Revenue Management in the airline industry: problems and solutions

Guillaume Lanquepin-Chesnais

supervisors: Asmund Olstad and Kjetil K. Haugen

Høgskolen i Molde

23^{th} November 2012

Layout

1. Introduction
   - Characteristics of the airlines industry
   - A definition and some numbers
   - A historical perspective
   - The main components of RM
2. Overbooking
   - Overview of the problem
   - Initial model of Beckmann (1958)
   - Dynamic problem of Rothstein (1971)
3. Seat inventory control
   - Littlewood (1972): from overbooking to SIC
   - Nested fare class
   - Origin-Destination Fare (ODF)
   - Network revenue management
   - Bid-Price control for Network Revenue Management
4. Evidences of RM
   - The example of Ryanair

Layout

1. Introduction
   - Characteristics of the airlines industry
     - A definition and some numbers
     - A historical perspective
     - The main components of RM
2. Overbooking
   - Overview of the problem
   - Initial model of Beckmann (1958)
   - Dynamic problem of Rothstein (1971)
3. Seat inventory control
   - Littlewood (1972): from overbooking to SIC
   - Nested fare class
   - Origin-Destination Fare (ODF)
   - Network revenue management
   - Bid-Price control for Network Revenue Management
4. Evidences of RM
   - The example of Ryanair

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Characteristics of the airlines industry

- We can not "store" any seat: they are only available at the departure. Each empty seat is a revenue loss.
- The number of seats in a airplane is fixed.
- The cost of a flight is largely independent of the numbers of occupied seats.
- People who make their reservations early are more price sensitive: we can segment market by the *time of purchase*.

The motivation of Revenue Management

Given that capacity and cost are fixed for each flight, how to increase the profitability?

Increasing revenue by segmenting market

Layout

1. Introduction
   - Characteristics of the airlines industry
   - A definition and some numbers
   - A historical perspective
   - The main components of RM
2. Overbooking
   - Overview of the problem
   - Initial model of Beckmann (1958)
   - Dynamic problem of Rothstein (1971)
3. Seat inventory control
   - Littlewood (1972): from overbooking to SIC
   - Nested fare class
   - Origin-Destination Fare (ODF)
   - Network revenue management
   - Bid-Price control for Network Revenue Management
4. Evidences of RM
   - The example of Ryanair

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

A definition and some numbers

Definition (Smith et al., 1992)

A [. . .] description of yield management (YM) as it applies to airlines is the control and the management of reservations inventory in a way that increases (maximizes, if possible) company profitability, given the flight schedule and structure.

- American Airlines (AA) estimates the annual revenue contribution of YM around $500 millions at the beginning of 90's
- $8.4 billions benefice for the airline industry in 2011 (IATA, 2012)
- Operating margin is on average only 1.64% in US airlines (Bureau of Transportation Statistics, 2011)
- A medium-sized airline operates 1000 flight legs, 10 booking classes and its reservation system accepts booking 330 days in advance (Barnhart et al., 2003).

Layout

1. Introduction
   - Characteristics of the airlines industry
   - A definition and some numbers
   - A historical perspective
   - The main components of RM
2. Overbooking
   - Overview of the problem
   - Initial model of Beckmann (1958)
   - Dynamic problem of Rothstein (1971)
3. Seat inventory control
   - Littlewood (1972): from overbooking to SIC
   - Nested fare class
   - Origin-Destination Fare (ODF)
   - Network revenue management
   - Bid-Price control for Network Revenue Management
4. Evidences of RM
   - The example of Ryanair

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,. . .)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,. . .)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,. . .)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,. . .)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,. . .)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,. . .)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,...)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,. . .)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,...)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,. . .)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,...)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,...)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,. . .)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,. . .)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,. . .)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,...)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,...)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,...)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

A historical perspective: the example of AA

1966 Implementation of SABRE (semi-automated business research environment) a centralized and computerized reservation system at AA

1977 Introduction of *Super Saver* discount fare at AA

1978 Airline Deregulation Act in USA that deregulates schedules and prices. It increases the number and the variety of discount fares. Besides, airlines began offering connecting services, using centrally located airports as hubs.

1981 First Low Cost Carriers (PEOPLExpress,...)

1985 Introduction of *Ultimate Super Saver* at AA

1988 Implementation of new Revenue Management System (RMS) called DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that manages network effects with *virtual nesting*.

Layout

1. Introduction
   - Characteristics of the airlines industry
   - A definition and some numbers
   - A historical perspective
   - The main components of RM
2. Overbooking
   - Overview of the problem
   - Initial model of Beckmann (1958)
   - Dynamic problem of Rothstein (1971)
3. Seat inventory control
   - Littlewood (1972): from overbooking to SIC
   - Nested fare class
   - Origin-Destination Fare (ODF)
   - Network revenue management
   - Bid-Price control for Network Revenue Management
4. Evidences of RM
   - The example of Ryanair

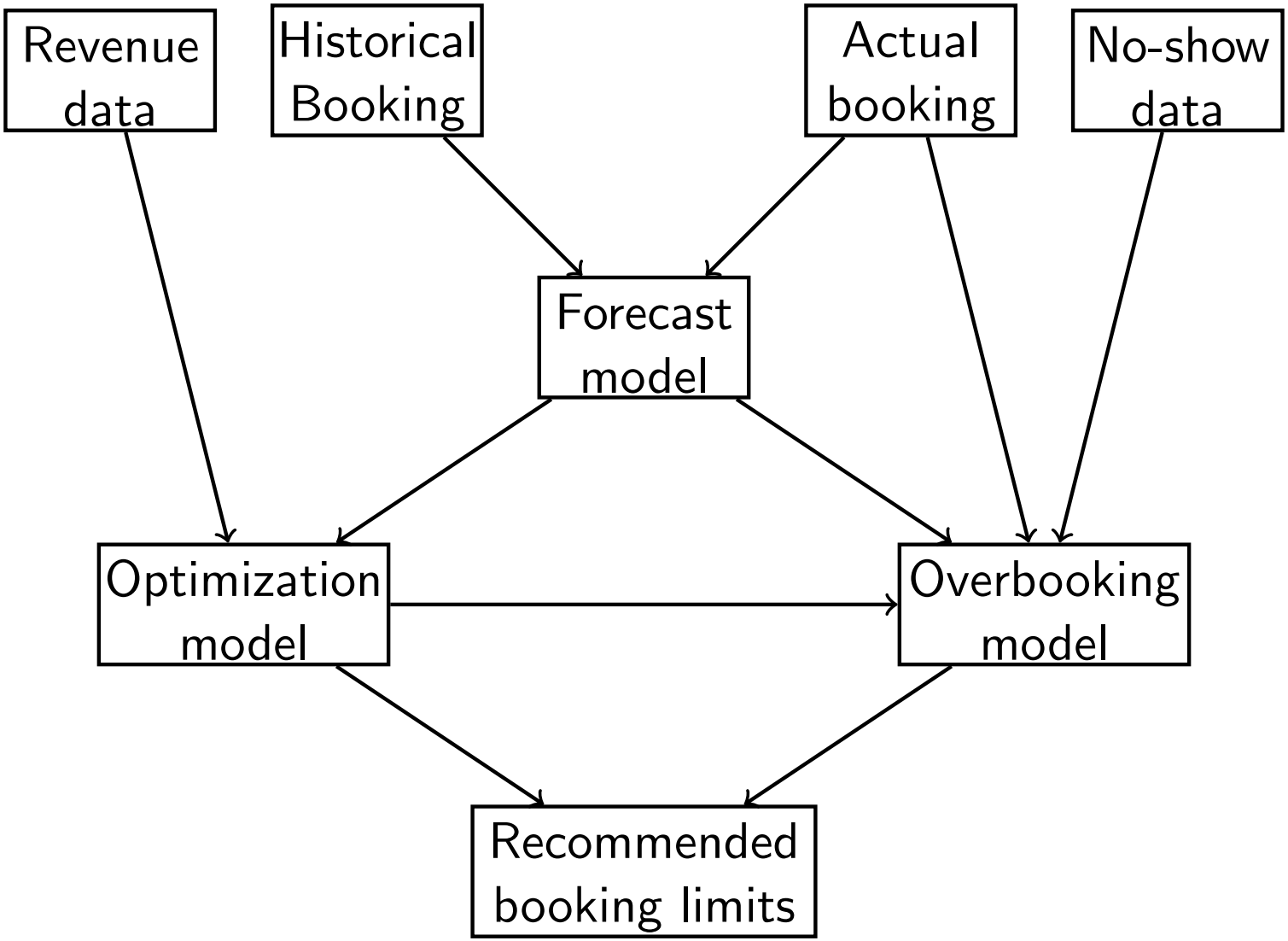


Figure 1: Third-generation airline RMS (Barnhart et al., 2003)

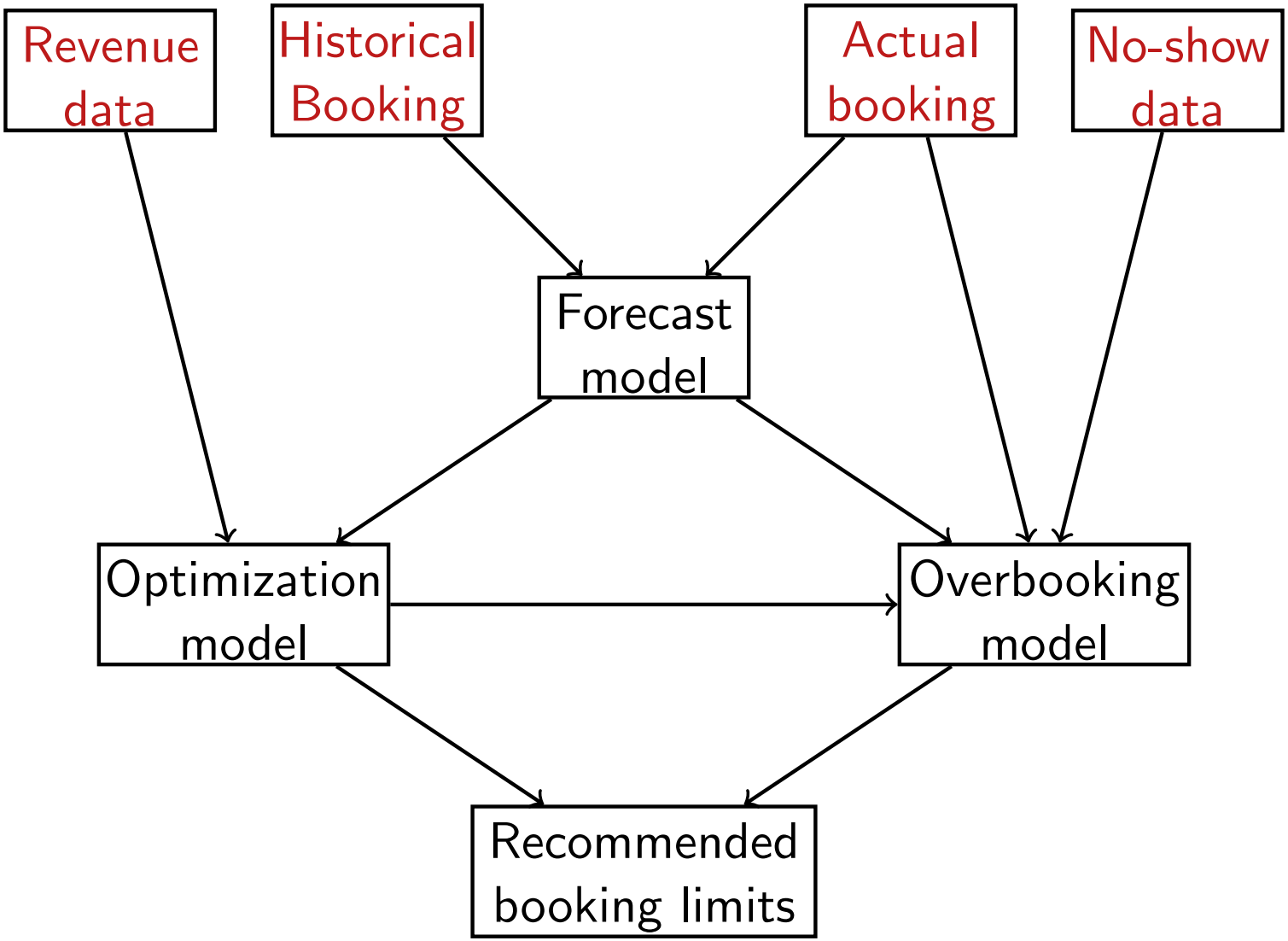


Figure 1: Third-generation airline RMS (Barnhart et al., 2003)

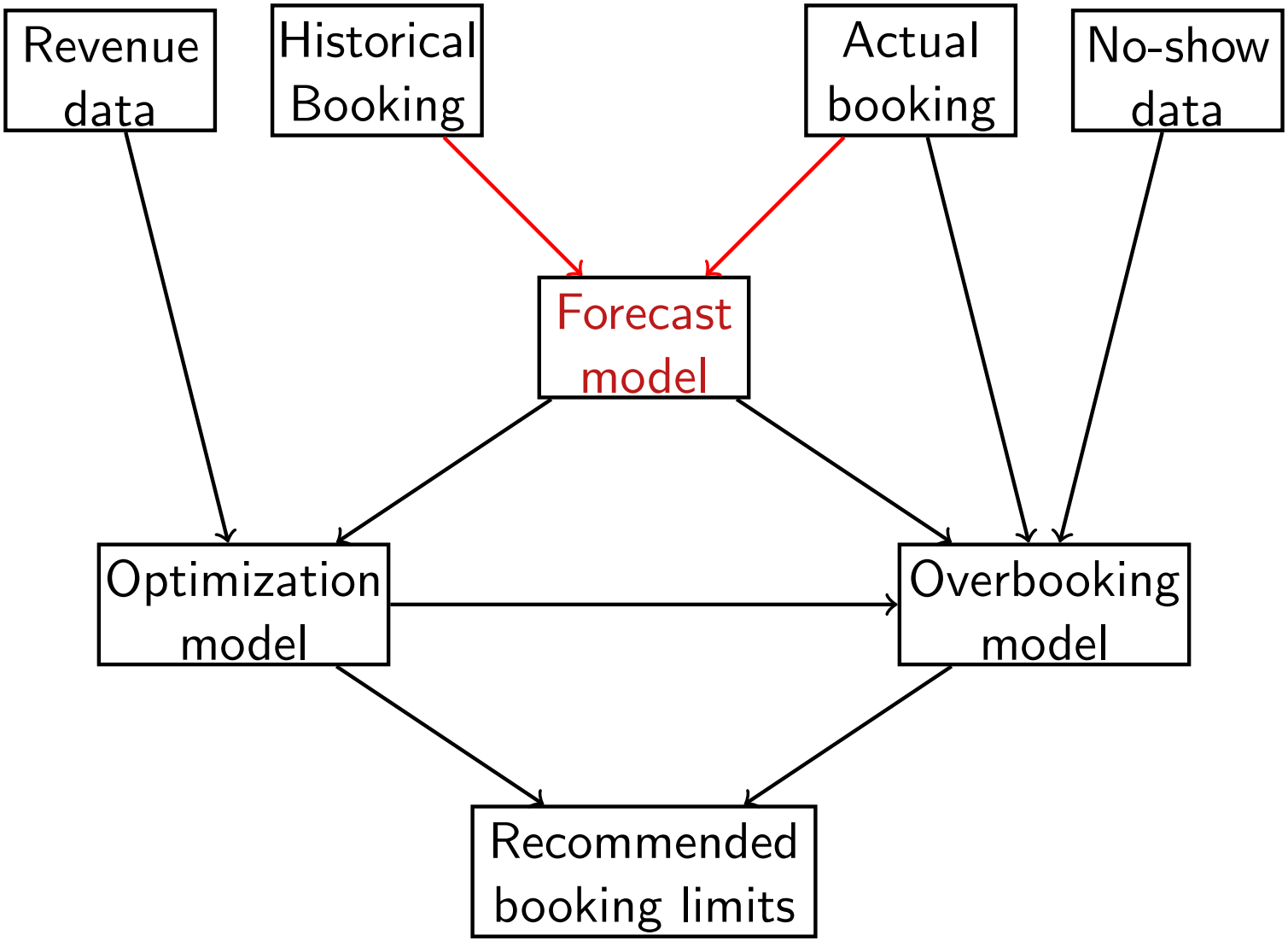


Figure 1: Third-generation airline RMS (Barnhart et al., 2003)

