Math. Appl. **11** (2022), 33–43
DOI: 10.13164/ma.2022.04

A GAME THEORY EXPLANATION FOR MENSTRUAL SYNCHRONY: THE HAREM PARADOX

KJETIL K. HAUGEN

Abstract. The 50th anniversary of the McClintock effect deserves a new view on the subject. This paper applies (evolutionary) game theory to gain further insight. Among interesting results are strong indications of Nash equilibria in mixed strategies, indicating that the effect depends on parameters characterizing both females and males in the group. As such, much of the empirical research conducted on the subject over the last 50 years may be questioned. Furthermore, the article predicts that the effect's potential presence depends strongly on female envy/jealousy as well as male preferences on female attractiveness.

1. Introduction

Martha McClintock [7] published her famous paper on menstrual cycle synchronisation within close female groups back in 1971. Now, more than 50 years have passed, and research is still unclear on whether the effect[1] actually exists and, more specifically, if it exists, why does it exist?

In the time period after 1971, there has been considerable criticism in research literature on whether the effect actually exists. Some of the criticism is related to McClintock's method. Both statistical and logical flaws have been pointed out. See, for instance, [12] and [13].

Another brand of criticism relates to the reproduction or replication of McClintock's experiments often with inconclusive or (lately) even opposing results. See, for instance, Harris and Vitzthum [5], who conclude: (quote)

> *"In light of the lack of empirical evidence for MS [menstrual synchrony] sensu stricto, it seems there should be more widespread doubt than acceptance of this hypothesis."*

As the title of this article should indicate, the main focus here is on evolution. That is, the question of why, in an evolutionary perspective, should (or could) such an effect exist? In her original work, McClintock overlooks this dimension and, in general, discusses few potential reasons for the effect. However, evolutionary explanations or attempts to give such have been discussed by other researchers. Of special interest for this paper are perhaps contributions by Foley and Fitzgerald [2] and Knight [6]. Knight argues that menstrual synchronization provides

MSC (2020): primary 91A05, 91A22; secondary 91A80.
Keywords: game theory, menstrual synchronization.
This research is dedicated to Martha McClintock.

[1]Today, the effect is often referred to as the McClintock effect – see, for instance, [3].

opportunities for the males, for instance for hunting or other resource acquisition, instead of guarding females. If all females are inactive in breeding at the same time, this opens up for male group activity. Foley and Fitzgerald disagree, and argue that menstrual synchronization increases competition among females for male company. In fact, they establish a simulation model to prove their point, concluding that menstrual synchronization (under almost any parameter assumptions) is not an evolutionary stable strategy. That is, it will not maximise offspring.

Applying game theory to explain evolutionary processes (evolutionary game theory[2]) has grown in popularity in latter years. The concept, introduced by Maynard Smith and Price [10] and [11], is based on the (logical) assumption that intelligence is not necessary to play games. Existence of strategies is enough. As long as you know what to do, it is (of course) unnecessary to know why you do it to actively participate in game play. Normally, understanding why you do something is helpful in successful game play, but if nobody understands why they do things, a fair play situation seems reasonable to assume. Given this recognition, game theory can be applied in games without rational players, for instance, in Biology. The case at hand is somewhat special. Females are intelligent, but menstrual synchronization is, at least in this area, assumed not to be under female intelligent control. Presumably, evolution has selected this ability without females controlling it or being aware of it.

As opposed to other research in this area[3], this article suggests some actual game theory modelling with the purpose of gaining further insight into female menstrual synchronization. The game models are (consciously) simplified, with the aim of identifying how various reasonable parameters may drive the existence or non-existence of menstrual synchronization.

In Section 2, some necessary notation is defined. In Subsections 2.1, 2.2 and 2.3, three different game models, with extending complexity, are established and analysed. The results indicate that envy/jealousy as well as male sexual preferences (perhaps not very surprising) play a vital role in the potential existence of menstrual synchrony. Section 3 concludes, while 2 appendices, introduced to ease the reader experience, complete the article.

2. Some simple game models

In consecutive subsections, a game model is developed. The model starts out as simple as possible, and develops gradually to a final model. Some definitions of notation are necessary:

The *players* of an imperfect information two-player simultaneous game are:

$\mathcal{F}_A$: An attractive female belonging to some group of two.
$\mathcal{F}_U$: A less attractive (unattractive) female belonging to the same group.

