



Master's degree thesis

LOG950 Logistics

**Forecasting and Inventory Management Optimization
in Stokke AS**

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Number of pages including this page: 69

Molde, 2011



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Preface

This document presents research work performed to obtain a MSc degree in Logistics at Molde University College, Specialized University in Logistics, Norway. It is the main result of my academic work as a master student during December 2010 – May 2011.

This thesis was supervised by Associate Professor Johan Oppen from Molde University College where the work has been conducted. In addition to my supervisor I have worked with Karolis Dugnas, researcher and project leader from Møreforsking Molde AS, and Nils Laugesen, Supply Chain Management and Logistics Manager from Stokke AS, Ålesund, Norway.

The subject area of this thesis is forecasting and inventory management. In particular, the thesis considers theoretical and optimization issues related to forecasting and inventory management optimization in continuous process supply chain. Moreover, an existing supply chain of Stokke is presented, underlying the set of real problem. This thesis contributes to research by bridging the gap between theory and practice. On one hand, it describes an unexplored problem and solves it. On other hand, it tries to apply the theoretical findings to a real-world problem.

The evaluation committee for this work has been associate professor Johan Oppen (Molde University College, Specialized University in Logistics, Molde, Norway) and Sigrid Nonås (NHH, Norwegian School of Economics and Business Administration, Bergen, Norway).

Summary

This master thesis deals with problems of forecasting and inventory management. A case of Stokke AS, a company designing and distributing children products, has been reviewed. The company operates in the environment of long lead times from supply side and short lead times from demand side. In such environment the precise planning of inventory level plays an important role.

To evaluate possible ways of inventory management results improvement the master thesis was divided in two parts. The first of them deals with problems of forecasting and the seconds with problems of inventory management optimization.

Three methods of forecasting were used to develop and evaluate a forecasting model capable to give accurate forecasts of expected demand. The Moving Average approach, the Holt-Winters exponential smoothing approach and the Box-Jenkins approach were evaluated. It was shown that none of examined approaches give reasonably better results compared to an assumption about demand randomness.

Based on this assumption a multi-item periodic review system for inventory and transportation management was developed and modeled. The model was tested on two instances of supply chain design: current and suggested supply chains of the company. Test results are reported, confirming that changes in supply chain design could lead to a significant improvement of inventory management.

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1. Introduction

The current master thesis is concerned with the fields of forecasting and inventory management. The topic appeared in cooperation between Stokke AS and the research institute Møreforsking Molde.

Stokke, a company focused exclusively on children's furniture and equipment, realized the need for improvement their performance. The main goal for the company is optimization of inventory management and transportation procedures in order to minimize logistical costs. The company contacted Møreforsking Molde to determine the critical points of flow of goods and warehouse management.

Karolis Dugnas and Oddmund Oterhals performed that task and presented results of their work in the report "Flow of goods and warehouse optimization for Stokke. Mapping and improvement of the logistics processes" (*Dugnas, Oterhals, 2010*). The findings presented in this report led to a few main possibilities to decrease the costs for the company.

They were:

- Development and usage of more sophisticated forecasting techniques;
- Usage of aggregation forecasting;
- Reduction of lead times.

The current master thesis is continuance of cooperation between Stokke and Møreforsking Molde. The research topic mainly belongs to the fields of "Inventory management" and "Time Series Forecasting". The main goal of the master thesis is to develop and analyze concrete methods for warehouse management and transportation optimization.

The master thesis consists of two parts. The first is devoted to development and evaluation of different forecasting methods. The second concerns development and solution of logistical costs minimization problem.

Most authors agree that accurate forecasting is essential part of inventory planning (*Lee et al, 1997; Silver et al, 1998; Waters, 2003*). Due to this fact the starting point for elaboration of concrete advices for Stokke is development of forecasting technique which will give the most accurate and reliable forecast of demand. To solve this task simple and weighted moving average approaches, the Holt-Winters exponential smoothing approach and the Box-Jenkins approach were evaluated. As well the situation when demand is just a random value following one of the known distributions was evaluated. This analysis gave

an opportunity to fulfill the second part of research which was valuable not only from the practical (company's) point of view but also academically. On one hand, it describes an unexplored problem and solves it. On other hand, it tries to apply the theoretical findings to the real-world problem.

Based on the results of best approach for demand forecasting, the inventory level and transportation optimization models were designed and evaluated. This topic is covered in the second part of the master thesis. The benefit for the company from this work lies in the possibility to give answers to several managerial questions:

1. What are the main possibilities to increase outcome from inventory and transportation management?
2. What can be done to decrease the costs if it is possible?

From the academic point of view the most important issue is development and solution of unexplored problem. In most cases the inventory level and transportation optimization problems were discussed either from a strategic point of view (for ex.: *Nozick, Turnquist, 2000; Sen et al, 2010*) or from an inventory management point of view (*Silver et al, 1998; Waters, 2003; Zhou et al, 2007*). In the first case the inventory costs are simplified to the holding cost per unit disregarding the volume of inventory. In the second case the transportation costs are not taken into consideration when determining the optimal order quantities and service level.

In this thesis the author tried to combine known models for inventory management with detailed consideration of transportation costs and limitations. In case of the application of such models, the whole process of ordering, transportation and inventory holding could be optimized simultaneously on the item level.

To carry out the research the author has used following data sources:

1. Primary data: Stokke's database of sold products and demand for the years 2008-2010, information about contract details with suppliers, estimation of cost of delivery from the suppliers and ordering costs, focus group with participation of the Stokke SC Manager, the Thesis supervisor, and the Møreforsking Molde representative.
2. Secondary data: the report from Møreforsking Molde, journal articles, master- and PhD-dissertations, books.

2. Company profile and problem description

Stokke AS is a Norwegian company with headquarters located at **Ålesund**, in the northwestern part of Norway. Stokke provides worldwide distribution of own designed and developed products through **selected retailers represented in around fifty countries**. The company does not operate the production facilities, but rather has long term agreements with vendors located in Europe and China.

The company distributes a small variety of high quality unique products. Each product has a number of possible modifications. The company is presented in the highchair (Tripp Trapp), stroller (Stokke Xplory) and nursery (Stokke Care) market segments. Equipment and furniture distributed by Stokke are targeted on families with income above average.



Figure 1. Tripp Trapp children chair.

Source: Dugnas, Oterhals, 2010.



Figure 2. Stokke Xplory children stroller.

Source: Dugnas, Oterhals, 2010.

To provide better service to distributors the company operates three warehouses: one in Asia, one in Europe and one in the United States. The company itself does not run the retail store system, but all the products are distributed under the Stokke trademark through specialist children's stores.

In the current thesis one part of the Stokke supply chain is discussed. The flow of textile from a single Asian vendor to a single European warehouse will be analyzed (Figure 3).

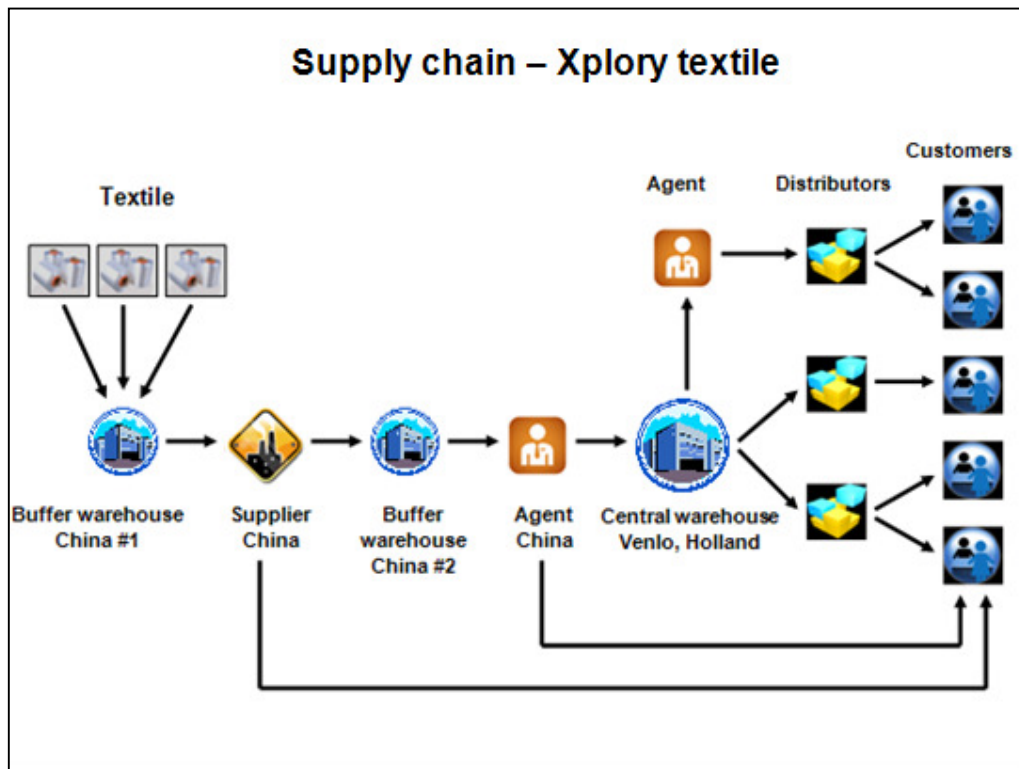


Figure 3. Supply chain – Xplory textile.

Source: Dugnas, Oterhals, 2010.

Stokke is one of the leaders in targeted market niches and develops its business in current and some new locations. The growth of the company's business over the last several years has indicated some weaknesses in inventory management. The company operates in an environment of long lead times (up to three months) from supply side and short lead times (a few days) from demand side. The second issue is increasing production costs and inventory holding costs. In combination with a top management desire to decrease the logistical costs these issues led to the necessity of inventory management optimization.

The company has decided to rely on scientific methods of problem solving. Hence its representatives have contacted Møreforsking Molde. The cooperation between Stokke and Møreforsking Molde has led to development of the report “Flow of goods and warehouse optimization for Stokke. Mapping and improvement of the logistics processes” which indicates the main directions for inventory management improvement in the company.

The analysis done in this report lead to a few conclusions:

- The quality of forecasts should be improved by using some more sophisticated techniques than the company uses;
- The possibilities to decrease the length of the order cycle should be regarded;
- The service level and level of inventory should be balanced precisely.

The last point means that some traditional service level measurement should be applied. For example it can be probability of having no stockouts (P1 service level). Currently Stokke uses the share of orders satisfied with a single delivery as service measurement. But it can appear that customers ask themselves to split the delivery in several parts. Hence it is difficult to measure the actual value of the service level.

Following these findings, Stokke has changed forecasting methods to more sophisticated ones. Unfortunately the comparison of the forecasted and the actual demand for CC Textile Set in June-December 2010 (Figure 4) shows that the forecasted quantities can still be misleading.

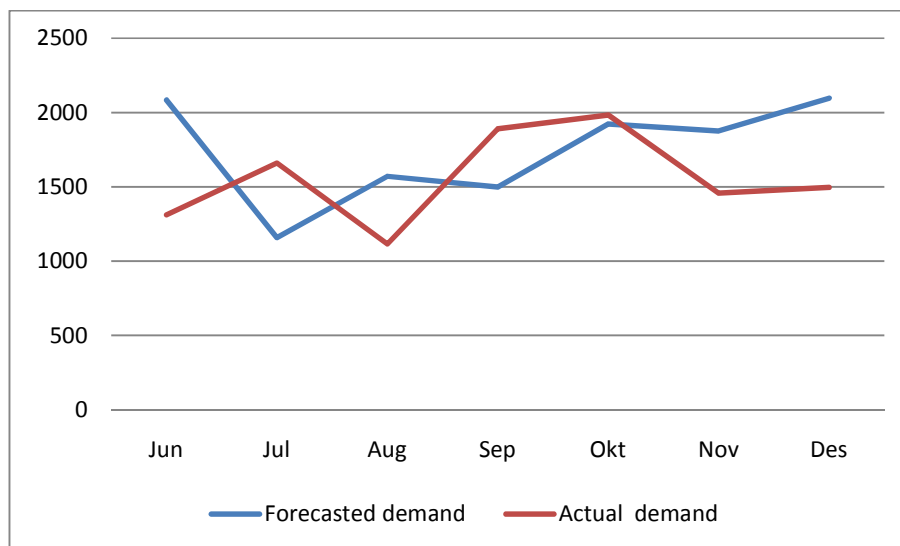


Figure 4. Actual and Forecasting demand for CC Textile Set products group in June-December 2010.

Besides the products options changed completely in the beginning of 2010. It means that sophisticated methods of forecasting such as Holt-Winters exponential smoothing or Box-Jenkins autoregressive models cannot be used to make item-level forecasts. Due to this fact the company should use aggregate forecasting which makes the process of determining the order quantities more difficult.

The task for the current master thesis is to develop and evaluate opportunities to solve earlier found problems. The research outcome should be a system capable to deal with all issues mentioned above and able to optimize logistical costs in the current situation.

3. Aggregation of products into groups

Demand forecasting is one of the most essential parts of logistical planning and in some cases can be the most time consuming task. Wrong forecasts of demands are listed among the main reasons of resource wastage in supply chains (*Lee et al., 1997*). According to *Dugnas, Oterhals, 2010*, development of correct forecasts can be one of the ways to improve operations management performance at Stokke. Due to this fact I will pay a lot of attention to demand forecasting in the current thesis.

Forecasting is a time consuming task, especially when it comes to sophisticated techniques such as Box-Jenkins approach, which will be discussed later in this thesis. The purpose of aggregation in this case is to decrease the number of forecasting models. A number of product modifications within the group of products which will be analyzed come up to 42 (current data). This scope of the problem makes the forecasting difficult and time consuming.

Another reason to aggregate products into groups is structural changes within the groups. In case of Stokke 7 product groups will be analyzed. These groups were present among the product variety for the three last years. But the structure of each of the groups (i.e. number and variance of products) has changed completely during March-April 2010. Hence forecasting methods incorporating seasonal or trend parameters could not be applied for single products as such. Thus it is reasonable to aggregate items into product groups.

3.1. An approach for aggregation of item-level demands

In most businesses, forecasting is not only made on the operational level as in our case, but is also used for making strategic decisions. At this level forecasts usually consider not every option of the product but the product family as a whole. Nowadays two main approaches of forecasting at the strategic level are discussed: bottom-up and top-down (*Schwarzkopf et al., 1988; Dangerfield and Morris, 1992; Zotteri et al., 2005; Widiarta et al., 2008*). According to the bottom-up approach the forecasts are made individually for every item and then aggregated. Under the top-down methodology the forecasts are made at an aggregated level. Some researchers argue that top-down forecasting is better because of its lower cost and better accuracy (*Schwarzkopf et al., 1988; Kahn, 1998; Lapide, 1998*). Besides aggregated forecasting is regarded as one of the

“risk pooling” strategies to reduce demand fluctuation (*Dekker et al., 2004*). Hence in our case aggregated forecasts could be used, not only for the operational planning of demands, but also for strategic purposes.

From the point of view of the operational planning, where decisions are made for each product separately, aggregate forecasting is not the best option (*Chen, Blue, 2010*). From the other side, in many cases the difference in results of the bottom-up and top-down approaches is insignificant and does not influence the quality of single products forecasts (*Widiarta et al., 2008*).

The discussion of aggregation approaches in the literature is sparse. Most authors do not pay too much attention to the technique of aggregation. Others say that in almost every case the correlation of product demands does not influence the quality of prognostication if products are related to the same product family (*Chen, Blue, 2010, Widiarta et al., 2009*). Thus the idea to use top-down approach in forecasting item-level demands for textile for Stokke Xplory children stroller seems to be good enough as the products are related to the same product family.