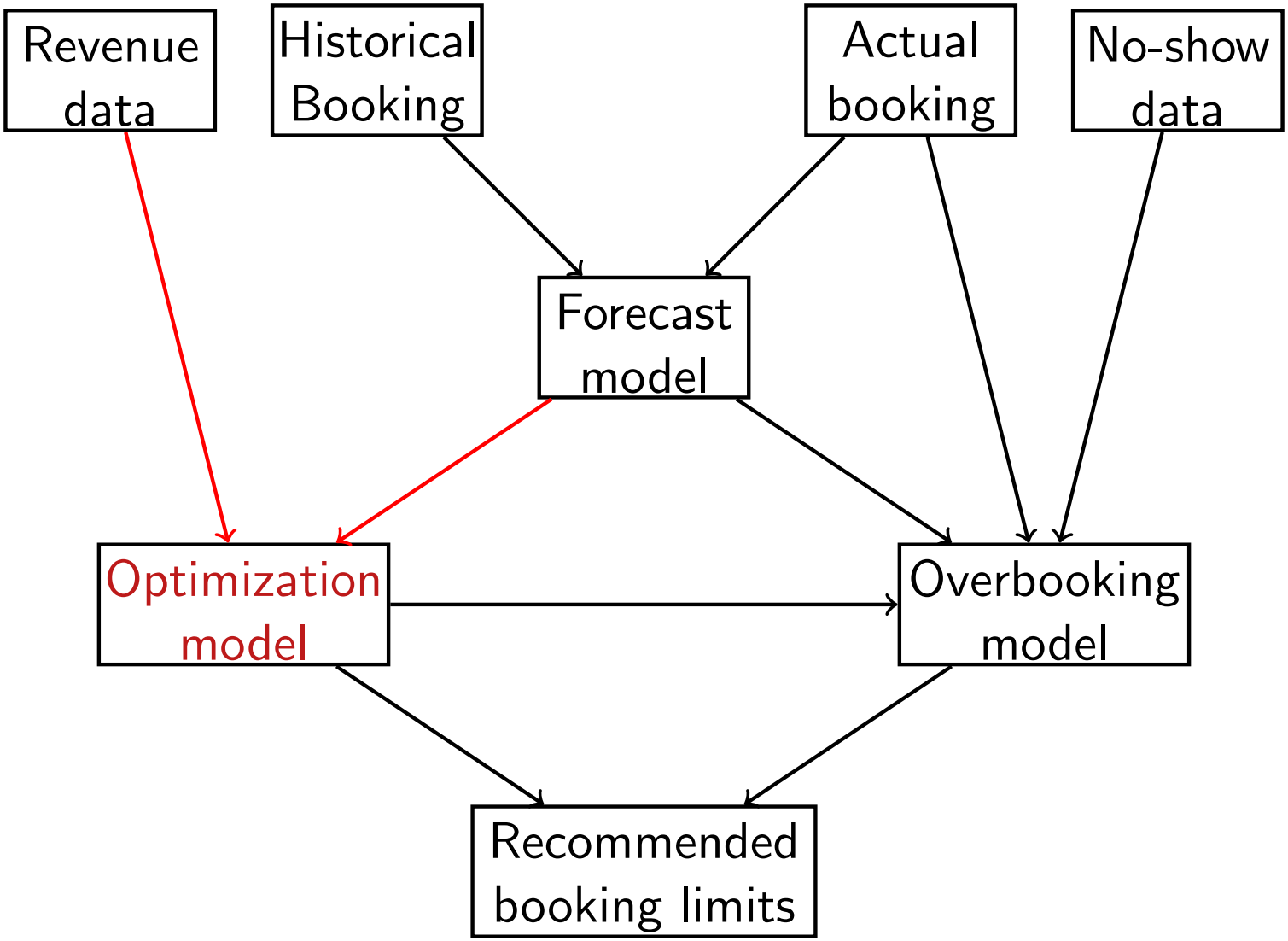


Figure 1: Third-generation airline RMS (Barnhart et al., 2003)

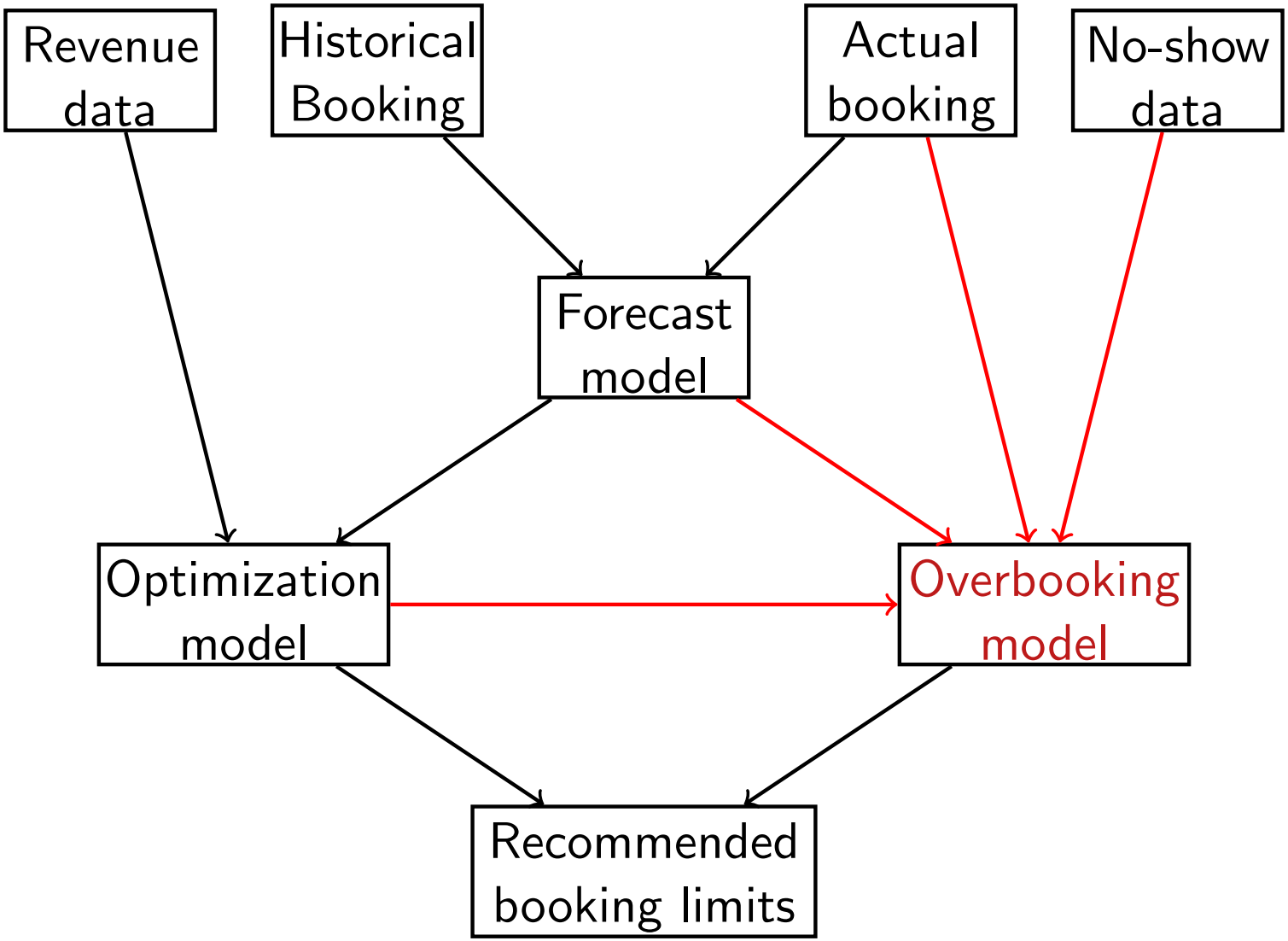


Figure 1: Third-generation airline RMS (Barnhart et al., 2003)

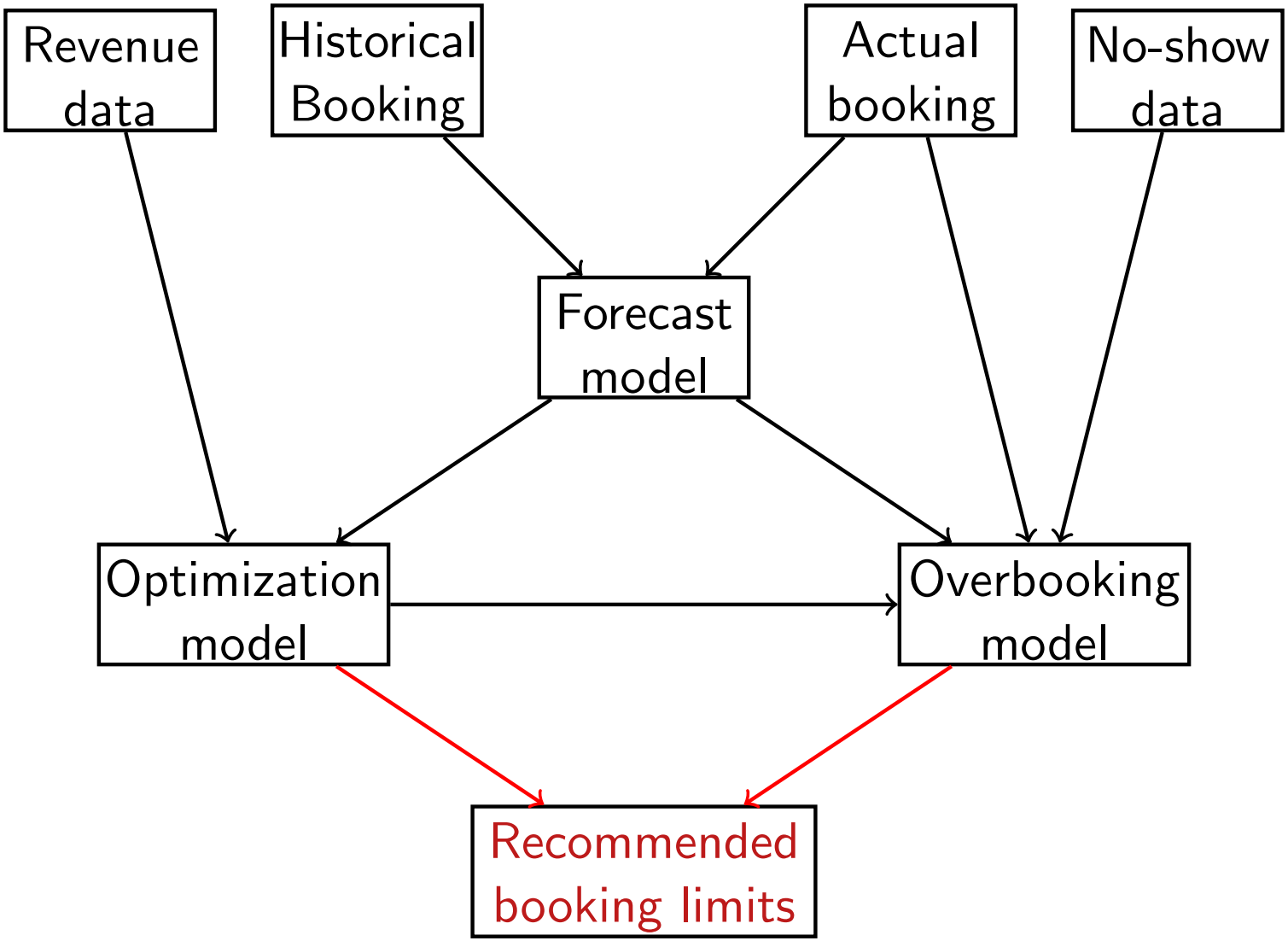


Figure 1: Third-generation airline RMS (Barnhart et al., 2003)

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Main problematics of RM in airlines

Overbooking

No-show rates average 5%-25% of final predeparture bookings. Hence, overbooking is a trade off between revenue losses of *spoiled seats* and costs of *denied boarding*.

Seat inventory control

SIC is a trade off between revenue losses of excessive number of low-fare seats and empty seats. Models go from the initial *expected marginal seat revenue* (EMSR) to *optimal booking limits* by *Origin-Destination-fare (ODF) control*.

Pricing

There is a natural duality between price and seat allocation decisions (McGill and Van Ryzin, 1999). *Bid-prices* can replace multiple booking limits and complex nesting schemes.

Barnhart, C., Belobaba, P., and Odoni, A. (2003). Applications of operations research in the air transport industry. *Transportation science*, 37(4):368–391.

Beckmann, M. (1958). Decision and team problems in airline reservations. *Econometrica: Journal of the Econometric Society*, pages 134–145.

Belobaba, P. (1989). Or practice—application of a probabilistic decision model to airline seat inventory control. *Operations Research*, 37(2):183–197.

Curry, R. (1990). Optimal airline seat allocation with fare classes nested by origins and destinations. *Transportation Science*, 24(3):193–204.

Littlewood, K. (1972). Forecasting and control of passenger bookings. *AGIFORS Proceedings*, 12(2):95–117.

McGill, J. and Van Ryzin, G. (1999). Revenue management: Research overview and prospects. *Transportation science*, 33(2):233–256.

Rothstein, M. (1971). An airline overbooking model. *Transportation Science*, 5(2):180–192.

Smith, B., Leimkuhler, J., and Darrow, R. (1992). Yield management at american airlines. *Interfaces*, 22(1):8–31.

Layout

1 Introduction
- Characteristics of the airlines industry
- A definition and some numbers
- A historical perspective
- The main components of RM

2 Overbooking
- Overview of the problem
- Initial model of Beckmann (1958)
- Dynamic problem of Rothstein (1971)

3 Seat inventory control
- Littlewood (1972): from overbooking to SIC
- Nested fare class
- Origin-Destination Fare (ODF)
- Network revenue management
- Bid-Price control for Network Revenue Management

4 Evidences of RM
- The example of Ryanair

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Overview of overbooking

Dilemma of overbooking

Overbooking is the fact of overselling seats to compensate losses from cancellations and *no-shows*. But *denied boarding* has a cost as well as *spoiled seats*.
This Civil Aeronautics Board recognized and controlled this practice.

- Booking are accepted up to 330 days in advance
- Cancellations or new reservations can occur during this period
- Even with a full booked flight, *no-shows* can lead to empty seats
- *Standby* may appear at flight time
- It may possible to reassign excess passengers

Layout

1. Introduction
   - Characteristics of the airlines industry
   - A definition and some numbers
   - A historical perspective
   - The main components of RM
2. Overbooking
   - Overview of the problem
   - Initial model of Beckmann (1958)
   - Dynamic problem of Rothstein (1971)
3. Seat inventory control
   - Littlewood (1972): from overbooking to SIC
   - Nested fare class
   - Origin-Destination Fare (ODF)
   - Network revenue management
   - Bid-Price control for Network Revenue Management
4. Evidences of RM
   - The example of Ryanair

Initial model of Beckmann (1958)

A simple model

The model of Beckmann (1958) yields the *booking level* for reservations that minimizes expected costs.

$$a \int_0^{x-c} (x - \kappa - c) dP(\kappa \mid x) + b \int_{x-c}^{x} \mathcal{N}(\kappa) dP(\kappa \mid x) \qquad (1)$$

- The *booking level*, x, is given
- Model assume a unique fare
- Model does not give any *probability of denied boarding*.

Initial model of Beckmann (1958)

A simple model

The model of Beckmann (1958) yields the *booking level* for reservations that minimizes expected costs.

$$a \int_0^{x-c} (x - \kappa - c) dP(\kappa \mid x) + b \int_{x-c}^{x} \mathcal{N}(\kappa) dP(\kappa \mid x) \qquad (1)$$

- The *booking level*, x, is given
- Model assume a unique fare
- Model does not give any *probability of denied boarding*.

Initial model of Beckmann (1958)

A simple model

The model of Beckmann (1958) yields the *booking level* for reservations that minimizes expected costs.

$$a \int_0^{x-c} (x - \kappa - c) dP(\kappa \mid x) + b \int_{x-c}^{x} \mathcal{N}(\kappa) dP(\kappa \mid x) \qquad (1)$$

- The *booking level*, x, is given
- Model assume a unique fare
- Model does not give any *probability of denied boarding*.

Initial model of Beckmann (1958)

A simple model

The model of Beckmann (1958) yields the *booking level* for reservations that minimizes expected costs.

$$a \int_0^{x-c} (x - \kappa - c) dP(\kappa \mid x) + b \int_{x-c}^{x} \mathcal{N}(\kappa) dP(\kappa \mid x) \qquad (1)$$

- The *booking level*, x, is given
- Model assume a unique fare
- Model does not give any *probability of denied boarding*.

