Brice Assimizele

Models and algorithms for optimal dynamic allocation of patrol tugs to oil tankers along the northern Norwegian coast

Molde University College
Specialized University in Logistics

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A dissertation submitted to Molde University College - Specialized University in Logistics for the degree of Philosophiae Doctor

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Molde University College - Specialized University in
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Preface

This document is submitted as partial fulfillment of the requirements for the degree of *Philosophiae Doctor* (PhD) in Logistics at Molde University College - Specialized University in Logistics, Molde, Norway.

This work is carried out from October 2012 to September 2016, during this period I have been employed as a Research Fellow at the Norwegian University of Science and Technology (NTNU), Ålesund Campus. One year of this period has been dedicated to teaching and administrative duties.

Associate Professor Johan Oppen from Molde University College has been the main advisor for this research, with Associate Professor Robin T. Bye from NTNU, Ålesund Campus, Norway and Professor Johannes O Royset from the Naval Postgraduate School, California, USA as co-supervisors.

The research is conducted in close collaboration with the Norwegian Coastal Administration (NCA), where the primary objective is to develop models and algorithms that optimally reduce the environmental costs from oil tankers grounding accidents. This includes optimal positioning of patrol tugs in a highly dynamic and stochastic environment. We propose a flexible and efficient decision support tool to the NCA, validated with historical events, that significantly reduce environmental risk associated with drifting vessels. The thesis consists of a brief introduction to the problem and related literature, followed by four papers that present the theoretical and practical contribution of this research.

The evaluation committee for this work has been Professor Anton Kleywegt, Georgia Institute of Technology, Georgia, Atlanta, USA, Professor Ingrid Schjølberg, NTNU, Trondheim, Norway and Professor Lars Magnus Hvattum from Molde University College - Specialized University in Logistics, Norway.

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I am grateful to my main supervisor, Associate Professor Johan Oppen for his modeling and writing assistance. I owe him many thanks for his research network and, for his good humor and jokes during our meeting that make math models less frustrating. I am happy for not taking many bullets during these four years of hunting, and being one of the surviving deers.

Special thanks to Professor Johannes O Royset for the inputs and comments that have improved the quality of this research. The intensive and fruitful research work with him during my visit at the Naval Postgraduate School in California, USA has opened the way to the successful achievement of this thesis.

I would like to express my thanks and appreciation to Associate Professor Robin T. Bye for giving me the opportunity to take this PhD program. His excellent communication and writing skills have been of great help in the completion of this work. I wish to thank him, as a leader of the DRAMA research group at the Norwegian University of Science and Technology (NTNU), Ålesund campus, for financing part of my research trip to California, USA.

This research has been conducted in collaboration with the Norwegian Coastal Administration (NCA). I am thankful to Trond Ski at the NCA for the great collaboration and support in obtaining input data. I also wish to thank Ståle Sveinungsen for providing insight into the vessel traffic service (VTS) surveillance and all the staff at the VTS center in the town of Vardø for their warm welcoming during our visit and the monthly update of the oil tankers traffic in the High North.

I also want to acknowledge the Department of Ocean Operations and Civil Engineering at NTNU for all the encouragement and financial support. Particular thanks to the football team at NTNU for all the good and relaxing time.

My sincere thanks to my wife Larissa Assimizele and my daughter Kiara Assimizele for their patience and love, for being my mentor in all these four years of hard work. A special gratitude to my late grand-mother Biome Assimizele who has always been there for me, but unfortunately could not assist in my graduation.

Last but not the least, many thanks to the almighty God who allowed me to take this PhD program.

Molde, Norway
September, 2016

Brice Assimizele

Contents

Introduction

Introduction

The increasing technological developments create unprecedented potentially harmful effects on public health and the environment, which make the challenge of dealing with uncertain risk increasingly urgent. Oil spills are some of those potential hazards that present a global dimension because of multinational companies and therefore may irreversibly harm the health and physical integrity of the entire population and environment. Maritime crude oil transportation began in the end of the nineteenth century and has steadily increased over the last decade. Today, oil tankers transport more than 1.86 billion tonnes of crude oil across the seas each year. The main driving force for crude oil transportation is refinery requirements, which use crude oil to derive various petroleum products. The type and volume of a petroleum product that can be produced depend on refinery capabilities and the type of crude oil available (Hennig et al., 2012). The demand for crude oil and petroleum products, and consequently, worldwide oil tanker traffic, has considerably increased during the last decade, which inevitably results in accidental oil releases. Oil tanker grounding accidents result in environmental and socio-economic consequences. The extent of these impacts is mostly determined by a diverse set of factors : (a) the volume, rate and type of oil spilled; (b) the location that comprises geographical position as well as political and legal issues; (c) the vicinity to sensitive resources; (d) the choice and effectiveness of cleanup strategies. Once the oil is released in the sea, it undergoes complex physical and chemical transformations, including spreading, drifting, dispersion, stranding, and weathering. Millions of tonnes of oil and other petroleum products are transported by tankers every year. This represents a significant threat to the marine environment in the event of ship collisions or grounding accidents (Gong et al., 2014) and the extent of this threat is still unknown.

Concerns over human error and poor management have motivated the foundation of the International Maritime Organization (IMO) and the International Safety Management (ISM) code. The IMO and ISM regulations are directly related to crew competence and general operational aspects of maritime transportation. Ships transporting dangerous cargo are covered by IMO regulations intended to protect the marine environment (Burgherr, 2007). Despite all these regulations, the public has always been carrying the burden of proving that a given activity such as oil tanker transport is dangerous, while those creating these activities or products are considered innocent until proven guilty. This burden of scientific proof is a monumental barrier to health and environmental protection. In fact, action to prevent harm are generally taken after a significant proof of harm is established, which may be too late.

In Norway, preventive actions against potential oil spills from oil tanker grounding accidents are being taken by the Norwegian coastal administration (NCA). This thesis presents the results from a research project conducted in close collaboration with the NCA that addresses the oil tanker grounding accident described as the optimal tugboats positioning (OTP) problem in the next section.

Optimal Tugboats Positioning Problem

Marine transportation of crude oil and petroleum products and its associated risk to the environment have increased significantly during the last decades. To safeguard the marine environment from potential oil spills and other damage from grounding of vessels, the NCA operates a center for vessel traffic service (VTS) in the town of Vardø in northeastern Norway. The region is environmentally sensitive due to important fisheries and increasing tourism and drift grounding accidents from oil tankers might lead to devastating consequences to the marine ecosystem (Lecklin et al., 2011). In addition to the potential human casualties, the clean-up costs from oil spills are prohibitive.

The VTS center is responsible for the coastline from the Russian boarder in the Barent Sea to Rørvik near Trondheim, a distance of more than 600 nautical miles. In this region of interest, about five to six high risk vessels (see Table 1) receive special attention daily by the VTS center. These high risk vessels include oil tankers and all ships over 5000 gross tonnage (GT). Although very large oil spills come from oil tankers, many bulk carriers and container ships carry bunker fuel of about 10,000 tonnes, which are large quantities that some oil tankers carry as cargo. Additionally, bunkers usually consist of heavy fuel oils, which are highly viscous and more persistent than light crude oil carried by some oil tankers. These vessels are required by law to sail along a predefined

Table 1: Number of high risk vessels and total volume of petroleum product in transit in the High North during seven years

	2010	2011	2012	2013	2014	2015	2016
High risk vessels	1610	1508	1522	1584	1642	1899	1831
Oil volume (million tonne)	14.8	11.9	10.9	12.1	11.9	23.1	24.1

corridor about 27 nautical miles from the coast. At any time, there is a risk that a vessel moving in the region loses its maneuverability, e.g., through steering or propulsion failure. Thus, the tugboats have to be sufficiently close to hook-up with any drifting vessel before it runs aground. Depending on weather conditions (wind, currents), towing power of each tugboat, size and shape of tanker, a single tugboat can at least stop or slow the drift for a sustained period of time until nearby vessels or other tugboats can assist. A vessel grounding can be categorized as being soft or hard. A soft grounding occurs when a vessel is stuck in sand or silt while a hard grounding occurs when a vessel is stuck on a fixed object or hard rock.

The large number of high risk vessel traffic as well as the increasing petroleum volume in the northern Norwegian coast (see Table 1) make it difficult to dynamically position the tugboats to locations where they can be the most effective. Presently, the operators at the VTS center do not use computer algorithms or mathematical models to solve this OTP problem but rely on their own knowledge and experience when faced with constantly changing weather and traffic conditions. Under the current operational strategy, the tugboats are often in an escort capacity, following a moving vessel, or on hold in a given position as sentinels. In an effort to improve the dynamic positioning of tugboats, we were invited by the NCA to visit the VTS center in Vardø and suggest improvements in the process. This thesis focuses on the operational strategy and proposes a policy that minimizes the risk of vessel grounding accident with the available resource.

