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Stability metrics for a maritime inventory routing problem under sailing time uncertainty

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Abstract

We study a multi-product maritime inventory routing problem (MIRP) with sailing time uncertainty. We explicitly consider the replanning that happens after uncertainty is revealed. The objective is to determine the stability of the adjusted plans after the occurrence of an uncertain event and to evaluate the effect of incorporating different stability metrics in the rescheduling process. Five stability metrics are introduced, and mathematical formulations of the MIRP incorporating each metric are presented. A reoptimization framework is then used to analyze the impact of each stability metric. Calculations are performed using 270 instances. The main result is that adjustments to the original plan occur at no additional cost almost 50% of the time. If decision makers want a more stable plan, they should accept a 5% cost deterioration, resulting in 20% more stable solutions.

Keywords: reoptimization, uncertainty, stability metrics, maritime inventory routing

1. Introduction

In 2020, more than 80% of the volume of goods in international trade was carried by maritime transport, corresponding to 10.7 billion tons. It is expected that the volume of maritime trade will grow by 2.4% annually between 2022 and 2026 (UNCTAD, 2021), and therefore optimized maritime transportation is of great importance. We study the maritime inventory routing problem (MIRP) which is a particular maritime transportation planning problem (Papageorgiou et al., 2014). The MIRP is a variant of the inventory routing problem (IRP) in a maritime context. The IRP integrates inventory management decisions with routing decisions under a vendor-managed inventory (VMI) system, where the supplier is responsible for determining the delivery schedule for a given customer, the delivery quantity for that customer, and the assignment of customers to vehicle routes.

In the MIRP there are five time elements, shown in Figure 1 along with six event points labelled from "a" to "f". The routing is between points "a" and "b" and after "f". Although the vessel is stationed at a port between points "b" and "f", the temporal status of this interval affects the routing after "f" to reach "b" at the next port.

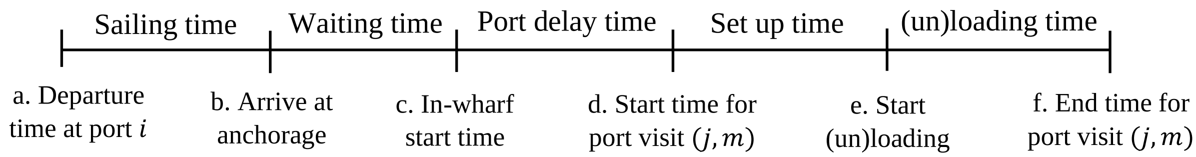


Figure 1. Five time elements

There are always several uncertain parameters in maritime trade. According to UNCTAD (2021), supply chain disruptions, changes in globalization patterns, transportation costs, port congestion, and pandemics are the main uncertain elements. In the MIRP, there are several problem features that could be influenced by uncertainty, some of which are listed below:

- The sailing time can be uncertain due to reasons such as bad weather conditions (Rodrigues & Agra, 2022), mechanical failure of vessels (Rodrigues & Agra, 2022), or the ice conditions in the Arctic region (Choi et al., 2015).
- The waiting time can be uncertain due to port congestion (Agra et al., 2015).
- The port delay time can be uncertain for reasons such as strikes and equipment failure at ports (Christiansen & Nygreen, 2005).
- Demand, which is the main feature with uncertainty in inland IRPs (Touzout et al., 2021), can also be uncertain in a maritime setting (Cheng & Duran, 2004; Soroush & Al-Yakoob, 2018).

Previous research on the MIRP under uncertainty has mostly focused on sailing time as an uncertain parameter. Papageorgiou et al. (2014) stated that the sailing time is one of the primary features influenced by uncertainty in maritime applications. Accordingly, sailing time is considered as the only source of uncertainty in the current study.

Three different approaches for dealing with uncertainty in an optimization problem have been proposed in the literature (Rodrigues & Agra (2022), De Maio et al. (2021), Aytug et al. (2005)), defined in Table 1.

Table 1. Three approaches for dealing with uncertainty

Approaches	Which decisions are made before uncertainty is revealed		Which decisions are made after uncertainty is revealed	Notes
	Considering uncertain information explicitly	Considering deterministic parameters		
Proactive	All decisions	-	No adjustment	These approaches are better suited for problems with low uncertainty and where the original plan can be maintained without any adjustment.
Reactive	-	An initial plan	All the decisions are recourse actions	These approaches are better suited for problems with high uncertainty.
Mixed	An initial plan	-	Some of the decisions are recourse actions	This is known as a priori optimization, a concept introduced by Bertsimas et al. (1990)

In a MIRP with reactive approaches, frequent adjustments to the original plan can lead to inefficiencies from the perspective of the port planner (Liu et al., 2017), since these adjustments can trigger a series of changes in subsequent decisions such as staff scheduling and container storage (Xu et al., 2012). It is therefore important to know how stable the adjusted plans are, i.e., how large the deviation from the original plan is when reactive actions are applied. Hence, the research question of the current study is: how to measure the stability of solutions to a MIRP under the uncertainty of sailing time? We introduce stability metrics that examine the sequence of routes, which port visits are made, and the quantities loaded and unloaded in each visit.

The only paper having used a reactive approach for the MIRP is by Dong et al. (2018) who solved an uncertain MIRP using a mixed integer linear programming model and then reviewed the information revealed in each period. Whenever the solution obtained after considering this information is infeasible, a reoptimization is performed for the entire planning horizon, ignoring the amount of deviation from the original plan. After the reoptimization, the horizon is rolled forward and the procedure is iterated until the end of the horizon.

Touzout et al. (2021) attempted to measure the stability of solutions to the IRP under uncertain demand using reoptimization models. They stated that their method could be extended to other applications and proposed to consider other sources of uncertainty. In this regard, the current study aims to introduce a reoptimization framework for the MIRP under sailing time uncertainty in which stability metrics are introduced. The main contribution of this paper is threefold:

1. Unlike Dong et al. (2018) and Touzout et al. (2021), who performed reoptimization at specific time intervals, in the current study reoptimization can occur at any point in time. This allows us to respond to uncertainties whenever they are revealed. This is explained in Section 5.
2. Relevant stability metrics for the MIRP are identified, and mathematical formulations for each of these metrics are proposed. This is explained in Section 6.
3. Each of the formulations is tested by performing computational experiments to determine the impact of each stability metric. This is explained in Section 7.

The remainder of the paper is organized as follows. Section 2 reviews the literature on MIRPs with uncertainties and classifies the papers according to uncertain parameters, approaches, and models. Section 3 provides a description of the problem. Mathematical notations are explained in Section 4. Section 5 is devoted to the reoptimization framework. Stability metrics are introduced in Section 6, followed by their analysis in Section 7, where numerical results and findings are presented. Finally, Section 8 provides concluding remarks.

2. Literature review

The most recent review of the MIRP was presented by Papageorgiou et al. (2014), who studied a deterministic single-product MIRP. The authors stated that robustness is a challenge for MIRP and therefore recommended the development of approaches that can deal with uncertainty. Ksciuk et al. (2022) provided a review of uncertainty in maritime ship routing and scheduling, examining uncertainty in eight different problems, including the MIRP. The authors mentioned that in the MIRP, there are no fixed pickup and delivery port pairs, and no predetermined number of port calls. Therefore, they concluded that this makes the MIRP a challenging problem even without uncertainty.

The current section focuses on MIRPs with uncertainty. The reviewed papers are summarized in Table 2, which indicates the uncertain parameters, the approaches used to deal with uncertainty, and the employed model. The remainder of this section first introduces some of the commonly used modelling techniques in an uncertain environment and then discusses each of the approaches used to deal with uncertainty.

Some of the commonly used techniques for modelling of optimization problems under uncertainty are the following:

- Stochastic programming is a modeling framework for optimization problems under uncertainty (Klein Haneveld et al., 2020), in which the uncertain parameters are assumed to follow known (or partially known) probability distributions (Rodrigues & Agra, 2022).
- Recourse models are a class of models in stochastic programming, including two-stage and multistage models. When the true value of an uncertain parameter is observed, corrective actions can be taken in recourse models (Klein Haneveld et al., 2020).
- Chance-constrained programming, introduced by Charnes & Cooper (1959), provides a tool for solving optimization problems under uncertainty. This method optimizes the problem in such a way that the constraints are satisfied with a given probability. The minimum required reliability should be set by the decision maker to a value between zero and one. If this value is zero, the decision maker is extremely risk seeking, and if it is one, it indicates an extremely conservative attitude (risk averse).
- Robust optimization accounts for uncertainty sets where the probability distribution is unknown or does not exist (Rodrigues & Agra, 2022). The decision maker constructs a solution that is feasible for each realization of uncertainty in the given set (Bertsimas et al., 2011). In other words, it optimizes the problem based on the worst possible outcome within the uncertainty set. Unlike stochastic programming, where the uncertain parameter is assumed to be a random variable that follows a known (or partially known) probability distribution, the uncertainty model in robust optimization is usually deterministic and set-based (Bertsimas et al., 2011). Therefore, the uncertain parameters can take any value within the uncertainty set.
- Reoptimization can be used to deal with uncertainty or in situations where the planning horizon is shorter than the horizon of the actual problem (Dong et al., 2018).
- The integration of simulation and optimization can be used to deal with uncertainty. Zhou et al. (2021) studied different types of integration approaches in maritime logistics.

In terms of proactive approaches, Cheng & Duran (2004) considered a decision support system that uses a simulation model and an optimization model. The simulation model represents the inventory and transportation system, and the optimization model is formulated as a discrete-time Markov decision process that deals with the uncertainty of sailing time and of demand. A deterministic model with penalty costs is used as a proactive approach in two studies. First, Christiansen & Nygreen (2005) considered the uncertainty of sailing time and waiting time for a single-product MIRP. They applied soft inventory constraints, where levels should lie within a certain interval, and introduced lower and upper alarm intervals with artificial penalty costs to increase the robustness of their model. Second, Rakke et al. (2011) introduced a deterministic model with penalty costs for deviating from long-term customer contracts, maximizing revenue based on the spot market price and the quantity of sales in that market.

In another proactive approach by Soroush & Al-Yakoob (2018) for a single-product MIRP, demand was assumed to be a normally distributed random variable, and penalties for understocking or overstocking were considered. The authors proposed a stochastic optimization model with linear constraints and a convex objective function. They used DICOPT as a commercial solver to solve the problem.

