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THE TRADE-OFF BETWEEN GOALS AND UNCERTAINTY OF OUTCOME IN PROFESSIONAL TEAM SPORTS

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Abstract. Based on an assumption of an existing scoring probability difference between two teams engaged in a football match, the conditional probability distribution given the number of goals in the match for the low quality team beating the high quality team, is derived. Furthermore, similar distributions for the high quality team beating the low quality team as well as a draw are also derived. Based on a Poisson distribution of goals, we discuss a potential for optimizing expected uncertainty of outcome (UO) by adjusting intra-match rules or league rules for team sports. The main variables in this regard are (i) the typical number of goals scored, and (ii) the evenness of competing teams. We identify a curve defining the optimal expected number of goals as a function of team quality difference.

1. Introduction

In [20], the author discusses scoring rates in sports. It is argued that the probability of the weaker football team beating a better team decreases with an increasing number of goals in a match[1]. This relates to an important general "design problem" in sports. How should you design a sport with an optimal number of goals given that spectator demand is positively dependent on both uncertainty of outcome and goals? Unfortunately, Wesson [20] does not develop necessary analytical expressions for some important probability distributions. As such analytical expressions will be necessary for a formal mathematical modelling of the problem, this article derives several of these distributions. These are the conditional distribution given the total number of goals in a match for a victory for the low quality team, victory for the high quality team and the conditional draw distribution.

In the second part of the article, using a match model based on Poisson distributed goals, we investigate the (perhaps surprising) properties related to the existence of an optimal number of expected goals with respect to the weak team winning probability. That is, we contradict Wesson's claim in [20]. Some results regarding this optimality are also outlined. The main contribution is the identification of a curve defining the optimal expected scoring rate λ for a match, given

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Thanks to J. Wesson and his excellent book: *The science of soccer*, a great inspiration for this work.

[1]As our analysis will show, we dispute this allegation – at least to some extent.

the ratio r of each teams scoring rates. Optimality here refers to maximal uncertainty of outcome (UO) interpreted as the probability of the weaker team winning the match. We assume that demand for a team sport is positively correlated with uncertainty of outcome. Based on this curve, we discuss how the demand for a given team sport can be increased by a redesign targeting increased uncertainty of outcome. In this discussion, we point to two directions where a sport can "move" to become more interesting for spectators: (i) changing the intra-match rules to change the expected total goal score (e.g. change the size of the goals or change the rules to make penalty kicks easier or harder to get), (ii) changing the league rules to make the teams more (or less) even (e.g., introduce salary caps or redistribution of ticket value from good to less good teams).

2. A model based on actual goal score

In this section, we deduce the probability distributions for weak win, draw and strong win given a certain number of goals scored. Two teams, T_H and T_L, are engaged in a football match. The two teams are assumed unequal in performance quality. T_H is assumed to have higher quality while T_L has lower quality. This quality difference is formalized through a difference between the teams in scoring probability. It is assumed that whenever a goal is scored:

$$\begin{aligned} p &= Pr(T_H \text{ scores against } T_L) \quad \left(p > \frac{1}{2}\right), \\ 1-p &= Pr(T_L \text{ scores against } T_H). \end{aligned}$$

These probabilities are assumed independent of the goals scored up to any time-point in the match.

Initially, the number of goals scored in the match is restricted to the set of positive odd numbers; $\{1, 3, 5, \ldots\}$. This is a simplification, as the draw option is ruled out[2].

So, the output to be established is an analytical expression containing all probabilities that the low quality team, T_L, beats the high quality team conditioned on the number of all possible end-results. That is[3]:

$$w^o(g) = Pr(T_L \text{ beats } T_H | \text{ number of goals} = g)\ .$$

Let us look at some simple examples, $(w^o(1), w^o(3))$, to establish the general distribution. If $g = 1$ $(w^o(1))$, there is only one goal scored. The probability that T_L wins is then simply $1 - p$. It becomes slightly more complex in the $w^o(3)$-case. Now, there are exactly two options; either T_L wins 3–0 or 2–1. The 3–0 situation is simple. Then, T_L scores three goals in succession with a probability of $(1-p)^3$. In the 2–1 case, there are three possibilities; either HLL, LHL or LLH, where H and L denote the scoring sequence. The probability is the same, $p(1-p)^2$ for each of these options, and the total probability is hence $3p(1-p)^2$. As a consequence:

[2]A similar distribution for even number of goals results is necessary and will be developed later on.