The VTS center operates a fleet of tugboats with the purpose of intercepting any potentially drifting vessel. In 2014, the VTS center reduced its fleet from three to two tugboats, which makes the efficient use of the available fleet highly important. Dynamic information (e.g., position, heading, speed over ground, rate of turn) and static information (e.g., identity, dimensions, cargo, flag) of the ships entering the region are obtained every two seconds on average through the Automatic Identification System (AIS). In addition to AIS information, weather forecast, real-time measurements of ocean currents, wave height, and wind are available to predict drift trajectories. Presently, this fleet of only two tugboats is fortunately assisted by a unique system for reducing the speed of drifting vessels called “ship arrestor”. It consists primarily of a lasso, a towline and a large para sea anchor. The towline is connected to the vessel, with the help of a helicopter and the para sea anchor attached to the towline is then dropped into the water.

An average of 38 drifting of high risk vessels is registered by the VTS center every year, of which about 11 ran aground and 27 managed to make self-repair or received assistance from the tugboats (see Table 2). Despite this low frequency of vessel grounding accidents, their consequences could be serious, including the loss of life, oil spill and long term ecological and environmental impact, which is a major concern to the NCA. Thus, the main objective of this research is to reduce the environmental risk or cost from vessel grounding accidents. Moreover, maximizing the efficiency of the expensive available resources is an additional motivation for this thesis.

Table 2: Frequency of vessel drifting and grounding accidents during four years

	2013	2014	2015	2016
Drifting	43	34	35	38
Grounding	8	12	14	9

The assignment of tugboats to vessels is done dynamically in a highly uncertain environment. Thus, the OTP problem is modeled as a multiple time periods dynamic and stochastic convex nonlinear and linear binary integer programs integrated in a receding horizon control framework, discussed in the solution methods, to allow the change of information about vessels, probabilities of failure, drift trajectories and its associated expected environmental cost or risk, and probabilities of successful hook-up. This approach is a major contribution of the thesis as it avoids the formulation of a multistage stochastic program and the associated high-computational cost. It is, however, important to note that the model formulations in this thesis look similar to that of a multi-stage stochastic problem. This thesis reports models for the OTP problem as well as computational and simulation results with real world data.

The thesis proposes a better tugboats decision policy that reduces the environmental risk from vessel grounding accidents along the northern Norwegian coast. Specifically, we develop models that dynamically move tugboats to positions where the probabilities of successful rescue operations are higher and minimize the overall expected environmental consequences. In addition, we propose preliminary models for obtaining the drift trajectories and the probabilities of successful hook-up of tugboats with potentially drifting high risk vessels. Moreover, we give flexibility to the managers at the VTS center by modeling difference level of risk averseness. Numerical experiments with realistically sized test instances present a great potential for environmental cost reduction. Test cases with real-world instances, run with more than 50,000 scenarios, indicate a total clean-up and socioeconomic cost saving opportunity of 45%. We demonstrate that on a single day in 2014, decision support by the proposed models might have reduced the expected cost from grounding accidents that day from NOK 0.75 million to NOK 0.34 million. Additionally, we are 95% confident that in case of grounding accident, the average environmental cost is less than NOK 6.646 millions and the expected worst case scenarios is NOK 8.670 millions, which are promising results for risk-averse managers at the NCA. Details on the models and results with historical events are presented in the four papers that constitute the main part of this dissertation.

Related Literature

In this section, we present a review on safety organization and emergency response models followed by maritime search and rescue operations literature that specifically relates to the optimal tugboats positioning problem.

Common models on resource location/allocation and patrol routing problems in the literature include p-median problems (p-MP) (Church and ReVelle, 1976; Church et al., 2004; Campbell, 1996; Ishfaq and Sox, 2010), p-center problems (p-CP) (Espejo et al., 2015; Drezner, 1984; Suzuki and Drezner, 1996; Davidović et al., 2011), covering problems that are categorized into maximal covering location problems (MCLP) (Davari et al., 2011; Church and ReVelle, 1974; Balcik and Beamon, 2008), set covering problems (SCP) (Caprara et al., 2000; Beasley and Jørnsten, 1992; Badri et al., 1998), maximum coverage patrol routing problems (MCPRP) (Çapar et al., 2015; Keskin et al., 2012; Li and Keskin, 2013; Dewil et al., 2015) and police districting problems (PDP) (D'Amico et al., 2002; Camacho-Collados and Liberatore, 2015).

General Emergency Location Problem

The p-MP consists of finding the location of p facilities with an objective of minimizing the total weighted sum of travel distances between demands and facilities, whereas the p-CP, known as a minimax problem seeks to locate p facilities such that the maximum distance of any demand to its closest facility is minimized. In the SCP, the main objective is to minimize the cost of facility location as well as the number of required facilities for a predefined level of coverage. The MCLP is concerned with the cases where the available resource cannot meet the desired level of coverage within a predefined threshold (Church and ReVelle, 1974; Davari et al., 2011). In addition, the patrol routing problem consists of maximizing the coverage of critical highway stretches subject to constraints on available resources and feasible routes. As in the vessel traffic service, ambulance emergency medical services (EMS), fire stations, police patrols and freeway services are cases of emergency response systems, where resources are allocated to demand points in a short response time. The OTP problem, however, differs from other emergency response systems because of the higher environmental consequences, human casualties and the possibility of extremely bad weather conditions, which require dynamic allocation and positioning of resources in a highly stochastic and dynamic environment.

The primary goal of the EMS is to save lives by ensuring quick response to emergencies of which the performance is affected by the ambulance locations and their deployment. Tavakoli and Lightner (2004) use a mathematical modeling approach to solve the locating/allocating of emergency vehicles and facilities. Moreover, preparedness is a way of assessing the ability to serve current and future potential patients with ambulances. Accordingly, Andersson and Värbrand (2007) propose new algorithms for the ambulance deployment and the dynamic ambulance relocation problems, with the primary goal of finding new locations for some of the ambulances that increases the preparedness in the area of interest. Their problem is similar to the one studied in Gendreau et al. (2001), with the major difference of relocating ambulances to any zone in the region of interest, not just to vacant stations. The objective function of their dynamic relocation model minimizes the maximum travel time for any of the relocated ambulances. Lee (2011) investigates the role of preparedness by developing a dispatching algorithm that takes into account future calls. It is found in their study that the consideration of preparedness in ambulance deployment considerably reduce the response time. Indeed, a greedy minimization of each current call might increase the response time of future calls that have high call rates.

To account for uncertainty on the ambulance availability, the hypercube model is found to be useful in determining the EMS systems performance (Jarvis, 1985). However, it is computationally expensive as the number of simultaneous equations for m servers (ambulances) would be equal to 2^m. To address this issue, approximations of the hypercube model have been developed with some assumptions on the distribution of the service time (Rajagopalan et al., 2008). A mixed integer programming (MIP) model that finds best locations of highway incidents at a minimum cost is proposed by Pal and Bose (2009). Their approach considers fixed and variables costs of vehicles and depots, but they do not consider the deployment of the response segments as in Iannoni et al. (2009). These latter authors develop a method to optimize the configuration and operation of EMS on Brazilian highways. The method consider the location of ambulance bases along the highway and the districting of the response segments. Their approach embed a spatially distributed hypercube model into a hybrid genetic algorithm. Similarly, Toro-Díaz et al. (2013) propose a mathematical formulation that combines an integer programing model representing location and dispatching decisions, with a hypercube model that represents the congestion phenomena and queuing elements. Their results, obtained with genetic algorithms, show that minimizing the response time and maximizing the coverage can be achieved by using a common closest dispatching rule. Majzoubi et al. (2012) consider the problem of dispatching and relocating EMS vehicles with possibility of rerouting a vehicle transporting a low-priority patient to pick-up one more patient. They propose a solution based on integer nonlinear and linear programming models with an approximation algorithm. Although the OTP problem is different from the EMS, they share a common goal of quick response to distress calls.

Maximum Coverage Patrol Routing Problem

The MCPRP, introduced by Keskin et al. (2012), is used to assist traffic enforcement. The authors claim that a heuristic solution is preferred instead of an exact technique and developed a local search and tabu search heuristic, which give good quality solution in short time period. Moreover, Li and Keskin (2013) develop a bi-objective multi-period patrol routing problem, where intermediate temporary stations in the patrol routes are introduced. The problem is solved with a heuristic algorithm that decomposes the problem into location and routing problem. Çapar et al. (2015) present an improved formulation of the MCPRP by investigating the structural properties of the optimal solution and formulate a new MIP that solves real life instances within seconds, where previous methods in the literature failed to find a near optimal solution within an hour. Dewil et al. (2015) model the MCPRP as a minimum cost network flow problem (MCNFP) and use a network simplex algorithm to obtain solutions faster than that of the local search/tabu search heuristic proposed by Keskin et al. (2012). Their algorithm is however not faster than the improved MIP developed by Çapar et al. (2015), but can be solved in polynomial time if very large instances are constructed.