Initial model of Beckmann (1958)

A simple model

The model of Beckmann (1958) yields the *booking level* for reservations that minimizes expected costs.

$$a \int_0^{x-c} (x - \kappa - c) dP(\kappa \mid x) + b \int_{x-c}^{x} \mathcal{N}(\kappa) dP(\kappa \mid x) \qquad (1)$$

- The *booking level*, x, is given
- Model assume a unique fare
- Model does not give any *probability of denied boarding*.

Initial model of Beckmann (1958)

A simple model

The model of Beckmann (1958) yields the *booking level* for reservations that minimizes expected costs.

$$a \int_0^{x-c} (x - \kappa - c) dP(\kappa \mid x) + b \int_{x-c}^{x} \mathcal{N}(\kappa) dP(\kappa \mid x) \qquad (1)$$

- The *booking level*, x, is given
- Model assume a unique fare
- Model does not give any *probability of denied boarding*.

Initial model of Beckmann (1958)

A simple model

The model of Beckmann (1958) yields the *booking level* for reservations that minimizes expected costs.

$$a \int_0^{x-c} (x - \kappa - c) dP(\kappa \mid x) + b \int_{x-c}^{x} \mathcal{N}(\kappa) dP(\kappa \mid x) \qquad (1)$$

- The *booking level*, x, is given
- Model assume a unique fare
- Model does not give any *probability of denied boarding*.

Initial model of Beckmann (1958)

A simple model

The model of Beckmann (1958) yields the *booking level* for reservations that minimizes expected costs.

$$a \int_0^{x-c} (x - \kappa - c) dP(\kappa \mid x) + b \int_{x-c}^{x} \mathcal{N}(\kappa) dP(\kappa \mid x) \qquad (1)$$

- The *booking level*, x, is given
- Model assume a unique fare
- Model does not give any *probability of denied boarding*.

Layout

1. Introduction
   - Characteristics of the airlines industry
   - A definition and some numbers
   - A historical perspective
   - The main components of RM
2. Overbooking
   - Overview of the problem
   - Initial model of Beckmann (1958)
   - Dynamic problem of Rothstein (1971)
3. Seat inventory control
   - Littlewood (1972): from overbooking to SIC
   - Nested fare class
   - Origin-Destination Fare (ODF)
   - Network revenue management
   - Bid-Price control for Network Revenue Management
4. Evidences of RM
   - The example of Ryanair

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

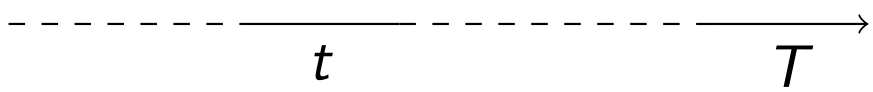


Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

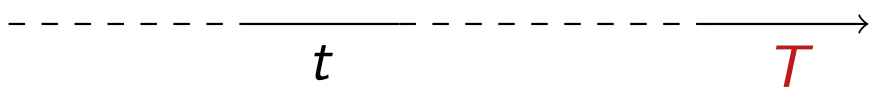


Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

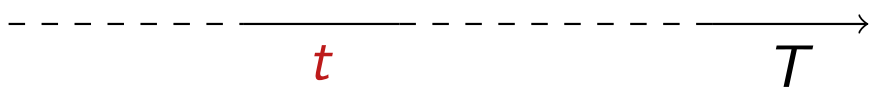


Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

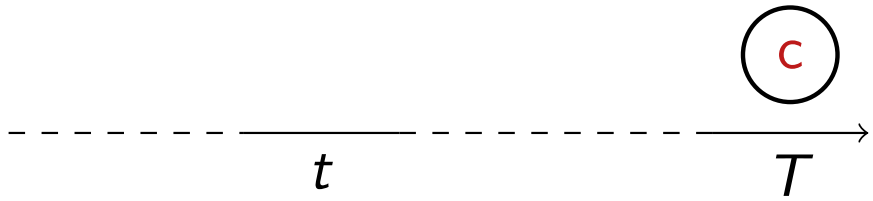


Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

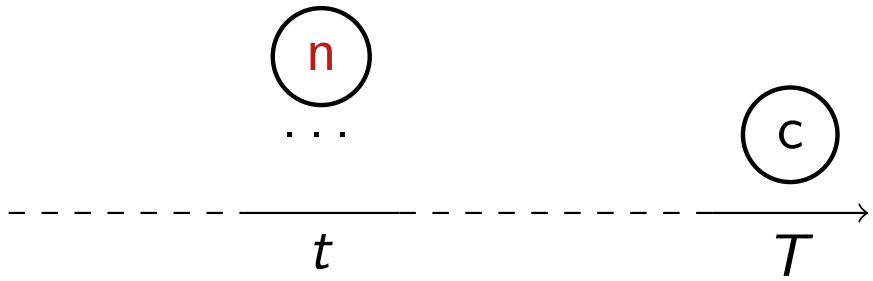


Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

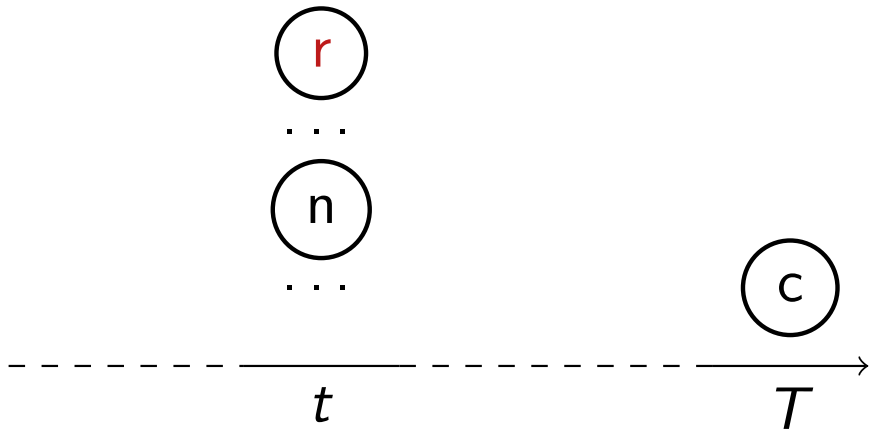


Figure 2: Markovian decision process described by Rothstein (1971)

Dynamic problem of Rothstein (1971)

- Time is discrete
- Cancellation and booking probabilities are time-dependent
- Cancellation and booking processes have no memory
- Define a *denied boarding indicator*, d, so the maximum allowed reservation is $r = d + c$

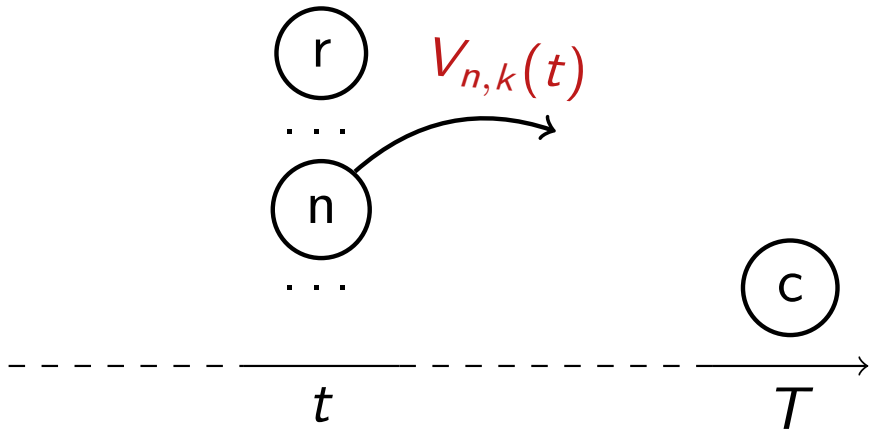


Figure 2: Markovian decision process described by Rothstein (1971)

Layout

1 Introduction
- Characteristics of the airlines industry
- A definition and some numbers
- A historical perspective
- The main components of RM

2 Overbooking
- Overview of the problem
- Initial model of Beckmann (1958)
- Dynamic problem of Rothstein (1971)

3 Seat inventory control
- Littlewood (1972): from overbooking to SIC
- Nested fare class
- Origin-Destination Fare (ODF)
- Network revenue management
- Bid-Price control for Network Revenue Management

4 Evidences of RM
- The example of Ryanair

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \tag{2}$$

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \quad (2)$$

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \quad (2)$$

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \quad (2)$$

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \quad (2)$$

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \quad (2)$$

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \quad (2)$$

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \quad (2)$$

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \tag{2}$$

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \tag{2}$$

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \tag{2}$$

Littlewood (1972): from overbooking to SIC

Overbooking: When prices were regulated, maximizing profit was equivalent to maximize the number of passengers carried by flight.

Seat Inventory Control: When prices are unregulated, maximizing profit leads also to optimize mix of fares.

Single leg Seat Inventory Control

Littlewood (1972) proposed to protect p high-fare seats so that the probability of *denial boarding* for high-fare is bounded by $Pr(D_{high} \geq p)$. The Expected Marginal Seat Revenue (EMSR) rule is the following:

$$f_{low} \geq f_{high} Pr(D_{high} \geq p) \tag{2}$$

Layout

1. Introduction
   - Characteristics of the airlines industry
   - A definition and some numbers
   - A historical perspective
   - The main components of RM
2. Overbooking
   - Overview of the problem
   - Initial model of Beckmann (1958)
   - Dynamic problem of Rothstein (1971)
3. Seat inventory control
   - Littlewood (1972): from overbooking to SIC
   - Nested fare class
   - Origin-Destination Fare (ODF)
   - Network revenue management
   - Bid-Price control for Network Revenue Management
4. Evidences of RM
   - The example of Ryanair

Nested fare class

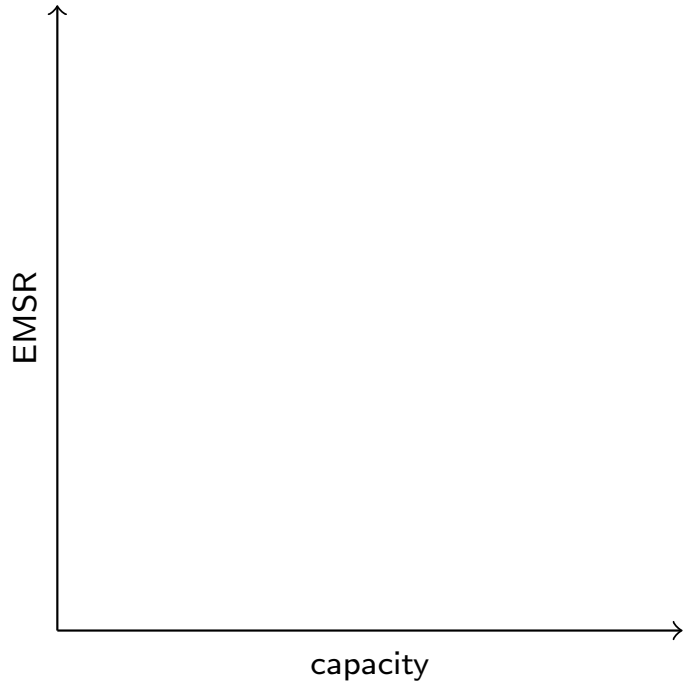


Figure 3: Expected Marginal Seat Revenues for 2-class (Belobaba, 1989)

Nested fare class

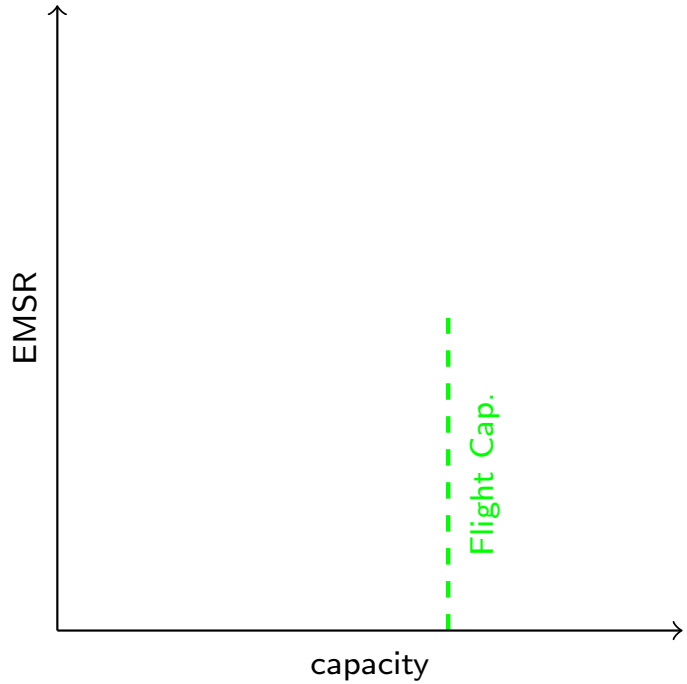


Figure 3: Expected Marginal Seat Revenues for 2-class (Belobaba, 1989)

Nested fare class

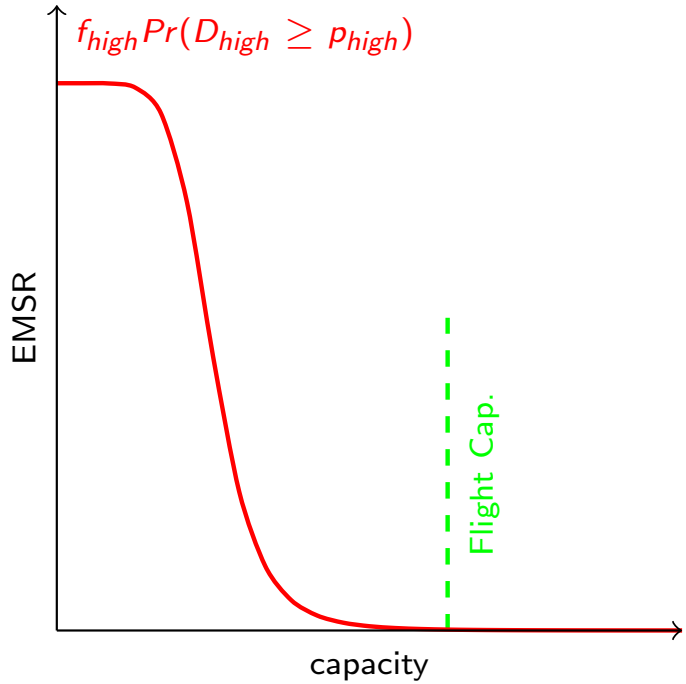


Figure 3: Expected Marginal Seat Revenues for 2-class (Belobaba, 1989)

Nested fare class

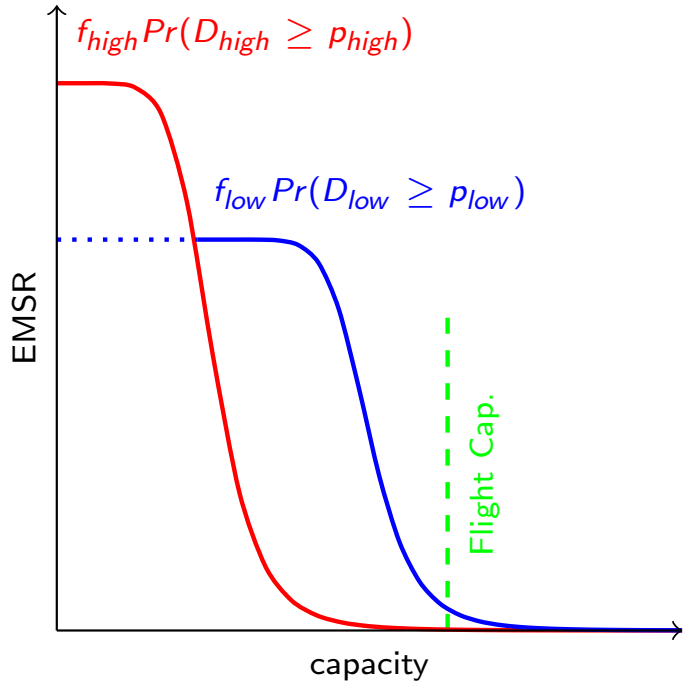


Figure 3: Expected Marginal Seat Revenues for 2-class (Belobaba, 1989)

