Master's degree thesis

LOG950 Logistics

Internal transportation and inventory management in Asak Miljøstein AS

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Preface and acknowledgements

This Master thesis sums up and represents the main results of our academic work as MSc students in Logistics at Molde University College – Specialized University in Logistics from December 2012 until June 2013.

First of all, we would like to thank Professor Irina Gribkovskaia for making our studies at Molde University College possible and for her constant support during the last two years. Furthermore, we deeply appreciate Associate Professor Johan Oppen for his guidance, help, high responsiveness and his perfect sense of humor that helped us stay motivated and avoid stress during the whole process of working on this paper. We also want to thank Jørn-Andre Hammer, managing director of Asak Miljøstein AS, for the real case data provided to us and for his attention to all of our questions. Last but not least we would like to thank all of our families' members and all our friends for their support and understanding during the last two years.

The subject area of the present paper is combinatorial optimization and mathematical modeling. This Master thesis is focused mainly on specification and mathematical description of the real-world problem and on construction of its mathematical model that could be used later on as a basis for solving the problem with the use of any available optimization tools.

This Master thesis was evaluated by Associate Professor Johan Oppen and Professor Lars Magnus Hvattum.

Summary

This paper is devoted to the analysis of the problem of optimization of production, inventory management and internal transportation policies of the Norwegian company Asak Miljøstein AS. This problem can be related to a class of combined production-inventory-transportation problems, which are nowadays already relatively deeply analyzed and described in the literature.

The main focus of this work was set on design and development of the mathematical model capable of dealing with optimization of the presented combined problem. A classical Capacitated Vehicle Routing Problem (CVRP) was selected as a basis for the transportation model, which was further extended and combined with inventory and production sub-problems.

This work should be of interest not only for Asak Miljøstein AS, but also for a wide range of production companies facing similar problems as the one scrutinized in this Master thesis.

In the first part of this paper the complex case problem is described, analyzed and specified, and a set of assumptions for modeling purposes is made. Further on, literature overview and problem description are provided, followed by a stepwise mathematical model formulation and construction. In the last part of the work some additional recommendations for production, inventory and transportation policies improvement are made. Finally, possible ways of solution and application of the model are suggested.

Contents

1. Introduction

This Master thesis deals with a real-world combined production-inventory-transportation problem faced by Asak Miljøstein AS, a sales organization for three producers of concrete products in Norway.

The main goal of the present paper was to develop a mathematical model reflecting as detailed as possible the given real-world problem of Asak Miljøstein AS in order to create a basis for solving it later on with the use of programming tools. Thus this work can be related mainly to a field of Mathematical modeling.

In Chapter 2 a detailed description of the problem is provided.

A general overview of the literature related to a wide range of combined optimization problems, with the main focus set on combined production-inventory-transportation problems and on their sub-problems taken separately, is done in Chapter 3.

In Chapter 4 the preliminary classification and description of the mathematical model of the problem is performed. A Capacitated Vehicle Routing Problem (CVRP) was chosen to be the basis of the mathematical model. In order to make it possible for the model to reflect the specified problem characteristics related to transportation, CVRP is further extended to the periodic vehicle routing problem with pick-ups and deliveries and time windows (PVRPPDTW). The resulting transportation model is further used as a basis for the combined production-inventory-transportation model.

In order to make it possible to model the problem, it is specified and narrowed by introduction of a set of assumptions. Assumptions applied to the discussed problem are listed and explained in Chapter 5.

Chapter 6 represents the main part of this work – specification, construction and description of the mathematical model. The model is built in two stages. In the first stage the model for PVRPPDTW is developed for solving the transportation sub-problem. In the second stage PVRPPDTW is extended to a multi-product model with split demands and pick-ups, which is then combined with inventory and production sub-problems. A comprehensive description of parameters, variables, objective functions and constraints for models on both stages is also provided in this chapter.

Mainly due to quite a high complexity and non-linearity of the resulting model and to the limitation of time, programming of the model, its testing and application for optimization of the production-inventory-transportation problem based on the real data provided by Asak Miljøstein AS, was not implemented in this work and thus left for further research and development.

In Chapter 7 some additional theoretical suggestions of improvement of current production, inventory management and transportation policies of Asak Miljøstein AS are made.

Finally, in Chapter 8, all the main results of the present work are summed up and recommendations for further development of the problem this Master thesis deals with are made. The authors believe that the carried out detailed description and specification of the problem and the constructed mathematical model provide a very good basis for future optimization of the problem with use of any of the existing solution methods.

2. Problem description

The problems considered in this Master thesis are related to the transportation of raw materials and finished products between factories and between factories and warehouses of Asak Miljøstein AS (hereinafter – Asak), a sales organization for three producers (who own five factories) of concrete products, mainly pavement blocks and facing stone.

Among the major customers of Asak are such companies as Byggmakker Norge AS, Coop Norge SA, Optimera AS, MAXBO (Løvenskiold Handel AS), Bygger'n Norge, Nordek AS, BYGGtorget and Gausdal Landhandleri AS.

The main competitors of Asak are Aaltvedt Betong AS, Multiblokk AS and Benders Norge AS.

Asak Miljøstein AS's operational results for the years 2007-2011 are reflected in Table 2.1 (all values are in Norwegian krones):

Table 2.1. Asak Miljøstein AS operational results for 2007-2011

	2011	2010	2009	2008	2007
Revenue	252 040 000	232 904 000	241 922 000	215 132 000	188 378 000
Profit	797 000	198 000	-966 000	-341 000	441 000

At present, Asak Miljøstein's personnel consists of 18 workers who are occupied with marketing and sales of the products. The total number of employees at all of the five factories is approximately 100.

The policy of Asak is based on the principle that all factories, for each of which the production capacities and demands are different, should be self-sufficient with all products in the company's product line, so Asak has to plan and carry out internal transportation between five factories and four warehouses. Most products are very heavy with a low price per ton, so the logistic costs are high and represent a large portion of the products total cost. The main problem Asak faces in this respect is organization of as cost-effective transportation as possible between factories and between factories and warehouses. To this issue the problem of organization and maintenance of cost-effective inventory management system is closely related.

As already mentioned, Asak represents five factories today: Kristiansand, Fetsund, Hønefoss, Stjørdal and Bodø (see Figure 2.1).

Figure 2.1. Geographical locations of factories and warehouses of Asak Miljøstein AS

(Source: Google maps)

Due to one common brand, Asak, each factory must be able to deliver a complete assortment, normally within 3 days. Many customers (dealers and professionals) place the order on site and expect to bring the goods with them as they leave the premises. This means that each factory needs to stock a sufficient volume of the complete assortment. All the factories try to be self-sufficient with the complete assortment, but due to different production facilities and/or machinery this is not possible. Some products represent a small volume nationally or are specialized, and therefore it is not economic to produce them at all locations. This means that some products are only produced at one factory and then transported to the other locations. This is especially typical for Fetsund, where they have very specialized machinery. Hønefoss and Fetsund serve the same market, and Fetsund, due to their specialized machinery, cannot produce all of the larger products (in volume) and are therefore supplied by Hønefoss. In addition, one major ingredient in concrete products is sand. It is crucial for Asak to maintain similar colors on similar products in the same market, and since Fetsund and Hønefoss are delivering to the same market they need to be supplied from the same quarry to ensure similarity. The sand is therefore extracted from a neighboring site of the Hønefoss factory to supply both factories. Fetsund depends on purchasing both sand and volume products from the Hønefoss area and the costs then are higher at Fetsund than at Hønefoss. At the same time the sales price must be similar at both factories in the same market. Due to specialized production or products with smaller volume, there is a lot of transportation between the factories, but not equally divided between the factories.

3. Literature review

This Master thesis deals with a combined production-inventory-transportation problem. In this problem such functions as production planning, inventory management, transportation and distribution are integrated into a single optimization model and are simultaneously solved. Traditionally these types of functions are optimized separately where an output of optimization of one of the functions becomes an input for optimization of another (for example, inventory levels are found first, and then a transportation model is solved). Nowadays different types of combined models and integrated analyses can be found in the literature.

In case of integrated production-inventory-distribution system Mak and Wong (1995) formulate a genetic search algorithm to solve a total cost minimization problem in the whole system. Blumenfeld *et al.* (1985) explore interconnections and trade-offs between inventory, transportation and production set-up costs, and based on these links they determine optimal shipping policies. Issues like integrated distribution and inventory problems can be found among works of Speranza and Ukovich (1994), Bertazzi and Speranza (1999), Burns *et al.* (1985). Martin et al. (1993) develop a linear-programming model based on one year planning horizon called FLAGPOL that combines production, distribution, and inventory operations in order to optimize them. Integration between production and distribution processes and the value of coordination of these two problems were analyzed by Chandra and Fisher (1994). Flumero and Vercellis (1999) proposed an integrated optimization model for production and distribution planning in which such decisions as capacity management, inventory allocation and vehicle routing are optimally coordinated.

Combined production-inventory-transportation problems can be split into three separate sub-problems which are widely observed in the literature.

The first sub-problem is a production planning problem. The task of production planning is to decide what type of product and how much of it to produce in each period of planning horizon (for example each day/week/year) in order to minimize costs. In the given Master thesis the production planning problem would consist in determination of amount of production of stones of different types for each factory for each day of the planning horizon in the “high season” and for each week or month of the planning horizon in the “low season”. This problem can be seen as a deterministic production planning problem which was observed by Florian et al. (1980).

The second sub-problem is a transportation problem that is observed under the class of Vehicle Routing Problem (VRP) which can be defined as a combinatorial optimization and integer programming problem. VRP was proposed by Dantzig and Ramser (1959). The problem considered in this Master thesis represents an extended version of the classical VRP and can be referred to as the periodic multi-depot vehicle routing problem with pick-ups and deliveries and time windows (PMDVRPPDTW). Solution of this sub-problem is supposed to identify the optimal sequence of factories and warehouses that should be visited in the planning horizon with simultaneous pick-ups and deliveries of different types of stones, as well as it should identify the optimal vehicle fleet.

The third sub-problem is an inventory management problem where we are facing a problem of limited storing capacity at each factory. With respect to this constraint, reorder points, order quantities and stock levels at each factory should be specified. In the given problem demand for products varies during the year, therefore a planning horizon would be cut into intervals representing periods in which demand has common behavior and more or less stable level. In our model the third sub-problem will be represented only as constraints under transportation cost minimization function. As a basis for determining order quantities the Economic order quantity (EOQ) Model, which was developed by Ford W. Harris (1915) and was widely used by R. H. Wilson (1934), can be used.