[3]Obviously, $w^o(g)$ also is a function of p, $w^o(g, p)$. We do, however, omit this notation for the time being, for simplistic reasons.

$$w^o(3) = (1-p)^3 + 3p(1-p)^2 \ .$$

By some relatively simple inductive arguments[4], it should be straightforward to realize that:

$$w^o(g) = \sum_{i=1}^{\frac{g+1}{2}} \binom{g}{i-1} p^{i-1}(1-p)^{g-i+1}, g \in \{1,3,\ldots\} \ . \tag{2.1}$$

We are (of course) also interested in the distribution for an even number of goals. We name the distribution $w^e(g)$. The case of $g = 0$ provides a certain draw, so the probability of T_L beating T_H is zero given a goalless match.

It turns out that the distribution is quite similar to (2.1). Suppose we investigate the case of $g = 4$. Then, either a 4–0 or a 3–1 victory for T_L are the only possibilities. A 4–0 victory has a probability of $(1-p)^4$, while a 3–1 victory is established either by $LLLH$, $LLHL$, $LHLL$ or $HLLL$, all with a probability of $p(1-p)^3$. Consequently, the total probability in this case is $(1-p)^4 + 4p(1-p)^3$ or

$$\binom{4}{0} p^0(1-p)^4 + \binom{4}{1} p^1(1-p)^3 \ .$$

Hence, the structure for the even goal-case is similar to the odd goal-case, apart from the simple fact that the summation end subscript must be changed from $\frac{g+1}{2}$ to $\frac{g}{2}$. Then, the distribution can be formulated as:

$$w^e(g) = \sum_{i=1}^{\frac{g}{2}} \binom{g}{i-1} p^{i-1}(1-p)^{g-i+1}, g \in \{2,4,\ldots\}.$$

Now, it is straightforward to see that the two expressions $w^o(g)$ and $w^e(g)$ can be combined into a common expression, $w(g)$, by:

$$w(g) = \sum_{i=1}^{\lfloor \frac{g+1}{2} \rfloor} \binom{g}{i-1} p^{i-1}(1-p)^{g-i+1}, g \in \mathbb{N} \ . \tag{2.2}$$

The conditional distribution (2.2) is simply a cumulative binomial distribution. This is perhaps more easily seen by a simple substitution. Setting $k = i - 1$, (2.2) can be reformulated as:

$$w(g) = \sum_{k=0}^{\lfloor \frac{g-1}{2} \rfloor} \binom{g}{k} p^k(1-p)^{g-k}, g \in \mathbb{N} \ . \tag{2.3}$$

As the forthcoming parts of the article will show, the remaining conditional distributions for a draw and victory of the better team, T_H, are also relevant. It turns out that these two distributions are easily derived. In the case of a victory

[4]For example, a match with 5 goals must then include terms of the type $(1-p)^5, p(1-p)^4$ and $p^2(1-p)^3$ with binomial coefficients defining constants in front of these terms

for T_H, it is obvious that a simple interchange of p and $1-p$ in the expressions will do the trick. That is,

$$v(g) = Pr(T_H \text{ beats } T_L|g) = \sum_{k=0}^{\lfloor \frac{g-1}{2} \rfloor} \binom{g}{k} (1-p)^k p^{g-k}, g \in \mathbb{N}\,.$$

Finally, the conditional distribution for a draw is simply found by the probabilistic “norm to one” condition:

$$d(g) = Pr(T_H \text{ and } T_L \text{ plays a draw}|g) = 1 - (v(g) + w(g))\,, g \in \mathbb{N}\,.$$

To get a taste of what the functions $w(g)$, $v(g)$ and $d(g)$ look like, a plot is given in Figure 1.

