avenues appear promising to enhance the proposed study. The consideration of larger test instances is one such avenue. A different one would take into account even longer planning horizons. A third promising research avenue is the utilization of heuristic mechanisms to derive IRP decisions. The exploration of this latter direction could provide a significant impetuous to solve IRP models with long planning horizons.

Acknowledgments

This research was carried out with financial support from the AXIOM project, partly funded by the Research Council of Norway. We thank the associate editor and the two reviewers for their valuable comments on our manuscript, which have helped us to improve it.

References

Agra, A., Christiansen, M., Delgado, A., Simonetti, L., 2014. Hybrid heuristics for a short sea inventory routing problem. *European Journal of Operational Research* 236, 924–935.

Al-Ameri, T.A., Shah, N., Papageorgiou, L.G., 2008. Optimization of vendor-managed inventory systems in a rolling horizon framework. *Computers & Industrial Engineering* 54, 1019–1047.

Andersson, H., Christiansen, M., Desaulniers, G., 2015. A new decomposition algorithm for a liquefied natural gas inventory routing problem. *International Journal of Production Research* 54, 564–578.

Andersson, H., Hoff, A., Christiansen, M., Hasle, G., Løkketangen, A., 2010. Industrial aspects and literature survey: combined inventory management and routing. *Computers & Operations Research* 37, 1515–1536.

Anily, S., Federgruen, A., 1990. One warehouse multiple retailer systems with vehicle routing costs. *Management Science* 36, 92–114.

Archetti, C., Bertazzi, L., Laporte, G., Speranza, M.G., 2007. A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science* 41, 382–391.

Archetti, C., Coelho, L.C., Speranza, M.G., 2019. An exact algorithm for the inventory routing problem with logistic ratio. *Transportation Research Part E: Logistics and Transportation Review* 131, 96–107.

Archetti, C., Desaulniers, G., Speranza, M.G., 2016. Minimizing the logistic ratio in the inventory routing problem. *EURO Journal on Transportation and Logistics* 6, 289–306.

Bard, J.F., Huang, L., Jaillet, P., Dror, M., 1998. A decomposition approach to the inventory routing problem with satellite facilities. *Transportation Science* 32, 189–203.

Bazaraa, M.S., Sherali, H.D., Shetty, C.M., 2005. Linear complementary problem, and quadratic, separable, fractional, and geometric programming. In *Nonlinear Programming*. John Wiley & Sons, Hoboken, NJ, pp. 655–749.

Chien, T.W., Balakrishnan, A., Wong, R.T., 1989. An integrated inventory allocation and vehicle routing problem. *Transportation Science* 23, 67–76.

Chitsaz, M., Divsalar, A., Vansteenwegen, P., 2016. A two-phase algorithm for the cyclic inventory routing problem. *European Journal of Operational Research* 254, 410–426.

Coelho, L.C., Cordeau, J.F., Laporte, G., 2014a. Heuristics for dynamic and stochastic inventory-routing. *Computers & Operations Research* 52, 55–67.

Coelho, L.C., Cordeau, J.F., Laporte, G., 2014b. Thirty years of inventory routing. *Transportation Science* 48, 1–19.

Dai, Z., Gao, K., Giri, B., 2020. A hybrid heuristic algorithm for cyclic inventory-routing problem with perishable products in VMI supply chain. *Expert Systems with Applications* 153, 113322.

Dror, M., Ball, M., Golden, B., 1985. A computational comparison of algorithms for the inventory routing problem. *Annals of Operations Research* 4, 1–23.

Ekici, A., Örsan Özener, O., Kuyzu, G., 2015. Cyclic delivery schedules for an inventory routing problem. *Transportation Science* 49, 817–829.

Fagerholt, K., Christiansen, M., Hvattum, L.M., Johnsen, T.A., Vabø, T.J., 2010. A decision support methodology for strategic planning in maritime transportation. *Omega* 38, 465–474.

Goyal, S., Gunasekaran, A., 1995. An integrated production-inventory-marketing model for deteriorating items. *Computers & Industrial Engineering* 28, 755–762.

Grønhaug, R., Christiansen, M., 2009. Supply chain optimization for the liquefied natural gas business. In Fandel, G., Trockel, W. (eds) *Lecture Notes in Economics and Mathematical Systems*. Springer, Berlin–Heidelberg, pp. 195–218.

Grønhaug, R., Christiansen, M., Desaulniers, G., Desrosiers, J., 2010. A Branch-and-price method for a liquefied natural gas inventory routing problem. *Transportation Science* 44, 400–415.

Hemmati, A., Stålhane, M., Hvattum, L.M., Andersson, H., 2015. An effective heuristic for solving a combined cargo and inventory routing problem in tramp shipping. *Computers & Operations Research* 64, 274–282.

Jaillet, P., Bard, J.F., Huang, L., Dror, M., 2002. Delivery cost approximations for inventory routing problems in a rolling horizon framework. *Transportation Science* 36, 292–300.

Kleywegt, A.J., Nori, V.S., Savelsbergh, M.W.P., 2002. The stochastic inventory routing problem with direct deliveries. *Transportation Science* 36, 94–118.

Kleywegt, A.J., Nori, V.S., Savelsbergh, M.W.P., 2004. Dynamic programming approximations for a stochastic inventory routing problem. *Transportation Science* 38, 42–70.

Krugman, P., Wells, R., 2015. *Microeconomics*. Macmillan Learning, New York, NY.

Lefever, W., Aghezzaf, E.H., Hadj-Hamou, K., 2016. A convex optimization approach for solving the single-vehicle cyclic inventory routing problem. *Computers & Operations Research* 72, 97–106.

Liu, S.C., Lee, W.T., 2011. A heuristic method for the inventory routing problem with time windows. *Expert Systems with Applications* 38, 13223–13231.

