



Master's degree thesis

LOG950 Logistics

Generation of supply vessels schedules with models

Victor Rubinat

Number of pages including this page: 62

Molde, May 2013



Mandatory statement

Each student is responsible for complying with rules and regulations that relate to examinations and to academic work in general. The purpose of the mandatory statement is to make students aware of their responsibility and the consequences of cheating. Failure to complete the statement does not excuse students from their responsibility.

Please complete the mandatory statement by placing a mark <u>in each box</u> for statements 1-6 below.		
1.	I/we hereby declare that my/our paper/assignment is my/our own work, and that I/we have not used other sources or received other help than is mentioned in the paper/assignment.	<input type="checkbox"/>
2.	I/we hereby declare that this paper <ol style="list-style-type: none"> 1. Has not been used in any other exam at another department/university/university college 2. Is not referring to the work of others without acknowledgement 3. Is not referring to my/our previous work without acknowledgement 4. Has acknowledged all sources of literature in the text and in the list of references 5. Is not a copy, duplicate or transcript of other work 	Mark each box: 1. <input type="checkbox"/> 2. <input type="checkbox"/> 3. <input type="checkbox"/> 4. <input type="checkbox"/> 5. <input type="checkbox"/>
3.	I am/we are aware that any breach of the above will be considered as cheating, and may result in annulment of the examinaion and exclusion from all universities and university colleges in Norway for up to one year, according to the Act relating to Norwegian Universities and University Colleges, section 4-7 and 4-8 and Examination regulations section 14 and 15.	<input type="checkbox"/>
4.	I am/we are aware that all papers/assignments may be checked for plagiarism by a software assisted plagiarism check	<input type="checkbox"/>
5.	I am/we are aware that Molde University college will handle all cases of suspected cheating according to prevailing guidelines.	<input type="checkbox"/>
6.	I/we are aware of the University College`s rules and regulation for using sources	<input type="checkbox"/>

Publication agreement

ECTS credits: 30

Supervisor: Irina Gribkovskaia

Agreement on electronic publication of master thesis

Author(s) have copyright to the thesis, including the exclusive right to publish the document (The Copyright Act §2).

All theses fulfilling the requirements will be registered and published in Brage HiM, with the approval of the author(s).

Theses with a confidentiality agreement will not be published.

I/we hereby give Molde University College the right to, free of charge, make the thesis available for electronic publication: yes no

Is there an agreement of confidentiality? yes no

(A supplementary confidentiality agreement must be filled in)

- If yes: **Can the thesis be online published when the period of confidentiality is expired?** yes no

Date: May 2013

Preface

Firstly and most importantly, I would like to express my most special appreciation and thanks to my supervisor Professor Dr. Irina Gribkovskaia for all her insights. Without you this work would not have been possible, literally. Thank you for your advices professionally and outside of the box.

I would also thank for the ideas the PhD student in HiMOLDE Eugene Moysyuk has given to me. Your tips have been really useful and encouraging. In addition, I would also recognize the contribution to Yevgheni Petkevich, IT advisor and my neighbor for all the drafting process. We have shared the ups and downs during the whole process of doing a long work. The staff of HiMOLDE has been also very helpful from the beginning to the end, specially you Anette Kristin Myrstad.

Finally, a special thanks to my family. Mom and Dad you have encourage me through Skype to keep going and pursue my dreams. Many thanks also to my sister Alicia and my grandmother Carmen for their thoughts and good willingness while I am abroad. Special thanks too, to my uncle, Fernando, for his advices as a university professor and his help in many other aspects.

Summary

The main aim of this thesis is to create an exact mathematical tool that allows to solve the problem of vessels scheduling to offshore platforms with multiple time windows. The tool has been tested with several instances and some extended experiments have been carried out with the use of real data.

The problem can be summarized as a number of platforms that need to be supplied frequently via sea from a single base. What we want to obtain is an assignation of vessels and times to visit those platforms. At the same time, it is needed to verify a number of constraints such as: opening times, maximum duration of the voyages or frequency of the visits.

The solution model consists of 2 parts. In the first part we have to generate a number of possible voyages and in the second part we have to choose some of those voyages and assign them to vessels and times under some restrictions and preferences. It has been validated with two data sets: one small and one medium. The experiments and the results has been done with a large data set.

The tool has been proved functional, although not fast. To reduce the number of voyages and accelerate the process a procedure has been applied and suggested. Furthermore, some experiments on the data has been undergone and one of the conclusions is that the slacks at the end of the voyages are less costly than to distribute them along the voyages.

KEY WORDS: SCHEDULING, MULTIPLE TIME WINDOWS, SUPPLY VESSELS, PERIODIC VEHICLE PROBLEM, OPTIMIZATION.

Contents

1	Introduction	8
1.1	Problem description	9
1.2	Field of study	10
1.3	Motivation	11
2	Objectives	12
3	Literature review	14
4	Theoretical background	17
5	Research plan	22
6	Solution method description	24
6.1	Route generation model	25
6.2	Set covering model	30
6.3	Validating the method	31
6.4	Illustrative example	33
7	Extensions of the method	38
7.1	Robust schedules	38
7.2	Spread departures	39
7.3	Supply base policy	40
7.4	Collision constraints	41

8 Experiments and results	42
8.1 Test instance	42
8.2 Basic solution	44
8.3 Robust schedule: slacks at the end of the voyages	47
8.4 Robust schedule: slacks after each visit	48
8.5 Collision constraints	49
8.6 Other policies: several working times for the supply base	50
8.7 Other policies: no departures on weekends	51
9 Comparative analysis of the obtained schedules	52
10 Conclusions	55
Bibliography	58

Chapter 1

Introduction

In offshore installations there is a need for regular supplies of commodities from land. The cargo to be shipped ranges from food and drinking water to drilling liquid. These commodities are transported using vessels due to high volumes and weights. They are a costly resource, so their routes and schedules should be planned carefully. Different mathematical models and computation techniques have been developed for optimizing and simulating the operation of transport fleets in order to serve the customers demand with the cost efficiency and service maximization in mind.

Many fleet management problems correspond to combinatorial optimization problems, such as vehicle routing and scheduling, that are notoriously difficult to solve, even in a static context [1]. Moreover, there are many other parameters and difficulties in those problems to be considered such as service restrictions, fluctuating demands, limited capacity offshore, limited distance/time travel and capacity for vessels or bad weather that prevent operations.

The research in improving the planning procedure in different situations is important because of the need of avoiding shortage cuts, for safety reasons, reducing emissions and obviously for reducing operational cost. This subject is of special relevance for the Norwegian government as it gets 78% of taxes of the revenues of the oil industry and many installations have reached the peak oil. Consequently, tail production starts to become a reality [20]. Therefore, in this kind of production

environment logistics gets even more relevant as they suppose more percentage of the costs.

1.1 Problem description

The problem can be summarized as a number of platforms that need to be supplied frequently via sea from a base. The solution is an assignation of vessels and times to visit the platforms verifying a number of constraints such as periodic opening times of the platforms, maximum duration of the voyages, vessel's capacity, etc. The objective is to reduce the number of vessels and the consumed fuel. So, as it has been hinted so far, there are three main Stakeholders in these problems which are expanded below:

The platforms / rigs / offshore installations: There are two types of platforms. The first one are the production platforms which are offshore factories. Their demand is easily to predict for longer periods. Basically, they need routine production material (like pipes for example) and provisions of food and water. The second type are the research platforms: Exploration rigs. They move every some month and need intensively new equipment. Therefore demands are uncertain and fluctuate more. They need more frequent service as they cannot predict in advance. It is variable due to they need unpredictable quantity of pipes and other equipment, which is usually expensive and rented for short periods. Urgent calls are produced more often as more incidents happened and they work with uncertainty.

Their location matters, how far are from the base and how they are spread will change the problem substantially. They have different time windows, some open 24hours a day others close at night. They are more concerned more in service more than cost. Usually they need small part as the space available at the platforms is very limited; Decks of ships are bigger than platforms decks usually crowded with different items. In this work the demands are deterministic and with a known frequency.

The supply vessels: In this work only supply vessels are considered not oil tankers. They are rented for some periods or with permanent contracts. Those used in permanent contract are thought to be used as much as possible, including uses they were not intended for like sharing them with other bases or emergencies. Sometimes even security vessels are used as temporary storages for example and need to be supplied too. A uniform fleet will be considered.

The main parameters of supply vessels are their speed and they have limited cargo capacity. Their lead time to delivery and service requirement would change with different locations of installations. They transport two main kinds of cargo: Deck and bulk. Deck is often considered alone because it has 84% of returns and more frequent delivery. However, bulk commodities have a different pattern with 20% of pick up and can affect the solution too [20].

The supply base: They are places inshore from where the vessels depart with the cargo to the installations. They have limited capacity that influences in the number of vessel handling process at the same time. Additionally, opening times affect time of departure and time of arrive of the vessels. They are affected by long supply chain of some components as some supplies are very unique and delivery to the base can be delayed. We must play with slacks.

1.2 Field of study

In this research we have studied the problem of scheduling of supply vessels to a determined number of rigs. This problem is grouped inside the process of planning of the procurement logistics of finished goods to the customers (rigs). The main field of these kind of problem is periodic vehicle routing involving multiple time windows.

The resulting plan is what it is called as "tactical planning" or guide schedule. This is due to it provides an average schedule that will be modified in practice for the operational handling of the installations. This is so as demands are variable and weather conditions are uncertain for more than some days and affect heavily

the performance of the system. The weekly schedule provided should include slacks to deal with those uncertainties placed into the foreseeable future and being used as a guide.

