



# Master's degree thesis

**LOG950 Logistics**

**Combined Scheduling-Transportation Model. Veidekke  
Industri AS Case Study.**

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## **Preface**

The current paper is a Master Thesis, presenting the main results of my academic work on a position of MSc student in logistics at Molde University College during December 2011-May 2012.

I would like to appreciate the help of Associate Professor Johan Oppen from Molde University College, who supervised my research work and management of Veidekke Industri AS, which provided me with real case data.

The subject area of the research is mathematical modeling and combinatorial optimization. The problem, taken as a case in the Master Thesis, presents a combination of transportation and scheduling tasks needed to be fulfilled with minimum costs. It was decided to design, describe and solve a mathematical model capable to deal with all aspects of the real case problem. The current Master Thesis is devoted mainly to the model design and development.

The models presented in the Master Thesis are closely correspondent to the real case data, thus can be directly applied as a part of decision support system. At the same time there are no combined models of this type, which author is aware of. Thus the Thesis should enrich academic society as well.

The evaluation committee for this work has been Associate Professor Johan Oppen (Molde University College, Specialized University in Logistics, Molde, Norway) and Associate Professor Lars Magnus Hvattum (NTNU, Norwegian University of Science and Technology, Trondheim, Norway).

## Summary

The Master Thesis presented in this paper is devoted to design and development of a model capable to deal with the Combined Scheduling-Transportation Problem. There can be found a wide range of real cases corresponding to this type of problem. It should present a special interest for the fields of Construction Engineering, Oil and Gas Industry, Road Construction, etc. In this Master Thesis a case of Veidekke Industri AS, a company providing asphalt road building in Møre og Romsdal region, is regarded.

The model designed to deal with this type of problems represents a combination of models for the Vehicle Routing Problem with extensions and the Scheduling Problem with sequence dependent set-up times.

The first part of the Master Thesis is devoted to the case description. Then a precise description of the extensions needed to adjust the model for the real case data is given. Then the model is presented together with formulation and description.

At first a two-stage synchronized model is presented. Using this model one shall create a schedule for project fulfillment first and then solve a routing problem for asphalt transportation. Then a combined model dealing with all aspects of the real case problem is presented.

Possible ways of model application and solution are suggested together with recommendation for further research.

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## 1. Introduction

This Master Thesis can be related to the field of Mathematical Modeling. It is concerned with description and formulation of a mathematical model, suitable to describe a real world problem and applicable to be solved by any of the existing solution methods.

The case analyzed in the Master Thesis was presented by the Norwegian company Veidekke Industri AS, a company providing asphalt road building in Møre og Romsdal region. After the preliminary analysis it was defined that the developed mathematical model shall be capable to deal with scheduling, transportation and production aspects of road building.

The second chapter of the Master Thesis presents a precise description of the real case, received from Veidekke Industri. It will provide almost all information needed both for model design and solution stages.

The supposed model is related to a wide range of combined optimization problems, some of which will be discussed in Chapter 3 of this Master Thesis. Even though each of the sub-problems analyzed here is well described in the literature, the combined models of this type are not so well known. Thus further development in this region can be beneficial both for academic and business society. From the academic side, the new type of combined models will be introduced for analysis. From business side, the model creates a basis for decision making in complicated and interdependent supply chain, thus making cost reduction possible.

The fourth part of the Master Thesis is devoted to the description of the basic model for developing a combined scheduling-transportation problem. From the author's point of view, the routing aspect will add most complexity to the problem. Thus Capacitated Vehicle Routing Problem (Golden et al, 2008) was chosen as the basis for model development. Several necessary extensions were described to adjust the model according to the real case requirements.

Finally, the fifth part presents a model development stage. The starting point for model development was description and adjusting of existent models to handle scheduling and transportation aspects (production aspect of the problem is left for further research). Unfortunately, the usage of separated sub-models to handle the case is not sufficiently



convenient and can hardly allow cost minimization in comparison to the current way of problem solution.

The next step is to solve the problem in two connected steps. As the natural starting point, a development of the scheduling sub-model was selected. Then, based on the assumptions that the schedule of project fulfillment is obtained, the transportation problem was modeled. On this stage only truck routing aspect was taken into consideration. Assuming that the transportation model is solved one can get the solution of the whole problem with a number of limitations. The obtained solution will not require adjusting, but the total costs will probably not be optimal. To avoid this issue the combined model was developed.

It has taken into consideration project scheduling, boat and truck routing aspects. Now the objective was to optimize the total costs connected to project fulfillment. The development of the model required a number of simplifications and assumptions to make it easier to implement. These simplifications will be fully described in the fifth chapter.

The conclusion and recommendation chapter present the analysis of achieved modeling goals and describes direction for further improvement and development. The author suggests that this Master Thesis can be used as a basis for the application of the model on a real case of Veidekke Industri.

## 2. Case description

The research problem is a combined problem containing both project scheduling and transportation planning. A typical situation would be a company which has several projects to perform at different locations, where each project requires resources (machinery, personnel, and materials) to be transported to the location of the project.

The current Master Thesis will be devoted to the description and modeling of such types of problems based on the case of Veidekke Industri AS, a Norwegian company performing asphalt road building in Møre og Romsdal region.

The company makes a precise plan for a one week period. It means that each Wednesday the plan for the coming week is settled. During a normal working week the company has to perform 7 governmental projects (large projects) and 105 private projects on average. Projects can vary in size significantly, from 500 kilos of asphalt per project to 5000 tons. The date and time to perform some private projects are preliminary decided (for 35 projects out of 105), while for others they are flexible. Some jobs should be performed only during night hours. Each project requires a single type of asphalt, but the type can vary from project to project. Currently the company supplies 5 different types of asphalt.

All the machinery needed to perform every single project can be loaded on (unloaded from) special transport within 20 minutes. If the transportation distance between two projects is less than 2 km all the machinery can be transported within 15 minutes without loading-unloading.

All projects during a week are performed by 5 constant teams. Normally, one of them is assigned to all governmental projects, while 4 others are working on private ones. Each team consists of 5 people. The working day lasts 8 hours and there are 5 working days during a week. Average labor costs per hour are 400 NOK per worker. The normal working day starts at 8 in the morning and finishes at 4 in the evening. If a worker has to work between 6 am and 8am or between 4 pm and 8 pm, he or she gets paid 50% more per hour in this period. If the job shall be done during the rest of the day (night) or during public holidays, the hourly increase in the salary is 100%. It is not allowed to have more than 8 hours of overtime per week for each worker. The productivity of each team on average is estimated to be 35 tons of asphalt per hour.

The company operates 3 asphalt factories supplying the asphalt for all projects. The largest of them is situated in Kristiansund. Its capacity is 240-280 tons of asphalt per hour (depending on the type of asphalt produced). The inventory capacity at the factory is 1100 tons. Due to technological issues the asphalt cannot be kept for more than 24 hours within this storage. The factory is situated nearby the harbor which can serve one boat at the time. The loading capacity in the harbor is estimated to be 240 tons per hour. The factory has also capacity to load trucks. Loading a truck of 13 tons or a trailer of 14 tons capacity takes 3 minutes.

Another factory is situated in Ålesund. It has the same characteristics as the factory in Kristiansund. The asphalt produced in Ålesund is mostly transported by boat to a harbor situated close to the project location. Relatively small amounts of asphalt can be transported by trucks directly, as the company does not carry out many projects close to the asphalt factory in Ålesund.

The third factory is situated at Røvika, about 10 km from Åndalsnes. It has no connection to harbor and can supply asphalt to nearby locations. The factory can produce a limited variety of asphalt types with a production capacity of 50 tons per hour. It has no storage capacity, thus all the asphalt produced shall be loaded to trucks immediately.

The transportation of asphalt is usually conducted in two ways. The first option is to load trucks directly at a factory. The capacity of one truck is 13 tons, and the capacity of one trailer is 14 tons. Each truck or trailer can transport only one type of asphalt at a time. The company operates 15 trucks and 10 trailers, which can be connected to trucks. The cost of one truck is 700 NOK per hour, the cost of a truck with trailer is estimated to be 900 NOK per hour.

The second option is to hire a boat, which will transport the asphalt from the factory harbor to a harbor nearby the project location. There is a possibility to hire either 650 tons' boat or 1200 tons' boat. Each of the boats is able to transport two types of asphalt simultaneously. The fixed cost of hiring a boat is 35000 NOK and 40000 NOK per day depending on the capacity. In addition, there are variable costs per ton of asphalt. This cost is assigned to each boat and each geographical zone in the region. The asphalt can be unloaded from the boat only on a truck. The time to load a truck (a trailer) from a boat is estimated to be 3 minutes.

The company may use more than 40 harbors in the region. Given that the distribution of harbors is relatively even, this means that every project location can be situated not more than 40 kilometers away from the closest harbor.

The task of a developed optimization model (or models) will be to minimize the total costs of transportation and project execution as well as waiting time of trucks (both at harbors and project locations) and teams. This will require a combination of the transportation and scheduling problems. The connections between two concrete project locations, between a project location and a harbor and between two harbors will be regarded as direct links.

One possible way is to solve two problems separately. This means that after solving the problems the planner has to adjust both of the solutions to make them compatible. This way of problem solving is very similar to the manual planning and thus does not correspond to the task of developing a sophisticated method. Thus I will not develop separated solutions for the problems.

The second option is to solve the project scheduling problem or the transportation planning problem, assuming the second problem is solved. This would for example mean one would solve the project scheduling problem assuming everything will be brought to where it is needed in time. The transportation planning problem would then be solved, using the given project schedule as input data. This option is regarded as an easiest way to solve the complete problem in a sophisticated manner.

The third possibility is to combine two problems into one complex problem where decisions on both transportation and scheduling aspects will be made simultaneously. The developing of a combined model is regarded as a main task of the current Master Thesis.

Such types of complex problems can easily be found in the real world. The proposed model is based on the example of a road construction company from Møre og Romsdal region of Norway, but it is not concerned with the solution of this specific company's problem. In adverse it will present a generalized model applicable to different situations. For example, it can be applied to construction industry, oil and gas industry (offshore construction and maintenance), etc.

## 2.1 Assumptions

The real problem discussed in the previous section is an example of a real life rich problem, which can be hard to describe mathematically. Thus there will be implemented a set of assumptions for the mathematical modeling simplification:

1. The planning horizon is one week.

2. Each project is supplied from not more than one harbor or factory during the day. It means that there will be only one supply source for each project.
3. Each harbor can be supplied by at most one boat and from at most one factory during the day. It means that there will be at most one boat visiting the harbor during the day.
4. Each project is performed by exactly one team from the start till the end without breaks. It means that if team 1 has started to work on project 1 it will not start to work on project 2 until project 1 is finished. In addition, no other team will work on project 1.
5. All projects require the same type of asphalt.
6. Each truck is connected to a single harbor during a day. If a truck is assigned to a harbor it can serve only projects which are assigned to the same harbor during that day.
7. Each truck can travel several times during a day to the same project from a harbor. In addition a truck can visit several projects on the same route until the truck capacity is utilized completely.