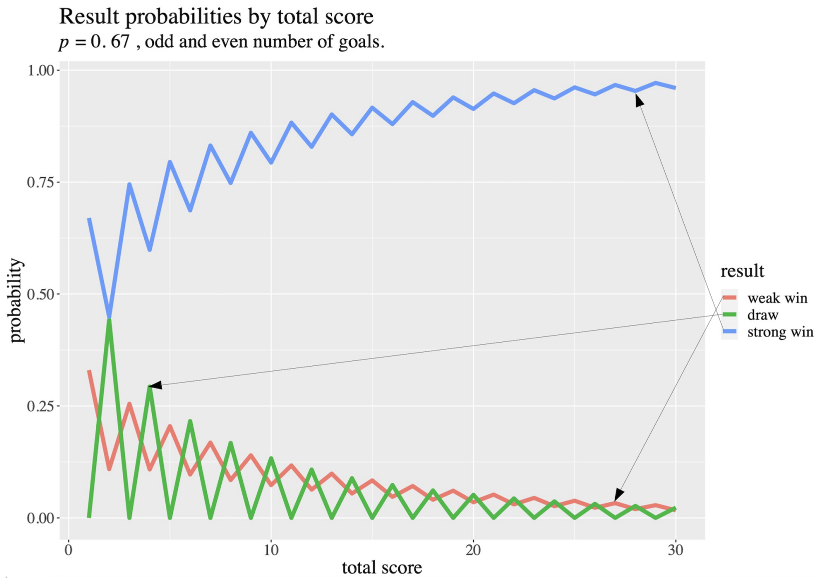

Figure 1. Plots of $w(g)$ (weak win), $v(g)$ (strong win) and $d(g)$ (draw).

Figure 1 shows a characteristic saw-tooth pattern for all functions $w(g)$, $v(g)$ and $d(g)$. Initially, this may seem a bit strange. After all, why should the probability of the weaker team beating the better team be higher with an odd rather than even number of goals? The explanation is banal. Given an odd number of goals in a football match, the draw option is ruled out and the probability for either team winning increases when g increases by 1 from an even to an odd number. As Figure 1 only shows the situation for a given $p = 0.67$, it still seems worthwhile to investigate the general situation. We write $w^o(g,p)$ and so on to underline the fact that the functions also depend on p in what follows.

Proposition 2.1. *If* $p > \frac{1}{2}$ *and* $g \in \mathbb{N}$ *is odd, then* $w^o(g,p) > w^e(g+1,p)$.

The proof of 2.1 is left for Appendix A. Hence, the saw-tooth pattern observed in Figure 1 is not a special case for the given probability, but holds for any $p > \frac{1}{2}$.

Another striking feature of Figure 1 is the decaying behaviour of two of the saw-tooth patterns, (($w(g)$ and $d(g)$), as well as the increasing pattern of $v(g)$. One could attempt a proof[5] proposing $\lim_{g\to\infty} v(g) = 1$. However, as a similar proof already exists in [17], and the close connection between the two mathematical models applied here is shown in equation (5.1), such a proof is omitted.

3. A preliminary discussion

The fact that many goals lead to few draws is interesting if sports practice is taken into consideration. A clear difference is easily observed if we compare American football with association football[6]. In association football, there is only one type of goal, which leads to goal difference as a simple match determination. However, in American football, at least two different types of goals, field goals and touch downs, lead to a need for a conversion system between goals and points. As such, comparing the number of goals in association football with American football is not straightforward. Still, some simple approximation may be enlightening. According to [6], the average point score in NFL in the time period from 1976 to 1994 was 40.15 points. In order to convert points to goals, it seems reasonable to use the share of field goals in NFL. According to [18], roughly 25% of points are earned as field goals. Given 6 points for a touch down and 3 points for a field goal[7], this leads (roughly) to 10+5 = 15 "goals".