1.3 Motivation

We have to take into account the great number of variables involved. They can be grouped in three groups. Being the first one related to tactical parameters of the rigs and the base (distances, demands, frequency of the visits), operational parameters of the rigs and the base (opening times, service time, spread of arrivals) and parameters of the vessels (capacity, speed, visits in a single voyage).

This complex problem has been solved with heuristics and with exacts methods from different authors (see literature review), but never solved using a single optimization tool such as AMPL for the whole process. This is what is solved here. Then, several changes are made into the parameters and several experiments are carried out which allow to give recommendations for future research.

It is an important field as it is a current topic. The better it is studied the biggest the savings in costs and risks. So as to keep the rigs and supply vessels working properly.

Chapter 2

Objectives

The main aim of this thesis is to create an exact mathematical tool that allows to solve the problem of scheduling of vessels for the rigs, test the tool and do some experiments to include different circumstances with the use of real data.

The solution model consists in 2 parts. In the first part we have to generate a number of possible voyages and in the second part we have to choose those voyages, assign them to vessels and times under some restrictions and preferences.

The first model has been created from scratch as it uses AMPL for this propose and is one of the main contributions. In this model a vessel departs from the base and visits a number of installations and then return to base. All the possible voyages combinations are initially explored. Because the number of voyage to be generated is quite large for larger models, it is necessary to reduce the number of voyages. To do so, the models discard those voyages that visiting a determined range of installations, have too much demand for the vessels and/or have too much or too few slack.

After selecting those selected voyages a traveling salesman problem with time windows (TSPTW) is solved iteratively for each of the them. For each voyages the model yields the installations visited on each voyage in the form of a binary parameter, the details of each voyage, the total cost and the total duration. It allows to consider vessels with different capacities and different departure time (Both options multiply the number of voyages considerably)

The second model needs of the data for the voyages obtained in the first model. It is in practice what is called as "voyage-based model" in the paper [14]. It selects the voyage that require a minimum cost while fulfilling the constraints of number of visits to installations, spread departures, depot capacity and it assigns a vessel and a time for each of those voyage. This is what is called scheduling and is the anticipated result of the model.

Chapter 3

Literature review

Following the definitions given in [18], routing can be defined as the assignment of sequences of ports to be visited by the ships. The term scheduling is used when the temporal aspect is brought into routing.

The vehicle routing problem is a well-studied problem. In contrast, the supply vessel planning problem is more complex than that as it includes opening times and multiple visits during the planing horizon of one week. In addition, there is a fleet composition problem with the possible inclusion of different vehicles and the creation of schedules.

In industry, a fleet of vehicles is not homogeneous. It varies in dimension, compatibility in cargo and costs. Nevertheless, the majority of the bibliography deals with homogeneous vehicles to simplify calculations [6] and there is the necessity of dealing with a few problems characteristics at a time. In [2], [17] and [1] presented a wide survey of the different specific problems and the state of art of the main contributions in the relevant literature for maritime optimization problems among others.

One type of problem is the fleet size and mix problem routing problem (FS-MVRP). It tries to minimize the cost of managing the vehicles and the routing costs. It was first studied by [10]. Later it was expanded with time windows and multiple vehicles by [7]. Then several heuristics and metaheuristics methods have been developed to tackle the problem such as [5] where even heterogeneous fleets

are considered in addition to the fleet sizing problem with a column generation algorithm.

Another related problem is the Periodic vehicle routing. There, routes are constructed to visit customers one or more times and the problem is to decide when. [8] treated this extension problem among others with a exact methodology. In these kind of problems, application of exact methods is still limited to about 100 customers for vehicle routing problems, less if more parameters are taking into account such as scheduling or multicommodity. Therefore, it is desirable to obtain approximate solutions instead of optimal ones. Some of the great amount of references in periodic that use heuristics for PVRP are [16] or [12]. In [14] we can find an exact method that solves a combination of both periodic vehicle routing and fleet composition. The method give exact solutions, consists of two phases and is detailed in Theoretical background. Nevertheless, it is limited to twelve installation due to high computational time.

A point that deserve mention are some policies and tools developed so far. In [9] several scenarios were developed varying opening times and weekly service to a group of installations. There they conclude that instead of closing during nights it would produce more savings to have two vessels visiting the installations four times per week. This high frequency would reduce the need of hiring extra vessels for rush orders.

According to [15] weather will affect in the following way: if waves are more than 4,5m it is not possible to undergo load/unload operations due to safety regulations. If there is rough weather with waves less than 4,5m operations may take more time and the sail. speed may be reduced as well. This is the most important element of uncertainty in for vessel planning therefore some robustness approach should be taken into account.

In [14] it is shown the robustness approach involves adding an slack to each voyages in a way that voyages that have less slack are discarded. In [15] it is proposed a method of valuing the different voyages according to a robustness profit obtained through the idle days of the vessels. The issue is further studied in [13] where several robustness strategies are compared using a simulator for a 3

to 12 months period.

An additional related feature that could be considered for those kind of problems is the pick up and delivery. In them, the cargo travels both ways from depots to customers and vice-versa. The capacities are what determines the solution routes. It is considered at [11] in the service of offshore installations with capacitated customers. It is also studied in combination with different commodities at [22].

The last point to mention is that, as the author in [6] sees it, there is an exponential growth in the articles published in the vehicle routing field showing the high interest in the topic.

Chapter 4

Theoretical background

First we need to present how it looks like a basic vehicle routing of a vessel visiting a number of installations from a base, considering the existence of time windows:

minimize:

$$\sum_{(i,j) \in A} C_{ij} X_{ij} \quad (4.1)$$

subject to:

$$\sum_{(0,j) \in A} X_{0j} = 1 \quad (4.2)$$

$$\sum_{(i,j) \in A} X_{ij} = 1 \quad (4.3)$$

$$\sum_{(i,j) \in A} X_{ij} = \sum_{(i,j) \in A} X_{ji} \forall i \in N \quad (4.4)$$

$$T_i \geq l; \forall i \in N \quad (4.5)$$

$$T_i + s_i \leq u; \forall i \in N \quad (4.6)$$

$$T_i + t_{i,j} + s_j \leq T_j + M(1 - x_{i,j}); \forall i \in N, j \in N : i \neq j \quad (4.7)$$

TSPTW (Traveling salesman problem with time windows)

Variable	Description
X	Binary variable to select which arcs i-j are selected in the final route solution.
T	Time of visiting each customer needed in order to satisfy the time windows constraints.
Parameter	Description
A	Arcs from i to j.
N	Number of customers to be visited.
i, j	Origin and destination parameters, 0 for the depot.
t	Travel distance between customers.
s	Service time spent in each customer.
M	Big number.
u	Upper boundary time to visit the customer i.
l	Lower boundary time to visit the customer i.

Table 4.1: Parameters and variables of the TSPTW model

In (4.1) it is minimized the total cost traveled for all the arcs selected.

In (4.2) there should be an arc in the route that departs from depot which is considered as the node 0.

In (4.3) This impose the condition of visiting all the customers (at least one arc leaving each installation). That is the reason why it is called a traveling salesman problem as all the customers are visited.

In (4.4) if we visit a installation we should depart from it, this constraint is called route continuity.

In (4.5) we have the lower time window constraints.

In (4.6) There is the upper time window constraint considering the service time for the customer.

In (4.7) The time of visit for one costumer has to be more then the time for the customer visited before. There is the need of using the variable X to distinguish among the customers who are visited along the route.

Secondly it is presented a basic industrial ship scheduling model similarly to what is shown in[2], which allows to select among a number of routes and creating a schedule:

minimize:

$$\sum_{v \in V} \sum_{r \in R_v} C_{vr} X_{vr} + \sum_{i \in N} C_{SPOT_i} S_i \quad (4.8)$$

subject to:

$$\sum_{v \in V} \sum_{r \in R_v} a_{ivr} X_{vr} + s_i = 1, \forall i \in N \quad (4.9)$$

$$\sum_{r \in R_v} X_{vr} = 1 \quad (4.10)$$

$$X_{vr} \in [0, 1], \forall v \in V, r \in R_v, S_i \in [0, 1], \forall i \in N \quad (4.11)$$

Basic scheme of scheduling through route selection

Variable	Description
X_{vr}	Binary variable which is equal to one if ship v sails the schedule r and zero otherwise.
S_i	Binary variable which is equal to one if cargo i is serviced by a spot charter and zero otherwise.
Parameter	Description
V	Ships to be scheduled.
N	the cargoes
R_v	the candidate routes available for each ship v
a_{ivr}	1 if schedule r for ship v services cargo i and 0 otherwise.
c_{vr}	Cost of the route r for the vessel v.
c_{SPOT_i}	Cost of chartering a spot.

Table 4.2: Parameters and variables of the route selection model

In (4.8) minimizes the sum of the costs of operating the fleet and the costs of the spot shipments, while constraints(4.9) ensure that all cargoes are serviced either by a ship in the fleet or by a spot charter. Constraints (4.10) assure that each ship in the fleet is assigned exactly one schedule (column). Constraints (4.11) impose the binary requirements on the variables.

Then, we present the methodology to follow. In [9] a first algorithm for the problem of multi trip vehicle routing problem is proposed. It regards two steps. Firstly, a number of feasible candidate routes are generated for each vessel (using a vehicle routing model similar to what is shown before. Secondly, the vessels to be used and their weekly schedule is determined by solving an integer programming model which is closely related to the basic industrial ship scheduling model shown in equations (4.8-4.11).