Based on these assumptions, the real world problem of Veidekke Industri will be described using mathematical modeling.

### 3. Literature overview

This Master Thesis deals with a combined scheduling-transportation problem. Nowadays it can be found a wide variety of literature on the topics of combined problems. The coordination of production and distribution systems is discussed in works of Glover et al. (1979), Thomas and Griffin (1996), and Tayur et al. (1999). Issues like production and transportation are combined in the works of Blumenfeld et al. (1991), Fumero and Vercellis (1999); issues of production and inventory planning are discussed by Williams (1981) and Cohen and Lee (1988); the integration of transportation and inventory are discussed in works of Speranza and Ukovich (1994) and Bertrazzi and Speranza (1999). These works provide an evidence of the growing importance of the complex decision support systems for supply chain management.

Each of the sub-problems which are going to be combined in this research is quite well known and developed.

The assignment problem is the simplest among the reviewed. The task is to assign the resources to the consumption points. The problem discussed in this Master Thesis requires an assignment of teams to projects, harbors to projects and factories to harbors. In addition, these assignments should be done within a multi-period time horizon. This is an example of a dynamic assignment problem. Zhang and Bard (2006) have described an example of a dynamic assignment problem for machine scheduling.

The second sub-problem presents a transportation problem, which can be related to the class of Vehicle Routing Problems (VRP). There are several VRP aspects in the Master Thesis problem. Firstly, an optimal route for visiting projects shall be found for each of the teams. Secondly, the number of routes and the sequence of the projects in each route shall be found for each vehicle. And thirdly, the sequence of the visited harbors shall be found for each of the boats.

The VRP was first described by Danzig and Ramser (1959). Problems of this type are usually hard to solve and at the same time they are important to industry.

The model used in the current Master Thesis is based on the classical VRP model formulation with a few extensions. Generally it can be related to the multi-period vehicle routing problem with split deliveries (MPSDVRP).. The integer programming formulation of this type of problem was presented by Dror, Laporte and Trudeau (1994).

The routing aspect of the problem discussed in the Master Thesis is closely connected to the third sub-problem, the scheduling. A sequence of projects shall be developed for each of the teams. This sequence can also be described as a route. At the same time, a sequence of project locations shall be defined for each of the vehicles. The problem is that the team can not start up the project until the vehicles deliver the asphalt, and the vehicle cannot leave the project location until the team utilizes all supposed asphalt. In this case each project can be regarded as a job, and a team and a vehicle can be regarded as two machines, which shall be synchronized over time. To solve the scheduling problem the time constraints will be added to the VRP problem discussed above. An example of such constraints can be found in many “rich” (real world) VRP problems (Oppen et al, 2007).

## 4. Problem description

The well-known VRP problem will be used as a basis to formulate the real-world problem of Veidekke Industri AS. The scheduling aspects of the problem will be incorporated in the basic model as additional constraints and extensions.

The resulting model is expected to be hard to solve, thus decomposition can be required as one method of the solution. In this case it is supposed that the assignment problem can be separated from the vehicle routing problem. Then one of the models will be solved on the first stage and another on the second.

### 4.1 The basic Vehicle Routing Problem (VRP)

The classical VRP was first presented in 1959 (Dantzig and Ramser). It is defined on a graph  $G = (N, A)$  where  $N = \{0, \dots, n\}$  is a set of nodes and  $A = \{(i, j) : i, j \in N\}$  is an arc set. Node 0 is a depot, the other nodes are the customers. The travel cost between nodes  $i$  and  $j$  is defined as  $c_{ij} > 0$ , and  $d_i$  is the demand of customer  $i$ . The model assumes that the vehicles are of the same type with identical capacity  $q$ . The objective is to minimize the costs associated with the customer service (demand satisfaction), when all the vehicle routes start and end in the depot and each customer is visited exactly once. The total demand of all customers on a route must be less than or equal to the vehicle's capacity  $q$ . This type of vehicle routing problem is usually referred to as the capacitated VRP (CVRP). Many types of extensions to this problem can be found in (Golden et al, 2008).

### 4.2 Extensions to the basic CVRP

Further on a set of extensions to the classical CVRP problem will be presented. They are needed to describe the combined scheduling-transportation problem (STP). Some of these extensions can be found in other VRP problems and some are developed specially for STP model.



### **4.2.1 Multi-period planning horizon**

In contrast to a standard CVRP problem, the planning horizon of the STP problem is not limited to one period. The model takes into account that the planning horizon consists of one week (7 days), and for each of the days an independent set of routes shall be constructed.

Such problems are related to the class of periodic VRP (PVRP) problems, a few examples of which are described by Francis et al (2008).

### **4.2.2 Tour length and duration**

Theoretically the tour length is not restricted, but the maximal working day length limits the number of possible route options. The normal working day cannot exceed 8 hours, in addition it is allowed to use at most 6 hours of overtime. The cost of the overtime hours is presented by a pricewise function. The first three hours of the overtime cost 150% of a normal wage rate, and the rest of the overtime hours cost a double wage rate. At this point we assume that all the jobs shall be performed during the day.

This constraint is present on all levels of planning. It means that there are time limits both for project teams' members and for truck and boat drivers.

### **4.2.3 Heterogeneous vehicle fleet**

The company operates two types of vehicles. There are 5 vehicle of the first type with 13 tons capacity and 10 vehicles of the second type with 27 tons capacity. The second type of the vehicles are trucks with trailers. The trailers can be disconnected from the trucks, thus reducing the vehicle capacity to 13 tons. For the reasons of simplification it will be assumed that the capacity is constant over the planning horizon (one week).

### **4.2.4 Time windows**

We assume that there are no explicit time windows for any of the customers or any of the harbors, but at each project location a visit by a vehicle should be synchronized with a visit of the team. In the same way vehicles shall be synchronized with boats at a harbor

location. It means that the routes for teams, vehicles and boats are interdependent. Thus to find the optimal solution of the problem (minimizing the cost) it is required to create all routes simultaneously.

This aspect of the STP is related to the scheduling sub-problem of the model.

#### **4.2.5 Multiple tours per vehicle per day**

In the basic VRP model a vehicle can be used to perform only one trip per day. In contrast, in the STP model multiple routes for a vehicle within a day are allowed. The first tour of the vehicle usually starts from the depot and the last tour ends in the depot. On the first tour the second node after depot shall be one of the harbors. All other routes can be started at any of the harbors and ended at any of them. The last route shall be finished at the depot

Boats can also perform multiple tours per day (even though it is unlikely). Each of them shall be started from one of the factories and ended there. To simplify the modeling task it is assumed that the first route for must be started at the depot, and the last route during the day shall be finished at the depot.

#### **4.2.6 Multiple depots**

Each of the boats has a possibility to start and end the day at any of the two factories, thus a VRP model for boats is extended to a multiple-depot VRP (MDVRP). As well each of the vehicles during the day can be loaded at any of the 40 harbors, which can be regarded as depots. Thus a vehicle routing aspect also can be tackled as a MDVRP model.

Such types of problems are well known and a number of solution methods for them have been proposed (Cordeau et al, 1997; Francis et al, 2008).

#### **4.2.7 High demand projects**

Some projects' demand can exceed the capacity of any single vehicle and the capacity of all vehicles serving the project. Thus a basic VRP model can be extended to the VRP with split deliveries (VRPSD) model. In such a model it is assumed that the demand of a single customer can be served by several vehicles, splitting the total demand into parts.

## 5. Mathematical model for the Scheduling-Routing Problem

This part of the Master Thesis will be devoted to the development of the mathematical model, suitable for a description of the Veidekke Industri AS problem. At first the sub-problems will be presented:

- Scheduling problem;
- Transportation problem.

It is proposed to start from developing the schedule of projects fulfillment, minimizing the maximum makespan (the completion time of the last scheduled projects) for each of the days. This will allow minimizing the overtime hours, and total costs consequently.

On the second stage the routing problem for trucks will be solved to satisfy the demand of the teams according to the project fulfillment schedule.

On a later stage these sub-problems will be combined into one, optimizing all processes connected to the project fulfillment, i.e. asphalt transportation and team scheduling.

### 5.1 Scheduling problem

At first we will try to formulate a scheduling problem, assuming that the projects are jobs and the teams are machines. Another necessary assumption is that everything is delivered when it is needed and where it is needed in time. It means that the trucks will deliver the asphalt to the project location in time, when it is needed for the teams.

The description of the problem can be done in the following way:

- Five identical parallel machines (teams)
- 135 jobs to be done
- permutation-dependent set-up time (time to go from one project to another is dependent on the permutation of the projects in a team schedule)
- 5 days' time horizon
- The objective is to minimize total costs

Further on the model for a scheduling problem is presented together with the formulation's description.

Table 1. Notation used in the scheduling model

<b>Sets:</b>	
$\mathcal{N}$	set of jobs (projects and a depot)
$0 \in \mathcal{N}$	depot job
$\mathcal{D}$	set of time periods within a planning horizon (number of working days)
<b>Parameters:</b>	
$M$	number of teams
$U$	maximum working day duration (including overtime hours)
$T_i$	time to perform job $i$ , $i \in \mathcal{N}$
$S_{ij}$	set up time from job $i$ to job $j$ , $i, j \in \mathcal{N}$
<b>Variables:</b>	
$\lambda_{ij}^d \in \{0,1\}$	1 if a team fulfill job $j$ after job $i$ on a day $d$ , $i, j \in \mathcal{N}, d \in \mathcal{D}$
$t_i^d$	time left after completion of job $i$ at day $d$ , $i \in \mathcal{N}, d \in \mathcal{D}$

$$\min \max_{d \in \mathcal{D}} t_0^d \quad (1)$$

subject to

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N}} \lambda_{ij}^d = 1, \quad i \in \mathcal{N} \setminus \{0\} \quad (2)$$

$$\sum_{i \in \mathcal{N}} \lambda_{ij}^d = \sum_{k \in \mathcal{N}} \lambda_{jk}^d, \quad j \in \mathcal{N}, d \in \mathcal{D} \quad (3)$$

$$\sum_{j \in \mathcal{N}} \lambda_{0j}^d \leq M, \quad d \in \mathcal{D} \quad (4)$$

$$U(1 - \lambda_{ij}^d) + t_i^d \geq t_j^d + T_j + S_{ij}, \quad i \in \mathcal{N}, j \in \mathcal{N} \setminus \{0\}, d \in \mathcal{D} \quad (5)$$

$$0 \leq t_i^d \leq U, \quad i \in \mathcal{N}, d \in \mathcal{D} \quad (6)$$

$$\lambda_{ij}^d \in \{0,1\}, \quad i \in \mathcal{N}, j \in \mathcal{N}, d \in \mathcal{D} \quad (7)$$

In this model, (1) is the objective function that minimizes the maximum working time during all days of the week for any of the teams. Constraint (2) guarantees that all the jobs except the depot job were done exactly once, and (3) is a constraint that guarantees that after each job follows the next one. Constraint (4) set up the limit on the number of machines per day, which is equal to the number of teams available. (5) is a constraint which enables the limit on the total working time available during a day per machine. The first term in the constraint uses the maximum day duration  $U$  to deactivate the constraint if two jobs are not

performed at the same machine sequentially at the same day ( $\lambda_{ij}^d=0$ ). If the jobs follow each other on the same machine within a day ( $\lambda_{ij}^d=1$ ), the constraint forces the time available after job  $j$  is finished to be at least equal to the available time after job  $i$  is finished reduced by the time needed to fulfill job  $j$  ( $T_j$ ) and time needed to set up the machine from job  $i$  to job  $j$  ( $S_{ij}$ ). In constraints (6) and (7) the domains for variables  $t$  and  $\lambda$  are given. In this model we do not refer to a specific machine, but we impose the limit to the number of used machines.