According to [1], the average goal score in association football has been steady around 2.75 goals per match in the time period from 1980 to 2020. That is, American football averages on almost 7 times more goals than association football. If we look at another typical American team sport, basketball, the number of goals is even higher. Without going into details, 50 goals is quite common in a match.

The interesting observation comes if we compare association football with US sports rule-wise. US sports explicitly lack the draw option. The typical mechanism used is the "sudden-death-principle". That is, if a match ends in a draw, time is added until one of the teams scores to define a winner and a loser. This proposes a kind of paradox. As we have seen already, US sports have far more goals than association football. However, many goals lead to almost zero draw probabilities. Why are US sports designed so that draw (rule-wise) is illegal? In practice, it will almost never happen. The following quote from [14] says it all:

> "*Donovan McNabb and other Philadelphia Eagles players said – when the team and Cincinnati Bengals tied 13-13 in 2008 – that they did not know that a game could end in a tie.*"

It does not seem unreasonable to suggest some "sociological-like" explanations. Perhaps the winner is more important in the US than in Europe? We will leave this

[5] It follows trivially that $\lim_{g=\infty} d(g) \to 0$.

[6] American football (sometimes also named gridiron football) is mainly performed in the US, in the National Football League (NFL), while association football is performed in Europe, Latin America and almost everywhere else.

[7] The actual point rules in NFL are a bit more complex. There are, for instance, options for extra points. As we are basically interested in magnitude comparisons, we accept this approximation.

discussion here and will merely point to some research discussing the differences between US and European sports along similar lines – see, for instance, [8].

The fact that the winning probability, $v(g)$, of the better team approaches 1 as g approaches ∞ is also interesting. In fact, we could also here identify a somewhat paradoxical situation. As pointed out above, US sports are characterized with many more goals than the most popular European team sport counterpart, association football. Then, with the knowledge of many and varied US means to enhance competitive balance (increase UO) – Reverse drafting [22], salary caps [12] and gate revenue sharing [19] – it may seem slightly odd that US sports contain many goals. Why design sports with many goals in the first way, producing low UO (as argued above) and then introduce regulations to increase UO? Of course, this is not the true story. The design process of US sports was most probably not done with notion of the UO concept. Still, it may be of interest today when sport redesign may be necessary. After all, it is not practically complicated to reduce the number of goals/points, for instance, in basketball. A simple lift of the basket will do the trick.

4. The probability of the weaker team beating the stronger team is not overall decreasing

One can be led to believe, by Wesson [20], that the probability of the weaker team beating the strong team is generally decreasing as the number of goals increase. Some basic calculations based on our simple model show that this is only true in an asymptotic sense; for a given strong team scoring probability p there is a goal score $g > 0$ that maximizes the weak win probability. We can see this in Figure 2, where it is readily observed that the probability function is uni-modal and hence both increasing and decreasing.[8] This is an interesting point, which deserves some additional comments.

This fact is relevant for the Uncertainty of Outcome (UO) concept. This concept, formally introduced by Rottenberg [16], simply states that sports spectators should (logically) be less interested in competitions where the predictability of the result is high. After all, who would pay to watch a football match if one knew the end-result beforehand? This so-called *Uncertainty of outcome hypothesis* is a favourite subject for many sports economists, and has consequently gained a lot of attention in sports economic research. See, for instance, [2] for an excellent survey on relevant research.

The reason for this point's relevance here is the obvious fact that the probability that a weaker team beats a stronger team may be a good micro (or ex-ante) definition of the concept itself.

In traditional sports economic literature, the UO concept is largely defined ex-post. That is, most attempts to measure UO is based on information available after the competition or the match (or the matches in a league situation) is finished. See, for instance, [3] for a good review of various ex-post UO measurement techniques. However, here, as we have developed a probability distribution for the weaker team beating the stronger team, depending on the initial (ex-ante) probabilistic quality

[8] We have only included even scores g here to avoid cluttering the picture with saw-teeth.