Nested fare class

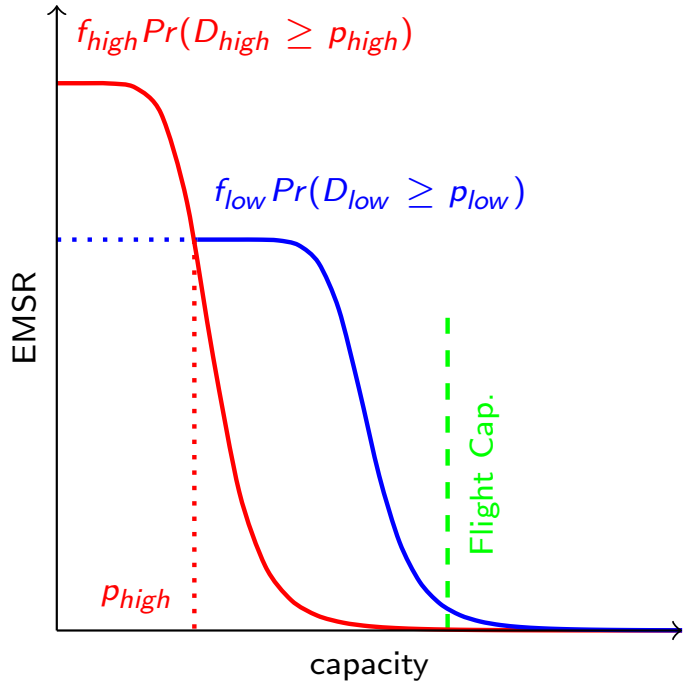
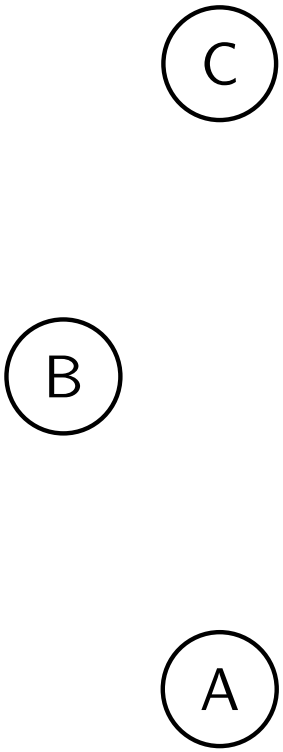


Figure 3: Expected Marginal Seat Revenues for 2-class (Belobaba, 1989)

Layout

1. Introduction
   - Characteristics of the airlines industry
   - A definition and some numbers
   - A historical perspective
   - The main components of RM
2. Overbooking
   - Overview of the problem
   - Initial model of Beckmann (1958)
   - Dynamic problem of Rothstein (1971)
3. Seat inventory control
   - Littlewood (1972): from overbooking to SIC
   - Nested fare class
   - Origin-Destination Fare (ODF)
   - Network revenue management
   - Bid-Price control for Network Revenue Management
4. Evidences of RM
   - The example of Ryanair

Origin-Destination Fare (ODF)



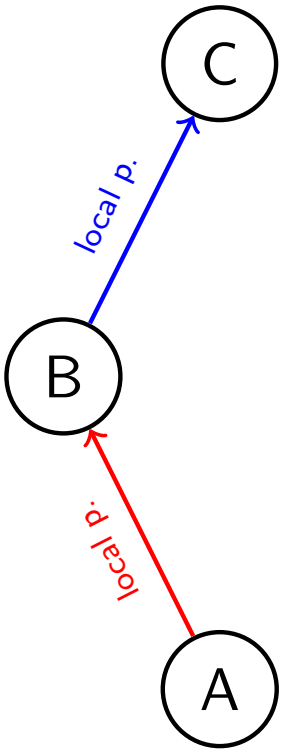
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



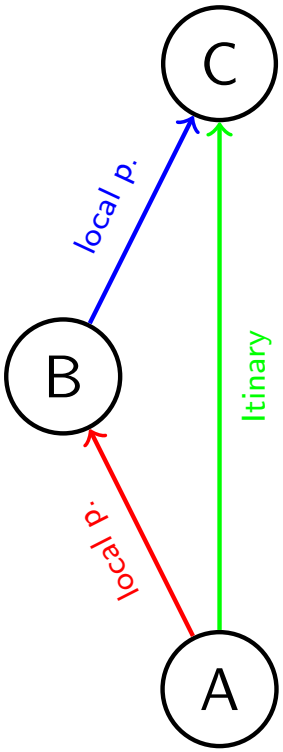
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



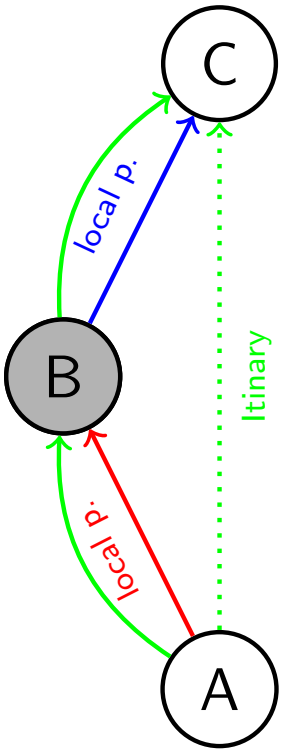
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



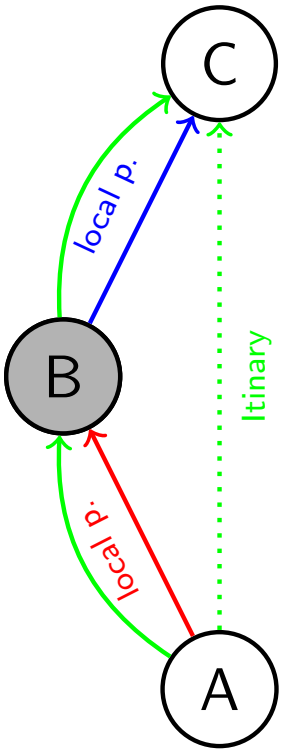
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



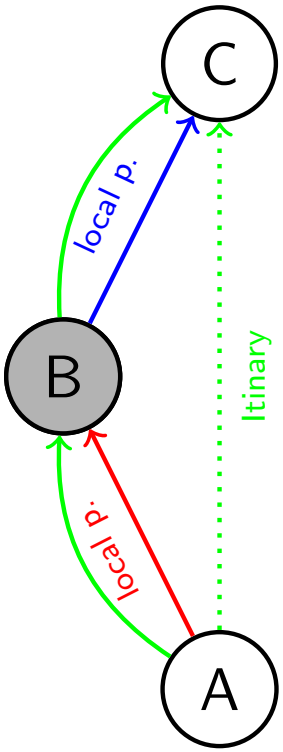
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



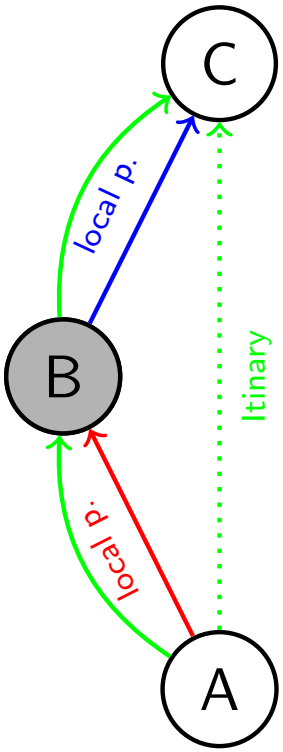
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



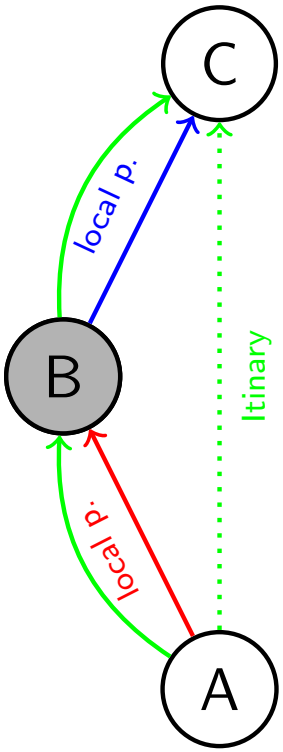
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



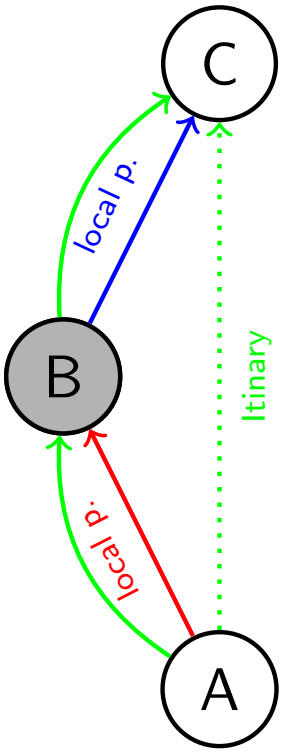
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



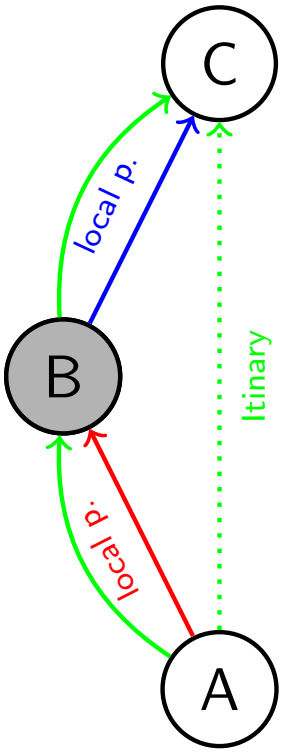
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



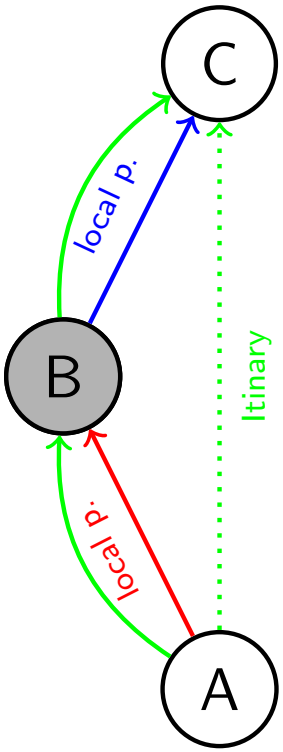
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



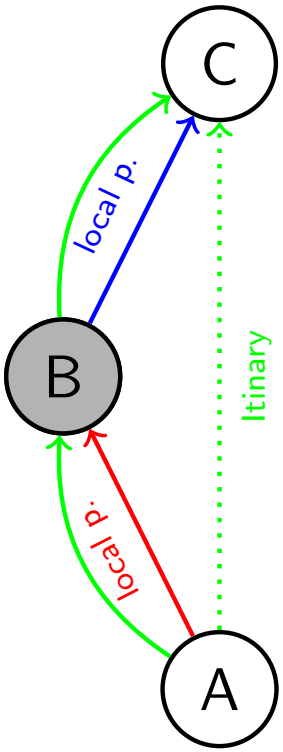
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



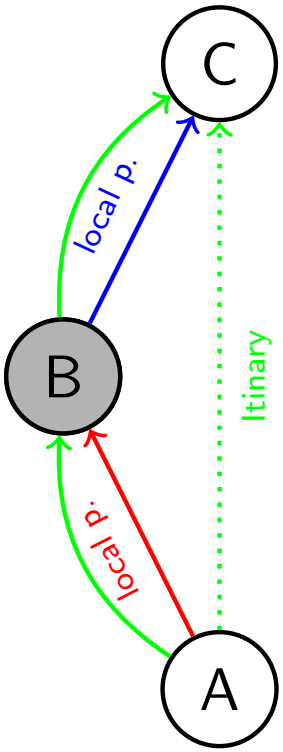
Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Origin-Destination Fare (ODF)



Virtual Nesting

We assume two fares H,L and 2 itineraries AB,AC

Virtual Class	Leg AB	
	Itina.	Fare
Y_1	AB	H
Y_2	AB	L
	AC	H
Y_3	AC	L

Table 1: Virtual class for the leg AB

Layout

1. Introduction
   - Characteristics of the airlines industry
   - A definition and some numbers
   - A historical perspective
   - The main components of RM
2. Overbooking
   - Overview of the problem
   - Initial model of Beckmann (1958)
   - Dynamic problem of Rothstein (1971)
3. Seat inventory control
   - Littlewood (1972): from overbooking to SIC
   - Nested fare class
   - Origin-Destination Fare (ODF)
   - Network revenue management
   - Bid-Price control for Network Revenue Management
4. Evidences of RM
   - The example of Ryanair

Network revenue management

A leg-based seat inventory control system can not discriminate between AB and AC booking on the AB leg: it does not consider itinerary.

Optimal seat allocation with ODF (Curry, 1990)

Curry (1990) proposed the following program to maximize the expected revenue for all itineraries j.

$$\max_{A} \sum_{j} \sum_{k} R_{jk}(A_{jk}) \tag{3}$$

subject to

$$\sum_{j} \sum_{k \in l} A_{jk} \leq C_l \tag{4}$$

Network revenue management

A leg-based seat inventory control system can not discriminate between AB and AC booking on the AB leg: it does not consider itinerary.

Optimal seat allocation with ODF (Curry, 1990)

Curry (1990) proposed the following program to maximize the expected revenue for all itineraries j.

$$\max_{A} \sum_{j} \sum_{k} R_{jk}(A_{jk}) \tag{3}$$

subject to

$$\sum_{j} \sum_{k \in i} A_{jk} \leq C_i \tag{4}$$

Network revenue management

A leg-based seat inventory control system can not discriminate between AB and AC booking on the AB leg: it does not consider itinerary.

Optimal seat allocation with ODF (Curry, 1990)

Curry (1990) proposed the following program to maximize the expected revenue for all itineraries j.

$$\max_{A} \sum_{j} \sum_{k} R_{jk}(A_{jk}) \tag{3}$$

subject to

$$\sum_{j} \sum_{k \in i} A_{jk} \leq C_i \tag{4}$$

Network revenue management

A leg-based seat inventory control system can not discriminate between AB and AC booking on the AB leg: it does not consider itinerary.

Optimal seat allocation with ODF (Curry, 1990)

Curry (1990) proposed the following program to maximize the expected revenue for all itineraries j.

$$\max_{A} \sum_{j} \sum_{k} R_{jk}(A_{jk}) \tag{3}$$

subject to

$$\sum_{j} \sum_{k \in i} A_{jk} \leq C_i \tag{4}$$

Network revenue management

A leg-based seat inventory control system can not discriminate between AB and AC booking on the AB leg: it does not consider itinerary.

Optimal seat allocation with ODF (Curry, 1990)

Curry (1990) proposed the following program to maximize the expected revenue for all itineraries j.

$$\max_{A} \sum_{j} \sum_{k} R_{jk}(A_{jk}) \tag{3}$$

subject to

$$\sum_{j} \sum_{k \in i} A_{jk} \leq C_i \tag{4}$$

Network revenue management

A leg-based seat inventory control system can not discriminate between AB and AC booking on the AB leg: it does not consider itinerary.

Optimal seat allocation with ODF (Curry, 1990)

Curry (1990) proposed the following program to maximize the expected revenue for all itineraries j.

$$\max_{A} \sum_{j} \sum_{k} R_{jk}(A_{jk}) \tag{3}$$

subject to

$$\sum_{j} \sum_{k \in i} A_{jk} \leq C_i \tag{4}$$

Network revenue management

A leg-based seat inventory control system can not discriminate between AB and AC booking on the AB leg: it does not consider itinerary.

Optimal seat allocation with ODF (Curry, 1990)

Curry (1990) proposed the following program to maximize the expected revenue for all itineraries j.

$$\max_{A} \sum_{j} \sum_{k} R_{jk}(A_{jk}) \tag{3}$$

subject to

$$\sum_{j} \sum_{k \in i} A_{jk} \leq C_i \tag{4}$$

Network revenue management

A leg-based seat inventory control system can not discriminate between AB and AC booking on the AB leg: it does not consider itinerary.