Another related patrol problem studied in the literature is the freeway service patrol (FSP) problem, which consists of deploying freeway service patrols to detect, respond to and clear traffic incidents. A network of freeways are divided into a set of patrol beats (connected freeway segments) and tow trucks are assigned to patrol each of these beats, moving back and forth to clear any possible incident, which consists of changing flat tires, offering gasoline and moving vehicles. Lou et al. (2011) develop two nonlinear mixed integer programming (MINLP) models for deterministic and stochastic integrated beat design and fleet allocation of FSP. Their main objective is to minimize the expected total response time over the high-consequence scenarios.

Police Districting Problem

The PDP involves the design of patrol sectors in terms of performance attributes such as response time and workload that result in crime reduction and better service. Camacho-Collados and Liberatore (2015) develop a decision support system based on mathematical algorithms that incorporates predictive policing capabilities with patrolling districting model. The aim of their system is to reduce the probability of criminal acts. Similarly, Mišković et al. (2015) propose a mathematical model for the emergency service network of Police Special Forces Units (PSFUs) in the Republic of Serbia and solve the problem with a variable neighborhood search (VNS) algorithm. In addition, Camacho-Collados et al. (2015) investigate crime prevention by increasing the effectiveness of the deterrent effect of the agent's presence on the territory. This is achieved by concentrating the agents in the areas with a higher risk of crime. Other approaches such as dynamic programming are also used to optimally deploy crime preventive police patrol teams to areas of higher risk of crime (Oghovese and Olaniyi, 2014). D'Amico et al. (2002) and Zhang and Brown (2013) on the other hand focus on the reaction to crime incidents with the objective of minimizing the response to emergency calls.

Maritime Search and Rescue Operations

The OTP problem is closely related to maritime search and rescue (SAR) operations. SAR operations consist of searching missing or distressed vessels followed by their rescue. Basdemir (2004) proposes an MCLP that allocates SAR helicopters to candidate bases to satisfy predefined incidents regions. A combination of p-MP and p-CP models are used in Dawson et al. (2007) to determine the locations of security teams over a geographic area to maintain security for the United States air force intercontinental ballistic missile systems. The combined model minimizes both the distance traveled and the maximum distance from any missile site to required security forces. An optimization and simulation method is used in Afshartous et al. (2009) to determine the locations of the United States coast guard air stations to respond to emergency distress calls. They model the problem as a p-Uncapacitated Facility Location Problem (p-UFLP). The authors assume the demand for each client to be equal and served with a single resource. A similar problem is presented in Razi and Karatas (2016), but the demand for each incident varies and each demand can be covered from multiple resources. Radovilsky and Koermer (2007) develop an integer linear programming model to optimally allocate small boats to the United States coast guard (USCG) stations. Their objective function minimizes the shortage or excess capacity at the stations. An improved formulation called boat allocation tool (BAT) is developed by Wagner and Radovilsky (2012), but do not consider actual locations of incidents and the corresponding response time. Chircop et al. (2013) address the fleet sizing problem faced by the Royal Australian Navy (RAN) with a column generation algorithm incorporated into a branch-and-price framework. A fleet of patrol boats should be able to provide complete coverage of a set of specified patrol regions. Moreover, Millar and Russell (2012) develop a binary integer programming model (BIP) for the fisheries surveillance patrol routing problem in the Canadian Atlantic offshore groundfish fishery. The primary goal of the fisheries patrol routing problem is to maximize the deterrent effect of a patrol vessel through routing over a network of fishing grounds. They are the first to formulate this problem, which relates to the selective traveling salesman problem, where the fishing grounds represent the cities, and all or a subset of grounds is visited on a given trip. Their model, however, focuses more on scheduling than boat positioning. Pelot et al. (2015) categorize SAR boats based on their capabilities and use historical incident data to solve the allocation problem for the Canadian coast guard. In their study, incidents are classified based on their severity and a response time requirement is established for each type. Similarly, Eide et al. (2007) develop a dynamic risk model that prioritize oil tankers based on their potential oil spill volume in case of grounding accidents and subdivide the northern Norwegian coastline in segments, where each segment has an associated risk level. The model estimates the environmental risk of a drift grounding accident occurring with a specific tanker, at a given location, and under current weather conditions. Drift trajectories with high risk can then be prioritized in the planning of tugboat positions. Abi-Zeid and Frost (2005) develop a geographic decision support tool (SRAPlan) based on search theory to assist the Canadian forces in the planning of search missions for missing aircrafts. A similar system is also developed for the

Polish SAR teams (Wysokiński et al., 2014).

Despite the importance of all these models and tools, they do not suggest how and where the fleet of tugboats should move in order to minimize risk. Razi and Karatas (2016) on the other hand develop a tactical model for determining the optimal placement of SAR boats, however their model do not account for uncertainty related to vessel incidents and the dynamic nature of the SAR resources positioning, which are the primary concern of the OTP problem. In this thesis, we propose models simulated with historical events that effectively address the OTP problem.

Solution Methods

Most of the problems related to the OTP, described in the literature review section, have been addressed with exact and heuristic algorithms. These solution methods are the main approaches used in the operations research and management science. Exact methods provide optimal solutions to an optimization problem, while heuristic methods refer to experience-based techniques for problem solving and are used to improve the computational time with satisfactory solution, where an exhaustive search is not practical. Due to the extremely high potential environmental and socio-economic consequences related to oil tankers grounding accidents as well as the stochastic and dynamic nature of the OTP problem, we integrate model predictive control (MPC) algorithms into linear and nonlinear mixed integer/binary programs with a risk-averse measure of risk.

Risk management is generally considered as a scientific approach to address risks, because it draws on empirical evidence obtained from risk assessment studies. In this thesis, we use superquantile risk measure, which is presently one of the widely used risk measures suggested by theoreticians and market practitioners (Huang et al., 2008; Sarykalin et al., 2014) and is recognized as an important approach in risk analysis and stochastic optimization (Rockafellar and Royset, 2013). The concept is used in various portfolio optimization problems (Tong et al., 2010) and is known under a variety of names such as ”conditional value-at-risk,” ”average value-at-risk,” ”tail value-at-risk,” and ”expected shortfall,” with some minor variations in definitions. In many applications, distributional information about a random variable are most of the time incomplete, which makes superquantiles especially important for stability (Rockafellar and Royset, 2013). This is why superquantile is well suited for risk-averse decision making and optimization. It has been applied in financial engineering (Alexander et al., 2006; Balbás et al., 2010; Uryasev et al., 2010; Wang and Uryasev, 2007), structural engineering (Haukaas and Mahsuli, 2013; Rockafellar and Royset, 2010; Minguez et al., 2013), military operations (Commander et al., 2007; Kalinchenko et al., 2011; Molyboha and Zabarankin, 2012), supply chains (Tomlin, 2006; Verderame and Floudas, 2011; Ansaripoor et al., 2014), and energy systems (Carrion et al., 2007; Conejo et al., 2008). We refer to Krokhmal et al. (2011); Rockafellar (2014); Sarykalin et al. (2014); Rockafellar and Royset (2015) for reviews of risk measures and superquantiles.

We make use of the MPC, also named as receding horizon control (RHC), features by dynamically solving the OTP problem, implementing the first stages, then moving forward one time step and resolving. This solution procedure is similar to a very common rolling horizon procedure for solving sequential decision problems. In each period, the decision maker (a) obtains a forecast and updated parameter values, (b) solves an optimization problem, (c) implements a decision policy for the immediate period, and (d) observes additional input parameters for the next forecast. This approach has been widely applied in the literature (He et al., 2012; Beraldi et al., 2011; Rakke et al., 2011; Mayne, 2014; Bye, 2012; Samà et al., 2013; Kostin et al., 2011; Nielsen et al., 2012; Guigues and Sagastizábal, 2012; Silvente et al., 2015; Balakrishnan and Cheng, 2009). The MPC has good robustness properties with regards to mismatches between parameters and the model. It also easily integrates the available forecasts on uncertain quantities into the problem.

Scientific contribution

The thesis consists of four papers that present detailed contribution of this work. All these papers address the environmental consequences related to oil tanker grounding accidents along the northern coast of Norway. A brief summary of the scientific contribution of the four papers are described as follows.