4. Problem structure analysis

The basis of the model that would represent the considered combined production-inventory-transportation problem of Asak Miljøstein AS will be formed by the Vehicle Routing Problem (VRP). The VRP will be extended by the limitation of the capacity of vehicles (making it a Capacitated VRP, or CVRP), the possibility to carry out simultaneous pick-ups and deliveries (VRPPD), the limitation of working hours of the factories (VRP with time windows), the limitation of the maximum duration of the working day of a truck driver, the fact that each factory may be used as a depot (multi-depot VRP, or MDVRP) and the possibility to satisfy customer node's demand with more than one vehicle (split delivery VRP, or SDVRP). Production and inventory sub-problems will be represented in the model as additional constraints: limitations of the storing and production capacities at each factory, inventory level balance constraints, initial inventory levels at each factory, and others.

4.1 Vehicle Routing Problem (VRP)

The Vehicle Routing Problem (VRP) is a combinatorial optimization and integer programming problem dealing with least-cost satisfaction of demands of a number of customers by a fleet of vehicles. VRP was first introduced by Dantzig and Ramser in 1959. In general, the problem can be represented in the following way. A graph $G=(N,A)$ represents two sets: a set of nodes $N=\{0, \ldots, n\}$, where node 0 is a depot, and other nodes are customers, and a set of arcs $A=\{(i, j): i,j \in N\}$. The travel cost between nodes i and j is denoted by $c_{ij} > 0$. Each customer i has a demand d_i. All vehicles are assumed to have the same capacity C. The objective is to satisfy all customers' demands while minimizing the total sum of travel costs. Each vehicle starts and ends its route in the depot, and each customer should be visited only once (Gribkovskaia 2011). With capacity constraints only, i.e. with a condition that the vehicle capacity C may not be exceeded during any of the routes, VRP is often referred to as a Capacitated Vehicle Routing Problem (CVRP).

4.2 Extensions to VRP

Several extensions to the classical CVRP will be applied in order to represent the combined production-inventory-transportation problem Asak faces.

4.2.1 VRP with pick-up and deliveries (VRPPD)

In case of VRPPD the classical CVRP is complicated by the condition that some or all of the customers in addition to the delivery demands d_i have pick-up demands p_i which need to be brought back to the depot. All customers have either delivery demands or pick-up demands, or both. It is assumed that neither the sum of all delivery demands d_i nor the sum of all pick-up demands p_i may exceed the total capacity of all vehicles taken together.

There are three alternative cases that can be met in any VRPPD (Gribkovskaia 2011):

- For each customer $d_i \geq p_i$. In this case the capacity of any vehicle cannot be violated in any point during the route, since after each visit of a customer the vehicle's load will either decrease or remain constant, and such a problem may be solved as an ordinary VRP with delivery demands as input parameters.
- For each customer $d_i \leq p_i$. In this case the vehicle's capacity may either remain constant or increase along the route. But since the total pick-up load of all of the customers may not exceed the vehicle's capacity, there will never appear a problem of overload in this case. Such a problem may be solved as an ordinary VRP with pick-up demands as input parameters.
- For some customers $d_i \geq p_i$, and for some of them $d_i \leq p_i$. In this case the vehicle's capacity may be violated along the route depending on the sequence of visiting customers, and this makes a problem much more complex than in previous two cases. Such a problem should be solved with both delivery and pick-up demands as input parameters. In general, there exist two approaches for solution of such problems: a simultaneous service (both delivery and pick-up services are done during a single visit), and a split service (two visits of a customer are allowed, in the first of which the delivery service is being performed, and during the second visit the pick-up service is done). Both mentioned approaches are widely described in the literature, including (Chen and Wu 2006), (Wassan, Nagy, and Ahmadi 2008), (Hoff et al. 2009).

4.2.2 Time Windows

Basically, in this problem we have the same time window for all of the factories, i.e. time when they are available for loading and unloading, namely a period between 07.00 and 16.00 every working day. The Vehicle Routing Problem with Time Windows (VRPTW) and different approaches to its solution are comprehensively described in (Bräysy and Gendreau 2005a) and (Bräysy and Gendreau 2005b).

4.2.3 Multiple depots

Asak does not operate its own vehicle fleet. Instead, it buys transportation services from third-party logistical operators. This means that there is no particular location that could be referred to as the vehicles' "depot" in this problem. Thus any vehicle may start its route at any location (factory) and end it in any other location without obligation to return to the initial location. This makes the problem a Multi-depot VRP (or MDVRP), since all factories in this case may be treated as depots. MDVRP is deeply researched in (Crevier, Cordeau, and Laporte 2007), (Nagy and Salhi 2005), (Liu et al. 2010).

4.2.4 Tour length limitation

There is no explicit limitation of the length of any tour, but there exists a limitation in the truck driver's driving time – it should not exceed nine hours per day, overtimes are not allowed. This condition will be treated as one of the constraints in the model.

4.2.5 Split Delivery VRP

One of conditions in the basic Vehicle Routing Problem is that for each customer pickup and delivery demands do not exceed vehicle capacity. In addition, each customer can be visited only once. In the given problem this Master thesis deals with internal factories' demands for products are not restricted and thus may exceed vehicles' capacities. We assume that each factory can be visited as many times as it is needed in order to satisfy its demand under total cost minimization objective. Therefore Split Delivery VRP (SDVRP) is applicable for the considered problem of Asak. In SDVRP the restriction that each customer is visited only once is removed. Moreover, the demand of each customer can be greater than the capacity of the vehicles. The SDVRP is NP-hard, and can be solved to optimality in a systematic way only on instances with less than 30 customers (Speranza and Archetti 2012).

5. Model assumptions

In the previous chapter a general structure of the model was designed and described. In order to be able to proceed to the stage of actual development of the model, several assumptions and simplifications have to be applied to the considered problem. A set of assumptions to the real-world problem is listed in part 5.1 below. One of the major assumptions made is that levels of customer demands in different periods for different products at each of the factories are used as input parameters in the model. The way of determination of customer demand for this problem is discussed in part 5.2.

5.1 Assumptions

As it was already mentioned above, the real world problem that we are facing in the given Master thesis is hard to be represented by a mathematical model, therefore we will need to use a set of assumptions to simplify the existing problem:

1. All trucks that are used for transportation of end-products or raw materials have equal capacities;
2. Transportation of raw materials and end-products are separated into two different problems, because raw materials and end-products are transported by vehicles with different body types, that are designed for transportation either of raw materials (e.g. sand) or final products (e.g. stones);
3. End-products are aggregated into groups with similar characteristics in order to avoid overloading of the model with too many parameters/variables;
4. As demand varies highly during the year, it will be specified under certain distribution and planning horizon and will be cut into intervals with a similar behavior of demand;
5. Product specialization of factories cannot be changed;
6. Only Hønefoss, Fetsund and Kristiansand factories are considered. There are two main reasons for this:
    - The major part of Asak's internal transportation takes place between these three factories;
    - The current internal transportation policy of Asak is based on the principle of direct carriage of raw materials and final products between only two

locations. The purpose of our research is to analyze possible improvements of the current transportation policy of Asak that could be reached by including additional locations into trucks' routes. Due to the long distances between Stjørdal and Bodø factories and the three other locations (see Table 5.1), and to limitation of the vehicle's driving time to nine hours per day, it is not possible to include Stjørdal and Bodø factories into daily routes of vehicles that would serve Hønefoss, Fetsund or Kristiansand factories.

Table 5.1. Driving distances and approximate driving times between factories

(Source: Google maps)

Distance, km/ Driving time	Fetsund	Kristiansand	Stjørdal	Bodø
Hønefoss	91,2/ 1h 21min	355/ 4h 21min	536/ 7h 4min	1210/ 16h
Fetsund	-	350/ 4h 11min	515/ 6h 37min	1189/ 15h 35min
Kristiansand	-	-	845/ 10h 22min	1519/ 19h 19min
Stjørdal	-	-	-	675/ 9h

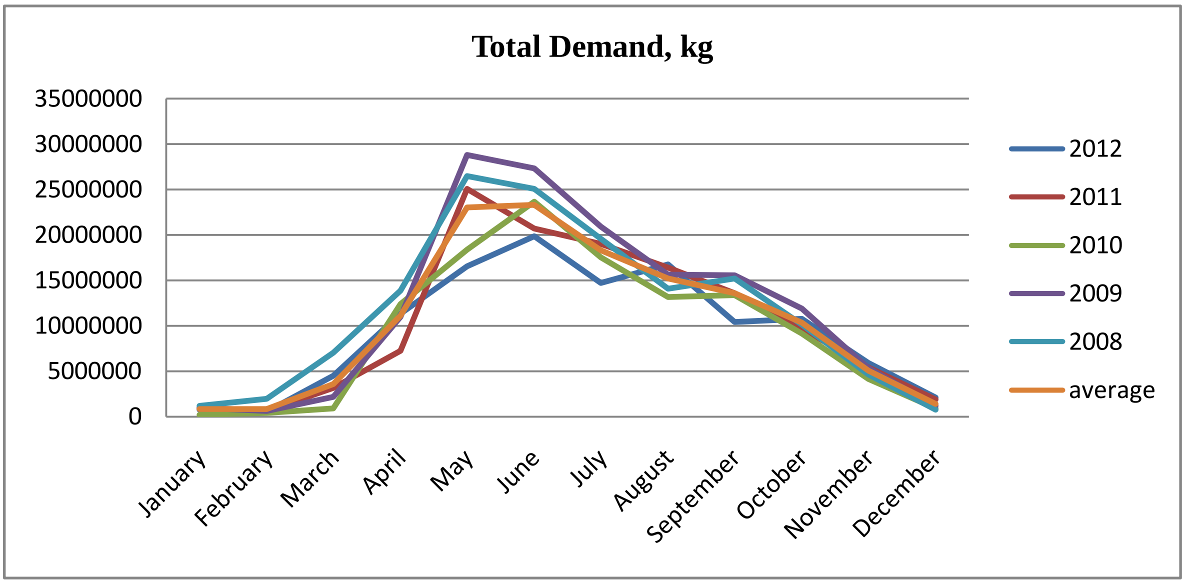
7. Boats are not considered as an alternative way of transportation of final products or raw materials.
8. Service time of loading/unloading a vehicle at a specific factory does not depend on initial load of a vehicle and a volume of products to be loaded/unloaded on/from a vehicle at this factory.

5.2 Determination of customer demand

According to the historical data provided by Asak for total demand for each month for each factory, it is possible to define distribution of the total annual demand per each month. For each product we are provided only with data for a yearly demand for each factory, so when distribution pattern is determined it is possible to find demand per month per each product in each factory. We assume that during any month demand is constant, but from month to month it varies according to the distribution pattern.