Optimal seat allocation with ODF (Curry, 1990)

Curry (1990) proposed the following program to maximize the expected revenue for all itineraries j.

$$\max_{A} \sum_{j} \sum_{k} R_{jk}(A_{jk}) \tag{3}$$

subject to

$$\sum_{j} \sum_{k \in i} A_{jk} \leq C_i \tag{4}$$

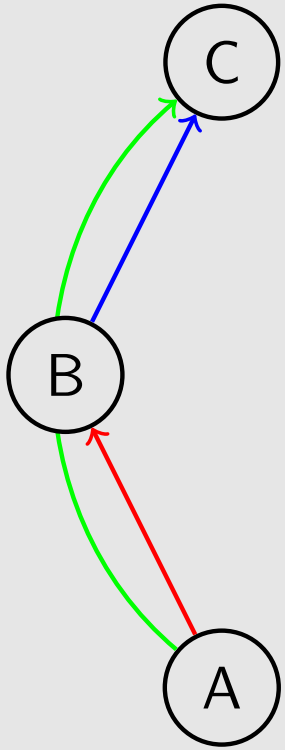
Virtual Nesting

We optimize over all legs with new fare class

Network RM

We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting

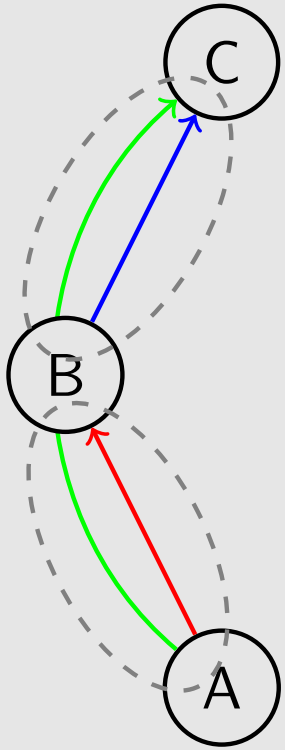


We optimize over all legs with new fare class

Network RM

We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting

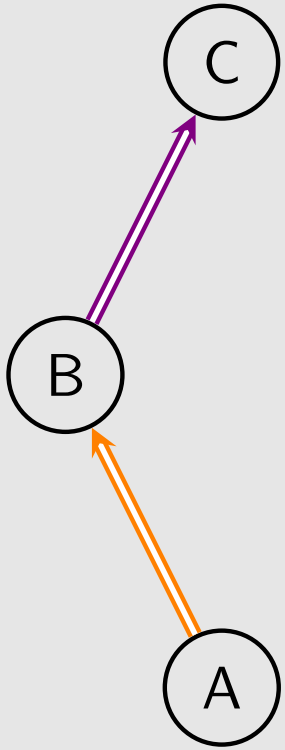


We optimize over all legs with new fare class

Network RM

We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting

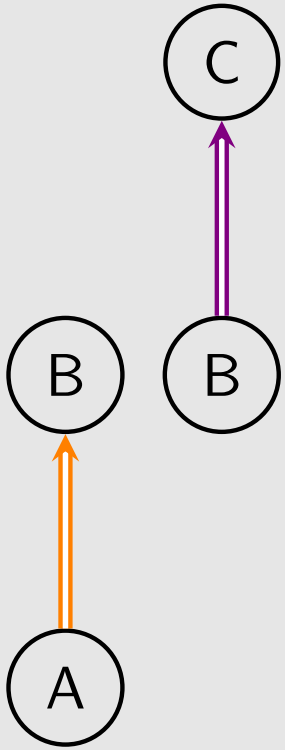


We optimize over all legs with new fare class

Network RM

We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting

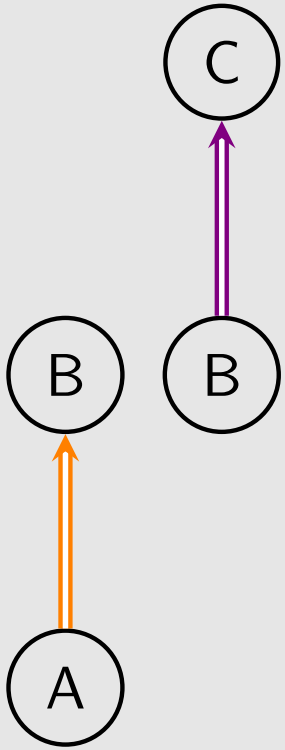


We optimize over all legs with new fare class

Network RM

We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting

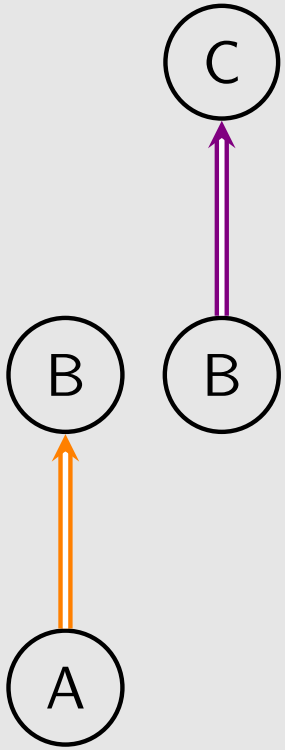


We optimize over all legs with new fare class

Network RM

We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting

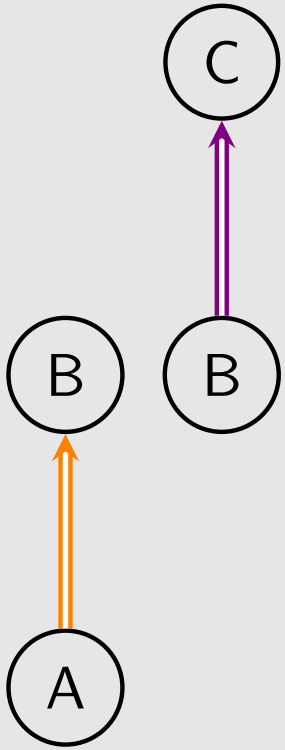


We optimize over all legs with new fare class

Network RM

We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting

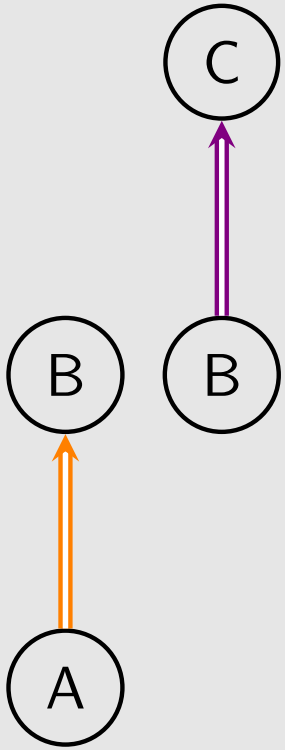


We optimize over all legs with new fare class

Network RM

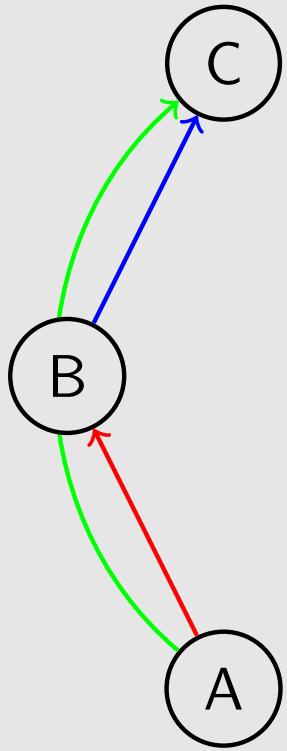
We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting



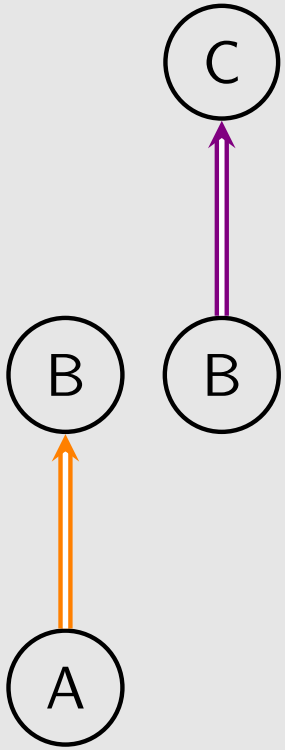
We optimize over all legs with new fare class

Network RM



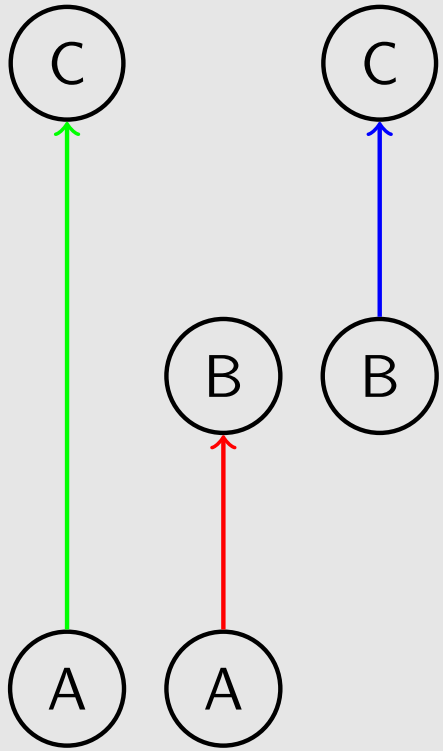
We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting



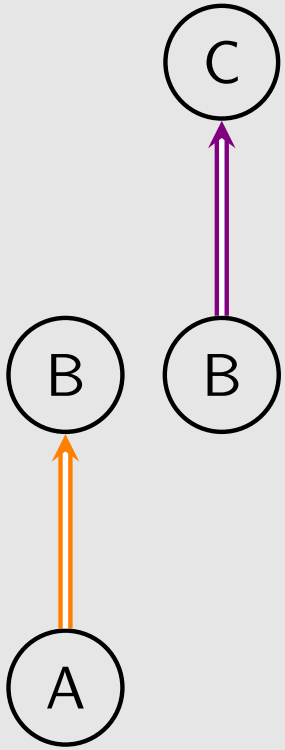
We optimize over all legs with new fare class

Network RM



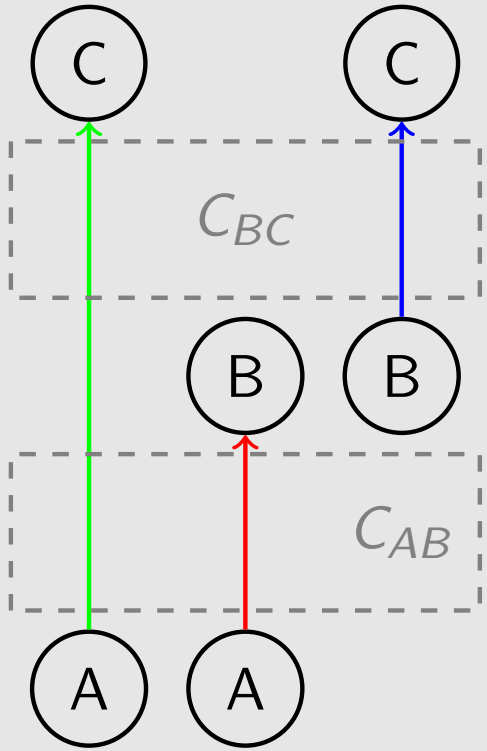
We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting



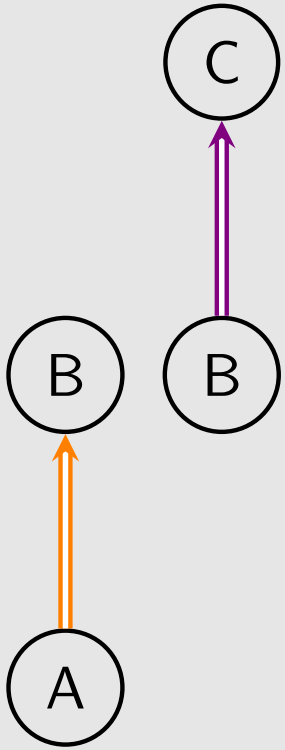
We optimize over all legs with new fare class

Network RM



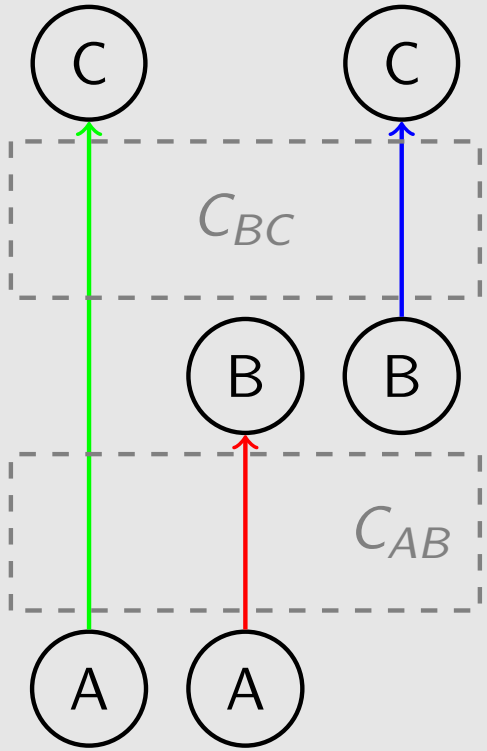
We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting



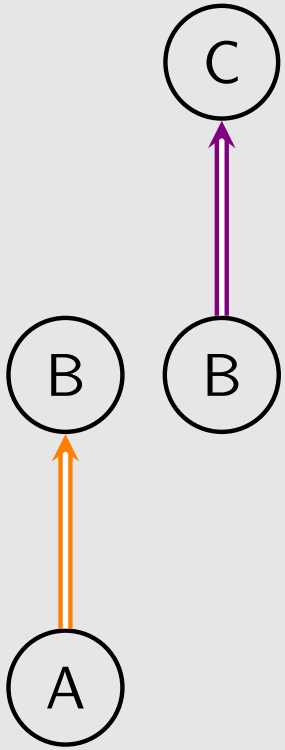
We optimize over all legs with new fare class

Network RM



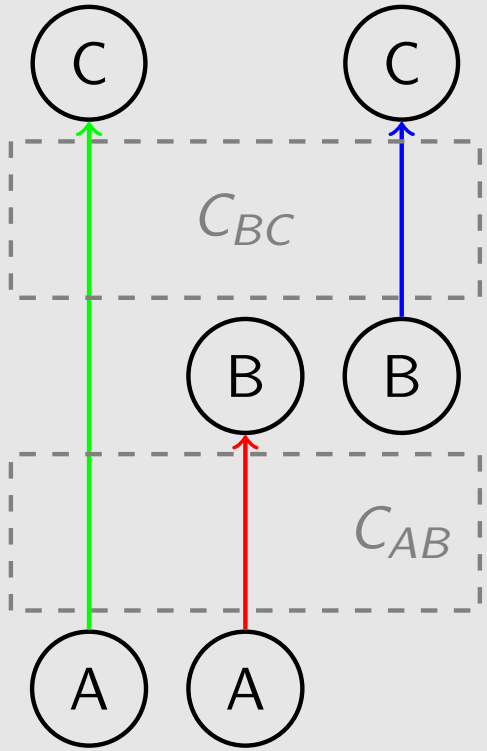
We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting



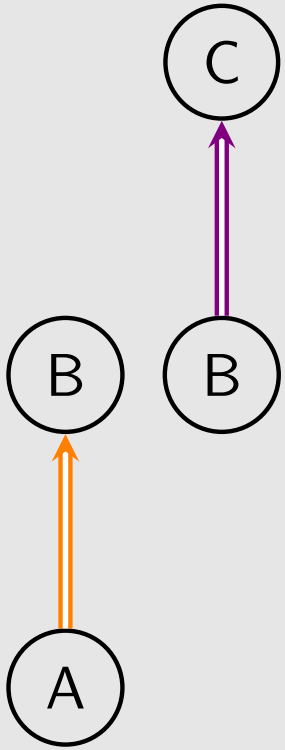
We optimize over all legs with new fare class

Network RM



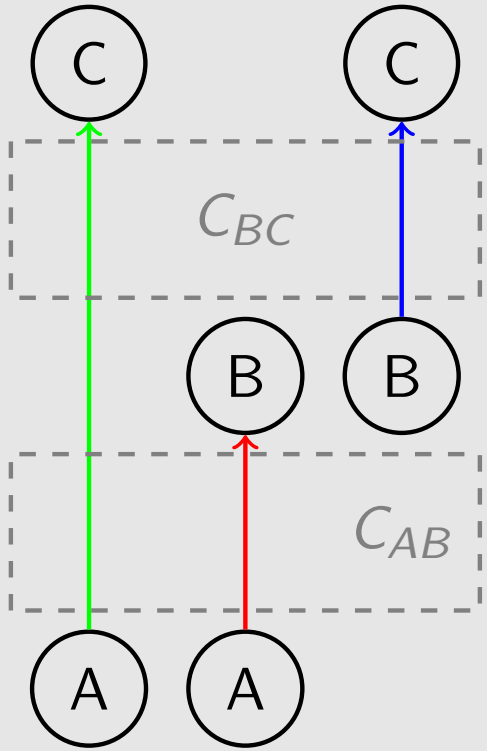
We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting



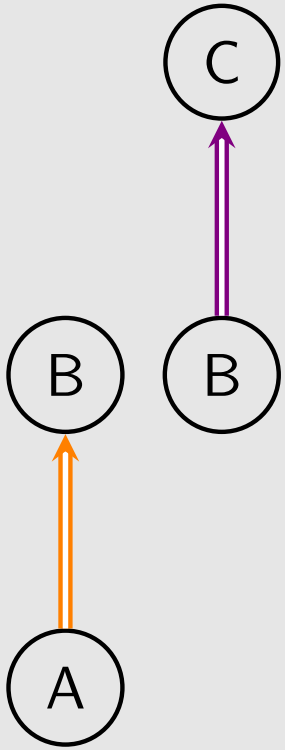
We optimize over all legs with new fare class

Network RM



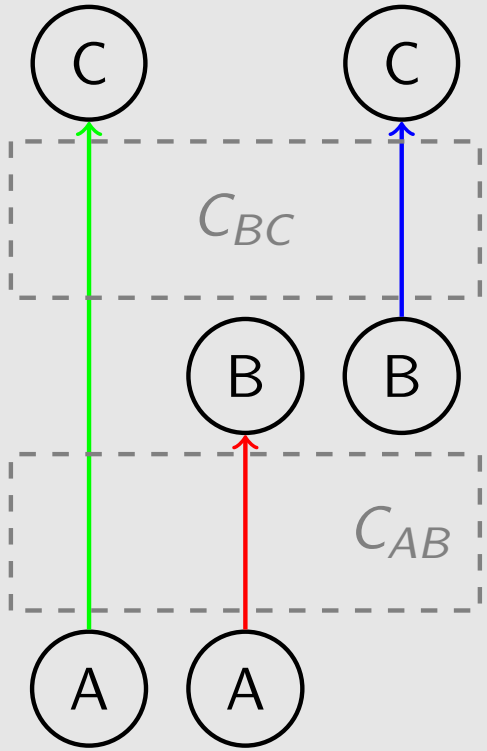
We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Virtual Nesting



We optimize over all legs with new fare class

Network RM



We optimize over all O-Ds with constraint per leg and fares nesting per O-D

Layout

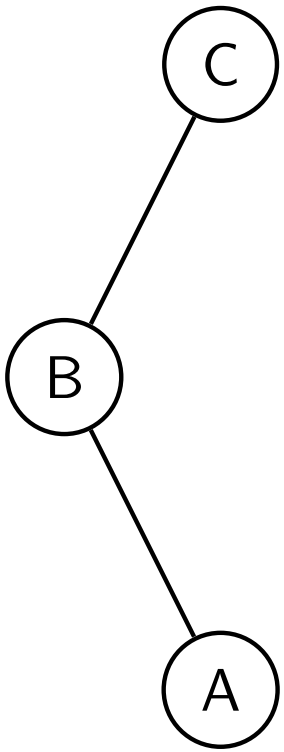
1 Introduction
- Characteristics of the airlines industry
- A definition and some numbers
- A historical perspective
- The main components of RM

2 Overbooking
- Overview of the problem
- Initial model of Beckmann (1958)
- Dynamic problem of Rothstein (1971)

3 Seat inventory control
- Littlewood (1972): from overbooking to SIC
- Nested fare class
- Origin-Destination Fare (ODF)
- Network revenue management
- Bid-Price control for Network Revenue Management

4 Evidences of RM
- The example of Ryanair

Bid-Price control for Network Revenue Management



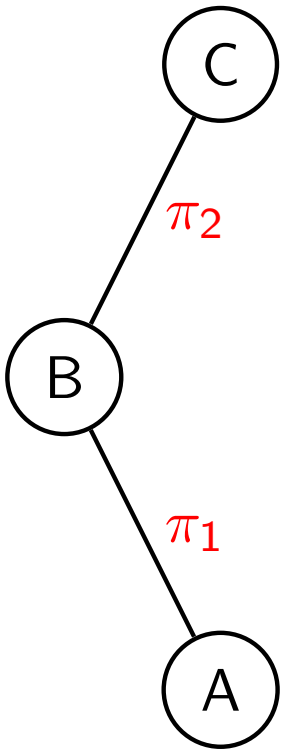
Bid-price

We accept a fare only when R exceeds the *opportunity cost* of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 1 & R_j \geq \sum_j \pi_j(X_j) \\ & X_j \leq C_t \\ 0 & \text{otherwise} \end{cases}$$

- Similair to dual prices of (4).
- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing π (LP,Prorated EMSR,...)

Bid-Price control for Network Revenue Management



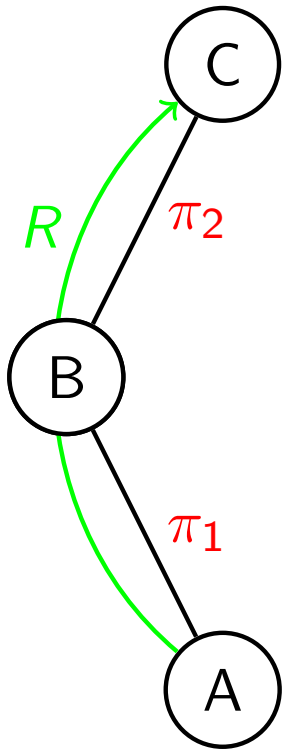
Bid-price

We accept a fare only when R exceeds the *opportunity cost* of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 1 & R_j \geq \sum_j \pi_j(X_j) \\ & X_j \leq C_t \\ 0 & \text{otherwise} \end{cases}$$

- Similair to dual prices of (4).
- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing π (LP,Prorated EMSR,...)

Bid-Price control for Network Revenue Management



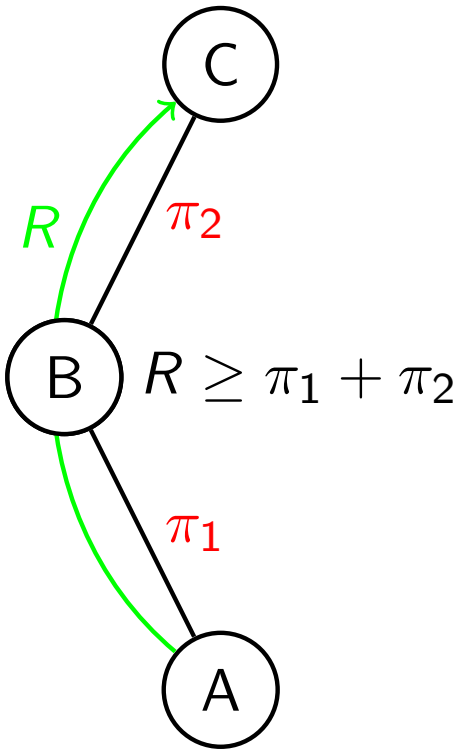
Bid-price

We accept a fare only when R exceeds the *opportunity cost* of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 1 & R_j \geq \sum_j \pi_j(X_j) \\ & X_j \leq C_t \\ 0 & \text{otherwise} \end{cases}$$

- Similair to dual prices of (4).
- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing π (LP,Prorated EMSR,...)

Bid-Price control for Network Revenue Management



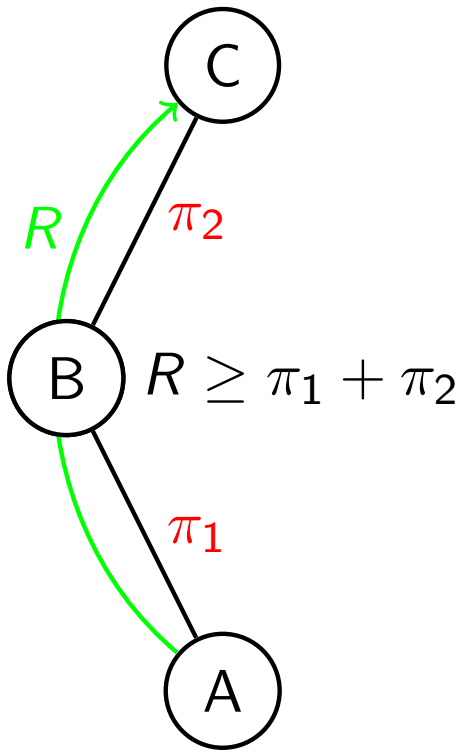
Bid-price

We accept a fare only when R exceeds the *opportunity cost* of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 1 & R_j \geq \sum_j \pi_j(X_j) \\ & X_j \leq C_t \\ 0 & \text{otherwise} \end{cases}$$

- Similair to dual prices of (4).
- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing π (LP,Prorated EMSR,...)

Bid-Price control for Network Revenue Management



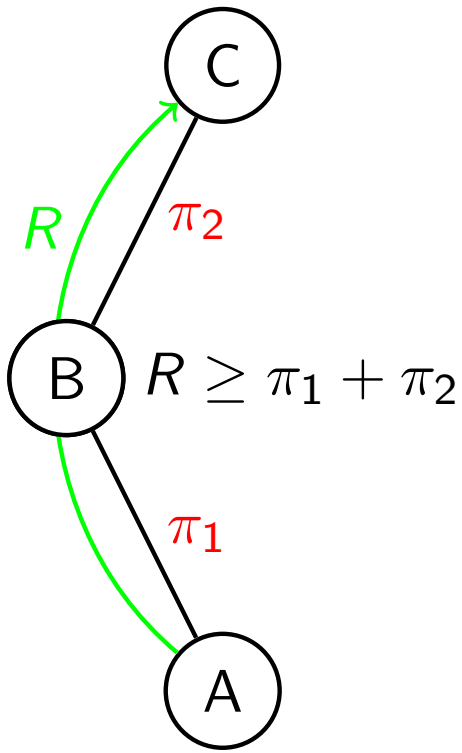
Bid-price

We accept a fare only when R exceeds the *opportunity cost* of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 1 & R_j \geq \sum_j \pi_j(X_j) \\ & X_j \leq C_t \\ 0 & \text{otherwise} \end{cases}$$

- Similair to dual prices of (4).
- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing π (LP,Prorated EMSR,...)

Bid-Price control for Network Revenue Management



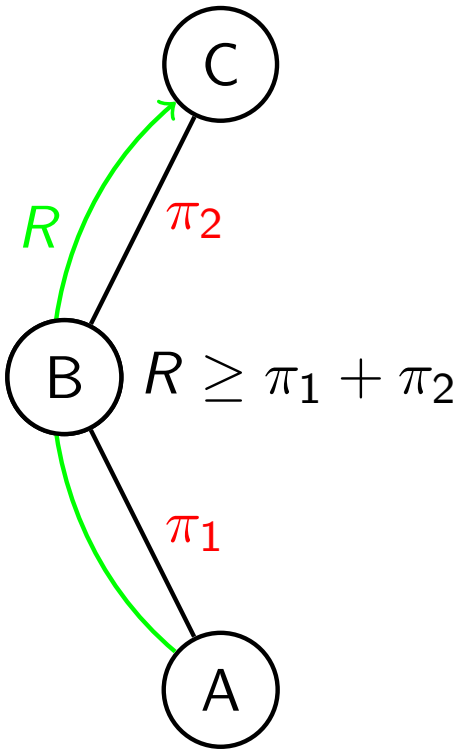
Bid-price

We accept a fare only when R exceeds the *opportunity cost* of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 1 & R_j \geq \sum_j \pi_j(X_j) \\ & X_j \leq C_t \\ 0 & \text{otherwise} \end{cases}$$

- Similair to dual prices of (4).
- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing π (LP,Prorated EMSR,...)

Bid-Price control for Network Revenue Management



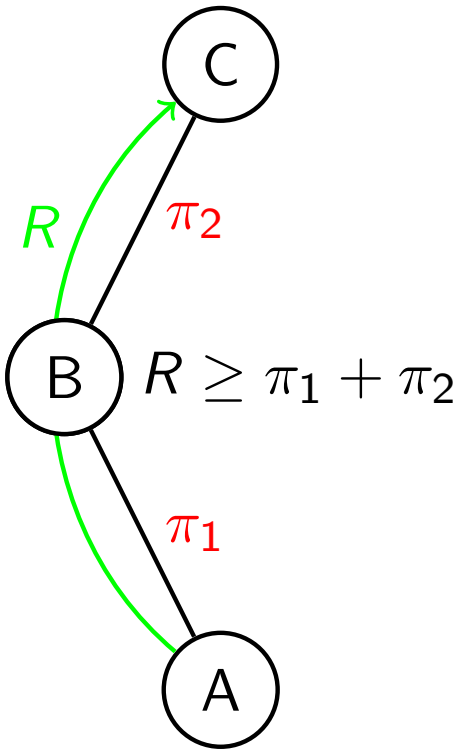
Bid-price

We accept a fare only when R exceeds the *opportunity cost* of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 1 & R_j \geq \sum_j \pi_j(X_j) \\ & X_j \leq C_t \\ 0 & \text{otherwise} \end{cases}$$

- Similair to dual prices of (4).
- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing π (LP,Prorated EMSR,...)

Bid-Price control for Network Revenue Management



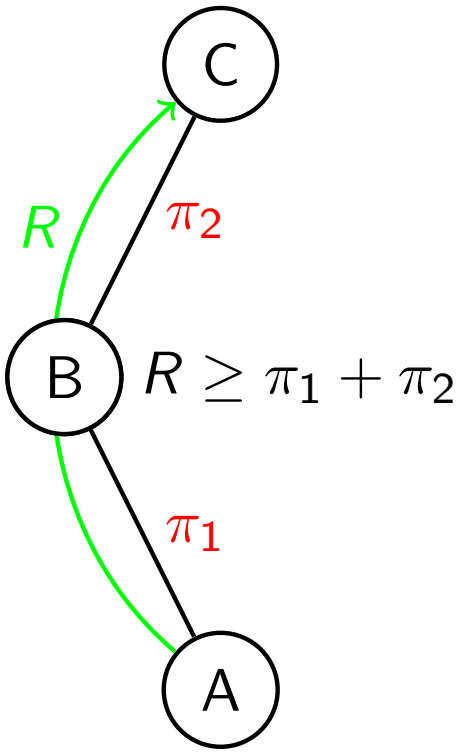
Bid-price

We accept a fare only when R exceeds the *opportunity cost* of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 1 & R_j \geq \sum_j \pi_j(X_j) \\ & X_j \leq C_t \\ 0 & \text{otherwise} \end{cases}$$

- Similair to dual prices of (4).
- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing π (LP,Prorated EMSR,...)

Bid-Price control for Network Revenue Management



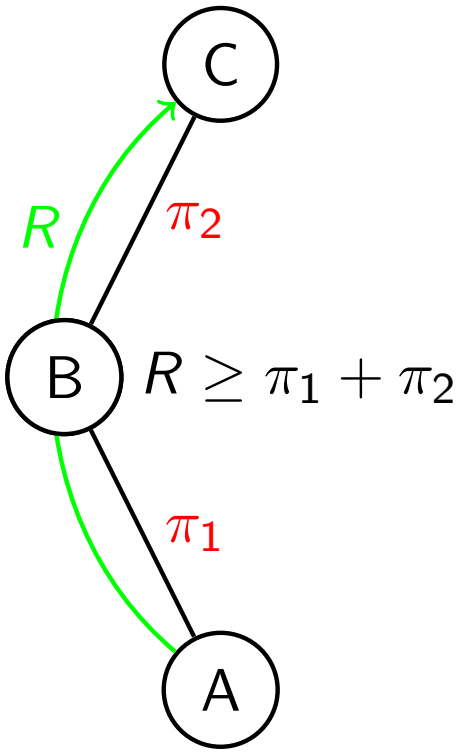
Bid-price

We accept a fare only when R exceeds the *opportunity cost* of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 1 & R_j \geq \sum_j \pi_j(X_j) \\ & X_j \leq C_t \\ 0 & \text{otherwise} \end{cases}$$

- Similair to dual prices of (4).
- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing π (LP,Prorated EMSR,...)