In the first paper, we develop a new mathematical model based on the one-dimensional formulation approach from previous work in the literature. The model simultaneously minimizes the distances between tugboats and the nearest potential drifting oil tankers and dynamically allocate patrol tugboats to vessels at each time period of the planning horizon. Additionally, a receding horizon control algorithm is integrated in the model to address the dynamic and stochastic nature of the problem. Numerical results with similar randomly generated instances to those reported in the literature present a significant improvement in the solution quality. The computational experiments also highlight the benefits derived from the combination of the receding horizon control algorithm with the developed mathematical model.

The second paper gives a two-dimensional formulation of the optimal tugboat positioning (OTP) problem. A nonlinear mathematical formulation of the OTP is developed with detailed description of how the input parameters, such as probabilities of successful hook-up, drift trajectories and environmental costs from oil tankers grounding accidents are generated. The model is linearized with two different approaches that give good bounds of the objective function. Analysis with historical events, where vessels actually ran aground, present considerably higher probabilities of successful rescue operations if the model had been in use by the VTS. In addition, the results obtained with realistic and real-world cases suggest significant environmental costs reduction from the tugboat positioning policies proposed by the model.

The third paper extends the second paper by examining different levels of risk-averseness of the OTP problem and uses superquantile optimization to minimize the expectation of the worst case drifting vessel scenarios. Additionally, this paper gives a more accurate estimation of the grounding locations of potentially drifting vessels by predicting more than one trajectory for each vessel scenario. The numerical results with historical events describe the flexibility of switching between different levels of risk-averseness which is of great importance to the managers at the NCA.

Finally, the last paper includes further examinations of the solutions quality and performance of the OTP problem by proposing two alternative mathematical models. The paper discretizes the region of interest with smaller cell size that significantly improves the decisions on tugboat positions and reduces the expected environmental risk. The computational experiments with real world data show that the proposed models are well suited for practical implementation.

Suggestions for Future Research

Despite the theoretical and practical contributions of these four years of research work, which are of great interest to the NCA, there is still a room for improvements. We recommend the following future research directions that will better address the oil tanker grounding accidents problem in the High North.

We have solved the problem at the operational level with the main objective of optimally utilizing the available resources. Further research could address the drift grounding accidents problem at the tactical and strategical levels by determining the optimal fleet size and capacity, and including the daily operational costs of the tugboats as well as scheduling of the fleet and workers in the model. In addition, one could assess whether it is better to leave the tugboats static for certain time in the planning or to have a sufficient fleet size that will operate as in the emergency medial service system. Moreover, the CO_2 emission from the fleet of patrol tugs could be included in the objective function of the model.

A finer discretization of the region of interest will result in a more accurate representation of the real world, but will dramatically increase the complexity of the problem. This raises the need of sophisticated solution methods such as metaheuristic optimization algorithms. Additionally, empirical research and field tests on the proposed probability of successful hook-up and other input parameters are needed for better decision policies.

Summary of the papers

This section contains a listing of the four papers that constitute the main part of this thesis. For each paper, the contributions of the different authors are stated as well as the list of conferences and workshops where materials from the papers have been presented. The information on the publication status of each paper is also given in this section.

Paper 1 – A Sustainable Model For Optimal Dynamic Allocation of Patrol Tugs to Oil Tankers

This paper is co-authored with Associate Professors Johan Oppen and Robin T. Bye. The mathematical model in this paper is developed by the candidate with the assistance of Associate Professor Johan Oppen. Ideas on minimizing the distances to drifting oil tankers and integrating the model predictive control algorithm are proposed by Associate Professor Robin T. Bye. All implementation, computational testing, result analysis and writing are done by the candidate. The co-authors contributed in improving the language, style and clarity of the paper with their comments.

The paper is published at the *Proceedings of the 27th European Conference on Modeling and Simulation (ECMS)*, pages 801-807, 2013. The paper was presented by the candidate at the 27th ECMS conference, Ålesund, Norway, May 2013 and, at the PhD seminars at Molde University college - Specialized University in Logistics and the Norwegian University of Science and Technology, Norway.

Paper 2 – Preventing Environmental Disasters from Grounding Accidents: A Case Study of Tugboat Positioning along the Norwegian Coast

Preliminary work on this paper is done by Professor Johannes O Royset in cooperation with Associate Professor Johan Oppen and the candidate, during the visit at the Naval Postgraduate School, Monterey, California, USA. The paper is co-authored with Professor Johannes O Royset, Associate Professors Johan Oppen and Robin T. Bye. Two linearizations of the objective function are proposed by the candidate and Professor Johannes O Royset. The real world data collection, implementation, numerical results analysis and discussion, and output visualization are done by the candidate. Comments and suggestions from the four anonymous referees helped to improve the quality of the paper. The results, analysis and writing are further improved with the help of the co-authors.

The paper is accepted for publication in the *Journal of the Operational Research Society*. Material from the paper was presented by the candidate at the Institute for Operations Research and the Management Sciences (INFORMS) Annual Meeting 2013, Minneapolis, USA, October 2013 and, at the PhD seminars at Molde University college - Specialized University in Logistics and the Norwegian University of Science and Technology, Norway.

Paper 3 – A risk-averse decision-making tool for dynamic positioning of patrol tugs along the northern Norwegian coastline

The third paper in this thesis is co-authored with Professor Johannes O Royset, Associate Professors Johan Oppen and Robin T. Bye. Ideas on superquantile optimization in this paper are elaborated by Professor Johannes O Royset during the visit of the candidate at the Naval Postgraduate School, Monterey, California, USA. In addition, the structure of the computational results are proposed in Skype meeting by Professor Johannes O Royset. Associate Professor Robin T. Bye, with the other co-authors significantly contributed to the writing quality of the paper. The implementation and writings of the entire paper are done by the candidate.

The paper is ready for submission in the *European Journal of Operational Research*. Parts of this paper are presented by the candidate at the Canadian Operational Research Society (CORS)/INFORMS 2015 Joint International Meeting, Montreal, Canada, June 2015.

Paper 4 – Minimizing the Environmental Risk from Oil Tanker Grounding Accidents in the High North

The work on the final paper of this thesis is done by the candidate. Alternative improvement models of the optimal tugboat positioning problem are developed and implemented by the candidate. Constructive comments from the supervisors improved the clarity of the paper.

The paper is ready for submission in the *Marine Pollution Bulletin* journal. Parts of this paper are presented by the candidate at the APL (Association pour la Promotion de la Logistique au Cameroun) conference, Douala, Cameroon, June 2016 and at the PhD seminars at Molde University college - Specialized University in Logistics and the Norwegian University of Science and Technology, Norway.

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Paper 1

A Sustainable Model For Optimal Dynamic Allocation of Patrol Tugs to Oil Tankers

A Sustainable Model For Optimal Dynamic Allocation of Patrol Tugs to Oil Tankers

Brice Assimizele *[a,b], Johan Oppen[b], and Robin T. Bye[c]

[a]Ålesund University College P O Box 1517 NO-6025 Ålesund Norway

[b]Planning, Optimization and Decision Support Group Molde University College P O Box 2110 NO-6402 Molde Norway

[c]Faculty of Engineering and Natural Sciences Ålesund University College P O Box 1517 NO-6025 Ålesund Norway

Abstract

Oil tanker traffic constitutes a vital part of the maritime operations in the High North and is associated with considerable risk to the environment. As a consequence, the Norwegian Coastal Administration (NCA) administers a number of vessel traffic services (VTS) centers along the Norwegian coast, one of which is located in the town of Vardø, in the extreme northeast part of Norway. The task of the operators at the VTS center in Vardø is to command a fleet of tug vessels patrolling the northern Norwegian coastline such that the risk of oil tanker drifting accidents is reduced. Currently, these operators do not use computer algorithms or mathematical models to solve this dynamic resource allocation problem but rely on their own knowledge and experience when faced with constantly changing weather and traffic conditions. We therefore propose a novel sustainable model called the receding horizon mixed integer programming (RHMIP) model for optimal dynamic allocation of patrol vessels to oil tankers. The model combines features from model predictive control and linear programming. Simulations run with real-world parameters highlight the performance and quality of our method. The developed RHMIP model can be implemented as an operational decision support tool to the NCA.

Keywords: Dynamic Resource Allocation; Mixed Integer Programming; Receding Horizon Control; Maritime Operations.

1 INTRODUCTION

Maritime shipping is an important channel of international trade. More than seven billion tonnes of goods are carried by ships every year (UNCTAD, 2007). In Norway, several hundred

*Corresponding author. Email: bras@hials.no

oil tankers transit each year along its northern coastline (Bye, 2012). This traffic is associated with potential grounding accidents due to oil tankers losing control of steering or propulsion, a problem that is highly underreported and likely occurs almost every day[1]. Such accidents can have severe environmental consequences from oil spill and may even lead to loss of lives.