The data provided by Asak is confidential, so we changed it in a way that does not influence the distribution pattern. On the figures 5.1-5.3 below variations of the demand for each month from the year 2008 to the year 2012 and distribution patterns for Fetsund, Hønefoss and Kristiansand factories, respectively, can be seen.

Figure 5.1. Monthly demand distribution, Fetsund factory



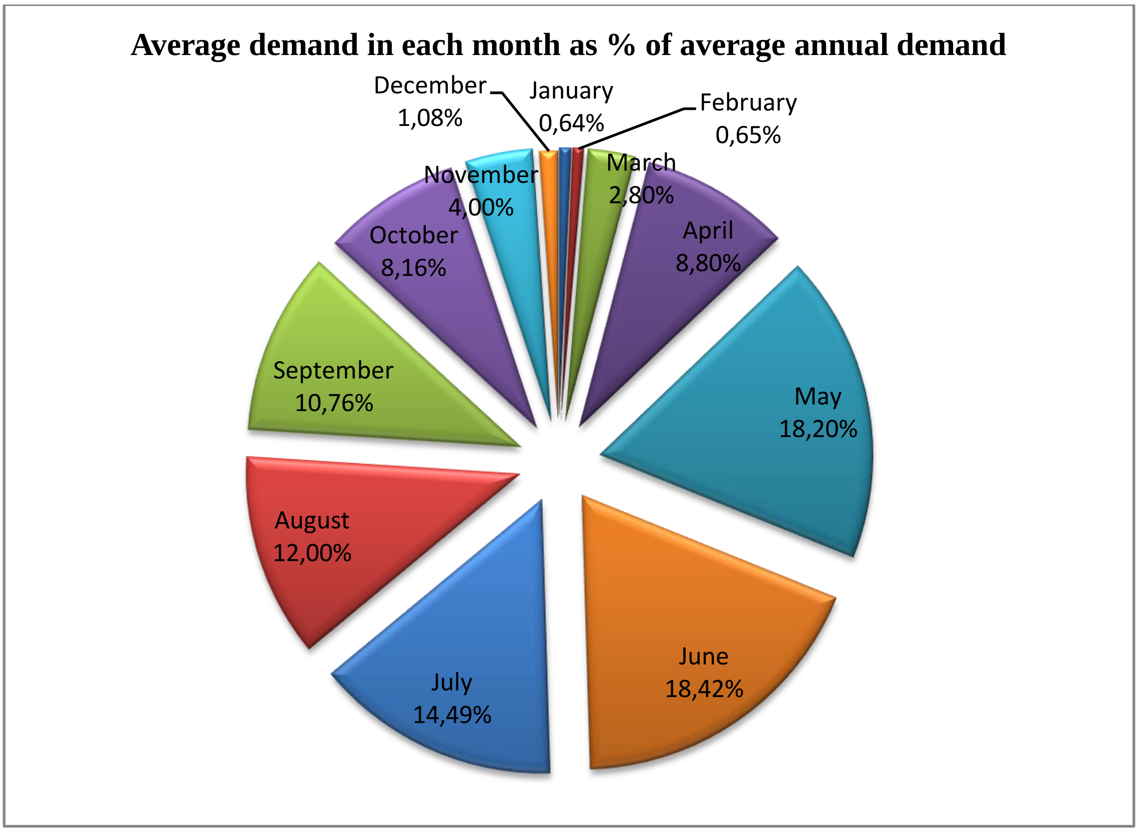
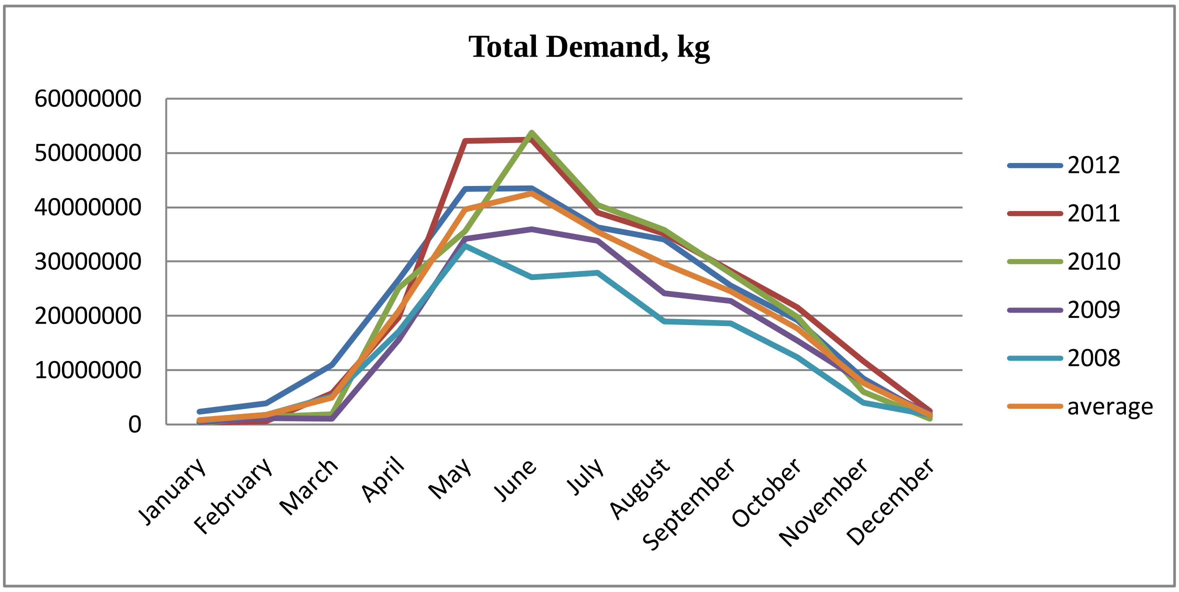


Figure 5.2. Monthly demand distribution, Hønefoss factory



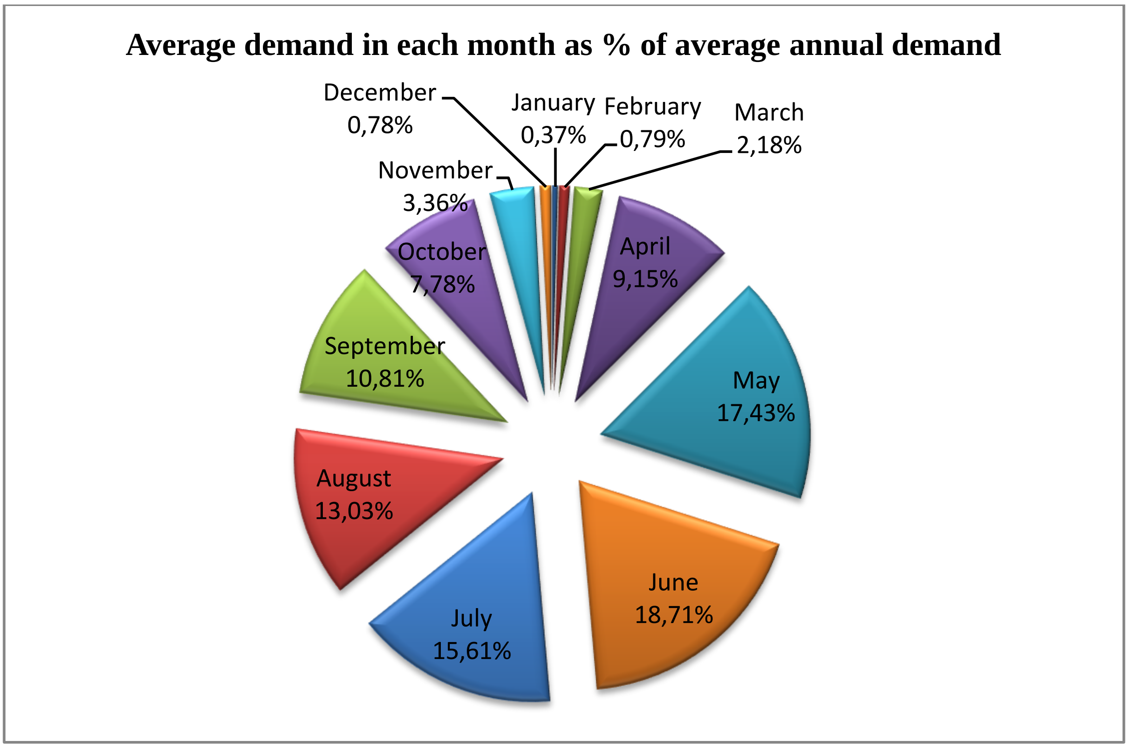
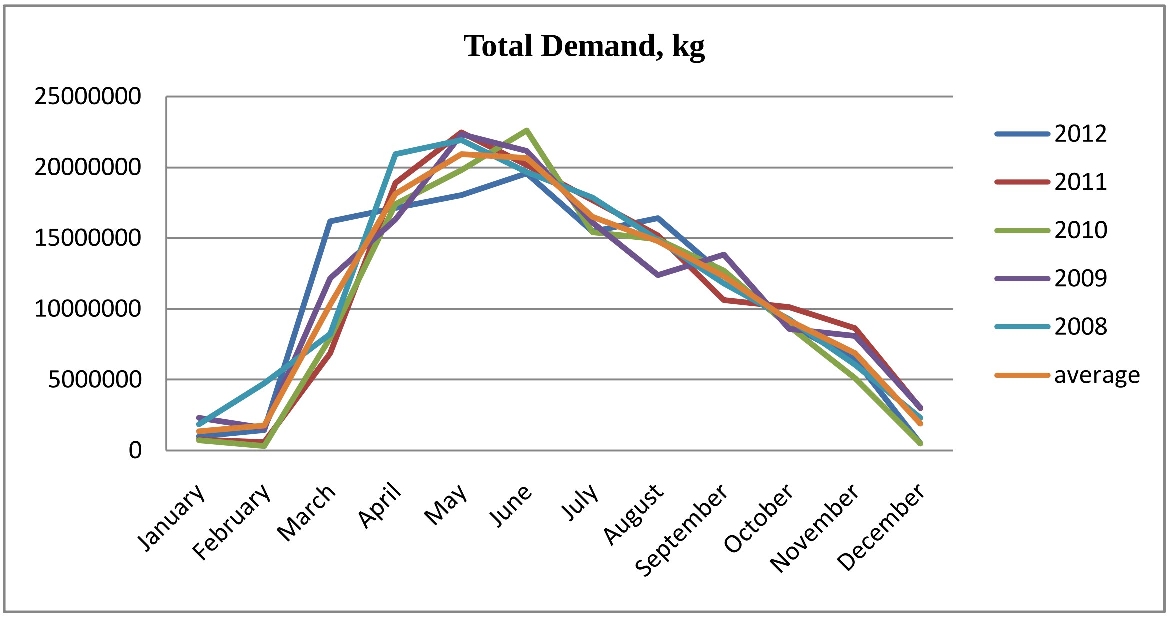
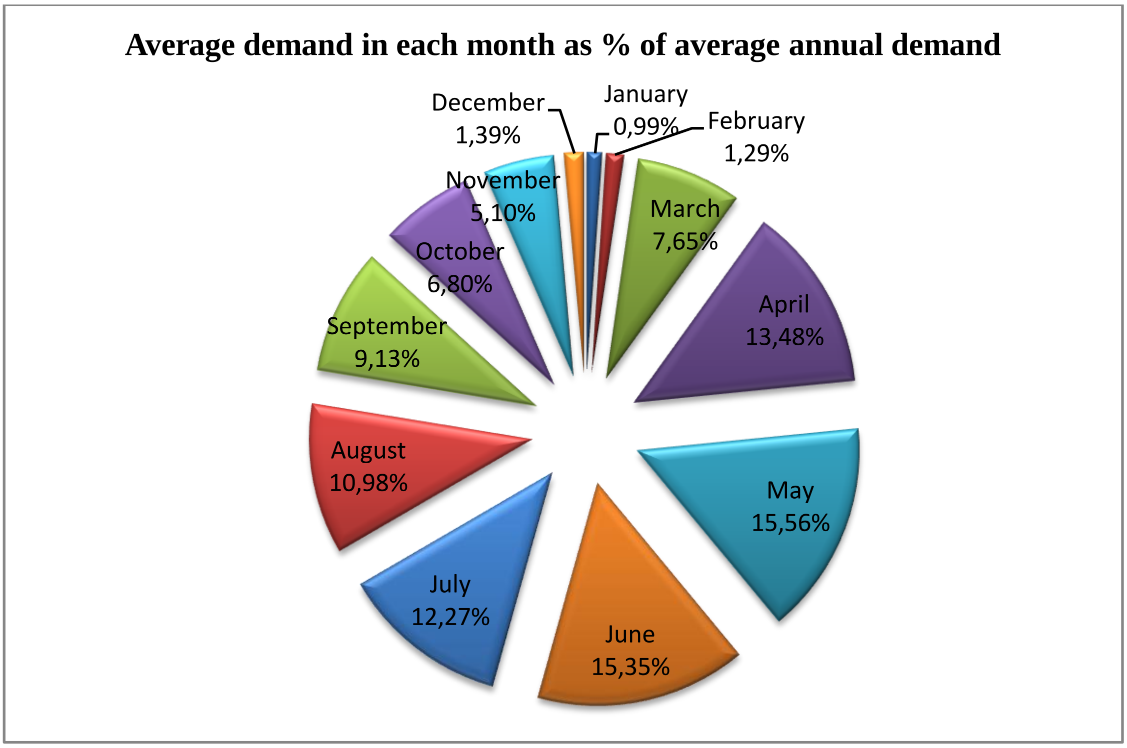