In the model presented above the variable  $t_i^d$  presents the working time left after the completion of job  $i$  during the day  $d$ . For each of the projects there is just one possible value of the  $t_i^d$  variable. The depot is visited several time (there  $M$  teams), but it is also characterized by one variable  $t_0^d$ . This is the longest working time length of one of the teams during the day  $d$ . For example, if there are 5 teams, and one of them needs 7,5 hours to performed projects assigned to it, while other teams needs less time, the value of  $t_0^d$  will be equal to 7,5. It means that after leaving the depot all teams have at most 7,5 hours to perform all assigned to them tasks. The model presented above is trying to minimize this time, thus minimizing the working time of all of the teams during the week.

This problem is NP-hard and nonlinear. Thus it cannot be solved using linear solver. Therefore it should be solved using heuristical method or an approximation algorithm. To do this we need to reformulate the problem. Now we can assume that each five machines working during 5 days are equal to 25 machines. Assuming that there is no set up times we can formulate the problem in the following way:

$$P_m || C_{max}$$

$P_m$  means that there are  $m$  identical machines working in parallel and  $C_{max}$  means that we are minimizing maximum makespan (longest processing time). The known List Scheduling (LS) algorithm with LPT (largest processing time) ordering for this problem is:

1. Sort the list of jobs in the non-increasing order of the processing time (LPT);
2. At any time that a machine becomes available, take the first job in the list and assign it to the machine. Remove the assigned job from the list.
3. Repeat Step 2 until all jobs are assigned.

In case of job sequence-dependent setup times the problem shall be reformulated in the following way (Ying et al, 2010):

$$P_m | S_{ij} | C_{max}$$

We will try to develop the algorithm based on the LS with LPT algorithm:

1. On the first available machine sort the list of jobs in the non-increasing order of the following ratio:

$$\frac{p_j}{s_j}$$

where  $p_j$  is the processing time of job  $j$ , and  $s_j$  is the set up time, needed to prepare the chosen machine for fulfillment of job  $j$ ;

2. Take the first job in the list and assign it to the available machine. Remove the assigned job from the list.
3. Repeat Steps 1 and 2 until all jobs are assigned.

This algorithm can give an initial (not very close to optimum) solution, which can be improved by applying one of the well known Metaheuristics. An example can be found in Ying et al. (2010).

## 5.2 Transportation problem

When the scheduling problem for the teams and projects is solved, the second problem should be addressed. On this stage a plan for transportation of asphalt from the harbors to the project locations shall be developed.

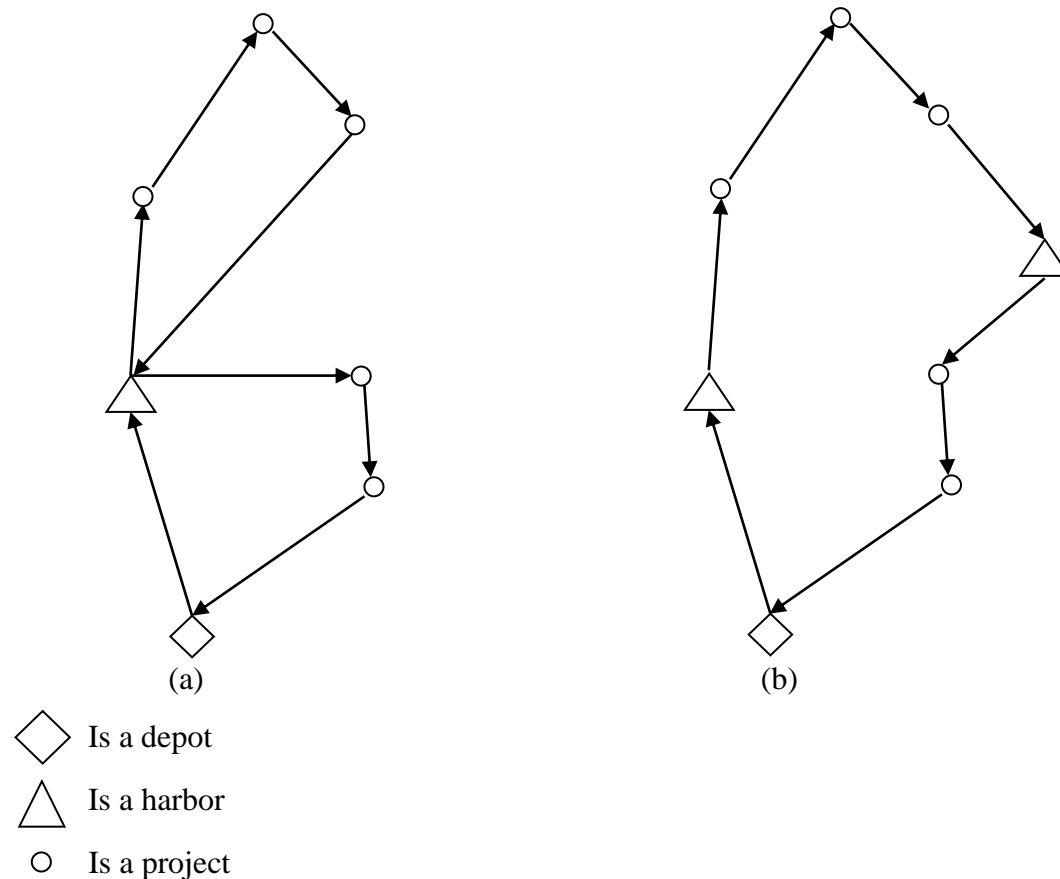
The transportation problem is defined on a set of harbors, set of projects and one depot. Two types of vehicles (trucks and trailers) are leaving the depot in the morning, then pick up asphalt in one of the harbors and deliver it to one of the projects, then if possible they visit the second project or go back to one of the harbors for reloading. In the end of the day all the trucks go back to the depot.

The routes of the trucks can be drawn in two ways (Figure 1).

On the previous stage it was assumed that everything is delivered where it is needed in time. This implies additional restrictions on the transportation. Now the delivery shall be performed within certain time limits, i.e. no later than the team is ready to fulfill the project.

Some projects are too large to be satisfied with one delivery. The reason for that can be the capacity (limited load) of the asphalt spreading equipment. Typically only 10 tons of asphalt can be loaded onto this equipment. This amount of asphalt can be utilized within 20 minutes (the average speed is 30 tons per hour). In addition 3 minutes is needed to load asphalt onto the spreading machine. Further loadings for the same projects can be done

during the spreading, thus the next truck should arrive 17 minutes after departure of the previous. For example, to fulfill a project, a team needs 54 tons of asphalt, there should then be 6 deliveries (5 of 10 tons and one of 4 tons) with an interval of 17 minutes between each delivery. Such a condition adds complexity to the model. To deal with this complexity, we assume that each loading represents a separate project. Thus instead of one large project we will analyze 6 smaller projects with the same location. Each of them will be characterized by its own time window and demand (needed quantity of asphalt).



**Figure 1. Two possible types of the truck's routes: (a) with reloading in the same harbor; (b) with reloading in a different harbor.**

Time windows add another type of complexity. For each node (including projects, harbors and depot) the arrival time (departure time) of each used truck shall be computed. The same truck can visit the same project (if it is large) and the same harbor several times during a day. But the time of arrival in a linear model can be computed just once for each node. In case of projects this problem is avoided by splitting large projects into smaller subprojects, as explained above. Then the arrival time is computed for each of the subprojects separately. In case of harbors this problem can be solved by adding a set of artificial harbors. Assume that each truck will not visit the same harbor more than 5 times per day (from real life experience). Then for each harbor we will introduce 5 artificial duplicates. Another

option is to limit the number of routes per vehicle per day, and calculate the arrival time for each of the routes separately.

**Table 2. Notation used in the transportation model**

<b>Sets:</b>	
$\mathcal{N}$	set of projects
$\{0\}$	depot
$\mathcal{H}$	set of harbors
$\mathcal{K}$	set of vehicles
$\mathcal{D}$	set of time periods within a planning horizon (number of working days)
<b>Parameters:</b>	
$U$	maximum working day duration (including overtime hours)
$Q_k$	loading capacity of vehicle $k$ , $k \in \mathcal{K}$
$C_i$	asphalt demand at location $i$ , $i \in \mathcal{N}$
$T_i$	time to load/unload at location $i$ , $i \in \mathcal{N} \cup \mathcal{H} \cup \{0\}$
$S_{ij}$	traveling time between locations $i$ and $j$ , $i, j \in \mathcal{N} \cup \mathcal{H} \cup \{0\}$
$A_i^d$	time bound to arrive at location $i$ at day $d$ , $i \in \mathcal{N}$ , $d \in \mathcal{D}$ (if the location should not be visited at day $d$ , $A_i^d = 0$ )
$R$	continuous number between 0 and 1, such that for any $A_i^d > 0$ the sum $A_i^d + R \geq 1$ and for any $A_i^d = 0$ the sum $A_i^d + R < 1$
<b>Variables:</b>	
$\beta_{ij}^{dk} \in \{0,1\}$	1 if arc between location $i$ and $j$ is traversed by vehicle $k$ on day $d$ , $i, j \in \mathcal{N} \cup \mathcal{H} \cup \{0\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$
$t_i^{dk}$	departure time of vehicle $k$ from location $i$ at day $d$ , $i \in \mathcal{N} \cup \mathcal{H} \cup \{0\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$
$w_i^{dk}$	waiting time of vehicle $k$ at location $i$ at day $d$ , $i \in \mathcal{N} \cup \mathcal{H} \cup \{0\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$
$q_i^{dk}$	load of vehicle $k$ after visiting location $i$ at day $d$ , $i \in \mathcal{N} \cup \mathcal{H} \cup \{0\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$

One more necessary assumption is that the asphalt is available at any harbor, when the truck is going there. This can be hard to achieve in real life, because trucks can be loaded



only from boats. There are only two boats available and they require time to travel from harbor to harbor. Thus the timetable of boats should be synchronized with the timetable of trucks. The possible solution can be to solve the boat routing problem simultaneously with truck routing problem. This task will be left for further research.