The VTS center in Vardø, located in the extreme northeast part of Norway, controls a fleet of tugs patrolling the coastline. By means of the automatic identification system (AIS) used by ships and VTS centers all over the world, the VTS center in Vardø obtains static (e.g., identity, dimensions, cargo, destination) and dynamic (e.g., position, speed, course, rate of turn) information about vessels along the coast. In addition to the AIS information, weather forecasts and dynamic models of wind, wave heights, and ocean currents can be used to predict possible drift trajectories and grounding locations of tankers that lose maneuverability. The aim of the patrolling tug vessels is to move along the coastline in a collectively intelligent manner such that potential drift trajectories can be intercepted. The closest tug vessel will then intercept the drifting oil tanker before it runs aground (Eide et al., 2007).

The number of oil tanker transits off the northern coastline of Norway is predicted to increase rapidly in the coming years (IMR, 2010). In addition, the number of patrolling tug vessels may increase as a response to the increase in oil tanker traffic. Consequently, the VTS operators' task of manually commanding the fleet of patrol tugs is becoming unmanageable without the aid of a decision support tool. Addressing this need, Bye et al. (2010) and Bye (2012) used a heuristic and suboptimal receding horizon genetic algorithm (RHGA) to dynamically allocate patrolling tug vessels to oil tankers along the northern coastline of Norway. Our aim here is to present a receding horizon mixed integer programming (RHMIP) model to optimally solve the same fleet optimization problem. The remainder of this paper is organized as follows: Section 2 explicitly describes the problem whereas Section 3 presents a methodology for the solution. Section 4 reports some computational experiments. Finally, discussions and propositions for future research are made in Section 5.

2 PROBLEM DESCRIPTION

Oil tankers move, by law, along piecewise-linear corridors well defined in advance and approximately parallel to the coastline. We adopt the problem description used in Bye (2012) and Bye et al. (2010). Accordingly, we assume a set C of oil tankers moving in one dimension along a line of motion z. Moreover, we assume a set P of tug vessels moving along a line of motion y parallel to z and close to shore. An illustration of the problem is presented in Figure 1, where patrol tug vessels are represented as black circles, oil tankers as white cycles, predicted oil tanker positions as dashed circles, and cycles with a cross represent points where the predicted drift trajectories cross the patrol line y. We refer to these points as cross points.

[1]Information provided by the operators at the VTS center in vardø.

Based on real-time information of oil tankers from the AIS and on a set of forecasting models developed previously by the NCA and partners, we assume that it is possible to predict future oil tankers positions as well as the corresponding potential drift trajectories.

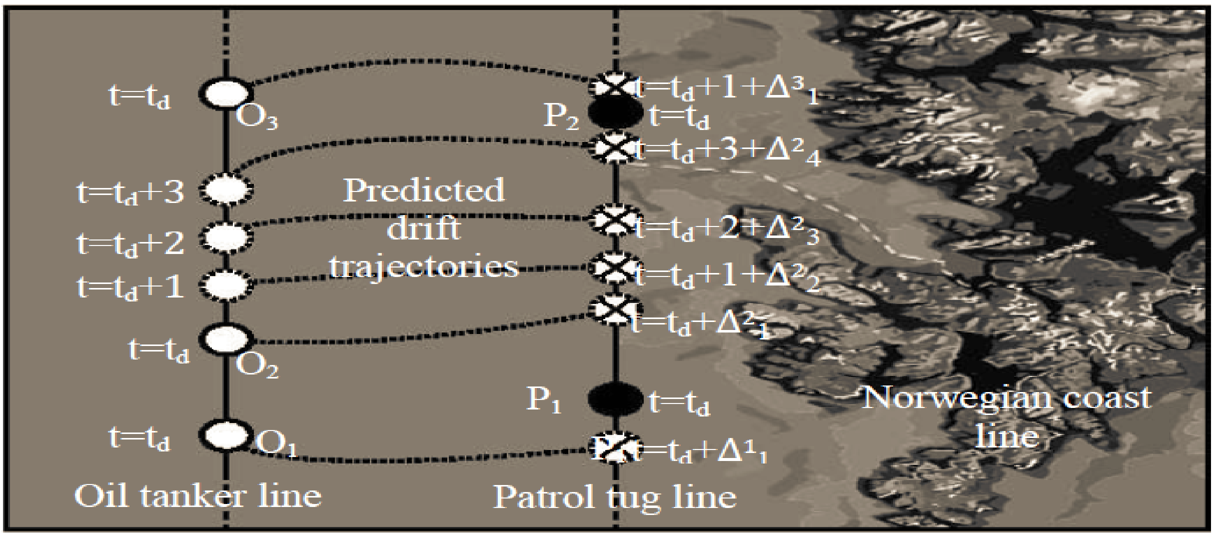


Figure 1: Problem illustration

Specifically, for each current position of a given oil tanker $z(t)$, there is a corresponding predicted drift trajectory which crosses the line y at a cross point $y(t^{'})$ where $t^{'} - t = \Delta$ represents the estimated drift time. Thus, the main goal is to make sure there is always a tug vessel at a position $y^{'}(t^{'})$ close enough to any potential cross point $y(t^{'})$ to rescue the drifting oil tanker. That is, what is the optimal positioning of tug vessels along the coastline for a minimum rescue time of potentially drifting oil tankers?

3 METHODOLOGY

Task allocation in real-time systems in order to meet certain deadlines is known to be an NP-hard problem (Gertphol and Prasanna, 2003). In addition, the highly uncertain weather conditions and the dynamic environment add to the complexity of the problem. To overcome these challenges, we propose using a combination of different methods that complement each other. An iterative solution approach for different types of problems that integrate optimization and simulation methodologies have been developed by several researchers in the literature (Acar et al. 2009). Here, we make use of the receding horizon control principle together with a linear optimization approach to develop our novel RHMIP model. Whilst this approach can be used to solve the specific problem presented in this paper, our model can likely be extended to solve other problems such as dynamic fleet optimization of platform supply vessels (PSVs) or other resource allocation problems both offshore and on land.

Model predictive control (MPC) or receding horizon control (RHC) is a class of control algorithms that uses explicit process models to predict the future response of a system and guide a system to a desired output using optimization as an intermediate step (Park et al., 2009). Receding horizon optimization is widely recognized as a highly practical approach with high performance (Zheng et al., 2011). It has become a very successful strategy in real-time control problems (Goodwin et al., 2006). Morari and Lee (1999) showed that many important practical and theoretical problems can be formulated in the RHC framework. The RHC algorithm consists of two main steps: (1) prediction of future system behavior on the basis of current measurements and a system model and (2) solution of an optimization problem for determining future values of the manipulated variables, subject to constraints (Wang et al., 2007). For a given planning time horizon T, with step $k \geq 0$ corresponding to the time instant $k\lambda$ with λ the sampling time, the future control sequence $\mu(k), \mu(k+1), ..., \mu(k+T-1)$ is computed by solving a discrete-time optimization problem over the period $[k\lambda, k+T\lambda)$ in a way that a performance index defined over the considered period is optimized subject to some operational constraints. For our problem, tug vessels are constrained to move no faster than their maximal speed, which leads to a limitation on the number of oil tankers allocated to a given tug vessel. Once the optimal control sequence is computed, only the first control sample is implemented, and then the horizon is shifted. Subsequently, the new state of the system is estimated, and a new optimization problem at time $k+1$ is solved using this new information (Tarău et al. 2011). In effect, the RHC principle introduces feedback control, and thus robustness to changes in the environment.

Mixed-integer linear programming (MIP) problems are optimization problems with a linear objective function, subject to linear equality and inequality constraints and where some variables are constrained to be integers. The advantage of using this approach is the availability of efficient solvers that can compute the global optimal solution within reasonable time (Tarău et al., 2011).

Despite the high uncertainty related to weather, wave heights and ocean currents, we have decided to develop a deterministic MIP model. This decision is justified by the fact that the model is run dynamically and parameters are updated at every time step, thus implicitly handling the stochasticity of the problem.

3.1 RHMIP model

Previous work done by Bye (2012) and Bye et al. 2010) aimed to reduce the distances between all cross points and the nearest patrol points in the planning horizon (which is equivalent to minimizing rescue time if all patrol tugs have the same maximal speed). Indeed, this is a logical choice that tries to maximize the number of oil tankers that can be successfully assisted. The following cost function was used as a minimization objective:

$$\sum_{t=t_d}^{t_d+T} \sum_{c=1}^{N_0} \sum_{p \in P} \min |y_t^c - y_t^p|$$

Here, y_t^c represents the cross point of the drift trajectory of oil tanker c at time t , y_t^p is the position of patrol tug p at time t ; N_0 is the number of oil tankers, P is the set of patrol tugs, and T is the planning horizon. The above cost function can be rewritten as a linear cost function by adding some extra variables and can therefore be solved optimally using linear MIP.