On the other hand there are two main weak sides of the research results proposed by now. At first, authors examine the behavior just of two demand series and their aggregate, and mention that behavior of the aggregate of three and more should be studied further (*Chen, Blue, 2010*). Besides, authors assume that demands are changing in the same way. They use either AR(1) or MA(1) models as subject for their studies. In case of AR(1) models it is assumed that time series is dependent on its level in previous period, and in case of MA(1) process it is dependent on its stochastic error in previous period (*Gujarati and Porter, 2009*). In our case we are going to build the ARIMA (p, d, q) model for aggregate series of three and more products. This model incorporates both components AR and MA of order p and q correspondingly and can be integrated of order d , i.e. deviation in series at time t is dependent on the value of variable in time $t-d$. Thus the results should be examined more carefully.

Secondly, while disaggregating the forecasts for the groups into forecasts for single items, most authors do not consider the dispersion of the option percentage¹. This increases the risk of stock failures. To avoid this mistake in the current project, the deviation of item-level demand will be regarded as the combination of the group demand deviation and

¹ The **option percentage** is frequency with which a variant item is used within a product family (*Schonsleben, 2007*)

the option percentage deviation. Formulas 1 and 2 will be used for each periodic demand (Schonsleben, 2007).

$$E(OD)=E(PFD)*E(OPC) \quad (1)$$

$$s^2(OD)=s^2(PFD)+s^2(OPC)+s^2(PFD)*s^2(OPC) \quad (2)$$

where OD – option demand

PFD – product family demand

OPC – option percentage

E() – expected mean value

s² – sample variance

In this case, the behavior of the product group time series will not lead to stock failures, because they will be secured by the safety stock, dependent on the standard deviation of demand. From the other side, if the optional percentage is very volatile, the standard deviation of the option demand will increase significantly.

Another question, which is not fully discovered in the literature, is the problem of grouping items into families of products to run aggregated forecasting. Most authors usually consider two time series with identical behavior. In case of a real situation, demands for different options of a product family could possess quite different characteristics. To avoid this problem it could be reasonable to run a two stage aggregation procedure:

1. At the first stage, the family is split in several parts according to the mean yearly demand.
2. At the second stage, each of the groups from the previous stage is split in two parts according to the coefficient of variation² or relative standard deviation (an absolute value of the coefficient of variation).

Relative standard deviation can be computed as the ratio of standard deviation to average value of demand series:

$$Relative\ standard\ deviation = \frac{Standard\ deviation}{Average}$$

A small value of relative standard deviation (<0,1) indicates that demand (time series) is stable. This means that it can be easier to predict its future values.

The first stage of such an aggregation will ensure that items with comparable demands are in the same group. This will lead to a decrease of option percentage

² The **coefficient of variation** is a measure of relative dispersion that expresses the standard deviation as a percentage of the mean (provided the mean is positive) (Newbold et al., 2009).

violations, because items with relatively large demand will not influence items with relatively small demand.

The second stage will ensure that demand forecasts for items with relatively small deviation of demand will not be affected by the high deviation of demand for other items in the group.

Unfortunately (from the forecasting's point of view), Stokke AS has completely changed the product line in March-April, 2010. Thus the time series of single products are not long enough to run the forecasting procedures for any of them using sophisticated techniques such as Holt-Winters exponential smoothing or Box-Jenkins models. Thus in this master thesis the product families will be considered to make aggregate forecasts.

3.2. Aggregation of the item-level demands into groups

In the current project we will analyze only textile products for the Stokke Xplory children stroller. The range of the products counts for 56 single products (March 2011). The forecasts will be built using the demand data set from January 2008 to December 2010. As some of the items (14 products) were not sold in this period, they will not be included into the forecasts. The procedure of determining order quantity for these items will be discussed later. The list of the analyzed products and characteristics of their demands is presented in Appendix 1.

The statistical characteristics of demand for each item were based on demand data from May 2010 to December 2010. The reason for the exclusion of two first months of sales (March and April 2010 or April and May 2010) is the significant disturbance of demand pattern. It can be explained by the launching of new products when each of the buyers would like to make the initial stock of new items. At the same time the rests of discontinued products are sold out during the launching period. These two factors make the demand pattern behave irregularly.

After the two first months of sale the buyers made necessary inventory and the sales of discontinued products were almost stopped. Hence they could not affect the pattern of demand for new products.

All of the analyzed products (42 items) are combined in 7 groups. They are:

- CC Textile Set;

- CC Textile Set for UK;
- Seat Textile Set;
- Seat Textile Set for UK;
- Foot Muff;
- Parasol;
- Changing bag.

Each of the groups consists of six items that differ in color. It is natural that the relationships between the items within the group are close and negative. If the demand for one item is growing the demand for others should decrease.

Table 1 presents correlations between the shares of the items within the CC Textile Set group.

Table 1 The correlation of the items' options percentage within the CC Textile Set group

		CC_Dark_Navy	CC_Blue	CC_Red	CC_Beige	CC_Purple	CC_Green
CC_Dark_Navy	Pearson Correlation	1	0.122	-0.706	0.112	0.01	0.258
	Sig. (2-tailed)		0.774	0.05	0.791	0.982	0.537
CC_Blue	Pearson Correlation	0.122	1	-0.48	-0.186	-0.498	.773*
	Sig. (2-tailed)	0.774		0.229	0.66	0.209	0.024
CC_Red	Pearson Correlation	-0.706	-0.48	1	-0.505	0.134	-0.323
	Sig. (2-tailed)	0.05	0.229		0.202	0.751	0.436
CC_Beige	Pearson Correlation	0.112	-0.186	-0.505	1	-0.122	-0.497
	Sig. (2-tailed)	0.791	0.66	0.202		0.773	0.21
CC_Purple	Pearson Correlation	0.01	-0.498	0.134	-0.122	1	-0.332
	Sig. (2-tailed)	0.982	0.209	0.751	0.773		0.422
CC_Green	Pearson Correlation	0.258	.773*	-0.323	-0.497	-0.332	1
	Sig. (2-tailed)	0.537	0.024	0.436	0.21	0.422	

Note: Correlation is significant at the 0.05 level (2-tailed).

It can be seen from table 1 that demands for most items are negatively correlated with others. It means that the increase of demand for one product should lead to the decrease of demand for others. From the other side, the correlation coefficients are not significant in most cases. Hence the conclusions about interdependency of demands are not reliable. This can be explained by relatively small number of entries (8 months data). It can

also be seen that the demand for Green and Blue CC Textile Sets is strongly positively correlated. And the correlation is significant. The reason could be that these two colors traditionally become fashionable together. Hence if the fashion for one of them is increasing the popularity of another grows up as well.

Table 2 will be used to compare the relative standard deviation of demand for single items within the CC Textile Set group and for the group in total. All the values are calculated based on the 8-months (May-December 2010) demand pattern.

Table 2 Characteristics of demand for items within CC Textile Set group

Product Name	Average Demand	Standard deviation of demand	Relative standard deviation
XPLORY Style Kit CC Dark Navy	374.6	113.2	0.302
XPLORY Style Kit CC Blue	176.9	90.8	0.513
XPLORY Style Kit CC Red	285.0	104.5	0.367
XPLORY Style Kit CC Beige	529.4	189.6	0.358
XPLORY Style Kit CC Purple	331.5	128.7	0.388
XPLORY Style Kit CC Green	92.8	60.6	0.654
Total	1790.1	687.5	0.384
CC Tex Set Group	1832	527.6	0.288

From Table 2 it can be seen that the average demand of **the group** is almost the same as the sum of average demands for **items taken separately**. But the value of relative standard deviation for the group is significantly less the sum of standard deviation of items in group. Hence we can conclude that the hypothesis about negative relationship between the demands for single items is approved and the variation of the group in total is less than sum of the single items variations.

Table 3 Aggregated groups of products

Group	January 2008-December 2010			May (June) -December 2010		
	Average	Standard deviation	Relative standard deviation	Average	Standard deviation	Relative standard deviation
CC Tex Set	1220.8	469.9	0.385	1832.0	527.6	0.288
CC Tex Set UK	195.6	90.3	0.462	333.6	65.3	0.196
Seat Tex Set	1595.4	538.1	0.337	2046.6	702.6	0.343
Seat Tex Set UK	245.6	105.8	0.431	330.4	181.4	0.549
Changing Bag	724.4	335.4	0.463	1241.0	286.6	0.231
Foot Muff	925.3	347.4	0.375	1174.4	371.6	0.316
Parasol	1107.4	691.6	0.625	1663.8	936.5	0.563

Knowing this fact we can conclude that if the variation of option percentage within the group is relatively small, then the forecasts of demand for single items will not differ significantly from disaggregated forecasts of demand for the group as a whole.

The next step is the evaluation of the demand characteristics of all seven groups calculated both for the whole demand pattern (January 2008 – December 2010) and for the period since launch of new products (March-December 2010). While calculating average demand and standard deviation of demand, the data for March and April 2010 were excluded for groups: CC Textile Set, Seat Textile Set, Foot Muff, Parasol. For the groups CC Textile Set UK, Seat Textile Set UK and Changing Bag the data for April and May 2010 were excluded from calculations. The launching process was going during these months; hence the demand behavior was unusual. If the data for these periods were taken into consideration these could lead to misleading conclusions and unnecessary forecasts errors.

As it can be seen from Table 3 the average demand for all the groups is much higher in May (June) – December 2010 than in January 2008- December 2010. This can be an indicator of significant increasing trend in sales. At the same time the relevant standard deviation in last 8 months is smaller than it in the whole period. Hence we can conclude that sales became more stable and predictable. This fact can be an additional indicator that inventory policy should be developed based on standard deviation of demand within May (June) – December 2010.

4. Development and evaluation of forecasting methods

This section is devoted to the development of forecasting methods which can be used to build the forecasts of demand for Stokke AS. The first step will include preliminary data analysis. It will consist of data plotting in order to evaluate the presence of trend and seasonality in demand patterns. The monthly demands for each of the product groups will be regarded as time series.

Then, based on the results of plotting, three methods of forecasting will be examined. The first one is a simple moving average method previously used in the company. The second one is Holt-Winters exponential smoothing widely used in business praxis. Since 2011 Stokke AS has started to use this technique. The third one is seasonal ARIMA (Box-Jenkins) methodology.

All of the methods referred to above are going to be used to make forecasts of demand for the second half of 2010. Then the results of forecasting will be compared with the real data. Afterwards the forecasting methods will be evaluated based on mean squared prediction error (MAPE)³ and some other measures of error.

4.1. Moving average model.

A moving average is a time series constructed by taking averages of several sequential values of another time series (*Hyndman, 2009*). It is one of the simplest techniques for business forecasting. In case of using a moving average model the forecasted value is determined as some combination of the previous values of the examined variable.

In general form, a moving average smoothing model can be presented in the following way:

$$Y_t = \sum_{j=1}^k \beta_j Y_{t-j} \quad (3)$$

Where Y_t - is the forecasted value in period t , k - is a number of smoothed periods and β_j - is the coefficient of smoothing for Y_{t-j} value of examined variable.

³ $MAPE = \frac{1}{n} \sum_{i=1}^n \frac{|y_i - \hat{y}_i|}{y_i} \cdot 100\%$, where y_i is the actual value of the dependent variable in period i , \hat{y}_i is the estimated value of the dependent variable in period i , n is the number of periodical observations.

In the simplest case it is assumed that values at each of the past periods has the same influence on forecasted value. Then all the coefficients are equal:

$$\beta_j = \frac{1}{k}, \forall j \in (\overline{1, k}) \quad (4)$$

In case of decaying importance of past values the coefficient β_j can be determined using following formulas:

$$\beta_1 = 1 - \sum_{j=2}^k \left(\frac{1}{k}\right)^{j-1} \quad (5)$$

$$\beta_j = \left(\frac{1}{k}\right)^{j-1}, \forall j \in (\overline{2, k}) \quad (6)$$

The number of lags to obtain moving average can be determined iteratively using one of the fitting techniques (for example, minimizing MAPE or any other error measurement).

As it can be seen, such techniques are not among the most sophisticated ones and hence they could give unpredictable errors in forecasting.

4.2. Holt-Winters exponential smoothing model

The family of exponential smoothing techniques is one of the most widely used in the business world (*Goodrich, 1989*). It includes a wide range of forecasting methods from most simple, single parameter methods, to the most sophisticated three parameter methods, such as Holt-Winters technique.

The Exponential smoothing technique is similar to the moving average methods. But whereas the simple moving average assigns equal coefficients to past observation, the exponential smoothing uses exponentially decreasing coefficients (*Goodrich, 1989*).

This chapter is devoted to the Holt-Winters methodology of model building. These models include three parameters: level, trend and seasonal component (*Janacek and Swift, 1993*).

The basic structure of this methodology was provided by C.C. Holt (1957) and P. Winters (1960) and is widely used up to date (*Goodwin, 2010*). There are two main types of Holt-Winters models: additive and multiplicative (*Kalekar, Bernard, 2004*).

In additive models the value of seasonal changes are presented as an absolute value and are added to the level irrespectively of level changes. Multiplicative models

incorporate the seasonality as the coefficient to which the level is multiplied at a certain period. It means the seasonal changes are related to the general level changes.

In the current master thesis I will use the multiplicative models as the pattern of demand data seems to be significantly changed over the time horizon. Hence the changes in demand caused by seasonality could also be volatile.

The basic forecasting equation for the multiplicative Holt-Winters model is (Goodrich, 1989):

$$\hat{Y}_{t+m} = (L_t + mT_t)\hat{I}_{t+m} \quad (7)$$

And the smoothing equations are:

$$L_t = \alpha \frac{Y_t}{I_{t-s}} + (1 - \alpha)(L_{t-1} + T_{t-1}) \quad (8)$$

$$T_t = \gamma(L_t - L_{t-1}) + (1 - \gamma)T_{t-1} \quad (9)$$

$$I_t = \delta \frac{Y_t}{L_t} + (1 - \delta)I_{t-s} \quad (10)$$

Where:

Y_t is the observation of demand at period t .

L_t is the smoothed observation of demand at period t .

T_t is the trend factor at period t .

I_t is the seasonal factor at period t .

m is the number of forecasted demand period.

α, γ, δ are the constant parameters of the Holt-Winters model.

To determine parameters α, γ, δ one of the software packages minimizing the MAPE of the forecasting model is usually used. In our case the Excel Solver will be used to determine the parameters of the model.

To determine all the parameters of the equations at least two periods' data are needed (in our case it is 2 years' demand observations). The initial values of the trend and seasonal factors are determined according to following formulas (NIST/SEMATECH, 2006):

$$T = \frac{\sum_{i=1}^s (Y_{s+i} - Y_i)}{s^2} \quad (11)$$

$$I_s = \frac{1}{n} \sum_{i=1}^n \sum_n \frac{Y_{n*i+s}}{A_n} \quad (12)$$

$$A_n = \frac{1}{s} \sum_{i=1}^s Y_s, \quad (13)$$

Where:

n is the number of periods

s is the number of seasons in one period.

A_n is the total demand in period n .

The initial value of smoothed observation L is set equal to actual observation Y in previous period.

4.3. Seasonal ARIMA (Box-Jenkins) model

The second type of proposed forecasting techniques is based on Box-Jenkins approach of model composition. This approach was first presented in the book of Box and Jenkins (1970) *Time Series Analysis, Forecasting, and Control*.

In this book the authors introduced a methodology to create autoregressive moving average model (ARMA). These models possess one essential requirement for the data set. The Box-Jenkins model assumes that the time series is *stationary*.

