Transportation Research Part C 104 (2019) 305–316

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

Supply vessel routing and scheduling under uncertain demand

Yauheni Kisialiou[a,*], Irina Gribkovskaia[a], Gilbert Laporte[b]

[a] Molde University College, Specialized University in Logistics, Postbox 2110, NO-6402 Molde, Norway
[b] HEC Montréal, 3000, chemin de la Côte-Sainte-Catherine, Montréal H3T 2A7, Canada

ARTICLE INFO

Keywords:
Maritime logistics
Supply vessel planning
Recourse
Reliable vessel schedules
Metaheuristic
Simulation

ABSTRACT

We solve a supply vessel planning problem arising in upstream offshore petroleum logistics. A fleet of supply vessels delivers all the necessary equipment and materials to a set of offshore installations from an onshore supply base, according to a delivery schedule or sailing plan. Supply vessels, being the major cost contributor, are chartered on a long-term basis. The planning of supply vessels implies resolving the trade-off between the cost of the delivery schedule and the reliability of deliveries on the scheduled voyages, i.e. the service level. The execution of a sailing plan is affected by stochastic demands at the installations since a high demand fluctuation quite often leads to insufficient vessel capacity to perform a voyage according to the sailing plan. In addition, the average demand level at the installations may change over time, while the number of vessels in the sailing plan remains the same. Maintaining a reliable flow of supplies under stochastic demand therefore leads to additional costs and reduced service level. We present a novel methodology for reliable supply vessel planning and scheduling, enabling planners to construct delivery schedules having a low expected total cost. The methodology involves the construction of delivery schedules with different reliability levels using an adaptive large neighborhood search metaheuristic algorithm combined with a discrete event simulation procedure for the computation of the expected solution cost.

1. Introduction

In the problem under study, a fleet of supply vessels performs deliveries to offshore installations on a regular basis from an onshore supply base, according to a sailing schedule constructed for a time horizon of D days, and applicable for a period of one to three-six month. Supply vessels represent the major cost contributor in offshore upstream logistics. The daily supply vessel charter cost varies around 25,000 USD and therefore planners always try to minimize the fleet size needed to serve the installations. The execution of the vessel schedule is quite often disrupted by uncertain demand at the installations, so that the total demand of a voyage may exceed the capacity of the vessel performing it. Such situations require changes in the schedule, which entail additional costs, and may even require the use of an extra vessel. Therefore, uncertain demand conditions should be taken into account during the construction of the schedule. There already exist several studies dedicated to different variations of the studied problem in its deterministic variant (Borthen et al., 2017; Kisialiou et al., 2018a) and in its stochastic variant under uncertain weather conditions (Halvorsen-Weare and Fagerholt, 2011; Norlund et al., 2015; Kisialiou et al., 2018b), and as well as rich literature on the vehicle routing problem with uncertain demand (Gendreau et al., 2014, 2016). Other sources of uncertainty, such as bunker prices, have also been considered in the maritime transportation literature (Wang et al., 2018). However, our problem has not previously been studied.

* Corresponding author.
E-mail addresses: yauheni.kisialiou@himolde.no (Y. Kisialiou), irina.gribkovskaia@himolde.no (I. Gribkovskaia), gilbert.laporte@cirrelt.ca (G. Laporte).

https://doi.org/10.1016/j.trc.2019.04.011
Received 10 September 2018; Received in revised form 7 April 2019; Accepted 13 April 2019
Available online 23 May 2019

Here we introduce a methodology for the generation of reliable vessel schedules with respect to the total expected cost for problem instances of realistic sizes and uncertain demand.

1.1. Problem description

In the upstream petroleum industry, equipment and materials for the offshore installations are delivered by a set of supply vessels from an onshore supply base. Every supply base serves a predefined set of installations where the delivery of cargo takes place. The research focus of this paper is the problem of supply vessel tactical planning and scheduling which is known as the Periodic Supply Vessel Planning Problem (PSVPP), an extension of the well-known periodic vehicle routing problem (PVRP) which is NP-hard. In the PSVPP each installation requires a certain number of deliveries from the base over the *planning period* (one week in our case) and has a cargo demand to be supplied evenly across the deliveries. Deliveries take place according to the planned vessels schedule constructed for the planning period.

For each installation, the service time for the loading operations performed by a supply vessel is defined as a function of the delivered amount of cargo. Some installations have opening hours only during the day and are closed at night for servicing. Each supply vessel possesses technical characteristics such as its daily charter cost (the main cost contributor), deck capacity, sailing speed, fuel consumption rate (when sailing, servicing and waiting at an installation, and at the base), and the time required for the loading operations at the base (hereinafter referred to as the turnaround time). Similarly to the installations, the supply base has opening hours (for example, from 8:00 to 18:30) which define a time window during which a vessel can perform loading operations. Each vessel has a predefined set of possible daily departure time options from the base (referred to as flexible departures). In addition, each vessel belongs to a certain type which is defined by the vessel's capacity and its ability to serve a certain set of installations. Both the installations and the supply base require predictability of departures to installations, which is why the vessels schedule is performed repeatedly over the period of several month or a season referred to as the schedule *execution horizon.* The repetitive schedule execution implies that the planning period is cyclic, i.e., the vessel may depart on a voyage at the end of a planning period (Saturday) and arrive at the beginning of the next one (Monday). Since the planning period is circular, any two vessels of the same type may swap their voyages on every subsequent week (in general this depends on the length of the planning horizon). Vessels swapping their schedules are referred to as coupled (for a mathematical formulation of the PSVPP with flexible departures and coupled vessels see in Kisialiou et al., 2018a). The specificity of the delivery process and of the service requirements at the installations stems from practical restrictions such as the base capacity (defined by the maximum number of departures that can be performed by all vessels per day), as well as the delivery lead time constraints which limit the maximum voyage duration and the number of installations that can be served during a voyage. The departure days of the vessels that serve a given installation should be evenly spread throughout the planning period to ensure a steady supply of cargo. In addition, voyage non-overlap constraints mean that a vessel cannot start loading operations at the base until it has returned from its previous voyage.

A vessel delivery schedule is defined by a set of charter vessels and their associated voyages to be performed within the planning period. A vessel voyage starts and ends at the supply base and is defined by a set of installations to be visited in a certain sequence, a departure time from the base and duration. Fig. 1 depicts a vessel schedule for a weekly planning period plus one day (Monday) of the next one. This schedule involves three vessels (marked V1, V2 and V3), each having a set of voyages (marked with bold lines and containing a set of installations marked with three capital letters) with turnaround times at the base (marked as "B"). The last voyage of vessel V2 violates the *non-overlap* constraint. Such an infeasibility is eliminated by the coupling of vessel V2 with either vessel V1 or V3 when the coupled vessels may swap their voyages in the subsequent periods.

A new schedule is constructed for a new season or when there are changes in the number of installations, their locations, planned activities and demand. The construction of the vessels schedule involves decisions on the fleet composition, the assignment of voyages to vessels, the voyages departure time, the assignment of installations to voyages and their sequencing. The aim of the PSVPP is to construct a feasible vessels schedule with minimized vessel charter and fuel costs, subject to constraints such as base capacity, vessel capacity, spread of departures, voyage non-overlap, maximum voyage duration, and maximum number of visits on a voyage. Each installation must receive the required number of visits within the planning period.