Zhang et al. (2018) employed time windows to model sailing time uncertainty for a single-product MIRP. They defined flexible solutions as those that can accommodate unplanned disruptions by adjusting routing solutions where delivery dates and total delivery quantities cannot be changed. Furthermore, a Lagrangian heuristic was implemented to find flexible solutions using soft constraints, and a simulator was introduced that generates a disruption in each simulation run to evaluate the robustness of the solutions. Diz et al. (2019) considered the uncertainty of the total time vessels spend in ports due to delays in vessel operations for a single-product MIRP. They developed a robust optimization scheme using more vessels to protect the solution against delays. The risk of infeasibility was quantified for different levels of robustness and Gurobi used to solve the problem.

Regarding the mixed approaches, three studies have considered recourse models that take into account routing, the quantities to be loaded and unloaded, the order of port visits in the first stage, as well as visit times to ports and inventory decisions in the second stage, which can be adjusted to the scenario. The first study, by Agra et al. (2015), introduced a two-stage stochastic programming model with recourse and solved this model using a decomposition algorithm in which optimality cuts are added dynamically. The second study, by Agra et al. (2016), used a model similar to that of the previous study, but solved it with a combination of a commercial solver and local search heuristics. In the third study, by Agra et al. (2018), robust optimization was used and a decomposition algorithm was suggested. Also, an iterated local search heuristic was introduced to improve the decomposition algorithm.

Several techniques to handle uncertainty in MIRPs were compared by Rodrigues et al. (2019), who considered uncertain sailing times for a single-product MIRP and employed different models and algorithms to handle uncertainty. They discovered that three methods provide a good trade-off between the amount and probability of inventory limit violations and routing costs. These methods are 1) deterministic modeling with inventory buffers, 2) stochastic programming with high penalties for inventory bounds violations, and 3) a hybrid algorithm that solves a deterministic approach with inventory buffers derived from a conditional value-at-risk approach.

Another mixed methodology applied to MIRP under uncertainty comes from Cho et al. (2018), who proposed a two-stage stochastic programming model in which production inventory schedule decisions are made in the first stage and the production rate is adjusted for each scenario in the second stage. Liu et al. (2021) applied a two-stage distributionally robust optimization algorithm in which the routing decisions are made in the first stage, while decisions regarding quantities to be loaded and unloaded, visit time to ports, and inventory levels are made in the second stage after observing uncertainties.

A reactive methodology was applied once in the context of MIRP by Dong et al. (2018). The authors developed stochastic simulations to account for several sources of uncertainty, presented in Table 2, and an algorithm that integrates reoptimization and stochastic simulation results. They reoptimized the model at a specified frequency, typically once per day. At each stage, the parameters are updated as uncertainties are observed, and the optimization problem is solved. This procedure is repeated for each day of the time horizon of the current problem.

Table 2. Summary of MIRP papers with uncertainty.

Year	Authors	Uncertain parameters	Approach			Models
			P*	R**	M†	
2004	Cheng & Duran	• Sailing times • Demand	✓			Simulation and optimization
2005	Christiansen & Nygreen	• Sailing times • Port delay times	✓			Deterministic model with penalty cost for inventory violation
2011	Rakke et al.	• Spot market price	✓			Deterministic model with penalty cost for deviation from the customer long-term contracts
2015	Agra et al.	• Sailing times • Waiting times			✓	Stochastic programming
2016	Agra et al.	• Sailing times			✓	Stochastic programming
2018	Soroush & Al-Yakoob	• Demand	✓			Chance-constrained programming
2018	Agra et al.	• Sailing times • Port delay times			✓	Robust optimization
2018	Cho et al.	• Sailing times			✓	Stochastic programming
2018	Zhang et al.	• Sailing times	✓			Stochastic programming
2018	Dong et al.	• Vessel availability • Trip delays • Pick-up window information • Consumption and production rates		✓		Reoptimization

2019	Diz et al.	• Waiting times • Port delay times	✓			Robust optimization
2019	Rodrigues et al.	• Sailing times	✓		✓	• Deterministic models with inventory buffers • Robust optimization • Stochastic programming • Conditional value-at-risk
2021	Liu et al.	• Sailing times • Waiting times			✓	Two-stage distributionally robust optimization
2023	Current study	• Sailing times		✓		Reoptimization including stability metrics

* P: Proactive approaches ** R: Reactive approaches † M: Mixed approaches

3. Problem description

The MIRP considers the transportation of products between multiple ports while meeting inventory requirements. Different ports produce and consume multiple products at a given production and consumption rate. Initial inventories, minimum inventory levels, and maximum inventory levels are specified for each port.

A heterogeneous fleet of vessels with a given capacity, a fixed speed, and a daily operating cost is given. The position of a vessel at the beginning of the planning horizon is referred to as its origin, which can be a port or any location at sea. Sailing times from the origin to each port and between each pair of ports are determined based on the given distance and speed of the vessel. The sailing costs are also derived from the sailing time multiplied by the daily cost of a vessel. The maximum unloading quantities are determined by the consumption ports based on the vessel capacity and the maximum inventory of the port. The maximum number of visits to each port is predetermined. The holding cost and penalty cost for each product in each port are known. The objective of the problem is to minimize the sum of three components: sailing costs, inventory holding costs, penalty costs for backlogs and overstocks.

The sailing times are assumed to be subject to uncertainty due to weather conditions. Although a planning horizon is specified, the uncertainty in sailing times may cause the planning horizon to be exceeded. The problem is solved under deterministic conditions and whenever the uncertainty is revealed, reoptimization is performed.

4. Mathematical notations

This section explains some of the most frequently used notations throughout this paper, whereas the complete list of notations can be found in Appendix A. The problem consists of some ports represented by i and j, and each port can be visited at most $\overline{m}$ times. There is a set of products denoted by K and a set of vessels denoted by V. We define a network in which the nodes are represented by (i, m), denoting the visit m to port i. The vessels movement from node (i, m) to node (j, n) are represented by (i, m, j, n). The set of possible port arrivals (i, m) is defined as S^A and the set of port arrivals that may be made by vessel v is defined as S_v^A. The set of all possible vessel movements (i, m, j, n) is defined as S^X and the set of all possible moves for vessel v is defined as S_v^X.

The binary variable o_{imvk} is one if and only if product *k* is loaded onto or unloaded from vessel *v* at the port visit (i, m). The amount of product *k* loaded onto or unloaded from vessel *v* at port visit (i, m) is denoted by q_{imvk}. The amount of product *k* that vessel *v* transports from port visit (i, m) to port visit (j, n) is denoted by f_{imjnvk}. Let s_{imk} represent the inventory level of product *k* at the start of port visit (i, m) and s_{imk}^E represent the inventory level of product *k* at the end of port visit (i, m).

The sailing of vessel v from port arrival (i, m) directly to port arrival (j, n) is denoted by x_{imjnv}, sailing of vessel v from its initial position to port arrival (i, m) is denoted by x_{imv}^O, the port visit (i, m) is denoted by y_{im}, the visit to port i by vessel v at port arrival (i, m) is denoted by w_{imv}. Let t_{im} be the start time for port arrival (i, m) and t_{im}^E be the end time for port arrival (i, m).

Figure 2 depicts the route of one vessel as an example of this network, where O_v is the origin of v.

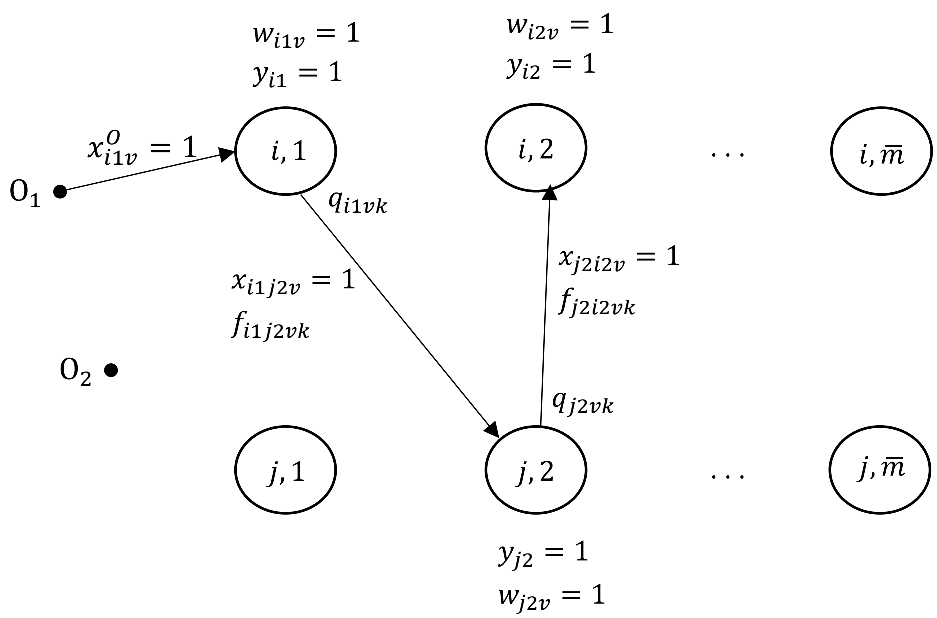


Figure 2. Example of the network

5. Reoptimization framework

The mathematical formulation of the MIRP given in Appendix A is the same as that of Shaabani et al. (2023). It is used as the basic model of the reoptimization framework. The solution of this deterministic model is considered as the initial plan for the reoptimization.

Periodic reoptimization considers the problem periodically at fixed time intervals. Continuous reoptimization, on the other hand, solves the problem throughout the day and whenever data change; a procedure collects the information up to that point and then starts the reoptimization (Pillac et al., 2013). In the current study, unlike Dong et al. (2018) and Touzout et al. (2021), a continuous-time model is used that reacts to uncertainties as soon as they appear, hence continuous reoptimization is performed. In this context, T^U is defined as the time at which an uncertain event occurs. Therefore, the nominal sailing times are used until T^U and then the sailing times are changed due to the uncertain event. Since T^U is an uncertain event, it can occur at any time in $[0, T]$. We assume that there is only one T^U value in each planning horizon. Whenever the T^U value is revealed, the following changes are made to the basic model to prepare the model for reoptimization.