There are two types of stationarity. A time series is called *strictly stationary* when the joint probability distribution of its values is independent of the time of origin (Goodrich, 1989).

Wide sense stationarity exists when the first and the second order statistics (i.e. mean and covariences) are independent of time origin (Goodrich, 1989).

In common language, if the behavior of demand is not dependant on the time of observation, then the time series representing this demand is called stationary.

In the real world stationary processes are quite rare, thus in many cases the data should be preprocessed to become stationary times series. To obtain stationary series from non-stationary one, the last should be differentiated one or several times (NIST/SEMATECH, 2006). Models which will use this preprocessed data are called autoregressive integrated moving average models (ARIMA).

ARIMA models describe the current behavior of variables in terms of linear relationships with their past values. An ARIMA model can be decomposed in three parts. First, it has an Integrated (I) component, which represents the amount of differencing to be performed on the series to make it stationary. The second Autoregressive (AR) component

explains the correlation between the current value of the time series and some of its past values. The third Moving Average (MA) component represents the duration of the influence of a random (unexplained) shock (Weisang, Awazu, 2008). The general view of ARIMA (p, d, q) model is following:

$$Y_t^d = \theta + a_1 Y_{t-1}^d + \dots + a_p Y_{t-p}^d + \beta_0 u_t^d + \dots + \beta_q u_{t-q}^d \quad (14)$$

Where Y_t^d – is the d th difference of the observation at period t .

u_t^d is the white noise stochastic error of d -differenced observation of time series at period t .

θ is the constant term.

a_i, β_j are the coefficients of regression, $i \in (\overline{1, p}), j \in (\overline{0, q})$.

For example the ARIMA (1,0,1) model will look as follows (Gujarati, Porter, 2009):

$$Y_t = \theta + a_1 Y_{t-1} + \beta_0 u_t + \beta_1 u_{t-1} \quad (15)$$

The second weakness of the original ARMA models is that they do not fit seasonal time series. There are two ways of solving this problem. The first way is to deseasonalize data first and then apply the ARMA model. The second way is to incorporate the seasonal component in the model directly (Janacek and Swift, 1993). In this thesis we will use the second approach. The general view of ARIMA (p, d, q) model seasonal at lag s is:

$$Y_t^d = \theta + a_1 Y_{t-1}^d + \dots + a_p Y_{t-p}^d + a_1^s Y_{t-s}^d + \dots + a_p^s Y_{t-p-s}^d + \beta_0 u_t^d + \dots + \beta_q u_{t-q}^d \quad (16)$$

Where Y_{t-s}^d is the value of the d th difference of the observation at period $t-s$,

a_i^s - is the coefficient of seasonality at period s , $i \in (\overline{1, p})$

To use the Box-Jenkins approach of model building we should pass the three stages (NIST/SEMATECH, 2006):

1. Model Identification
2. Model Estimation
3. Model Validation

The first step includes the tests for stationarity and seasonality of the time series, determination of the I(d), the AR(p) and the MA(q) orders.

The second stage includes evaluation of coefficients in the model. The SPSS Inc. software will be used to run this stage in the current master thesis.

The model validation stage includes the analysis of the residuals. They should satisfy the assumptions of a stationary univariate process. It means that residuals should be independent on each other, be normally distributed and have constant mean and variance.

4.4. The order of ARIMA model identification, estimation and validation

The first step of ARIMA models building is model identification. It is based on the tests of stationarity and seasonality of data patterns.

The stationarity of time series will be determined based on Graphical Analysis and the Unit Root Test.

The graphical analysis could be useful as an initial procedure to determine the stationarity of time series. To run this analysis the data set should be plotted on a time based axis. If the deviation of the demand changes corresponding to changes in average value of demand this indicates that time series is not stationary.

To give more precise evaluation of stationarity the unit root test can be used. This kind of test of stationarity became widely popular over the past years (*Gujarati and Porter, 2009*). In the current master thesis the Augmented Dickey-Fuller (ADF) test will be used. The ADF test gives the possibility to test stationarity of time series which incorporates trend and seasonality. The general equation for the ADF test is the following (*Gujarati and Porter, 2009*):

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum_{i=1}^m \alpha_i \Delta Y_{t-i} + \varepsilon_t \quad (17)$$

Where $\Delta Y_t = Y_t - Y_{t-1}$, β_1 and β_2 are the drift coefficients, t is the trend, α_i are the coefficients of correlation error term to ΔY_{t-i} , and ε_t is the pure white noise error term.

If the value of δ coefficient is less than 0, than the time series is said to be stationary. Otherwise it is not. At the same time, the t value (the Dickey-Fuller statistics) of δ coefficient in absolute term is less than critical value of τ -statistics the time series is said to be nonstationary (*Gujarati and Porter, 2009*). In other words the value of δ coefficient should be significantly less than 0 to make conclusion that observed increase in demand is not dependent on previous increase of demand.

To determine the level of stationarity (i.e the period length between two dependent demand values) one should run the ADF test for first, second etc. differenced time series until the time series becomes stationary. In real world one could hardly find a logical sense of, for example, fourth difference. Nevertheless, the differencing is required to precede the process of model building, and on the last stage one can apply the reverse process of addition to get the value of demand.

The seasonality in the time series will be determined according to the graphical analysis of:

1. A run sequence plot.
2. Seasonal stacked line plots.
3. The autocorrelation plot.

All these techniques can be easily explained on a practical example. Thus we will leave detailed explanation till the data analysis chapter.

The next stage of the model identification is determining the order of autoregressive and moving average term. The preliminary stage will include analysis of autocorrelation (ACF) and partial autocorrelation functions (PACF). The rules for determining orders of AR(p) and MA(q) components are described in Table 4.

In practice it can be difficult to determine orders of autocorrelation and moving average terms, especially if models include seasonality. Hence the process of model building is full of trials and failures. The quality of the model depends on the experience of the forecaster. Thus ARIMA models based on the same data pattern could vary significantly depending on the expertise of the model builder.

Table 4 Theoretical patterns of ACF and PACF

Type of Model	Typical pattern of ACF	Typical Pattern of PACF
AR(p)	Decays exponentially or with damped sine wave pattern or both	Significant spikes through lags p
MA(q)	Significant spikes through lags q	Declines exponentially
ARMA(p, q)	Exponential decay	Exponential decay

Source: Gujarati and Porter, 2009

As in our case models are expected to be mixed, i.e include both AR(p) and MA(q) components, we will use the Akaike Information Criterion (AIC) to determine the lag length in our model (*NIST/SEMATECH, 2006*).

AIC criterion is defined as (*Gujarati and Porter, 2009*):

$$\ln AIC = \left(\frac{2k}{n}\right) + \ln\left(\frac{RSS}{n}\right) \quad (18)$$

Where k is the number of regressors, n – is the number of observation, RSS is regression sum of squares.

This criterion is used to compare several different models. The model with the smallest value of AIC test is the best one, i.e. it will give the most precise forecast.

The second step includes model estimation. The main approach for fitting the Box-Jenkins models is non-linear least squares. This technique is quite complicated hence it will be not included here. Fortunately, it is incorporated in most software packages, so it will be done automatically using EViews 4 package.

The third step is the model validation. It includes diagnosis to find out if the residuals follow the assumptions for a stationary univariate process (*Janacek and Swift, 1993*):

1. They are normally distributed. To check this hypothesis the Jarque-Bera test of normality will be used (*Newbold et al, 2009*). Here we will not describe the mathematical implementation of this test and will use automatically computed p-value. If it is reasonably high, then we cannot reject the normality assumption for residuals series.
2. They are independently distributed and have constant mean and variation. To test this assumption the ACF and PACF plots will be used. Residuals will be regarded as time series. The methodology of analysis is the same as that on the stage of model identification.

Afterwards, the MAPE of the model forecast will be checked both for known (January 2008 – December 2010) and unknown (January-March 2011) demand patterns.

4.5.Data analysis

The starting point for demand forecasting is preliminary data analysis which can be made using graphical techniques. The purpose of such analysis is to determine whether demand patterns are likely to be seasonal, trendy, have structural changes or any other shocks.

In the current master thesis graphical analysis will consist if line and bar charts, seasonal stacked line plots, and analysis of ACF and PACF plots.

Figure 5 shows the line charts of all seven product groups demand series. As it can be seen from these graphs all the lines incorporate significant peak of demand either in March 2010 or in April 2010. The reason for such sharp increase of demand is changes in

the structure of the product groups. Increased demand in these two months can be explained by initial inventory building processes at direct customers of Stokke AS.

For some groups with relatively low average demand (i.e. CC Textile Set for UK and Seat Textile Set for UK), significant increase in demand was followed by a sharp decrease in orders in May or June 2010. This decrease can mean that initial inventory of products within these two groups was too high, and that demand in UK was overestimated.

Line charts for CC Textile Set, CC Textile Set for UK, Seat Textile Set and Seat Textile Set for UK indicate that these demand series have some increasing trend. This hypothesis will be checked later during the process of model identification and validation.

Figure 6 includes bar charts of demand series and Figure 7 includes seasonal stacked line chart. They can be more helpful in determining the seasonality of demand series. The bar charts will visualize the changes in demand. In the seasonal stacked line charts the data pattern is divided in twelve seasons, for each of them the average value of demand is represented by horizontal line and the actual value of demand during three different years is represented by a broken line.

It can be seen from these figures, that CC Textile Set, CC Textile Set for UK, Seat Textile Set and Seat Textile Set for UK demand patterns are not likely to be significantly seasonal. But demand series of three other groups are seemed to be seasonal. The more precise evaluation will be done during the estimation of seasonality coefficients significance.

Time series for Changing Bag group has its peak values of demand in September, then is decaying till December and then begins to grow again. The peaks of average sales in April and May are seemed⁴ to be the results of the group structure changes in 2010.

Time series for Foot Muff has its peak demand in October, after this peak it is decaying till June and then starts to grow again. The reason for high average demand in April is the same as for the Changing Bag group. It is structural changes.

The demand pattern of the Parasol group has the most significant evidence of seasonality. It seems that peak sales of parasols are observed in April and May (even taking in consideration the shocks of 2010). Then average demand begins to decrease and is decaying till January. After that it is growing again.

⁴ The precise evaluation will be done during the process of coefficients estimation.