Montagné, R., Gamache, M., Gendreau, M., 2019. A shortest path-based algorithm for the inventory routing problem of waste vegetable oil collection. *Journal of the Operational Research Society* 70, 986–997.

Papageorgiou, D.J., Cheon, M.S., Nemhauser, G., Sokol, J., 2015. Approximate dynamic programming for a class of long-horizon maritime inventory routing problems. *Transportation Science* 49, 870–885.

Papageorgiou, D.J., Keha, A.B., Nemhauser, G.L., Sokol, J., 2014a. Two-stage decomposition algorithms for single product maritime inventory routing. *INFORMS Journal on Computing* 26, 825–847.

Papageorgiou, D.J., Nemhauser, G.L., Sokol, J., Cheon, M.S., Keha, A.B., 2014b. MIRPLib – a library of maritime inventory routing problem instances: survey, core model, and benchmark results. *European Journal of Operational Research* 235, 350–366.

Raa, B., Aghezzaf, E.H., 2008. Designing distribution patterns for long-term inventory routing with constant demand rates. *International Journal of Production Economics* 112, 255–263.

Raa, B., Aghezzaf, E.H., 2009. A practical solution approach for the cyclic inventory routing problem. *European Journal of Operational Research* 192, 429–441.

Raa, B., Dullaert, W., 2017. Route and fleet design for cyclic inventory routing. *European Journal of Operational Research* 256, 404–411.

Rakke, J.G., Stålhane, M., Moe, C.R., Christiansen, M., Andersson, H., Fagerholt, K., Norstad, I., 2011. A rolling horizon heuristic for creating a liquefied natural gas annual delivery program. *Transportation Research Part C: Emerging Technologies* 19, 896–911.

Stålhane, M., Andersson, H., Christiansen, M., Fagerholt, K., 2014. Vendor managed inventory in tramp shipping. *Omega* 47, 60–72.

Trudeau, P., Dror, M., 1992. Stochastic inventory routing: route design with stockouts and route failures. *Transportation Science* 26, 171–184.

Vansteenwegen, P., Mateo, M., 2014. An iterated local search algorithm for the single-vehicle cyclic inventory routing problem. *European Journal of Operational Research* 237, 802–813.

Zaitseva, A., Hvattum, L.M., Urrutia, S., 2018. Profit maximization in inventory routing problems. In *2018 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, IEEE, Piscataway, NJ, pp. 1230–1234.

Zenker, M., Emde, S., Boysen, N., 2016. Cyclic inventory routing in a line-shaped network. *European Journal of Operational Research* 250, 164–178.

Appendix A: Linearization process of IRP model under monopoly

To enhance the solvability of the proposed nonlinear IRP under monopoly, we first apply a transformation technique to reduce the model to a separable form. Next, we use a piecewise linear approximation to derive an equivalent linear formulation. We start by recalling the nonseparable term in the objective (3.1) needed throughout:

Furthermore, we define the ensuing substitutions:

$$\alpha_{it} = \frac{1}{2}(p_i + w_{it}), \quad i \in V', t \in T, \tag{A.1}$$

$$\beta_{it} = \frac{1}{2}(p_i - w_{it}), \quad i \in V', t \in T, \tag{A.2}$$

$$\alpha_{it}, \beta_{it} \geq 0, \quad i \in V', t \in T. \tag{A.3}$$

Without loss of generality and for simplicity of description, we assume that the domains for (z) and (p) are used to define the auxiliary variables (α) and (β). Strictly speaking, given [0, p_{max}], respectively, [0, Q], the domain of (p), respectively, of (z), then (α) and (β) are defined over [l_α, u_α], respectively, [l_β, u_β], such that

$$[l_\alpha, u_\alpha] = \left[0, \frac{1}{2}(p_{max} + Q)\right], \tag{A.4}$$

$$[l_\alpha, u_\alpha] = \left[0, \frac{1}{2}(p_{max} - Q)\right]. \tag{A.5}$$

We note the product $p_i z_{it}$ can be replaced by $\alpha_{it}^2 - \beta_{it}^2$, as long as the domains are preserved. Using the identities (A.1) and (A.2), we obtain

$$\sum_{i \in V'} \sum_{t \in T} p_i w_{it} = \sum_{i \in V'} \sum_{t \in T} (\alpha_{it}^2 - \beta_{it}^2). \tag{A.6}$$

The expression (A.6) contains now nonlinear functions α_{it}^2 and β_{it}^2 of single variables and is separable.

The separable problem defined in (A.6) can be approximated into a linear form by replacing each nonlinear function with a piecewise linear approximation. For illustration, consider the function θ defined by $\theta(\mu) = \mu^2$. In general, the function θ can be approximated for any finite interval [a, b] via the grid points $\mu_1, \ldots, \mu_n$ by the piecewise linear function $\hat{\theta}$

$$\hat{\theta}(\mu) = \sum_{j=1}^{n} \lambda_j \theta(\mu_j), \sum_{j=1}^{j} \lambda_j = 1; \lambda_j \geq 0, \quad j = 1, \ldots, n, \tag{A.7}$$

where at most two λ_j-variables are positive, and they must be adjacent. This condition can be modeled using binary variables s_j for $j = 1, \ldots, n-1$ (where $s_j = 1$ if $\mu_j \leq \mu \leq \mu_{j+1}$ and $s_j = 0$

otherwise) and the constraints:

$$\lambda_1 \leq s_1,$$
$$\lambda_i \leq s_{j-1} + s_j, \quad j = 1, \ldots, n-1,$$
$$\lambda_n \leq s_{n-1},$$
$$\sum_{j=1}^{n-1} s_j = 1,$$
$$s \in [0, 1].$$

This set of variables and constraints represents a special-ordered set of type 2 (SOS 2), and this representation is called the λ-form approximation (Bazaraa et al., 2005). In many modern MILP solvers, it is possible to declare special ordered sets explicitly.