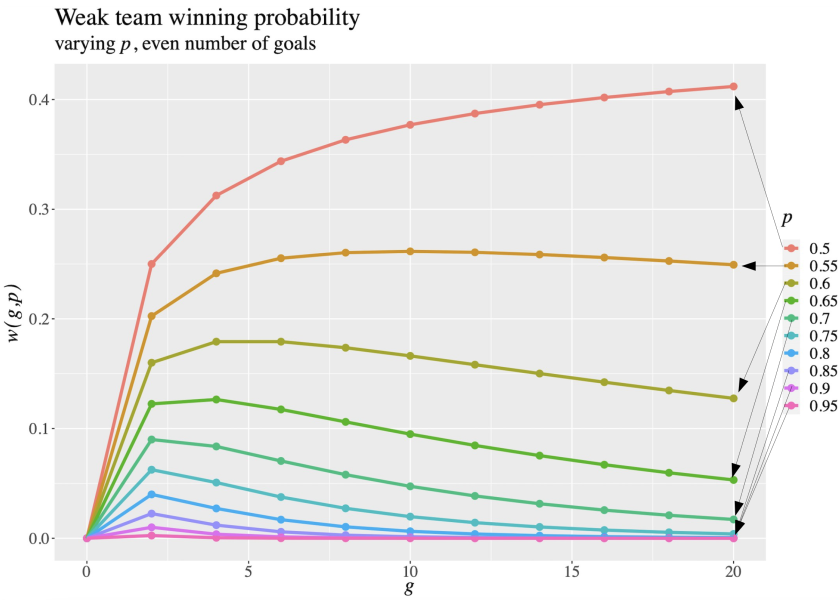

Figure 2. Plots of $w^e(g)$ for various p's. (Note that points identifying probabilities are piecewise linearly connected only for presentation purposes.).

difference between the teams, $\{p, 1-p\}$ and the number of goals in a match g, we can think of an ex-ante definition of UO. That is, UO is high if $w^e(g,p)$ is high; low if $w^e(g,p)$ is low.

The fact that this probability, according to Figure 2, has a maximum as a function of g is by itself interesting. After all, if UO is important for demand of a given sport, the existence of a maximal UO for a given goal number may be of relevance in sport design. Hence, it is of relevance to investigate this possible optimality further.

Unfortunately, our initial discrete model is not well suited for such an investigation, so we shift to a more traditional mathematical modelling approach for the continued analysis. It is well worth pointing out that the proposed optimality discussed above is mentioned briefly in [17], but not explicitly analysed.

5. A formulation using Poisson distributed goal scores

We can model the number of goals scored for two teams in a match by independent Poisson distributed random variables X_1, X_2 with parameters λ_1, λ_2. We assume team 1 is the weaker team so that $\lambda_2 = r\lambda_1$ for some $r > 1$. We let $\lambda = \lambda_1 + \lambda_2$ denote the expected total number of goals in the match. From this, we trivially get

$$\lambda_1 = \frac{\lambda}{1+r}, \quad \lambda_2 = \frac{r\lambda}{1+r} \, .$$

This modelling approach corresponds to the model used previously, where the parameters are related by

$$p = \frac{\lambda_2}{\lambda_1 + \lambda_2} = \frac{r}{1+r} \ . \tag{5.1}$$

We can write $Z = X_1 - X_2$ for the goal difference, so that $Z > 0$ represents a win for the weak team, $Z = 0$ represents a draw and so on. The variable Z is known to have a Skellam distribution with parameters λ_1, λ_2 given by discrete probabilities

$$Pr[Z = z] = \exp(-(\lambda_1 + \lambda_2)) \left(\frac{\lambda_1}{\lambda_2}\right)^{z/2} I_z\big(2\sqrt{\lambda_1 \lambda_2}\big), \quad z \in Z^9 \ ,$$

where $I_z(x)$ denotes the modified Bessel function of the first kind, see [11] and references therein for more details. For simplicity, we will stick with the assumption of independent goal scores here. However, we note that the Skellam distribution for Z is still valid under certain forms of correlated scores, as demonstrated in [11].