- The set S^A in the basic model, is replaced with S^B, excluding port arrivals visited before T^U. In the same manner, the set S_v^A is replaced with S_v^B, the set S^X is replaced with S^Y, the set S_v^X is replaced with S_v^Y.
- The solution to the deterministic problem is extracted and defined as the data set for the reoptimization model. The sailing of vessel v from port arrival (i, m) directly to port arrival (j, n) is denoted by X_{imjnv}, the sailing of vessel v from its initial position to port arrival (i, m) is denoted by X_{imv}^O, the visit to port arrival (i, m) is denoted by Y_{im}, the visit of port i by vessel v at port arrival (i, m) is denoted by W_{imv}, and the amount of product *k* loaded onto or unloaded from vessel *v* at port visit (i, m) is denoted by Q_{imvk}.
- Due to the occurrence of the uncertain event, two new time constraints are defined:

$$t_{im} \geq T^U \qquad (i, m) \in S^B \qquad (1)$$

$$t_{im}^E \geq T^U \qquad (i, m) \in S^B. \qquad (2)$$

  If a vessel was on route from (i, m) to (j, n) when time hit T^U, then the vessel is forced to visit (j, n), but the planned arrival time may be affected by the updated sailing times.
- The initial inventory levels are updated when the new problem starts after T^U. If $J_{ik} = -1$, the amount of inventory consumed up to T^U is subtracted from the initial inventory, and if $J_{ik} = 1$, the amount of inventory produced up to T^U is added to the initial inventory.
- The values of the decision variables visited before T^U are fixed. These decision variables are as follows: x_{imjnv}, x_{imv}^O, o_{imvk}, q_{imvk}, f_{imjnvk}, s_{imk}, s_{imk}^E, t_{im}^E.
- Constraints (A28) and (A29) are deleted because the uncertainty of the sailing time is considered, which may lead to exceeding the planning horizon.

Now the modified model is ready, and we call it "Model 0", which represents the metric "*cost*". The reoptimized solution represents the sailing costs and the port operation costs, plus penalty costs for backlogs and overstocks without stability metrics. The sailing costs and port operation costs are called C^*. Therefore, the reoptimized solution may differ from the initial solution. In this context, stability metrics are introduced in the next section to reduce this discrepancy.

6. Stability metrics

In this section we present five stability metrics for the MIRP and new constraints added to the model are then given for each metric. The objective function of the mathematical model for each of the stability metrics consists of two parts: first it minimizes the violation of the metrics, second it minimizes the penalty cost for backlogs and overstocks. Based on the values of consumption and production rates, vessel capacity, and minimum and maximum inventories, the sizes of the different elements in the objective function are such that we implicitly prioritize the first part of the objective function before the second part.

6.1. Sequence preservation

The sequence preservation metric, called SP, means that the sequence of the reoptimized solution should not differ significantly from that of the original solution (Dettenbach & Ubber, 2015). Applications of the sequence preservation metric mostly belong to routing and scheduling problems (Touzout et al., 2021).

In the MIRP, traveling times are typically much longer than in an inland IRP, and because the uncertain event can occur at any time, changing the sequence and rerouting may be costly. If the sequence of shipments has changed, more lifting operations are required at the new port in order to reach the unscheduled product unloads, resulting in higher costs. Another case where sequence preservation is critical in maritime transport occurs on transshipment routes where another vessel is waiting at a port of transshipment.

The mathematical formulation of the SP metric contains two new binary variables. The binary variable z^{SP}_{imjnv} is defined to indicate whether or not there is a sequence change, and z^{SPO}_{imv} is a binary variable equal to one if and only if there is a change in the first visit made by the vessel. Therefore, two new constraints are defined as follows:

$$z^{SP}_{imjnv} = \left|X_{imjnv} - x_{imjnv}\right| \qquad v \in V, (i,m,j,n) \in S^Y_v \qquad (3)$$

$$z^{SPO}_{imv} = \left|X^O_{imv} - x^O_{imv}\right| \qquad v \in V, (i,m) \in S^B_v. \qquad (4)$$

Constraints (3) and (4) are nonlinear but can be linearized into constraints (6) to (9) as shown by Touzout et al. (2021). Constraints (3) count a sequence change when an arc from (i,m) to (j,n) is visited by vessel v in the original solution but not in the reoptimized solution, and vice versa. Constraints (4) count a sequence change if vessel v sails directly from its initial position to port arrival (i,m) in the original solution but does not in the reoptimized solution, and vice versa. The mathematical formulation for the SP metric is as follows:

Model 1: Reoptimization based on sequence preservation (SP) metric

$$\text{Minimize} \sum_{v\in V} \sum_{(i,m,j,n)\in S_v^X} z_{imjnv}^{SP} + \sum_{v\in V} \sum_{(i,m)\in S_v^A} z_{imv}^{SPO} \tag{5}$$
$$+ \sum_{(i,m)\in S^A} \sum_{k\in K_v|J_{ik}=-1} C_{ik}^P \left(r_{imk} + r_{imk}^E\right) + \sum_{i\in N} \sum_{k\in K_v|J_{ik}=-1} C_{ik}^P\, r_{ik}^T$$
$$+ \sum_{i\in N} \sum_{k\in K_v|J_{ik}=1} C_{ik}^{PP}\, r_{ik}^{PT}$$

subject to

(1) and (2)

(A2) to (A27) and (A30) to (A45)

$$z_{imjnv}^{SP} \geq X_{imjnv} - x_{imjnv} \qquad v\in V, (i,m,j,n)\in S_v^Y \tag{6}$$
$$z_{imjnv}^{SP} \geq x_{imjnv} - X_{imjnv} \qquad v\in V, (i,m,j,n)\in S_v^Y \tag{7}$$
$$z_{imv}^{SPO} \geq X_{imv}^O - x_{imv}^O \qquad v\in V, (i,m)\in S_v^B \tag{8}$$
$$z_{imv}^{SPO} \geq x_{imv}^O - X_{imv}^O \qquad v\in V, (i,m)\in S_v^B \tag{9}$$
$$z_{imjnv}^{SP} \in \{0,1\} \qquad v\in V, (i,m,j,n)\in S_v^Y \tag{10}$$
$$z_{imv}^{SPO} \in \{0,1\} \qquad v\in V, (i,m)\in S_v^B. \tag{11}$$

6.2. Sequence preservation with vessel replacement

The sequence preservation with vessel replacement metric, called SPV, is similar to the SP metric with one difference. This metric measures the preservation of the sequence of the original solution for the reoptimized solution, even if the vessels are replaced with others. For example, an arc from (i,m) to (j,n) that was traversed by vessel 1 in the original solution can be traversed by vessel 3 in the reoptimized solution. This metric is especially helpful when it is important to retain the sequence even with a new vessel. In addition, the SPV metric may make a correct measurement when the fleet is homogeneous, whereas SP metric may make a correct measurement when the fleet is heterogeneous.

The mathematical formulation of the SPV metric uses two new binary variables. The binary variable z_{imjn}^{SPV} is defined to indicate whether or not there is sequence change with vessel replacement, and z_{im}^{SPVO} is a binary variable equal to one if and only if there is sequence change with vessel replacement for the initial position of the vessels. Therefore, two new constraints are defined as follows:

$$z_{imjn}^{SPV} = \left|\sum_{v\in V} X_{imjnv} - \sum_{v\in V} x_{imjnv}\right| \qquad (i,m,j,n)\in S^Y \tag{12}$$
$$z_{im}^{SPVO} = \left|\sum_{v\in V} X_{imv}^O - \sum_{v\in V} x_{imv}^O\right| \qquad (i,m)\in S^B. \tag{13}$$

Like constraints (3) and (4), constraints (12) and (13) can be linearized, as is the case in constraints (15) to (18). Constraints (12) count a change of sequence with vessel replacement if an arc from (i, m) to (j, n) was visited in the original solution but not in the reoptimized solution and vice versa. Constraints (13) count a change of sequence with vessel replacement if a vessel sails directly from its initial position to port arrival (i, m) in the original solution but not in the reoptimized solution and vice versa. The mathematical formulation for the SPV metric is as follows:

Model 2: Reoptimization based on sequence preservation with vessel replacement (SPV) metric

$$\text{Minimize} \sum_{(i,m,j,n)\in S^X} z_{imjn}^{SPV} + \sum_{(i,m)\in S^A} z_{im}^{SPVO} + \sum_{(i,m)\in S^A} \sum_{k\in K_v | J_{ik}=-1} C_{ik}^{P} \left(r_{imk} + r_{imk}^{E}\right) + \sum_{i\in N} \sum_{k\in K_v | J_{ik}=-1} C_{ik}^{P} r_{ik}^{T} + \sum_{i\in N} \sum_{k\in K_v | J_{ik}=1} C_{ik}^{PP} r_{ik}^{PT} \qquad (14)$$

subject to

(1) and (2)

(A2) to (A27) and (A30) to (A45)

$$z_{imjn}^{SPV} \geq \sum_{v\in V} X_{imjnv} - \sum_{v\in V} x_{imjnv} \qquad (i,m,j,n) \in S^Y \qquad (15)$$

$$z_{imjn}^{SPV} \geq \sum_{v\in V} x_{imjnv} - \sum_{v\in V} X_{imjnv} \qquad (i,m,j,n) \in S^Y \qquad (16)$$

$$z_{im}^{SPVO} \geq \sum_{v\in V} X_{imv}^{O} - \sum_{v\in V} x_{imv}^{O} \qquad (i,m) \in S^B \qquad (17)$$

$$z_{im}^{SPVO} \geq \sum_{v\in V} x_{imv}^{O} - \sum_{v\in V} X_{imv}^{O} \qquad (i,m) \in S^B \qquad (18)$$

$$z_{imjn}^{SPV} \in \{0,1\} \qquad (i,m,j,n) \in S^Y \qquad (19)$$

$$z_{im}^{SPVO} \in \{0,1\} \qquad (i,m) \in S^B. \qquad (20)$$

6.3. Visit deviation

The visit deviation metric, called VD, compares port visits in the reoptimized solution to those in the original solution and counts visit violations which should be minimized (Touzout et al., 2021). This metric does not consider the vessel number. As an example, port i at visit m that is visited by vessel 1 in the original solution, may be visited by vessel 3 in the reoptimized solution. This metric is helpful in situations where it is very costly to miss a scheduled visit. If a planned port visit is omitted in the reoptimized solution, this may result in wasted time and resources, and if a new port visit occurs in the reoptimized solution that was not planned in the original solution, this may result in higher operating costs due to the unavailability of resources at a port.