The model for the transportation problem will be presented together with the formulation's description (Table 2).

$$\min \max_{d \in \mathcal{D}, k \in \mathcal{K}} \left( \max_{i \in \mathcal{N} \cup \mathcal{H}} t_i^{dk} + \sum_{i \in \mathcal{N} \cup \mathcal{H}} (\beta_{i0}^{dk} \cdot S_{i0}) \right) \quad (8)$$

subject to

$$\sum_{k \in \mathcal{K}} \sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \mathcal{H} \cup \{0\}} \beta_{ij}^{dk} = 1, \quad i \in \mathcal{N} \quad (9)$$

$$\sum_{i \in \mathcal{N} \cup \mathcal{H} \cup \{0\}} \beta_{ij}^{dk} = \sum_{m \in \mathcal{N} \cup \mathcal{H} \cup \{0\}} \beta_{jm}^{dk}, \quad j \in \mathcal{N} \cup \mathcal{H} \cup \{0\}, d \in \mathcal{D}, k \in \mathcal{K} \quad (10)$$

$$\sum_{j \in \mathcal{H}} \beta_{0j}^{dk} \leq 1, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (11)$$

$$\sum_{j \in \mathcal{N}} \beta_{0j}^{dk} = 0, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (12)$$

$$\sum_{j \in \mathcal{H} \cup \{0\}} \beta_{ij}^{dk} = 0, \quad i \in \mathcal{H}, d \in \mathcal{D}, k \in \mathcal{K} \quad (13)$$

$$U(1 - \beta_{ij}^{dk}) + t_j^{dk} \geq t_i^{dk} + w_j^{dk} + T_j + S_{ij}, \quad i \in \mathcal{N} \cup \mathcal{H} \cup \{0\}, j \in \mathcal{N} \cup \mathcal{H}, \quad (14)$$

$$d \in \mathcal{D}, k \in \mathcal{K}$$

$$0 \leq t_i^{dk} \leq U, \quad i \in \mathcal{N} \cup \mathcal{H}, d \in \mathcal{D}, k \in \mathcal{K} \quad (15)$$

$$t_0^{dk} = 0, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (16)$$

$$Q_k(1 - \beta_{ij}^{dk}) + q_i^{dk} \geq q_j^{dk} + C_j, \quad i \in \mathcal{N} \cup \mathcal{H}, j \in \mathcal{N}, d \in \mathcal{D}, k \in \mathcal{K} \quad (17)$$

$$0 \leq q_i^{dk} \leq Q_k, \quad i \in \mathcal{N} \cup \mathcal{H}, d \in \mathcal{D}, k \in \mathcal{K} \quad (18)$$

$$q_0^{dk} = 0, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (19)$$

$$A_j^d \cdot \sum_{i \in \mathcal{N} \cup \mathcal{H}} \beta_{ij}^{dk} = t_j^{dk}, \quad j \in \mathcal{N}, d \in \mathcal{D}, k \in \mathcal{K} \quad (20)$$

$$(A_j^d + R) \geq \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N} \cup \mathcal{H}} \beta_{ij}^{dk}, \quad j \in \mathcal{N}, d \in \mathcal{D} \quad (21)$$

$$\beta_{ij}^d \in \{0,1\}, \quad i \in \mathcal{N}, j \in \mathcal{N}, d \in \mathcal{D} \quad (22)$$

$$w_j^{dk} \geq 0, \quad j \in \mathcal{N}, d \in \mathcal{D}, k \in \mathcal{K} \quad (23)$$

In this model, (8) is the objective function that minimizes the maximum drivers working time during all days of the week. For each vehicle and each day the completion time of the last job is defined  $\max_{i \in \mathcal{N} \cup \mathcal{H}} t_i^{dk}$ . Then the time to travel from this job to the depot is calculated  $(\sum_{i \in \mathcal{N} \cup \mathcal{H}} (\beta_{i0}^{dk} \cdot S_{i0}))$ . There is at most one route to the depot for each of the vehicles within each day (according to constraints (10) and (11)), thus the sum  $\sum_{i \in \mathcal{N} \cup \mathcal{H}} (\beta_{i0}^{dk} \cdot S_{i0})$  is equal to the time needed to travel from the last job  $i$  to the depot. Then the objective is to minimize the maximum of the working times for any of the drivers during any day of the week. I.e. the optimal value of the objective function will represent the longest working day of one of the drivers during the week. Any working day of all other drivers will be not longer than the objective function value.

Constraint (9) guarantees that all project locations are visited just once during the week, and (10) is a constraint that guarantees that any vehicle arrives in location as many times as it leaves it. Constraints (11) set a limit of routes from the depot to the harbors. Each vehicle shall leave the depot not more than one time per day. (12) forbids the direct routes from the depot to project locations, i.e. before going to any of the projects the vehicle shall visit any of the harbors to pick up asphalt. (13) forbids the direct routes from harbors to other harbors or to the depot. This restriction shall be avoided by the model automatically, but to ensure absence of unnecessary moves it is included.

(14) is a constraint which enables the limit on the total working time available during a day per vehicle. The first term in the constraint uses the maximum day duration  $U$  to deactivate the constraint if the distance between two locations is not traversed by a vehicle during the day ( $\beta_{ij}^{dk}=0$ ). If the distance between two locations is traversed by a vehicle during the day ( $\beta_{ij}^{dk}=1$ ), the constraint forces the departure time from location  $j$  to be at least equal to the sum of the departure time from location  $i$ , the time needed to unload a vehicle at location  $j$  ( $T_j$ ), waiting time at location  $j$  ( $w_j^{dk}$ ), and time needed to travel from location  $i$  to location  $j$  ( $S_{ij}$ ). In constraint (15) the domains for variables  $t$  are given. Constraint (16) set up the departure time from the depot for each vehicle. It means that all vehicle leaves the depot in the beginning of the day.

(17) is a constraint which enables the limit on the total capacity of a vehicle on a route. The first term in the constraint uses the maximum vehicle capacity ( $Q_k$ ) to deactivate the constraint if the distance between two locations is not traversed by a vehicle during the day ( $\beta_{ij}^{dk}=0$ ). If the distance between two locations is traversed by a vehicle during the day

( $\beta_{ij}^{dk}=1$ ), the constraint forces the load after leaving location  $i$  to be at least equal to the load after leaving location  $j$  summarized with the demand at location  $j$  ( $C_j$ ). In constraint (18) the domains for variables  $q$  are given. (19) imposes zero load when vehicles are leaving depot.

(20) represent a set of constraints on time windows for visiting each of the projects. If vehicle  $k$  visits location  $j$  at day  $d$  ( $\sum_{i \in \text{NUH}} \beta_{ij}^{dk} = 1$ ), the time of the vehicle departure from the location ( $t_j^{dk}$ ), should be equal to the starting time of the work on the project fulfillment (i.e. truck leaves location when the spreading equipment is loaded and team can start the job). Constraint (21) limits number of days when the location of the project can be visited to one. If there is no time windows for day  $d$  ( $A_j^d = 0$ ), there will be no vehicles visiting location  $j$  at this day ( $R$  cannot be higher than 1). In other case there will be at least one. The  $R$  value is needed to ensure that the project will be visited in case of relatively early starting time. For example if  $A_j^d = 0,1$  hours (i.e. 6 minutes), then  $R$  shall be at least 0,9 to ensure that the sum of the connections ( $\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N} \cup \mathcal{H}} \beta_{ij}^{dk}$ ) can still be equal to 1. In constraints (22) and (23) the domains for variables  $\beta$  and  $w$  are given.

This model implies that each project and each harbor is visited just once. Thus to solve the initial problem, one should predefine artificial projects and harbors as described above.

### 5.3 Combined Scheduling-Transportation Problem

The models presented above are regarded as two separate steps. The solutions of the two problems can produce a relatively good, but probably not optimal, result for the whole problem. I.e. even if both problems can be solved to optimality the total cost is not necessarily optimal.

The combined problem will allow optimizing all the costs connected to the project fulfillment, thus there optimal solution to this problem will give the minimal possible total costs.

The combination of sub-problems in one combined problem will require a few modifications. At first the scheduling problem was solved, assuming that the projects are not split in smaller parts, while the transportation problem provides a solution for split projects. In the combined problem projects will be subdivided in smaller sub-projects, which shall be

fulfilled one after the other. It means that the working team will be required to fulfill sub-project  $i$  after sub-project  $j$ , if they are both parts of the same real project. Thus some additional constraints will be introduced in the model.

The second modification will also be made with the scheduling sub-problem. The waiting time of the teams will be introduced. In real life it can happen that a team needs to wait until the asphalt is delivered, and in some cases it can even lead the reduction of the total cost. Thus introduction of the waiting time for teams also seems to be reasonable.

The third difference of the combined model compared to the separated models is that now the starting time of project fulfillment will be calculated for the teams. In the separated scheduling problem the length of the day left after project fulfillment was calculated.

And finally to make the combined model linear, a node representing the duplicated depot will be used. At this node the length of all working days (both for truck drivers and for the team members) will be calculated. It will allow minimization of the costs in a linear manner.

Now the combined scheduling-routing problem will be presented together with formulation.

**Table 3. Notation used in the scheduling-transportation model**

<b>Sets:</b>	
$\mathcal{N}$	set of projects
$\{0\}$	starting depot
$\{1\}$	ending depot
$\mathcal{H}$	set of harbors
$\mathcal{K}$	set of vehicles
$\mathcal{D}$	set of time periods within a planning horizon (number of working days)
<b>Parameters:</b>	
$U$	maximum working day duration (including overtime hours)
$Q_k$	loading capacity of vehicle $k$ , $k \in \mathcal{K}$
$C_i$	asphalt demand at location $i$ , $i \in \mathcal{N}$
$T_k^i$	time to load/unload vehicle $k$ at location $i$ , $k \in \mathcal{K}, i \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$
$S_{ij}$	traveling time between locations $i$ and $j$ , $i, j \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$
$M$	number of teams
$Z_i$	time to perform job $i$ , $i \in \mathcal{N} \cup \{0\} \cup \{1\}$