To do the first step a full enumeration is undergone. Therefore, we start enumerating the visit of each single offshore installation, then in all the possible pairs, groups of three and so forth.

When we have generated all the possible routes we proceed with a model that has as an objective function to minimize the total cost of using the vessels plus the cost of hiring each supply vessel to generate the solution schedule. The author considers only three main restrictions:

- Each offshore installation is serviced at least the minimum number of times.
- The total time of the schedule for a vessel does not exceed 168 h or 1 week.
- Selection of a schedule means that the binary variable for the vessel selection must be one

In a similar but more advanced way [14] approaches the problem. It includes the same two steps but with more constraints, parameters and using resources in a more clever way.

For the first step it proposes an algorithm to be programmed in any computer language such as C++. In this paper it is called "voyage generator" and consist of Creating sets of vessels ordered from larger to smaller loading capacity. Then,

a enumeration of all sets of installations that fulfill minimum and maximum requirements and capacity is done. Eventually, the voyage is determined by solving a traveling salesman problem with time windows starting and ending at the supply depot where all installations of the set are visited exactly once

The second step is an upgrade of [9] as shown in (6.3) and it is called "voyage-based model". In this model there are the same binary variables about ships selected and voyages selected, but the variable X for voyages selected depends also on vessels and the day of the week.

The objective is as the previous model to minimize the combined cost of chartering a vessel and the cost of the service, but expressed in a different way. The first constraint ensures that all installations get their required number of visits similarly to [9]. The second constraint also ensures that the duration of all voyages sailed by a vessel does not exceed the maximum number of days instead of a fixed amount of hours. As in the previous model this constraint also determines the variable δ which specifies the vessels chosen. Meaning that include two constraints of the previous models.

There are three important contribution to the previous model apart for rewriting in a more convenient way the previous constraints Firstly, is that it adds a capacity for the depot. Secondly it ensures that a vessel does not start on a new voyage before it has returned to the depot. Finally, it suggest a way to include spread of departures and avoid simultaneous visits to a installation what they called: collision constraints.

The main problem with this procedure is that in the model of second step the method could not find or prove optimal solutions to the largest problem instances within a not too big CPU time. Because of this the same author developed in [14] and [15] a tool for robust scheduling with a simulator for larger models such as:

- Add slacks at the end of each of the voyages.
- Add slacks after each visit to a installation.
- Include a robustness profit in the objective function of the second step that weights the advantages of a voyage for having idle time.

Chapter 5

Research plan

The objective of this research work is to develop a tool using only exact mathematical optimization that allows to build routes with shortest duration and then select a number of them that fulfill the demands and the number of visits required. The tool as a consequence is divided in two parts: The route generation model and the set-covering model.

In the first part there are generated all the routes/voyages that comply with the capacity of the vessel and the demands. Of those routes, only those that last for several days and visit a predefined range of installations in a single route are taking into account. Then we have a pool of candidate routes determined by different parameters such as cost, installations visited and duration of the voyage.

From all the candidate routes a limited number is selected and there are many restrictions to be considered. For example a limitation in time to service, the way the departures spread, how large are the demands, if there are time windows restrictions, policies of the base (opening times, vessels served,..). Depending on this restrictions there would be one or another result that need to be studied.

Therefore the plan for this Thesis has been the following:

- Develop the tool and validate it with a small (4 installations) and medium instance (8 installations) with known results.
- Implement the tool with a larger model (13 installations) and get a basic solution.
- Evaluate what happen to the results for the larger model after several experiments are carried out.
- Understand what is the contribution of this tool and what conclusions can we draw from it.

Chapter 6

Solution method description

The procedure for solving the problem starts with defining which voyages are possible. Then solving a multiple time windows TSP with the route generator model obtaining all the costs and duration of those voyages. Finally with the help of the model found in [14] we are able to chose which are the voyages selected obtaining the final schedule. A resulting schedule would look like this:

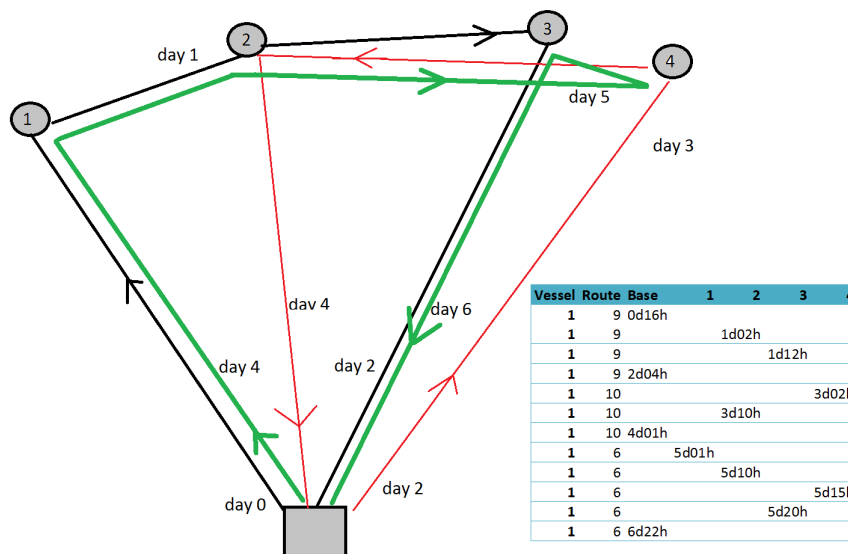


Figure 6.1: Schedule of a small instance with four installations to be visited, solved with the model designed for this work

6.1 Route generation model

This model consists of enumerating a list of possible voyages a vessel can take in order to select a limited number of them later using the model in [14].

The process consists in a vessel that departs from the base and visits a number of installations and then return to base. Once they are generated a traveling salesman problem with time windows (TSPTW) is solved iteratively for each of the voyages selected.

The number of voyages to be generated may be quite large: $\sum_{i=1}^n C_{i,n}$ and a program written in C has been used to generate all the possible combinations as the number is quite large. For a small instance of 4 installations it would be only 15, but 255 for 8 installations and 8191 for 13 installations.

This number would increase dramatically with vessels with different capacities as it would be multiplied by the number of vessels, it can be also increased if we want to considerate other different departure time for example. In addition, it has to be taken into consideration that for each voyage several parameters are accounted: installations visits, times of arrives and departures, cost and duration. Furthermore the second model has to assign those voyages to a vessels and to a time. If the number is to big the required memory would be big as well. For all those reasons, it is necessary to reduce the number of voyages. It has been done considering the following assumptions that have been included into the model designed:

- visiting a determined range of installations discarding the other voyages. For example only voyages that visit between 3 and 7 installations could be taken into account.
- The demands covered in a voyage do not surpass the vessels capacities. In order to consider the demand it is considered as a demand per visit (i.e. demand/frequency).
- Only voyages that last 2 or 3 days are taken int account, discarding the others.

- The model allows to unselect voyages that have a too small (or too big) slack time. So for example voyages that last exactly 48 hours would be discarded as there is no slack for operations.

As an input we have the enumeration of installation groups combinations to be visited, all the data for installations such as opening times, distances, demands, average service time or the beginning of the shift and the vessels capacity and speed. As an output, we obtain the cost, the times of arrive to each installation, the order of visits and the total duration of each of the possible routes.

In order to calculate the distances when given in coordinates the Harvensine formula has been used (where R is the Earth radius of 6,371km.

$$a = \sin^2(\Delta\phi/2) + \cos(\phi_1).\cos(\phi_2).\sin^2(\Delta\lambda/2) \quad (6.1)$$

$$d = 2R \arctan \sqrt{\frac{(1 - a)}{a}} \quad (6.2)$$

The objective function consists of minimizing the time of arrive back to the base. It has been tried another the possibility of minimizing the total cost, understood as fuel consumption. That means reducing the wait. time for each of the installations visited in the route. This second possibility has been discarded as it gives more total time per voyage, i.e. worse results.

The restrictions of the model include the restrictions for a traveling salesman problem with time windows combined with a limitation in the duration, departure time and imposing the route between the base and the virtual base. See model from equation (6.3) to (6.14) and list of parameters at table 6.1.

In the model a virtual base (N+1) is used placed in the same place as the original with a traveling cost between them of 0. In addition, voyages should not exceed three days and are not less than two days. An exhaustive list of all the possible combinations of installations visited R is generated. (For example $r=14$ is 1-2, $r=904$ is 4-5-9-13 or $r = 5229$ is 2-3-6-7-9-11-12 for a model with 13 installations and one type of vessel). In order to allow this model to generate all the voyage as

we want them some programming have been done. It is shown in the table 6.2. as a pseudocode where $N(k)$ are the installations visited in the voyage k . This has been implemented in the .run file in AMPL.

minimize:

$$24 * D[N + 1] + T[N + 1] \quad (6.3)$$

subject to:

$$\sum_{j=1}^N X_{0,j} = 1 \quad (6.4)$$

$$X_{0,N+1} = 1 \quad (6.5)$$

$$\sum_{X_{i,j} \in A}^N X_{i,j} = \text{card}[M_n] \quad (6.6)$$

$$\sum_{i,j \in A} X_{i,j} - \sum_{j,i \in A} X_{j,i} = 0; \forall i \in N \quad (6.7)$$

$$T_i \geq l; \forall i \in N \quad (6.8)$$

$$T_i + s_i \leq u; \forall i \in N \quad (6.9)$$

$$24D_i + T_i + \frac{C_{i,j}}{v} + s_j \leq T_j + 24D_j + M(1 - x_{i,j}); \forall i \in N, j \in N : i \neq j \quad (6.10)$$

$$T_i + 24D_i + s_i + C_{i,0} \leq 24k + a \quad (6.11)$$

$$T_0 = 16 \quad (6.12)$$

$$D_0 = 0 \quad (6.13)$$

$$d \in \mathcal{N}; X_{i,j} \in 0, 1; 0 \leq T_j \leq 24 \quad (6.14)$$

Multi-period TSPTW

In (6.3) we minimize total cost traveled for each voyage

In (6.4) There should be an arc in the route that departs from depot

In (6.5) 0 is the base and $N+1$ a Duplicate of the base. In order to make a cycle between all the bases we need that the arc between bases is selected in the way we want. This could also be done with a cost 0 and infinite.