The meaning of the term attraction here is straightforward. If the (present) male is given the choice of accompanying either $\mathcal{F}_A$ or $\mathcal{F}_U$, he will always choose $\mathcal{F}_A$.

Each player in the game can choose from the following *action set*:

[2]A nice introduction to the topic can be found in [4].
[3]To the best of this authors knowledge.

T_1: Some time period in a month containing the time period of a menstrual cycle.

T_2: The remaining part of the month, also contaning a menstrual cycle.

That is, we assume that both females $\mathcal{F}_A$ and $\mathcal{F}_U$ have been given the (unusual) gift of deciding the timing of the menstrual cycle. For simplistic reasons, but without loss of generality, it is assumed that 2 discrete possible choices are given, early (T_1) or late (T_2) in a month.

Related to various *Pay-off* outcomes the following notation is also needed:

V : The "value" of male company.

ϵ : Positive, not necessarily small number.

p : $P(\mathcal{F}_A$ is more attractive than $\mathcal{F}_U)$.

The value of male company, V, is a necessity to be able to perform some sensible mathematical analysis in these games. In practice, such a value is perhaps hard to establish. p is added at a later stage in order to handle possible male uncertainty regarding female attractiveness. The ϵ is simply a parameter used to adjust Pay-offs in the game formulations.

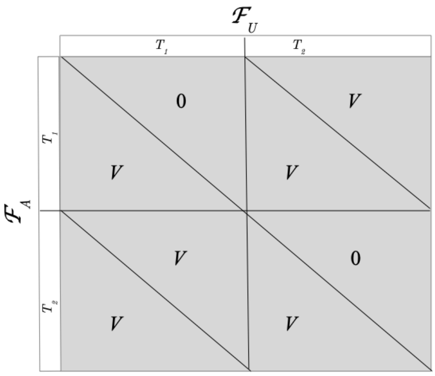
2.1. The simplest possible game model

Now, in order to apply game theory, it is necessary to assume that the females $(\mathcal{F}_A, \mathcal{F}_U)$ are able to choose the time period of their menstrual cycle. Today, with birth control pills, this is easy to do. However, it was not easy (impossible) in older days and, from an evolutionary perspective, it seems safe to assume this to be an impossible option. The underlying point here (model wise) is that "the invisible hand of evolution" performs the choice; the female is just an instrument observing when her period takes place.

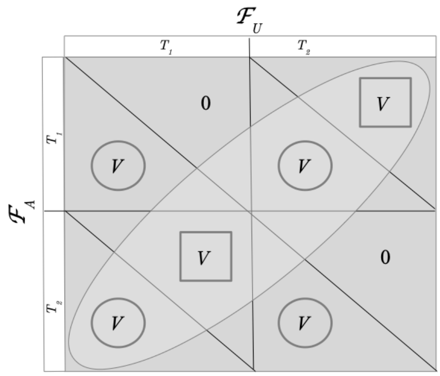
At this point, it may be sensible to look at a special group of females, the Harem. It makes the arguments easier to follow. Hence, we assume that a group (of two) females are joining each other in a harem with the purpose of maximising their offspring. Such an objective is of course in itself questionable, but it seems reasonable from an evolutionary perspective. The females, now present in the Harem, can increase their chances of offspring by maximal contact with the sheik over the month. That is, the more time they are able to entertain the sheik, the higher their probability of offspring success will be. Furthermore, we assume that the Sheik will not entertain a female within her menstrual cycle. To simplify, we have defined only two possible time periods for choice of menstrual cycle (T_1, T_2). Now, each female can either choose an early timing of their menstrual cycle (T_1) or a late one (T_2). Finally, we assume that this information is available to both females, and that they make their choice simultaneously. That is, each month the females choose, unobserved by each other, either T_1 or T_2. Given these choices, 4 possible outcomes can take place: $\{T_1, T_1\}$, $\{T_1, T_2\}$, $\{T_2, T_1\}$ or $\{T_2, T_2\}$.

Given the above assumptions, both females will receive sheik company, and hence the value V, if their choices turn out to be $\{T_1, T_2\}$ or $\{T_2, T_1\}$ (Separation). If, however, their choices coordinate, $\{T_1, T_1\}$ or $\{T_2, T_2\}$, only the attractive female will get male company and hence the value V. This of course holds given an assumption that the male (sheik) chooses the most attractive company when he makes a choice between $\mathcal{F}_A$ and $\mathcal{F}_U$.

