Revenue Management in the airline industry: problems and solutions

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- The number of seats in an airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the time of purchase.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?
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- Historical Booking
- Actual booking
- No-show data
- Forecast model
- Optimization model
- Overbooking model
- Recommended booking limits

Figure 1: Third-generation airline RMS (Barnhart et al., 2003)
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Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*. This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
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A simple model

The model of Beckmann (1958) yields the booking level for reservations that minimizes expected costs.

\[ a \int_{0}^{x-c} (x - \kappa - c) dP(\kappa | x) + b \int_{x-c}^{x} \mathcal{N}(\kappa) dP(\kappa | x) \]  

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G.LC (himolde)  
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- Cancellation and booking probabilities are time-dependent
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Figure 2: Markovian decision process described by Rothstein (1971)
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Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

**Single leg Seat Inventory Control**

Littlewood (1972) proposed to protect $p$ high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{\text{high}} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{\text{low}} \geq f_{\text{high}} Pr(D_{\text{high}} \geq p)$$

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座席インベントリ制御

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Virtual Nesting

We assume two fares H, L and 2 itineraries AB, AC

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<tr>
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<th>Leg</th>
<th>Itinerary</th>
<th>Fare</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_1 )</td>
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We optimize over all legs with new fare class

Network RM

We optimize over all O-Ds with constraint per leg and fares nesting per O-D
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Bid-price

We accept a fare only when $R$ exceeds the *opportunity cost* of the reduction in leg capacities.

\[ u_{jt}(R_j, X_j) = \begin{cases} 
1 & R_j \geq \sum_j \pi_j(X_j) \\
0 & \text{otherwise}
\end{cases} \]

- Similar to dual prices of (4).
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- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing $\pi$ (LP, Prorated EMSR, ...)

$\pi_2 \geq \pi_1$
Bid-price control for Network Revenue Management

Bid-price

We accept a fare only when $R$ exceeds the opportunity cost of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 
1 & R_j \geq \sum_j \pi_j(X_j) \\
0 & \text{otherwise}
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Figure 4: Comparison between the daily average price and the estimated price on CIA–STN route.
Average price and demand

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Figure 5: Prices on the *Rome Ciampino–London Stansted* route for two specific dates.
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Thanks you for your attention