In this study, we implement two variants of our model, one using static tug vessels (Static MIP) and the other with dynamic tug vessels (RHMIP). The definition of sets, parameters and variables are as follows.

Sets

$\mathcal{P}$	set of tug vessels
$\mathcal{C}$	set of oil tankers

Parameters

v_{max}^p	maximal speed of tug vessel p
v_{max}^c	maximal speed of oil tanker c
y_t^c	cross point of the cth oil tanker's predicted drift trajectory at time t. Note that t represent the time at which the drifting oil tanker crosses the patrol line
y_0^p	initial position of tug vessel p
y_0^p	initial position of oil tanker c
Δ_{min}	drift time of oil tankers
λ	length of each time period
T_{RHMIP}	number of simulation steps
T	length of the planning horizon of the MIP optimization model
M	a large number (See constraints (4) and (9).)

Decision variables

Y_t^p	position of tug vessel p in period t
I_t^p, J_t^p	direction of tug vessel p in period t. If $I_t^p - J_t^p > 0$ the tug vessel will move forward and backward if $I_t^p - J_t^p < 0$, otherwise it will remain static.
Ψ_t^{cp}	distance from potentially drifting oil tanker c to its allocated tug vessel p in period t. Specifically, t represent the time period where the potentially drifting oil tanker c cross the patrol line.
W_t^{cp}	1 if tug vessel is allocated to oil tanker in period t, 0 otherwise

3.2 Algorithm

Below is an algorithm implementing the RHMIP model. The basic idea in Step 2 is that the maximal speed of oil tankers and tug vessels may vary over time due to changing weather conditions such as ocean currents, wave heights, and wind, or change in cargo weight after loading or unloading. In addition, some tug vessels may be unavailable due to maintenance or change of crew. Finally, an oil tanker leaving the defined protection zone should be removed from the set for the next planning period, whereas other oil tankers may enter the zone and should be included in the next planning period.

Step 1:
a- Let $t := 0$; let $y_t^p :=$ initial value $\forall p \in \mathcal{P}$
b- Compute predicted drift trajectories, cross points of oil tankers and the predicted maximal. speed of oil tankers and tug vessels.
c- Run MIP model to obtain the optimal position and allocation of patrolling tug vessels over the planning horizon $[t, t+1, ..., T]$.
d- Implement the first period of the MIP solution.
Step 2:
a- Let $t := t+1$;
$y_0^p := y_0^p + v_{max}^p t (I_{\Delta_{min}}^p - J_{\Delta_{min}}^p) \forall p \in \mathcal{P}$ (Obtain current position of tug vessels).
b- Update the predicted drift trajectories, cross points and the predicted new maximal speed of oil tankers and tug vessels. In addition, update the current number of oil tankers moving along the coastline as well as the available number of tug vessels.
c- Run MIP model to obtain the optimal position and allocation of patrolling tug vessels over the planning horizon $[t, 1+t, ..., T+t]$.
d- Implement the first period of the new MIP solution.
Step 3: Go back to Step 2 or stop if $t = T_{RHMIP} + 1$.

The MIP model is used as an optimization phase in the algorithm:

Minimize $\sum_{c \in C, p \in P, t \in \{\Delta_{min}..T\}} \Psi_t^{cp}$
Subject to

$$Y_t^p = Y_{t-1}^p + v_{max}^p \lambda (I_t^p - J_t^p) \qquad \forall p \in \mathcal{P}, t \in \{(\Delta_{min}+1)..T\} \tag{1}$$

$$I_t^p + J_t^p \leq 1 \qquad \forall p \in \mathcal{P}, \forall t \in \{\Delta_{min}..T\} \tag{2}$$

$$M(1 - W_t^{cp}) + \Psi_t^{cp} \geq y_t^c - Y_t^p \qquad \forall c \in \mathcal{C}, \forall p \in \mathcal{P}, \forall t \in \{\Delta_{min}..T\} \tag{3}$$

$$M(1 - W_t^{cp}) + \Psi_t^{cp} \geq Y_t^p - y_t^c \qquad \forall c \in \mathcal{C}, \forall p \in \mathcal{P}, \forall t \in \{\Delta_{min}..T\} \tag{4}$$

$$\sum_{p \in P} W_t^{cp} = 1 \qquad \forall c \in \mathcal{C}, \forall t \in \{\Delta_{min}..T\} \tag{5}$$

$$Y_{\Delta_{min}}^p = y_0^p + v_{max}^p \Delta_{min} (I_{\Delta_{min}}^p - J_{\Delta_{min}}^p) \qquad \forall p \in \mathcal{P} \tag{6}$$

$$Y_t^p \geq 0, I_t^p, J_t^p \in [0,1] \qquad \forall p \in \mathcal{P}, \forall t \in \{\Delta_{min}..T\} \tag{7}$$

$$W_t^{cp} \in \{0,1\}, X_t^{cp} \geq 0, Z_t^{cp} \geq 0, \Psi_t^{cp} \geq 0 \qquad \forall c \in \mathcal{C}, \forall p \in \mathcal{P}, \forall t \in \{\Delta_{min}..T\} \tag{8}$$

$$M \geq \max_{p \in P, c \in C} |y_0^p - (y_0^c + T.v_{max}^c)| \tag{9}$$

Constraints (1), (2), and (6) determine the optimal speed and direction of each tug vessel at every time period. Because there are no cross points for $t \in [0, (\Delta_{min} - 1)]$, the model determines, in constraint (6), the speeds and directions of tug vessels at these time periods for an optimal allocation in period Δ_{min} . Constraints (3) through (5) optimally allocate each oil tanker to one tug vessel, while constraints (7) and (8) define bounds on the decision variables.

For each time period of length λ in the planning horizon T, there is a predicted drift trajectory for each oil tanker which is expected to cross the patrol line after Δ_{min} periods ahead in time. Thus, there will be cross points at every time period starting from Δ_{min}. In case an oil tanker starts drifting in period t, the model gives direction and speed to its allocated tug vessel at each time period such that their distance, after min $t+\Delta_{min}$ time period, is minimized. A tug vessel in period t will be allocated to cross point(s) in period min $t+\Delta_{min}$and possibly different cross point(s) at the next time period. The result is that the tug vessels will proactively move to make sure there is enough time to rescue any drifting oil tanker.

3.3 Static MIP model

The variant of the model with static tug vessels is obtained by replacing the variable Y_t^p by y_0^p constraints (3) of the MIP model. In addition, only constraints (3), (4), (5) and (8) are kept and the rest are removed. The model is then run once for each time period in the planning horizon. This variant simply gives optimal allocation of static tug vessels to oil tankers and is only used for comparison.

4 COMPUTATIONAL SIMULATION STUDY

In this section, we present the simulation settings and results of the computational experiment used to evaluate the performance of our solution method to the tug vessels allocation problem.

4.1 Simulation settings

The mathematical models and algorithms for simulations were coded in AMPL and the MIP models were optimally solved with CPLEX 9.0. All the experiments were executed on a personal computer with an Intel Pentium IV 3.0 GHz CPU and 4.0 GB of RAM, with the operating system Microsoft Windows 7.

Notwithstanding the expected increase in oil tanker traffic along the northern coast of Norway, we have decided to use 6 oil tankers and 3 tug vessels for the simulation. These realistic numbers were provided by the NCA in 2010 and are reasonable choices for comparison with previous work done by Bye (2012).

The typical maximum speed of tug vessels in this region is about 28 km/h and the normal operating speed of oil tankers is about 18 to 26 km/h (Bye 2012). Based on this information, we conservatively chose to use a random velocity of each oil tanker generated in the interval $\pm[20,30]$(km/h), whereas a maximum velocity of ± 30 km/h was used for the tug vessels. In both cases, a positive speed denotes a northbound movement and a negative speed a southbound movement. A drift time of only 10 hours is considered fast drift, while slow drift means most tankers will not run ashore before having drifted for 20 to 30 hours (Eide et al. 2007). For this reason, Bye (2012) used a conservative estimate in the interval$[8,12]$ (hours). Note that this interval represents the possible values of Δ presented in section 2. To be even more risk averse, we decided to use a constant value of $\Delta_{min} = 8$ hours for each oil tanker in this study.