Using these definitions, we introduce now the grid points $\hat{\alpha}_{itj}$ for $i \in V'$, $t \in T$ and $j = 1, \ldots, n_\alpha$, where $\hat{\alpha}_{,1} = 0$ and $\hat{\alpha}_{it,n_\alpha} = \frac{1}{2}(p_{max} + Q)$. Hence, the therm α_{it}^2 could be replaced for each $i \in V'$, $t \in T$ by its linear approximation

$$\alpha_{it}^2 = \sum_{j=1}^{n_\alpha} \lambda_{itj}^{\alpha} \hat{\alpha}_{itj}^2, \quad i \in V', t \in T, \tag{A.8}$$

$$\alpha_{it} = \sum_{j=1}^{n_\alpha} \lambda_{itj}^{\alpha} \hat{\alpha}_{itj}, \quad i \in V', t \in T, \tag{A.9}$$

$$\sum_{j=1}^{n_\alpha} \lambda_{itj}^{\alpha} = 1, \quad i \in V', t \in T, \tag{A.10}$$

$$\lambda_{itj}^{\alpha} \geq 0, \quad j = 1, \ldots, n_\alpha, i \in V', t \in T. \tag{A.11}$$

Similarly, we can derive a linear representation of β_{it}^2 using the grid points $\hat{\beta}_{itj}$ for $i \in V'$, $t \in T$, and $j = 1, \ldots, n_\beta$, where $\hat{\beta}_{,1} = 0$ and $\hat{\beta}_{it,n_\beta} = \frac{1}{2}(p_{max} - Q)$. Hence, the corresponding linear approximation is described by

$$\beta_{it}^2 = \sum_{j=1}^{n_\beta} \lambda_{itj}^{\beta} \hat{\beta}_{itj}^2, \quad i \in V', t \in T, \tag{A.12}$$

$$\beta_{it} = \sum_{j=1}^{n_\beta} \lambda_{itj}^{\beta} \hat{\beta}_{itj}, \quad i \in V', t \in T, \tag{A.13}$$

$$\sum_{j=1}^{n_\beta} \lambda_{itj}^{\beta} = 1, \quad i \in V', t \in T \tag{A.14}$$

$$\lambda_{itj}^{\beta} \geq 0, \quad j = 1, \ldots, n_\beta, i \in V', t \in T. \tag{A.15}$$

Bazaraa et al. (2005) maintain that the SOS constraints become redundant when minimizing a strictly convex function. Hence, the ones associated with the β-variables can be omitted, as we are maximizing the negation of a convex function ($\sum_{i\in V'}\sum_{t\in T}(-\beta_{it}^2)$). Finally, by using the foregoing definitions, the IRP under monopoly can be restated in an equivalent compact linear form as follows.

$$\text{Maximize} \sum_{j=1}^{n_\alpha} \lambda_{itj}^{\alpha}\hat{\alpha}_{itj}^2 + \sum_{j=1}^{n_\beta} \lambda_{itj}^{\beta}\hat{\beta}_{itj}^2 - \sum_{k\in K}\sum_{t\in T} c_k y_{kt}$$

$$-\sum_{i\in V}\sum_{t\in T} h_i I_{it} - \sum_{i\in V'}\sum_{t\in T} b_i(g(p_i) - w_{it}) - \sum_{t\in T} f(r_{0,t})r_{0,t}, \tag{A.16}$$

subject to

$$(1.2)-(1.5), \quad (1.7)-(1.10), \quad (2.2), \quad (3.2)-(3.5),$$

$$\alpha_{it} = \frac{1}{2}(p_i + w_{it}), \quad i \in V', t \in T, \tag{A.17}$$

$$\beta_{it} = \frac{1}{2}(p_i - w_{it}), \quad i \in V', t \in T, \tag{A.18}$$

$$\sum_{j=1}^{n_\alpha} \lambda_{itj}^{\alpha}\hat{\alpha}_{itj} = \alpha_{it}, \quad i \in V', t \in T, \tag{A.19}$$

$$\sum_{j=1}^{n_\beta} \lambda_{itj}^{\beta}\hat{\beta}_{itj} = \beta_{it}, \quad i \in V', t \in T, \tag{A.20}$$

$$\sum_{j=1}^{n_\alpha} \lambda_{itj}^{\alpha} = 1, \quad i \in V', t \in T, \tag{A.21}$$

$$\sum_{j=1}^{n_\beta} \lambda_{itj}^{\beta} = 1, \quad i \in V', t \in T \tag{A.22}$$

$$\lambda_{itj}^{\alpha} \geq 0, \quad j = 1, \ldots, n_\alpha, i \in V', t \in T, \tag{A.23}$$

$$\lambda_{itj}^{\beta} \geq 0, \quad j = 1, \ldots, n_\beta, i \in V', t \in T, \tag{A.24}$$

At most, two adjacent λ^α are nonzero. (A.25)