1.2. Uncertainty in supply vessel scheduling

In practice, the repetitive execution of the planned schedule is hardly possible due to the influence of stochastic factors. Uncertain weather conditions affect the sailing and service times durations, which often results in delays and schedule disruptions. Delayed deliveries of material and equipment from the suppliers to the base may also create uncertainty in the voyage departure time from the

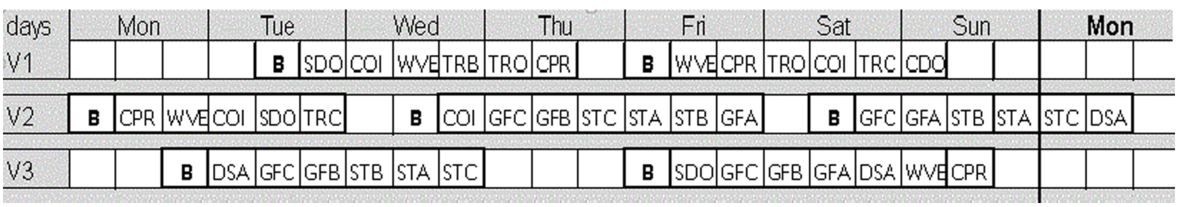

Fig. 1. Example of a weekly vessels schedule.

base. The frequency of deliveries to the installations and the weekly volume of cargo to be delivered may vary from week to week due to the changing level of activities at the installations and ad hoc situations. To keep the scheduled departure times, logistics planners modify the planned voyages and if necessary plan additional voyages which require extra resources.

In this study, we deal specifically with the uncertain demand at the installations. The demand at an installation varies from delivery to delivery, especially on drilling rigs where the processes are rather unstable due to the relatively unpredictable exploration activity. Therefore, there is always a risk that the total demand of the installations to be visited on a voyage will exceed the vessel's capacity. The probability of such an event depends on the average load of a vessel and on the level of demand fluctuation at the installations. In addition, there may be ad hoc demands, i.e. extra calls for unplanned deliveries, which also make the number of deliveries uncertain. Uncertainty in demand may result not only in the vessels' deck capacity violations, but also in service time variations at the installations since the duration of the loading operations depends on the delivered amount of cargo. For this reason, in the event of demand surges, the voyage duration may increase, so that the non-overlap constraints may become violated, especially when there are some night-closed installations to visit during the voyage. For this reason, it is essential that the schedule be constructed with some incorporated measures accounting for the uncertainty to reduce the use of extra resources.

When the voyage non-overlap constraints or the capacity constraints are violated, the logistics planners perform operational modifications or *recourse actions*, by relocating part of visits (i.e., part of demand) from infeasible voyages to some other feasible voyages. There are several alternatives of recourse actions. First, the excess demand may be delivered on other planned voyages having some free deck space. Another way to deliver the excessive demand is to use an available charter vessel to perform an unplanned voyage. In this case, the charter vessel must be available and all the constraints inherent to the PSVPP must be satisfied. Finally, a spot vessel hired on a short term basis at a higher cost may be used to perform some unplanned deliveries. It is worth noting that logistics planners do not require that deliveries on voyages be always performed by the vessels used in the planned schedule and according to the visit sequence defined in that schedule. For them it is more important to stick to the planned voyages departure days and planed daily fleet of vessels. The main problem is that several alternatives of recourse actions may be applied simultaneously in different combinations, and for each combination there are many ways to reinsert visits. Recourse actions generate additional costs, the largest of which is considered to be the hiring of spot vessels since their rates are significantly higher than those of the charter vessels. Since the installations require predictable and stable delivery schedules, these should be constructed so as to be reasonably immune to uncertain phenomena and so that the cost after recourse is minimized.

The objective of the PSVPP with uncertainty is then to construct a vessels schedule with minimized expected vessels and fuel costs after application of recourse actions. The problem can be modeled through a simple modification of the deterministic model presented in Kisialiou et al. (2018a). First, the demands are now random variables instead of being known constants. Second, a term must be added to the objective function to account for the expected recourse value.

1.3. Literature review

In this section we describe solution approaches to the vehicle routing problem with stochastic demand (VRPSD) which are relevant to the study of the stochastic PSVPP, and provide an overview of the literature dedicated to the deterministic and stochastic PSVPP under weather uncertainty.

1.3.1. VRPSD

In a more general perspective, solution approaches to the VRPSD are subdivided into two-stage stochastic programming with recourse, reoptimization, robust optimization, and chance-constrained programming.

Most solution approaches to the VRPSD rely on two-stage stochastic programming with recourse and are based on *a priori optimization* (Bertsimas et al., 1990). There exist several exact algorithms (e.g., Gendreau et al., 1995; Laporte et al., 2002; Christiansen and Lysgaard, 2007; Jabali et al., 2014; Salavati-Khoshghalb et al., 2018), as well as heuristics (e.g., Lei et al., 2011; Pandelis et al., 2012; Mendoza and Villegas, 2013). Most of these approaches apply the so-called classical recourse action which assumes that in case of failure, the vehicle returns to the depot to replenish its capacity and resumes its route starting at the point of failure, although other recourse actions exist (see Gendreau et al., 2014, 2016; Oyola et al., 2016, 2017 for recent surveys).

In contrast to a priori optimization, reoptimization algorithms (e.g., Dror et al., 1989; Zhu et al., 2014) reoptimize the routes while information on demand is dynamically revealed during their execution. Reoptimization outperforms a priori optimization in terms of efficiency, being a proactive approach, but loses in terms of operational convenience (Gendreau et al., 2016).

Robust optimization (see, e.g., Ben-Tal and Nemirovski, 1999; Bertsimas and Sim, 2004; Bertsimas et al., 2011) offers solutions yielding protection against the worst-case outcome within an uncertainty set. It enables the construction of high-quality solutions without the need to know the distribution of stochastic parameters. Unlike two-stage stochastic programming with recourse, it does not offer a solution with minimized expected cost and does not apply any recourse actions, which are frequently present in real-life problems. Applications of robust optimization to vehicle routing can be found in Bertsimas and Simchi-Levi (1996), Sungur et al. (2008) and Gounaris et al. (2013).

Chance-constrained programming is similar to robust optimization but does not work with an uncertainty set. Instead it ensures that the probability of satisfying constraints with uncertain parameters is above a certain *reliability level* (Birge and Louveaux, 1997). The main drawback of chance-constrained programming is the difficulty of computing general distributions, which is usually done via analytical methods or Monte Carlo approximations. An application to routing problems can be found in Laporte et al. (1989).

1.3.2. PSVPP

There exist several studies dedicated to the PSVPP under weather uncertainty that apply a robust optimization methodology. Thus, Halvorsen-Weare and Fagerholt (2011), Norlund et al. (2015) and Kisialiou et al. (2018b) introduced robust approaches involving combination of optimization and simulation. Halvorsen-Weare and Fagerholt (2011) suggested computing a robustness measure for voyages, Norlund et al. (2015) generated schedules with given robustness requirements and Kisialiou et al. (2018b) considered voyage specific slacks in construction of vessel delivery schedules.