Table3. Notation used in the scheduling-transportation model

<b>Parameters:</b>	
$G_{ij}$	set up time from job $i$ to job $j$ , $i, j \in \mathcal{N} \cup \{0\} \cup \{1\}$ "
$V_{ij} \in \{0,1\}$	1 if project $j$ is required to be performed immediately after project $i$ , $i, j \in \mathcal{N}$
$P$	piecewise cost parameter per unit of time.
$R$	continuous number between 0 and 1, such that for any $x_j^d > 0$ the sum $x_j^d + R \geq 1$ and for any $x_j^d = 0$ the sum $x_j^d + R < 1$
$L$	maximum number of working hours during a week
<b>Variables:</b>	
$\beta_{ij}^{dk} \in \{0,1\}$	1 if arc between location $i$ and $j$ is traversed by vehicle $k$ on day $d$ , $i, j \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$
$\lambda_{ij}^d \in \{0,1\}$	1 if a team fulfill job $j$ after job $i$ on a day $d$ , $i, j \in \mathcal{N} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$
$t_i^{dk}$	departure time of vehicle $k$ from location $i$ at day $d$ , $i \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$
$w_i^{dk}$	waiting time of vehicle $k$ at location $i$ at day $d$ , $i \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$
$q_i^{dk}$	load of vehicle $k$ after visiting location $i$ at day $d$ , $i \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$
$x_i^d$	starting time of job $i$ fulfillment at day $d$ , $i \in \mathcal{N} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$
$y_i^d$	waiting time before job $i$ fulfillment at day $d$ , $i \in \mathcal{N} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$

$$\min \sum_{d \in \mathcal{D}} \left( x_1^d \cdot P + \sum_{k \in \mathcal{K}} (t_1^{dk} \cdot P) \right) \quad (24)$$

subject to

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \{1\}} \lambda_{ij}^d = 1, \quad i \in \mathcal{N} \quad (25)$$

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \{0\}} \lambda_{1j}^d = 0 \quad (26)$$

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \{1\}} \lambda_{j0}^d = 0 \quad (27)$$

$$\sum_{i \in \mathcal{N}} \lambda_{ij}^d = \sum_{k \in \mathcal{N}} \lambda_{jk}^d, \quad j \in \mathcal{N}, d \in \mathcal{D} \quad (28)$$

$$\sum_{j \in \mathcal{N}} \lambda_{0j}^d = \sum_{j \in \mathcal{N}} \lambda_{j1}^d, \quad d \in \mathcal{D} \quad (29)$$

$$\sum_{j \in \mathcal{N}} \lambda_{0j}^d \leq M, \quad d \in \mathcal{D} \quad (30)$$

$$\sum_{d \in \mathcal{D}} \lambda_{ij}^d \geq V_{ij}, \quad i \in \mathcal{N}, j \in \mathcal{N} \quad (31)$$

$$U(1 - \lambda_{ij}^d) + x_j^d \geq x_i^d + Z_i + G_{ij} + y_j^d, \quad i \in \mathcal{N} \cup \{0\}, j \in \mathcal{N} \cup \{1\}, d \in \mathcal{D} \quad (32)$$

$$0 \leq x_i^d \leq U, \quad i \in \mathcal{N} \cup \{0\} \cup \{1\}, d \in \mathcal{D} \quad (33)$$

$$x_0^d = 0, \quad d \in \mathcal{D} \quad (34)$$

$$y_1^d = 0, \quad d \in \mathcal{D} \quad (35)$$

$$\sum_{d \in \mathcal{D}} x_1^d \leq L \quad (36)$$

$$\sum_{k \in \mathcal{K}} \sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \mathcal{H} \cup \{1\}} \beta_{ij}^{dk} = 1, \quad i \in \mathcal{N} \quad (37)$$

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \mathcal{H} \cup \{0\}} \beta_{1j}^{dk} = 0 \quad (38)$$

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \mathcal{H} \cup \{1\}} \beta_{j0}^{dk} = 0 \quad (39)$$

$$\sum_{i \in \mathcal{N} \cup \mathcal{H} \cup \{0\}} \beta_{ij}^{dk} = \sum_{m \in \mathcal{N} \cup \mathcal{H} \cup \{1\}} \beta_{jm}^{dk}, \quad j \in \mathcal{N} \cup \mathcal{H}, d \in \mathcal{D}, k \in \mathcal{K} \quad (40)$$

$$\sum_{j \in \mathcal{H}} \beta_{0j}^{dk} \leq 1, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (41)$$

$$\sum_{j \in \mathcal{N}} \beta_{0j}^{dk} = 0, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (42)$$

$$\sum_{j \in \mathcal{H} \cup \{1\}} \beta_{ij}^{dk} = 0, \quad i \in \mathcal{H}, d \in \mathcal{D}, k \in \mathcal{K} \quad (43)$$

$$\sum_{j \in \mathcal{H}} \beta_{j0}^{dk} = \sum_{j \in \mathcal{N}} \beta_{j1}^{dk}, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (44)$$

$$U(1 - \beta_{ij}^{dk}) + t_j^{dk} \geq t_i^{dk} + w_j^{dk} + T_j^k + S_{ij}, \quad i \in \mathcal{N} \cup \mathcal{H} \cup \{0\}, \quad (45)$$

$$j \in \mathcal{N} \cup \mathcal{H} \cup \{1\}, \quad d \in \mathcal{D}, k \in \mathcal{K}$$

$$0 \leq t_i^{dk} \leq U, \quad i \in \mathcal{N} \cup \mathcal{H} \cup \{1\}, d \in \mathcal{D}, k \in \mathcal{K} \quad (46)$$

$$t_0^{dk} = 0, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (47)$$

$$w_1^{dk} = 0, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (48)$$

$$\sum_{d \in \mathcal{D}} t_1^{dk} \leq L, \quad k \in \mathcal{K} \quad (49)$$

$$Q_k(1 - \beta_{ij}^{dk}) + q_i^{dk} \geq q_j^{dk} + C_j, \quad i \in \mathcal{N} \cup \mathcal{H}, j \in \mathcal{N} \cup \{1\}, d \in \mathcal{D}, k \in \mathcal{K} \quad (50)$$

$$0 \leq q_i^{dk} \leq Q_k, \quad i \in \mathcal{N} \cup \mathcal{H}, d \in \mathcal{D}, k \in \mathcal{K} \quad (51)$$

$$q_0^{dk} = q_1^{dk} = 0, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (52)$$

$$U \cdot \left(1 - \sum_{i \in \mathcal{N} \cup \mathcal{H}} \beta_{ij}^{dk}\right) \geq (x_j^d - t_j^{dk}), \quad j \in \mathcal{N}, d \in \mathcal{D}, k \in \mathcal{K} \quad (53)$$

$$(x_j^d + R) \geq \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N} \cup \mathcal{H}} \beta_{ij}^{dk}, \quad j \in \mathcal{N}, d \in \mathcal{D} \quad (54)$$

$$\lambda_{ij}^d \in \{0,1\}, \quad i \in \mathcal{N} \cup \{0\} \cup \{1\}, \quad j \in \mathcal{N} \cup \{0\} \cup \{1\}, \quad d \in \mathcal{D} \quad (55)$$

$$y_j^d \geq 0, \quad j \in \mathcal{N} \cup \{0\} \cup \{1\}, \quad d \in \mathcal{D} \quad (56)$$

$$\beta_{ij}^{dk} \in \{0,1\}, \quad i \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}, j \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}, d \in \mathcal{D}, k \in \mathcal{K} \quad (57)$$

$$w_j^{dk} \geq 0, \quad j \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}, d \in \mathcal{D}, k \in \mathcal{K} \quad (58)$$

In this model, (24) is the piecewise objective function that minimizes the total cost of transportation and cost associated with the longest working day of one of the teams, fulfilling the projects. Variable  $x_1^d$  represents the longest duration of the working day at day  $d$ . It means that none of the project teams works longer than this amount of time. Product  $x_1^d \cdot p$  represents a piecewise function of costs connected to longest working day. If  $x_1^d \leq 8$ , then the product equals to  $x_1^d \cdot p$ . If  $8 < x_1^d \leq 11$ , then the product equals to  $8 \cdot p + (x_1^d - 8) \cdot 1,5p$ . And if  $x_1^d > 11$ , then the cost of one working day is calculated in the following way  $8 \cdot p + 3 \cdot 1,5p + (x_1^d - 11) \cdot 2p$ . In the model presented above, it is assumed that each working day begins at 8 in the morning.

Constraints (25) guarantee that each project is fulfilled exactly one time. Constraints (26) and (27) forbid fulfillment of any of the jobs after the end depot job is fulfilled or before the start depot job is fulfilled. It means that the teams cannot leave the end depot, or arrive at

the start depot. (28) are the connectivity constraints for each of the project jobs. It means that if the job is started during day  $d$ , it should be finished during the same day. Constraint set (29) requires the number of teams leaving the start depot to be equal to the number of teams arriving at the end depot during each day. (30) limit a number of teams which can work during the day.

Constraint set (31) guarantees that the subprojects representing parts of one large project are fulfilled one after the other. Together with constraints (28) it guarantees, that subprojects of a project will be done during the same day. For example, if sub-project  $i$  is visited during day  $d$  ( $\sum_{i \in N} \lambda_{ki}^d = 1$ ) and it is required to go to the next subproject  $j$  ( $V_{ij} = 1$ ), then  $\lambda_{ij}^d$  will be equal to 1. The constraints (31) are needed to satisfy the requirements of the real world problem. It is required that the projects were fulfilled from the start till the end without interruptions. For the purposes of modeling the projects were divided into several parts, thus these parts shall be fulfilled one after the other.

(32) are constraints which enable the limit on the total working time available during a day per team. The first term in the constraint uses the maximum day duration  $U$  to deactivate the constraint if two jobs are not performed by the same team sequentially at the same day ( $\lambda_{ij}^d=0$ ). If the jobs follow each other within a day ( $\lambda_{ij}^d=1$ ), the starting time of job  $j$  fulfillment to be at least equal to sum of the starting time of job  $i$  fulfillment, the time needed to fulfill job  $i$  ( $Z_i$ ), the time needed to set up the machine from job  $i$  to job  $j$  ( $G_{ij}$ ) and the waiting time at before job  $j$  is started ( $y_j^d$ ). Any starting time of the job fulfillment cannot exceed lower or upper bound (constraints (33)) and the starting time of the start depot job fulfillment is equal to 0 (constraint (34)). The time to fulfill the depot jobs ( $F_0$  and  $F_1$ ) is equal to 0, thus the job fulfillment starts and end at the same time point. Limiting the starting time of the end depot job fulfillment  $x_0^d$  we restrict the maximum working day length (because all the teams must return to the end depot each day if they left the start depot). The constraints (35) requires that teams were not waiting before fulfilling the end depot job. Finally constraint (36) limits the total duration of the working week for any of the teams. According to Norwegian law it is not allowed to have more than 8 hours of overtime during working week. Constraint (36) allows satisfying this real world requirement.

The next batch of constraints set a number of requirements to fulfill the transportation. Constraints (37) limit the number of deliveries to each of project locations. It means that each of the project locations is visited just once per week by one of the vehicles.



None of the routes is allowed to be started at the end depot (constraint (38)) and ended in the start depot (39). According to constraints (40), the number of vehicles arriving at harbor or project location shall be equal to the number of vehicle leaving it the same day.

A set of constraints (41) indicates that a vehicle can leave the start depot not more than once, when going to harbors. The direct routes from the start depot to any of the projects are forbidden by constraints (42). No routes between harbors are allowed, according to constraints (43). Constraint set (44) guarantees that, if vehicle  $k$  leaves the start depot at day  $d$ , it should arrive to the end depot the same day.