Appendix B: Linearization process of IRP model with variable production costs

To enhance the solvability of the IRP model with variable production costs, we propose to apply a λ-form piecewise linear approximation to derive an equivalent linear model representation of this problem. Toward this end, first consider the objective function (4.1) that include the nonlinear term:

$$\sum_{t\in T} u_1' R_{0,t}^2 + u_2' R_{0,t} + u_3'.$$

Following Bazaraa et al. (2005), we first consider the interval of interest [0, r_{max}]. We next define the grid points $\hat{R}_{jt}$ for $j = 1, \dots, n, t \in T$, where $\hat{R}_{1t} = 0$ and $\hat{R}_{nt} = r_{max}$. Accordingly, the functions for which $R_{0,t}$ is an argument could be replaced by their linear approximations:

$$R_{0,t}^2 = \sum_{j=1}^{n} \lambda_{jt} \hat{R}_{jt}^2, \quad j = 1, \dots, n, t \in T, \tag{B.1}$$

$$R_{0,t} = \sum_{j=1}^{n} \lambda_{jt} \hat{R}_{jt}, \quad j = 1, \dots, n, t \in T, \tag{B.2}$$

$$\sum_{j=1}^{n} \lambda_{jt} = 1, \quad t \in T, \tag{B.3}$$

$$\lambda_{jt} \geq 0, \quad j = 1, \dots, n, t \in T. \tag{B.4}$$

This yields the following equivalent reformulation of the IRP problem with variable production costs:

$$\text{Maximize} \sum_{i\in V'} \sum_{k\in K} \sum_{t\in T} p_0 c_{ikt} - \sum_{k\in K} \sum_{t\in T} c_k y_{kt} - \sum_{i\in V} \sum_{t\in T} h_i I_{it}$$
$$- \sum_{i\in V'} \sum_{t\in T} b_i (r_{it} - w_{it}) - \sum_{j=1}^{n} (u_1' \lambda_{jt} \hat{R}_{jt}^2 + u_2' \lambda_{jt} \hat{R}_{jt} + u_3'), \tag{B.5}$$

subject to

$$(1.2)-(1.6), \quad (1.8), \quad (2.2)-(2.3), \quad (4.2)-(4.6),$$

$$R_{0,t} = \sum_{j=1}^{n} \lambda_{jt} \hat{R}_{jt}, \quad j = 1, \dots, n, t \in T, \tag{B.6}$$

$$\sum_{j=1}^{n} \lambda_{jt} = 1, \quad t \in T, \tag{B.7}$$

$$\lambda_{jt} \geq 0, \quad j = 1, \dots, n, t \in T. \tag{B.8}$$

The SOS 2 variables and constraints are deliberately omitted from this formulation as the adjacency restriction inherently holds (see Bazaraa et al., 2005).

Appendix C: Summary of computational results

In Tables C.1–C.3, we summarize the results obtained for the cost minimization model, where columns 1 and 2 give the parameters of the test. In particular, the first column displays the selected time horizon, and the second column shows the allowed length of vehicle routes. Columns 3–9 show the results. More specifically, column 3 provides the CPU time in seconds. Column 4 gives the average gap, $\sum_{\Delta t=1}^{S} \frac{Z^D - Z}{Z^D}$, where Z refers to the cost of the incumbent solution at each simulation instant Δt, and Z^D is the dual bound found within the time limit. Column 5 shows the total incurred costs TC, and column 9 the total profits TP. Finally, columns 6–8 provides a summary of transportation and inventory holding costs. We denote by VC, IHS, and IHC the transportation costs, the inventory holding costs at the supplier, and the inventory holding costs at customers.

Table C.1
Summary of cost minimization results for 24 tests with $n = 5$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	TP
2	1	2.5	0.0	49,612	30,107	6750	526	97,964
2	2	2.9	0.0	47,409	28,013	6080	1087	100,167
2	3	3.7	0.0	55,621	36,317	5554	1521	91,955
2	4	5.7	0.0	47,644	28,210	5964	1241	99,932
2	5	9.6	0.0	43,608	24,186	5627	1566	103,968
2	6	12.7	0.0	40,653	21,205	5969	1250	106,923
2	7	17.3	0.0	40,925	21,476	5717	1504	106,651
2	8	34.2	0.0	41,279	21,845	5883	1323	106,297
3	1	2.3	0.0	49,667	30,138	6853	447	97,909
3	2	3.2	0.0	47,283	27,903	5957	1195	100,293
3	3	6.8	0.0	46,326	26,951	5546	1600	101,250
3	4	9.2	0.0	42,808	23,388	5889	1302	104,768
3	5	14.0	0.0	43,395	23,992	5609	1566	104,181
3	6	28.9	0.0	42,724	23,298	5854	1343	104,852
3	7	39.4	0.0	41,157	21,723	5629	1576	106,419
3	8	76.5	0.0	41,864	22,441	5888	1613	105,712
4	1	2.9	0.0	49,730	30,191	6894	416	97,846
4	2	4.1	0.0	52,081	32,678	5657	1518	95,495
4	3	7.3	0.0	44,486	25,089	5614	1554	103,090
4	4	11.2	0.0	43,375	23,957	5879	1311	104,201
4	5	19.0	0.0	42,974	23,535	5825	1386	104,602
4	6	31.4	0.0	42,130	22,711	5846	1344	105,446
4	7	48.5	0.0	41,182	21,744	5637	1571	106,394
4	8	78.4	0.0	41,016	21,578	5961	1249	106,560

Table C.2
Summary of cost minimization results for 24 tests with $n = 10$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	TP
2	1	*	*	*	*	*	*	*
2	2	5.9	0.00	126,226	59,074	19,037	3876	191,246
2	3	9.0	0.00	129,633	61,932	17,123	6340	187,839
2	4	11.9	0.00	119,465	52,463	19,260	3504	198,007
2	5	18.4	0.00	115,893	48,674	18,778	4203	201,579
2	6	33.5	0.00	113,421	46,099	18,397	4688	204,051
2	7	87.6	0.00	114,718	47,447	18,432	4601	202,754
2	8	129.4	0.00	114,341	47,084	18,676	4342	203,131
3	1	6.1	0.00	126,570	59,925	20,951	1455	190,902
3	2	14.3	0.00	119,131	52,220	19,093	3580	198,341
3	3	40.9	0.00	115,009	47,540	17,638	5593	202,463
3	4	182.5	0.00	113,777	46,529	18,518	4492	203,695
3	5	519.8	0.00	111,927	44,555	18,394	4739	205,545
3	6	3,511.1	0.00	110,360	43,037	18,596	4488	207,112
3	7	9,639.7	0.00	110,379	43,074	18,563	4504	207,093
3	8	22,920.6	0.03	109,950	42,672	18,692	4348	207,522
4	1	14.7	0.00	123,372	56,716	20,955	1463	194,100
4	2	74.1	0.00	122,724	55,995	18,018	4473	194,748
4	3	218.3	0.00	113,696	46,210	18,016	5232	203,776
4	4	1,374.6	0.00	112,490	45,224	18,483	4545	204,982
4	5	12,806.0	0.00	111,217	43,868	18,427	4683	206,255
4	6	34,445.3	0.17	110,892	43,641	18,453	4559	206,580
4	7	46,185.8	0.34	110,938	43,649	18,580	4471	206,534
4	8	50,676.8	0.51	109,360	42,056	18,615	5036	208,112

*This setting led to infeasible IRP instances.