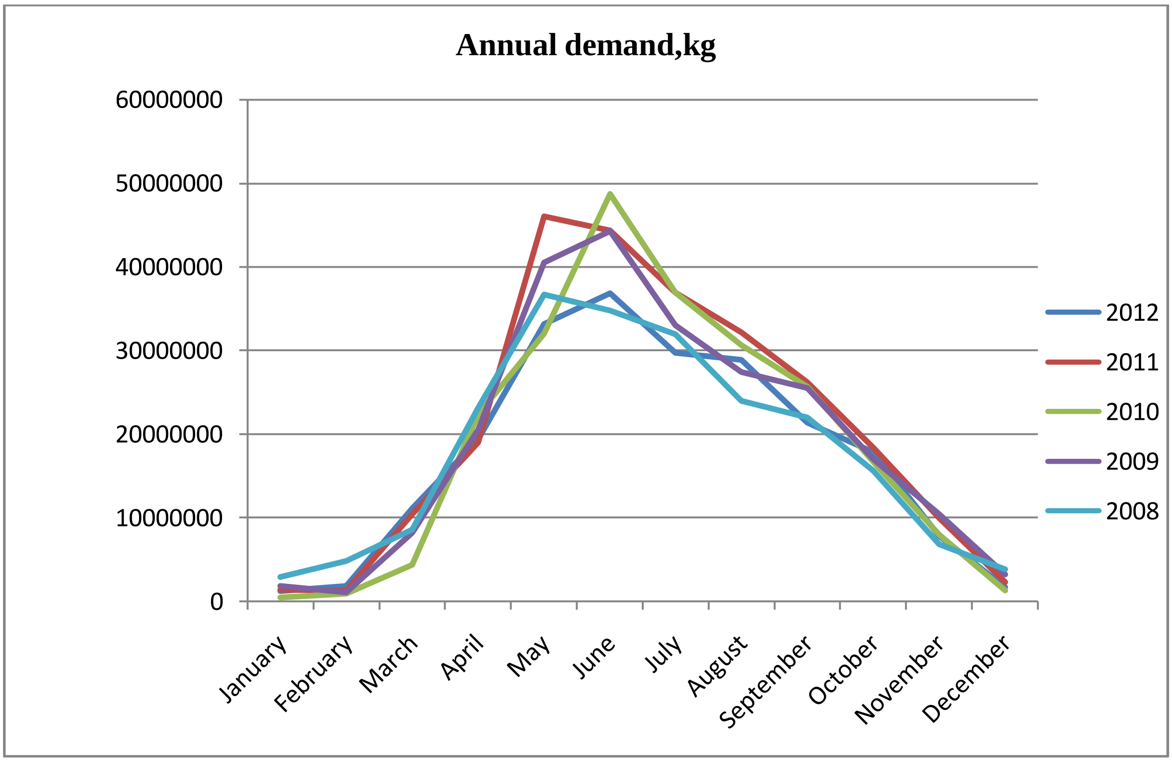
Figure 5.3. Monthly demand distribution, Kristiansand factory





As it can be seen from Figure 5.4 below (the real data is camouflaged), during the year total annual demand of the three factories (Hønefoss, Fetsund and Kristiansand) has a low season from December to February, high season from May to August and intermediate seasons from March to April and from September to November.

Figure 5.4. Monthly total demand distribution



The planning horizon of the model represented further in Chapter 6 of this Master thesis consists of 6 weeks: 2 weeks (14 days) from the low season, 2 weeks from the high season and 2 weeks from the intermediate season. This problem can be classified as a Periodic Vehicle Routing Problem (PVRP), which was observed and classified by Mourgaya and Vanderbeck (2006).

6. Mathematical model construction

In this chapter of the Master thesis a mathematical model for solving the combined production-inventory-transportation problem of Asak is developed.

In the first stage the model for Periodic Pick-ups and Delivery Vehicle Routing Problem with Time Windows (PPDVRPTW) is developed for solving the transportation sub-problem.

In the second stage PPDVRPTW is extended to a multi-product model with split demands and pickups, which is then combined with inventory and production sub-problems.

6.1 First stage: Transportation model

We will start with formulation of a Periodic Pick-ups and Delivery Vehicle Routing Problem with Time Windows, assuming that demands and pickups for each factory are already specified. Since Asak does not have any own vehicle depots (the company uses transportation services of third-party logistics operators), for modeling purposes we need to set up an artificial depot. As far as in reality there will not be any physical movements of vehicles to this artificial depot, we assume that traveling times and traveling distances from the depot to each factory are equal to zero. All vehicles start and finish their routes at the depot. Moreover, demands and pickups at the depot are always equal to zero. We also assume that there is only one product transported between factories. Cost of transportation of one ton of a product between different locations depends on the load of the vehicle. Asak’s transportation policy is built on the principle of having preferably only full truckloads because per-ton transportation costs are inversely proportional to the actual load of the vehicle. In addition, we assume that the total volume of delivery demands is equal to the total volume of pickup demands in the system.

Additional description of the problem:

- Planning horizon is 14 days;
- Objective function is to minimize total transportation costs;
- Four nodes (one depot and three factories as customer nodes);
- Homogeneous vehicle fleet with 30 available trucks with 30 tons carrying capacity;

- Delivery and pickup demands for each factory do not exceed the capacity of the vehicle;
- Deliveries and pickups are made simultaneously at each factory;
- Transportation pricing policy: if load of a vehicle is more than 20 tons, then Asak pays for transportation of actually carried products (actual load of a vehicle multiplied by per-ton cost of transportation between locations); if load of a vehicle is less than 20 tons, then Asak pays anyway for transportation of 20 tons (20 tons multiplied by per-ton cost of transportation between locations).

Below the formulation and description of the model for transportation sub-problem is presented.

Table 6.1. Notation for the transportation model

Sets:	
$\mathcal{K}$	set of vehicles
$\mathcal{A}$	set of edges
$\mathcal{T}$	set of time periods (days) within a planning horizon
$\mathcal{N}$	set of factories
$\{0\}$	depot
Parameters:	
$G_i^t \in \{0,1\}$	1, if delivery and/or pickup demands are more than zero in period t at factory i, $t \in \mathcal{T}, i \in \mathcal{N}$
PK_i^t	pickup demand at factory i in period t, $t \in \mathcal{T}$, $i \in \mathcal{N}$
D_i^t	delivery demand at factory i in period t, $t \in \mathcal{T}, i \in \mathcal{N}$
C_{ij}	cost of transporting one ton of products between locations i and j , $i, j \in \mathcal{N} \cup \{0\}$
TR_{ij}	traveling time between locations i and j , $i, j \in \mathcal{N} \cup \{0\}$
TR_{max}	maximum available traveling time