(45) is a set of constraints which enables the limit on the total working time available during a day per vehicle. The first term in the constraint uses the maximum day duration  $U$  to deactivate the constraint if the distance between two locations is not traversed by a vehicle during the day ( $\beta_{ij}^{dk}=0$ ). If the distance between two locations is traversed by a vehicle during the day ( $\beta_{ij}^{dk}=1$ ), the constraint forces the departure time from location  $j$  to be at least equal to the sum of the departure time from location  $i$ , the time needed to unload a vehicle at location  $j$  ( $T_j$ ), waiting time at location  $j$  ( $w_j^{dk}$ ), and the time needed to travel from location  $i$  to location  $j$  ( $S_{ij}$ ). In constraint (46) the domains for variables  $t$  are given. The departure time from the start depot is equal to 0, according to constraints (47). And the waiting time at the end depot is also equal to 0 (constraints (48)). Thus all the vehicles do not need to wait when they are coming to the depot. Constraints (49) limit the total duration of the working week, as well as in case of team members.

(50) is a constraint set which enables the limit on the total capacity of a vehicle on a route. The first term in the constraint uses the maximum vehicle capacity ( $Q_k$ ) to deactivate the constraint if the distance between two locations is not traversed by a vehicle during the day ( $\beta_{ij}^{dk}=0$ ). If the distance between two locations is traversed by a vehicle during the day ( $\beta_{ij}^{dk}=1$ ), the constraint forces the load after leaving location  $i$  to be at least equal to the load after leaving location  $j$  summarized with the demand at location  $j$  ( $C_j$ ). In constraints (51) the domains for variables  $q$  are given. (52) impose the zero load requirement when vehicles are leaving the start depot or arriving at the end depot.

The next two constraint sets provide the connectivity in work of project teams and trucks. None of the teams can start the project fulfillment until a truck brings asphalt. As well no truck can leave a project location until the team utilizes the load. Thus the trucks' routes shall be synchronized with the teams' schedules.

Constraint set (53) requires that the time when a vehicle leaves project  $j$  equals the start of the project fulfillment, if this vehicle serves the project. If vehicle is serving the project  $j$  during day  $d$  ( $\sum_{i \in \mathcal{N} \cup \mathcal{H}} \beta_{ij}^{dk} = 1$ ) then the difference between time of project fulfillment start and time of the truck departure should be at most 0. It means that the truck can leave the project location no earlier the start of the project fulfillment by a team. If the project is not served by the truck, then the left hand side is equal to the length of the day, meaning that the time of the truck departure can be equal to 0 even if the project has to be fulfilled during the same day.

Constraints (54) limit number of days when the location of the project can be visited to one. If there is no time windows for day  $d$  ( $x_j^d = 0$ ), there will be no vehicles visiting location  $j$  at this day ( $R$  cannot be higher than 1). In other case there will be at least one. The  $R$  value is needed to ensure that the project will be visited in case of relatively early starting time. For example if  $x_j^d = 0,1$  hours (i.e. 6 minutes), then  $R$  shall be at least 0,9 to ensure that the sum of the connections ( $\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N} \cup \mathcal{H}} \beta_{ij}^{dk}$ ) can still be equal to 1. In constraints (55) – (58) the domains for variables  $\lambda, \beta, \gamma$  and  $w$  are given.

As it can be seen the model representing combined Scheduling-Routing Problem is defined in a linear manner, making it possible to be solved by a linear solver. The next step is to add the boat routing aspect into the problem. It is expected that such extension should not be difficult to implement.

## 5.4 Full Scheduling-Transportation Problem

This part of the Master Thesis is devoted to the description and formulation of the full model for Scheduling-Transportation problem. The model is capable of dealing with all aspects of the real world problem presented above. It takes care about simultaneous optimization of boat routes, truck routes and project fulfillment. It means that all these tasks are synchronized. The current model is an extended version of the previous one. It includes the boat routing aspect. Each of the boat routes shall be put in correspondence with truck routes. I.e. a boat shall be available at a harbor no later than the first truck arrives there to pick up asphalt, and a boat shall leave a harbor no earlier than the last truck picked asphalt. In addition a boat shall carry enough asphalt to load all trucks, which must be loaded. Truck routes are synchronized in time with project fulfillment schedule in the same way as in the previous model.

To simplify the model several assumptions are implemented:

- each boat shall spend the night (or time between two working days) at a depot. It means that each boat starts and ends each working day at a depot;
- each boat visits any of the harbors not more than once per day;
- the time to load a boat is not dependent on the amount of asphalt needed to load.

Some other changes were implemented without affecting the correspondence of the model with reality. To simplify the model it is assumed that there are two depots (starting and ending), even though they are situated in the same place. Each of the factories was also duplicated. The reason for that is that any of the boats can have a few routes during a day going through the same factory (to refill the capacity), but to each location only one time of arrival and departure can be assigned. Here is used the same approach as in the previous model, when modeling truck routes going through the same harbor several times during one day.

Now the full combined scheduling-routing problem will be presented together with formulation.

**Table 4. Notation used in the full combined model**

<b>Sets:</b>	
$\mathcal{N}$	set of projects
$\{0\}$	starting depot
$\{1\}$	ending depot
$\{2\}$	starting depot for boat departure
$\{3\}$	ending depot for boat arrival
$\mathcal{H}$	set of harbors
$\mathcal{K}$	set of vehicles
$\mathcal{D}$	set of time periods within a planning horizon (number of working days)
$\mathcal{F}$	set of factories, producing asphalt
<b>Parameters:</b>	
$U$	maximum working day duration (including overtime hours)
$Q_k$	loading capacity of vehicle $k$ , $k \in \mathcal{K}$
$C_i$	asphalt demand at location $i$ , $i \in \mathcal{N}$
$T_k^i$	time to load/unload vehicle $k$ at location $i$ , $k \in \mathcal{K}, i \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$

Table4. Notation used in the full combined model

<b>Parameters:</b>	
$S_{ij}^K$	traveling time between locations $i$ and $j$ , when using a truck $i, j \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$
$S_{ij}^B$	traveling time between locations $i$ and $j$ , when using a boat $i, j \in \mathcal{F} \cup \mathcal{H} \cup \{2\} \cup \{3\}$
M	number of teams
$Z_i$	time to perform job $i$ , $i \in \mathcal{N} \cup \{0\} \cup \{1\}$
$G_{ij}$	set up time from job $i$ to job $j$ , $i, j \in \mathcal{N} \cup \{0\} \cup \{1\}$
$V_{ij} \in \{0,1\}$	1 if project $j$ is required to be performed immediately after project $i$ , $i, j \in \mathcal{N}$
P	piecewise cost parameter per unit of time.
R	continuous number between 0 and 1, such that for any $x_j^d > 0$ the sum $x_j^d + R \geq 1$ and for any $x_j^d = 0$ the sum $x_j^d + R < 1$
L	maximum number of working hours during a week
$H_b$	maximal capacity f boat $b$ , $b \in \mathcal{B}$
$E_i^b$	time needed to load/serve boat $b$ at location $i$ , $i \in \mathcal{F} \cup \{2\} \cup \{3\}$ , $b \in \mathcal{B}$
<b>Variables:</b>	
$\beta_{ij}^{dk} \in \{0,1\}$	1 if arc between location $i$ and $j$ is traversed by vehicle $k$ on day $d$ , $i, j \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$
$\lambda_{ij}^d \in \{0,1\}$	1 if a team fulfill job $j$ after job $i$ on a day $d$ , $i, j \in \mathcal{N} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$
$t_i^{dk}$	departure time of vehicle $k$ from location $i$ at day $d$ , $i \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$
$w_i^{dk}$	waiting time of vehicle $k$ at location $i$ at day $d$ , $i \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$
$q_i^{dk}$	load of vehicle $k$ after visiting location $i$ at day $d$ , $i \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$ , $k \in \mathcal{K}$
$x_i^d$	starting time of job $i$ fulfillment at day $d$ , $i \in \mathcal{N} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$
$y_i^d$	waiting time before job $i$ fulfillment at day $d$ , $i \in \mathcal{N} \cup \{0\} \cup \{1\}$ , $d \in \mathcal{D}$

Table4. Notation used in the full combined model

$\gamma_{ij}^{db} \in \{0,1\}$	1 if arc between location $i$ and $j$ is traversed by boat $b$ on day $d$ , $i, j \in \mathcal{F} \cup \mathcal{H} \cup \{2\} \cup \{3\}$ , $d \in \mathcal{D}$ , $b \in \mathcal{B}$
$a_i^{db}$	Arrival time of boat $b$ at location $i$ at day $d$ , $i, \in \mathcal{F} \cup \mathcal{H} \cup \{2\} \cup \{3\}$ , $d \in \mathcal{D}$ , $b \in \mathcal{B}$
$o_i^{db}$	departure time of boat $b$ from location $i$ at day $d$ , $i, \in \mathcal{F} \cup \mathcal{H} \cup \{2\} \cup \{3\}$ , $d \in \mathcal{D}$ , $b \in \mathcal{B}$
$h_i^{db}$	load of boat $b$ after visitng location $i$ at day $d$ , $i, \in \mathcal{F} \cup \mathcal{H} \cup \{2\} \cup \{3\}$ , $d \in \mathcal{D}$ , $b \in \mathcal{B}$

$$\min \sum_{d \in \mathcal{D}} \left( x_1^d \cdot P + \sum_{k \in \mathcal{K}} (t_1^{dk} \cdot P) + \sum_{b \in \mathcal{B}} (o_3^{db} \cdot P) \right) \quad (59)$$

subject to

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \{1\}} \lambda_{ij}^d = 1, \quad i \in \mathcal{N} \quad (60)$$

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \{0\}} \lambda_{1j}^d = 0 \quad (61)$$

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \{1\}} \lambda_{j0}^d = 0 \quad (62)$$

$$\sum_{i \in \mathcal{N}} \lambda_{ij}^d = \sum_{k \in \mathcal{N}} \lambda_{jk}^d, \quad j \in \mathcal{N}, d \in \mathcal{D} \quad (63)$$

$$\sum_{j \in \mathcal{N}} \lambda_{0j}^d = \sum_{j \in \mathcal{N}} \lambda_{j1}^d, \quad d \in \mathcal{D} \quad (64)$$

$$\sum_{j \in \mathcal{N}} \lambda_{0j}^d \leq M, \quad d \in \mathcal{D} \quad (65)$$

$$\sum_{d \in \mathcal{D}} \lambda_{ij}^d \geq V_{ij}, \quad i \in \mathcal{N}, j \in \mathcal{N} \quad (66)$$

$$U(1 - \lambda_{ij}^d) + x_j^d \geq x_i^d + Z_i + G_{ij} + y_j^d, \quad i \in \mathcal{N} \cup \{0\}, j \in \mathcal{N} \cup \{1\}, d \in \mathcal{D} \quad (67)$$

$$0 \leq x_i^d \leq U, \quad i \in \mathcal{N} \cup \{0\} \cup \{1\}, d \in \mathcal{D} \quad (68)$$

$$x_0^d = 0, \quad d \in \mathcal{D} \quad (69)$$

$$y_1^d = 0, \quad d \in \mathcal{D} \quad (70)$$

$$\sum_{d \in \mathcal{D}} x_1^d \leq L \quad (71)$$

$$\sum_{k \in \mathcal{K}} \sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \mathcal{H} \cup \{1\}} \beta_{ij}^{dk} = 1, \quad i \in \mathcal{N} \quad (72)$$