In (6.6) Number of arcs in the voyage is equal to to the number of installations to be visited

In (6.7) If we visit a installation we should depart from it.

In (6.8) and (6.9) time windows should be fulfilled.

In (6.10) time continuity before and after visiting a installation.

$$\text{arrive}(i) + \text{service}(i) + (i \rightarrow j) = \text{arrive}(j)$$

In (6.11) total duration of the voyage less than a limit. That could mean that the voyage is infeasible.

In (6.12) usually vessels depart at 16:00 from installation

In (6.13) The day the vessels depart is the day 0

model	AMPL name	Description
N	N	Number of installations to be visited
P	P	Period of planing (7 days f.ex.)
C	dist	travel distance between factory and customers
s	service	service time spent in each customer
d	departure	departure from depot (departure at 4pm)
a	arrive	arrive to depot (maximum, arrive at 8am)
l	lowerTW	Initial time of service wanted. format of time decimal p.ex.14,5 \rightarrow 2:30pm
u	upperTW	Final time of service wanted.
v	v	speed in km/h
k	duration	maximum duration of the voyage 2 or 3 days (the rest is slack)
$X_{i,j}$	X	Which arcs would be part of the Xs
T_i	T	hour(0-24) we visit each customer
U_i	U	order at we visit each customer (subcycle elimination)
D_i	D	day we visit each customer

Table 6.1: Parameters and variables of the first model

```

for v in 1..V (vessels)
for k in 1..R (voyages)
  if sum (i in N(k)) demand(i)/ frequency(i) <= capacity.vessel (v)
  then solve
    if total.cost >= 24*1 and <= 24*3
    then calculate and print: origin(k,v), dest.(k,v), arrive(k,v), start(k,v), sail.(k,v), wait.(k,v)
      for (i in 1..N(k))
        calculate A (k,v)
        calculate Arrive (k,v)
        calculate Departure (k,v)
      end for
    calculate duration (k,v)
    calculate duration.round (k,v)
    calculate wait..time(k,v)
    calculate sail..time(k,v)
    calculate service.time(k,v)
    calculate total.cost(k,v): wait..time(k,v)*wait..time+
    sail..time(k,v)*sail..cost+ service.time(k,v)*service.cost
    calculate slack(k,v)
      If duration.round =2
      then Assign voyae to 2.days voyage
      else Assign voyae to 3.days voyage
    else
    else
    print "skip"
  end for
end for
display duration-voyages-round,total-cost, A,arrive, departure
display 2-days duration voyages, 3 days duration voyages

```

Table 6.2: Pseudo-code of the run file used to generate all the voyages

6.2 Set covering model

This second phase of the model can be found in [14]. The model consists in minimizing the total cost combined of chartering the vessels and the voyages selected, it is an extension of the model shown in the equations (4.8 to 4.11). The model has 4 main constraints, but others such as spread of departures may be added.

The first of them correspond to installations visited the number of times specified (6.16). The second one is related with the duration of the voyages avoiding to use the vessels more than possible and defining the variable δ_v (6.17). The third one is related with the base capacity of assisting vessels for departure, if it is unknown it can be not used (6.18). The last one is used to account that a vessel returns to depot before departing again, and there are as many constraints as how many different duration voyages we have. In our case, with voyages of 2 and 3 days (R_2 and R_3 respectively) those constraints can be rewritten as (6.19) and (6.20).

minimize:

$$\sum_{v \in V} \sum_{r \in R_v} \sum_{t \in T} C_{vr} X_{vrt} + \sum_{v \in V} C'_v \delta_v \quad (6.15)$$

subject to:

$$\sum_{v \in V} \sum_{r \in R_v} a_{ivr} X_{vrt} \geq s_i, \forall i \in N \quad (6.16)$$

$$\sum_{v \in V} \sum_{r \in R_v} D_{vr} X_{vrt} \leq F_v \delta_v, \forall v \in V \quad (6.17)$$

$$\sum_{r \in R_v}^N X_{vrt} \leq B_t, \forall t \in T \quad (6.18)$$

$$\sum_{r \in R_2} X_{vrt} + \sum_{r \in R_2} X_{vr, (t+1) \bmod |T|} \leq 1 \quad (6.19)$$

$$\sum_{r \in R_3} X_{vrt} + \sum_{r \in R_3} \sum_{n=1}^2 X_{vr, (t+n) \bmod |T|} \leq 1 \quad (6.20)$$

$$X_{vr} \in [0, 1] \forall v \in V, r \in R_v, \forall t \in T \quad (6.21)$$

$$S_i \in [0, 1] \forall i \in N \quad (6.22)$$

Variable	Description
X_{vr}	Binary variable that is 1 when we assign the vessel $v \in V$ to one of the voyages generated in the phase 1 $r \in R$ ans to a time 1 to 7 or 0 to 6 if the schedule must be weekly as it is our case.
δ_i	Binary variable that is 1 if we select the vessel $v \in V$, 0 otherwise.
Parameter	Description
V	Ships to be scheduled.
N	the cargoes
R_v	the candidate routes available for each ship v
a_{ivr}	1 if schedule r for ship v services cargo i and 0 otherwise.
C_{vr}	Cost of the route r for the vessel v .
C'_v	Cost of chartering a spot.
D_{vr}	Duration of the voyages.
F_v	Duration of the voyages.
s_i	Number of visits need by installation i .
B_t	Maximum number of vessels departures from the base on day t .

Table 6.3: Parameters and variables of the route selection model

6.3 Validating the method

Before starting with larger models we checked the model with a small instance (4 installations) and a medium instance (8 installations). Days start at 8:00 for both instances. Voyages can last 2 or 3 days and the preparation time for the voyages in the base is 8 hours. Therefore vessels departs at 16:00 from base.

For the small instance of 4 installations, 11 possible routes are generated in the first stage out of the 15 possible combinations as explained in 6.4. A vessel is able to visit the installations the required number of times in three voyages and

one week. The solving time is about a few seconds for both stages. See the results in figure 6.2.

This model has allowed to check that it works and that the voyages generated are correct. In the table there is a comparison of times for the route "Base-1-2-4-3-Base" for the model developed here and the results supplied by the supervisor of this work which are identical showing that the model is correct at least in the first part of generating voyages. This can be seen in the figure 6.2. The voyage shown is the voyage 1 that involves all the 4 installations: 1 2 3 4. Numbers 0 and 5 are the base and the virtual base respectively.

Obtained in the model:						Results to compare:		
origin	dest.	arrival	start	sailing	waiting	Voyage	Arrival	Sart service
0	1	25.25	31	33.25	5.75	0		8
1	2	34.98	34.98	38.98	0	1	25.25	31
2	4	39.4	39.4	42.9	0	2	34.9833	34.9833
3	5	70.35	70.35	70.35	0	4	39.4	39.4
4	3	44.29	55	59.9	10.71	3	44.29	55
						5	70.35	70.35

Figure 6.2: Comparison of the same route between the model created and the results obtained.

In addition to validate the model we have compared with the results obtained in [19] by an heuristic procedure. We had a problem with 8 installations the comparative analysis is given in the table 6.4. There we can see that the voyages are not the same as the procedure to generate them is not the same. However the cost is the same with a difference of 0.054% the same number of voyages, vessels and visits. Thus, we can conclude that the model designed for this work is correct. In addition the visits are distributed differently because different procedures of spreading departures may have been used by [19].

	Current model	Shyshou heuristics
Cost	2778270	2779770
Visits	36	36
Vessels	3	3
Routes	6	6
Voyages generated	225	-
Solving time of the first model	1795.93	-
Solving time of the second model	269.25	-
Total solving time	2065.18	-
Installations (visits)	visits	visits
1 (6)	1 2 3 5 6 7	1 2 3 4 5 6
2(5)	1 2 3 4 7	1 2 4 5 6
3 (5)	1 2 3 5 7	1 2 3 5 6
4 (5)	2 3 5 6 7	1 2 3 4 6
5(2)	4 6	3 6
6(2)	1 3	1 5
7 (5)	1 2 3 5 7	1 2 3 5 6
8 (6)	1 2 3 4 6 7	1 2 3 4 5 6

Table 6.4: Comparison of the results obtained in this work and by [19]

6.4 Illustrative example

In the beginning to check that the model worked properly a small instance of 4 installations was tried. This could be used as an example to understand how it works.

We have 4 installations with a service time (2.25 4 4.9 3.5) and the time windows for the visits (7:00-19:00 all). Each installation require the following number of visits: 1:1, 2:3, 3:1, 4:1. We have also a matrix of distances between them and that the day starts at 8:00 a.m and 8 hours are required in the base for preparation of each voyage. The average planned speed is 12knots/M and the capacity of the vessels (900 tones). We want to visit between 2 and 4 installations in a single voyage. The fuel consumption per hour sailed is 0.43 ton but when is wait. is 0.20.