Bid-Price control for Network Revenue Management



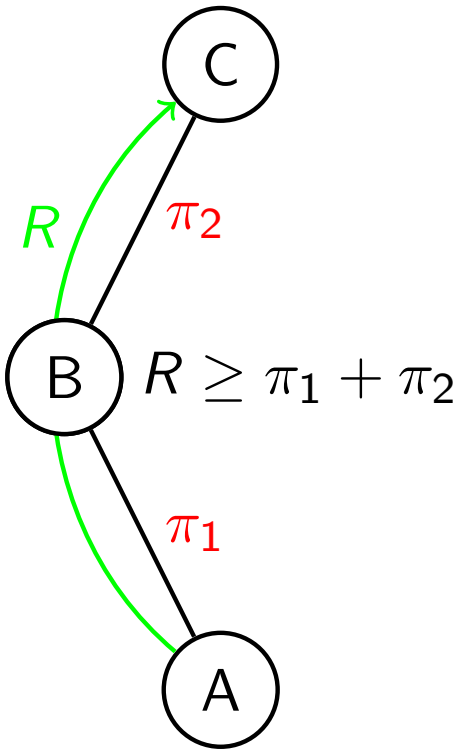
Bid-price

We accept a fare only when R exceeds the *opportunity cost* of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 1 & R_j \geq \sum_j \pi_j(X_j) \\ & X_j \leq C_t \\ 0 & \text{otherwise} \end{cases}$$

- Similair to dual prices of (4).
- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing π (LP,Prorated EMSR,...)

Bid-Price control for Network Revenue Management



Bid-price

We accept a fare only when R exceeds the *opportunity cost* of the reduction in leg capacities.

$$u_{jt}(R_j, X_j) = \begin{cases} 1 & R_j \geq \sum_j \pi_j(X_j) \\ & X_j \leq C_t \\ 0 & \text{otherwise} \end{cases}$$

- Similair to dual prices of (4).
- Suboptimal when $\pi_j \geq \sum_{i \in j} \pi_i$
- How computing π (LP,Prorated EMSR,...)

Layout

1. Introduction
   - Characteristics of the airlines industry
   - A definition and some numbers
   - A historical perspective
   - The main components of RM
2. Overbooking
   - Overview of the problem
   - Initial model of Beckmann (1958)
   - Dynamic problem of Rothstein (1971)
3. Seat inventory control
   - Littlewood (1972): from overbooking to SIC
   - Nested fare class
   - Origin-Destination Fare (ODF)
   - Network revenue management
   - Bid-Price control for Network Revenue Management
4. Evidences of RM
   - The example of Ryanair

Average price and demand

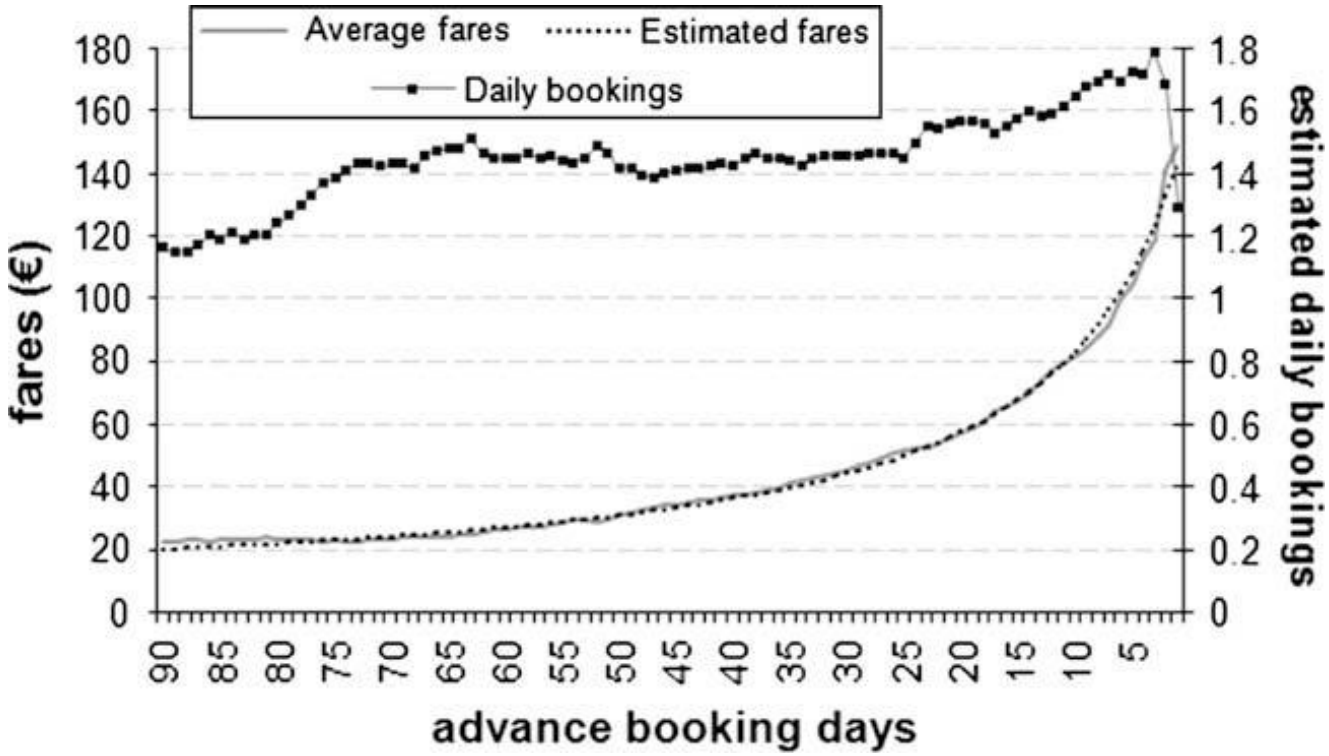


Figure 4: Comparison between the daily average price and the estimated price on CIA–STN route.

Average price and demand

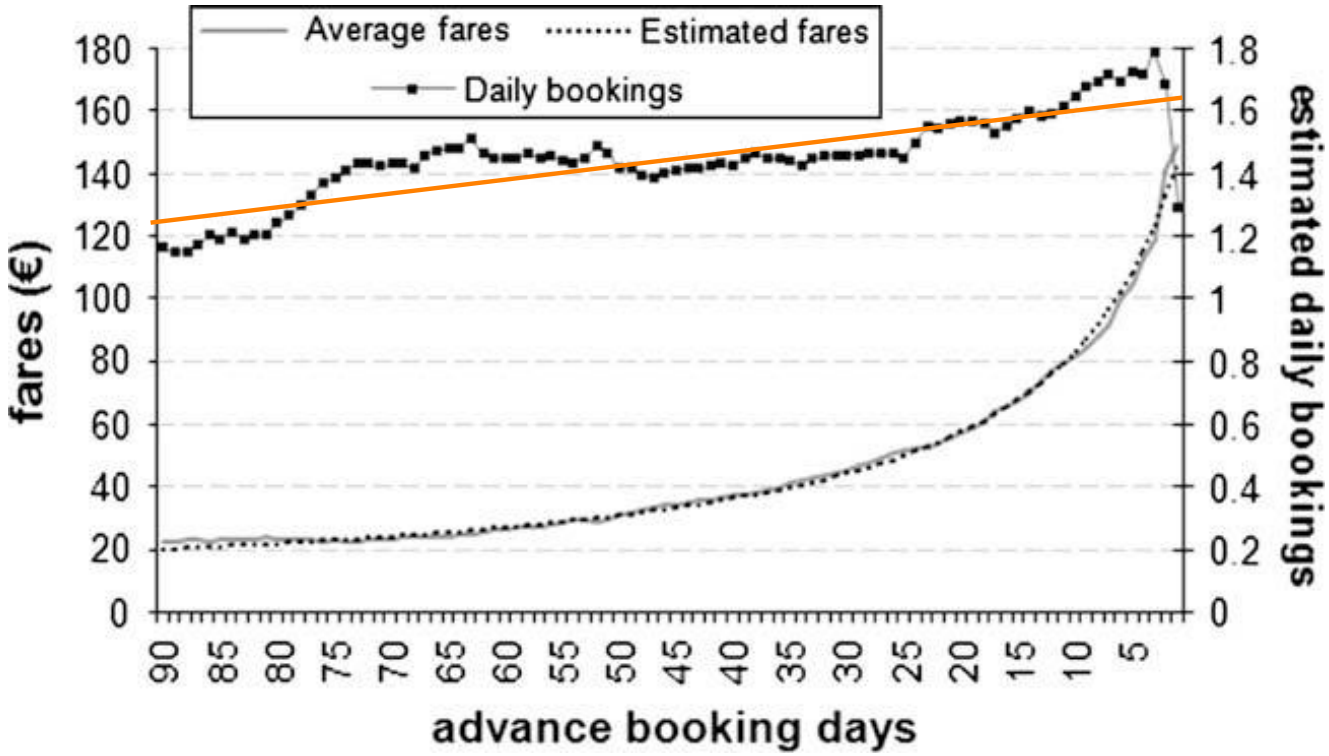


Figure 4: Comparison between the daily average price and the estimated price on CIA–STN route.

Average price and demand

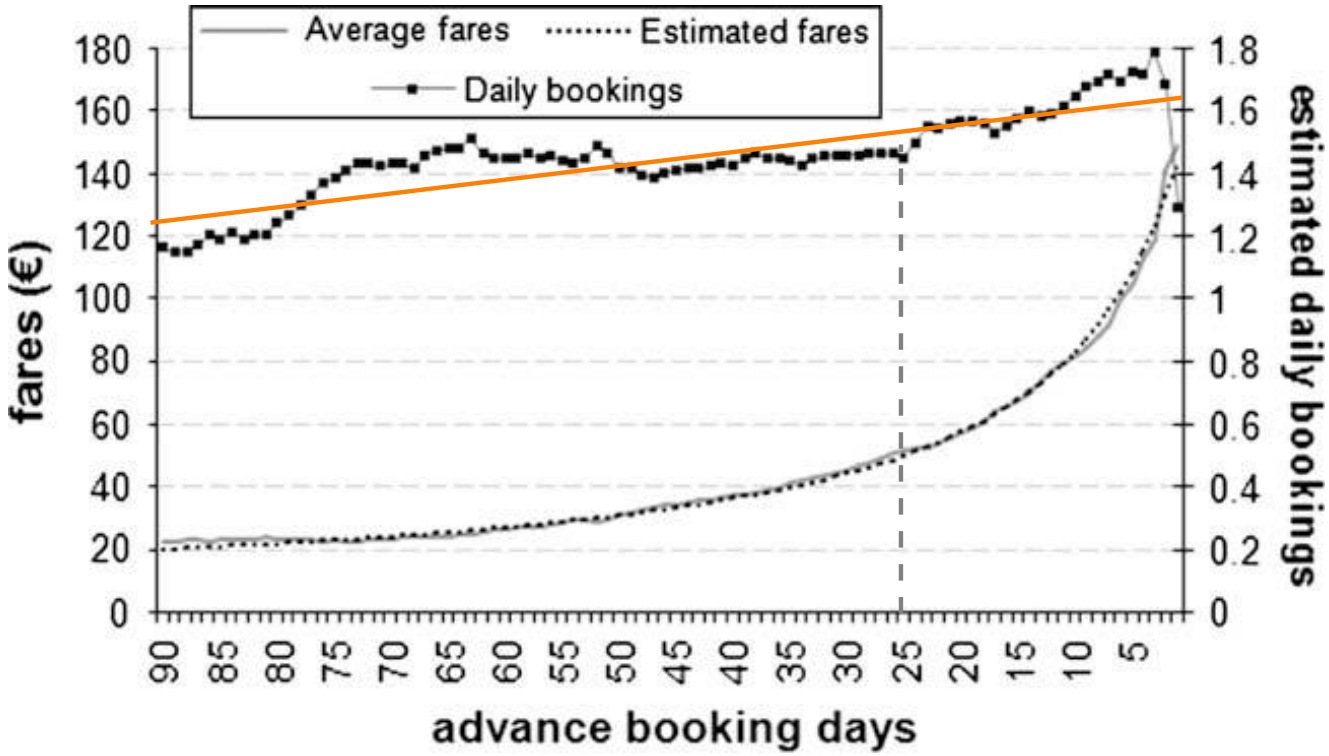


Figure 4: Comparison between the daily average price and the estimated price on CIA–STN route.

Fares according to booking days

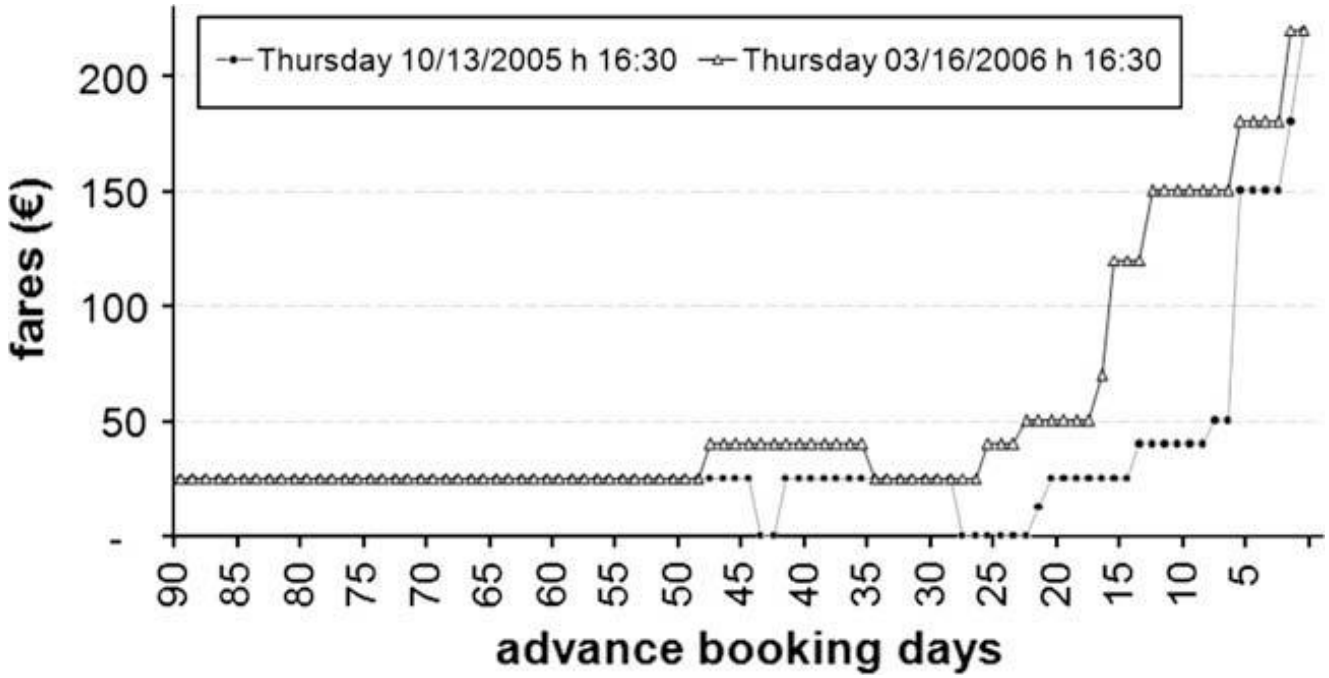


Figure 5: Prices on the *Rome Ciampino–London Stansted* route for two specific dates.

Fares according to booking days

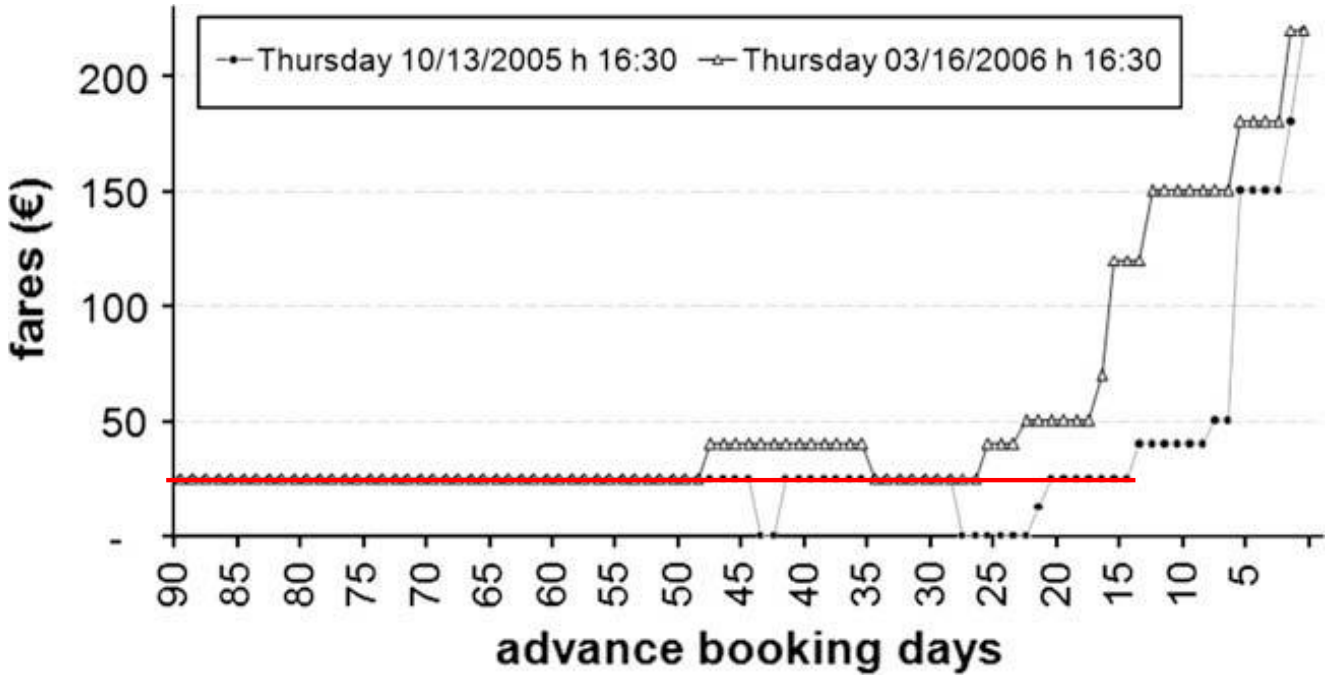


Figure 5: Prices on the *Rome Ciampino–London Stansted* route for two specific dates.