Pure robust and chance-constrained optimization do not suit us since they do not imply any recourse actions. Reoptimization does not fit tactical planning. The only approach in our case is two-stage stochastic programming with recourse which, however, cannot be applied in the classical way. In our problem, the voyages in the schedule last more than one day and the recourse actions performed on any day depend on the recourse actions performed on previous days i.e., for a given first-stage solution the second-stage problem should be solved dynamically for each day of the planning period (sequentially, starting from Monday). Recourse actions applied to infeasible voyages of a certain day are performed simultaneously and are interdependent, which involves approximating the cost of recourse (for a given day) simultaneously for all the voyages departing on this day. Furthermore, the possibility of coupling vessels involves swapping their voyages from week to week (in the case of a one-week planning period) and the demand scenario for each visit of each installation is created for the full execution horizon of the schedule. All this makes it almost impossible to solve a mathematical formulation (in the context of the two-stage programming with recourse). A heuristic coupled with simulation modelling seems to be a good alternative to approximate the cost of recourse (provided that there are no serious simplification assumptions).

1.4. Contribution and organization of the paper

The objective of this research is to propose a methodology for the periodic supply vessel routing problem with uncertain demands. We apply a chance-constrained programming to control the probability of generating infeasible solutions, which we embed within a metaheuristic capable of constructing cost-effective and reliable vessel schedules with respect to demand fluctuations. Our metaheuristic is based on the adaptive large neighborhood search (ALNS) metaheuristic developed by Kisialiou et al. (2018a) for the PSVPP with flexible departures and coupled vessels. It accounts for the uncertain demand by incorporating chance constraints. Finally, we develop a simulation model allowing for the assessment of the schedule's performance over a certain time horizon and for the computation of its expected cost. To this end, our simulation model incorporates several recourse actions alternatives aimed at preventing schedule infeasibility. Both the optimization metaheuristic algorithm and the simulation model are integrated within a single decision support tool enabling the generation of reliable schedules with minimized expected cost for large-size PSVPP instances. To demonstrate how the proposing methodology works, we performed computational experiments using two real-life instances provided by the Norwegian offshore oil and gas operator Equinor.

To our knowledge we are the first to solve the large-scale PSVPP with stochastic demand. Our methodology is not limited to the periodic supply vessel planning, but may also be applied to stochastic large-scale VRPs with uncertain demand requiring reliable solutions. A special feature of the developed method is that it enables the generation of schedules of minimum expected cost possessing a certain degree of reliability against the uncertainty. In addition, our approach does not impose any limitations on the types of demand distributions. It can also handle multiple and complex route-dependent recourse actions which are applied simultaneously in various combinations. Our study exemplifies the application of emerging operations research technologies to the solution of a complex transportation planning problem which cannot be solved by means of traditional exact algorithms. We have combined optimization and simulation to improve the performance of a transportation system, namely in what concerns efficiency, level of service and reliability. To our knowledge, our methodology enables, for the first time, the solution of a highly complex and significant stochastic logistics problem arising in oil and gas offshore operations.

The remainder of this paper is organized as follows. In Section 2 we introduce a methodology allowing the control of voyage reliability under uncertain demand. We present a heuristic for the construction of vessels schedules with incorporated reliability control, and a discrete-event simulation model for the assessment of the schedule's performance. In Section 3 we conduct experiments and we analyze the behavior of the algorithm with respect to the reliability and expected cost of the solution. Conclusions are derived in Section 4.

2. Solution methodology

In this section, we present a methodology for the reliable supply vessel routing and scheduling problem with uncertain demand. Our algorithm heuristically solves a two-stage stochastic programming problem with recourse, and instead of exploring the whole set of feasible solutions in the first phase, it reduces the search to several solutions generated with a preset minimal reliability level. We combine optimization and simulation (e.g., Sörensen and Sevaux, 2009; Takes and Kosters, 2010), by which several solutions are generated and evaluated within a simulation process. The key idea of such an approach is to (1) use a search algorithm to find strategy settings and (2) use a discrete event simulation to model the operations as close as possible to the real world and to evaluate the performance of the selected strategy (Zhou et al., 2018). Our simulation model accounts for the schedule's feasibility, after the demand at the installations is revealed, by applying recourse actions to infeasible voyages. Such an approach allows a reduction of the computational complexity of the heuristic and is capable of handling the specific stochastic structure of the problem (Bierlaire, 2015), which is heavy for traditional stochastic programming algorithms.

We first address the issue of approximating the demand probability distribution at the installations. We then develop a chance-

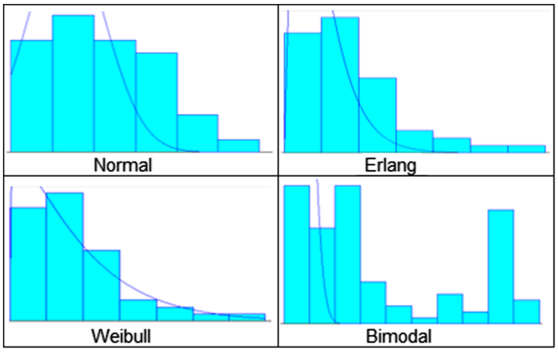

Fig. 2. Four examples of demand distributions.

constrained based metaheuristic that allows for the construction of solutions in which voyages have some degree of protection against demand uncertainty. This is followed by a simulation model for the computation of the expected cost of the generated delivery schedule for a certain value of the reliability level. Finally, we integrate both the optimization algorithm and the simulation model into a single method for the controlled generation of reliable schedules with minimized expected cost after recourse.

2.1. Approximating the demand probability distribution

Here we propose a methodology to approximate the demand probability distribution at the installations. We assume that there is no correlation between the demands i.e., the demands at the installations are independent random variables following some probability distribution. Although there may be some correlation caused by weather conditions we omit it assuming that the demand is only dependent on the processes at the installation having their own production plan. In our test instances, the number of installations to be served fluctuates around 30 and their demands follow various distributions such as normal, Erlang or Weibull, and they can even be bimodal (Fig. 2). In other words, they exhibit a high degree of variation. In this context, a reasonable methodology to estimate the demand probability distribution is to approximate the cumulative distribution function (CDF) by means of the fast Fourier transform (FFT) algorithm (Van Loan, 1992). Often applied in signal processing, the FFT methodology is very efficient for the computation of the convolution products. There are several benefits to this approach. First, we deal directly with the CDF (although discretizing), which increases the accuracy, and second, there is no need to generate a large-size sample as in case of Monte Carlo sampling, which leads to a higher computing speed. As demonstrated by several authors (Sakamoto et al., 1997; Rao and Rajib, 2008; Pillay and O'Hara, 2011; Bormetti and Cazzaniga, 2011; Ibrahim et al., 2014), the FFT algorithm significantly outperforms Monte Carlo simulation in terms of speed and accuracy.