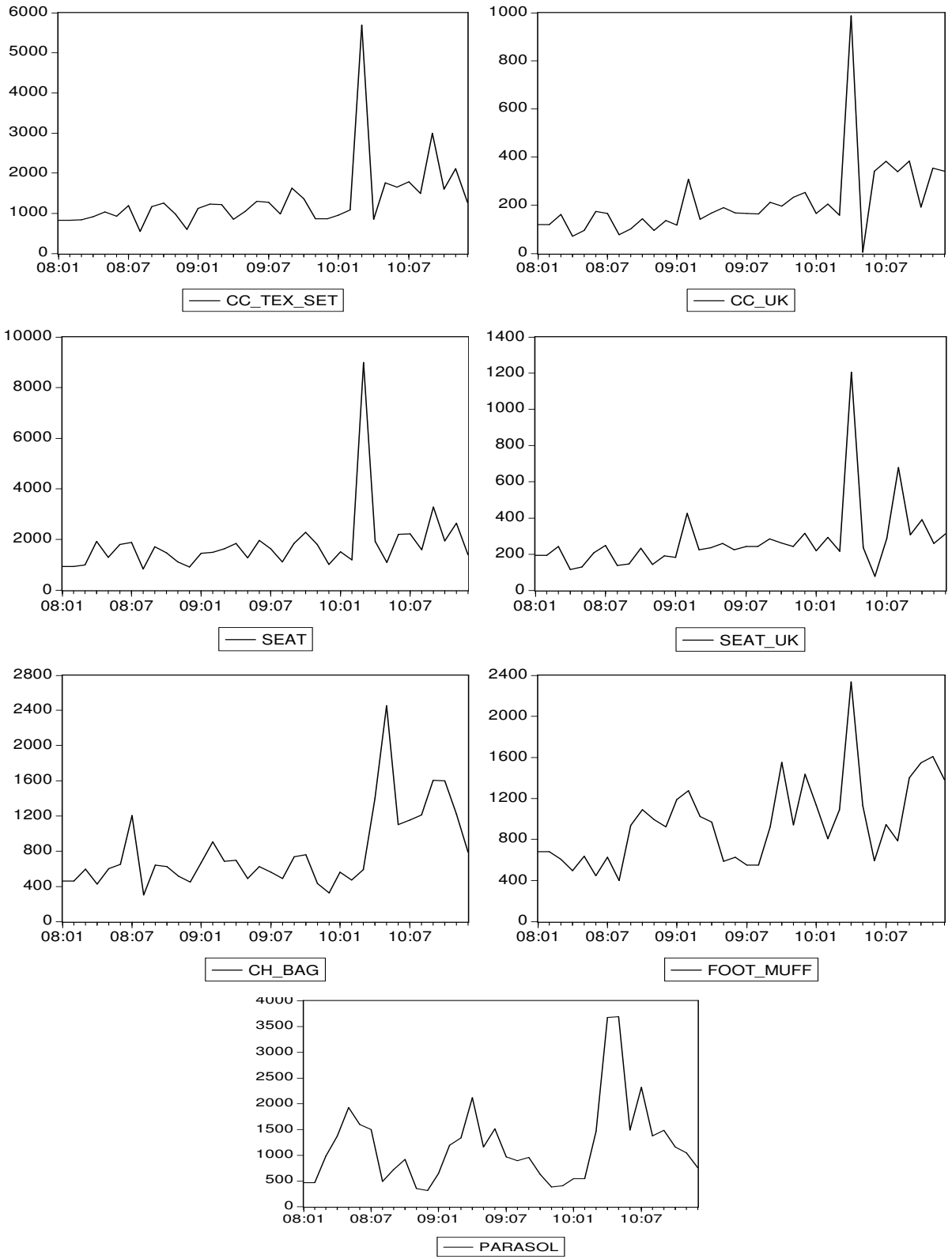


Figure 5 Line graphs of the groups demand series

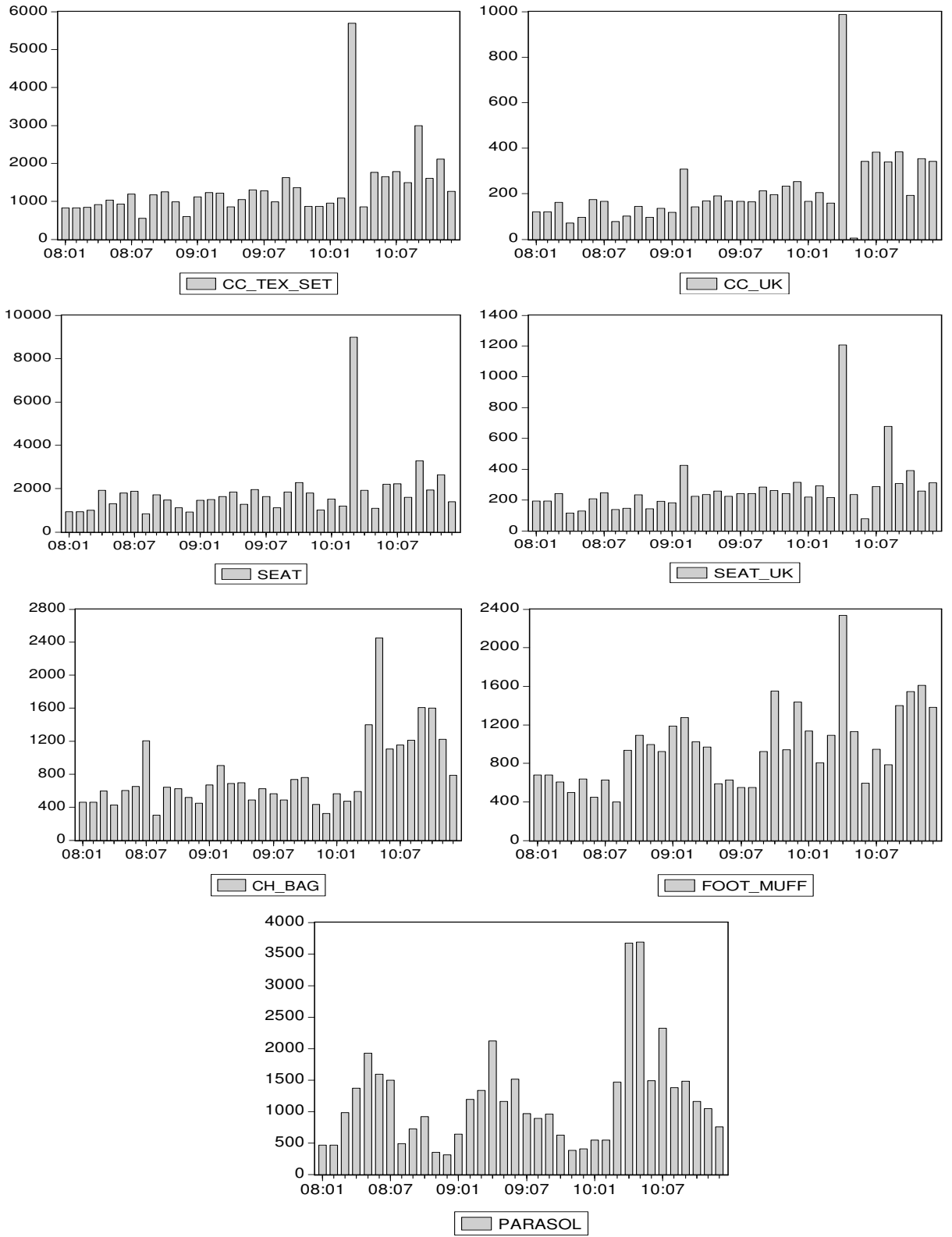


Figure 6 Bar graphs of the groups demand series

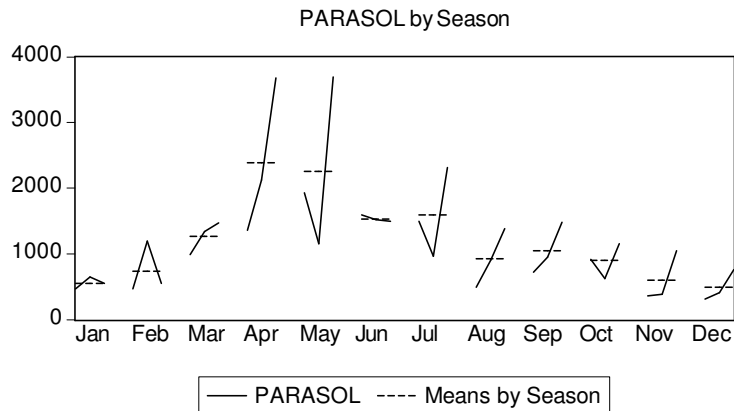
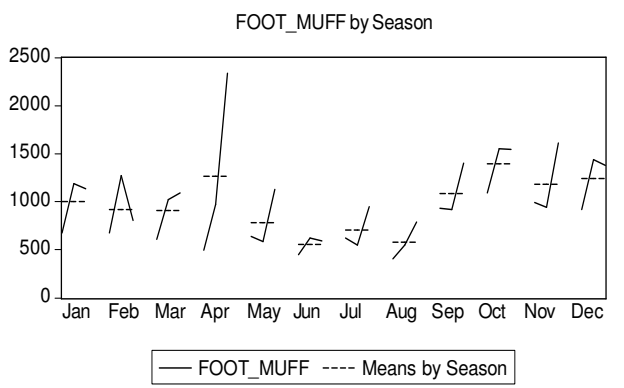
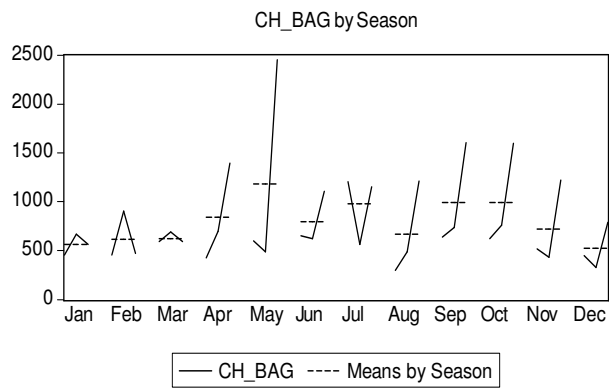
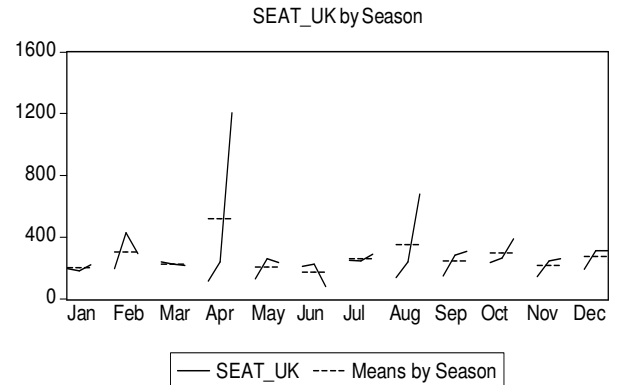
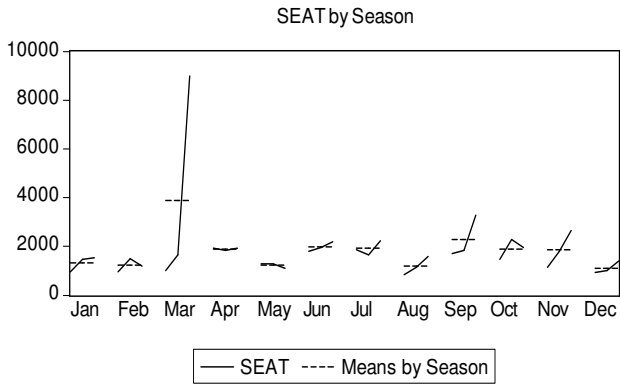
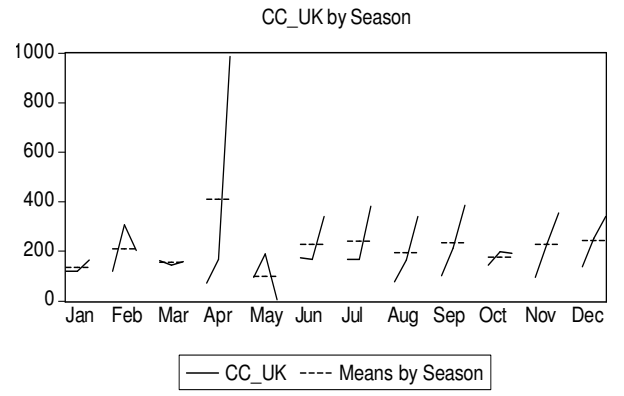
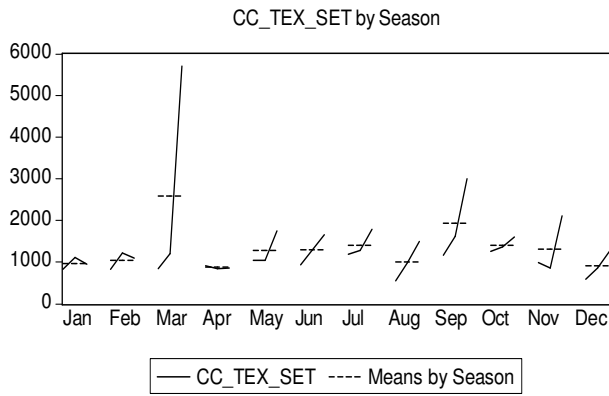


Figure 7 Seasonal stacked line plots of demand series

Preliminary data analysis shows that demand for most of the groups seems to be trendy, and that there are significant seasonal changes in sales of Changing Bags, Foot Muffs and Parasols.

The next step of data analysis will consist of test for **the model specification stage** of ARIMA models building.

The starting point of model specification is the determining of the stationarity order of time series. To determine the order of stationarity the Augmented Dickey-Fuller (ADF) test (*Gujarati and Porter, 2009*) will be used. The results of testing are presented in the table 5.

Table 5 ADF test of stationarity for demand series

Group	τ -value observed	τ -value critical (5%)	Level of stationarity
CC Tex Set	-7.15	-3.54	(0) with trend and intercept
CC Tex Set UK	-8.77	-3.54	(0) with trend and intercept
Seat Tex Set	-6.44	-3.54	(0) with trend and intercept
Seat Tex Set UK	-6.66	-3.54	(0) with trend and intercept
Changing Bag	-3.04	-2.95	(0) with intercept
Foot Muff	-4.41	-3.54	(0) with trend and intercept
Parasol	-2.96	-2.95	(0) with intercept

As it can be seen from the table 5, all the time series appeared to be stationary at level. This means that all the forecasting models for them can be described as ARIMA (p, 0, q) or ARMA(p, q) processes.

Additional evidence, that series are stationary, is the shapes of autoregressive function (ACF) presented in Figure 8. As it can be seen ACF is decaying to zero, hence we can conclude that time series are stationary.

Another outcome from the analysis of autocorrelation (ACF) and partial autocorrelation functions (PACF) is that some time series (CC Textile Set, CC Textile Set for UK, Seat Textile Set and Seat Textile Set for UK) follows the random process. Nevertheless, we will try to model this processes.

Using Figure 8 and rules from Table 4 we can determine expected order of AR(p) and MA(q) processes. Unfortunately, as it was said before, mixed models are usually difficult to identify. Thus we will use the AIC criterion (*Gujarati and Porter, 2009*) to determine the model which fits best.

Significant spikes at lag 6 of CC Textile Set series ACF indicate that model incorporates either AR(6) or MA(6) seasonal component. It would be valuable to check existence of year seasonality.

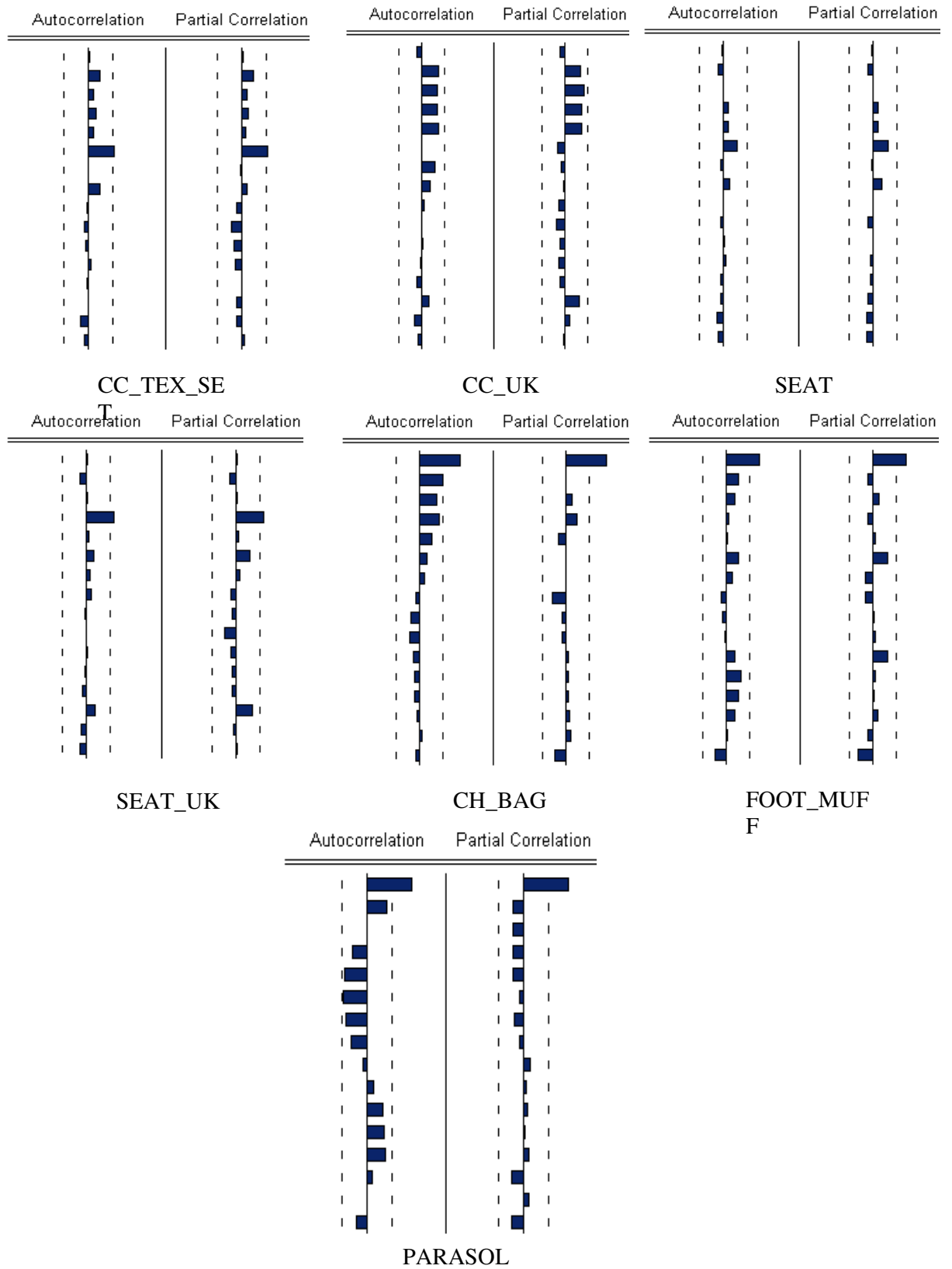


Figure 8 ACF and PACF graphs of demand series

CC Textile Set for UK series seems to behave as a random process but it can be assumed that it follows the same ARMA(p, q) model as CC Textile Set.

Seat Textile Set series also reminds random process, but theoretically we can assume that Seat Textile Sets are bought together with CC Textile Sets. The evidence for such assumption is the shape of ACF functions for these two series. The values of ACF and PACF for Seat Textile Set series are also increasing at lag 6 but do not reach the critical value.

Significant spikes at lag 4 of Seat textile Set for UK series ACF indicate the model incorporates seasonal component at lag 4.

The rest of time series most probably follow AR(1) process with seasonal component at lag 12.

We could not yet make final conclusions about ARIMA(p, d, q) model specifications. But this preliminary data analysis could be used as a starting point for forecasting models building.

4.6. Forecasting using Moving Average models

For each of the groups six types of moving averages forecasting models were examined: three simple moving averages models with 3, 6 and 12 smoothed periods and three weighted moving averages models with 3, 6 and 12 smoothed periods.

For each of the product group all six type of moving average models were built and evaluated using MAPE. Then the best of them was chosen to run forecast for January-February 2011. The forecasting equation for a simple moving average is:

$$F_{t+k} = \frac{1}{n} \left(\sum_{j=1}^{n-k-1} F_{t+j} + \sum_{i=0}^{n-k} Y_{t-i} \right), \text{ if } k \leq n \quad (19)$$

$$F_{t+k} = \frac{1}{n} \sum_{i=k-n}^{k-1} F_{t+i}, \text{ if } k > n \quad (20)$$

Where n is the number of smoothed periods, k is the number of forecasted period, F_i is the forecast for period i , and Y_i is the actual value of variable in period i .