We will first consider the question of how the probability of a weak win depends on the *total expected score* λ and the *ratio of the teams' scoring rates* r. So now, let $w(\lambda, r) = Pr[Z > 0]$ denote this probability. In terms of λ, r, the probability distribution for Z can be expressed by

$$Pr[Z = z] = \exp(-\lambda) r^{-z/2} I_z\Big(\frac{2\lambda\sqrt{r}}{1+r}\Big), z \in Z \ .$$

As it appears challenging to derive analytical results based on the above formula, we can use numerical computations[10] to visualize the function w. In Figure 3, we see w as a function of λ for a set of fixed values of r. What we see is that the function has a maximum value for some $\lambda(r)$ depending on the ratio r of the scoring rates. In particular, when the teams are relatively even in strength, the maximum appears for a fairly large value of λ, e.g. when $r = 1.20$, the strong team has a 20% higher scoring rate, and in this case the weak team has a maximal winning chance when the expected number of goals is about 6. It is therefore not true that an increasing number of goals generally reduces the UO. Roughly speaking, if one were to maximize the UO (in the sense of the weak team winning probability) when r is about 1.20, one should design the game so as to have about 6 expected goals. The figure moreover shows that when λ is close to 0, the weak team almost never wins. This is, of course, because almost all games end 0–0. In the other direction, for any value $r > 1$, the probability $w(\lambda, r)$ will ultimately decrease and approach 0, because with an increasing number of goals, the law of large numbers will benefit the stronger team.

In the same context, it may be interesting to look for the λ value that maximizes $w(\lambda, r)$ as a function of r, i.e., the function

$$\lambda^*(r) = \arg\max_{\lambda} w(\lambda, r) \ .$$

This function is shown in Figure 4, where we have used $r = 1.10, 1.20, \ldots, 5.0$ and then found the maximizing λ numerically. We can read from this picture that when the teams are fairly even in strength (r close to 1), the weak team

[9] Set of integers.

[10] Using the *skellam* [13] and *tidyverse* [21] packages for the programming language R [15].

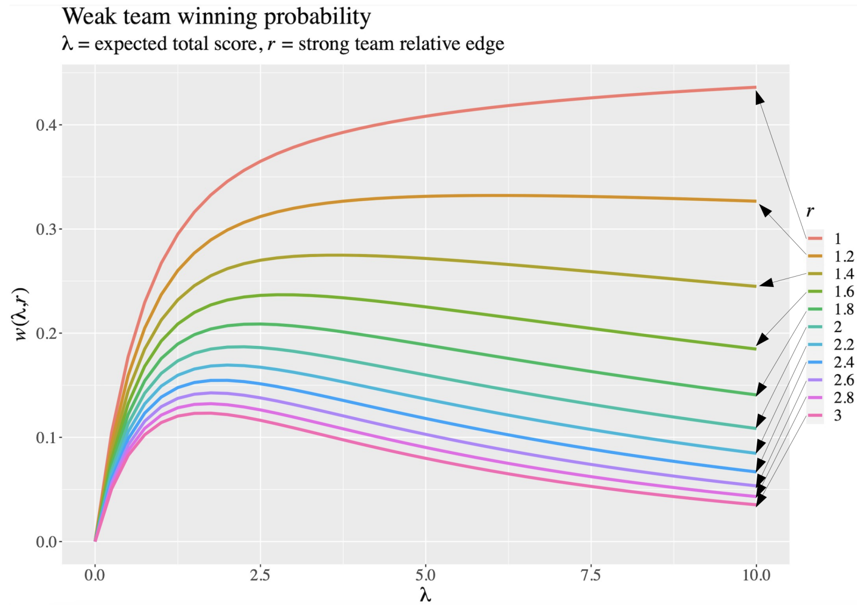

Figure 3. Plots of $w(\lambda, r)$ for various r's.

winning probability is maximized in a game with a relatively high number of goals. When the strong team is substantially stronger, the maximizing λ decreases. Our computations indicate that

$$\lim_{r \to \infty} \lambda^*(r) = 1 \ .$$

The interpretation of this is that even in a grossly uneven match, there has to be some expectancy of goals being scored for the weak team to maximize its winning probability.