S_i	service time of unloading-loading a vehicle at factory i, $i \in \mathcal{N}$
L	latest time an unloading-loading service may begin at a factory
E	earliest time an unloading-loading service may begin at a factory
F	20 ton's load of the vehicle
W	capacity of a vehicle
M	a very big number
m	a very big negative number
Variables: $n_{ij}^{tk} \in \{0,1\}$	1 if load of vehicle k traveling between location i and j in period t is at least equal to F, $i, j \in \mathcal{N} \cup \{0\}$, $k \in \mathcal{K}, t \in \mathcal{T}$
$x_{ij}^{tk} \in \{0,1\}$	1 if vehicle k used edge $\{i, j\}$ in period $t, i, j \in \mathcal{N} \cup \{0\}, t \in \mathcal{T}$, $k \in \mathcal{K}$
u_i^{tk}	actual starting unloading-loading service time for vehicle k in period t at factory i, $t \in \mathcal{T}, i \in \mathcal{N}, k \in \mathcal{K}$
pl_i^{tk}	pickup load of vehicle k after leaving factory i in period t, $t \in \mathcal{T}, i \in \mathcal{N}, k \in \mathcal{K}$
dl_i^{tk}	delivery load of vehicle k after leaving factory i in period t, $t \in \mathcal{T}, i \in \mathcal{N}, k \in \mathcal{K}$

Objective function:

$$min \sum_{k \in \mathcal{K}} \sum_{(i,j) \in \mathcal{A}} \sum_{t \in \mathcal{T}} \left[\begin{matrix} \left(C_{ij} \cdot (dl_i^{tk} + pl_i^{tk}) + C_{ij} \cdot (F - dl_i^{tk} - pl_i^{tk} \right) \cdot \\ (1 - n_{ij}^{tk})) \cdot x_{ij}^{tk} \end{matrix} \right] \tag{1}$$

subject to

$$\sum_{j \in \mathcal{N}} x_{0j}^{tk} = 1, k \in \mathcal{K}, t \in \mathcal{T} \tag{2}$$

$$\sum_{j\in\mathcal{N}} x_{ij}^{tk} = \sum_{j\in\mathcal{N}} x_{ji}^{tk}, i \in \mathcal{N}, k \in \mathcal{K}, t \in \mathcal{T} \tag{3}$$

$$pl_j^{tk} \geq pl_i^{tk} + PK_j^t - M(1 - x_{ij}^{tk}), i, j \in \mathcal{N} \cup \{0\}, k \in \mathcal{K}, \ t \in \mathcal{T} \tag{4}$$

$$dl_j^{tk} \leq dl_i^{tk} - D_j^t + M(1 - x_{ij}^{tk}), i, j \in \mathcal{N} \cup \{0\}, k \in \mathcal{K}, \ t \in \mathcal{T} \tag{5}$$

$$dl_i^{tk} + pl_i^{tk} \leq W, i \in \mathcal{N}, \ k \in \mathcal{K}, \ t \in \mathcal{T} \tag{6}$$

$$dl_0^{tk} = 0, k \in \mathcal{K}, \ t \in \mathcal{T} \tag{7}$$

$$pl_0^{tk} = 0, k \in \mathcal{K}, \ t \in \mathcal{T} \tag{8}$$

$$(dl_i^{tk} + pl_i^{tk}) - F \geq m \cdot (1 - n_{ij}^{tk}), i, j \in \mathcal{N} \cup \{0\}, k \in \mathcal{K}, \ t \in \mathcal{T} \tag{9}$$

$$(dl_i^{tk} + pl_i^{tk}) - F \leq M \cdot n_{ij}^{tk}, i, j \in \mathcal{N} \cup \{0\}, k \in \mathcal{K}, \ t \in \mathcal{T} \tag{10}$$

$$G_i^t \leq \sum_{j\in N} \sum_{k\in K} x_{ij}^{tk}, i \in \mathcal{N}, t \in \mathcal{T} \tag{11}$$

$$u_i^{tk} + S_i + TR_{ij} - \mathrm{M}\left(1 - x_{ij}^{tk}\right) \leq u_j^{tk}, i, j \in \mathcal{N}, k \in \mathcal{K}, \ t \in \mathcal{T} \tag{12}$$

$$E \cdot \sum_{j\in\mathcal{N}} x_{ij}^{tk} \leq u_i^{tk} \leq L \cdot \sum_{j\in\mathcal{N}} x_{ij}^{tk}, i \in \mathcal{N}, k \in \mathcal{K}, t \in \mathcal{T} \tag{13}$$

$$\sum_{(i,j)\in A} x_{ij}^{tk} \cdot TR_{ij} \leq TR_{max}, k \in \mathcal{K}, t \in \mathcal{T} \qquad (14)$$

$$x_{ij}^{tk} \in \{0,1\}, (i,j) \in \mathcal{A}, k \in \mathcal{K}, t \in \mathcal{T} \qquad (15)$$

$$n_{ij}^{tk} \in \{0,1\}, (i,j) \in \mathcal{A}, k \in \mathcal{K}, t \in \mathcal{T} \qquad (16)$$

$$dl_i^{tk} \geq 0, i \in \mathcal{N} \cup \{0\}, k \in \mathcal{K}, t \in \mathcal{T} \qquad (17)$$

$$pl_i^{tk} \geq 0, i \in \mathcal{N} \cup \{0\}, k \in \mathcal{K}, t \in \mathcal{T} \qquad (18)$$

In this model, (1) is the objective function that minimizes total traveling costs for all periods and all vehicles. If vehicle *k* traveled from location *i* to location *j* in period *t* then $x_{ij}^{tk} = 1$, therefore it is needed to calculate traveling costs from *i* to *j*, which are correlated with vehicle load. If vehicle load is at least 20 tons then $n_{ij}^{tk} = 1$ and traveling costs will be equal to load (delivery and pickup load) multiplied by costs of transporting of 1 ton from location *i* to *j*: $C_{ij} \cdot (dl_i^{tk} + pl_i^{tk})$. If vehicle's load is less than 20 tons then $n_{ij}^{tk} = 0$ and traveling costs will be calculated for 20 tons load. These costs consist of costs of transportation of the exact load $C_{ij} \cdot (dl_i^{tk} + pl_i^{tk})$ and the difference between exact load and 20 tons $\left(C_{ij} \cdot (F - dl_i^{tk} - pl_i^{tk})\right)$. If vehicle *k* does not travel between locations *i* and *j* in period *t* then $x_{ij}^{tk} = 0$ and traveling costs are equal to zero.

Constraints (2) guarantee that each route for each vehicle in each period starts at the depot and (3) ensure that if a vehicle enters node *i* it leaves it as well.

Constrains (4) and (5) specify demand load and pickup load for each vehicle for each period moving from node *i* to node *j*.

Constraints (6) ensure that vehicle capacity is not exceeded while (7) and (8) set up delivery load and pickup load equal to zero in the depot.

Constraints (9) and (10) define load level when each vehicle travels from location i to location j in each period. When vehicle load $(dl_i^{tk} + pl_i^{tk})$ is more than 20 tons then $(dl_i^{tk} + pl_i^{tk}) - F > 0$, and in constraint (10) the product $M \cdot n_{ij}^{tk}$ should also be more than zero, and this will force n_{ij}^{tk} to be equal to 1; $n_{ij}^{tk} = 1$ identifies that vehicle's load is at least equal to 20 tons. If vehicle's load $(dl_i^{tk} + pl_i^{tk})$ is less than 20 tons then $(dl_i^{tk} + pl_i^{tk}) - F < 0$, then in constraint (9) the product $m \cdot (1 - n_{ij}^{tk})$ should also be less than zero, and this will force n_{ij}^{tk} to be equal to 0; $n_{ij}^{tk} = 0$ identifies that vehicle's load is less than 20 tons.

Constraints (11) ensure that node i is visited if it has positive delivery and/or pickup demands.

Constraints (12) guarantee time feasibility: vehicle k cannot start unloading-loading service at factory j before finishing it at the previous factory i and traveling from i to j, and (13) is the constraint that ensures feasibility of time windows restrictions, while (14) constrains traveling time.

Constraints (15)-(18) guarantee that variables are not negative.

According to the structure of the objective function of this model it can be seen that transportation costs will be optimized. In addition, by solving the problem using this model and analyzing the results, the optimal vehicle fleet can be found. The model provides a possibility to identify how many vehicles are needed in each season (high, low and intermediate seasons). The output of the model will also show the design of optimal transportation routes.

The described model is a pure transportation problem and it doesn't take into consideration inventory management and production planning sub-problems, which are important constituents of the considered problem. In connection with this a transportation policy that can be found with the help of this model cannot be implemented directly by the company since there exists a possibility of violation of storing and production capacities, and shortage of products in stock may occur. Therefore the described model has to be extended in order to be able to deal with the real problem of Asak.

6.2 Second stage: Combined model

In this stage a combined production-inventory-transportation model will be developed. The transportation model that was developed above will be used as a basis for the combined model. It will be extended to a multi-product problem with possibility of splitting delivery and pickup demands. Moreover, additional constraints and variables will be introduced to describe inventory and production sub-problems.

The output of the resulting model should be represented in the form of the hourly vehicles' transportation schedules for each day from each of the periods of two weeks from the low season, the high season and the intermediate season. These schedules should represent the optimal or close-to-optimal (with respect to the transportation costs) typical transportation patterns for the vehicles serving the three factories – Hønefoss, Fetsund and Kristiansand.

Table 6.2. Notation for the combined production-inventory-transportation model

Sets:	
$\mathcal{K}$	set of vehicles
$\mathcal{A}$	set of edges
$\mathcal{T}$	set of time periods within planning horizon
$\mathcal{P}$	set of products
$\mathcal{N}$	set of factories
$\{0\}$	depot
Parameters:	
PPR_{pi}^{t}	production capacity of product p in period t at factory i, $i \in \mathcal{N}$, $t \in \mathcal{T}, p \in \mathcal{P}$
SS_{pi}	safety stock required for product p at factory $i, p \in P, i \in N$
W	capacity of a vehicle
B_i	inventory capacity of factory i, $i \in N$
C_{ij}	cost of transporting of one ton between locations i and j, $i, j \in \mathcal{N} \cup \{0\}$
F	20 ton's load of the vehicle