Fares according to booking days

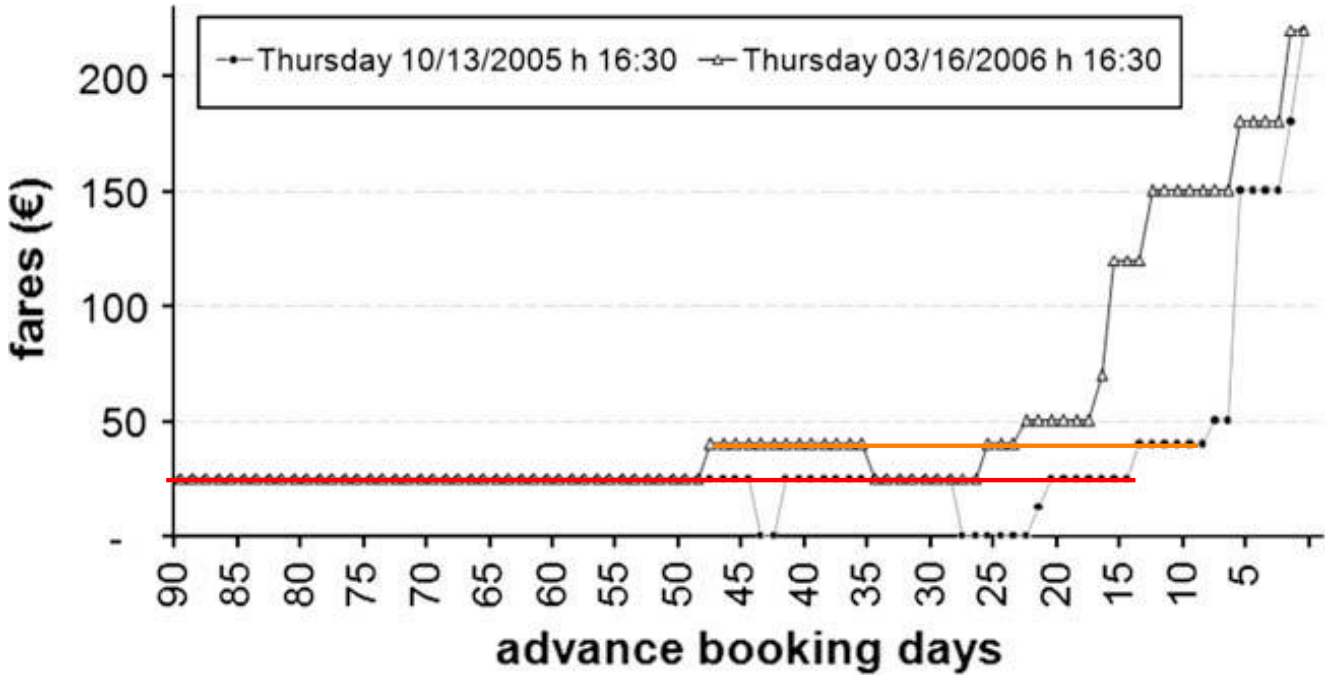


Figure 5: Prices on the *Rome Ciampino–London Stansted* route for two specific dates.

Fares according to booking days

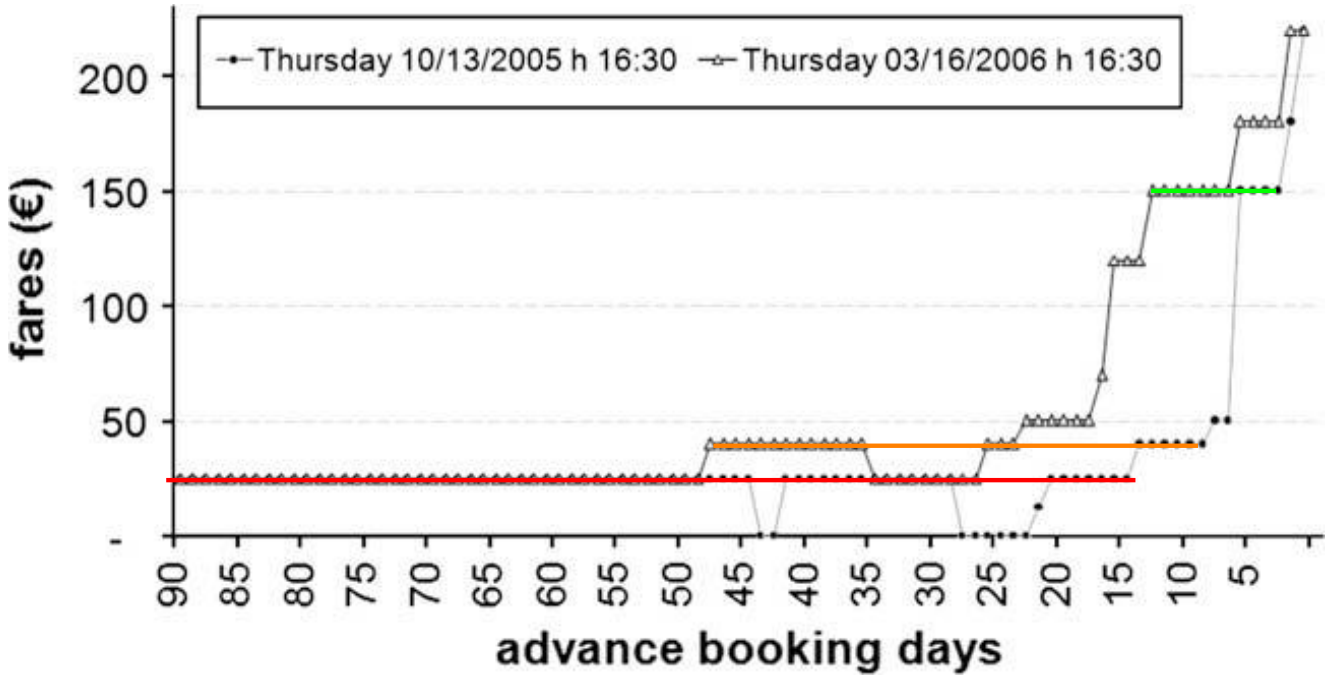


Figure 5: Prices on the *Rome Ciampino–London Stansted* route for two specific dates.

Fares according to booking days

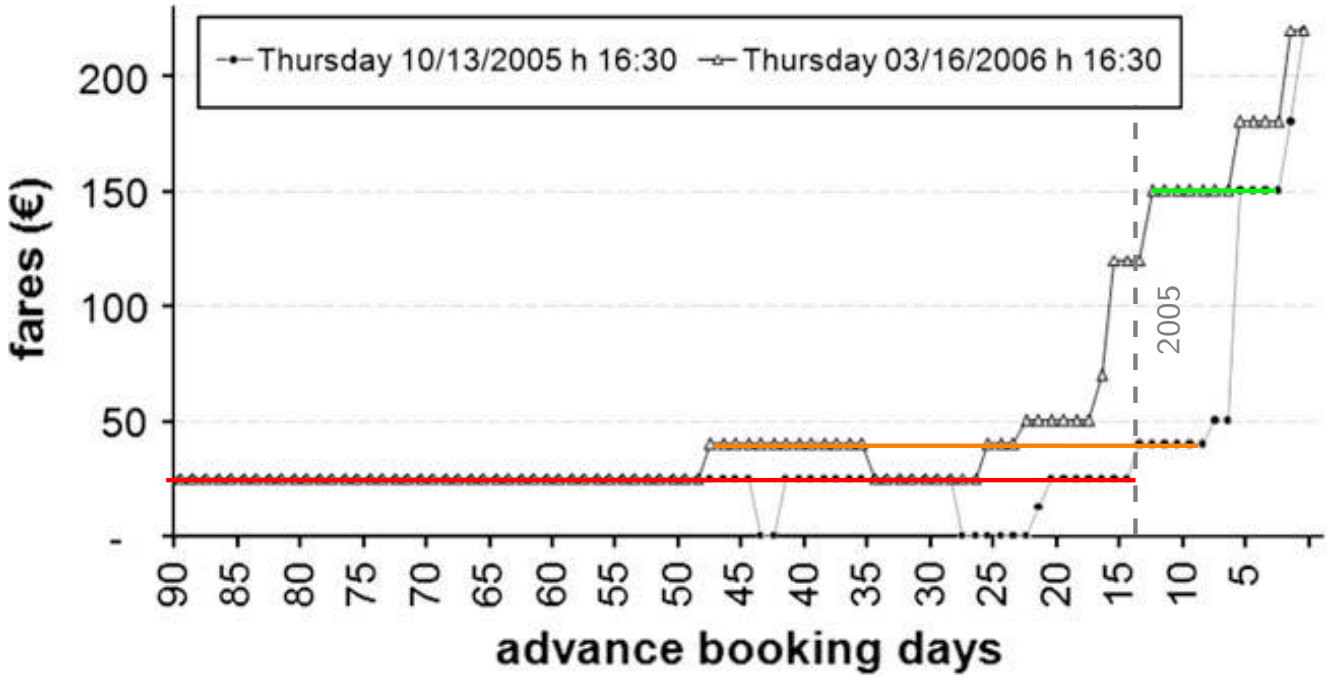


Figure 5: Prices on the *Rome Ciampino–London Stansted* route for two specific dates.

Fares according to booking days

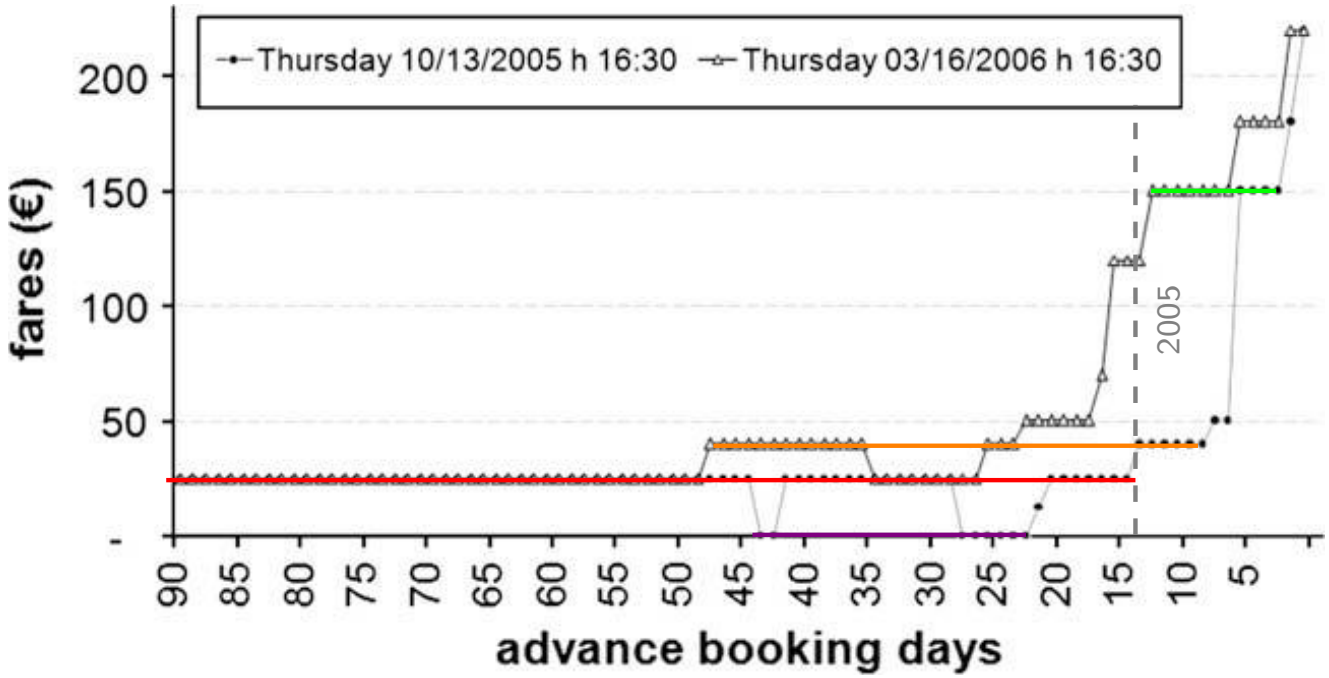


Figure 5: Prices on the *Rome Ciampino–London Stansted* route for two specific dates.

Fares according to booking days

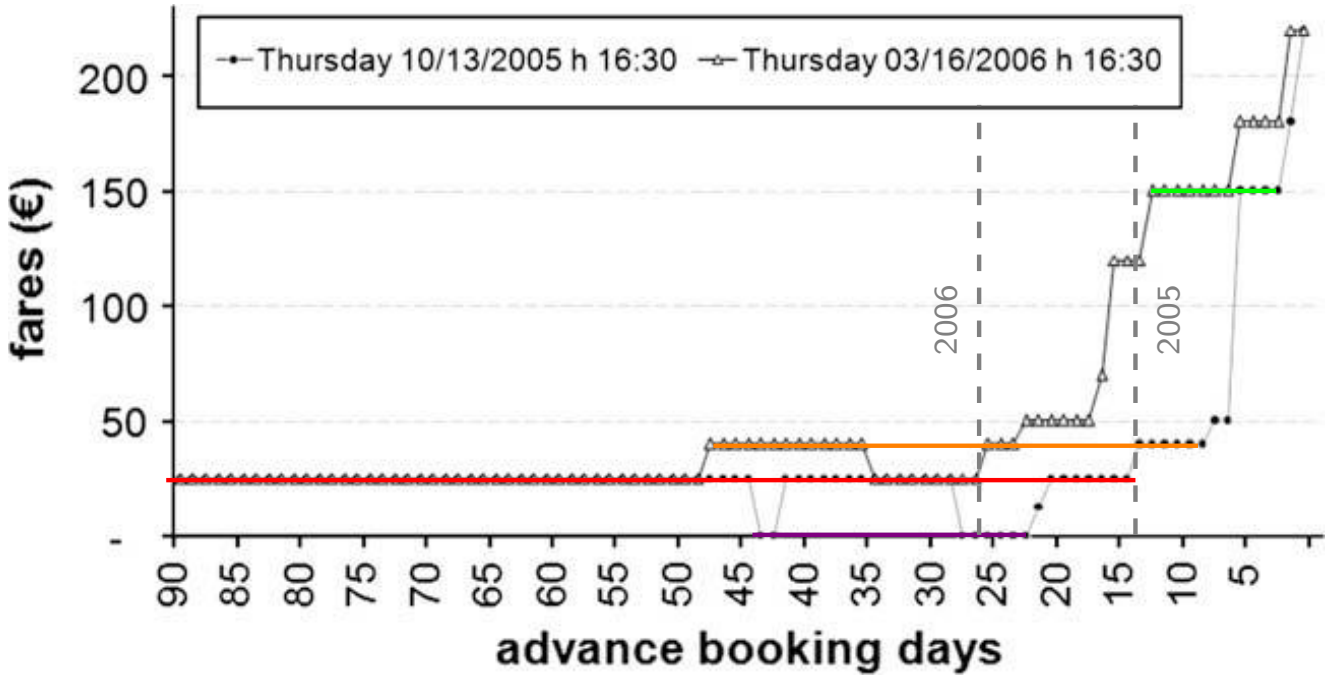


Figure 5: Prices on the *Rome Ciampino–London Stansted* route for two specific dates.

Thanks you for your attention

Høgskolen i Molde