The mathematical formulation of the VD metric has a new binary variable, z_{im}^{VD} which is defined to denote whether or not there is a visit deviation for port arrival (i, m). Thus, the new constraint is defined as follows:

$$z_{im}^{VD} = |Y_{im} - y_{im}| \qquad (i, m) \in S^B. \qquad (21)$$

Like constraints (3) and (4), constraint (21) can be linearized, resulting in constraints (23) and (24). Constraints (21) count a visit violation if port arrival (i, m) was visited in the original solution and not in the reoptimized solution, and vice versa. The mathematical formulation for the VD metric is as follows:

Model 3: Reoptimization based on visit deviation (VD) metric

$$\text{Minimize} \sum_{(i,m)\in S^A} z_{im}^{VD} + \sum_{(i,m)\in S^A} \sum_{k\in K_v|J_{ik}=-1} C_{ik}^{P} (r_{imk} + r_{imk}^{E}) + \sum_{i\in N} \sum_{k\in K_v|J_{ik}=-1} C_{ik}^{P} r_{ik}^{T} + \sum_{i\in N} \sum_{k\in K_v|J_{ik}=1} C_{ik}^{PP} r_{ik}^{PT} \qquad (22)$$

subject to

(1) and (2)

(A2) to (A27) and (A30) to (A45)

$$z_{im}^{VD} \geq Y_{im} - y_{im} \qquad (i, m) \in S^B \qquad (23)$$

$$z_{im}^{VD} \geq y_{im} - Y_{im} \qquad (i, m) \in S^B \qquad (24)$$

$$z_{im}^{VD} \in \{0,1\} \qquad (i, m) \in S^B. \qquad (25)$$

6.4. Visit deviation without vessel replacement

The visit deviation without vessel replacement metric, called VDV, computes the number of ports that are not visited in the reoptimized solution but are visited in the original solution with a certain vessel number. This metric differs with previous metric in terms of vessel number. For example, if $W_{221} = 1$ and $w_{223} = 1$ then VDV metric is equal to 2 but VD metric is equal to 0 because $Y_{22} = 1$ and $y_{22} = 1$.

The mathematical formulation of the VDV metric contains a new binary variable. Let z_i^{VDV} denote whether there is a visit deviation without vessel replacement for port i or not. Hence, the new constraint is defined as follows:

$$z_{imv}^{VDV} = |W_{imv} - w_{imv}| \qquad v \in V, (i, m) \in S_v^B. \qquad (26)$$

Constraints (26) count a violation of visit without vessel replacement if the number of visits to port i in the original solution is not the same as in the reoptimized solution. Like constraints (3) and (4), constraints (26) can be linearized as shown in constraints (28) and (29). The mathematical formulation for the VDV metric is as follows:

Model 4: Reoptimization based on visit deviation without vessel replacement (VDV) metric

Minimize $\sum_{(i,m)\in S_v^A}\sum_{v\in V} z_{imv}^{VDV} + \sum_{(i,m)\in S^A}\sum_{k\in K_v|J_{ik}=-1} C_{ik}^P\,(r_{imk}+r_{imk}^E)$ (27)

$+\sum_{i\in N}\sum_{k\in K_v|J_{ik}=-1} C_{ik}^P\, r_{ik}^T + \sum_{i\in N}\sum_{k\in K_v|J_{ik}=1} C_{ik}^{PP}\, r_{ik}^{PT}$

subject to

(1) and (2)

(A2) to (A27) and (A30) to (A45)

$z_{imv}^{VDV} \geq W_{imv} - w_{imv}$ $\quad v\in V, (i,m)\in S_v^B$ (28)

$z_{imv}^{VDV} \geq w_{imv} - W_{imv}$ $\quad v\in V, (i,m)\in S_v^B$ (29)

$z_{imv}^{VDV} \in \{0,1\}$ $\quad v\in V, (i,m)\in S_v^B.$ (30)

6.5. Quantity deviation

The quantity deviation metric, called QD, calculates the difference between the quantity of product loaded onto or unloaded from a vessel at a port in the original solution and the loaded or unloaded quantity in the reoptimized solution. Minimizing this metric leads to fewer planning issues (Touzout et al., 2021) and it is significant because it is the only metric that addresses the inventory component of the MIRP.

The mathematical formulation of the QD metric contains a new variable. Let z_{imvk}^{QD} be the difference in the quantity of product k loaded onto or unloaded from vessel v upon arrival at port (i,m) in the original solution and the reoptimized solution. Thus, the new constraint is defined as follows:

$z_{imvk}^{QD} = |Q_{imvk} - q_{imvk}|$ $\quad v\in V, (i,m)\in S_v^B, k\in K_v: J_{ik}\neq 0.$ (31)

Constraints (31) calculate the difference between the loaded or unloaded quantity in the original solution and the reoptimized solution. Like constraints (3) and (4), constraints (31) can be linearized as in constraints (33) and (34). The mathematical formulation for the QD metric is as follows:

Model 5: Reoptimization based on quantity deviation (QD) metric

Minimize $\sum_{(i,m)\in S_v^A}\sum_{v\in V}\sum_{\substack{k\in K_v\\ J_{ik}\neq 0}} z_{imvk}^{QD} + \sum_{(i,m)\in S^A}\sum_{k\in K_v|J_{ik}=-1} C_{ik}^P\,(r_{imk}+r_{imk}^E)$ (32)

$+\sum_{i\in N}\sum_{k\in K_v|J_{ik}=-1} C_{ik}^P\, r_{ik}^T + \sum_{i\in N}\sum_{k\in K_v|J_{ik}=1} C_{ik}^{PP}\, r_{ik}^{PT}$

subject to

(1) and (2)

(A2) to (A27) and (A30) to (A45)

$z_{imvk}^{QD} \geq Q_{imvk} - q_{imvk}$ $\quad v\in V, (i,m)\in S_v^B, k\in K_v: J_{ik}\neq 0$ (33)

$$z_{imvk}^{QD} \geq q_{imvk} - Q_{imvk} \qquad v \in V, (i,m) \in S_v^B, k \in K_v: J_{ik} \neq 0 \qquad (34)$$

$$z_{imvk}^{QD} \geq 0 \qquad v \in V, (i,m) \in S_v^B, k \in K_v: J_{ik} \neq 0. \qquad (35)$$

7. Analysis of the stability metrics

In this section, each stability metric is analyzed. First, the problem instances are presented in Section 7.1. The evaluation procedure is explained in Section 7.2 and numerical results and findings are presented in Section 7.3.

7.1. Problem instances

There are 270 instances in this paper. Instances are divided into three groups called A, B and C. Group A consists of instances with one product, three vessels and eight ports; Group B consists of instances with two products, four vessels and 16 ports; and group C is similar to group A but with four products. In group A and B, each port is limited to at most one product, but in group C, there are sometimes more than one product in a given port. The input parameters of instances are derived from base cases I1 to I10 of Shaabani et al. (2023) where each case differs by the initial inventory of product k at port i, the maximum inventory of product k at port i, and the demand rate of port i for product k. Two different time horizons ($T = 30, 60$ days) are considered for the problem.

The sailing times are assumed to be subject to uncertainty due to weather conditions. As was done by Agra et al. (2015), in the current study we introduce two possible changes in sailing times, considering that sailing times can also remain unchanged. In the first change, sailing times are increased to 1.5 times the original value, and in the second change, they are increased to twice the original value. Based on any of these changes, the sailing times for each port may change. Since uncertainty may affect an area at sea, sailing times are selected based on the combination of arrival and departure ports. Therefore, for example, if an event occurs in the area of port 1, all sailing times from port 1 to other ports and from other ports to port 1 are affected. Table 3 shows the three probability distributions considered, where each column represents one of the possible changes that can occur for sailing times and each row represents a probability distribution and gives the probability for each of the three possible changes. If all sailing times are assigned to the first column, then no uncertainty has occurred. However, since the current problem examines uncertainty in sailing times, this is not considered, and another change is created based on the same probability distribution until at least one sailing time is assigned to the second or third column. Besides, three scenarios are generated for each probability distribution.

Table 4 shows the characteristics of the problem instances where the total number of instances is 270 (3 groups $\times$ 5 base cases $\times$ 2 time horizons $\times$ 3 probability distributions $\times$ 3 scenarios). Let g be a group of instances and $G = \{A, B, C\}$ be the set of groups. Also, let u be a base case and U a set of base cases, then

$$U = \begin{cases} I1, \ldots, I5 & , \quad g \in \{A, B\} \\ I6, \ldots, I10 & , \quad g \in \{C\} \end{cases}.$$

According to Shaabani et al. (2023), the structure of instances can make the MIRP difficult to solve; therefore, these instances are selected such that the optimal solutions can be found in less than 21,600 seconds, thereby enabling a fair analysis of the stability metrics.

Table 3. Probability distributions for sailing time

Probability distributions	Sailing time		
	$\times 1$	$\times 1.5$	$\times 2$
$P = 1$	0.85	0.10	0.05
$P = 2$	0.50	0.30	0.20
$P = 3$	0.15	0.45	0.40

Table 4. Characteristics of the problem instances

Groups g	Number of products $\|K\|$	Number of vessels $\|V\|$	Number of ports $\|N\|$	Base cases U	Time horizon T	Probability distributions P	Scenarios S
A	1	3	8	$\{I1,\dots,I5\}$	$\{30, 60\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$
B	2	4	16	$\{I1,\dots,I5\}$			
C	4	3	8	$\{I6,\dots,I10\}$			

7.2. Evaluation procedure

A set of metrics which represents the "$cost$" metric and five introduced stability metrics in Section 6 is defined as $\Theta = \{\theta_0,\dots,\theta_5\}$ where $\theta_0 = cost$, $\theta_1 = SP$, $\theta_2 = SPV$, $\theta_3 = VD$, $\theta_4 = VDV$, $\theta_5 = QD$.

All instances are solved directly by CPLEX 20.1. First the deterministic model given in Section 5 is solved according to Algorithm 1. Then, the evaluation procedure for an instance is given in Algorithm 2 and matrix R represents the structure of the outcomes for all metrics for an instance and is shown in Table 5.

The details of the evaluation procedure are as follows. After solving the deterministic model the modifications to the basic model explained in Section 5 are applied. Now the modified model can be solved, minimizing the "$cost$" metric, and the reoptimized solution is obtained, which includes the sailing costs and port operation costs, plus penalty costs for backlogs and overstocks, but only the sailing costs and port operation costs, which is called C^*, is reported. Since all models include penalty cost terms and the values obtained for these terms are mostly identical, the solutions for all models are reported without the value for penalty costs.