To introduce nonlinearity of drift trajectories, such as that caused by wave heights, wind, ocean currents, and oil tanker size and shape, we used the same simple formula as Bye (2012), which has no physical relation to real drift trajectories but was merely chosen for its nonlinearity:

$$y(t') = z(t) + v\sin(\frac{2\pi}{T}\Delta_{min}).$$

Hence, any oil tanker will follow an eastbound sinusoidal trajectory with period equal to T scaled by its velocity v. The initial positions of oil tankers on the z line were randomly chosen in the range $[-750,750]$(km). For simplicity in the implementation, this interval was translated to $[1000,2500]$(km) to obtain only positive values. In order to compare the dynamic RHMIP model with the static MIP model, we decided to divide the above interval into 3 equal subintervals and place one tug vessel, at a “tug base”, at the center of each segment for the static model.

The RHMIP and static MIP models were simulated for$T_{RHMIP} = 26$ hours, a duration picked somewhat randomly, although we emphasize that the models should be simulated for at least a duration long enough to allow the tug vessels to move from initially bad to good positions and thenceforth remain in good positions, where “good” and “bad” positions refer to how well the tugs collectively optimize the cost function presented above.

At every step of $\lambda = 1$, the associated MIP model was run for a planning period of $T = 24$ hours, but only the solution for the first hour was implemented. A total number of 30 scenarios were simulated. Details on the simulation settings are presented in Table 1.

Table 1: Simulation settings

Number of oil tankers	6
Random initial position (km)	$[1000,2500]$
Random velocity (km/h)	$[20,30]$
Minimal drift time Δ_{min} (hours)	8
Number of tug vessels	3
Initial tug positions	$\{1250,1750,2250\}$
Maximal velocity (km/h)	± 30
Planning horizon T (hours)	24
Simulation step length λ (hours)	1
Simulation steps T_{RHMIP} (hours)	26
Number of scenarios	30

4.2 Results

The static tug vessel policy resulted in an average total distance of 22234, with a high standard deviation of 5880. The best case scenario had a total minimal distance of 12915 while the worst case scenario had a maximal distance of 27325. Unsurprisingly, using the static policy, a considerable number of potentially drifting oil tankers will not be rescued even at a maximal speed of the nearest tug vessel. However, the developed static MIP model can at least provide an optimal allocation of tug vessels to oil tankers, which cannot easily be achieved manually or using heuristic methods. The average running time for the MIP of this model was 10 sec for each time step.

The average total distance of the RHMIP model was 7702 with a standard deviation of 2912. The best case scenario had a minimal total distance of 2989, whereas the worst case scenario had a maximal total distance of 10609. The average performance improvement in terms of mean total distance of the RHMIP solution compared to the static policy was 66%. This dynamic variant of the model had a MIP average running time of 20 min at each time step, which is in the order of two magnitudes greater than the static MIP but is still acceptable for real-time implementation, since the calculation only needs to finish before the beginning of the next hourly receding horizon control step. The results are summarized in Table 2.

The parameters of our simulations were the same as those used in Bye (2012) except for the length of the drift trajectories, where our model was implemented as a worst case analysis with the minimum of 8 hours instead of random drift times in the interval $[8,12]$ (hours). As a consequence, a few of our simulated scenarios will have a slightly higher number of cross points. Nevertheless, the results from the two studies are still comparable, since the scenarios were randomly drawn from the same population but with different random samples. Moreover, the main comparison is based on the performance improvement from the static tug vessel policy. Comparison of RHGA vs. RHMIP simulation results are presented in Table 3. The RHGA

Table 2: Simulation results

	Static	MIP	RHMIP Reduction by RHMIP
Mean	22234	7702	66%
STD	5880.7	2911.6	50.5%
Min	12915	2989	-
Max	27325	10609	-
Step time	10 sec	20 min	

and RHMIP approaches used the same planning horizon of 24 hours at the optimization step. Compared with a static policy, the RHMIP showed a 66% improvement, whereas the RHGA showed 57.5%, thus the RHMIP outperformed the RHGA by 8.5% (see Table 3).

Figure 2 highlights the difference between the MIP models. The two models were run once, with the same parameters, for $T = 24$ hour time periods. The straight lines and piecewise linear

Table 3: Improvement from static policy

	RHGA	RHMIP
Improvement	57.5%	66%

functions represent the dynamic allocation of patrolling tug vessels over the planning horizon. The cross points, starting in period eight, are represented by circles in the figure. Compared to the MIP for static tug vessels, our dynamic MIP model cleverly and optimally allocates and tracks the potential drifting oil tankers for the given fixed planning horizon.

We recall that only the first period of the MIP model solution is implemented at each step in the simulation of the RHMIP model. At every step, the parameters (current numbers of oil tankers, maximal speeds, cross points and tug vessel positions) are updated. This allows tackling the weather uncertainty at each simulation step and coping with the variation of the parameters.

Another advantage of using the receding horizon approach is that of better tug vessels allocation at each time period. In fact, if we assume a situation where the weather is stable or accurately predicted for T=24 hours planning horizon, one may be tempted to implement the entire solution planned from a single MIP optimization, which will not allow better allocation of tug vessels at each period. The RHMIP model, run in the same conditions for $T_{RHMIP} = 24$ hours, will give better allocation because the planning at each period will not be influenced by that of the previous, which is not the case in the MIP model. This is illustrated in Figure 3, where the letters A to F represent the oil tankers and columns for distances represent the sum of the distances between a tug vessel and its allocated oil tankers. The total distances demonstrate the advantage of using the RHMIP model although the weather is accurately forecasted and the parameters constant.