According to the convolution theorem, the Fourier transform of a convolution of two signals (functions) is the pointwise product of their corresponding Fourier transforms (for details on the Fourier Transforms properties, see Bracewell, 2000). A Fourier transform provides the decomposition of a signal (represented as function of time) into the frequencies that make it up. We use the term "signal" as an example since originally the Fourier transform was applied to signal processing. The result of the Fourier transform is a complex-valued function of frequency showing the components of a signal at any given frequency. The convolution property of the Fourier transform states that a convolution in the time domain equals to the pointwise multiplication in the frequency domain (Broughton and Bryan, 2008). It worth noting that a Fourier transform is not limited to functions of time, i.e., this is just a matter of representations, and the domain of the original function (CDF in our case) is referred to as the "time" domain (just to have a unified language). To perform a Fourier transform is used the FFT algorithm which computes a discrete Fourier transform (DFT) of a sequence (transforms a discretized function into its frequency domain). Thus, according to the convolution theorem, taking any two CDFs F_x and F_y and descritizing them we have

$$\mathrm{F}(F_x * F_y) = \mathrm{F}(F_x)\cdot\mathrm{F}(F_y), \tag{1}$$

where F denotes Fourier transform operator, the dot (·) denotes point-wise multiplication, and the asterisk (∗) denotes a convolution. Performing the inverse Fourier transform F^{-1} by using the inverse FFT, we compute the inverse DFT (transform into the "time" domain) which provides the convolution product of F_x and F_y:

$$F_x * F_y = \mathrm{F}^{-1}(\mathrm{F}(F_x)\cdot\mathrm{F}(F_y)). \tag{2}$$

The computation of a general convolution S using such point-wise multiplication can be performed for any number of functions simultaneously:

$$F_S = F_1 * F_2 * \ldots * F_n = \mathrm{F}^{-1}(\mathrm{F}(F_1)\cdot\mathrm{F}(F_2)\cdot\ldots\cdot\mathrm{F}(F_n)). \tag{3}$$

Thus, utilizing the property of the convolution theorem, for any voyage we can define the CDF of its total demand. Such approach, applied in the context of the heuristic used for the schedule construction, results in a high accuracy of the demand convolution and at the same time, yields a relatively fast performance.

2.2. Metaheuristic algorithm for the construction of reliable vessel schedules

We now briefly describe a metaheuristic for the construction of reliable PSVPP solutions. We adapt the ALNS heuristic developed by Kisialiou et al. (2018a) to incorporate into it chance constraints accounting for stochastic demand.

The brief pseudo-code of our ALNS metaheuristic with incorporated reliability control is provided in Algorithm 1. The algorithm is applied for n restarts of η iterations each. The reliability level α, defining the lower bound on the probability that the demand exceeds the vessels capacity is used as an input to the algorithm. The best known solution found by the algorithm during the search is defined as g^*. At the beginning of each restart an initial solution g^0 is randomly constructed and an attempt is then made to reduce its cost for η iterations. At the beginning of each iteration, the current solution g is partially destroyed by randomly removing q visits from it. The removal of the visits is performed by using a destroy operator which is selected according to a discrete probability distribution from a set ψ: *random voyage removal*, *Shaw removal* (Shaw, 1997) and *worst removal*. Each of the operators has its own logic, which ensures the diversity of visits removal from solution g and an efficient search of the solution space. All the visits removed from the current solution g are stored in a pool S of removed visits and the partially destroyed solution is defined as g''. An attempt is then made to reinsert the visits contained in S back into g'' by using a repair operator which is selected from a set φ: *deep greedy insertion*, *2-regret insertion* and *3-regret insertion*. If the reinsertion is successful, a new solution g' is then obtained and the algorithm resumes its search for a better solution in the neighbourhood of solution g'. To this end, several improvement operators are applied to solution g' in a given sequence while its cost decreases. The list of improvement operators we use is *deep greedy relocation*, *deep greedy swap, number of voyages reduction*, *daily departure relocation* and *reduce the fleet size*. Using the Eq. (3) introduced in Section 2.1 we can compute the CDF F_v of the total demand for any voyage v in either g' or g''. Whenever a voyage is constructed or modified (a visit is added or reassigned to another vessel), the condition

$$F_v(Q_v) \geqslant \alpha. \tag{4}$$

is checked (lines 3, 7 and 10), where F_v is the CDF of voyage's v total demand, Q_v is a capacity of the vessel sailing voyage v and α is the reliability level used in chance-constrained programming. If the condition is not satisfied, the voyage modification is rejected. Such checks are implemented within each repair and improvement operator. After the application of the improvement operators, the solution g' is compared with the best known solution g^* and with the current solution g. We denote by $\lambda(g)$ the cost of solution g. If $\lambda(g') > \lambda(g)$, a simulated annealing acceptance criterion is applied (line 17) which accepts solution g' with a certain probability depending on $\lambda(g')$, $\lambda(g)$ and a parameter τ called the temperature. The initial value of τ is equal to $\lambda(g^0)$. At each iteration, τ is multiplied by a cooling rate $\mu \in (0, 1)$, which is dependent on the number of iterations η and a user-defined parameter θ (line 4). After the last iteration of the last restart, the algorithm returns the best known solution g^*. For more details on the general ALNS principles, see Ropke and Pisinger (2006).

Algorithm 1. ALNSα for the PSVPP with chance-constraints

1: State Set the cost of the best known solution $\lambda(g^*) = \infty$;
2: **for** n restarts **do**
3: Construct initial solution g^0 satisfying (4);
4: $g^* \leftarrow g^0$; $g \leftarrow g^0$; $\tau \leftarrow \lambda(g^0)$; $\mu \leftarrow 1 - \theta/\eta$;
5: **for** η iterations **do**
6: $g'' \leftarrow \psi(g, q, S)$, remove q visits;
7: $g' \leftarrow \varphi(g'', q, S)$, insert q visits while satisfying (4);
8: **if** $S = \varnothing$ and g' is feasible **then**
9: **while** g' improves **do**
10: Run the set of improvement operators while satisfying (4);
11: **end while**
12: **if** $\lambda(g') \leqslant \lambda(g^*)$ **then**
13: $g^* \leftarrow g'$; $g \leftarrow g'$;
14: **else if** $\lambda(g') \leqslant \lambda(g)$ **then**
15: $g \leftarrow g'$;
16: **else**
17: $g \leftarrow g'$ with probability $e^{-(\lambda(g')-\lambda(g))/\tau}$;
18: **end if**
19: **end if**
20: $\tau \leftarrow \mu\tau$;
21: **end for**
22: **end for**
23: **return** g^*

2.3. Evaluation of a schedule

We now describe the discrete event simulation model that enables the assessment of a schedule's expected cost and incorporating recourse actions, in order to eliminate potential schedule infeasibilities.

2.3.1. Simulation model

In discrete event simulation a process is represented as a flow of events occurring at certain points in time. We simulate vessel schedule execution for the period the schedule is to be applied. In our case, discrete events are defined by the scheduled voyages' starts from the base. The demand for each installation visit on a voyage is sampled from the corresponding probability distribution. We distinguish between two types of vessel schedule costs: the planned schedule cost, i.e., the cost of the schedule generated by the ALNS heuristic, and the expected schedule cost calculated as an average for all simulation replications. In addition, we distinguish between planned and simulated installation's demands.