The model is then solved considering each stability metric. Therefore, the five models given in Section 6 are solved, and the objective values are given in the main diagonal part of Table 5. Then, similarly to the procedure introduced by Touzout et al. (2021), an evaluation of the stability metrics is performed, where one metric is fixed at its optimal value by some additional constraints presented in Table 6, and the model is solved to optimality. For example, in the second row in Table 5, the SP metric was fixed at its optimal value (SP^*) except for the main diagonal element, and then the optimal value of the other metrics was found. For example, $QD^*(SP^*)$ reports the optimal solution of the model with the QD metric, while SP is kept at its optimal value.

Algorithm 1: Solving the deterministic model

Inputs: A group $g \in G$, a base case $u \in U$, a planning horizon $T \in \{30, 60\}$
Solve the deterministic model in Section 5
Output: Initial plan for the reoptimization

Algorithm 2: The evaluation procedure for an instance

Inputs: One probability distribution $P \in \{1,2,3\}$, one of its scenarios $S \in \{1,2,3\}$, an uncertain event occurs at T^U, an empty matrix R
Modifications to the model (according to Section 5)
for $\theta_i \in \Theta$
    Solve the "Model i" (according to Section 5 and Section 6.1 to 6.5)
end for
for $\theta_i \in \Theta$
    for $\theta_j \in \Theta, \theta_j \neq \theta_i$
        Add the additional constraints α_j to the "Model i" (according to Table 6)
        Solve the model
    end for
end for
Output: Matrix R (Shown in Table 5)

Table 5. Matrix R showing the structure of outcomes for an instance

	Cost	SP	SPV	VD	VDV	QD
Cost	C^*	$SP^*(C^*)$	$SPV^*(C^*)$	$VD^*(C^*)$	$VDV^*(C^*)$	$QD^*(C^*)$
SP	$C^*(SP^*)$	SP^*	$SPV^*(SP^*)$	$VD^*(SP^*)$	$VDV^*(SP^*)$	$QD^*(SP^*)$
SPV	$C^*(SPV^*)$	$SP^*(SPV^*)$	SPV^*	$VD^*(SPV^*)$	$VDV^*(SPV^*)$	$QD^*(SPV^*)$
VD	$C^*(VD^*)$	$SP^*(VD^*)$	$SPV^*(VD^*)$	VD^*	$VDV^*(VD^*)$	$QD^*(VD^*)$
VDV	$C^*(VDV^*)$	$SP^*(VDV^*)$	$SPV^*(VDV^*)$	$VD^*(VDV^*)$	VDV^*	$QD^*(VDV^*)$
QD	$C^*(QD^*)$	$SP^*(QD^*)$	$SPV^*(QD^*)$	$VD^*(QD^*)$	$VDV^*(QD^*)$	QD^*

Table 6. Additional constraints

Additional constraints indicator	Added constraints
α_0	$\sum_{v \in V} \sum_{(i,m,j,n) \in S_v^X} C_{ijv}^T \, x_{imjnv} + \sum_{v \in V} \sum_{(i,m) \in S_v^A} C_{oiv}^{TO} \, x_{imv}^O + \sum_{v \in V} \sum_{(i,m) \in S_v^A} \sum_{k \in K_v \mid J_{ik} \neq 0} C_{ik}^O \, o_{imvk} = C^*$
α_1	Constraints (6) to (11), and $\sum_{v \in V} \sum_{(i,m,j,n) \in S_v^X} z_{imjnv}^{SP} + \sum_{v \in V} \sum_{(i,m) \in S_v^A} z_{imv}^{SPO} = SP^*$
α_2	Constraints (15) to (20), and $\sum_{(i,m,j,n) \in S^X} z_{imjn}^{SPV} + \sum_{(i,m) \in S^A} z_{im}^{SPVO} = SPV^*$

α_3	Constraints (23) to (25), and	$\sum_{(i,m)\in S^A} z_{im}^{VD} = VD^*$
α_4	Constraints (28) to (30), and	$\sum_{(i,m)\in S_v^A} \sum_{v\in V} z_{imv}^{VDV} = VDV^*$
α_5	Constraints (33) to (35), and	$\sum_{(i,m)\in S_v^A} \sum_{v\in V} \sum_{\substack{k\in K_v \\ J_{ik}\neq 0}} z_{imvk}^{QD} = QD^*$

7.3. Numerical results and findings

This section contains the results for the three groups, A, B, and C. Each group includes 90 instances and the aggregate result of these 90 instances is given for each group. To create an aggregate result, the equivalence rate must be defined.

Two metrics are said to be equivalent when optimizing one does not prevent the other from being optimal and vice versa (Touzout et al., 2021). This is evaluated using the output of Algorithm 2. Also, two metrics are said to be divergent if optimizing one worsens the value of the other metric and vice versa. A new matrix is created corresponding to matrix R in which each cell has a value of 1 when two metrics are equivalent and has a value of 0 when two metrics are divergent.

Here is an example, Table 7(a) shows the results for the first scenario of the first probability distribution of group A with time horizon 60 for base case $I1$, where SP and VD are equivalent since $SP^*(VD^*) = SP^*$ and $VD^*(SP^*) = VD^*$ but SP and QD are divergent because $SP^* = 6$ and by fixing QD at the optimal value, the SP^* deteriorates so that $SP^*(QD^*) = 7$ and $QD^* = 464.37$ and by fixing SP at the optimal value, the QD^* deteriorates such that $QD^*(SP^*) = 746.32$. Table 7(b) shows the new matrix corresponding to the matrix in Table 7(a) showing which metrics are equivalent.

The matrix shown in Table 7(b) is an example of a single instance, and this type of matrix is created for all instances. Thus, there are 90 such matrices for each group and the ratio of instances in which each pair of metrics is equivalent is called the equivalence rate of these metrics. Table 8 shows the equivalence rate between the stability metrics for the three groups.

Table 7. Results for an instance of group A

(a) Results							(b) Metrics are: equivalent = **1** or divergent = **0**						
	Cost	SP	SPV	VD	VDV	QD		Cost	SP	SPV	VD	VDV	QD
Cost	632.99	9	9	7	8	900.46	Cost	-	0	0	0	0	0
SP	732.79	6	6	4	5	746.32	SP	0	-	1	1	1	0
SPV	740.52	6	6	4	5	746.32	SPV	0	1	-	1	1	0
VD	732.52	6	6	4	5	746.32	VD	0	1	1	-	1	0
VDV	732.52	6	6	4	5	746.32	VDV	0	1	1	1	-	0
QD	673.78	7	7	5	6	464.37	QD	0	0	0	0	0	-

Table 8. The percent equivalence rate between the stability metrics

Group A	Cost	SP	SPV	VD	VDV	QD
Cost	-	40.00	40.00	45.56	53.33	42.22
SP	40.00	-	94.44	78.89	76.67	62.22
SPV	40.00	94.44	-	78.89	88.89	56.67
VD	45.56	78.89	78.89	-	100.00	73.33
VDV	53.33	76.67	88.89	100.00	-	73.33
QD	42.22	62.22	56.67	73.33	73.33	-
Group B	**Cost**	**SP**	**SPV**	**VD**	**VDV**	**QD**
Cost	-	40.00	36.67	57.78	70.00	60.00
SP	40.00	-	93.33	70.00	70.00	80.00
SPV	36.67	93.33	-	78.89	70.00	88.89
VD	57.78	70.00	78.89	-	91.11	83.33
VDV	70.00	70.00	70.00	91.11	-	88.89
QD	60.00	80.00	88.89	83.33	88.89	-
Group C	**Cost**	**SP**	**SPV**	**VD**	**VDV**	**QD**
Cost	-	51.11	51.11	50.00	47.78	58.89
SP	51.11	-	96.67	66.67	64.44	91.11
SPV	51.11	96.67	-	64.44	58.89	82.22
VD	50.00	66.67	64.44	-	38.89	35.56
VDV	47.78	64.44	58.89	38.89	-	75.56
QD	58.89	91.11	82.22	35.56	75.56	-

Figure 3 illustrates the information from Table 8 and Figure 4 shows the average value of the equivalence rate for the three groups. Based on these two figures, the following observations can be made.

- As shown in Figure 4, cost is the only metric that is always below a 60% equivalence ratio. This is not surprising, since it is costly to keep a plan unchanged when it is subject to uncertainty, but there is a slight difference when comparing the cost with the five stability metrics. On average, keeping SP or SPV at the optimal value is almost divergent with optimizing cost (equivalence rate below 45%). Looking at the different groups, it can be seen that SP or SPV with cost metric are less divergent in group C compared with group A and B, which is due to the structure of group C, where each port is not limited to one product; therefore, it is less difficult to keep the sequence.
- The equivalence rate between VD and VDV for group A is 100%. There may be two reasons for this. First, this is a single-product problem where it is not difficult to keep a scheduled visit when uncertainties arise, so it is not necessary to replace a vessel to keep a scheduled visit. For group B, where there are two products, this value was reduced by 10%. Second, in these two cases, each port is limited to at most one product, so it is easy to keep a scheduled visit.
- The largest difference in the equivalence rate of similar pairs of metrics between three groups belongs to two pairs (VD and VDV, VD and QD), where there is a high equivalence

rate (over 70%) for groups A and B, but it is very low (under 40%) for group C. The main reason for this large difference is the structure of the groups, where each port is limited to at most one product for groups A and B but in group C, there is no such limitation; however, it could still be that there is only one product. Therefore, the results state these two pairs (VD and VDV, VD and QD) are divergent when each port is not limited to one product.

- The SP and SPV are the only two metrics shown to be strongly equivalent for all groups, with an average of about 95%. As shown in Figure 4, the equivalence rate of all other pairs of metrics averages less than 80%.
- Although the highest equivalence rate is between SP and SPV, eight other pairs of metrics also have high equivalence rates. As shown in Figure 4, equivalence rate for eight pairs of metrics is between 70% and 80%.
- On average, as seen in Figure 4, keeping QD at its optimal value is more likely to lead to convergence in the other metrics than keeping VD at its optimal value. However, based on Figure 3, it is the other way around when considering only group A, since there is only one product, which leads to less flexibility in terms of quantity delivered, but it is much easier to keep the visit unchanged since it always includes one product.
- Keeping VD or VDV at the optimal value leads to almost the same equivalence rate in all three groups with SP. This is because minimal changes in scheduled port visits contribute to a smaller number of changes in visit order optimization.