I.e. each unknown value of variable Y_i is replaced by forecasted value F_i of this variable. The same technique is used when a weighted moving average model is applied.

Table 6 Forecasting models and their quality using moving average approach

Group	Moving Average Type	MAPE past	max APE	MAPE future
CC Tex Set	Simple Moving Average 12 months	22.53	56.84	8.08
CC Tex Set UK	Simple Moving Average 12 months	21.54	60.23	46.89
Seat Tex Set	Simple Moving Average 12 months	22.75	59.38	18.56
Seat Tex Set UK	Simple Moving Average 12 months	24.50	222.2	31.13
Changing Bag	Weighted Moving Average 12 months	25.26	74.42	11.24
Foot Muff	Weighted Moving Average 3 months	25.71	78.89	48.25
Parasol	Weighted Moving Average 12 months	32.61	76.94	35.33

The results of the forecasts were compared to real demand values using MAPE. Results of forecasting processes are presented in Table 6.

It can be seen that the demand for most of the product groups can be better predicted using 12 months moving average. The demand for CC and Seat Textile Sets is more stable. Thus models with equal weights resulted in smaller MAPE value. The demand for changing bags, foot muffs and parasols is more volatile. Hence weighted average models performed better for these product groups.

MAPE of forecasts for past values of time series (used for modeling) are varying between 20% and 33%. But the maximum absolute percentage error of forecasts is exceeding 50% for all of the groups and reaches 222,2% for Seat Textile Set for UK group. It means that forecasts built using moving averages techniques could lead to severe problems in inventory management operations.

4.7. Forecasting using Holt-Winters exponential smoothing models

The next method used to make forecasts of demand for product groups is Holt-Winters exponential smoothing. The parameters α, γ, δ were determined using the Excel Solver application while minimizing MAPE past of forecast.

Equation 7 was then used to make forecasts for January-February 2011 and MAPE of these forecasts was calculated. The results of calculations are presented in Table 7.

Table 7 Forecasting models and their quality using Holt-Winters exponential smoothing approach

Group	Parameter of Holt-Winters model			MAPE past	MAPE future	max APE
	α	γ	δ			
CC Tex Set	0.003	0	0.56	22.6	7.7	54.5
CC Tex Set UK	0	0	0.24	16.5	33.3	48.8
Seat Tex Set	0.024	0	0.52	22.6	11.8	53.3
Seat Tex Set UK	0.048	0	1	21.3	12.1	50.0
Changing Bag	0.000002	1.0	0.88	12.6	4.2	54.7
Foot Muff	0.034	0	1	20.3	8.2	59.4
Parasol	0.000125	0.06	0.67	34.3	18.3	80.5

These models give better results than simple or weighted moving average models presented in the previous section. But still MAPE varies between 12% and 34%. This means that on average, forecasts are different from actual demand patterns up to 35%. Besides, maximum absolute percentage error could reach 80,5% for parasol and is varying for other groups from 50% to 60%. It means that in worst case there could be significant shortage of 80% of expected demand should be kept in storage to avoid stock outs. We will pay more attention to this issue in the following chapters.

4.8. Forecasting using Box-Jenkins approach

The process of ARIMA model building is quite complicated and requires a lot of practical experience from a forecaster. Hence we will not claim that models discussed in this chapter are the best ones for examined demand patterns. But the author took as much efforts as he could to build the most appropriate regressions to fit demand time series.

Table 8 presents final models for each of the demand groups and their characteristics.

As it can be seen from the table almost all of the forecasting models (except the model for CC Textile Set Group) incorporate wither AR(p) or MA(q) component. Besides, all the models are either AR or MA seasonal.

High level of R^2 indicates that specified models can explain more than 82% (in most cases more than 90%) of demand series variation. F-statistic indicates that all the R^2 are significant.

Table 8 ARIMA models specifications for product group series

Group	ARIMA model specification	R squared	F statistic	AIC	Residuals normality test	
					JB	P-value
CC Tex Set	(2,0,6) ¹²	0.9	68.69	14.35	1.7759	0.4114
CC Tex Set UK	(1,0,0) ¹²	0.889	60.01	11.09	2.04	0.3603
Seat Tex Set	(0,0,0) ¹²	0.944	179.91	14.54	0.79	0.67
Seat Tex Set UK	(4,0,0) ¹²	0.8259	44.29	11.89	0.73	0.69
Changing Bag	(0,0,3) ¹²	0.935	68.6	12.88	0.821	0.663
Foot Muff	(4,0,0) ¹²	0.86	32.03	13.27	4.099	0.128
Parasol	(5,0,0) ¹²	0.909	64.96	14.16	2.69	0.2597

Table 9 presents values of MAPE and max APE of forecasts made by these models.

Table 9 Forecasting models and their quality using Box-Jenkins approach

Group	MAPE past	MAPE future	max APE
CC Tex Set	20.48	14.65	54.30
CC Tex Set UK	47.19	29.52	538.71
Seat Tex Set	21.09	22.17	70.31
Seat Tex Set UK	26.49	33.16	274.28
Changing Bag	23.68	38.51	67.80
Foot Muff	28.70	19.26	83.75
Parasol	32.14	45.13	106.08

It can be seen that values of MAPE are higher than 20% for all of the models and could reach up to 50% for some of the groups (CC Textile Ste for UK Group). Hence even such sophisticated methods as Box-Jenkins could not provide the company with reliable forecasts.

The following step of forecasting techniques evaluation will be devoted to the comparison of the result of three different approaches.

4.9. Forecasting models comparison and conclusions

To make final conclusions about possibilities to use one or several of concerned forecasting methods we compared the MAPE received from the forecasts built using each of the approaches (table 10).

The smallest values of the forecasts' MAPE for each of the product groups are marked with bold font.

As it can be seen, forecasts built using Moving Average approach gave the worst results. For four out of seven groups the Holt-Winters exponential smoothing approach

demonstrated the best result. For three others groups the ARIMA model (Box-Jenkins approach) gave the best results.

Table 10 The comparison of the forecasting methods

Group	MAPE			
	Moving Average approach	Holt-Winters approach	Box-Jenkins approach	Best
CC Tex Set	22.53	22.6	20.48	20.48
CC Tex Set UK	21.54	16.5	47.19	16.5
Seat Tex Set	22.75	22.6	21.09	21.09
Seat Tex Set UK	24.5	21.3	26.49	21.3
Changing Bag	25.26	12.6	23.68	12.6
Foot Muff	25.71	20.3	28.7	20.3
Parasol	32.61	34.3	32.14	32.14

On the other hand, neither the results of the ARIMA approach nor the results of the Holt-Winters approach are significantly higher than the results of the Moving Average approach.

At the same time, the ARIMA model building is time consuming and requires a lot of theoretical and practical experience from forecaster. The Holt-Winters exponential smoothing and the Moving Averages approaches are easier to understand and apply.

On the other side, the distribution of the forecast error while using the Holt-Winters exponential smoothing and the Moving Averages approaches cannot be modeled. Hence these models could not give the clear prediction of time series behavior in the future. Thus there is no way to examine theoretically how large could be the deviation of the real demand from the forecasted one.

Another issue is the value of MAPE. For most of the forecasting techniques it is above 20%. The usage of such high value of MAPE will not lead to significant benefits comparing to the usage of methodology based on assumption of the random behavior of demand.

Thus instead of using the values of forecasted demand and MAPE term to determine the order quantity and needed safety stock, it can be easier to use the assumption of demand randomness.

These findings lead to a conclusion that the difference between the real demand and the forecasted by Stokke professionals occurs not due to usage of wrong forecasting technique but due to behavior of demand pattern.

Stokke AS is specialized on sales of children products. The demand for this product is determined by the birth rate in market countries, and the birth rate is determined by many economical, social, ecological factors etc. At the same time there is no clear dependency between the birth rate in two adjacent years. Thus the usage of the autoregressive techniques of demand forecasting will not be beneficial in determining the expected demand.

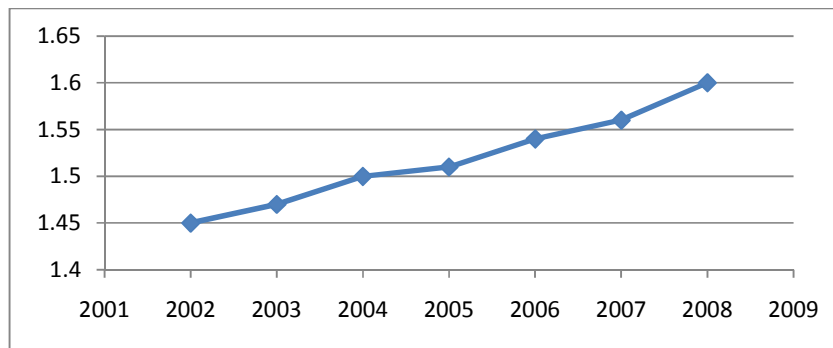


Figure 9. Total fertility rate in European Union (27 countries). Number of children per woman

Source: Eurostat, 2011

From the other side the birth rate in Europe is more or less stable (see Figure 9). The increase in birthrate during 2002-2008 was 0,15 children per women. Hence the only possibility to increase the sales is the enlarging of the market share which is not easy to do.

Thus from the inventory management point of view the demand for children product in Europe can be regarded as random varying about constant mean with constant standard deviation. Having this assumptions in mind one could use one of the known techniques for order quantity determination. One of such model will be examined and extended in the second part of the master thesis.

5. Determining the expected item-level demand

5.1. Test of normality for demand groups' time series

The expected item-level demand can be determined (Waters, 2003):

- based on qualitative methods, such as marketing forecasts, subjective evaluation of single specialist and so on:
- based on quantitative methods, such as projective (based on past data about demand) or casual (based on factors which are determining the demand in future) forecasting.

Due to restrictions on data availability we decided to focus on projective demand forecasting. A few methods of forecasting were examined but unfortunately none of them gave reasonably good results.

Having this and some other issues in mind it was decided to consider the demand as a random variable following one of the known distributions.

I will start the analysis from determining whether the demand data follows the normal distribution.

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of cc is normal with mean 758.028 and standard deviation 355.083.	One-Sample Kolmogorov-Smirnov Test	.088	Retain the null hypothesis.
2	The distribution of cc_uk is normal with mean 195.611 and standard deviation 87.653.	One-Sample Kolmogorov-Smirnov Test	.156	Retain the null hypothesis.
3	The distribution of seat is normal with mean 1,604.389 and standard deviation 525.165.	One-Sample Kolmogorov-Smirnov Test	.708	Retain the null hypothesis.
4	The distribution of seat_uk is normal with mean 245.278 and standard deviation 102.708.	One-Sample Kolmogorov-Smirnov Test	.169	Retain the null hypothesis.
5	The distribution of ch_bag is normal with mean 758.028 and standard deviation 355.083.	One-Sample Kolmogorov-Smirnov Test	.088	Retain the null hypothesis.
6	The distribution of foot_muff is normal with mean 930.056 and standard deviation 338.5.	One-Sample Kolmogorov-Smirnov Test	.558	Retain the null hypothesis.
7	The distribution of parasol is normal with mean 1,042.028 and standard deviation 510.23.	One-Sample Kolmogorov-Smirnov Test	.947	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure 10 Kolmogorov-Smirnov test of time series distribution

But as it is known, each demand series incorporates shock of demand in March (April) 2010. Thus we at first replaced the actual values of demand at these months by average values of demand for whole time series excluding this months. Then the Kolmogorov-Smirnov test was used to check if the time series follow normal distribution (Newbold et al, 2009). The results of test are presented in the Figure 10.

As it can be seen from Figure 10, all the time series are normally distributed. Hence we can assume the demand data to be random variables following the normal distribution.

5.2. Average item level demand and standard deviation

In case if we assume that the data are normally distributed there is no need to aggregate single items in product groups. The reason for that is that 7-8 months demand pattern is enough to check if the demand series follows the normal distribution. Then if the hypothesis is approved we can simply use the average demand and standard deviation of demand as input parameters for determining the necessary order quantities.

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of CC_DR_NAVY is normal with mean 374.625 and standard deviation 113.171.	One-Sample Kolmogorov-Smirnov Test	.817	Retain the null hypothesis.
2	The distribution of CC_BLUE is normal with mean 176.875 and standard deviation 90.8.	One-Sample Kolmogorov-Smirnov Test	1.000	Retain the null hypothesis.
3	The distribution of CC_RED is normal with mean 285 and standard deviation 104.544.	One-Sample Kolmogorov-Smirnov Test	.728	Retain the null hypothesis.
4	The distribution of CC_BEIGE is normal with mean 529.375 and standard deviation 189.612.	One-Sample Kolmogorov-Smirnov Test	.827	Retain the null hypothesis.
5	The distribution of CC_PURPLE is normal with mean 331.5 and standard deviation 128.718.	One-Sample Kolmogorov-Smirnov Test	.900	Retain the null hypothesis.
6	The distribution of CC_GREEN is normal with mean 92.75 and standard deviation 60.644.	One-Sample Kolmogorov-Smirnov Test	.888	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure 11. Kolmogorov-Smirnov test for CC Textile Set items demand

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of CC_UK_NAVY is normal with mean 78.286 and standard deviation 18.839.	One-Sample Kolmogorov-Smirnov Test	.772	Retain the null hypothesis.
2	The distribution of CC_UK_BLUE is normal with mean 32.429 and standard deviation 39.042.	One-Sample Kolmogorov-Smirnov Test	.430	Retain the null hypothesis.
3	The distribution of CC_UK_RED is normal with mean 69.429 and standard deviation 41.117.	One-Sample Kolmogorov-Smirnov Test	.942	Retain the null hypothesis.
4	The distribution of CC_UK_BEIGE is normal with mean 56.286 and standard deviation 34.355.	One-Sample Kolmogorov-Smirnov Test	.938	Retain the null hypothesis.
5	The distribution of CC_UK_PURPLE is normal with mean 74.857 and standard deviation 22.967.	One-Sample Kolmogorov-Smirnov Test	.984	Retain the null hypothesis.
6	The distribution of CC_UK_GREEN is normal with mean 14.429 and standard deviation 23.107.	One-Sample Kolmogorov-Smirnov Test	.532	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure 12. Kolmogorov-Smirnov test for CC Textile Set for UK items demand

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of SEAT_NAVY is normal with mean 450.125 and standard deviation 222.061.	One-Sample Kolmogorov-Smirnov Test	.776	Retain the null hypothesis.
2	The distribution of SEAT_BLUE is normal with mean 216.375 and standard deviation 137.632.	One-Sample Kolmogorov-Smirnov Test	.997	Retain the null hypothesis.
3	The distribution of SEAT_RED is normal with mean 282.625 and standard deviation 127.006.	One-Sample Kolmogorov-Smirnov Test	.859	Retain the null hypothesis.
4	The distribution of SEAT_BEIGE is normal with mean 494.625 and standard deviation 328.685.	One-Sample Kolmogorov-Smirnov Test	.492	Retain the null hypothesis.
5	The distribution of SEAT_PURPLE is normal with mean 408.75 and standard deviation 150.329.	One-Sample Kolmogorov-Smirnov Test	.999	Retain the null hypothesis.
6	The distribution of SEAT_GREEN is normal with mean 139.25 and standard deviation 182.007.	One-Sample Kolmogorov-Smirnov Test	.755	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure 13. Kolmogorov-Smirnov test for Seat Textile Set items demand