Table C.3
Summary of cost minimization results for 24 tests with $n = 15$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	TP
2	1	*	*	*	*	*	*	*
2	2	*	*	*	*	*	*	*
2	3	*	*	*	*	*	*	*
2	4	*	*	*	*	*	*	*
2	5	*	*	*	*	*	*	*
2	6	*	*	*	*	*	*	*
2	7	*	*	*	*	*	*	*
2	8	*	*	*	*	*	*	*
3	1	*	*	*	*	*	*	*
3	2	67.3	0.00	137,539	53,606	27,404	1492	337,565
3	3	139.6	0.00	138,602	50,120	22,885	5713	336,502
3	4	533.0	0.00	143,205	54,529	24,307	4486	331,899
3	5	1,089.6	0.00	140,446	51,768	24,038	4755	334,658
3	6	1,940.1	0.00	136,114	47,574	23,183	5473	338,990

Continued

Table C.3
(Continued)

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	TP
3	7	2,004.4	0.00	135,264	46,658	23,723	4999	339,840
3	8	3,986.0	0.00	136,114	47,451	23,368	5412	338,990
4	1	*	*	*	*	*	*	*
4	2	238.9	0.00	142,561	39,311	27,202	1337	332,543
4	3	566.5	0.00	13,316	46,129	22,686	6003	340,586
4	4	5,370.9	0.00	13,316	45,387	23,703	4953	341,178
4	5	12,277.6	0.01	132,856	44,226	23,540	5206	342,248
4	6	23,589.0	0.11	129,946	41,375	23,419	5268	345,158
4	7	40,719.8	0.20	131,142	42,537	23,507	5214	343,962
4	8	51,496.8	0.55	131,136	42,676	22,920	5656	343,968

*This setting led to infeasible IRP instances.

Tables C.4–C.6 display the numerical results of the simulation study conducted for the case of profit maximization with lost sales. Columns 1 and 2 give the parameters of the test, and columns 3–10 show the results. An additional column is included to accommodate the cost of lost sales.

Table C.4
Summary of profit maximization with lost sales results for 24 tests with $n = 5$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	Lost sales	TP
2	1	2.6	0.0	50,129	21,901	7377	1185	1528	96,461
2	2	3.9	0.0	45,097	25,694	6113	1062	0	102,479
2	3	8.0	0.0	50,924	31,613	5460	1622	0	96,652
2	4	21.5	0.0	48,507	29,074	5942	1262	0	99,069
2	5	37.6	0.0	43,463	23,901	5762	1489	82	103,705
2	6	59.5	0.0	41,131	21,693	5926	1283	0	106,445
2	7	140.1	0.0	41,078	21,646	5662	1541	0	106,498
2	8	438.6	0.0	40,815	21,362	5962	1262	0	106,761
3	1	2.5	0.0	48,898	29,175	6955	446	94	98,210
3	2	6.7	0.0	47,242	27,857	5979	1178	0	100,334
3	3	21.3	0.0	44,076	24,470	5791	1459	128	102,862
3	4	47.7	0.0	42,936	23,498	5929	1274	7	104,604
3	5	163.8	0.0	42,503	22,944	5758	1468	104	104,553
3	6	215.4	0.0	42,724	23,298	5854	1343	0	104,852
3	7	387.9	0.0	40,982	21,539	5648	1566	0	106,594
3	8	1132.1	0.0	41,850	22,428	5923	1271	0	105,726
4	1	4.8	0.0	50,720	30,841	7114	360	177	95,971
4	2	7.8	0.0	52,081	32,678	12,229	1518	0	95,495
4	3	28.9	0.0	44,492	24,746	5747	1546	225	101,961
4	4	66.3	0.0	41,802	22,375	5752	1446	0	105,774
4	5	168.9	0.0	43,211	23,544	5982	1334	122	103,753

Continued

Table C.4
(Continued)

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	Lost sales	TP
4	6	408.7	0.0	41,306	21,883	5867	1328	0	106,270
4	7	502.4	0.0	41,294	21,831	5617	1603	14	106,211
4	8	1352.7	0.0	41,001	21,556	5937	1279	0	106,575