First we need to generate the candidate voyages. The initial possible voyages would be like those shown in table 6.5. However, due to voyages with one single visit are not wanted, resulting in the voyages highlighted with a '*' are discarded in the beginning. Voyages are neither affected by the capacity of the vessels nor the slacks in this small instance. Therefore, more voyages are removed for the pool of candidates and we continue with 11 out of the 15 initial voyages. In bigger models this is needed to reduce the computing time as the number of candidates is quite big.

When all the data is processed correctly we have the voyages with the following features described in the tables 6.6 and 6.7. In addition we have the details of the voyages described in the table 6.8

voyage [12*] =1	voyage [13*] =2	voyage [14*] =3
voyage [15*] =4	voyage [1] =1 2	voyage [2] =1 3
voyage [3] =1 4	voyage [4] =2 3	voyage [5] =2 4
voyage [6]=3 4	voyage [7]=1 2 3	voyage [8]=1 2 4
voyage [9]=1 3 4	voyage [10]=2 3 4	voyage [11]=1 2 3 4

Table 6.5: Example of initial voyages

Voyage number	total wait.	total service	total sail.	Total cost
1	16.4583	14.65	23.2417	13.286
2	14.7417	11.15	24.6083	13.53
3	4.85	9.75	21.55	10.236
4	16.7583	10.65	23.0917	13.281
5	16.675	12.4	23.575	13.472
6	4.75	6.25	21.2333	10.08
7	4.55	7.15	23.025	10.811
8	4.85	5.75	21.4	10.172
9	4.55	8.9	22.4833	10.578
10	4.75	7.5	20.8167	9.901
11	4.55	8.4	21.9917	10.366

Table 6.6: Example of initial voyages: costs of the voyages

Voyage number	Duration in days	Duration in hours	Ins1	Ins2	Ins3	Ins4
1	3	62.35	1	1	1	1
2	3	58.5	1	1	1	0
3	2	44.15	1	1	0	1
4	3	58.5	1	0	1	1
5	3	60.65	0	1	1	1
6	2	40.233	1	1	0	0
7	2	42.725	1	0	1	0
8	2	40	1	0	0	1
9	2	43.933	0	1	1	0
10	2	41.067	0	1	0	1
11	2	42.942	0	0	1	1

Table 6.7: Duration and installations visits in the format of the matrix A

Voy. 1	Voy. 5	Voy. 9
i j arrive start sail wait 0 1 25.25 31.00 33.25 5.75 1 2 34.98 34.98 38.98 0.00 2 4 39.40 39.40 42.90 0.00 3 5 70.35 70.35 70.35 0.00 4 3 44.29 55.00 59.90 10.71	i j arrive start sail wait 0 2 26.25 31.00 35.00 4.75 2 3 36.78 36.78 41.68 0.00 3 4 43.07 55.00 58.50 11.93 4 5 68.65 68.65 68.65 0.00	i j arrive start sail wait 0 3 26.45 31.00 35.90 4.55 2 5 51.93 51.93 51.93 0.00 3 2 37.68 37.68 41.68 0.00
Voy. 2	Voy. 6	Voy. 10
i j arrive start sail wait 0 2 26.25 31.00 35.00 4.75 1 5 66.50 66.50 66.50 0.00 2 3 36.78 36.78 41.68 0.00 3 1 45.01 55.00 57.25 9.99	i j arrive start sail wait 0 2 26.25 31.00 35.00 4.75 1 5 48.23 48.23 48.23 0.00 2 1 36.73 36.73 38.98 0.00	i j arrive start sail wait 0 2 26.25 31.00 35.00 4.75 2 4 35.42 35.42 38.92 0.00 4 5 49.07 49.07 49.07 0.00
Voy. 3	Voy. 7	Voy. 11
i j arrive start sail wait 0 4 26.15 31.00 34.50 4.85 1 5 52.15 52.15 52.15 0.00 2 1 40.65 40.65 42.90 0.00 4 2 34.92 34.92 38.92 0.00	i j arrive start sail wait 0 3 26.45 31.00 35.90 4.55 1 5 50.73 50.73 50.73 0.00 3 1 39.23 39.23 41.48 0.00	i j arrive start sail wait 0 3 26.45 31.00 35.90 4.55 3 4 37.29 37.29 40.79 0.00 4 5 50.94 50.94 50.94 0.00
Voy. 4	Voy. 8	
i j arrive start sail wait 0 3 26.45 31.00 35.90 4.55 1 5 66.50 66.50 66.50 0.00 3 4 37.29 37.29 40.79 0.00 4 1 42.79 55.00 57.25 12.21	i j arrive start sail wait 0 4 26.15 31.00 34.50 4.85 1 5 48.00 48.00 48.00 0.00 4 1 36.50 36.50 38.75 0.00	

Table 6.8: Details of the voyages. Time and visits of the installations

When all the information of the voyages is gathered and generated in phase 1 we introduce this data in the second model. This model selects a number of those voyage and assign times to them. Therefore, in this model is when we can adjust spread departures, collision constraints and other concerns about times like visits on weekends banned, etc. It assigns also voyages to vessels. In this example it assigns all the voyage to a single vessel and peaks the voyages 6, 9 and 10 in the days Monday (0), Wednesday (2) and Saturday (5). It has to be taken into account that the voyages are generated in a modular way. Therefore, it is possible starting a voyage on Saturday (5) and finish it on Monday (0). The total cost of the solution is 1030.56. The results are summarized in the table 6.9

voyage	Instalaltion1	Instalaltion2	Instalaltion3	Instalaltion4
6	Tue	Mon		
9		Sat	Fri	
10		Wed		Thu
Total visits	1	3	1	1
X (r, t, v)		Vessels	Total Cost	
6 0 1	r:voyage	1	1030.56	
9 4 1	t:day			
10 2 1	v:vessel			

Table 6.9: Summary of the final results that highlight which voyages are selected and when each installation is visited

Chapter 7

Extensions of the method

Once the model was completely functional we tried with different changes in parameters and parts of the model to study how this affects the results.

7.1 Robust schedules

In order to achieve more robust schedules there are several ways. One of them is to add slacks of time. That means increasing the time for operations in order to cope with uncertainties such as bad weather that makes operations slower or unforeseen handling problems. It also helps to deal with demands as we have considered an example with deterministic demands but they usually tend to be very variable.

To do so we could add an extra time on the end of each voyage. Each vessel would have to arrive before closing time adding a buffer to the last arrive. When trying this many voyages that were possible would not be anymore, durations and cost would increase and more routes are needed. To do this change we just need to add time to the virtual base for service or as it has been done in the model limiting the time for the voyage to 4 hours less.

Another way would be adding time after each service spreading the slack along the voyage. This would be similar to the other slack but presumably results would be different and more close to reality. To do so we add extra service handling time for each installation or we can reduce the time windows closing time. We have

chosen the first option of adding extra service. The time added can be proportional to the service or a fixed amount.

7.2 Spread departures

Due to rigs want the deliveries the most spread possible we have to considerate how to spread them. Which policies should be considered among them. In this part the suggestions of [14] is considered. There a variation of the equation (7.1) is suggested among other variants. That equation works as following:

$$\sum_{v \in V} \sum_{r \in R} \sum_{k \in K} A_{vir} X_{vr, (t+k) \bmod |T|} \leq 1; \forall i \in n \subseteq N, t \in T \quad (7.1)$$

$$\sum_{v \in V} \sum_{r \in R} \sum_{k \in K} A_{vir} X_{vr, (t+k) \bmod |T|} \geq M; \forall i \in n \subseteq N, t \in T \quad (7.2)$$

- For installations with two visits, there will at maximum one departure every 3 days (K=2, equation (7.1), n= subgroup of installations with two visits N_2 . Pattern as: [* X *] [* X *] *)
- For three visits, there will be at least one departure in any 3 days periods (K=2, equation (7.2), n= N_3 , M=1. Pattern as: [* X *] [X * X] *)
- For four visits, there will be at least two departure in any 4 days periods (K=3, equation (7.2), n= N_4 , M=2. Pattern as: [X X * *] [X * X])
- For five visits, there will be at least one departure in any 2 days periods (K=3, equation (7.2), n= N_5 , M=2. Pattern as: [X *] [X X] [X *] [X])
- For six and seven visits, there will beat maximum one departure in any days (K=0, equation (7.1), n= N , M=2. Pattern as: X * X X X X X)

The spread of departure conditions will be used in all the models at is a pre-requisite of the industry.

7.3 Supply base policy

In this section changes in departures will be undertaken. What would happen if the base starts to work later than 8 so as to accommodate longer trips. For example: One vessel start loading at the base at 8:00 and departs at 16:00. After the voyage it arrives at 11:00 to the base again and the next trip departs at 19:00. In the case studied before that was not allowed. However, in this section it is allowed. To do so, trips that start later are included in the process of voyages generation allowing a more flexible policy. So, there would be trips starting at 16:00, 17:00, 18:00 and 19:00 multiplying per four the number of candidate routes for example. In the experiments only one other possibility has been added: base service type 1 from 8:00 to 16:00 and type 2 from 11:00 to 19:00.

In order to add this policy several changes in the model should be implemented. First of all we should generate $p \cdot R$ voyages, where R is the original combination of voyages and p is the number of policies. Those voyages would be different in the start and end. As an example a vessel could start a normal voyage at 16:00 do it for 2 days and arrive before 8:00 with some slack. That is arrive before the hour 56. Then it would be possible to start one voyage with the modified policy starting at 19:00 (hour 67) and do another route this time for 3 days arriving before 11:00 (hour 131). Then we have to decide if it is possible to start the next voyage at 16:00 or it must be at 19:00 or later.