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of SEAT_UK_NAVY is normal with mean 77 and standard deviation 57.364.	One-Sample Kolmogorov-Smirnov Test	.884	Retain the null hypothesis.
2	The distribution of SEAT_UK_BLUE is normal with mean 35.857 and standard deviation 47.995.	One-Sample Kolmogorov-Smirnov Test	.595	Retain the null hypothesis.
3	The distribution of SEAT_UK_RED is normal with mean 64.714 and standard deviation 69.057.	One-Sample Kolmogorov-Smirnov Test	.965	Retain the null hypothesis.
4	The distribution of SEAT_UK_BEIGE is normal with mean 46.429 and standard deviation 52.69.	One-Sample Kolmogorov-Smirnov Test	.572	Retain the null hypothesis.
5	The distribution of SEAT_UK_PURPLE is normal with mean 67.143 and standard deviation 51.867.	One-Sample Kolmogorov-Smirnov Test	.850	Retain the null hypothesis.
6	The distribution of SEAT_UK_GREEN is normal with mean 8.286 and standard deviation 9.895.	One-Sample Kolmogorov-Smirnov Test	.862	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure 14. Kolmogorov-Smirnov test for Seat Textile Set for UK items demand

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of FOOTMUFF_NAVY is normal with mean 239.875 and standard deviation 92.185.	One-Sample Kolmogorov-Smirnov Test	.993	Retain the null hypothesis.
2	The distribution of FOOTMUFF_BLUE is normal with mean 123.875 and standard deviation 30.012.	One-Sample Kolmogorov-Smirnov Test	.658	Retain the null hypothesis.
3	The distribution of FOOTMUFF_RED is normal with mean 190.375 and standard deviation 75.934.	One-Sample Kolmogorov-Smirnov Test	.560	Retain the null hypothesis.
4	The distribution of FOOTMUFF_BEIGE is normal with mean 285.25 and standard deviation 116.288.	One-Sample Kolmogorov-Smirnov Test	.625	Retain the null hypothesis.
5	The distribution of FOOTMUFF_PURPLE is normal with mean 205.125 and standard deviation 120.39.	One-Sample Kolmogorov-Smirnov Test	.750	Retain the null hypothesis.
6	The distribution of FOOTMUFF_GREEN is normal with mean 97.375 and standard deviation 35.689.	One-Sample Kolmogorov-Smirnov Test	.965	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure 15. Kolmogorov-Smirnov test for Foot Muff items demand

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of CH_BAG_NAVY is normal with mean 261.714 and standard deviation 55.641.	One-Sample Kolmogorov-Smirnov Test	.998	Retain the null hypothesis.
2	The distribution of CH_BAG_BLUE is normal with mean 127.429 and standard deviation 41.972.	One-Sample Kolmogorov-Smirnov Test	.999	Retain the null hypothesis.
3	The distribution of CH_BAG_RED is normal with mean 206.714 and standard deviation 60.046.	One-Sample Kolmogorov-Smirnov Test	.899	Retain the null hypothesis.
4	The distribution of CH_BAG_BEIGE is normal with mean 311.714 and standard deviation 82.52.	One-Sample Kolmogorov-Smirnov Test	.959	Retain the null hypothesis.
5	The distribution of CH_BAG_PURPLE is normal with mean 244.571 and standard deviation 83.843.	One-Sample Kolmogorov-Smirnov Test	.936	Retain the null hypothesis.
6	The distribution of CH_BAG_GREEN is normal with mean 86.429 and standard deviation 32.485.	One-Sample Kolmogorov-Smirnov Test	.926	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure 16. Kolmogorov-Smirnov test for Changing Bag items demand

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of PARASOL_NAVY is normal with mean 236.75 and standard deviation 78.234.	One-Sample Kolmogorov-Smirnov Test	.241	Retain the null hypothesis.
2	The distribution of PARASOL_BLUE is normal with mean 170.5 and standard deviation 87.25.	One-Sample Kolmogorov-Smirnov Test	.839	Retain the null hypothesis.
3	The distribution of PARASOL_RED is normal with mean 249.125 and standard deviation 115.74.	One-Sample Kolmogorov-Smirnov Test	.588	Retain the null hypothesis.
4	The distribution of PARASOL_BEIGE is normal with mean 314.5 and standard deviation 97.728.	One-Sample Kolmogorov-Smirnov Test	.893	Retain the null hypothesis.
5	The distribution of PARASOL_PURPLE is normal with mean 237 and standard deviation 86.308.	One-Sample Kolmogorov-Smirnov Test	1.000	Retain the null hypothesis.
6	The distribution of PARASOL_GREEN is normal with mean 112.75 and standard deviation 62.417.	One-Sample Kolmogorov-Smirnov Test	.888	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure 17. Kolmogorov-Smirnov test for Parasol items demand

Hence we need to check the hypothesis whether distribution of single items demand follows the normal distribution or not. To perform this analysis we will use the Kolmogorov-Smirnov test (*Newbold et al, 2009*). The results are presented in Figures 11-17.

The test results show that all the item-level demand series follow the normal distribution. Hence we could use the determined values of mean demand and standard deviation of demand to calculate the optimal order quantities. Besides there is no need to aggregate products into groups as the process of determining the mean demand and standard deviation of demand can be easily done using any statistical program or simple packages such as MS Excel for example.

The absence of necessity to aggregate products into groups makes the process of order quantity determination faster and easier. In this case the expected value of items demand will not be affected by the behavior of demand for other items in product groups.

Besides, the aggregate forecasts would be finally disaggregated in single-item forecasts. This means that it wouldn't be possible to decrease the inventory level due to the smaller deviation of demand for the product group as it was discussed in paragraph 3.2. Hence the item-level forecasting based on assumption of randomness could be the most appropriate decision for the company.

6. Transportation and Inventory Management Optimization

There are two main replenishment systems for managing individual-item inventories (*Silver et al, 1998*):

- based on determining the optimal pair of order quantity (Q) and re-ordering point (s) – continuous review systems;
- based on determining the optimal length of replenishment cycle (R) and the maximum inventory level (S) – periodic review systems.

From the practical point of view each of the systems has its negative and positive sides. In continuous review systems the order quantity always stays unchanged hence the producer always knows how much will be ordered. From the other side the period between orders can vary significantly.

In periodic review systems the situation is opposite. The order quantity can vary significantly, while the period between orders always stays unchanged. Most authors (*Silver et al, 1998; Zhou et al, 2008; Waters, 2003*) agree that in case of large stable demand it is better to use the periodic review inventory systems.

In case of Stokke AS the company has an agreement with its suppliers that orders are placed once a month. The same type of agreement is desired both by producers and purchasers due to managerial issues. Hence it was decided to use the periodic review system to determine the inventory level and minimize logistical costs.

6.1. Traditional periodic review system for individual items under probabilistic demand

In periodic review systems every R unit of time a replenishment order is placed to raise the inventory up to level S. As well as in other systems for managing individual-item inventories, an optimal pair of (R, S) parameters is determined while minimizing the function of total logistical costs (*Silver et al, 1998*):

$$TRC = \frac{D}{D_R} \cdot A + \left(\frac{D_R}{2} + k \cdot \delta_{R+L} \right) \cdot v \cdot r + \frac{D}{D_R} * B_2 * \delta_{R+L} * G_u(k) \quad (21)$$

where L is the lead time;

D is the total yearly demand;

D_R is the demand over replenishment period R;

δ_{R+L} is the standard deviation of demand over replenishment period R and lead time L;

v is a unit purchasing costs;

r is the internal rate of holding costs as a percentage of unit costs;

B_2 is the cost per unit stockout;

k is a safety factor and;

$G_u(k) = \int_k^\infty (u_0 - k) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u_0^2}{2}\right) du_0$, a specified function of the unit normal

distribution (mean 0, standard deviation 1) variable. It shows the total number of units short if the service level is set to be k , and demand has the mean equal to 0 and standard variation equal to 1.

Order-up-to-level value S is determined as:

$$S = D_R + ss = D_R + k \cdot \delta_{R+L} \quad (22)$$

where ss is a safety stock.

Specified charge (B_2) per unit short was chosen as a measurement for stock out costs due to the fact that this value has already been evaluated during the first stage of cooperation between Stokke AS and Møreforsking Molde. This value shows how much the company will lose if one unit of product will not be delivered in time to customer. As the process of shortage costs estimation could take long time and is quite difficult, it seems reasonable to use predefined values.

The following step is modifying the model presented in equation (21). The transportation costs will be incorporated into the function of total logistical costs. In addition the problem will be enriched with constraints concerning the capacities of transportation modes.

6.2. Modified periodic review system for inventory and transportation management

The primary goal of modifications, which will be implemented in the traditional periodic review system, is to regard all kind of logistical costs simultaneously. This means that ordering, holding and shortage costs, which are usually considered, will be complemented by transportation costs.

To do that we need to introduce several assumptions:

1. The smallest batch of any type of product can consist of one box of items.
2. Transportation costs stay unchanged throughout the year for any type of transport modes.
3. Unit costs, shortage unit costs and other parameters stay unchanged during the

planning horizon.

4. Utilization of transport modes is not influenced by any subjective factors (for example, by experience of workers packing the container).

The starting point will be a model for one item which is presented by equations 23–27.

$$\begin{aligned} \min \frac{1}{R} \cdot a + \left(\sum_{m \in M} \frac{DR_m}{2} + SD_{R+l} \cdot K \right) \cdot v \cdot i + \frac{1}{R} \cdot b_2 \cdot SD_{R+l} \cdot G_u(K) \\ + \frac{1}{R} \sum_{m \in M} Q_m \cdot c_m \end{aligned} \quad (23)$$

Subject to

$$DR_m \cdot u \leq Q_m * o_m, \forall m \in M \quad (24)$$

$$K \leq ub \quad (25)$$

$$K \geq lb \quad (26)$$

$$\sum_{m \in M} DR_m = d * R \quad (27)$$

$$DR_m, R \geq 0, \forall m \in M$$

$$Q_m \geq 0, \forall m \in M, \text{integer}$$

Notation:

Sets:

M is a set of transport modes.

Variables:

K is safety factor;

Q_m is number of containers (cargo spaces) of transport mode m , $m \in M$, integer.

DR_m is part of cyclical demand during period R, delivered by transport mode m , $m \in M$;

R is the length of replenishment cycle, defined as fraction of year;

SD_{R+l} is the standard deviation of demand during the replenishment cycle R and lead time l .

Parameters:

a is cost of placing one order;

l is length of lead time;

v is unit costs of product;

i is internal rate of holding costs as a percentage of unit costs;

b_2 is cost per unit of stockout;

$G_u(K) = \int_K^\infty (u_0 - K) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u_0^2}{2}\right) du_0$, a specified function of the unit normal distribution (mean 0, standard deviation 1) variable;

c_m is cost of using one unit of transport mode m , $m \in M$

u is volume in cbm utilized by one unit of product;

o_m is total volume (in cbm) of one unit of transport mode m , $m \in M$;

ub is upper bound on safety factor K corresponding to P1 service level⁵ of 99%;

lb is lower bound on safety factor K corresponding to P1 of 90%;

d is total yearly demand for product.

The first term in the objective function represents the ordering costs during the year. If R is a fraction of one year, then $\frac{1}{R}$ is the total number of orders per year.

The second term represents holding costs. $\sum_{m \in M} \frac{DR_m}{2}$ is the average number of units kept at warehouse during the replenishment cycle (it is assumed that in the beginning of the cycle the stock level is $\sum_{m \in M} DR_m$, and in the end it is equal to 0). $SD_{R+l} \cdot K$ is the safety stock needed to satisfy unpredictably high demand.

The third term represents the shortage costs. $SD_{R+l} \cdot G_u(K)$ is expected number of unit short during one replenishment cycle. Hence the total amount of unit short during the year is $\frac{1}{R} \cdot SD_{R+l} \cdot G_u(K)$.

Finally, the last term represents the total transportation costs during the year. $\sum_{m \in M} Q_m \cdot c_m$ is the costs of the transportation of one order using m different transportation modes.

Unfortunately this problem is not linear. Hence it could not be solved applying linear programming methodology. Thus we would propose to determine several reasonable values of R and K factor, and corresponding to them values of square root $\sqrt{R+l}$ and $G_u(K)$ safety function. If there are j different values of safety factor K and n possible length of replenishment cycles R , then using the binary variable $Y_{j,n}$ we can choose the optimal pair of these values and solve the problem linearly. For example, if $Y_{1,1}=1$, then we should choose the first option of safety factor K and the first option of replenishment cycle R length.

To do that, we have determined six options for R value. Assuming that one year consists of 52 weeks, and that the reasonable length of replenishment cycle can be one, two, three, four, six or eight weeks, we will get the following set of values for R :

⁵ P1 service level is probability of having no stockout per replenishment cycle (Silver et al, 1998).

$\frac{1}{52}, \frac{2}{52}, \frac{3}{52}, \frac{4}{52}, \frac{6}{52}$ or $\frac{8}{52}$. The value of safety factor K can take one of 106 options starting from value 1.28, corresponding to P1 service level of 90%, and finishing with value 2.33, corresponding to P1 service level of 99%, in increments of one hundredth. Corresponding value of $G_u(K)$ was taken from the table for normal distribution (*Silver et al, 1998*).

After all these assumptions the problem can be rewritten in the following way (equations 28 – 33)

$$\begin{aligned} \min \quad & \sum_{j \in J} \sum_{n \in N} \frac{Y_{j,n}}{r_{j,n}} \cdot a + \left(\frac{D}{2} + \sum_{j \in J} \sum_{n \in N} Y_{j,n} \cdot \delta \cdot x_{j,n} \cdot k_{j,n} \right) \cdot v \cdot p + \\ & + \sum_{j \in J} \sum_{n \in N} \frac{Y_{j,n}}{r_{j,n}} \cdot b_2 \cdot \delta \cdot x_{j,n} \cdot g_{j,n} + \sum_{m \in M} \sum_{n \in N} T_{m,n} \cdot c_m \end{aligned} \quad (28)$$

Subject to

$$DR_m \cdot u \leq Q_m * o_m, \forall m \in M; \quad (29)$$

$$\sum_{j \in J} \sum_{n \in N} Y_{j,n} = 1; \quad (30)$$

$$D = \sum_{j \in J} \sum_{n \in N} Y_{j,n} \cdot r_{j,n} \cdot d; \quad (31)$$

$$\sum_{m \in M} DR_m = D; \quad (32)$$

$$T_{m,n} \geq \frac{Q_m}{\sum_{j \in J} r_{j,n}} - (1 - \sum_{j \in J} Y_{j,n}) \cdot W; \quad (33)$$

$$DR_m, T_{m,n}, D \geq 0, \forall m \in M, n \in N$$

$$Q_m \geq 0, \forall m \in M, \text{integer}$$

$$Y_{j,n} = \{0,1\}, \forall j \in J, n \in N$$

Notation:

Sets:

M is a set of transport modes;

$J = 1, \dots, 106$ is a set of K safety factors;

$N = 1, \dots, 6$ is a set of replenishment cycles R ;

Variables:

Q_m is number of containers (cargo spaces) of transport mode m needed in one replenishment cycle, $m \in M, \text{integer}$.

$T_{m,n}$ is number of containers (cargo spaces) of transport mode m needed in total for all replenishment cycles $r_{j,n}, m \in M, n \in N$;

D is average demand during the replenishment cycle;
 DR_m is part of cyclical demand, delivered by transport mode m , $m \in M$;
 $Y_{j,n}$ is logical variable determining the optimal pair of replenishment cycle length $r_{j,n}$, and safety factor $k_{j,n}$, $j \in J$, $n \in N$.