S_i	service time of unloading-loading vehicle at factory i, $i \in \mathcal{N}$
E	earliest time unloading-loading service may begin at factory
L	latest time unloading-loading service may begin at factory
TR_{max}	maximum available traveling time
TR_{ij}	traveling time between locations i and j, $(i,j) \in \mathcal{A}$
CD_{pi}^{t}	customer demand for product p in period t at factory i, $i \in \mathcal{N}$, $p \in \mathcal{P}$, $t \in \mathcal{T}$
II_{pi}	initial inventory of product p at factory i, $i \in \mathcal{N}$, $p \in \mathcal{P}$
M	a very big number
m	a very big negative number
Variables: $n_{ij}^{tk} \in \{0,1\}$	1 if load of vehicle k traveling between locations i and j in period t is at least equal to F, $(i,j) \in \mathcal{A}$, $k \in \mathcal{K}$, $t \in \mathcal{T}$
$z_i^{tk} \in \{0,1\}$	1 if factory i has positive delivery and/or pickup demand in period t which is satisfied by vehicle k, $i \in \mathcal{N} \cup \{0\}$, $k \in \mathcal{K}$, $t \in \mathcal{T}$
$x_{ij}^{tk} \in \{0,1\}$	1 if vehicle k travels from i to j in period t, $(i,j) \in \mathcal{A}$, $k \in \mathcal{K}$, $t \in \mathcal{T}$
$h_i^{tp} \in \{0,1\}$	1 if need in ordering product p in period t at factory i occurs, $p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}$
$gap_{pi}^{t} \in \{0,1\}$	1 if customer demand for product p at the factory i in period t is not fully satisfied, $p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}$
$q_{pi}^{t} \in \{0,1\}$	1 if surplus of product p in period t exists at factory i, $p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}$
$v_{pi}^{t} \in \{0,1\}$	1 if there is a deficit of product p in period t at factory i, $p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}$
$pra_p^t \in \{0,1\}$	1 if supply for product p in the system exceeds demand for this product, t, $p \in \mathcal{P}, t \in \mathcal{T}$
dl_i^{tk}	delivery load of vehicle k after leaving factory i in period t, $i \in \mathcal{N}$, $k \in \mathcal{K}$, $t \in \mathcal{T}$

pl_i^{tk}	pickup load of vehicle k after leaving factory i in period t $i \in \mathcal{N}$,$k \in \mathcal{K}$, $t \in \mathcal{T}$
u_i^{tk}	actual starting unloading-loading service time for vehicle k in period t at factory i, $i \in \mathcal{N}$, $k \in \mathcal{K}$, $t \in \mathcal{T}$
in_{pi}^t	inventory of product p in the beginning of period t at factory i, $p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}$
scd_{pi}^t	satisfied customer demand for product p in period t at factory i, $p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}$
pr_{pi}^t	production of product p in period t at factory $i, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}$
$avpk_{pi}^t$	available pickup of product p at the factory i in period t, $p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}$
rd_{pi}^t	required demand for product p in period t at factory $i, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}$
dts_{pi}^t	delivery demand for product p in period t at factory i which will be satisfied, $p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}$
$pkts_{pi}^t$	pickup demand of product p at the factory i in period t which will be satisfied, $p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}$
$dtsv_{pi}^{tk}$	delivery demand for product p in period t at factory i which will be satisfied by vehicle k, $p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}, k \in \mathcal{K}$
$pktsv_{pi}^{tk}$	pickup demand of product p at the factory i in period t which will be satisfied by vehicle k, $p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}, k \in \mathcal{K}$

Objective function:

$$min \sum_{k \in \mathcal{K}} \sum_{(i,j) \in \mathcal{A}} \sum_{t \in \mathcal{T}} \left[\begin{matrix} \left(C_{ij} \cdot (dl_i^{tk} + pl_i^{tk}) + C_{ij} \cdot (F - dl_i^{tk} - pl_i^{tk}\right) \cdot \\ (1 - n_{ij}^{tk})) \cdot x_{ij}^{tk} \end{matrix} \right] \tag{1}$$

subject to

Inventory management constraints

$$\sum_{p\in\mathcal{P}} in_{pi}^{t} \leq B_i, i \in \mathcal{N}, t \in \mathcal{T} \tag{2}$$

$$in_{pi}^{t} = in_{pi}^{(t-1)} + pr_{pi}^{(t-1)} + dts_{pi}^{(t-1)} - pkts_{pi}^{(t-1)} - scd_{pi}^{(t-1)}, i \in \mathcal{N},\ t = 2 \dots \mathcal{T}, p \in \mathcal{P} \tag{3}$$

$$in_{pi}^{1} = II_{pi}, i \in \mathcal{N}, p \in \mathcal{P} \tag{4}$$

$$pr_{pi}^{t} \leq PPR_{pi}^{t}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{5}$$

$$CD_{pi}^{t} - in_{pi}^{t} - dts_{pi}^{t} - pr_{pi}^{t} + pkts_{pi}^{t} \leq M \cdot gap_{pi}^{t}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{6}$$

$$CD_{pi}^{t} - in_{pi}^{t} - dts_{pi}^{t} - pr_{pi}^{t} + pkts_{pi}^{t} \geq m \cdot (1 - gap_{pi}^{t}), p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{7}$$

$$scd_{pi}^{t} = CD_{pi}^{t} - (CD_{pi}^{t} - in_{pi}^{t} - dts_{pi}^{t} - pr_{pi}^{t} + pkts_{pi}^{t}) \cdot gap_{pi}^{t}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{8}$$

Time Windows constraints

$$u_i^{tk} + S_i + TR_{ij} - \mathrm{M}\left(1 - x_{ij}^{tk}\right) \leq u_j^{tk},\ i,j \in \mathcal{N}, k \in \mathcal{K},\ t \in \mathcal{T} \tag{9}$$

$$E \cdot \sum_{j\in\mathcal{N}\cup\{0\}} x_{ij}^{tk} \leq u_i^{tk} \leq L \cdot \sum_{j\in\mathcal{N}\cup\{0\}} x_{ij}^{tk}, i \in \mathcal{N}, k \in \mathcal{K},\ t \in \mathcal{T} \tag{10}$$

$$\sum_{(i,j)\in\mathcal{A}} x_{ij}^{tk} \cdot TR_{ij} \leq TR_{max},\ k \in \mathcal{K}, t \in \mathcal{T} \tag{11}$$

Pickup and Delivery constraints

$$\sum_{j\in\mathcal{N}} x_{0j}^{tk} = 1, k \in \mathcal{K}, t \in \mathcal{T} \tag{12}$$

$$\sum_{j\in\mathcal{N}} x_{ij}^{tk} = \sum_{j\in\mathcal{N}} x_{ji}^{tk}, i \in \mathcal{N}, k \in \mathcal{K}, t \in \mathcal{T} \tag{13}$$

$$in_{pi}^{t} + pr_{pi}^{t} - CD_{pi}^{t} \leq M \cdot q_{pi}^{t}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{14}$$

$$in_{pi}^{t} + pr_{pi}^{t} - CD_{pi}^{t} \geq m \cdot (1 - q_{pi}^{t}), p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{15}$$

$$avpk_{pi}^{t} = q_{pi}^{t} \cdot \left(in_{pi}^{t} + pr_{pi}^{t} - CD_{pi}^{t}\right), p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{16}$$

$$CD_{pi}^{t} - in_{pi}^{t} - pr_{pi}^{t} \leq M \cdot v_{pi}^{t}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{17}$$

$$CD_{pi}^{t} - in_{pi}^{t} - pr_{pi}^{t} \geq m \cdot (1 - v_{pi}^{t}), p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{18}$$

$$rd_{pi}^{t} = v_{pi}^{t} \cdot \left(CD_{pi}^{t} - in_{pi}^{t} - pr_{pi}^{t}\right), p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{19}$$

$$\sum_{i\in\mathcal{N}} avpk_{pi}^{t} - \sum_{i\in\mathcal{N}} rd_{pi}^{t} \leq M \cdot pra_{p}^{t}, p \in \mathcal{P}, t \in \mathcal{T} \tag{20}$$

$$\sum_{i\in\mathcal{N}} avpk_{pi}^{t} - \sum_{i\in\mathcal{N}} rd_{pi}^{t} \geq m \cdot (1 - pra_{p}^{t}), p \in \mathcal{P}, t \in \mathcal{T} \tag{21}$$

$$\sum_{i\in\mathcal{N}} dts_{pi}^{t} = \sum_{i\in\mathcal{N}} avpk_{pi}^{t} \cdot \left(1 - pra_{p}^{t}\right) + \sum_{i\in\mathcal{N}} rd_{pi}^{t} \cdot pra_{p}^{t}, p \in \mathcal{P}, t \in \mathcal{T} \tag{22}$$

$$dts_{pi}^{t} \leq rd_{pi}^{t}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{23}$$

$$\sum_{i\in\mathcal{N}} pkts_{pi}^{t} = \sum_{i\in\mathcal{N}} avpk_{pi}^{t} \cdot \left(1 - pra_{p}^{t}\right) + \sum_{i\in\mathcal{N}} rd_{pi}^{t} \cdot pra_{p}^{t}, p \in \mathcal{P}, t \in \mathcal{T} \tag{24}$$

$$pkts_{pi}^{t} \leq avpk_{pi}^{t}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{25}$$

$$\sum_{k \in \mathcal{K}} dtsv_{pi}^{tk} = dts_{pi}^{t}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{26}$$

$$\sum_{k \in \mathcal{K}} pktsv_{pi}^{tk} = pkts_{pi}^{t}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \tag{27}$$

$$pl_{j}^{tk} \geq pl_{i}^{tk} + \sum_{p \in \mathcal{P}} pktsv_{pj}^{tk} - M\left(1 - x_{ij}^{tk}\right) i, j \in \mathcal{N} \cup \{0\}, k \in \mathcal{K}, t \in \mathcal{T} \tag{28}$$

$$dl_{j}^{tk} \leq dl_{i}^{tk} - \sum_{p \in \mathcal{P}} dtsv_{pj}^{tk} + M\left(1 - x_{ij}^{tk}\right), i, j \in \mathcal{N} \cup \{0\}, k \in \mathcal{K}, t \in \mathcal{T} \tag{29}$$

$$dl_{i}^{tk} + pl_{i}^{tk} \leq W, i \in \mathcal{N},\ k \in \mathcal{K},\ t \in \mathcal{T} \tag{30}$$

$$dl_{0}^{tk} = 0\ , k \in \mathcal{K},\ t \in \mathcal{T} \tag{31}$$

$$pl_{0}^{tk} = 0, k \in \mathcal{K},\ t \in \mathcal{T} \tag{32}$$

$$(dl_{i}^{tk} + pl_{i}^{tk}) - F \geq m \cdot (1 - n_{ij}^{tk})\ , i, j \in \mathcal{N} \cup \{0\}, k \in \mathcal{K},\ t \in \mathcal{T} \tag{33}$$

$$(dl_{i}^{tk} + pl_{i}^{tk}) - F \leq M \cdot n_{ij}^{tk}, i, j \in \mathcal{N} \cup \{0\}, k \in \mathcal{K},\ t \in \mathcal{T} \tag{34}$$

$$(\textstyle\sum_{p \in \mathcal{P}} pktsv_{pi}^{tk} + \sum_{p \in \mathcal{P}} dtsv_{pi}^{tk}) \leq z_{i}^{tk} \cdot M, i \in \mathcal{N}, k \in \mathcal{K},\ t \in \mathcal{T} \tag{35}$$