In the case it should be at 19:00 or later we may do it adding a new parameter that indicates for each voyage what kind of starting of the voyage and then adding a constraint that does not allow it. As in equation 7.3 where r are the voyages in $1 \dots R_l$, v the vessels in $1 \dots V$ and t the days of the week form 1 to 7 as it is a weekly schedule. g in $1 \dots G$ is 1 for the earliest departure, 2 for the next and G for the last. In the equation G is 2 to simplify the explanation. 1 means departure of the voyage r at 16:00 and 2 at 19:00. L is 2 if the voyage is on the group R_2 (group of voyages that last 2 days) or 3 if it belongs to the group of voyages that last 3 days R_3 . If we have more possible departures more constraints should be added such as $g < h$ as shown in the equation 7.4.

$$\sum_{r \in R_L} X_{r,t,v,g=1} + \sum_{r \in R_L, l \in L} X_{r,(t+l) \bmod |T|,v,g=2} \leq 1; \forall v \in V, t \in T \quad (7.3)$$

$$\sum_{r \in R_L} X_{r,t,v,g} + \sum_{r \in R_L, l \in L} X_{r,(t+l) \bmod |T|,v,h} \leq 1; \forall v \in V, t \in T, g < h \quad (7.4)$$

7.4 Collision constraints

An important point of the model is to consider collision constraints as, so far, the model allows them. One way to avoid them is considering non-optimal voyages according to [14], so an heuristic method should be used.

The same author suggest another way, adding what is called in the same font "double voyage module" generating two voyages for each group of installations, duplicating the possibilities generated by the first model. The second voyage consist of placing the last installation in the beginning. For example the travel 0-5-8-7-1-0 would have another associated voyage that could be 0-1-5-8-7-0 but not necessarily. The only condition is that the last voyage goes first, so the optimal is chosen.

Those constraints due to the complexity added are not into the scope of this research work. Therefore, it is considered in the model later eliminating those solutions that are visits at the same moment to the same installations by several different vessels. This solution is not optimal but is a good approximation. if we wanted optimal solution we would need to use one of the two methods explained before.

Chapter 8

Experiments and results

In this chapter we deal with the model application for real data of a model of 13 installations. After validating with two smaller data sets of 4 and 8 installations a several basic solution have been found for a larger model of 13 installations. Then, several changes to the constraints of the problem have been added. The computer used is an *Intel (R) Core (TM) i7-2670QM CPU @2.20GHz and 4GB of RAM*, this is relevant for the computing time of the solutions.

8.1 Test instance

The problem consist of determining the delivering times of a determined number of vessels to a number of installations from a base. It has to be considered that the installations prefer frequent visits and that demands, weather conditions, etc. can change in order to take into account possible slacks for robustness.

There are a great number of parameters and considerations for this problem that are listed below giving numbers range:

- Number of installations (13)
- Capacity of vessels (1000 tones)
- Speed of vessels (12knots/hour)

- Distances between installations and base (from a few miles to hundreds of miles)
- Opening times of installations (all day or from 7 to 19:00)
- Opening times of base (8:00 to 16:00 Monday to Friday)
- Minimum and maximum installations to be visited in a single voyage (from 1 to 8)
- Frequency of visits (from 2 to 6 days in a week of 7 days)
- Demands highly variable(150-650 tones/week)
- Maximum departures per day (2)
- Lay time or service time spent in each installation for supplying (from 3 to 4 hours)
- Spread departures. Preferably to have visits along the week and not on two consequent days (f.ex.. Monday-Thursday-Saturday instead of Monday-Tuesday-Wednesday).
- Slacks to be considered for possible variations in demands, lay time and weather.
- Cost of chartering a vessel: 120000Kr/day.
- Cost of sailing that includes fuel cost:2423.86Kr/h .
- Cost of waiting and during service in installations that includes fuel cost:1211.93Kr/h

8.2 Basic solution

In this section the analyzed problem is solved for further comparisons with the other results in other experiments. Due to the great size of the problem an optimal solution is not possible to warranty with the available technology as there are many possible voyages and it is needed a choice among them.

First of all with 13 installations that could be picked in groups from 1 to 8 yields 7098 possible routes. If we took all those voyages for the second model we would have memory problems resulting in an unsolvable problem with the available tools. Therefore, different strategies and results have been tried. In all the possibilities it has been discarded those voyages that have more demand than the capacity of 1000 tons for all the vessels applicable for this data-set.

In the beginning a first non-optimal solution with only the voyages that comprehend groups of 1, 2, 3 and some of the 4 possible groups with 819 voyages. Solution called "Basic 1-3(5h) As those voyage have a small number of installations running the model takes less time than with bigger groups as solving a TSP takes greater time with larger models. It has been solved in 393.7 seconds for the voyage generation part and 444.9 seconds for the set-covering model giving a total cost of: 4512230Kr.

This approximation is not the best as it does not takes into account the voyages with larger number of installations which may be better to reduce the total cost. Therefore another solution have been found which considers all the 4-grouped installations (Basic-4). Of the 715 possibilities 697 are chosen and the rest discarded due to the demands exceed the vessels capacity. The first model takes 478.3 seconds to solve and the second model 540.5 seconds. The total cost is lower than the above solution: 4332100Kr.

Another approximation has been taken into account like only considering up to 2 installations on a voyage (Basic-2). At it was expected, those instances that take into consideration larger groups of installations to be visited in a single voyage are those with better results. Consequently, there are 3 solutions that yielded the same total cost of 4260220Kr. Those solution have been: Basic 4-7 (5h), Basic 1-7 (5h) and Basic 1-7 (10h).

Type of problem	Cost		Number of Vessels	Computing time	
	Total	% increase		phase 1	phase 2
Basic 1-2 (5h)	8138650	91.04%	8	185	82
Basic 1-3(5h)	4512230	5.92%	4	394	179
Basic-4 (5h)	4332100	1.69%	4	478	254
Basic 4-7 (5h)	4260220	0.00%	4	57403	754
Basic 1-7 (5h)	4260220	0.000%	4	57785	541
Basic 1-7 (10h)	4260220	0.000%	4	133514	1258

Table 8.1: Instances for obtaining the basic solution.

Type of problem	Voyages/routes					
	Solution	considered	selected	%	shortest	longest
Basic 1-2	22	91	91	100.00%	1	2
Basic 1-3	12	820	813	99.15%	2	3
Basic-4	11	715	697	97.48%	4	4
Basic 4-7	9	5434	697	12.83%	4	5
Basic 1-7(5h)	9	6247	1510	24.17%	4	5
Basic 1-7 (10h)	9	6247	2141	34.27%	4	5

Table 8.2: Instances for finding the basic solution (continuation)

Finally the solution chosen is that one that consider all the voyages but starts to discarded. From the experience acquired in the two trials with groups of 4 installations it sames reasonable not to include those voyage with less than 4 installations. In addition, after consulting how they are operated in the real life, they do not use visits larger than 7 installations in one route. Therefore, we could chose not to consider the 8- installations routes. Another reasonable way of discarding voyages is to avoid those that have too much slack. That is to say those voyage that allows too much vacant time for the vessels, different numbers have been tried and it has been seen that with 5 hours we get better results. Thus, the way to select those voyage has been as following:

- Groups of 1 to 7 installations instead of 1 to 8.
- Voyages that have less than 10 hours of idle time for the vessel.

- Voyages that the installations demand are within the vessel capacity.

After processing the initial 5434 candidate routes and discarding them according to the rules explained before it has been possible to reduce the possible voyages to 715 voyages for the set covering model. Anyway, we had to process almost all the candidate voyages, with the exception of capacity discarded voyages, to now how much free time the voyages had and their duration. To solved this problem it has been necessary to divide the total 5434 candidate routes in smaller groups due to memory problems. The total time for the first model has been: $1142.64+3096.81+12140.6+52264.7+64869.6=133514$ seconds or around 37 hours. With a 4-core computer it is in reality almost 10 hours which is in the scope of affordable.

That is quite a lot of time. Before arriving to try with so many voyages other possibilities have been studied.

The total cost of this solution is 1380220Kr. That is what we will call as *base solution* and for comparison will be called from now on as 100% solution. Some details of the solution are given in the tables 8.3 to 8.4. In the figure ?? we can see the details of the solution: vessel occupation, slack time and visits of each vessel.

Number of vessels:	4
Candidates voyages for the set-covering model:	715
Percentage of the initial voyages used:	13.2%
Number of voyages:	9
Number of visits to installations:	43
Total cost:	100%
Computing time of the whole model:	64962.57sec

Table 8.3: Some details of the base solution for a 13-installations real problem

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
1		1		1	2		1
2	1					1	
3			1			1	
4				1			1
5		1					1
6	1			1			
7	1				1		
8			2			1	2
9			1			1	
10			1			1	
11	1	1		2	1		
12		2		1	2		
13		1	2		1		1

Table 8.4: Visits of the basic solution for a 13-installations real problem

Vessel	Mon	Tue	Wed	Thu	Fri	Sat	Sun
v-1	6,7,11	1,11,12	9,10	11	1,5,12		8,13
v-2		2	3,4,8	8,13	1,4,12	1,11,12	9,10
v-3	5,12,13	1,8	13	2,3,8	5		
v-4		8,13	6,7,11				

Figure 8.1: Voyages distinguished by colors and the installations visited

Vessel	Mon	Tue	Wed	Thu	Fri	Sat	Sun
v-1	3.1201		4.079723		4.368098		
v-2			3.756515		1.875642		3.6928
v-3		1.82439			1.875642		
v-4			3.1201				

Figure 8.2: Voyages distinguished by colors and the slack time after each visit

8.3 Robust schedule: slacks at the end of the voyages

In this section it has been added to the basic result a slack of 4 hours. That means that the vessel have 4 additional hours to do each voyage. Therefore the voyages

are limited to 76 for the voyages of 3 days and 52 for the voyages that are planned for two days. Those number are obtained as:

$$24 * Days + Beginning - Slack \tag{8.1}$$

Where *Days* is 2 or 3 depending on the number of days, *Beginning* is 8 as the shift starts at 8:00 and they are expected to arrive at 8:00 as explained in the previous section. *Slack* is equivalent to 4 hours but could be another quantity if wanted. The 4 hours is chosen because it is a quantity used in practical operation [14]. The result is an increase of almost a 5% of the cost with the same number of vessels but a more distributed schedule with 10 voyages as shown in 8.4 and 8.5.