The pseudo-code describing the simulation model is provided in Algorithm 2. As an input, the simulation model uses the vessel schedule $g(\alpha)$, generated for a certain value of the reliability level α (see condition (4)) by Algorithm 1, and the modelled demand. The simulation is performed for ρ replications. Within each replication the schedule execution is simulated for N consecutive schedule planning periods which in total constitute a simulation time horizon. For each planning period $n = 1, ..., N$ the copy $g_n^{mdf}(\alpha)$ of schedule $g(\alpha)$, generated by Algorithm 1, is created to reflect recourse actions according to the available alternatives. For each day $d = 1, ..., D$ of the schedule planning period, the feasibility of voyages started on day d is ensured by applying recourse actions (see Section 2.3.2). It is assumed that the installations' demands on a voyage are known one day before the voyage starts.

After the demand simulation, the service times of the installations to be visited on voyages starting on days d and $(d + 1) \bmod D$ are recalculated and the voyages are rerouted to find a lower cost. The service time at an installation is defined by adjusting up or down the value of the planned service time proportionally to the increase or the decrease of the simulated demand relative to the planned demand. After the rerouting, the set V^d of all voyages departing on day d and infeasible in terms of the vessel capacity or duration constraints is defined.

The least cost recourse actions are performed by feasibly relocating visits from V^d to other voyages departing on either day d or $(d + 1) \bmod D$ or both (see Section 2.3.3). Rerouting and recourse actions are applied in planning period number n yield a modified schedule $g_n^{mdf}(\alpha)$ of cost λ_n. After each replication $i = 1, ..., \rho$, the algorithm computes the average cost λ_i^{av} of the modified vessels' schedule for this replication. After the last replication the algorithm returns the average expected schedule cost $\lambda^{exp}(\alpha)$ computed over all replications.

Algorithm 2. Schedule simulation

Modelled demand;
for $i = 1$ to ρ **do**
  for $n = 1$ to N **do**
    $g_n^{mdf}(\alpha) \leftarrow g(\alpha)$;
    for $d = 1$ to D **do**
      if $d = 1$ and $n = 1$ **then**
        Simulate demand for each visit on voyages started on days d and $(d + 1) \bmod D$;
        Recalculate service times and reroute planned voyages;
      end if
      Simulate demand for each visit on voyages started on day $(d + 1) \bmod D$;
      Recalculate service times and reroute planned voyages;
      $V^d \leftarrow$ Find infeasible voyages;
      if $V^d \neq \emptyset$ **then**
        $g_n^{mdf}(\alpha) \leftarrow$ Perform the least cost recourse actions for voyages in V^d (Algorithm 3);
      end if
    end for
    Calculate cost λ_n of modified schedule $g_n^{mdf}(\alpha)$;
  end for
  Calculate $\lambda_i^{av} = \sum_{n=1}^{N} \lambda_n / N$;
end for
return $\lambda^{exp}(\alpha) = \sum_{i=1}^{\rho} \lambda_i^{av} / \rho$

2.3.2. Recourse actions

We now describe the recourse actions alternatives applied to the schedules with the simulated demand where planned voyages may be rerouted. We consider three alternatives, each implying the relocation of the excess amount of cargo from the infeasible voyages in V^d to other feasible voyages so that they remain feasible in terms of vessel capacity and duration constraints. Feasible voyages used for the insertion of visits are referred to as target voyages. Below we provide the description of the recourse actions ensuring the feasibility of voyages for each day $d = 1, ..., D$:

(a) Relocation of visits into the planned voyages

For all infeasible voyages departing on day d, some visits are relocated into feasible target planned voyages departing on days d or

$(d + 1) \bmod D$.

(b) The use of available charter vessels

As an alternative to the relocation of visits from voyages in V^d into the feasible planned voyages, visits may be relocated to the newly created empty voyages starting on day d and performed by available charter vessels. A charter vessel is considered to be available for departure on day d if it has no planned departure on a voyage on that day and has returned from a planned voyage early enough to start an unplanned voyage on day d.

(c) Hiring of a spot vessel

A spot vessel is hired when it is impossible to achieve schedule feasibility with the insertion of visits into the planned and unplanned empty voyages performed by charter vessels. Similarly to what is done in the previous case, a relocation of visits from V^d is performed into an empty voyage of a spot vessel departing on day d.

When performing these recourse action a target voyage non-overlap and capacity constraints are checked. It is worth of noting that the simulation of the schedule for planning period n is partially dependent on the result of the schedule simulation in planning period $n - 1$. This is because some visits from voyages starting at the end of the planning horizon $n - 1$ may be relocated to the voyages departing at the beginning of the schedule of the planning period n.

2.3.3. Recourse action implementation

Algorithm 3 describes the procedure applied to implement recourse actions in case of voyage infeasibilities on day $d = 1, \ldots, D$. For all voyages in V^d the set U of all possible removal combinations of visits is generated, by full enumeration using a power set recursive procedure (Pinter, 2014) so that the removal of visits according to any removal combination $u \in U$ from voyages in V^d results in the elimination of their infeasibility in terms of capacity and duration. The generation of each combination is executed in such a way that it contains the minimal number of removed visits required to eliminate the infeasibility. The relocation of visits from infeasible voyages into possible target voyages, created according to available visits relocation alternatives, is performed for one of the visits removal combinations u of the set U. In our algorithm, several recourse actions may be applied simultaneously. For example, these can be the relocation of visits from infeasible voyages partially into the planned voyages, and partially into empty unplanned voyages of available charter vessels. For every visit removal combination $u \in U$ there may exist several combinations of visit insertions into target voyages based on the feasible visits relocations.

In our algorithm the schedule is first analyzed to identify feasible planned voyages departing on days d and $(d + 1) \bmod D$ (defined in the set V^{pl}). If there are any available charter vessels on day d, an empty voyage for each vessel is created with the departure on day d. We also create empty voyages of available spot vessels departing on day d. Thus, for each day d we define sets V^{ch} and V^{sp} including voyages of the available charter and spot vessels, respectively. The sets V^{pl}, V^{ch} and V^{sp} constitute the set V^{target} of target voyages for potential demand relocation from infeasible voyages. Further, for each visit removal combination u the least cost (with respect to voyages feasibility and base capacity) relocation option p^u is defined. Recourse actions are performed according to the best feasible insertion option p^*.

Algorithm 3. Perform recourse actions for day d

Generate set U of all possible visit removal combinations for voyages in V^d;
$V^{pl} \leftarrow$ Find feasible planned voyages departing on day d and $(d + 1) \bmod D$;
$V^{ch} \leftarrow$ Find available charter vessels and create empty voyages departing on day d;
$V^{sp} \leftarrow$ Create empty voyages of spot vessels departing on day d;
Define set of all target voyages for visits relocation $V^{target} \leftarrow V^{pl} \cup V^{ch} \cup V^{sp}$;
p^* - best feasible visits relocation option;
$c^* = \infty$ Cost of the best visit relocation;
for each $u \in U$ **do**
  $p^u \leftarrow$ Find a feasible insertion option of visits from u into voyages in V^{target} yielding the least cost $c(p^u)$;
  if $c(p^u) < c^*$ **then**
    $c^* \leftarrow c(p^u)$;
    $p^* \leftarrow p^u$;
  end if
end for
Perform relocation of visits according to the insertion option p^*;

2.3.4. The algorithm for the planning under uncertain demand

Algorithm 4 describes our optimization-simulation solution methodology for the construction of PSVPP schedules with different levels of reliability and allowing to define a schedule with the minimized expected cost. The algorithm takes as an input a set A of values for reliability parameter α, the parameter ρ defining the number of simulation replications, and the parameter N defining the number of planning periods for which the schedule is executed within a simulation time horizon. For each $\alpha \in A$ the ALNS metaheuristic constructs a solution $g(\alpha)$ which is saved in the list $R = \{g(\alpha): \alpha \in A\}$ of solutions. The solution $g(\alpha)$ and the parameters ρ and N are taken as inputs to the simulation model. The expected schedule cost $\lambda^{exp}(\alpha)$ computed after simulation is saved in the list of simulation costs $C = \{\lambda^{exp}(\alpha): \alpha \in A\}$. The algorithm returns the list of schedules with different reliability levels and corresponding expected schedules costs.