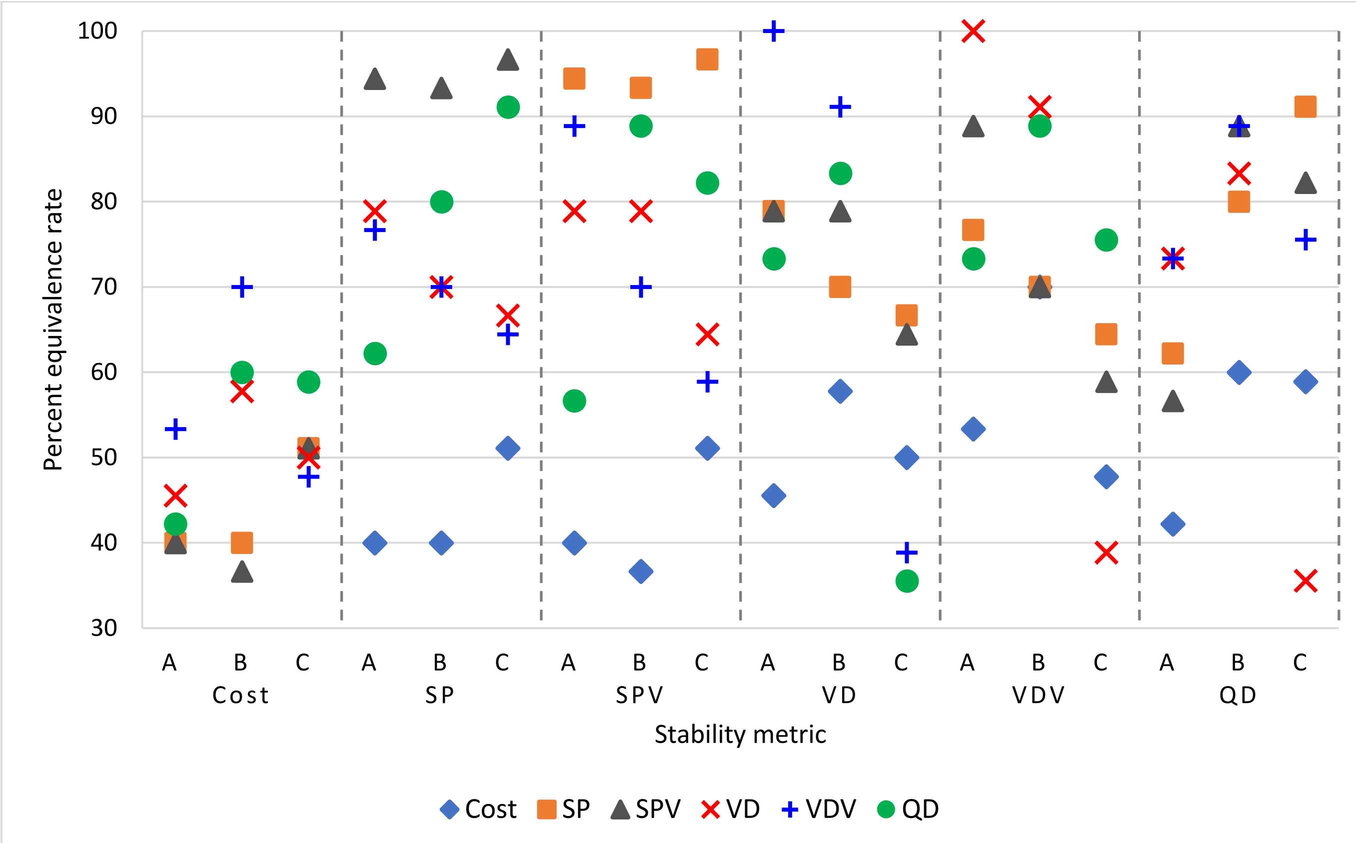


Figure 3. Equivalence rate between the stability metrics for groups A, B, and C

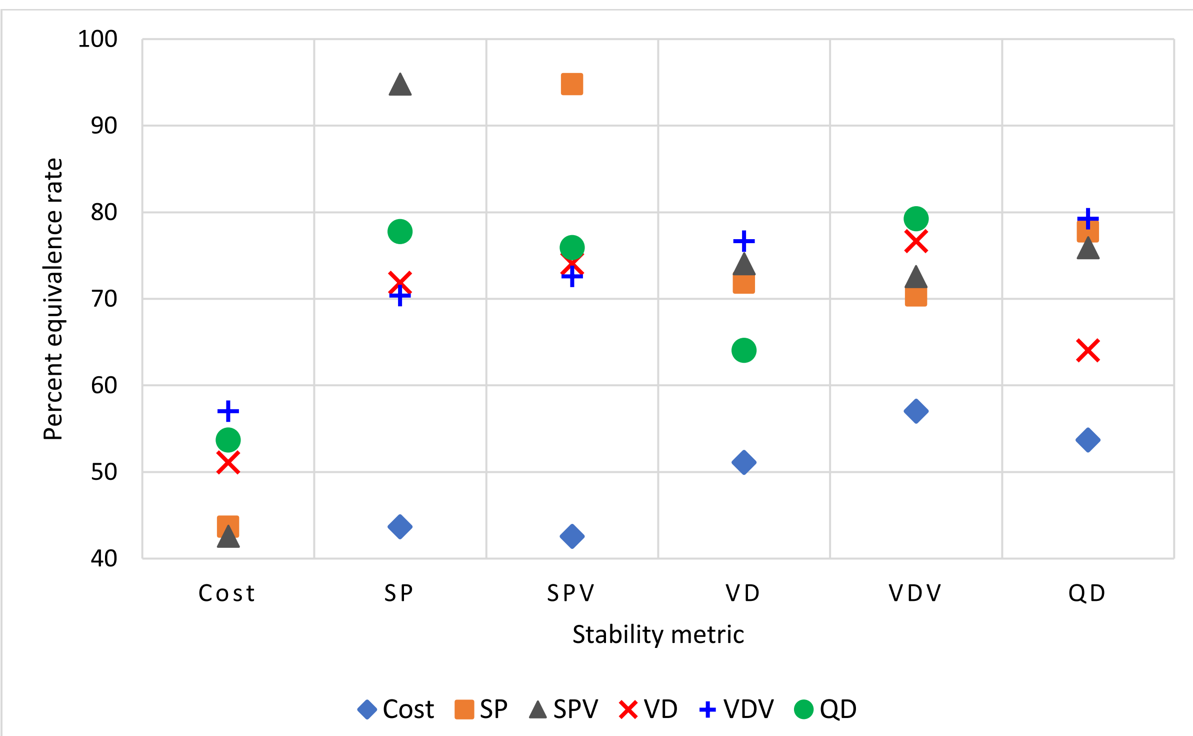
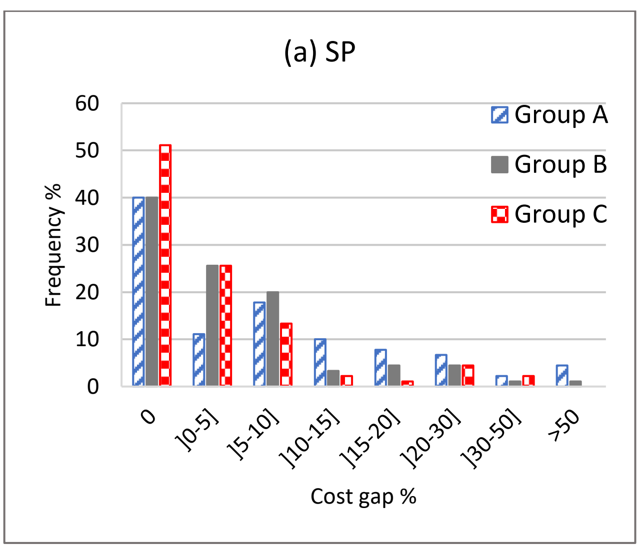
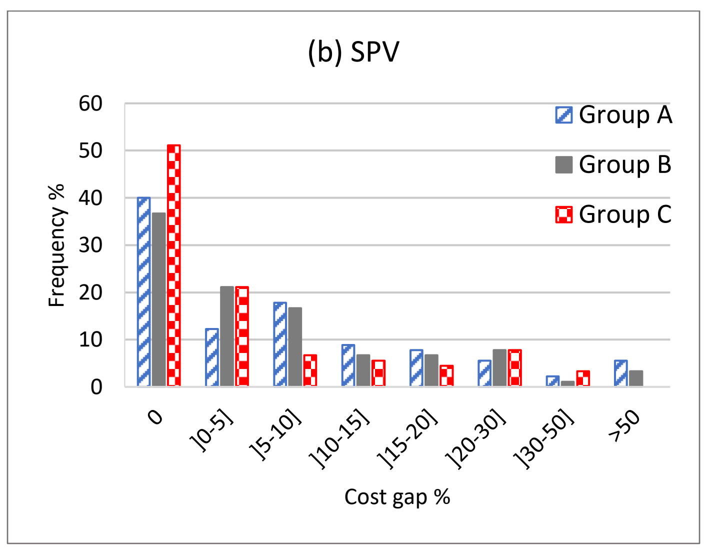


Figure 4. Average equivalence rate between the stability metrics

Since the cost metric is the most divergent metric, a more detailed analysis is performed for it, while the other metrics have their optimal value. In this context, the percentage of the cost gap of the optimal cost with respect to the other metrics is computed. For example, the cost gap of the cost metric with respect to the SP metric is calculated as $\frac{C^*(SP^*)-C^*}{C^*}$, where C^* is the optimal value for cost metric and $C^*(SP^*)$ is the optimal solution of the model with the cost metric, while SP is kept at its optimal value. Figure 5 shows the frequency of the cost gap in percent. Figure 5(a) through 5(e) show the cost gap for each of the three groups for SP, SPV, VD, VDV, and QD, and Figure 5(f) shows the mean of the cost gap for all groups and the cumulative percentage for the average of the five stability metrics. A cost gap of zero means that the cost metric is equivalent to the given metric.