6. Discussion and conclusions

In addition to the derivation of some potentially useful probability distributions, the main novelty in this paper is the content of Figure 4. In layman terms, this figure displays a relation between maximal uncertainty of outcome and the (expected) number of goals in any team sport. Hence, it gives information (and potential decision support) related to sports design and redesign. One can perhaps think of it as a means for deciding how many goals are needed to reach maximal uncertainty of outcome for a given sport, when the average ratio of scoring rates r is given.

In practice it is hard to deal with the term "optimal uncertainty of outcome". However, for the most important global team sports, most US team sports, as well as European handball and association football, the typical problem is rather lack

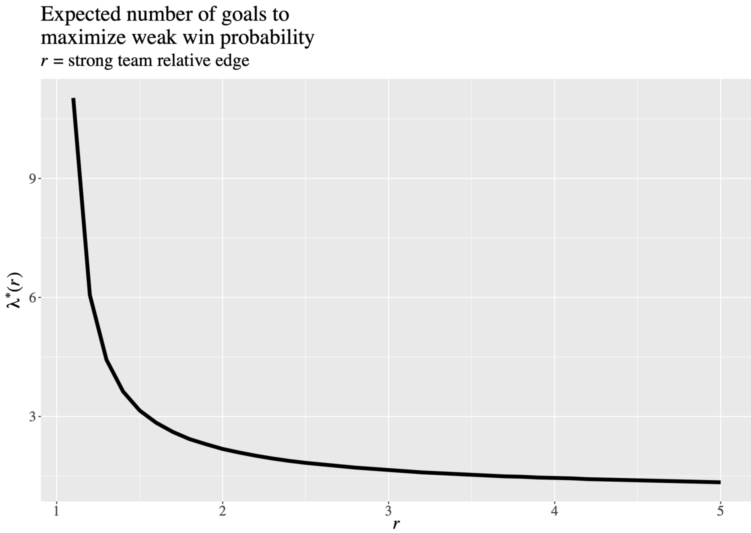

Figure 4. Plot of $\lambda^*(r)$.

of UO than too much UO[11]. In the US team sport case, the existence of regulative means obviously introduced to increase UO leads (empirically) to a conclusion of too low UO. In European handball as well as association football, several research papers point to a situation with diminishing UO, potentially with adverse demand effects. See, for instance, [9, 10].

To gain more insight into the potential decision support potential of Figure 4, it will prove worthwhile to note the following: If we move back to section 1, p was defined as the probability that the strong team scores against the weak team. In a UO-perspective, this leads to increased UO (higher $1-p$) if p decreases. By (5.1), a decrease in p leads to a decrease in r. That is, moving to the left in Figure 4 means increased UO.

In order to continue, we need to address demand for sports. According to [2], demand (or spectator willingness to pay) for sports products depends on many factors. Still, it seems evident that the two (perhaps) most important factors are UO and performance quality. That is, spectators consume more sports products if David may beat Goliath (UO) and if the performance is high (possible world record on 100 meters running). In the team sports we discuss here, a classical proxy for performance are goals – good goals, bad goals, many goals. Hence, it seems reasonable to assume that demand is positively related to both goals g (or λ in the last mathematical model) and UO. That is, $d = d(\lambda, r)$ and $\frac{\partial d}{\partial \lambda} > 0$, $\frac{\partial d}{\partial r} < 0$. As a consequence[12], moving to the left and upwards are "good moves" in Figure 4.