Vessel	Mon	Tue	Wed	Thu	Fri	Sat	Sun
v-1							
v-2							
v-3							
v-4							

Figure 8.3: Occupation of the vessels with four hours slack at the end of each voyage

8.4 Robust schedule: slacks after each visit

As it has been said in the previous chapter there are different options to consider the slacks in the middle of the voyage. We has opted for the solution of adding extra time service so every visit to every installation will need more time. We add a slack to the service for the installations.

Several possibilities has been tried adding different amounts of slack. The firs one consists of adding 1 hour to the service that in the original problem ranges in between 3 and 4 hours. Therefore, for a voyage that visits 4 installations the slack would be 4 hours (1 hour each).

The results of this policy are that the total cost is a 9,5% higher than the basic solution and 4.5% more than adding a single slack at the end of the voyage. It has

11 voyages that go between 3 and 4 installations visited and the occupation time of the 4 vessels is shown in the figure ??.

Another experiment has been trying with 2 hours added slack. Then the voyages are longer and an additional vessel is required, increasing the total cost to almost 24% more than the basic cost. The voyages are longer between 4 and 6 installations visited on a single voyage and the number of the selected voyages in the phase 1 is similar to that with the slack of 1 hour. The occupation of the vessels is shown on figures 8.4 and 8.5.

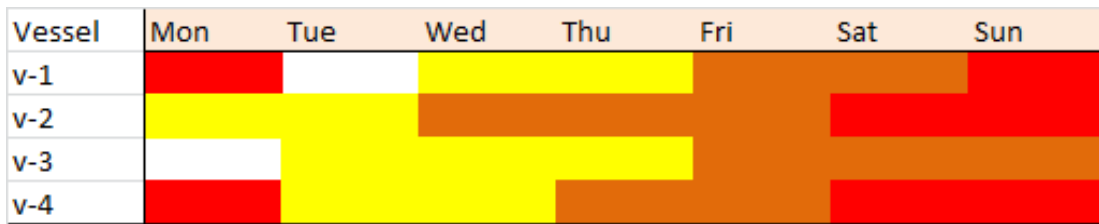


Figure 8.4: Occupation of the vessels with one hour slack in all installations

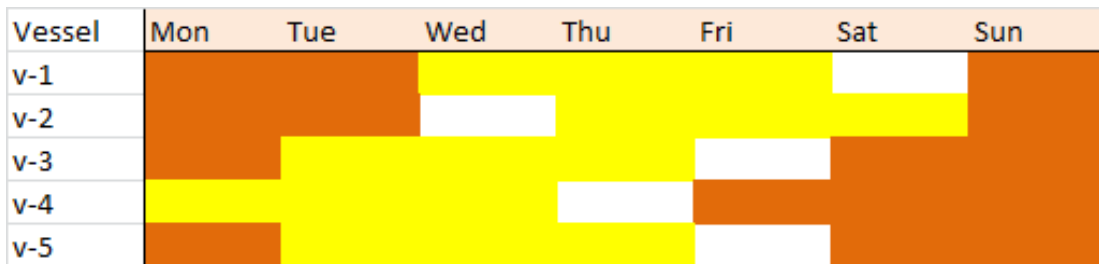


Figure 8.5: Occupation of the vessels with two hours slack in all installations

8.5 Collision constraints

As shown in the 8.4 the solution obtained has collision constraints. If we eliminate them by adding to the solution that the voyages with collisions are not selected at a determined time t after several trials may be possible to eliminate those collisions having the same total cost, adding the constraints $X_{V,R_1,T_1} + X_{V,R_2,T_2} \leq 1$ to the model where R_1 and R_2 are the voyages at a respective time T_1 and T_2 that have

a constraint. However this way would require manual work and is not advised. A better way to do it would be as explained at the previous chapter of extensions of the method, but as it has been said before this would be out of the scope of this work.

Nevertheless, after trying it has not been possible to achieve a solution with the available voyages what indicates that the additional voyages may be needed, making the solution not optimal. And this point has not been studied further, which can be an interesting research for the future.

8.6 Other policies: several working times for the supply base

In this section we study the possibility of adding a second policy for the supply base. It can work normally from 8:00 to 16:00 or by the new second policy from 11:00 to 19:00. To solve this it has been done as explained but simplifying the constraints. In the model studied instead of using G and change all the model R has been subdivided manually into 4 groups. R_{21} , R_{22} , R_{31} , R_{32} . Where R_{11} are the voyages of 2 days and $G=1$, i.e. departure at 16:00 which is virtually the same as explained in section 6.

The results of applying this policy to the model are slightly different from the basic result (voyages and times are different) but it was not possible to improve the solution in this case. However having to generate twice the number of voyages in this case 1859 out of 12494 makes the generation more slow, around 180000 second for the first phase and 4634 seconds for the second.

8.7 Other policies: no departures on weekends

Another possible policy is the case that the base closes during the weekends. Hence, no departures are possible during the days 6 and 7. This policy could be easily applied setting the departures on the last two days to 0 in the set-covering model. See equation 8.2 where r are the voyages in $1 \dots R$, v the vessels in $1 \dots V$ and t the days of the week from 1 to 7 as it is a weekly schedule.

$$\sum_{r \in R, v \in V} X_{r,t,v} \leq 0; \forall t \in 6, 7 \quad (8.2)$$

Applying this policy to our model it increases the cost on about 0,19% having the same number of routes selected:9 but slightly different schedule.

Chapter 9

Comparative analysis of the obtained schedules

As we can see in the tables 9.1 and 9.2 there are several solutions for the basic problem. That is like that because before trying with all the voyages it has been increased the range step by step. When it says *Basic i-j (t)* "i" is the minimum installations voyages accepted and "j" the maximum and "t" is the hours slack considered to select or discard a voyage. Thus, if it is 5 hours a voyage of 3 days would last at maximum 72 hours. So a voyage that lasts less than 67 is not counted. The reason beneath that is that those voyage has so much spare time that there are not candidates for the second phase, saving in computing time and memory.

As we can see if we compare the *Basic 1-7 (5h)* and *Basic 1-7 (5h)* the number of voyages for the first solution is a 30% lower than the second which reduces the computing time to more than a half while the final result is the same. This is so, because the voyages added are not meaningful for the second phase.

Type of problem	Cost		Number of Vessels	Computing time	
	Total	% increase		phase 1	phase 2
Basic	4260220	0.000%	4	133514	1258
Slack-4 (5h)	4327240	1.57%	4	100792	453
m-slack-1(5h)	4392450	3.10%	4	75477	341
m-slack-2 (5h)	5311050	24.67%	5	72829	211
No weekends	4262900	0.06%	4	57785	541
Flexible on departure times	4260220	0.00%	4	122624	4633.58

Table 9.1: Summary of all the solutions obtained in cost, number of vessels used and computing time.

Type of problem	Voyages/routes					
	Solution	considered	selected	%	shortest	longest
Basic	9	6247	2141	34.27%	4	5
Slack-4	10	6247	1278	20.46%	4	6
m-slack-1	11	6247	1004	16.07%	3	4
m-slack-2	10	6247	1113	17.82%	4	6
No weekends	9	6247	1510	24.17%	4	5
Flexible on departure times	9	12494	1859	14.88%	4	5

Table 9.2: Summary of the features of the voyages in the solutions obtained. Number of routes needed, considered previous phase 1, after phase 1 and minimum and maximum number of installations visited in the voyages used.