Figure 5 shows that almost 20% of the instances for all metrics have a cost gap of 0% to 5%, and almost 15% of the instances for the SP and SPV metrics and almost 10% of the instances for the VD, VDV, and QD metrics have a cost gap between 5% and 10%. In addition, almost 45% of the instances for the SP and SPV metrics and almost 55% of the instances for VD, VDV, and QD metrics are equivalent to the cost metric. Consequently, less than 20% of the instances for all metrics have a cost gap greater than 10%. This means that a decision maker has a choice between a more stable plan with some cost deterioration or minimizing costs by making frequent adjustments to the original plan. If the decision maker chooses to make frequent adjustments to the original plan, the cost deterioration threshold is set to 0%, which, based on the cumulative percentages shown in Figure 5(f), means that optimizing for stability results in no additional cost nearly 50% of the time. If the decision maker chooses a more stable plan, a certain threshold for cost deterioration can be set. The results show that if the decision maker accepts a cost deterioration of 5%, this leads to 20% more stable solutions. Thus, in almost 70% of the instances, optimizing stability resulted in a maximum of 5% additional cost. Even more stable solutions can be obtained where more than 80% of the instances lead to stable solutions, but the decision maker has to accept a cost deterioration of 10%.





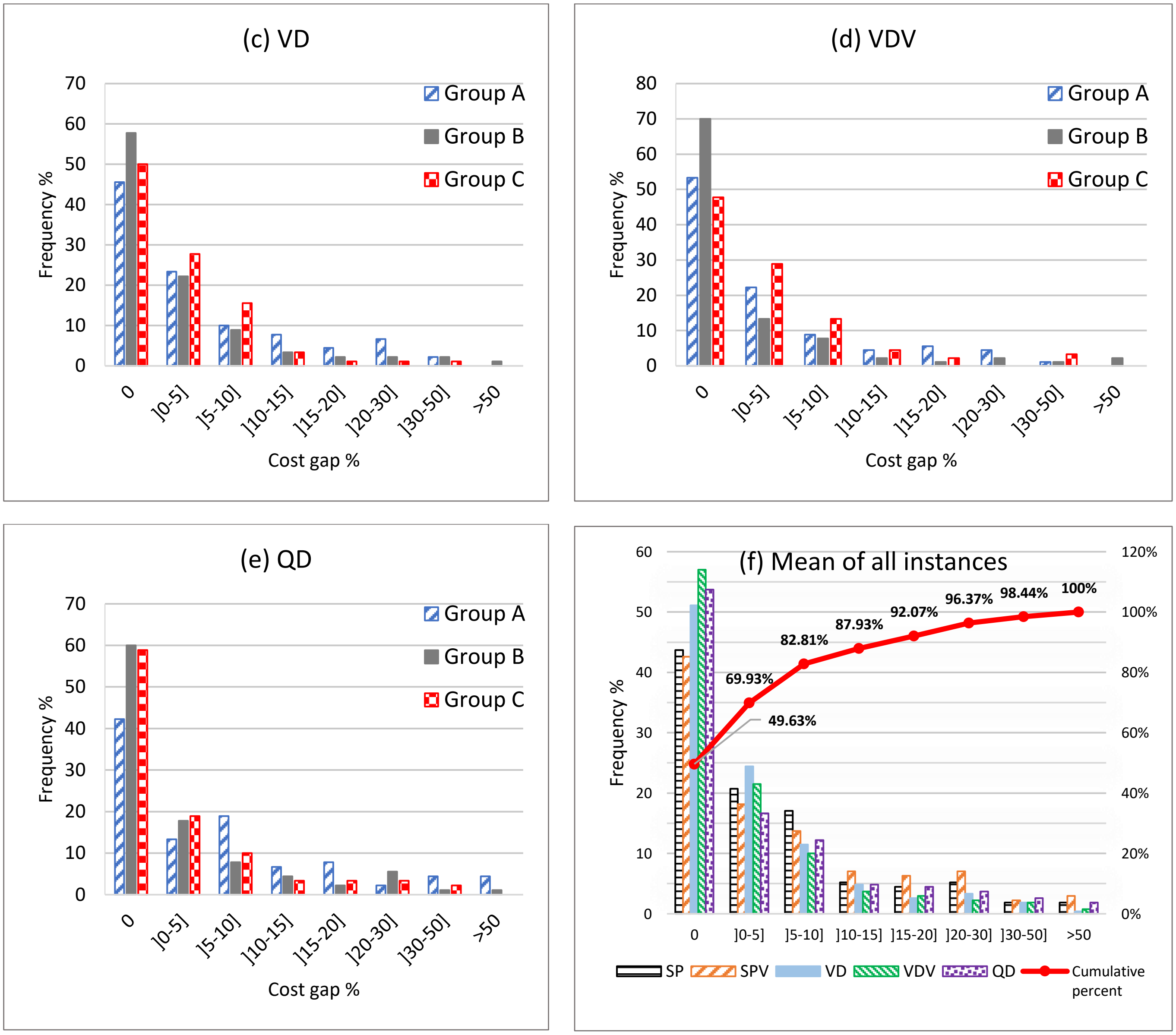


Figure 5. The frequency of the cost gap in percent

An absolute equivalence is defined here. An instance is said to be "absolutely equivalent" if all pairs of metrics are equivalent, meaning that optimizing any metric in one instance does not prevent the other from being optimal; thus, there are no conflicting metrics. The result shows that the number of instances with absolute equivalence for groups A, B, and C is 20, 9, and 14, respectively. Overall, almost 16% of the instances have absolute equivalence, which means that the problem involves conflicting decisions in about 84% of the instances. This may motivate the use of multiple-criteria decision analysis (MCDA) or multi-objective optimization to find trade-offs between different conflicting stability metrics.

8. Conclusion

We have studied the problem of maritime inventory routing under sailing time uncertainty. Five stability metrics were introduced and embedded in the mathematical formulations to determine the extent of changes to an original plan when uncertainties occur. Therefore, a reoptimization framework was implemented to determine the impact of each stability metric on the reoptimized plan.

The analyses have shown that it is costly to keep a plan unchanged. Therefore, the cost metric becomes worse when any other metrics have been set to their optimal values. The remaining pairs of metrics, including all pairs among SP, SPV, VD, VDV, QD can be divided into three groups. The first group, with the strongest equivalence rate of about 95%, includes only SP and SPV. The second group also contains only one pair, which includes VD and QD with the weakest equivalence rate of about 64%. The third group, where the equivalence rate is between 70% and 80%, includes the remaining eight pairs of metrics (SP and VD, SP and VDV, SP and QD, SPV and VD, SPV and VDV, SPV and QD, VD and VDV, VDV and QD). An analysis of the percentage of the cost gap showed that optimization of the stability metrics without additional costs occurs in almost 50% of the cases but accepting a 5% cost deterioration can lead to 20% more stable solutions.

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Appendix A. Mathematical model

This appendix first defines the notations and then presents the mathematical model.

Indices

i, j	Indices for ports, $i, j \in \{1, \ldots, \lvert N \rvert\}$
m, n	Indices for visits at each port, $m, n \in \{1, \ldots, \overline{m}\}$
v	Index for vessels, $v \in \{1, \ldots, \lvert V \rvert\}$
k	Index for products, $k \in \{1, \ldots, \lvert K \rvert\}$
(i, m)	Index for m^{th} visit of port *i*

Sets

S^A	Set of possible port arrivals (i, m)
S_v^A	Set of port arrivals that may be visited by vessel *v*
S^X	Set of all possible vessel movements (i, m, j, n)
S_v^X	Set of all possible movements of vessel *v*
V_i	Set of vessels that can visit port *i*
K_v	Set of products that vessel *v* can transport
N	Set of ports
V	Set of vessels
K	Set of products

Parameters

$\overline{m}$	Maximum number of visits
J_{ik}	1 if port *i* is a supplier of product *k*; -1 if port *i* is a consumer of product *k*; 0 if port *i* is neither a supplier nor a consumer of product *k*
R_{ik}	Demand rate of port *i* for product *k*
Q_{vk}^O	Load of vessel *v* of product *k* at the beginning of the planning horizon
C_{vk}	Capacity of the compartment of vessel *v* dedicated to product *k*
Q_{ik}^{MIN}	Minimum unloading quantities of product *k* at port *i*
Q_{ik}^{MAX}	Maximum unloading quantities of product *k* at port *i*
T	Length of the time horizon
T_{ik}^Q	Time required to (un)load one unit of product *k* at port *i*
T_{ik}^S	Set up time required to operate product *k* at port *i*
T_{ijv}	Sailing time between port *i* and *j* by vessel *v*
T_{iv}^O	Sailing time from initial position to port *i* by vessel *v*
T_i^B	Minimum interval between the departure of one vessel and the next arrival at port *i*
T_{im}^W	Waiting time at port arrival (i, m)
S_{ik}^O	Initial inventory level of product *k* at port *i*
S_{ik}^{MIN}	Minimum inventory level of product *k* at port *i*
S_{ik}^{MAX}	Maximum inventory level of product *k* at port *i*
C_{ijv}^T	Sailing cost of vessel *v* from port *I* to port *j*

C^{TO}_{oiv}	Sailing cost of vessel *v* from its initial port position to port *i*
C^{O}_{ik}	Operating cost of product *k* at port *i*
C^{P}_{ik}	Penalty cost for backlogging of product *k* at port *i*
C^{PP}_{ik}	Penalty cost for having more than maximum allowed level at port *i* for product *k*

Binary variables

x_{imjnv}	1 if and only if vessel *v* sails from port arrival (i,m) directly to port arrival (j,n)
x^{O}_{imv}	1 if and only if vessel *v* sails directly from its initial position to port arrival (i,m)
w_{imv}	1 if and only if vessel *v* visits port *i* at arrival (i,m)
z_{imv}	1 if and only if vessel *v* ends its route at port arrival (i,m)
y_{im}	1 if and only if vessel visit port arrival (i,m)
z^{O}_{v}	1 if and only if vessel *v* is not used
o_{imvk}	1 if and only if product *k* is loaded onto or unloaded from vessel *v* at port visit (i,m)