Parameters:

a is cost of placing one order;
 $k_{j,n}$ is option value of safety factor K , $j \in J$, $n \in N$;
 $r_{j,n}$ is option value of replenishment cycle R length, $j \in J$, $n \in N$;
 δ yearly standard deviation of demand;
 $x_{j,n} = \sqrt{r_{j,n} + l}$ is parameter determining the value of standard deviation of demand during the replenishment cycle $r_{j,n}$ and lead time l , $j \in J$, $n \in N$;
 v is unit costs of product;
 i is internal rate of holding costs as a percentage of unit costs;
 b_2 is cost per unit of stockout;
 $g_{j,n} = \int_{k_{j,n}}^{\infty} (u_0 - k_{j,n}) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u_0^2}{2}\right) du_0$, a value of specified function of the unit normal distribution (mean 0, standard deviation 1) variable, corresponding to safety factor $k_{j,n}$, $j \in J$, $n \in N$;
 c_m is cost of using one unit of transport mode m , $m \in M$
 u is volume in cbm utilized by one unit of product;
 o_m is total volume (in cbm) of one unit of transport mode m , $m \in M$;
 d is total yearly demand for product.
 W is relatively big number (for example, 10000).

In this model the length of replenishment cycle is determined as $Y_{j,n} \cdot r_{j,n}$. This expression can take $106 \cdot 6 = 636$ optional values. In 635 cases it will be equal to 0 and in one case to one of the six predetermined values of R .

The safety factor will be determined by the multiplication result of the expression $Y_{j,n} \cdot k_{j,n}$. This value also will be equal to 0 in 635 cases, and to one of 106 predetermined values K if $Y_{j,n}$ is equal to 1.

The value of the standard deviation of demand is determined by the expression $Y_{j,n} \cdot \delta \cdot x_{j,n}$. It will be equal to one of 6 predetermined values of $x_{j,n}$ multiplied by the standard deviation of monthly demand δ , if the value of $Y_{j,n}$ is 1, otherwise this expression will be equal to 0.

Finally, the value of safety function, determining the shortage will be determined by expression $Y_{j,n} \cdot g_{j,n}$. It will be equal to one of 106 predetermined values in case, if $Y_{j,n}$ is equal to 1, and to 0, if otherwise.

In addition to changes in the objective function, several changes in constraints were introduced. The mane of them concerns the total number of transport modes $T_{m,n}$ needed to use. This value is forced to be equal to 0, if $Y_{j,n}$ is equal to 0. And it is equal to number of transport modes needed to transport one order Q_m multiplied by number of orders $\frac{1}{\sum_{j \in J} r_{j,n}}$, otherwise.

The model presented by equations (28) – (33) can be used to determine optimal length of replenishment cycle and optimal value of safety factor k just for one item. In case of Stokke AS we need to develop a model for multi-product inventory management. Such model is presented below in equations (34) – (41):

$$\begin{aligned} \min \sum_{j \in J} \sum_{n \in N} \frac{Y_{g_1, p_1, j, n}}{r_{j, n}} \cdot a + \sum_{g \in G} \sum_{p \in P} \left(\frac{D_{g, p}}{2} + \sum_{j \in J} \sum_{n \in N} Y_{g, p, j, n} \cdot \delta_{g, p} \cdot x_{j, n} \cdot k_{j, n} \right) \cdot v_{g, p} \cdot i + \\ + \sum_{g \in G} \sum_{p \in P} \sum_{j \in J} \sum_{n \in N} \frac{Y_{g, p, j, n}}{r_{j, n}} \cdot b_{2, g, p} \cdot \delta_{g, p} \cdot x_{j, n} \cdot g_{j, n} + \sum_{m \in M} \sum_{n \in N} T_{m, n} \cdot c_m \end{aligned} \quad (34)$$

Subject to

$$\sum_{g \in G} \sum_{p \in P} DR_{g, p, m} \cdot u_{g, p} \leq Q_m \cdot o_m, \forall m \in M; \quad (35)$$

$$\sum_{j \in J} \sum_{n \in N} Y_{g, p, j, n} = 1, \forall g \in G, p \in P; \quad (36)$$

$$D_{g, p} = \sum_{j \in J} \sum_{n \in N} Y_{g, p, j, n} \cdot r_{j, n} \cdot d_{g, p}, \forall g \in G, p \in P; \quad (37)$$

$$\sum_{m \in M} DR_{g, p, m} = D_{g, p}, \forall g \in G, p \in P; \quad (38)$$

$$T_{m, n} \geq \frac{Q_m}{\sum_{j \in J} r_{j, n}} - \left(1 - \sum_{j \in J} Y_{g_1, p_1, j, n} \right) \cdot W, \forall m \in M, n \in N; \quad (39)$$

$$\sum_{j \in J} Y_{g, p, j, n} = \sum_{j \in J} Y_{f, g, j, n}, \forall g \in G, p \in P, n \in N, f \in G \setminus \{g\}; \quad (40)$$

$$\sum_{j \in J} Y_{g, p, j, n} = \sum_{j \in J} Y_{g, s, j, n}, \forall g \in G, p \in P, n \in N, s \in P \setminus \{p\}; \quad (41)$$

$$DR_{g, p, m}, T_{m, n}, D_{g, p} \geq 0, \forall g \in G, p \in P, m \in M, n \in N$$

$$Q_m \geq 0, \forall m \in M, \text{integer}$$

$$Y_{g,p,j,n} = \{0,1\}, \forall g \in G, p \in P, j \in J, n \in N$$

Where G is a set of product groups and P is a set of product options.

This problem is generally the same as the previous one. But the size of the problem increased significantly. In previous model the decision variable $Y_{j,n}$ could take on of 636 option values, determining the optimal pair of safety factor K and replenishment cycle R. In the current problem such decision variable should be assigned to each of the product. The number of products is defined by combination of product groups g and product options p . The total number of products is equal to $g*n=42$. Hence in the current model there are $636*42=26712$ decision variables.

Equation (35) defines a set of capacity constraints for each of the transport modes. It says that the total space $Q_m * o_m$ of each transport mode m should be larger or equal to the space utilized by total amount of products assigned to this transport mode $\sum_{g \in G} \sum_{p \in P} DR_{g,p,m} * u_{g,p}$.

Equation (36) defines a set of logic constraints for each of the products. It says that for each product (g, p) the value of decision variable $Y_{g,p,j,n}$ can be equal to 1 just in one case. It will guaranty that only one pair of safety factor K and replenishment cycle length R will be assigned to each of the products.

Equation (37) defines a set of demand constraints for each of the product. It says that the total amount of product (g, p) transported in one order should be equal to the demand during replenishment period.

Equation (38) represents a set of connectivity constraints. It says that the sum of product amounts, assigned to different transport modes, should be equal to the total amount of product needed to be transported.

Equation (39) defines the total number of all transport modes used during the year. This equation is similar to equation (33) in previous model. This value is forced to be equal to 0, if sum of $Y_{g_1,p_1,j,n}$ is equal to 0. And it is equal to number of transport modes needed to transport one order Q_m multiplied by number of orders $\frac{1}{\sum_{j \in J} r_{j,n}}$, otherwise. The reason why it can be used the decision variables for one of the items (g_1, p_1) is explained by equations (40) and (41).

Equations (40) and (41) set the same length of the replenishment cycle for all items. It means that if for product option 1 in product group 1 the length of replenishment cycle is set to be 4 weeks, the same length of replenishment cycle should be applied to all other

items. Hence we can regard just one item in equation (39) as the value of $\sum_{j \in J} Y_{f,g,j,n}$ for different products (g,p) will be the same for the n option of replenishment cycle length R .

The model includes 26712 decision variables, 3 single period transport variables, 18 total transport variables, 42 demand per replenishment cycle variables and 126 transportation quantity variables. Total number of constraints is equal to 3171. In addition there are simple non-negative requirements to variables $DR_{g,p,m}, T_{m,n}, D_{g,p}$, non-negative and integrality requirements for Q_m variables and binary requirements for $Y_{g,p,j,n}$ variables. Approximate time to solve this problem is ~ 3 min.

Solution to the problem shall also be supplemented by additional calculations to give all the answers to managerial questions. We will get the values of total costs and number of units of all transport modes which we need to order to get all the products delivered.

The value of order up to level S can be found using formula (42):

$$S_{g,p} = D_{g,p} + \sum_{j \in J} \sum_{n \in N} Y_{g,p,j,n} \cdot \delta_{g,p} \cdot x_{j,n} \cdot k_{j,n}, \forall g \in G, p \in P \quad (42)$$

And the order quantity for each of the products shall be calculated using formula (43):

$$Q = S - I - Q_{prev} \quad (43)$$

where I is inventory on-hand and Q_{prev} are previous orders which are on the way to warehouse but not yet delivered.

If necessary each component of logistical costs can also be calculated using formulas (44) – (47):

$$\text{Ordering costs: } \sum_{j \in J} \sum_{n \in N} \frac{Y_{g_1,p_1,j,n}}{r_{j,n}} \cdot a \quad (44)$$

$$\text{Holding costs: } \sum_{g \in G} \sum_{p \in P} \left(\frac{D_{g,p}}{2} + \sum_{j \in J} \sum_{n \in N} Y_{g,p,j,n} \cdot \delta_{g,p} \cdot x_{j,n} \cdot k_{j,n} \right) \cdot v_{g,p} \cdot i \quad (45)$$

$$\text{Shortage costs: } \sum_{g \in G} \sum_{p \in P} \sum_{j \in J} \sum_{n \in N} \frac{Y_{g,p,j,n}}{r_{j,n}} \cdot b_{2,g,p} \cdot \delta_{g,p} \cdot x_{j,n} \cdot g_{j,n} \quad (46)$$

$$\text{Transportation costs: } \sum_{m \in M} \sum_{n \in N} T_{m,n} \cdot c_m \quad (47)$$

6.3. Comparison of logistical costs when using two types of supply network

The multi-product periodic review model for inventory and transportation management will now be used to calculate and compare logistical costs when two types of supply network are used.

The first type of network is presented in Figure 3 (Chapter 2). It is the current supply network of Stokke for European warehouse in Venlo, Holland. In case of application this type of supply network all the textile products are delivered from the Chinese agent whom is connected with the producers supply network. The total lead time consist of 5-8 weeks of production and 5 weeks of transportation. The total lead time can vary from 10 to 13 weeks consequently. But in each order the priority can be given to different item. It means that, for example, Seat Textile sets can be delivered one time in 10 weeks and another time in 13 weeks. In these circumstances it can be assumed that total lead time for all items is 13 weeks.

Another type of supply network is presented in Figure 18. It is the network proposed by the author of this master thesis. It is suggested to accumulate all the items, needed to be delivered to Venlo, in Asian warehouse of Stokke. In this case Stokke can order all necessary items from its warehouse in Asia any time they are needed. As the items are ready, there will be no need to spend time on production and the total lead time will consist only of transportation time which is 5 weeks.

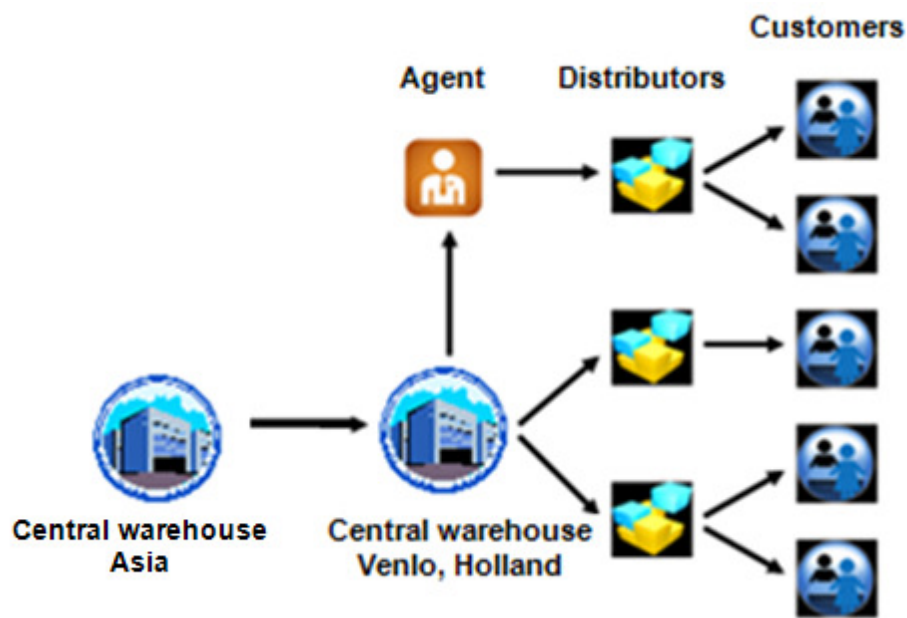


Figure 18. Suggested supply chain for Xplory textile.

As it can be seen from equations (45) and (46), the holding costs and shortage costs are determined by the standard deviation of demand during replenishment cycle and lead time (δ_{R+L}):

$$\delta_{R+L} = \delta \cdot \sqrt{R + L} \quad (48)$$

Where δ is the yearly standard deviation of demand, R is the replenishment cycle as fraction of year and L is the lead time as fraction of year.

Using equation (48), it can be proved, that the decrease in lead time will lead to decline of inventory level, holding costs and shortage costs.

The comparison of results when using the model for two types of supply network will be done based on calculations presented in Table 12 and data presented in Table 11.

Table 11. Input parameter for the model.

Parameters		Current Supply Network	Suggested Supply Network
Order cost, USD		200	200
Transportation costs (per cbm), USD	20 foot container	85	85
	40 foot container	60	60
	Air Freigh	600	600
Transport mode volume, cbm	20 foot container	25	25
	40 foot container	55	55
	Air Freigh	1	1
Stockout cost per unit of textile, USD		22.6	22.6
Holding cost, %		14	14
Lead time, weeks		13	5

Table 12. Comparison of logistical costs when using two types of supply network for Stokke AS

Parametr	Current Supply Network		Suggested Supply network	
	Sum	Share in Total	Sum	Share in Total
Ordering Costs, USD	1733.3	0.69	3466.7	1.52
Holding Costs, USD	93153.0	36.89	63208.1	27.76
Shortage Costs, USD	14655.3	5.80	9772.9	4.29
Transportation Costs, USD	143000.0	56.62	151233.0	66.42
Total logistical costs, USD	252541.6	100	227680.7	100.00
Length of Replenishment cycle, weeks	6		3	

As it can be seen the decrease of lead time from 13 to 5 weeks allows total logistical cost decrease from 252.5 thousand USD to 227.7 thousand USD (by 9.8%). The optimal length of replenishment cycle in the case of current supply chain is 6 weeks. Using suggested supply chain one should reduce the length of replenishment cycle to 3 weeks.

None of these replenishment cycles corresponds to current ordering policy of Stokke AS. Currently the company orders once a month, which can be put in correspondence to 4 weeks replenishment cycle. The reason for that are different managerial issues:

- It makes the system of order delivering easier;
- 4 weeks order cycles perfectly match calendar;
- This length of replenishment cycle corresponds to one production cycle at supplier plants etc.

It also should be mentioned that the length of optimal replenishment cycle depend mostly on cost of the acquisition of additional unit of transport mode and holding cost rates. The reason for this fact is that ordering costs are insignificant comparing to transport costs. Hence such lengths of replenishment cycles were determined by the model in order to utilize the transport modes as much as possible and at the same time use mostly 40 foot containers as it is the cheapest transport mode per cubic meter. The order quantity in the model is the same for all of the replenishments, but on practice such situation is not real because the demand is fluctuating during the time horizon. Hence the practical order quantity will be different at different order points. It means that transportation costs on practice can be higher than optimal and more 20 foot containers will be used than recommended by the model.