These arguments lead to the game formulation (a) in Figure 1.



(a) A maximally simplified game model.



(b) The game in (a) with best replies for $\mathcal{F}_A$ (circles) and $\mathcal{F}_U$ (squares). In addition, the diagonal ellipse indicating the Nash equilibria (NEs)

Figure 1. The maximally simplified game model

As can be observed on the right, (b) in Figure 1, this game contains three NEs – two in pure strategies and one in mixed strategies[4]. What characterizes these equilibria is that of separation. That is, both players choose the alternative strategy of her competitor. The fact that the base model produces this kind of equilibria is interesting, as it directly opposes the McClintock effect. McClintock proposed the opposite situation (equilibrium). A situation where females coordinate their menstrual cycles when joining groups such as a harem.

Furthermore, these equilibria seem sensible from an evolutionary perspective. They would maximise offspring, as both, instead of one female, spend time with the sheik.

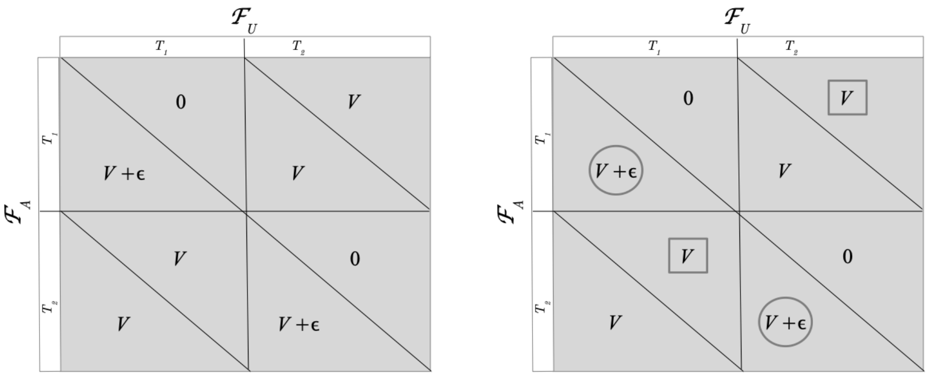
2.2. Envy between females

In this subsection, some more flavour is added to the model in Subsection 2.1. In a harem situation, or in any competitive group-situation involving a majority of females and a minority of males, envy and jealousy[5] in-between majority-group members is plausible. According to [1], jealousy is a basic emotion. As such, its importance in competitive situations cannot be underestimated. The fact that humans perform murder with jealousy as a motive indicates strong emotions.

[4]This game is often named a “chicken game“ in game theory – see, for instance, [8]

[5]In Psychology, envy and jealousy are distinctly different concepts [1]. In this setting, it is unnecessary to introduce this difference, so the terms are used as similar.

An easy way to introduce envy or jealousy into our model would be – in its absolutely simplest form – to add some extra value (say $\epsilon > 0$) to the female ($\mathcal{F}_A$) winning the competition of sheik-time. This happens when both choose the same strategy and is easily recognised on the left, (a) in Figure 2.



(a) A game model with envy

(b) The game in (a) with best replies for $\mathcal{F}_A$ (circles) and $\mathcal{F}_U$ (squares)

Figure 2. The game model with envy

As the best replies on the right, (b) in Figure 2, indicate, the NEs from Figure 1 change character radically. Now, the three-equilibrium situation in Figure 1 changes to a single unique NE in mixed strategies. So, after only this light, and highly reasonable model extension, the possibility of coordinated equilibria emerges. That is, the McClintock effect shows up as a NE or a solution to the game. As a consequence, it may be interesting to calculate this NE. The actual calculations are left for appendix A. However, the result is somewhat interesting. It turns out that the solution is:

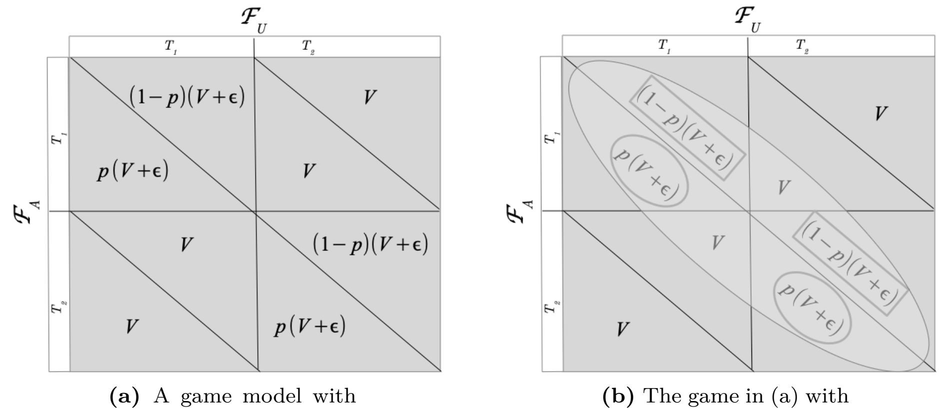
$$\{r^*, s^*\} = \left\{\frac{1}{2}, \frac{1}{2}\right\}. \tag{2.1}$$

The first interesting thing to note here, is that the NE in equation (2.1) is independent of the game parameters, V and ϵ. It may feel intuitive to expect that after introducing jealousy, the tendency for the NEs to shift may depend on the degree of jealousy – the value of ϵ. However, as the mixing probabilities in (2.1) are constant numbers, this is not the case. That is, it is the presence of jealousy as such that introduces the change in NEs, not the degree of jealousy.

Furthermore, and that is the main point here: given the unique mixed strategy NE in (2.1), the "Chicken" game from the previous subsection changes structure significantly. The "Chicken" game produced NEs with separation, the opposite of the McClintock effect, while now – at least with a certain probability, McClintock effect-type NEs are game solutions. In fact, it is extremely easy to predict the probability of occurrence of the McClintock effect. The NE in 2.1 means that both $\mathcal{F}_A$ and $\mathcal{F}_U$ choose T_1 and T_2 with equal probability of $\frac{1}{2}$. Consequently, the probability of a realization of either $\{T_1, T_1\}$ or $\{T_2, T_2\}$ must be $\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$. Hence, in 50 % of the cases, the coordination or, in this case, synchronized menstrual periods will occur. It may of course be tempting to use this type of result as an explanation for the somewhat unclear empirical results discussed in Section 1, but this may be overstepping. A more thorough discussion of these aspects is left for Section 3. Note also that even if the McClintock effect is predicted in 50 % of the cases, it is not predicted in the remaining 50 %. Hence, a slight re-modelling is performed in Subsection 2.3, giving some stronger predictions.

2.3. Male preference uncertainty

In the final game model, the possibility of male preference uncertainty is introduced. It seems reasonable to accept that a male may be uncertain about what female – among the harem inhabitants – to choose for company. After all, if the sheik always wanted the company of the most attractive female, why did he introduce other females into the Harem in the first place[6]? This is modelled through the probability p, introduced in Section 2. This probability is assumed exogenous to the game model and is common knowledge among the players[7]. It defines the probability that $\mathcal{F}_A \succ \mathcal{F}_U$. Consequently, as the indifference option is assumed practically non-existing, $1-p = P(\mathcal{F}_U \succ \mathcal{F}_A)$. Furthermore, it is reasonable to assume that $p > \frac{1}{2}$, in order to secure that the male still chooses $\mathcal{F}_A$ in the majority of cases.



(a) A game model with envy and male preference uncertainty.

(b) The game in (a) with best replies for $\mathcal{F}_A$ (circles) and $\mathcal{F}_U$ (squares).

Figure 3. The game model with envy and male preference uncertainty

Now, by extending the previous player objectives from $\max[V]$ (simple greed) to $\max E[V]$ (expected greed), expected values for the two players can be straightforwardly calculated, and the results are shown in Figure 3 on the left, (a).

[6]Underlying all game theory is of course the assumption of rationality.

[7]Both females know about p's existence and its value.