Table C.5
Summary of profit maximization with lost sales results for 24 tests with $n = 10$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	Lost sales	TP
2	1	3.5	0.00	125,774	54,632	22,502	3174	1228	185,561
2	2	4.4	0.00	126,226	59,074	19,037	3876	0	191,246
2	3	10.1	0.00	119,118	51,663	18,042	5175	0	198,354
2	4	23.5	0.00	120,925	53,865	19,332	3473	17	196,463
2	5	41.5	0.00	115,429	48,149	18,686	4342	14	201,975
2	6	62.6	0.00	112,754	45,365	18,358	4793	0	204,718
2	7	115.0	0.00	116,020	48,682	18,689	4374	37	201,265
2	8	161.2	0.00	113,407	46,049	18,697	4409	13	204,001
3	1	5.5	0.00	126,506	59,752	21,034	1450	32	190,807
3	2	16.8	0.00	118,782	51,874	18,721	3949	0	198,690
3	3	67.3	0.00	115,582	47,983	18,020	5282	58	201,599
3	4	260.8	0.00	112,090	44,729	18,375	4745	2	205,372
3	5	779.7	0.00	110,825	43,443	18,440	4695	9	206,603
3	6	3247.8	0.00	110,914	43,083	18,798	4209	5	207,111
3	7	12,399.8	0.01	110,727	43,328	18,450	4688	23	206,631
3	8	32,858.9	0.07	110,693	43,439	18,691	4325	0	206,779
4	1	12.6	0.00	120,973	53,982	21,260	1403	90	196,050
4	2	97.8	0.00	119,669	52,851	18,031	4548	0	197,803
4	3	380.1	0.00	114,928	47,288	17,564	5811	27	202,411
4	4	2741.2	0.00	112,868	45,605	18,603	4418	4	204,584
4	5	13,578.3	0.00	110,679	43,329	18,644	4467	0	206,793
4	6	35,790.7	0.10	110,325	43,069	18,647	4370	0	207,147
4	7	46,276.0	0.19	110,862	43,483	18,497	4633	11	206,554
4	8	53,698.9	0.30	110,337	43,090	18,630	4379	0	207,135

Table C.6
Summary of profit maximization with lost sales results for 24 tests with $n = 15$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	Lost sales	TP
2	1	6.5	0.00	186,781	43,869	56,756	1996	24,277	166,940
2	2	7.7	0.00	155,526	49,250	34,974	3797	7621	281,473
2	3	15.3	0.00	143,099	49,528	25,915	5013	2758	318,214
2	4	17.0	0.00	141,294	48,917	25,117	5069	2307	322,274

Continued

Table C.6
(Continued)

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	Lost sales	TP
2	5	54.3	0.00	141,986	49,630	25,112	4540	2307	321,582
2	6	71.3	0.00	140,262	47,905	25,112	5053	2307	323,306
2	7	136.5	0.00	140,330	47,952	25,117	5069	2307	323,238
2	8	179.4	0.00	140,309	47,952	25,112	5053	2307	323,259
3	1	12.5	0.00	148,126	54,587	28,891	2846	1918	317,387
3	2	31.2	0.00	137,892	49,346	23,495	5167	0	337,212
3	3	142.7	0.00	138,602	50,120	22,885	5713	0	336,502
3	4	362.6	0.00	141,142	52,357	24,381	4473	47	333,728
3	5	538.1	0.00	137,871	49,164	23,990	4024	37	337,048
3	6	1,489.8	0.00	136,114	47,574	23,183	5473	0	338,990
3	7	2,497.8	0.00	135,434	46,830	23,628	5091	0	339,670
3	8	5,880.9	0.00	134,324	45,564	23,968	4904	4	340,760
4	1	66.6	0.00	144,019	55,391	26,489	2099	156	330,303
4	2	215.7	0.00	142,561	54,215	22,911	5551	0	332,543
4	3	2,462.9	0.00	134,518	45,967	22,706	5960	0	340,586
4	4	6,038.6	0.01	133,341	44,744	23,669	5021	22	341,653
4	5	9,116.8	0.01	132,061	43,309	23,521	5304	43	342,829
4	6	16,284.0	0.01	130,115	41,583	23,344	5304	0	344,989
4	7	40,832.3	0.17	130,512	41,982	23,506	5138	2	344,584
4	8	43,013.2	0.14	130,090	41,522	23,553	5130	0	345,014

We provide in Tables C.7–C.9 the numerical results of the simulation study conducted for the case of a price-setting company. They report the parameters of the test case, the cost metrics, and the recorded profit. Column 9 displays the cost of lost sales.

Table C.7
Summary of profit maximization under monopoly results for 24 tests with $n = 5$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	Lost sales	TP
2	1	3.7	0.00	55,295	40,840	2171	55	0	116,253
2	2	7.4	0.00	52,771	33,317	3437	793	833	118,051
2	3	14.4	0.00	52,740	36,792	1697	607	224	118,326
2	4	30.4	0.00	50,064	37,395	2201	826	56	119,793
2	5	103.1	0.00	49,783	35,039	1389	983	142	120,073
2	6	309.9	0.00	48,301	33,109	1663	972	328	121,397
2	7	3690.9	0.00	49,476	34,332	1658	979	279	120,386
2	8	13,333.4	0.03	47,990	32,650	1738	1047	327	121,778
3	1	4.2	0.00	54,318	40,914	2151	59	0	117,513
3	2	10.3	0.00	52,987	36,954	2265	669	310	118,169
3	3	37.6	0.00	52,324	38,501	1744	601	188	118,564
3	4	147.9	0.00	49,529	36,228	1886	607	348	119,983

Continued

Table C.7
(Continued)

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	Lost sales	TP
3	5	880.7	0.00	49,118	37,344	1741	826	144	120,031
3	6	3355.6	0.00	48,392	33,223	1648	1010	283	120,944
3	7	27,166.4	0.10	48,999	33,796	1565	1132	277	120,569
3	8	52,479.8	0.70	49,245	33,988	1669	1157	202	120,527
4	1	4.4	0.00	54,579	39,724	2243	44	0	117,591
4	2	14.3	0.00	52,636	36,718	2166	508	345	118,514
4	3	55.2	0.00	52,763	36,838	1932	620	343	118,319
4	4	203.9	0.00	49,637	36,655	1880	767	104	120,278
4	5	1727.7	0.0	49,637	36,655	1880	767	104	120,278
4	6	8570.4	0.1	49,624	34,821	1447	929	198	120,064
4	7	41,264.8	0.23	48,999	33,796	1565	1132	277	120,569
4	8	54,281.1	0.99	49,145	33,913	1721	1069	213	120,695