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \mathcal{H} \cup \{0\}} \beta_{1j}^{dk} = 0 \quad (73)$$

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{N} \cup \mathcal{H} \cup \{1\}} \beta_{j0}^{dk} = 0 \quad (74)$$

$$\sum_{i \in \mathcal{N} \cup \mathcal{H} \cup \{0\}} \beta_{ij}^{dk} = \sum_{m \in \mathcal{N} \cup \mathcal{H} \cup \{1\}} \beta_{jm}^{dk}, \quad j \in \mathcal{N} \cup \mathcal{H}, d \in \mathcal{D}, k \in \mathcal{K} \quad (75)$$

$$\sum_{j \in \mathcal{H}} \beta_{0j}^{dk} \leq 1, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (76)$$

$$\sum_{j \in \mathcal{N}} \beta_{0j}^{dk} = 0, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (77)$$

$$\sum_{j \in \mathcal{H} \cup \{1\}} \beta_{ij}^{dk} = 0, \quad i \in \mathcal{H}, d \in \mathcal{D}, k \in \mathcal{K} \quad (78)$$

$$\sum_{j \in \mathcal{H}} \beta_{j0}^{dk} = \sum_{j \in \mathcal{N}} \beta_{j1}^{dk}, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (79)$$

$$U(1 - \beta_{ij}^{dk}) + t_j^{dk} \geq t_i^{dk} + w_j^{dk} + T_j^k + S_{ij}^k, \quad i \in \mathcal{N} \cup \mathcal{H} \cup \{0\}, \quad (80)$$

$$j \in \mathcal{N} \cup \mathcal{H} \cup \{1\}, \quad d \in \mathcal{D}, k \in \mathcal{K}$$

$$0 \leq t_i^{dk} \leq U, \quad i \in \mathcal{N} \cup \mathcal{H} \cup \{1\}, d \in \mathcal{D}, k \in \mathcal{K} \quad (81)$$

$$t_0^{dk} = 0, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (82)$$

$$w_1^{dk} = 0, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (83)$$

$$\sum_{d \in \mathcal{D}} t_1^{dk} \leq L, \quad k \in \mathcal{K} \quad (84)$$

$$Q_k(1 - \beta_{ij}^{dk}) + q_i^{dk} \geq q_j^{dk} + C_j, \quad i \in \mathcal{N} \cup \mathcal{H}, j \in \mathcal{N} \cup \{1\}, d \in \mathcal{D}, k \in \mathcal{K} \quad (85)$$

$$0 \leq q_i^{dk} \leq Q_k, \quad i \in \mathcal{N} \cup \mathcal{H}, d \in \mathcal{D}, k \in \mathcal{K} \quad (86)$$

$$q_0^{dk} = q_1^{dk} = 0, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (87)$$

$$U \cdot \left(1 - \sum_{i \in \mathcal{N} \cup \mathcal{H}} \beta_{ij}^{dk}\right) \geq (x_j^d - t_j^{dk}), \quad j \in \mathcal{N}, d \in \mathcal{D}, k \in \mathcal{K} \quad (88)$$

$$(x_j^d + R) \geq \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N} \cup \mathcal{H}} \beta_{ij}^{dk}, \quad j \in \mathcal{N}, d \in \mathcal{D} \quad (89)$$

$$\sum_{b \in \mathcal{B}} \sum_{i \in \mathcal{H} \cup \mathcal{F}} \gamma_{ij}^{db} \leq 1, \quad j \in \mathcal{H}, d \in \mathcal{D} \quad (90)$$

$$\sum_{i \in \mathcal{F} \cup \mathcal{H}} \gamma_{ij}^{db} = \sum_{m \in \mathcal{F} \cup \mathcal{H}} \gamma_{jm}^{db}, \quad j \in \mathcal{F} \cup \mathcal{H}, d \in \mathcal{D}, b \in \mathcal{B} \quad (91)$$

$$\sum_{j \in \mathcal{F} \cup \mathcal{H}} \gamma_{2j}^{db} \leq 1, \quad d \in \mathcal{D}, b \in \mathcal{B} \quad (92)$$

$$\sum_{j \in \mathcal{F} \cup \mathcal{H} \cup \{2\}} \gamma_{3j}^{db} = 0, \quad d \in \mathcal{D}, b \in \mathcal{B} \quad (93)$$

$$\sum_{j \in \mathcal{F} \cup \mathcal{H}} \gamma_{j2}^{db} = 0, \quad d \in \mathcal{D}, b \in \mathcal{B} \quad (94)$$

$$\sum_{j \in \mathcal{F} \cup \mathcal{H}} \gamma_{2j}^{db} = \sum_{j \in \mathcal{F} \cup \mathcal{H}} \gamma_{j3}^{db}, \quad d \in \mathcal{D}, b \in \mathcal{B} \quad (95)$$

$$a_i^{db} - U \cdot \left( 1 - \sum_{j \in \mathcal{F} \cup \mathcal{H}} \gamma_{ji}^{db} \right) \leq \min_k \{t_i^{dk} - T_i^k\}, \quad i \in \mathcal{H}, d \in \mathcal{D}, \quad (96)$$

$$U \cdot \left( 1 - \sum_{j \in \mathcal{F} \cup \mathcal{H}} \gamma_{ji}^{db} \right) + o_i^{db} \geq \max_k \{t_i^{dk}\}, \quad i \in \mathcal{H}, d \in \mathcal{D} \quad (97)$$

$$U \cdot (1 - \gamma_{ij}^{db}) + a_j^{db} \geq o_i^{db} + S_{ij}^B, \quad i \in \mathcal{F} \cup \mathcal{H} \cup \{2\}, \quad (98)$$

$$j \in \mathcal{F} \cup \mathcal{H} \cup \{3\}, \quad d \in \mathcal{D}, b \in \mathcal{B}$$

$$o_i^{db} = a_i^{db} + E_i^b, \quad i \in \mathcal{F} \cup \{2\} \cup \{3\}, d \in \mathcal{D}, b \in \mathcal{B} \quad (99)$$

$$a_2^{db} = 0, \quad d \in \mathcal{D}, b \in \mathcal{B} \quad (100)$$

$$0 \leq a_i^{db} \leq U, \quad i \in \mathcal{F} \cup \mathcal{H} \cup \{3\}, d \in \mathcal{D}, b \in \mathcal{B} \quad (101)$$

$$0 \leq o_i^{db} \leq U, \quad i \in \mathcal{F} \cup \mathcal{H} \cup \{3\}, d \in \mathcal{D}, b \in \mathcal{B} \quad (102)$$

$$H_b \cdot (1 - \gamma_{ij}^{db}) + h_i^{db} \geq h_j^{db} + \sum_{k \in \mathcal{K}} q_j^{dk}, \quad i \in \mathcal{H}, j \in \mathcal{H} \cup \{3\}, d \in \mathcal{D}, b \in \mathcal{B} \quad (103)$$

$$h_i^{db} = 0, \quad i \in \{2\} \cup \{3\}, d \in \mathcal{D}, b \in \mathcal{B} \quad (104)$$

$$0 \leq h_i^{db} \leq H_b, \quad i \in \mathcal{F} \cup \mathcal{H}, d \in \mathcal{D}, b \in \mathcal{B} \quad (105)$$

$$\lambda_{ij}^d \in \{0,1\}, \quad i \in \mathcal{N} \cup \{0\} \cup \{1\}, \quad j \in \mathcal{N} \cup \{0\} \cup \{1\}, \quad d \in \mathcal{D} \quad (106)$$

$$\gamma_j^d \geq 0, \quad j \in \mathcal{N} \cup \{0\} \cup \{1\}, \quad d \in \mathcal{D} \quad (107)$$

$$\beta_{ij}^{dk} \in \{0,1\}, \quad i \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}, \quad j \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}, \quad d \in \mathcal{D}, k \in \mathcal{K} \quad (108)$$

$$w_j^{dk} \geq 0, \quad j \in \mathcal{N} \cup \mathcal{H} \cup \{0\} \cup \{1\}, d \in \mathcal{D}, k \in \mathcal{K} \quad (109)$$

$$\gamma_{ij}^{dk} \in \{0,1\}, \quad i \in \mathcal{F} \cup \mathcal{H} \cup \{2\} \cup \{3\}, \quad j \in \mathcal{F} \cup \mathcal{H} \cup \{2\} \cup \{3\}, \quad d \in \mathcal{D}, \quad b \in \mathcal{B} \quad (110)$$

In this model, (59) is the piecewise objective function that minimizes the total cost of transportation by boats and trucks and cost associated with the longest working day of one of the teams, fulfilling the projects. The objective function is very similar to the one presented in formula (24). In addition to the project fulfillment costs and truck transportation cost the boat transportation costs are counted. They are calculated in the same way as truck costs were calculated. If a boat is used less than 8 hours, the costs are equal to  $o_3^{db} \cdot P$ , if a boat was used from 8 to 11 hours, the costs are  $8 \cdot P + (o_3^{db} - 8) \cdot 1,5P$ , and if boat was used more than 11 hours during the day, the costs are calculated using the following formula  $8 \cdot P + 3 \cdot 1,5P + (o_3^{db} - 11) \cdot 2P$ .

Constraints (60)-(89) correspond to constraints (25)-(54) of the previous model. Thus their description will be omitted here. (90) is a set of constraints guarantying that none of the factories or harbors will be visited more than once during a day by the same boat. Constraint set (91) guarantees that if a boat entered a factory or a harbor, it will leave it. And if it was not entering, it will not have possibility to leave.

Constraints (92) require that any of the boats leave a depot not more than once per day. Constraints (93) forbid departures from the ending depot and constraints (94) forbid arrival to the starting depot. Set of constraints (95) set up the equality between the number of routes from the starting depot and number of routes to the ending depot for each boat and each day. Thus if a boat leaves a depot, it should arrive to the depot in the end of the same day.

Constraint set (96) set limitation on the arrival time for any of the boat to a harbor. The second term  $(-U \cdot (1 - \sum_{j \in \mathcal{F} \cup \mathcal{H}} \gamma_{ji}^{db}))$  uses the maximum day duration  $U$  to deactivate the constraint if a boat does not visit the harbor during the day ( $\sum_{j \in \mathcal{F} \cup \mathcal{H}} \gamma_{ji}^{db} = 0$ ). If the boat is visiting the harbor then the arrival time of the boat shall be smaller or equal to than the arrival time of the first truck, which shall pick up asphalt there. As the arrival time for the truck is not calculated explicitly, the difference between the time of departure and time of loading is taken as the arrival time ( $\min_k \{t_i^{dk} - T_i^k\}$ ).