Algorithm 4. Optimization-simulation algorithm

```
1: Set A of α values;
2: ρ - the number of simulation replications;
3: N - the number of times the schedule is executed within the simulation horizon;
4: for each α ∈ A do
5:     g(α)← run ALNS(α);
6:     Save g(α) in a list R;
7:     λ^exp(α)← Schedule simulation (g(α), ρ, N);
8:     Save λ^exp(α) in a list C;
9: end for
10: return R and C;
```

3. Computational experiments

We now describe the computational experiments we have performed to study the behavior of the optimization-simulation algorithm and to assess the stability of the results in terms of the schedule's expected cost. The ALNS metaheuristic and the simulation model were coded in the C# programming language. Experiments were conducted on a computer with 16 GB of RAM and Pentium 8 Core processor of 2.3 GHz under the Windows operating system.

There are two types of input data for the algorithm: the data for the ALNS metaheuristic and the demand data for the simulation model. The PSVPP instances used in the experiments were provided by the Norwegian operator Equinor. The input for the ALNS includes data on the supply base, the installations, and the fleet of vessels. We used two instances with 14 and 26 installations and up to eight vessels including spot vessels. For each installation, we defined the PDF based on the historical demand data of the season corresponding to the simulation time horizon (winter 2016–2017). For those installations with insufficient demand observations we assumed that the demands follow a PERT probability distribution defined by the minimal, maximal and most likely values a random variable can take. The schedule planning period is one week. To assess the stability of the results we ran the algorithm 60 times for each value of the reliability level α to gather statistics. The simulation of a weekly vessel schedule was performed for 1000 replications. The values of the user-defined input parameters are provided in Table 1.

The results are presented in Figs. 3 and 4. These figures depict the mean cost of the planned schedules, and the mean and the box plot of the expected costs of the simulated schedules depending on the value of α. The width of the box corresponds to the inter-quartile range (IQR) and the whiskers are equal to 1.5×IQR. The red line connects the mean expected values of the simulated schedule costs. Both figures show that an increase in the value of α results in an increase of the planned schedule cost. For each instance we can see surges in the planned schedule cost. There are two surges for the instance with 14 installations corresponding to an α value changing from 0.7 to 0.8, and from 0.9 to 1.0. For the instance with 26 installations the surges correspond to α changing from 0.6 to 0.7 and from 0.9 to 1.0. These cost surges indicate the fleet size increase resulting from the higher reliability requirements imposed by the increase in the α value. Regarding the expected schedule costs, for low values of α (from 0.1 to 0.4), the simulated schedules use three more vessels on average compared with the corresponding planned schedules. The expected schedule cost goes down, gradually approaching the planned cost as the value of α increases. After the expected cost reaches the planned cost (for the instances with 14 and 26 installations corresponding α values are 0.8 and 0.6) both type of costs behave synchronously with some slight differences. The minimal expected costs for the instances are 3,601,251 and 6,068,985 with the corresponding α values of 0.9 and 0.6. Concerning the results' stability, the variability of the expected schedule's cost (see the box plots) is maximal for the smaller parameter value of α and reduces as α increases, becoming minimal when α reaches 1.0. At this point, the cost variability is defined purely by the variability of voyages' travel times due to the dependency of service time durations on the demand. On average, the variability of the expected schedule cost amounts to 0.5%.

The average computational time for both instances depending on the value of α is depicted in Fig. 5. As we can see from this figure, the computational time goes down as the value of α increases. This time reduction is due to the fact that the more reliable

Table 1
Input parameters.

Parameter	Description	Value
n	Number of restarts	2
η	Number of iterations	800
θ	Cooling rate control parameter	7
α	Set A of α values	[0.1, ..., 1.0]
ρ	Number of simulation replications	1000
N	Number of schedule executions within the simulation horizon	12

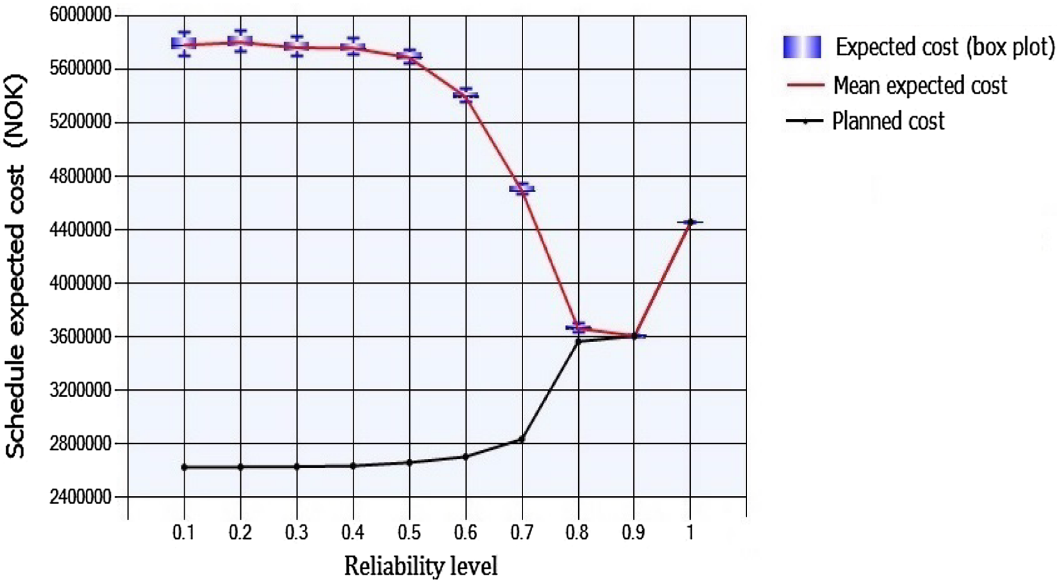

Fig. 3. Planned schedule costs and corresponding expected costs after simulation for the instance with 14 installations.