Next we are comparing the results of the basic solution with those of slack and with a basic not working on weekends in the graphic of the figure 9.1

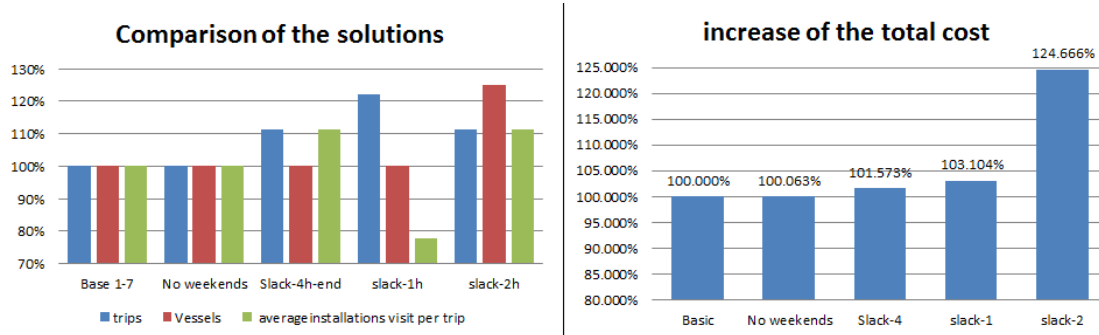


Figure 9.1: Occupation of the vessels with four hours slack at the end of each voyage

In conclusion, it has been used the created model with 4 different datasets and eleven sub-cases for the biggest instance analyzed with 13 installations. After comparing and contrasting the results the conclusions we can draw out are:

- It has been checked that generating with exact means all the possible takes a long time for problems as big as 13 installations. Therefore, for bigger instances other methods such be chosen such as heuristic methods. Furthermore, considering only optimal TSP solutions for the generation of the voyage adds difficulties to deal with the collision constraints needed in real life.
- In order to reduce the number of the generated voyage a procedure has been developed that consists of rejecting those voyage that do not comply with the capacity and the average demand and those voyage that has a more free time than 4 or 5 hours. In addition, there is the option of preselecting a previous range of minimum and maximum installations to be visited.
- In order to add slacks adding them after visiting each installations is more close to reality than at the end of each voyage but increases the cost substantially. Adding more than 1 hour of slack (25% of the service time) to each installation is not recommended as it reduces the number of visits.

Chapter 10

Conclusions

The objective of this research work was to create a functional tool that creates a schedule for several supply vessels to several customers from a base with the only use of exact optimization algebraic modeling language. The created tool selects a number of voyages and calculates the optimal routes. Then, it assigns some of the routes to the vessels and times of visit in order to create an optimal schedule. Thus, the main contributions may be summarized as the following:

- To create and validate the tool in the Mathematical Programming Language AMPL for the whole process.
- To determine a strategy to deal with high computational time and great amounts of data.
- To do several practical extensions for the tool in order to evaluate how to include slacks or changing the supply base operational policy.
- To suggest a functional way to apply several time operations for the base in a single problem.

From the experience of this work it may be relevant to highlight the next points:

- The tool is functional but it takes its time, more than 10.000 seconds for the 13 installations instance.
- To reduce the number of voyages a way is applied and suggested. It consists of discarding those solutions that the average demand of the costumers/installations exceed the capacity of the vehicles/vessels and those who has too much idle time.
- Considering slacks at the end of the voyages is less costly than distributing it at the end of each installations for the instances tried. With more than a 25% of slack the cost grows substantially for the tried instances.

Future research could be divided in two separate paths. The first one could be trying and investigating other solution tools, which not necessarily should be exact mathematical programming, such as simulating the process of creating robust schedules. While the second would be to improve and research about the solution tool in several facets, being the main ones detailed next.

- Automatize the whole process in a more user-friendly way, avoiding copy-paste between the step 1 and the step 2 of the algorithm for example.
- Explore not optimal voyages to maybe achieve a better solution.
- Investigate which time is ideal for discarding voyages due to large vessel idle time if this strategy is desired to be chosen. In this work 5 hours has been suggested.
- Deal with the problem of computing time and outrunning the memory by the use of heuristics or other means.
- Study and develop other strategies of reducing the available number of voyages.
- Expand the number of experiments to include other features such as multi-commodities or pickup and deliveries.

All in all, it has been created a functional tool that generates the schedule for a real instance of periodical supply vessel with time windows with the only use of a mathematical programming language. It has been possible despite the large quantities of data involved and the several constraints involved. The importance of this work has been that the tool is adaptable for different instances for cost reduction in the planning of schedules, the suggested strategy of preselecting the voyages and the creation of robust schedules that can be used and improved in the future.

Bibliography

- [1] Biellia M. , Biellib A., X Rossic R. (2011). "Trends in Models and Algorithms for Fleet Management" *Procedia Social and Behavioral Sciences* 20 (2011) 4–18.
- [2] Christiansen M., Fagerholt k., Ronen D. (2004). "Ship Routing and Scheduling: Status and Perspectives" *Transportation Science* Vol. 38, No. 1, February 2004, 1–18.
- [3] Cacchiani V., Hemmelmayr V.C., Tricoire F. (2012). "A set-covering based heuristic algorithm for the periodic vehicle routing problem" *Original Research Article. Discrete Applied Mathematics*, In Press, Corrected Proof, Available online 19 September 2012.
- [4] Cordeau J.-F., Gendreau M., Laporte G. (1997). "A tabu search heuristic for periodic and multi-depot vehicle routing problems". *Networks* 30(2), 105–119.
- [5] Choi E., Tcha D-W. (2007). "A column generation approach to the heterogeneous fleet vehicle routing problem". *Computers and Operations Research* 2007;34: 2080–95.
- [6] Eksioglu B., Vural A. V., Reisman A. (2009). "The vehicle routing problem: A taxonomic review" *Computers and Industrial Engineering*, Volume 57, Issue 4, November 2009, Pages 1472–1483.
- [7] Fagerholt, K. (1999) "Optimal fleet design in a ship routing problem". *International Transactions in Operational Research* 6(5), 453–464.

- [8] Francis P.M., Smilowitz K.R., Tzur M. (2008). "The period vehicle routing problem and its extensions". In: Golden, B., Raghavan, S., Wasil, E. (Eds.), *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer, Boston, 73–102.
- [9] Fagerholt K., Lindstad, H.(2000). "Optimal policies for maintaining a supply service in the Norwegian Sea". *Omega* 28, 269–275.
- [10] Golden B., Assad A., Levy L., Gheysens F., (1984). "The fleet size and mix vehicle routing problem". *Computers and Operations Research* 11 (1), 49–66.
- [11] Gribkovskaia I., Laporte G., Shlopak A. (2008). "A tabu search heuristic for a routing problem arising in servicing of offshore oil and gas platforms". *Journal of the Operational Research Society* 59 (11), 1449–1459.
- [12] Gulczynski D., Golden B., Wasil E. (2011). "The period vehicle routing problem: New heuristics and real-world variants" *Transportation Research Part E* 47 648–668.
- [13] Halvorsen-Weare E.E., Fagerholt K., Rönnqvist M. (2013). "Vessel routing and scheduling under uncertainty in the liquefied natural gas business". *Computers and Industrial Engineering*, Volume 64, Issue 1, January 2013, Pages 290–301.
- [14] Halvorsen-Weare E.E, Fagerholt K., Magne Nonås L., Egil Asbjørnslett B. (2010). "Optimal fleet composition and periodic routing of offshore supply vessels". *European journal of research* 223 (2012) 508-517.
- [15] Halvorsen-Weare E.E, Fagerholt K. (2011). "Robust Supply Vessel Planning" *Computer Science* Volume 6701, 2011, pp 559-573.
- [16] Hemmelmayr V.C., Doerner K.F., Hartl R.F. (2009). "A variable neighborhood search heuristic for periodic routing problems". *European Journal of Operational Research* 195, 791–802.

- [17] Hoff A., Andersson H., Christiansen M., Hasle G., Løkketangen A. (2010). "Industrial aspects and literature survey: Fleet composition and routing " , Computers and Operations Research, Volume 37, Issue 12, Pages 2041-2061.
- [18] Ronen D. (1993). "Ship scheduling: The last decade". Eur. J. Oper. Res. 71(3) 325–333.
- [19] Shyshou A. (2010). "Vessel Planning in Offshore Oil and Gas Operations". Thesis for the degree of philophiae doctor, Molde University College.
- [20] The Ministry of Petroleum and Energy& the Norwegian Petroleum Directorate (2012) "Facts 2012 - The Norwegian petroleum sector".
- [21] Tan, C.C.R., Beasley, J.E. (1984). "A heuristic algorithm for the period vehicle routing problem". Omega 12(5), 497–504.
- [22] Zouari, A., Akselvoll, A. F (2007). "Multi-Commodity Single Vehicle Pickup and Delivery Problem" Master's degree thesis Molde University College 2007.

List of Tables

4.1	Parameters and variables of the TSPTW model	18
4.2	Parameters and variables of the route selection model	19
6.1	Parameters and variables of the first model	28
6.2	Pseudo-code of the run file used to generate all the voyages	29
6.3	Parameters and variables of the route selection model	31
6.4	Comparison of the results obtained in this work and by [19]	33
6.5	Example of initial voyages	34
6.6	Example of initial voyages: costs of the voyages	35
6.7	Duration and installations visits in the format of the matrix A	35
6.8	Details of the voyages. Time and visits of the installations	36
6.9	Summary of the final results that highlight which voyages are selected and when each installation is visited	37
8.1	Instances for obtaining the basic solution.	45
8.2	Instances for finding the basic solution (continuation)	45
8.3	Some details of the base solution for a 13-installations real problem	46
8.4	Visits of the basic solution for a 13-installations real problem	47
9.1	Summary of all the solutions obtained in cost, number of vessels used and computing time.	53
9.2	Summary of the features of the voyages in the solutions obtained. Number of routes needed, considered previous phase 1, after phase 1 and minimum and maximum number of installations visited in the voyages used.	53

List of Figures

6.1	Schedule of a small instance with four installations to be visited, solved with the model designed for this work	24
6.2	Comparison of the same route between the model created and the results obtained.	32
8.1	Voyages distinguished by colors and the installations visited	47
8.2	Voyages distinguished by colors and the slack time after each visit .	47
8.3	Occupation of the vessels with four hours slack at the end of each voyage	48
8.4	Occupation of the vessels with one hour slack in all installations . .	49
8.5	Occupation of the vessels with two hours slack in all installations .	49
9.1	Occupation of the vessels with four hours slack at the end of each voyage	54