(97) is a set of constraints which sets limitation on the boat departure time from any of the harbors. The first term uses maximum day duration  $U$  to deactivate constraint if the boat does not visit the harbor during the day ( $\sum_{j \in \mathcal{F} \cup \mathcal{H}} \gamma_{ji}^{db} = 0$ ). If the boat visits the harbor then the departure time of the boat shall be later than the maximum departure time of any of the



trucks, which are loaded at the same harbor ( $\max_k \{t_i^{dk}\}$ ). It means that the boat can leave the harbor only when the last truck assigned to this harbor is loaded.

(98) is a set of connectivity constraints. The first term deactivate the restriction using the maximum day duration  $U$  if boat  $b$  does not traverse an arc between locations  $i$  and  $j$  ( $\gamma_{ij}^{db} = 0$ ) during day  $d$ . If the boat's route includes arc between location  $i$  and  $j$ , then the arrival time at location  $j$  ( $a_j^{db}$ ) shall be at least as large as the sum of the departure time from location  $i$  ( $o_i^{db}$ ) and traveling time between two locations ( $S_{ij}^B$ ). It means that the boat cannot arrive at location  $j$  earlier than it leaves location  $i$  and traveled the distance between two locations.

(99) put in correspondence arrival and departure time at factories and depots. Departure time is equal to the sum of arrival time and loading/service time. Constraints (100) give a starting point of the day. It is the time when boats are leaving the depot.

(101) and (102) set lower and upper bounds on the value of arrival and departure times for each boat within each day.

Set of constraint (103) enables the limit on the total capacity of a boat on a route. The first term in the constraint uses the maximum boat capacity ( $H_b$ ) to deactivate the constraint if the distance between two locations is not traversed by a boat during the day ( $\gamma_{ij}^{db}=0$ ). If the distance between two locations is traversed by a boat during the day ( $\gamma_{ij}^{db}=1$ ), the constraint forces the load after leaving location  $i$  to be at least equal to the load after leaving location  $j$  summarized with the demand at location  $j$  ( $\sum_{k \in \mathcal{K}} q_j^{dk}$ ). Total demand is calculated as the sum of truck loads, which are decided to be loaded at location  $i$ . (104) impose the zero load requirement when boats are leaving the start depot or arriving at the end depot. In constraints (105) the domains for variables  $h$  are given.

Sets of constraints (106)-(109) correspond to sets of constraints (55)-(58) in the previous model. They set the domains for  $\lambda$ ,  $y$ ,  $\beta$  and  $w$  variables. Constraint set (110) sets the domain for  $\gamma$  variable.

The full combined scheduling transportation model allows simultaneous optimization of routing and project fulfillment aspects of the problem. Nevertheless, it has a number of limitations. Firstly, it does not take into account that factories cannot load more than one boat simultaneously. Thus in some cases the solutions provided by the model should be adjusted to satisfy this additional constraint.

Secondly, in the real world the working day for boats, trucks and teams shall start at different time. In the model above it is assumed that everybody is leaving the depot at the same time. Another vital assumption which will affect the real costs, is that all the works are performed during the day time and do not start earlier than at 8 in the morning. In the real world, works can be performed during the night and early morning hours, thus affecting the hourly costs.

Thirdly, the model does not take in consideration the production capacity at factories. It relaxes the real complexity of the problem. From the problem description it is clear that factories have limited production and inventory capacity. Thus in case when boat will come one after another, some waiting time will be required because of production capacity. Including this aspect will require significant model modification.

Finally, the boat loading time is handled as a constant value disregarding the amount of asphalt to load. In the real world the loading time will be load dependent when speaking about boats (loading the trucks requires almost the same amount of time disregarding the amount to load). It means that if the full boat capacity will be utilized, the time spent at factory will increase.

Some other model characteristics shall be taken into account when solving it. The full combined scheduling-transportation model is now not-linear (boat arrival and departure times are dependent on the min and max functions of truck arrivals). Thus non-linear solver or Heuristics method shall be used to solve the problem.

## 6. Conclusions and recommendations

In the current Master Thesis a model for the Combined Scheduling-Transportation Problem (CSTP) is designed and developed. There are no existing combined models for problems of this type, which the author of this paper is aware of. Thus it should be of high interest to the academic society. At the same time the model was intended to be as close to the real case data as possible. Therefore it can be applied in decision support systems with small modifications.

The topic of the Master Thesis aroused in cooperation with Veidekke Industri AS, a company providing asphalt road building in Møre og Romsdal region of Norway. The company operates on a weekly time planning horizon, performing asphalt road construction. This process involves asphalt production, asphalt transportation (both from production facilities to transshipment nodes by sea and from transshipment nodes to the project locations by roads) and project fulfillment. The schedule for teams fulfilling the projects shall be synchronized with transportation on time to minimize the total costs connected to the process flow.

The idea to apply combined models for complex tasks is not new in the academic society. Nowadays it can be found a wide variety of literature on the topics of combined problems. The coordination of production and distribution systems is discussed in works of Glover et al. (1979), Thomas and Griffin (1996), and Tayur et al. (1999). Issues like production and transportation are combined in the works of Blumenfeld et al. (1991), Fumero and Vercellis (1999); issues of production and inventory planning are discussed by Williams (1981) and Cohen and Lee (1988); the integration of transportation and inventory are discussed in works of Speranza and Ukovich (1994) and Bertrazzi and Speranza (1999). These works provide an evidence of the growing importance of the complex decision support systems for supply chain management. Thus the idea to develop a combined model to deal with the complex problem Veidekke Industri seems to be obvious.

The classical Capacitated Vehicle Routing problem was chosen as a basis for CSTP. A number of extensions were presented to adjust the model for the real case data:

- 1) multi-period planning horizon;
- 2) tour length and duration limitations;
- 3) heterogeneous vehicle fleet;
- 4) time windows;

- 5) multiple tours per vehicle per day;
- 6) multiple depots;
- 7) high demand projects with split deliveries.

The model for extended routing problem was then combined with the model for scheduling problem with sequence dependent set-up times. Both of the problems present NP-hard combinatorial optimization tasks. Thus two possible ways of solution were presented.

At first a two-stage synchronized model was presented. As a natural starting point the scheduling problem is modeled and solved the first. Then based on the project fulfillment schedule the routing problem is handled. Such approach guarantees that both of the process flow tasks will be synchronized in time. As well it leaves a possibility (in case of small problem size) to find optimal schedule of project fulfillment. Nevertheless the total costs associated with the process are not necessary to be optimal (even in case of the small problem size and exact solution method). Thus the development and solution of one-stage combined model seemed to be needed.

The model for the combined problem is presented in part 5.3 of the Master Thesis. It takes into consideration the project fulfillment aspect, asphalt transportation from production facilities to transshipment nodes and from transshipment nodes to project locations. The problem is handled using a model with non-linear constraints, thus it cannot be applied and solved using a linear solver even in case of small instances.

The model presented in the Master Thesis highly corresponds to the real case data, nevertheless it is presented in a general form. Thus it can be applied to any problem of such types with little modification.

The further research in this field can follow different directions:

- the model can be simplified to make it more clear for understanding and studying purposes;
- the model can be enriched by inclusion of production aspect of supply chain;
- some simplifications and assumptions can be removed to make the result analysis easier;
- there can be developed an application (applications) based on the presented model.

The most interesting directions from the author's point of view is involvement of the production process in the model design and description and development of solution application to check the results obtained with and without use of the model.

The first task will require an additional analysis of the process flow in Veidekke Industri together with efforts to integrate the process description into the existing model for Scheduling-Transportation Problem.

The second task of model application and solution will require significant programming efforts. It is expected that the model cannot be solved using exact methods within reasonable time (the plan shall be ready each week). Thus an application of approximate or heuristic solution methods will be needed.

## References

- Bertazzi, L., M.G. Speranza. 1999. Models and algorithms for the minimization of inventory and transportation costs: A Survey, in *New Trends in Distribution Logistics* (Speranza, M.G. and Staehly, P., eds.), *Lecture Notes in Economics and Mathematical Systems*, Springer-Verlag, Berlin Heidelberg, Vol. 480, pp. 137-157.
- Blumenfeld, D.E., L.D. Burns, C.F.Daganzo. 1991. Synchronizing production and transportation schedules. *Transportation res. PartB* 25, pp. 23-37.
- Cohen, M.A., H.L. Lee. 1998. Strategic analysis of integrated production-distribution systems: Models and methods. *Operations Research*, 36, pp.216-227.
- Cordeau, J.-F., M., Gendreau, G., Laporte, 1997. A tabu search heuristic for periodic and the multi-depot vehicle routing problem, *Networks*, 30, pp. 105-119.
- Dantzig, G., J., Ramser, 1959. The truck dispatching problem, *Management science*, 6, pp. 80-91.
- Dror, M., G., Laporte, P.,Trudeau, P. 1994. Vehicle routing with split deliveries, *Discrete Appl. Math*, 50, pp. 239-254.
- Francis, P., M., Smilowitz, K., R., Tzur, M. 2008. The period vehicle routing problem and its extensions (Golden, B., Raghavan, S., Wasil, E., eds.), *Vehicle Routing Problem. Latest Advances and New Challenges*, Springer, New York, pp. 73-102.
- Fumero, F., C. Vercellis. 1999. Synchronized development of production, inventory and distribution schedule. *Transportation Sci.* 33, pp. 330-340.
- Glover, F., G. Jones, D. Karney. 1979. An integrated production, distribution and inventory planning system. *Interfaces* 9, pp. 21-35.
- Golden, B., Raghavan, S., Wasil, E. 2008. *The Vehicle Routing Problem. Latest Advances and New Challenges*. Springer, New York.
- Oppen, J., A., Løkketangen, D.L., Woodruff, 2007. Collecting animals for slaughter-adding inventory constraints to a real world routing problem. Working Paper. Molde University College.

- Speranza, M.G., W. Ukovich. 1994. Minimizing transportation and inventory costs for several products on a single link, *Operations Research*, 42, pp. 879-894.
- Tayur, S, R., M., Ganeshan, Magazine. 1999. *Quantitative Models for supply chain management*. Kluwer Academic Publisher, London, UK.
- Thomas, D. J., P. M. Griffin. 1996. Coordinated supply chain management. *Eur. J. Oper. Res.* 94, pp. 1-15.
- Williams, J.F. 1981. Heuristics techniques for simultaneous scheduling of production and distribution in multi-echelon structures: Theory and empirical comparisons. *Management Sciences*, 27, pp. 336-352.
- Ying K.-C., Lee Z.-J., Lin S.-W. 2010. Makespan minimization for scheduling unrelated parallel machines with setup times. *Journal of Intelligent Manufacturing*, published online.
- Zhang X., J. F. Bard. 2006. A multi-period machine assignment problem. *European Journal of Operational Research*, 170, pp. 398–415.