From the other side, from the managerial point of view it is more optimal to operate 4 weeks replenishment cycles. This will slightly increase logistical costs from the optimal level. But value of logistical costs will be more relevant to practice. Hence it was decided to resolve the problem for determined length of replenishment cycle (4 weeks). The results are presented in table 13.

Table 13. Comparison of logistical costs when using two types of supply network for Stokke AS and 4 weeks replenishment cycles

Parametr	Current Supply Network		Suggested Supply network	
	Sum	Share in Total	Sum	Share in Total
Ordering Costs, USD	2600.0	1.0	2600.0	1.10
Holding Costs, USD	87407.0	33.6	67844.0	28.67
Shortage Costs, USD	13542.6	5.2	9853.7	4.16
Transportation Costs, USD	156325.0	60.2	156325.0	66.07
Total logistical costs, USD	259874.6	100.0	236622.7	100.00
Length of Replenishment cycle, weeks	4		4	

The situation with Current Supply Network and 4 weeks replenishment cycle length represents current ordering policy at Stokke. The total cost of operating in such system is calculated to be 259.8 thousand USD. On practice this value can be a bit higher due to fluctuation of demand, and, consequently deviation of order quantities.

When using Suggested Supply Network, total logistical costs are decreased to 236.6 thousand USD (by 8.9%) compared to Current Supply Network. All the decrease was obtained due to decline of holding and shortage costs. The reason for that are decreased safety stocks which are needed to satisfy the demand during replenishment cycles (Table 14 and 15).

Table 14. Safety stocks when using Current Supply Network, units

Safety Stock	BEIGE	BLUE	DAR_NAV	GREEN	PURPLE	RED
CC Tex Set	841	402	503	269	572	463
CC_UK Tex Set	148	168	80	101	98	179
Changing Bag	388	328	240	171	466	549
Footmuff	537	138	426	166	557	352
Parasol	458	399	354	295	399	532
Seat Tex Set	1263	529	853	699	578	489
Seat UK Tex Set	201	183	219	38	198	263

Table 15. Safety stocks when using Suggested Supply Network, units

Safety Stock	BEIGE	BLUE	DAR_NAV	GREEN	PURPLE	RED
CC Tex Set	612	292	366	196	416	337
CC_UK Tex Set	107	123	58	74	72	130
Changing Bag	282	238	175	124	339	400
Footmuff	651	168	517	201	675	426
Parasol	333	290	258	215	290	387
Seat Tex Set	919	385	621	509	420	356
Seat UK Tex Set	146	133	159	28	144	191

It can be seen that safety stocks in case of Suggested Supply Network are much less than in Current Supply Network. At the same time the expected value of P1 service level stayed unchanged (Table 16).

Currently Stokke is using some other specific measurement of service level. It is the percentage of total amount of orders delivered in one batch. Unfortunately such measurement cannot give precise values of service level for each of the product group. In case of using P1 service level it can be predicted at least theoretically which percentage of orders will be delivered on time.

Table 16. P1 service value for product groups

Product Group	Current Supply Network	Suggested Supply Network
CC Tex Set	98.7	98.7
CC_UK Tex Set	98.5	98.5
Changing Bag	99	99
Footmuff	99	99
Parasol	99	99
Seat Tex Set	97.4	97.4
Seat UK Tex Set	97.2	97.2

It can be seen from Table 16 that expected value of service level P1 varies from 97 to 99% for different product groups. Hence the company can claim that with 97% probability all the orders from customers will be satisfied.

As it was said, changes in Supply Network can lead to the decrease in total logistical costs by 9% for European warehouse. At the same time such decision will lead to increase of costs at Asia warehouse. The time limit for writing this master thesis has made impossible evaluation of costs increase at Asia warehouse, but it should be mentioned that this value expected to be smaller than benefits at European warehouse. It can be achieved due to decrease of total demand deviation.

For example, let A and B denote two warehouses with inventory of the same product. The standard deviation of demand on the product from the A warehouse is δ_a , from the B warehouse – δ_b . If two warehouses will be merged then the standard deviation of demand on the product (δ_{ab}) will be equal to $\sqrt{\delta_a^2 + \delta_b^2}$, what is less then $\delta_a + \delta_b$. (Kulkarni et al, 2004).

As the inventory at Asian warehouse will be used for needs of two regions (Asia and Europe) the deviation of total demand will be less than sum of deviations. Hence the safety stocks at Asian warehouse will not increase that much.

It should be mentioned that Stokke AS operated one more warehouse in North America. If the same policy will be applied for all of warehouses it can lead to even more significant benefits for the company.

There are also some other reasons to consider changes in Supply Network:

- The order will come to the supplier not from three different warehouses but from one buffer (Asian) warehouse. This will make the process of communication with the supplier easier. At the same time the batches of order quantities will increase. Hence the supplier could spend less time on production set ups. This can lead to a

decrease in unit price.

- The handling of orders between the warehouses will be faster and will not require additional arrangements as all of the warehouses belong to one company.
- The combination of products in one container will not be restricted by the production capacity of producer. Even such small order as one box of product can be taken from a warehouse shelf.

All this managerial benefits will lead to more clear process of order handling.

Hence the company will spend less on the managerial control in addition to the decrease of logistical costs.

7. Conclusions and recommendations

In the current master thesis the problems of forecasting and inventory management were regarded. The data set was taken from the Norwegian company, Stokke, which competence lies in design and distribution of products for children.

During the last several years Stokke was constantly looking for the ways to improve operational processes. These efforts led to close cooperation between Stokke and Møreforsking Molde. During the first stage of this cooperation Karolis Dugnas and Oddmund Oterhals mapped the ways of possible improvement in the report “Flow of goods and warehouse optimization for Stokke AS. Mapping and improvement of the logistics processes” (*Dugnas, Oterhals, 2010*).

The current master thesis is the continuance of cooperation between Stokke and Møreforsking Molde. It is devoted to development and evaluation of forecasting techniques and models for inventory management. The research is based on the demand data for textile products for the Stokke Xplory children stroller.

The first part of the master thesis was concerned with problems of forecasting. Previously Stokke was using a simple forecasting technique based on the Moving Average approach. The demand data from Stokke for years 2008-2010 were used to evaluate the results of forecasting when using this technique. As well there were evaluated two more sophisticated techniques. One is based on the Holt-Winters approach, and another one is based on the Box-Jenkins approach. Evaluation of results was done using the MAPE measurement.

Development of forecasting models raised the problem of structural changes in demand pattern. The product groups which were analyzed in the master thesis have stayed unchanged during 2008-2010, but the structure of the product options within each group has changed completely in the beginning of 2010. This made impossible to apply the Holt-Winters approach and the Box-Jenkins approach on the item level. Thus the forecasts were developed for the product groups with possibility to decompose them into item-level forecasts on the last stage.

Evaluation of the forecasts led to a conclusion that the Holt-Winters approach outperforms two others in 4 cases out of 7, and the Box-Jenkins approach outperforms two others in 3 out of 7 cases. But the value of MAPE for most of the forecasting techniques was above 20%. The usage of forecasts with such high value of MAPE will not lead to

significant benefits comparing to the usage of methodology based on assumption of the random behavior of demand.

Thus instead of using the values of forecasted demand and MAPE term to determine the order quantity and needed safety stock, it can be easier to use the assumption of demand randomness.

These findings lead to a conclusion that the difference between the real demand and the forecasted by Stokke professionals occurs not due to usage of inappropriate autoregressive forecasting technique but due to behavior of demand pattern. The demand for children products is dependent on economical, social and other factors in the countries where these products are distributed. And the demand on these products in one period is hardly dependent on the demand during previous period. Hence none of the autoregressive forecasting models is expected to give reasonably good results.

On another hand, it is difficult to handle all the factors influencing demand in order to make a precise prediction of demand. Thus it can be reasonable to assume that demand series are following a random distribution. As the birth rate in European countries was slightly increasing during last years the mean demand could be recalculated monthly or yearly.

The Kholmogorov-Smirnov test was used to check the assumption that demand series for each of the product is normally distributed random variables. The positive results of these tests gave opportunity to apply one of the inventory management systems.

The second part of the master thesis is devoted to the development of inventory management system capable to deal within multi-items and multi transportation modes environment.

Due to managerial issues at Stokke it was decided to use the periodic review inventory management system for stochastic demand environment as the basis for model development.

The known (R,S) inventory management model was extended by incorporation of varying transportation costs and applied to multi-item environment. As the original model was not linear and thus required significant efforts to be solved, it was decided to limit the number of possible options for replenishment cycle length R to 6 and for safety factor K to 106. The lower bound for service level P1 was set to 90% and the upper one to 99%.

Using this and some other assumptions the model was solved for two options of supply network design. The first option corresponds to the current situation at Stokke when products are supplied to Stokke European warehouse from Chinese producer via the

Asian agent. The second type of Supply Network was proposed by the author. In this case Asian warehouse of Stokke accumulates the whole demand for its own needs and for the needs of European warehouses. The lead time decreases from 13 weeks in original supply network to 5 weeks in suggested one.

The decrease of total costs at European warehouse, when changing the supply network design, is equal to 24.8 thousand USD or 9.8% of current optimal total logistical costs for examined items. In the first case optimal length of replenishment cycle was decided to be 6 weeks, while in the second one the 3 weeks period was chosen.

Stokke operates in conditions when it is more convenient to handle orders once a month. This situation corresponds to a 4 weeks replenishment cycle length. Due to managerial issues the periodicity of order handling is desired to stay unchanged. Thus the total logistical costs were recalculated for the situation when the length of replenishment cycle is set to be 4 weeks.

The situation with Current Supply Network and 4 weeks replenishment cycle length represents current ordering policy at Stokke. The total cost of operating in such system is calculated to be 259.8 thousand USD. On practice this value can be a bit higher due to fluctuation of demand, and, consequently deviation of order quantities.

When using Suggested Supply Network, total logistical costs are decreased to 236.6 thousand USD (by 8.9% compared to Current Supply Network). All the decrease was obtained due to decline of holding and shortage costs.

The total number of product delivered in time to customers (P1 service level) in both cases will vary from 97% to 99% for different products.

In the current master thesis the author has evaluated just the decrease of costs for European warehouse. At the same time the increase of cost at Asian warehouse was not evaluated due to time limits for master thesis writing. Nevertheless, it is expected that the total increase of costs in Asia will be less than the decrease of costs in Europe. There are two main reasons for that. The first is the difference between costs of warehouse operations in Asia and Europe. The increase of workload in Asian warehouse should not lead to significant increase in costs. The second reason is the decrease of total demand deviation which will lead to decrease of safety stocks needed to keep the service level. Hence the costs of having an additional safety stock for Europe needs will be lower than the costs of having the safety stock in the European warehouse.

Moreover, Stokke operates a warehouse in the USA. If the same policy of “buffer warehouse” will be applied for all of the Stokke warehouses than the total savings could be much higher.

Finally, besides the costs decrease, there are some other managerial benefits:

- The process of communication with the supplier will be easier;
- The order handling between the warehouses of the same company will be faster than the order handling between the warehouse of Stokke and Chinese producer;
- The combination of products in one container will not be restricted by the production capacity of producer etc.

In this master thesis just a part of Stokke supply and distribution network was regarded. Thus there are certain possibilities for a future research. For example, one can take into consideration the optimization of the whole supply network of Stokke. Then the current multi-product, multi transportation mode problem can be solved for multi-warehouse environment.

Another possibility for a future research is to find the ways of optimal problem solution when it is not modified to be linear. In the current master thesis a limited number of options for safety factor and replenishment cycle length were considered. In some cases it could lead to situations when the solution is suboptimal for these options. Solving the problem without modifications for linearity could give better results.

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Appendixes

Appendix 1. List of textile products for the Stokke Xplory children stroller and characteristics of their demands for May (June) 2010-December 2010.

Group name	Product Name	Average Demand	Standard deviation of demand	Average Share in Group, %	Standard Deviation of Share in Group
CC Tex Set	XPLORY Style Kit CC Dark Navy	374,6	113,2	20,5	2,9
	XPLORY Style Kit CC Blue	176,9	90,8	9,5	3,7
	XPLORY Style Kit CC Red	285,0	104,5	15,7	5,1
	XPLORY Style Kit CC Beige	529,4	189,6	29,0	6,2
	XPLORY Style Kit CC Purple	331,5	128,7	17,8	2,0
	XPLORY Style Kit CC Green	92,8	60,6	5,2	3,5
CC Tex Set UK	XPLORY Style Kit CC UK Dark Na	78,3	18,8	24,7	9,3
	XPLORY Style Kit CC UK Blue	32,4	39,0	8,8	10,6
	XPLORY Style Kit CC UK Red	69,4	41,1	21,1	11,6
	XPLORY Style Kit CC UK Beige	56,3	34,4	16,5	8,4
	XPLORY Style Kit CC UK Purple	74,9	23,0	22,3	5,2
	XPLORY Style Kit CC UK Green	14,4	23,1	4,2	6,8
Seat Tex Set	XPLORY V3 Style Kit Seat DrkNv	450,1	222,1	21,6	7,0
	XPLORY V3 Style Kit Seat Blue	216,4	137,6	9,7	4,7
	XPLORY V3 Style Kit Seat Red	282,6	127,0	14,7	6,7
	XPLORY V3 Style Kit Seat Beige	494,6	328,7	23,8	9,9
	XPLORY V3 Style Kit Seat Prple	408,8	150,3	20,3	5,9
	XPLORY V3 Style Kit Seat Green	139,3	182,0	6,1	8,2
Seat Tex Set UK	XPLORY Style Kit Seat UK DrkNa	77,0	57,4	21,3	11,5
	XPLORY Style Kit Seat UK Blue	35,9	48,0	9,4	10,2
	XPLORY Style Kit Seat UK Red	64,7	69,1	19,0	23,9
	XPLORY Style Kit Seat UK Beige	46,4	52,7	12,9	15,4
	XPLORY Style Kit Seat UK Purpl	67,1	51,9	19,0	7,3
	XPLORY Style Kit Seat UK Green	8,3	9,9	2,1	2,2
Foot muff	XPLORY V3 Footmuff Dark Navy	239,9	92,2	20,5	4,3
	XPLORY V3 Footmuff Blue	123,9	30,0	11,0	2,5
	XPLORY V3 Footmuff Red	190,4	75,9	16,5	7,2
	XPLORY V3 Footmuff Beige	285,3	116,3	24,2	5,2
	XPLORY V3 Footmuff Purple	205,1	120,4	16,5	6,6
	XPLORY V3 Footmuff Green	97,4	35,7	8,3	1,5

Group name	Product Name	Average Demand	Standard deviation of demand	Average Share in Group, %	Standard Deviation of Share in Group
Changing Bag	XPLORY V3 Changing Bag DrkNavy	261,8	51,5	21,2	1,6
	XPLORY V3 Changing Bag Blue	148,4	70,9	10,2	2,1
	XPLORY V3 Changing Bag Red	243,8	118,6	16,8	4,6
	XPLORY V3 Changing Bag Beige	324,3	84,2	25,2	3,8
	XPLORY V3 Changing Bag Purple	267,6	101,4	19,5	3,0
	XPLORY V3 Changing Bag Green	94,3	37,3	6,8	1,3
Parasol	XPLORY V3 Parasol Dark Navy	236,8	78,2	16,3	5,0
	XPLORY V3 Parasol Blue	170,5	87,3	11,2	4,3
	XPLORY V3 Parasol Red	249,1	115,7	15,7	2,9
	XPLORY V3 Parasol Beige	314,5	97,7	22,2	8,1
	XPLORY V3 Parasol Purple	237,0	86,3	16,0	6,2
	XPLORY V3 Parasol Green	112,8	62,4	6,9	2,5