$$(\textstyle\sum_{p \in \mathcal{P}} pktsv_{pi}^{tk} + \sum_{p \in \mathcal{P}} dtsv_{pi}^{tk}) \geq z_{i}^{tk}, i \in \mathcal{N}, k \in \mathcal{K},\ t \in \mathcal{T} \tag{36}$$

$$z_{i}^{tk} \leq \textstyle\sum_{j \in N} x_{ij}^{tk}\ ,\ i, j \in \mathcal{N}, k \in \mathcal{K},\ t \in \mathcal{T} \tag{37}$$

$$n_{ij}^{tk} \in \{0,1\}, i,j \in \mathcal{N} \cup \{0\}, k \in \mathcal{K},\ t \in \mathcal{T} \quad (38)$$

$$z_i^{tk} \in \{0,1\}, i \in \mathcal{N}, k \in \mathcal{K},\ t \in \mathcal{T} \quad (39)$$

$$x_{ij}^{tk} \in \{0,1\}, (i,j) \in \mathcal{A}, k \in \mathcal{K},\ t \in \mathcal{T} \quad (40)$$

$$h_i^{tp} \in \{0,1\}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (41)$$

$$stpr_{pi}^{t} \in \{0,1\}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (42)$$

$$gap_{pi}^{t} \in \{0,1\}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (43)$$

$$q_{pi}^{t} \in \{0,1\}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (44)$$

$$v_{pi}^{t} \in \{0,1\}, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (45)$$

$$pra_{p}^{t} \in \{0,1\}, p \in \mathcal{P}, t \in \mathcal{T} \quad (46)$$

$$dl_i^{tk} \geq 0, i \in \mathcal{N} \cup \{0\}, k \in \mathcal{K},\ t \in \mathcal{T} \quad (47)$$

$$pl_i^{tk} \geq 0, i \in \mathcal{N} \cup \{0\}, k \in \mathcal{K},\ t \in \mathcal{T} \quad (48)$$

$$u_i^{tk} \geq 0, i \in \mathcal{N}, k \in \mathcal{K},\ t \in \mathcal{T} \quad (49)$$

$$in_{pi}^{t} \geq 0, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (50)$$

$$scd_{pi}^{t} \geq 0, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (51)$$

$$pr_{pi}^{t} \geq 0, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (52)$$

$$avpk_{pi}^{t} \geq 0, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (53)$$

$$rd_{pi}^{t} \geq 0, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (54)$$

$$dts_{pi}^{t} \geq 0, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (55)$$

$$pkts_{pi}^{t} \geq 0, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N} \quad (56)$$

$$dtsv_{pi}^{tk} \geq 0, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}, k \in \mathcal{K} \quad (57)$$

$$pktsv_{pi}^{tk} \geq 0, p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{N}, k \in \mathcal{K} \quad (58)$$

In this model, (1) is the objective function that minimizes total traveling costs for all periods and all vehicles. If vehicle k travels between locations i and j in period t then $x_{ij}^{tk} = 1$, so in this case cost for travelling between locations i and j appears. In our case traveling cost depends on the load of the vehicle: if vehicle's load is more than 20 tons then there is a fixed price per ton, and if vehicle's load is less than 20 tons, the cost of transportation of the load will anyway be the same as cost of transportation of 20 tons. For example, if vehicle's load is more than 20 ton then $n_{ij}^{tk} = 1$, and $\left(C_{ij} \cdot (dl_i^{tk} + pl_i^{tk}) + C_{ij} \cdot (F - dl_i^{tk} - pl_i^{tk}\right) \cdot \left(1 - n_{ij}^{tk}\right) = \left(C_{ij} \cdot dl_i^{tk} + C_{ij} \cdot (F - dl_i^{tk}\right) \cdot 0 = C_{ij} \cdot (dl_i^{tk} + pl_i^{tk})$, where C_{ij} is cost per ton and $(dl_i^{tk} + pl_i^{tk})$ is the load of the vehicle. If vehicle's load is less than 20 ton then $n_{ij}^{tk} = 0$, and $\left(C_{ij} \cdot (dl_i^{tk} + pl_i^{tk}) + C_{ij} \cdot (F - dl_i^{tk} - pl_i^{tk}\right) \cdot \left(1 - n_{ij}^{tk}\right) = \left(C_{ij} \cdot dl_i^{tk} + C_{ij} \cdot (F - dl_i^{tk}\right) \cdot 1 = C_{ij} \cdot (dl_i^{tk} + pl_i^{tk}) + C_{ij} \cdot (F - dl_i^{tk} - pl_i^{tk})$, where $C_{ij} \cdot (dl_i^{tk} + pl_i^{tk})$ is cost of transportation of actual load, and since all costs should be calculated (up to 20 tons), the second part of total costs is $C_{ij} \cdot (F - dl_i^{tk} - pl_i^{tk})$; all together the sum of these two parts give us cost of transportation of 20 tons.

Constraint (2) ensures that capacity of inventory at each factory is not violated.

Constraint (3) determines inventory level of each product for each period for each factory. Inventory level of product p in the beginning of the period t is equal to the sum of inventory level of this product in period $(t-1)$, actual volume of its production and volume of this product received from other factories in period $(t-1)$ minus number of this product delivered to other factories and sold to customers in period $(t-1)$.

Constraint (4) specifies initial inventory of each product for each factory.

Constraint (5) ensures that production level of product *p* is less than or equal to production capacity.

Constraints (6) and (7) identify if customer demand can be fully satisfied. Expression $(CD_{pi}^t - in_{pi}^t - dts_{pi}^t - pr_{pi}^t + pkts_{pi}^t)$ identifies which part of the customer demand can be satisfied, and $(in_{pi}^t + dts_{pi}^t + pr_{pi}^t - pkts_{pi}^t)$ identifies availability of product p in period *t* at factory *I* and equals to inventory, delivery and production of the product minus pickup of this product. If $(CD_{pi}^t - in_{pi}^t - dts_{pi}^t - pr_{pi}^t + pkts_{pi}^t)$ is less than or equal to zero, this means that there are enough products to satisfy customer demand fully, and variable $gap_{pi}^t = 0$. If $(CD_{pi}^t - in_{pi}^t - dts_{pi}^t - pr_{pi}^t + pkts_{pi}^t)$ is positive, this means that only this amount of customer demand can be satisfied and variable $gap_{pi}^t = 1$.

Constraint (8) defines amount of satisfied customer demand of product p at factory *i*. If $gap_{pi}^t = 0$, this means that satisfied customer demand is equal to customer demand $scd_{pi}^t = CD_{pi}^t$; if customer demand cannot be fully satisfied then $gap_{pi}^t = 1$ and satisfied customer demand $scd_{pi}^t = CD_{pi}^t$-$(CD_{pi}^t - in_{pi}^t - dts_{pi}^t - pr_{pi}^t + pkts_{pi}^t) \cdot gap_{pi}^t$ => $scd_{pi}^t = CD_{pi}^t$-$(CD_{pi}^t - in_{pi}^t - dts_{pi}^t - pr_{pi}^t + pkts_{pi}^t) \cdot 1$ => $scd_{pi}^t = in_{pi}^t + dts_{pi}^t + pr_{pi}^t - pkts_{pi}^t$, and this is the amount of product which is available in period *t* at factory *i*.

Constraint (9) guarantees time feasibility: vehicle *k* cannot start unloading-loading service at factory *j* before finishing it at previous factory i and traveling from i to j, and (10) is a constraint that ensures feasibility of time windows restrictions, while (11) constrains traveling time.

Constraint (12) guarantees that each route for each vehicle in each period starts at the depot and (13) ensures that if a vehicle enters node *i* it leaves it as well.

Constraints (14) and (15) define available pickup loads of product p in the system. If $(in_{pi}^{t} + pr_{pi}^{t} - CD_{pi}^{t})$ is less than or equal to zero then all available amount of product *p* at factory i will be used to satisfy customer demand and nothing will be left for pickup $(q_{pi}^{t} = 0)$; otherwise $q_{pi}^{t} = 1$, and this means that after satisfaction of the customer demand there are some products left which can be used as a pickup load.

Constraint (16) sets up available pickup for product *p* in the system.

Constraints (17) and (18) define level of demand for product p in the system. If $(CD_{pi}^{t} - in_{pi}^{t} - pr_{pi}^{t})$ is less than or equal to zero, then customer demand for product *p* can be fully satisfied with available amount of this product at this factory $(v_{pi}^{t} = 0)$. If amount of the available product cannot cover customer demand for this product, then the need for delivery of this product occurs.

Constraint (19) sets up needed demand for product *p* in the system.

Constraints (20) and (21) define product availability in the system. If available pickup in the system is bigger than needed demand for product p $(\sum_{i \in \mathcal{N}} avpk_{pi}^{t} - \sum_{i \in \mathcal{N}} rd_{pi}^{t})$, then all needed demand could be satisfied $(pra_{p}^{t} = 1)$. If available pickup in the system for product p is less than needed demand, the needed demand can be satisfied partly on the amount of available pickup $(pra_{p}^{t} = 0)$.

Constraint (22) sets up amount of the demand for product p which will be satisfied in period *t* in the whole system.

Constraint (23) ensures that demand for product *p* at the factory which will be satisfied in period *t* does not exceed needed demand for this product at the factory *i* in period *t*.

Constraint (24) sets up amount of pickup for product p at the factory which will be satisfied in period *t*.

Constraint (25) ensures that pickup demand for product p at factory i which will be satisfied in period *t* does not exceed available pickup load of this product at the factory *i* in period *t*.

Constraint (26) guarantees that all delivery demand for product *p* at factory *i* in period *t* which will be satisfied is actually satisfied by a set of vehicles.

Constraint (27) guarantees that all pickup demand for product *p* at the factory *i* in period *t* which will be satisfied is actually satisfied by a set of vehicles.

Constraints (28) and (29) define load level of each vehicle traveling from location *i* to location *j* in each period.

Constraint (30) ensures that vehicle capacity is not exceeded, while (31) and (32) set up delivery load and pickups load equal to zero at the depot.

Constraints (33) and (34) define load level of each vehicle traveling from location *i* to location *j* in each period.

Logical constraints (35) and (36) specify if node *i* has a positive delivery or/and pickup demand in period *t*. If expression $(\sum_{p \in \mathcal{P}} pktsv_{pi}^{tk} + \sum_{p \in \mathcal{P}} dtsv_{pi}^{tk})$ is positive then delivery or/and pickup is performed at factory i and z_i^{tk}=1. Otherwise, if $(\sum_{p \in \mathcal{P}} pktsv_{pi}^{tk} + \sum_{p \in \mathcal{P}} dtsv_{pi}^{tk})$ is equal to zero then factory i doesn't need any pickup or delivery and $z_i^{tk} = 0$.