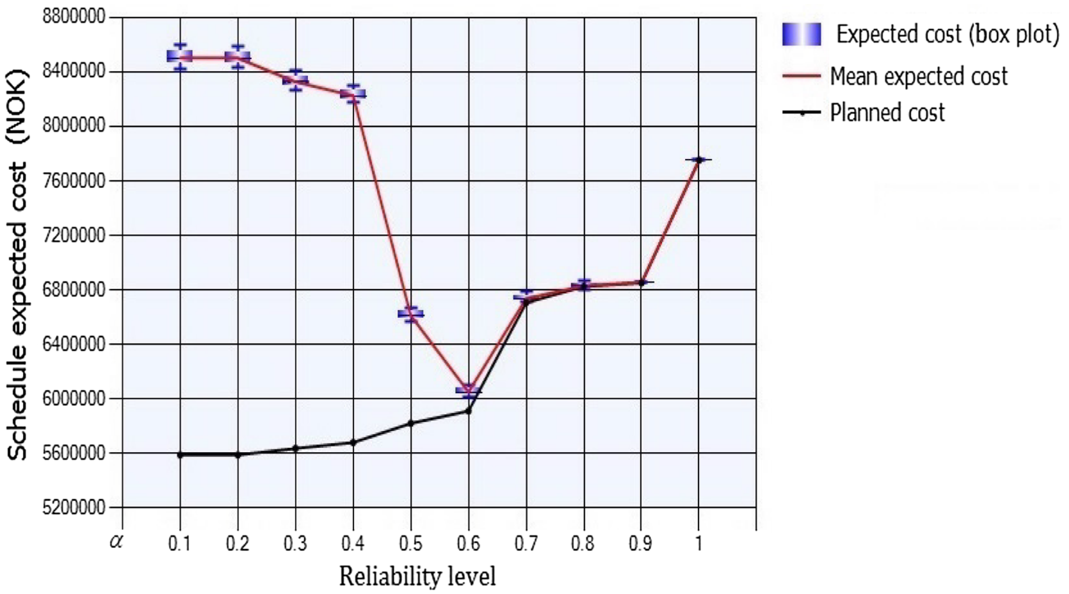

Fig. 4. Planned schedule costs and corresponding mean expected costs after simulation for the instance with 26 installations.

schedules require less computational effort in terms of the recourse actions assessment. The total computational time for all α values, for the instances with 14 and 26 installations, is 310 and 780 min, respectively. On average for the instance with 14 installations it is 2.5 faster to run the algorithm compared with the instance with 26 installations for the same α. It is worth noting that the total running time of the algorithm may be reduced by cutting the number of simulation replications by half with no significant impact on solution quality. In addition, there is no need to generate schedules with low values of α. Since we aim to generate reliable schedules, it is highly unlikely that schedules generated for α values from 0.0 to 0.4 will have a lower expected cost after the simulation, compared with those generated with higher α values. Therefore, the total computational time, for both instances, may be reduced by a factor of about four.

4. Conclusions

We have introduced a methodology for the solution of the periodic supply vessels planning problem with uncertain demand. The determination of vessel schedules is seriously affected by uncertain demand at the installations, especially at the drilling rigs.

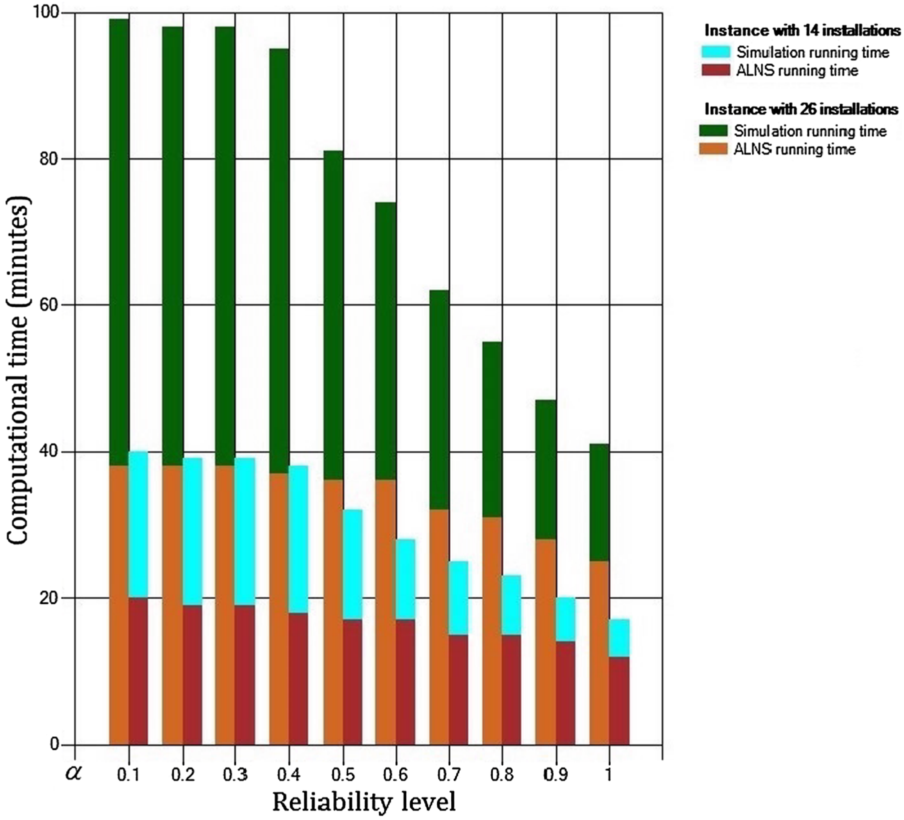

Fig. 5. Computational results for the PSVPP-FC (heterogeneous fleet, time windows) on the Equinor (2009) instances.

Schedule disruptions require modifications, which quite often leads to an unplanned use of charted vessels, or to the hiring of spot vessels, which results in additional costs. To cope with the influence of uncertain demand on the schedule performance, we have imposed requirements on the reliability of voyages during the construction of the vessel schedules by means of an ALNS metaheuristic. The ALNS algorithm was used to generate multiple vessel schedules with different reliability levels and low expected cost. To assess the performance of a schedule, we have developed a discrete-event simulation model incorporating several recourse actions alternatives aimed at eliminating schedule infeasibilities caused by uncertain demands. For each schedule, the simulation model computes the expected schedule after recourse and the schedule with the least expected cost is selected. Both the optimization algorithm and the simulation model were integrated into a single decision support tool. We have conducted extensive computational experiments to demonstrate how the methodology works in practice, to illustrate the behavior of the developed algorithm and to assess the stability of the results. Our methodology enables decision makers to construct vessel schedules for large-size instances with low expected cost and having some reliability level against stochastic demand.

Acknowledgements

This research was partially funded by the Canadian and Engineering Research Council under grant 2015-06189. Thanks are due to Ellen Karoline Nordlund from Equinor for her cooperation and for providing real PSVPP instances. We are also grateful to the referees for their valuable comments.

References

Ben-Tal, A., Nemirovski, A., 1999. Robust solutions to uncertain programs. Oper. Res. Lett. 25, 1–13.
Bertsimas, D., Brown, D.B., Caramanis, C., 2011. Theory and applications of robust optimization. SIAM Rev. 53 (3), 464–501.
Bertsimas, D., Jaillet, P., Odoni, A.R., 1990. A priori optimization. Oper. Res. 38, 1019–1033.
Bertsimas, D., Sim, M., 2004. The price of robustness. Oper. Res. 52, 35–53.
Bertsimas, D., Simchi-Levi, D., 1996. A new generation of vehicle routing research: robust algorithms, addressing uncertainty. Oper. Res. 44 (2), 286–304.
Bierlaire, M., 2015. Simulation and optimization: a short review. Transp. Res. Part C: Emerg. Technol. 55, 4–13.
Birge, J.R., Louveaux, F.V., 1997. Introduction to Stochastic Programming. Springer, New York.
Bormetti, G., Cazzaniga, S., 2011. Multiplicative noise, fast convolution and pricing. Quant. Financ. 14 (3), 481–494.
Borthen, T., Loennechen, H., Wang, X., Fagerholt, K., Vidal, T., 2017. A genetic search-based heuristic for a fleet size and periodic routing problem with application to offshore supply planning. Eur. J. Transp. Logist. 50 (4), 195–204.
Bracewell, R.N., 2000. The Fourier Transform and Its Applications, 3rd ed. McGraw-Hill, Boston.