Furthermore, it is easy to realize that to secure the situation[8] shown in Figure 3 on the right (b), the following two inequalities must be satisfied:

$$p(V+\epsilon) > V \tag{2.2}$$

and

$$(1-p)(V+\epsilon) > V. \tag{2.3}$$

Some simple algebraic manipulations of inequalities (2.2) and (2.3) lead to:

$$p(V+\epsilon) > V \Rightarrow pV + p\epsilon > V \Rightarrow p\epsilon > V - pV \Rightarrow \epsilon > V\frac{1-p}{p},$$

$$(1-p)(V+\epsilon) > V \Rightarrow V - pV + \epsilon - p\epsilon > V \Rightarrow \epsilon > V\frac{p}{1-p}.$$

Now, as ϵ must be bigger than both $V\frac{1-p}{p}$ and $V\frac{p}{1-p}$ to secure that both (2.2) and (2.3) are satisfied, the following can – by simple logic – be concluded:

$$\epsilon > V \cdot \max\left\{\frac{1-p}{p}, \frac{p}{1-p}\right\}. \tag{2.4}$$

Furthermore, it is extremely easy to prove the following[9]:

Proposition 1. *If* $p \in \left\langle \frac{1}{2}, 1\right\rangle$, *then* $\frac{p}{1-p} > \frac{1-p}{p}$.

Then, inequality (2.4) can be simplified to:

$$\epsilon > V\left\langle \frac{p}{1-p}\right\rangle. \tag{2.5}$$

Hence, if inequality (2.5) is satisfied, the NEs on the right provide the game solution. That is, the players choose the same strategy. Of course, we do not know which strategies they choose. The simple fact that the game contains three equilibria indicates this. However, the coordination structure of the "Stag-Hunt" game provides the necessary information. Rational females with the ability to choose menstrual timing end up synchronizing, even if such a solution is directly opposed to their individual preferences of maximising potential offspring.

In the previous situation in Subsection 2.2, the equilibrium conditions did not involve the parameter values. In this case, it is different, as the degree of jealousy/envy (ϵ) must reach a certain level. As the minimal value of $\frac{p}{1-p}$ is 1 – given $p > \frac{1}{2}$ – we see that ϵ, at least, must be bigger than[10] V. That is, this value must be somewhat considerable in order to achieve the given solution. This author, possibly alongside most other potential authors, has limited knowledge about such values, but most human beings have experience with such feelings, and their strength. I think we can all agree that casual empirics does not dispute potentially very large "values" related to envy or jealousy.

---

[8] This situation, with two pure coordinating NEs $\{T_1, T_1\}$ or $\{T_2, T_2\}$, and a third in mixed strategies is often referred to as a "Stag-Hunt" game. Refer, for instance, to [9] for a thorough discussion of these games. Note also that the "Stag-Hunt" game corresponds nicely with the McClintock effect, and may be considered the "opposite" of the "Chicken" game.

[9] The actual proof is left for appendix B. Note also that it seems reasonable to rule out the $p = 1$ option ($< \frac{1}{2}, 1 >$) as it returns the previous game of Subsection 2.2.

[10] Furthermore, $\frac{p}{1-p} \to \infty$ when $p \to 1$.

3. Discussion, conclusions and critique

In the previous sections, simplified games were established an analysed. All the games provide different answers (NEs) to the presence of the McClintock effect. In general, given belief in gradually more realistic models, a reasonable conclusion would be to state the presence of this effect. Or at least – conditional presence. As all the models contain mixed strategy NEs, all the models predict the (conditional) presence of the McClintock effect. Still, the development through the models, moving from a typical "Chicken game" through a game with a unique mixed strategy NE to a "Stag-Hunt" NE, indicates that, given an assumption of the models gradually moving towards greater real world realism, the McClintock effect seems more and more probable.

Much of the research following up McClintock's paper focuses on whether the McClintock effect exists or not. One interesting result of the games discussed above is that this might not be an either/or answer, but perhaps more of a both/and answer. As all the three models presented above provide mixed strategy NEs, the possibility of a realized separation (no McClintock effect) or coordination (McClintock effect) is real. From this perspective, the hunt for the truth related to the existence or non-existence may not be the right question to address. Perhaps a more correct question would be: Under what model assumptions would such an effect be present?

Most researchers involved in applying game theory would probably agree to a statement like: Game theory may provide all kinds of answers, especially (perhaps) those that the author wants. Or perhaps better; game theory applications are critically dependent on the assumptions underlying the model. This is most certainly true[11].

The question then drops down to the realism in the model parameters used here. Basically, apart from simple greed, envy or jealousy and male preference uncertainty on female attractiveness are the only model mechanisms introduced. Many may feel that human beings in general and females in particular are complex and the triplet greed, envy (on the female side) and preference uncertainty (on the male side) may provide a sparse description of humans. Still, others may perhaps agree that, given the setting here, these dimensions may be the most relevant to investigate.