Table C.8
Summary of profit maximization under monopoly results for 24 tests with $n = 10$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	Lost sales	TP
2	1	14.3	0.00	147,618	58,267	42,545	1844	723	177,988
2	2	31.0	0.00	149,058	56,774	43,944	4084	18	201,389
2	3	65.5	0.00	138,774	49,416	39,738	5368	14	218,324
2	4	530.7	0.00	132,263	40,499	41,543	5910	72	223,513
2	5	1458.4	0.00	135,312	40,176	45,668	5203	27	215,427
2	6	7133.9	0.00	132,125	37,337	45,115	5400	35	219,203
2	7	34,862.1	0.47	134,895	39,581	45,115	5931	29	214,861
2	8	54,292.3	0.62	132,193	37,248	44,771	5787	149	215,073
3	1	29.5	0.00	140,241	57,672	36,474	1857	0	219,086
3	2	108.0	0.00	133,039	40,992	44,014	3787	8	217,522
3	3	440.6	0.00	130,524	40,551	40,200	5514	20	224,531
3	4	12,415.3	0.00	128,642	36,359	44,238	5967	25	225,698
3	5	47,675.7	0.56	129,296	34,200	45,311	5537	9	221,555
3	6	54,197.4	1.06	128,472	33,757	44,648	5812	17	222,952
3	7	54,327.1	1.33	128,864	33,371	45,497	5722	36	220,531
3	8	54,139.7	1.33	128,248	33,671	44,693	5611	35	222,668
4	1	75.9	0.00	136,926	54,682	36,120	1887	0	222,682
4	2	336.2	0.00	131,415	39,549	43,932	3690	6	221,552
4	3	14,528.1	0.00	128,464	38,108	41,684	4430	3	227,858
4	4	49,884.4	0.82	127,536	34,673	43,742	4875	8	226,512
4	5	54,333.8	1.20	128,000	33,532	45,119	5108	3	223,428
4	6	54,142.6	1.41	128,088	33,997	44,133	5716	3	223,674
4	7	57,477.2	1.43	128,196	34,223	44,936	5569	25	221,769
4	8	54,118.7	1.63	129,367	34,289	44,909	5805	126	220,610

Table C.9
Summary of profit maximization under monopoly results for 24 tests with $n = 15$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	Lost sales	TP
2	1	33.2	0.0	188,022	37,487	81,416	1178	8055	128,277
2	2	153.8	0.0	174,115	46,369	58,585	3923	5355	300,531
2	3	13,018.8	0.1	175,728	32,996	67,646	5313	9889	252,798
2	4	40,772.0	2.1	168,627	30,922	65,819	6480	5521	270,476
2	5	54,223.6	7.8	169,110	32,229	65,102	6548	5347	272,752
2	6	54,191.2	9.3	169,289	33,001	64,950	6435	5019	271,178
2	7	54,274.4	10.1	170,284	32,573	65,715	6299	5813	262,675
2	8	54,242.3	10.1	169,512	31,003	66,292	6391	5943	258,550
3	1	98.9	0.0	174,198	57,482	52,088	1930	2814	296,325
3	2	6684.3	0.0	158,989	58,323	35,762	4213	808	390,277
3	3	53,780.5	1.4	158,739	58,885	34,121	5707	142	394,275
3	4	54,134.8	5.7	161,644	43,208	49,193	6211	3148	346,081
3	5	54,136.2	10.0	168,166	46,362	51,034	6325	4560	331,980
3	6	54,070.9	16.1	168,777	44,471	53,027	6096	5300	320,894
3	7	54,096.2	23.8	172,265	45,116	54,861	6397	6008	306,529
3	8	54,066.6	29.9	176,391	46,383	57,751	6524	5849	295,317
4	1	615.7	0.0	164,199	64,804	36,993	2511	6	387,450
4	2	36,141.0	0.1	153,808	54,436	34,182	4524	782	398,053
4	3	54,066.0	1.7	152,866	53,969	59,884	5486	536	401,086
4	4	54,097.9	8.7	156,268	52,198	37,858	5748	581	388,097
4	5	54,102.8	23.3	161,685	49,835	43,117	5231	3618	359,814
4	6	54,086.0	39.0	165,347	52,230	43,674	5996	3563	355,429
4	7	54,100.0	52.5	165,984	49,226	45,841	5952	5081	339,137
4	8	54,098.4	61.8	174,342	49,872	6311	52,203	6071	311,579

We provide in Tables C.10–C.12 the cost metrics pertaining to the profit maximization problem with variable production costs. In addition to the aforementioned cost metrics, we define *PC*, as displayed in column 9, a metric that maps the total production costs.

Table C.10
Summary of profit maximization with variable production costs results for 24 tests with $n = 5$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	PC	Lost sales	TP
2	1	1.9	0.0	49,599	38,607	361	55	10,458	118	97,385
2	2	2.3	0.0	36,460	24,751	286	350	9963	1110	105,567
2	3	4.0	0.0	36,352	25,056	247	461	10,589	0	111,224
2	4	5.0	0.0	35,480	23,958	256	633	10,633	0	112,096
2	5	8.1	0.0	35,336	23,627	357	682	10,614	56	111,960
2	6	12.4	0.0	34,339	22,729	342	634	10,634	0	113,237
2	7	25.3	0.0	34,161	22,049	310	897	10,645	261	112,112
2	8	46.8	0.0	33,812	22,045	319	748	10,639	61	113,457
3	1	2.3	0.0	47,178	36,248	364	8	10,468	90	99,949

Continued

Table C.10
(Continued)