Broughton, S.A., Bryan, K., 2008. Discrete Fourier Analysis and Wavelets: Applications to Signal and Image Processing. Wiley, New York.
Christiansen, C.H., Lysgaard, J., 2007. A branch-and-price algorithm for the capacitated vehicle routing problem with stochastic demands. Oper. Res. Lett. 35 (6), 773–781.
Dror, M., Laporte, G., Trudeau, P., 1989. Vehicle routing with stochastic demands: properties and solution frameworks. Transp. Sci. 23 (3), 166–176.
Gendreau, M., Jabali, O., Rei, W., 2014. Stochastic vehicle routing problems. In: Toth, P., Vigo, D. (Eds.), Vehicle routing: Problems, Methods, and Applications. MOS-SIAM Series on Optimization, chapter 8, second ed. SIAM, Philadelphia, pp. 213–239.
Gendreau, M., Jabali, O., Rei, W., 2016. 50th Anniversary invited article – future research directions in stochastic programming. Transp. Sci. 50 (4), 1163–1173.
Gendreau, M., Laporte, G., Séguin, R., 1995. An exact algorithm for the vehicle routing problem with stochastic demands and customers. Transp. Sci. 29 (2), 143–155.
Gounaris, C.E., Wiesemann, W., Floudas, C.A., 2013. The robust capacitated vehicle routing problem under demand uncertainty. Oper. Res. 61, 677–693.
Halvorsen-Weare, E.E., Fagerholt, K., 2011. Robust supply vessel planning. In: Pahl, J., Reiners, T., Voß, S. (Eds.), Network Optimization: 5th International Conference, INOC 2011, Hamburg, Germany, June 13–16, 2011. Springer, Berlin, Heidelberg, pp. 559–573.
Ibrahim, S.N.I., O'Hara, J.G., Constantinou, N., 2014. Pricing extendible options using the fast Fourier transform. Math. Probl. Eng. 7.
Jabali, O., Rei, W., Gendreau, M., Laporte, G., 2014. Partial-route inequalities for the multi-vehicle routing problem with stochastic demands. Discrete Appl. Math. 177, 121–136.
Kisialiou, Y., Gribkovskaia, I., Laporte, G., 2018a. The periodic supply vessel planning problem with flexible departure times and coupled vessels. Comput. Oper. Res. 94, 52–64.
Kisialiou, Y., Gribkovskaia, I., Laporte, G., 2018b. Robust supply vessel routing and scheduling. Transp. Res. Part C: Emerg. Technol. 90, 366–378.
Laporte, G., Louveaux, F.V., Mercure, H., 1989. Models and exact solutions for a class of stochastic location routing problems. Eur. J. Oper. Res. 39 (1), 71–78.
Laporte, G., Louveaux, F.V., Van Hamme, L., 2002. An integer L-shaped algorithm for the capacitated vehicle routing problem with stochastic demands. Oper. Res. 50 (3), 415–423.
Lei, H., Laporte, G., Guo, B., 2011. The capacitated vehicle routing problem with stochastic demands and time windows. Comput. Oper. Res. 38 (12), 1775–1783.
Mendoza, J.E., Villegas, J.G., 2013. A multi-space sampling heuristic for the vehicle routing problem with stochastic demands. Optimiz. Lett. 7 (7), 1503–1516.
Norlund, E.K., Gribkovskaia, I., Laporte, G., 2015. Supply vessel planning under cost, environment and robustness considerations. Omega 57, 271–281.
Oyola, J., Arntzen, H., Woodruff, D.L., 2016. The stochastic vehicle routing problem, a literature review, part I: models. EURO J. Transp. Logist. 5, 1–29.
Oyola, J., Arntzen, H., Woodruff, D.L., 2017. The stochastic vehicle routing problem, a literature review, part II: solution methods. EURO J. Transp. Logist. 6, 349–388.
Pandelis, D.G., Kyriakidis, E.G., Dimitrakos, T.D., 2012. Single vehicle routing problems with a predefined customer sequence, compartmentalized load and stochastic demands. Eur. J. Oper. Res. 217 (2), 324–332.
Pillay, E., O'Hara, J.G., 2011. FFT based option pricing under a mean reverting process with stochastic volatility and jumps. J. Comput. Appl. Math. 12, 3378–3384.
Pinter, C.C., 2014. A Book of Set Theory. Dover Publications, Mineola, New York.
Rao, B.N., Rajib, C., 2008. Probabilistic analysis using high dimensional model representation and fast Fourier transform. Int. J. Comput. Methods Eng. Sci. Mech. 9 (6), 342–357.
Ropke, S., Pisinger, D., 2006. An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. Transp. Sci. 40 (4), 455–472.
Sakamoto, J., Mori, Y., Sekioka, 1997. Probability analysis method using Fast Fourier transform and its application. Struct. Saf. 19 (1), 21–36.
Salavati-Khoshghalb, M., Gendreau, M., Jabali, O., 2018. A hybrid recourse policy for the vehicle routing problem with stochastic demands. EURO J. Transp. Logist. 1–30.
Shaw, P., 1997. A new local search algorithm providing high quality solutions to vehicle routing problems. Technical Report. APES Group, Department of Computer Science, University of Strathclyde, Glasgow, United Kingdom.
Sörensen, K., Sevaux, M., 2009. A practical approach for robust and flexible vehicle routing using metaheuristics and Monte Carlo sampling. J. Math. Model. Algorithms 8, 387–407.
Sungur, I., Ordóñez, F., Dessouky, M.M., 2008. A robust optimization approach for the capacitated vehicle routing problem with demand uncertainty. IIE Trans. 40, 509–523.
Takes, F., Kosters, W., 2010. Applying Monte Carlo techniques to the capacitated vehicle routing problem. In: Proceedings of 22nd Benelux Conference on Artificial Intelligence, BNAIC 2010.
Van Loan, C., 1992. Computational Frameworks for the Fast Fourier Transform. SIAM, Philadelphia.
Wang, Y., Meng, Q., Kuang, Q., 2018. Jointly optimizing ship sailing speed and bunker purchase in liner shipping with distribution-free stochastic bunker prices. Transp. Res. Part C: Emerg. Technol. 89, 35–52.
Zhou, C., Li, H., Lee, B.K., Qiu, Z., 2018. A simulation-based vessel-truck coordination strategy for lighterage terminals. Transp. Res. Part C: Emerg. Technol. 95, 149–164.
Zhu, L., Rousseau, L.-M., Rei, W., Li, B., 2014. Paired cooperative reoptimization strategy for the vehicle routing problem with stochastic demands. Comput. Oper. Res. 50, 1–13.