[11]A classical example of too much UO may be ski-jumping where the introduction of the so called V-style led to such wind dependence that many competitions resembled coin tossing. In that situation the excess UO had to be removed by introduction of a wind compensation system.

[12]This argument also needs an assumption that it is in the best interest of the sports manager to maximise d. In our opinion, a reasonable assumption.

Now, let us assume that we investigate some sport, say association football. Information to estimate λ and r (or p) is readily available both at global and more regional levels.

It will most likely not be the case that we land on the curve in Figure 4, i.e., the estimated point $(\bar{\lambda}, \bar{r})$ will be somewhere not on the curve. As indicated in Figure 5, two possible situations may occur, either below or above the curve, so we have O_1 or O_2.

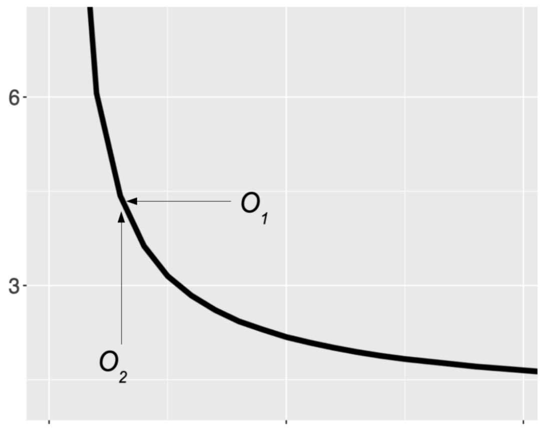

Figure 5. Parts of Plot of $\lambda^*(r)$ with (thought) empirical observations, O_1, O_2..

If O_1 is the case, the objective $(d(r, \lambda))$ can be improved by moving in the direction of the horizontal arrow. Here, the number of goals is kept constant while UO is increased. To achieve such a movement (in practice), some means like gate revenue sharing, salary caps or similar must of course be imposed to make teams more even (without altering the intra-match rules).

If alternatively O_2 is the case, a movement upwards can be achieved (the vertical arrow in Figure 5) by increasing the number of goals without changing relative team strengths. To actually increase the number of goals, some practical changes in match rules will typically be necessary. Lowering the basket in basketball or increasing the goal size in association football are some obvious possibilities.

In summary, we have relied on very simple models in our analyses and discussions, and it may be argued that such models are too simple to be of value – see, for instance, [4, 5, 7]. We still believe that the relationship depicted in Figure 4 exists, and that our model approximation may be reasonably close to the real function. We believe that a further study of this relationship can be valuable as an additional means to increase demand for team sports.

Appendix A. Proof of Proposition 2.1

Proof. If g is an odd integer, and $p > 1/2$, we want to prove that

$$w(g, p) > w(g + 1, p).$$

We will use the expression from equation (2.3), and since g is odd, we get

$$w(g,p) = \sum_{k=0}^{\frac{g-1}{2}} \binom{g}{k} p^k (1-p)^{g-k}$$

$$w(g+1,p) = \sum_{k=0}^{\frac{g-1}{2}} \binom{g+1}{k} p^k (1-p)^{g-k+1} .$$

The sums have the same number of terms, so we can compare term by term and show that for any $p > 1/2$,

$$\binom{g}{k} p^k (1-p)^{g-k} > \binom{g+1}{k} p^k (1-p)^{g-k+1} . \tag{A.1}$$

We can use the fact that

$$\binom{g+1}{k} = \frac{g+1}{g-k+1} \binom{g}{k} .$$

Substitute this into (A.1) to get the equivalent inequality

$$1 > \frac{g+1}{g-k+1}(1-p) . \tag{A.2}$$

The right hand side is maximized for any g by

$$p = \frac{1}{2}, k = \frac{g-1}{2} ,$$

so we get

$$\frac{g+1}{g-k+1}(1-p) < \frac{g+1}{g - \frac{g-1}{2} + 1} \cdot \frac{1}{2} = \frac{g+1}{g+3} < 1,$$

which demonstrates (A.2) and hence the proposition follows.

□

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