Constraint (37) ensures that node *i* is visited if it has positive delivery or/and pickup demands.

Constraints (38)-(59) guarantee that variables are not negative.

The combined production-inventory-transportation model is focused on optimizing total travelling costs with respect to requirements of inventory management and production planning sub-problems. This model is aimed at specification of internal transportation policy for Asak. With respect to minimization of total transportation costs an optimal vehicle fleet and optimal production levels can be found. However the constructed combined model represents a simplification of the real-world problem and does not cover some of its important aspects:

- Customer demand in the constructed model is assumed to be deterministic. In order to approximate the model to reality customer demand should be treated as stochastic;
- Warehouses and other factories may be included into the model;

- Boats may be introduced into the model as alternative means of transportation.

Addition of the listed characteristics into the model would increase its complexity.

As it can be seen from the form of the objective function and many of the constraints the model is non-linear. Basically there are two ways of solving such types of models:

- With usage of existing or especially developed non-linear solvers or heuristic methods. In this case, the designed model is solved without changes;
- Through transformation and simplification of a constructed model to the linear form.

The considered combined production-inventory-transportation model can be used to create daily transportation patterns and can be changed according to the requirements of additional conditions that could appear.

Results of solving the model with real data used as an input could be compared with the current "direct" transportation policy of Asak that implies only direct transportation of raw materials and final products from one location to another. Based on this comparison a conclusion about potential cost-savings of application of the resulting production-inventory-transportation model in practice could be made.

7. Additional measures of improvement of the current production, inventory management and transportation policies of Asak Miljøstein AS

In Chapter 6 of this Master thesis we introduced and described the mathematical model aimed at optimization of the production and inventory management policies used at Asak's Hønefoss, Fetsund and Kristiansand factories, and at optimization of transportation flows of final products between these factories. However, in spite of quite a high complexity of the model, it still leaves out several important issues that we would like to briefly discuss further in this chapter.

Firstly, the model uses the forecasted sales, or demand, figures of each factory as input parameters, and thus sets aside any analysis of reasons for these values. This consequently leads to the fact that the model overlooks the potential possibilities of improvement of internal transportation efficiency already on the stage of more equal distribution of customer demand between the factories. It is quite evident that a more even distribution of customer demands between the factories would lead to reduced levels of factories' internal demands for products in order to maintain the desired levels of their complete assortment in stock.

The forecasts of future sales are based on the data from the previous periods. Considering Hønefoss and Fetsund factories, both of which serve the same region, the historical data shows that among them the majority of demand for all types of products and, consequently, the biggest part of sales of products during all seasons take place at the Hønefoss factory mainly due to its bigger production capacity (almost twice as big as the production capacity of the factory in Fetsund). At the same time, the production capacity utilization rate of Fetsund factory is much higher than that of the factory in Hønefoss. One of the main reasons for such inequality in production utilization rates of the two considered factories is the convenient (for clients of the region) geographical location of the Fetsund factory compared to the location of the factory in Hønefoss, because the majority of Asak's customers of the region are situated closer to the Fetsund factory. Thus a high level of customer demand for the whole assortment of products and, consequently, a high level of production take place at the factory in Fetsund. As it was already mentioned above in chapter 2 "Problem description", Fetsund factory, due to its specialized machinery, cannot produce the most popular large-volume products and therefore depends on supplies of these products from the Hønefoss factory. This situation leads to mainly one-way transportation of products to Fetsund factory from the factory in Hønefoss. This problem is particularly acute in periods of high demand.

In order to decrease the volumes of costly internal transportation of final products between the factories, the most logical measure would be to try to stimulate customers to buy products at locations where they are actually produced. Considering Fetsund and Hønefoss factories, this could possibly be achieved by offering a certain discount off the standard sales price to the clients normally buying products from the Fetsund factory, in case they buy and collect products at the factory in Hønefoss. It seems to be reasonable for Asak to offer such a discount to their clients periodically, especially during the high sales seasons. The total amount of the discussed discount per ton of stones should not exceed the per-ton cost of transportation of stones from Hønefoss to Fetsund (which approximately amounts to NOK 100 currently).

Secondly, the developed model considers only the inter-factories transportation of *final* products, setting aside the major problem of internal transportation of raw materials (namely, sand) between Hønefoss and Fetsund factories. The causes of this problem have also already been touched upon above in the "Problem description" part of this work. In short, some of the products from Asak's product line are produced both at Fetsund and Hønefoss, and since both factories serve the same region of the market, it frequently happens that a client ordering products of a certain type is supplied with the mixture of pallets produced at both factories. This practice leads to the necessity for Asak to make sure that those types of products produced at both factories are totally identical, especially in terms of color, in order to avoid complaints from customers getting several pallets of products of incompatible colors. So, in order to ensure similarity in color of products, currently sand (the main ingredient used for production of concrete products) is supplied to both factories from the quarry situated nearby Hønefoss factory. This practice is the reason for high volumes of costly one-way transportation of sand from Hønefoss to Fetsund. The reason for non-inclusion of the described sand transportation problem into the model developed in Chapter 6 is the fact that for transportation of sand trucks with a specialized body type are used, which are not suitable for transportation of final products, and therefore it is not physically possible to combine transportation of sand and final products in the same route. However, we would like to propose here some actions that could possibly decrease the volume of transportation of sand between Hønefoss and Fetsund.

According to the information provided by Asak, the company possesses another quarry situated nearby Fetsund. However, the sand extracted from this quarry is not being currently used for production of those types of stones which are also produced at Hønefoss factory due to the sand color differences. Taking this information into consideration, we would suppose that the following actions could be useful for Asak:

- Consider splitting the customer region, currently served by Fetsund and Hønefoss factories, into two separate markets – one for each factory. This action would imply supplying each specific customer from one factory only and consequently would let Asak use the quarry situated nearby Fetsund for production of all types of products at the Fetsund factory. Provided that the capacity of the quarry nearby Fetsund and sand extraction rate from it are sufficient for Fetsund factory production needs, this measure would eliminate the necessity of transportation of sand between the two factories.
- Consider introduction of the new product sub-types clearly indicating the color difference of products produced at the Fetsund factory using sand from the neighboring quarry. This measure would give clients possibility to decide on their own whether to buy the products of just one color or, in case of lack of products of the same color, if the combination of different colors would be appropriate for them. This action would also eliminate the necessity of transportation of sand between the two factories.

Both of the proposed actions, however, have a significant drawback: even though they would not reflect on total volumes of production of those types of products manufactured at both factories, these measures would most probably lead to a decrease in production of popular products of the same color. This fact could especially become a problem during a high sales season and could reflect in a certain amount of lost sales. Still, we assume that the proposed measures are worth considering provided that proper estimations (comparison of the evaluated amounts of potential lost sales and benefits of the elimination of transportation of sand between the factories) could be carried out.

8. Conclusions

In this Master thesis a model for a real-world combined production-inventory-transportation problem was developed and constructed. The problem scrutinized in this work was provided by the Norwegian company Asak Miljøstein AS, a sales organization for three producers of concrete products in Norway.

In brief, the problem considered in this Master thesis consists in the following. According to the policy of Asak Miljøstein AS, all factories, for each of which the production capacities and customer demands are different, should be self-sufficient with all products in the company's product line. Therefore a need for planning and execution of internal transportation between five factories and four warehouses, owned by the producers of concrete products, appears. One of the main characteristics of the problem is that most products are very heavy with a low price per ton, and thus the logistic costs are high and represent a large portion of the products' total cost. The main task of Asak Miljøstein AS in this respect is organization of as cost-effective transportation as possible between factories and between factories and warehouses.

Combined production-inventory-transportation problems nowadays are relatively well researched and analyzed in the literature. Integrated production-inventory-distribution systems are examined in such papers as Blumenfeld *et al.* (1985) and Mak and Wong (1995). Speranza and Ukovich (1994), Bertazzi and Speranza (1999), Burns *et al.* (1985), Martin et al. (1993), Flumero and Vercellis (1999) and other authors deeply researched combined production and distribution problems. However, in spite of current quite a high level of development of the field of integrated production-inventory-transportation systems, the majority of the mathematical models developed by the authors specializing in this field are either highly customized for each specific problem and thus are hard to be generalized and applied to the problem considered in this Master thesis, or, conversely, are too general and consequently are also hard to be used for construction of a highly specified model. Therefore, the authors of this Master thesis made an attempt to design and construct a mathematical model that is not based on any of the already developed combined models.

Before the stage of design and construction of the mathematical model, the problem was narrowed by introduction of a set of assumptions and simplifications.

The classical Vehicle Routing Problem (VRP) was chosen by the authors to be the basis of the model representing the considered combined production-inventory-transportation problem of Asak Miljøstein AS. On the first stage of model construction, the VRP was extended by introduction of additional conditions: limitation of the capacity of vehicles, possibility to carry out simultaneous pick-ups and deliveries, limitation of working hours of the factories, limitation of the maximum duration of the working day of a truck driver and the fact that each factory may be used as a depot. On the second stage, the resulting model was further extended by allowance of the possibility to satisfy customer nodes' demands with more than one vehicle and also model was extended to become multi-product and multi-period. On this stage the resulting model was also combined with inventory and production sub-problems represented as additional constraints.

The model developed in this work was designed to reflect the real problem provided by Asak Miljøstein AS as detailed as possible. Consequently, the model turned out to be very complicated and non-linear. Therefore, the developed model cannot be solved using any linear solvers and requires usage of more sophisticated solution applications. Otherwise, the constructed model could possibly be turned into the linear form through transformation and simplification and then solved with usage of any of the existing linear solvers. Solution of the proposed mathematical model with usage of any of the mentioned approaches could be considered as a field for further research of the problem.

Finally, authors suggest considering some additional theoretical measures of improvement of current production, inventory management and transportation policies of Asak Miljøstein AS. In order to increase customer demand for products manufactured in Hønefoss (that would lead to a decrease of internal transportation of final products from the factory in Hønefoss to the Fetsund factory, which is more preferred by the customers due to its geographical location) introduction of a certain discount off the normal sales price for the products bought by customers at the factory in Hønefoss is offered by the authors. With respect to the problem of reduction of internal transportation of sand between factories in Hønefoss and Fetsund, two possible actions are offered: splitting the customer region and introduction of new product sub-types.

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