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	PC	Lost sales	TP
3	2	3.8	0.0	36,216	24,605	248	378	9970	1014	106,291
3	3	7.5	0.0	38,575	27,248	226	515	10,574	12	108,944
3	4	8.5	0.0	34,970	23,426	381	527	10,636	0	112,606
3	5	17.9	0.0	36,011	24,429	298	649	10,631	4	111,546
3	6	34.6	0.0	34,737	23,136	275	686	10,627	13	112,773
3	7	87.6	0.0	34,354	22,531	311	862	10,624	24	113,100
3	8	143.4	0.0	33,338	21,462	360	812	10,705	0	114,238
4	1	2.4	0.0	47,218	36,248	364	48	10,468	90	99,909
4	2	3.3	0.0	35,992	24,605	423	248	10,393	323	109,971
4	3	7.0	0.0	36,116	24,795	223	511	10,576	12	111,403
4	4	10.4	0.0	35,016	23,442	284	657	10,633	0	112,560
4	5	19.4	0.0	35,216	23,599	336	648	10,630	4	112,338
4	6	47.0	0.0	33,846	22,244	692	279	10,632	0	113,730
4	7	99.1	0.0	34,049	22,253	805	321	10,615	55	113,252
4	8	241.7	0.0	34,442	22,565	848	324	10,705	0	113,134

Table C.11
Summary of profit maximization with variable production costs results for 24 tests with $n = 10$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	PC	Lost sales	TP
2	1	2.7	0.00	97,110	56,672	923	242	21,235	18,038	130,173
2	2	9.0	0.00	98,215	56,114	761	2498	38,645	198	218,269
2	3	17.0	0.00	93,050	49,698	753	3594	38,743	261	223,118
2	4	60.4	0.00	93,568	49,524	1456	3384	39,102	102	223,395
2	5	133.5	0.00	92,103	48,399	1045	3677	38,880	103	224,855
2	6	288.3	0.00	90,225	46,224	1054	3765	39,142	40	227,049
2	7	333.7	0.00	91,002	46,911	1071	3814	39,129	77	226,085
2	8	385.7	0.00	89,042	44,443	1013	4260	39,312	15	228,354
3	1	6.7	0.00	113,594	72,214	929	210	37,437	2804	189,860
3	2	24.8	0.00	95,304	54,409	749	1680	37,893	572	219,308
3	3	199.0	0.00	90,623	47,250	898	3492	38,920	63	226,533
3	4	714.6	0.00	90,169	46,314	1184	3705	38,956	10	227,252
3	5	10,740.2	0.00	88,413	45,364	970	3133	38,909	36	228,877
3	6	16,857.0	0.01	86,513	42,894	865	3756	38,955	44	230,740
3	7	38,100.4	0.17	87,775	43,964	1069	3727	38,979	36	229,516
3	8	49,979.8	0.32	86,881	43,211	949	3645	39,038	38	230,399
4	1	18.3	0.00	106,627	67,130	954	192	38,315	35	210,668
4	2	193.0	0.00	99,453	57,313	654	2609	38,828	50	217,770
4	3	1522.6	0.00	91,483	49,266	749	2598	38,856	14	225,919
4	4	17,521.8	0.00	89,440	47,317	926	2407	38,726	63	227,718
4	5	51,758.7	0.4	88,303	45,996	946	2560	38,800	2	229,158
4	6	52,504.0	0.58	87,612	45,097	972	2741	38,786	17	229,774
4	7	54,032.6	0.81	87,443	44,722	956	2934	38,809	21	229,922
4	8	54,055.7	0.90	86,828	43,652	950	3386	38,823	18	230,555

Table C.12
Summary of profit maximization with variable production costs results for 24 tests with $n = 15$

L	H	CPU(s)	%Gap	TC	VC	IHSU	IHC	PC	Lost sales	TP
2	1	4.2	0.00	109,558	43,626	1379	296	18,412	45,845	136,321
2	2	7.1	0.00	105,321	46,436	1073	3109	43,583	11,121	314,177
2	3	15.7	0.00	106,472	45,631	965	4965	50,830	4082	348,223
2	4	27.2	0.00	109,908	51,269	1032	3656	52,799	38	353,580
2	5	66.6	0.00	111,390	50,063	1259	5055	52,580	2432	351,553
2	6	118.4	0.00	109,523	48,513	1203	4992	52,262	2554	352,811
2	7	180.3	0.00	109,061	47,952	1114	5069	52,618	2307	354,507
2	8	182.9	0.00	109,088	47,952	1132	5053	52,643	2307	354,480
3	1	10.8	0.00	111,674	52,695	1170	243	30,161	27,404	226,407
3	2	23.0	0.00	107,258	50,928	1019	2723	52,301	287	366,412
3	3	112.1	0.00	107,874	50,984	1019	3362	52,446	64	366,908
3	4	174.4	0.00	108,794	43,874	1186	4052	52,915	63	366,118
3	5	842.6	0.00	106,792	49,131	1142	3752	52,744	23	368,197
3	6	2438.3	0.00	104,300	46,360	1043	4103	52,720	74	370,435
3	7	7526.0	0.00	104,567	46,344	1243	4023	52,954	4	370,519
3	8	8764.0	0.00	102,250	44,149	1338	3820	52,943	0	372,854
4	1	61.5	0.00	120,344	65,344	1131	283	43,506	10,079	304,366
4	2	318.5	0.00	101,754	45,193	943	3026	52,180	413	371,286
4	3	899.6	0.00	103,532	48,917	1047	5061	52,560	2323	371,161
4	4	1260.5	0.00	102,090	46,843	993	3215	52,398	82	372,697
4	5	7064.2	0.01	100,744	43,073	1174	3769	52,725	4	374,343
4	6	38,107.1	0.07	97,550	39,996	925	3856	52,691	81	377,148
4	7	50,325.1	0.50	98,128	40,652	970	3750	52,734	23	376,861
4	8	52,299.1	1.56	98,146	41,405	10	2856	53,839	36	375,296