Continuous variables

q_{imvk}	Amount of product *k* loaded onto or unloaded from vessel *v* at port visit (i,m)
f_{imjnvk}	Amount of product *k* that vessel *v* transports from port visit (i,m) to port visit (j,n)
f^{O}_{imvk}	Amount of product *k* that vessel *v* transports from its initial position to port visit (i,m)
t_{im}	Start time for port arrival (i,m)
t^{E}_{im}	End time for port arrival (i,m)
t^{+}_{i}	Remaining time from the end of the last visit of port *i* until time *T*
s_{imk}	Inventory level of product *k* at the start of port visit (i,m)
s^{E}_{imk}	Inventory level of product *k* at the end of port visit (i,m)
s^{T}_{ik}	Inventory level of product *k*, above the minimum stock level for port *i* at the end of time *T* or at the end of the last visit (if this occurs after *T*)
s^{PT}_{ik}	Amount of product *k* below the maximum stock level for port *i* at the end of time *T* or at the end of last visit (if this occurs after *T*)
r_{imk}	Backlog of product *k* at the start of port visit (i,m)
r^{E}_{imk}	Backlog of product *k* at the end of port visit (i,m)
r^{T}_{ik}	Amount of product *k* below the minimum level for port *i* at the end of time *T*
r^{PT}_{ik}	Amount of product *k* above the maximum level for port *i* at the end of time *T*

$$\begin{aligned}\text{Minimize} \sum_{v\in V}\sum_{(i,m,j,n)\in S^X_v} C^T_{ijv}\, x_{imjnv} + \sum_{v\in V}\sum_{(i,m)\in S^A_v} C^{TO}_{oiv}\, x^O_{imv} \\ + \sum_{v\in V}\sum_{(i,m)\in S^A_v}\sum_{k\in K_v|J_{ik}\neq 0} C^O_{ik}\, o_{imvk} \\ + \sum_{(i,m)\in S^A_v}\sum_{k\in K_v|J_{ik}=-1} C^P_{ik}\,(r_{imk}+r^E_{imk}) + \sum_{i\in N}\sum_{k\in K_v|J_{ik}=-1} C^P_{ik}\, r^T_{ik} \\ + \sum_{i\in N}\sum_{k\in K_v|J_{ik}=+1} C^{PP}_{ik}\, r^{PT}_{ik}\end{aligned} \tag{A1}$$

subject to

routing constraints:

$$\sum_{(i,m)\in S_v^A} x_{imv}^O + z_v^O = 1 \qquad v \in V \qquad \text{(A2)}$$

$$w_{imv} - \sum_{\substack{(j,n)\in S_v^A \\ j\neq i}} x_{jnimv} - x_{imv}^O = 0 \qquad v \in V, (i,m) \in S_v^A \qquad \text{(A3)}$$

$$w_{imv} - \sum_{\substack{(j,n)\in S_v^A \\ j\neq i}} x_{imjnv} - z_{imv} = 0 \qquad v \in V, (i,m) \in S_v^A \qquad \text{(A4)}$$

$$\sum_{v\in V_i} w_{imv} = y_{im} \qquad (i,m) \in S^A \qquad \text{(A5)}$$

$$y_{i(m-1)} - y_{im} \geq 0 \qquad (i,m) \in S^A: m > 1 \qquad \text{(A6)}$$

$$x_{imv}^O, w_{imv}, z_{imv} \in \{0,1\} \qquad v \in V, (i,m) \in S_v^A \qquad \text{(A7)}$$

$$x_{imjnv} \in \{0,1\} \qquad v \in V, (i,m,j,n) \in S_v^X \qquad \text{(A8)}$$

$$z_v^O \in \{0,1\} \qquad v \in V \qquad \text{(A9)}$$

$$y_{im} \in \{0,1\} \qquad (i,m) \in S_v^A \qquad \text{(A10)}$$

loading and unloading constraints:

$$f_{imvk}^O + \sum_{(j,n)\in S_v^A} f_{jnimvk} + J_{ik} q_{imvk} = \sum_{(j,n)\in S_v^A} f_{imjnvk} \qquad \text{(A11)}$$

$$v \in V, (i,m) \in S_v^A, k \in K_v$$

$$f_{imvk}^O = Q_{vk}^O\, x_{imv}^O \qquad v \in V, (i,m) \in S_v^A, k \in K_v \qquad \text{(A12)}$$

$$f_{imjnvk} \leq C_{vk} x_{imjnv} \qquad v \in V, (i,m,j,n) \in S_v^X, k \in K_v \qquad \text{(A13)}$$

$$0 \leq q_{imvk} \leq C_{vk} o_{imvk} \qquad v \in V, (i,m) \in S_v^A, k \in K_v: J_{ik} = 1 \qquad \text{(A14)}$$

$$Q_{ik}^{MIN} o_{imvk} \leq q_{imvk} \leq Q_{ik}^{MAX} o_{imvk} \qquad v \in V, (i,m) \in S_v^A, k \in K_v: J_{ik} = -1 \qquad \text{(A15)}$$

$$\sum_{k\in K_v} o_{imvk} \geq w_{imv} \qquad v \in V, (i,m) \in S_v^A \qquad \text{(A16)}$$

$$\sum_{(i,m)\in S_v^A} \sum_{v\in V} \sum_{\substack{k\in K_v \\ J_{ik}=-1}} q_{imvk} \geq \sum_{i\in N} \sum_{\substack{k\in K \\ J_{ik}=-1}} R_{ik} T - \sum_{i\in N} \sum_{\substack{k\in K \\ J_{ik}=-1}} S_{ik}^O \qquad \text{(A17)}$$

$$o_{imvk} \leq w_{imv} \qquad v \in V, (i,m) \in S_v^A, k \in K_v \qquad \text{(A18)}$$

$$f_{imjnvk} \geq 0 \qquad v \in V, (i,m,j,n) \in S_v^A, k \in K_v \qquad \text{(A19)}$$

$$f_{imvk}^O \geq 0 \qquad v \in V, (i,m) \in S_v^A, k \in K_v \qquad \text{(A20)}$$

$$q_{imvk} \geq 0 \qquad v \in V, (i,m) \in S_v^A, k \in K_v: J_{ik} \neq 0 \qquad \text{(A21)}$$

$$o_{imvk} \in \{0,1\} \qquad v \in V, (i,m) \in S_v^A, k \in K_v : J_{ik} \neq 0 \tag{A22}$$

time constraints:

$$t_{im}^E \geq t_{im} + \sum_{v \in V} \sum_{k \in K_v} T_{ik}^S \, o_{imvk} + \sum_{v \in V} \sum_{k \in K_v} T_{ik}^Q \, q_{imvk} \qquad (i,m) \in S^A \tag{A23}$$

$$t_{im} - t_{i(m-1)}^E - T_i^B y_{im} \geq 0 \qquad (i,m) \in S^A : m > 1 \tag{A24}$$

$$t_{im}^E + \sum_{v \in V_i \cap V_j} T_{ijv} \, x_{imjnv} + T_{jn}^W y_{jn} - t_{jn} \leq 2T\left(1 - \sum_{v \in V_i \cap V_j} x_{imjnv}\right) \qquad (i,m,j,n) \in S^X \tag{A25}$$

$$\sum_{v \in V} T_{iv}^O \, x_{imv}^O + T_{im}^W y_{im} \leq t_{im} \qquad (i,m) \in S^A \tag{A26}$$

$$t_i^+ \geq T - t_{i\bar{m}}^E \qquad i \in N \tag{A27}$$

$$t_{im} \leq T \qquad (i,m) \in S^A \tag{A28}$$

$$t_{im}^E \leq T \qquad (i,m) \in S^A \tag{A29}$$

$$t_{im} \geq 0, t_{im}^E \geq 0 \qquad (i,m) \in S^A \tag{A30}$$

$$t_i^+ \geq 0 \qquad i \in N \tag{A31}$$

inventory constraints (consumption ports):

$$s_{i1k} = S_{ik}^O - R_{ik} t_{i1} + r_{i1k} \qquad i \in N, k \in K : J_{ik} = -1 \tag{A32}$$

$$s_{imk}^E + r_{imk} = s_{imk} + r_{imk}^E + \sum_{v \in V} q_{imvk} - R_{ik}(t_{im}^E - t_{im}) \qquad (i,m) \in S^A, k \in K_v : J_{ik} = -1 \tag{A33}$$

$$s_{imk} + r_{i(m-1)k}^E = s_{i(m-1)k}^E + r_{imk} - R_{ik}\left(t_{im} - t_{i(m-1)}^E\right) \qquad (i,m) \in S^A : m > 1, k \in K : J_{ik} = -1 \tag{A34}$$

$$s_{i\bar{m}k}^E + r_{ik}^T = r_{i\bar{m}k}^E + s_{ik}^T + R_{ik} t_i^+ + S_{ik}^{MIN} \qquad i \in N, k \in K : J_{ik} = -1 \tag{A35}$$

$$s_{imk}, S_{imk}^E \leq S_{ik}^{MAX} \qquad (i,m) \in S^A, k \in K : J_{ik} = -1 \tag{A36}$$

$$s_{imk}, s_{imk}^E, r_{imk}, r_{imk}^E \geq 0 \qquad (i,m) \in S^A, k \in K : J_{ik} = -1 \tag{A37}$$

$$s_{ik}^T, r_{ik}^T \geq 0 \qquad i \in N, k \in K : J_{ik} = -1 \tag{A38}$$

inventory constraints (production ports):

$$s_{i1k} = S_{ik}^O + R_{ik} t_{i1} \qquad i \in N, k \in K : J_{ik} = 1 \tag{A39}$$

$$s_{imk}^{E} = s_{imk} - \sum_{v \in V} q_{imvk} + R_{ik}(t_{im}^{E} - t_{im}) \qquad (i,m) \in S^{A}, k \in K_{v}: J_{ik} = 1 \qquad \text{(A40)}$$

$$s_{imk} = s_{i(m-1)k}^{E} + R_{ik}\left(t_{im} - t_{i(m-1)}^{E}\right) \qquad (i,m) \in S^{A}: m > 1, k \in K: J_{ik} = 1 \qquad \text{(A41)}$$

$$s_{i\bar{m}k}^{E} + R_{ik} t_{i}^{+} - r_{ik}^{PT} = S_{ik}^{MAX} - s_{ik}^{PT} \qquad i \in N, k \in K: J_{ik} = 1 \qquad \text{(A42)}$$

$$s_{imk}, S_{imk}^{E} \leq S_{ik}^{MAX} \qquad (i,m) \in S^{A}, k \in K: J_{ik} = 1 \qquad \text{(A43)}$$

$$s_{imk}, s_{imk}^{E} \geq 0 \qquad (i,m) \in S^{A}, k \in K: J_{ik} = 1 \qquad \text{(A44)}$$

$$s_{ik}^{PT}, r_{ik}^{PT} \geq 0 \qquad i \in N, k \in K: J_{ik} = 1 \qquad \text{(A45)}$$

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