Potential objections related to the choice of model structure, a one shot imperfect information game versus more players, repetition, incomplete and/or asymmetric information etc. are, as in the most applied game theory, evident and clearly relevant. Still, most relevant game modelling often has a tendency to grasp some fundamental points of the problem even if the game models may seem oversimplified. The fact that this is not meant to be a game played by humans but rather by evolution itself may of course also provide arguments for keeping things simple.

It would be wrong to conclude that this article provides decisive answers to why grouped females may produce menstrual synchrony. Still, it explains the

[11] This applies of course to all kinds of mathematical modelling and is nothing special in game theory.

effect as a potential outcome in several theoretical game constructs. Furthermore, it provides answers to why some researchers (empirically) may find it and others may not. In any case, it provides a different perspective to the problem, and sometimes different perspectives provide added insight.

Appendix A. The calculation of the mixed NE in Subsection 2.2

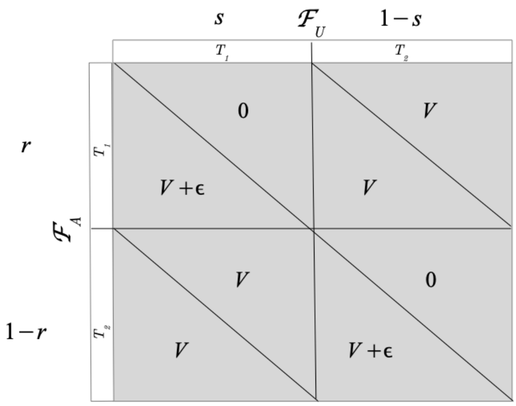


Figure 4. The envy/jealousy game with mixed strategy probabilities r and s

The standard procedure to find mixed strategy NEs involves calculating expected pay-off-functions, $\Pi_A^{\mathcal{F}}(r,s)$ and $\Pi_U^{\mathcal{F}}(r,s)$ for each player, based on the mixing probability distributions – $[r, 1-r]$ and $[s, 1-s]$ as follows:

$$\Pi_A^{\mathcal{F}}(r,s) = rs(V+\epsilon) + r(1-s)V + s(1-r)V + (1-s)(1-r)(V+\epsilon)$$

and

$$\Pi_U^{\mathcal{F}}(r,s) = r(1-s)V + (1-r)sV,$$

which, after a minimal amount of algebra, can be expressed as:

$$\Pi_A^{\mathcal{F}}(r,s) = r(1-2s)V + sV$$

and

$$\Pi_U^{\mathcal{F}}(r,s) = s(2r-1)\epsilon + V + \epsilon(1-r).$$

To progress, we need to find the best reply functions by solving $\max_r \Pi_A^{\mathcal{F}}(r,s)$ and $\max_s \Pi_U^{\mathcal{F}}(r,s)$. These two optimization problems, which are actually parametric Linear Programs, may in general be slightly challenging to solve. In this case, however, with only two players and two strategies, they are easily solved as follows: First, we form partial derivatives of $\Pi_A^{\mathcal{F}}(r,s)$ and $\Pi_U^{\mathcal{F}}(r,s)$, with respect to the players' decision variables, r and s, respectively. We get:

$$\frac{\partial \Pi_A^{\mathcal{F}}(r,s)}{\partial r} = (1-2s)V \quad \text{and} \quad \frac{\partial \Pi_U^{\mathcal{F}}(r,s)}{\partial s} = (2r-1)\epsilon. \tag{A.1}$$

As can be seen from both the expressions in (A.1), the lack of the r variable in $\frac{\partial \Pi_A^{\mathcal{F}}(r,s)}{\partial r}$ and similarly the lack of s in $\frac{\partial \Pi_U^{\mathcal{F}}(r,s)}{\partial s}$ confirms the linear programming situation. Hence, no inner optima exist, and we should expect piecewise constant best reply functions. The argument follows:

The key to the solution is the sign of the partial derivatives. Examining the leftmost part of (A.1), the expression $(1-2s)V$,we observe that if $s=\frac{1}{2}$, the partial derivative is zero. Hence, the optimal r solving $\max_r \Pi_A^{\mathcal{F}}(r,s)$ must be the whole interval – $r^*=[0,1]$. Furthermore, if $s>\frac{1}{2}$, $(1-2s)V<0$ and the best possibility for $\mathcal{F}_A$ is of course to minimize r, which yields $r^*=0$. Finally, if $s>\frac{1}{2}$, the partial derivative is positive and r should be maximized – $r^*=1$. Summing up, the best reply function for player $\mathcal{F}_A$ (naming it $r^*(s)$) is:

$$r^*(s)=\begin{cases} 1 & \text{if } s<\frac{1}{2},\\ [0,1] & \text{if } s=\frac{1}{2},\\ 0 & \text{if } s>\frac{1}{2}.\end{cases}$$

A perfectly analogous argument leads to the best reply function, $s^*(r)$, for player $\mathcal{F}_U$:

$$s^*(r)=\begin{cases} 0 & \text{if } s<\frac{1}{2},\\ [0,1] & \text{if } s=\frac{1}{2},\\ 1 & \text{if } s>\frac{1}{2}.\end{cases}$$

Now, if $r^*(s)$ and $s^*(r)$ are plotted in the same diagram, as in Figure 5, it is readily observed that the mixed strategy NE in equation (2.1) is correct.

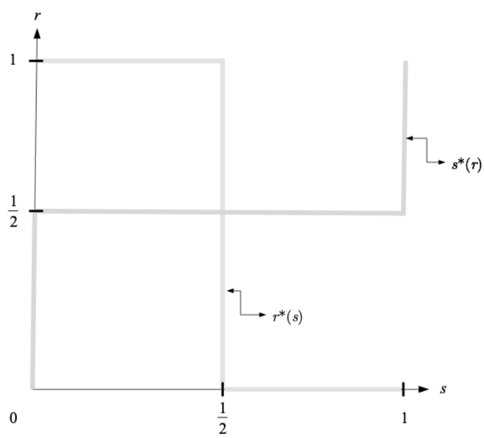


Figure 5. Best reply functions for the game in Subsection 2.2

Appendix B. Proof of Proposition 1

Proof. Starting with the original inequality

$$\frac{p}{1-p}>\frac{1-p}{p},$$

and multiplying on each side with $\frac{p}{1-p}$ gives:

$$p^2 > (1-p)^2.$$

Expanding the square and collecting items:

$$p^2 > (1 - 2p + p^2) \Rightarrow 0 > 1 - 2p \Rightarrow p > \frac{1}{2},$$

which concludes the proof.

□

References

[1] D. M. Buss, *Sexual jealousy*, Psychological Topics **22** (2013), 155–182.

[2] R. A. Foley and C. M. Fitzgerald, *Is reproductive synchrony an evolutionary stable strategy for hunger-gatherers?* Curr. Anthropol. **37**, 539–545.

[3] A. Gosline, *Do women who live together menstruate together?*, Scientific American, December 2007.

[4] P. Hammerstein and R. Selten, *Game theory and evolutionary biology*, in: R. J. Auman and S. Hart (eds.), Handbook of Game Theory with Economic Applications, vol. 2, Elsevier, Amsterdam, 1994, pp. 929–993.

[5] A. L. Harris and V. J. Vitzthum, *Darwin's legacy: An evolutionary view of women's reproductive and sexual functioning*, J. Sex Res. **50** (2013), 207–246.

[6] C. Knight, *Blood Relations: Menstruation and the Origins of Culture*, Yale University Press, New Haven, CT, 1991.

[7] M. K. McClintock, *Menstrual synchrony and suppression*, Nature **229** (1971), 244–245.

[8] A. Rapoport and A. M. Chammach, *The game of chicken*, Am. Behav. Sci. **10** (1966), 10–28.

[9] B. Skyrms, *The Stag Hunt*, Proc. Am. Philos. Soc. **75** (2001), 31–41.

[10] J. Maynard Smith, *On Evolution*, Edinburgh University Press, 1972.

[11] J. Maynard Smith and G. R. Price, *The logic of animal conflict*, Nature **246** (1973), 15–18.

[12] H. C. Wilson, *A critical review of menstrual synchrony research*, Psychoneuroendocrinology **6** (1992), 565–591.

[13] Z. Yang and J. C. Schank, *Women do not synchronize their menstrual cycles*, Hum. Nat. **17** (2006), 433–447.

Kjetil K. Haugen, Faculty of Business Administration and Social Sciences, Molde University College, Specialized University in Logistics, Britvegen 2, 6410 No-Molde, Norway
e-mail: `kjetil.haugen@himolde.no`