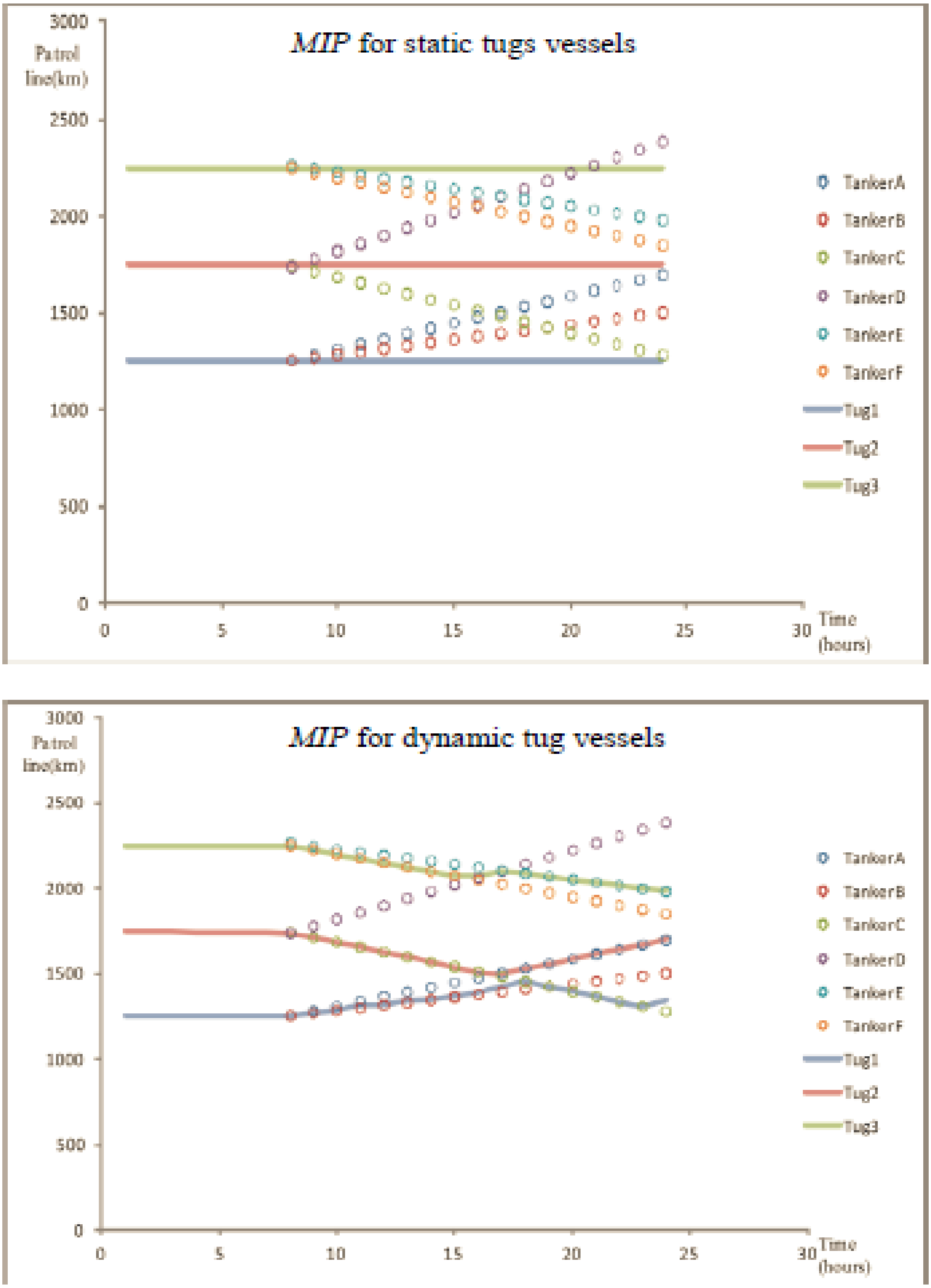
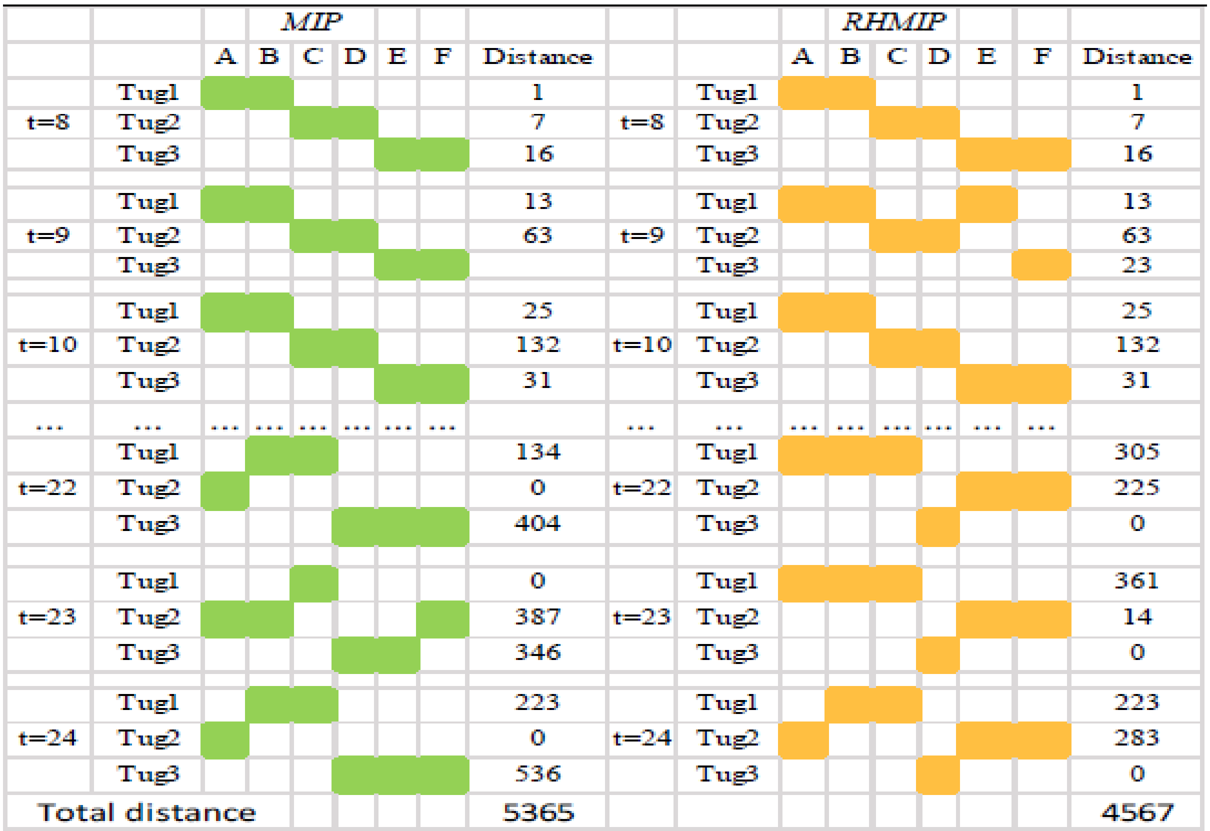


Figure 2: An illustration of employing static (top) and dynamic (bottom) tug vessels for a planning horizon of T=24 hours.



		MIP									*RHMIP*						
		A	B	C	D	E	F	Distance			A	B	C	D	E	F	Distance
	Tug1							1		Tug1							1
t=8	Tug2							7	t=8	Tug2							7
	Tug3							16		Tug3							16
	Tug1							13		Tug1							13
t=9	Tug2							63	t=9	Tug2							63
	Tug3									Tug3							23
	Tug1							25		Tug1							25
t=10	Tug2							132	t=10	Tug2							132
	Tug3							31		Tug3							31
...	...	...	...	...	...	...	...		...	...	...	...	...	...	...	...	
	Tug1							134		Tug1							305
t=22	Tug2							0	t=22	Tug2							225
	Tug3							404		Tug3							0
	Tug1							0		Tug1							361
t=23	Tug2							387	t=23	Tug2							14
	Tug3							346		Tug3							0
	Tug1							223		Tug1							223
t=24	Tug2							0	t=24	Tug2							283
	Tug3							536		Tug3							0
Total distance								5365									4567

Figure 3: Tug vessels allocations

4.3 Conclusions

The combined features of receding horizon control and mixed integer programming allow our model to optimally control tug vessels and allocate them to oil tankers in a dynamic and highly uncertain environment.

5 DISCUSSION AND FUTURE RESEARCH

Combining features from model predictive control and linear programming, this paper presents a novel sustainable model called the receding horizon mixed integer programming (RHMIP) model for optimal dynamic allocation of patrol vessels to oil tankers. Compared with previous work (Bye, 2012; Bye et al.,2010) that used a genetic algorithm to suboptimally minimize the proposed cost function, our model provides an exact (optimal) solution at every receding horizon time step. At the expense of slower computational evaluation, our optimal model outperforms the suboptimal heuristic method as well as providing a benchmark for future models.

5.1 Sustainability

International communities and government bodies such as NCA are expressing concern about the environmental impacts from shipping related activities. In fact, international shipping accounts for 2.7% of worldwide CO2 emissions (Psaraftis and Kontovas, 2013). One of the measures used, at a tactical or operational level, to address this issue is the speed reduction of ships. Accordingly, the RHMIP is a sustainable model as it explicitly reduces the speed of tugs vessels from the parameters settings and implicitly inside the model as well. Noticeably, the average operational speed of tug vessels was equal to 5 km/h for all scenarios. This is considerable slow-steaming compared to the 30 km/h maximum speed. In addition, a constraint on the maximal daily fuel consumption of tug vessels could be easily included in the model. However, limiting fuel consumption would cause a trade-off to be made between the short term CO2 emissions reduction plan with long term potential environmental impact caused by drifting oil tankers that could not be rescued on time.

5.2 Robustness

For each time period of one hour in the simulation, the initial speed of each tug vessel is determined by the MIP model and a tug vessel is supposed to move with the same speed through the whole time period. However, the wave heights, ocean currents and other factors may also affect the speed of the related tug vessel, thus causing deviations from the predicted future position of the tugs. This problem is overcome by the receding horizon control strategy, which at every planning interval will take into consideration the very latest current information about tug and tanker positions as well as updated weather forecasts. In addition, some tug vessels may not be available for some time periods due to maintenance or other possible reasons. It will be interesting to run the model with a variable number of tug vessels in the planning horizon.

The consequences of accidents will likely depend on the type and characteristics of oil tankers as well as the place or zone of accident in the coastline. Identifying the high risk zone and weighing the oil tankers will be of great benefit and can be easily included in the model.

Simulations with very large test instance size may highly increase the computational time. But one way of handling this issue is to subdivide the problem into small reasonable sizes. That is, the coastline can be divided into a few numbers of zones and each group of tug vessels will then patrol along its allocated coastline zone.

5.3 Future research

Although oil tankers are required by law to sail along predetermined piecewise linear corridors parallel to the coastline, more research can be done on a 2D dimensions. This paper aimed to minimize the distance between potential drifting oil tankers and their respective allocated tug vessels. Future research may be focused on other optimizations objective. For instance, one could decide to reduce the probability of an oil tanker running ashore. This can be achieved with probabilistic models or robust optimizations. The development of oil and gas fields in the Barents Sea will considerably increase the number of oil tankers transits along the coastline in the next $10-15$ years (Bye, 2012). Further research could be conducted to determine the optimal number of required tug vessels as well as deciding whether the vessels should be homogeneous or heterogeneous.

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Paper 2

Preventing Environmental Disasters from Grounding Accidents: A Case Study of Tugboat Positioning along the Norwegian Coast

Preventing Environmental Disasters from Grounding Accidents: A Case Study of Tugboat Positioning along the Norwegian Coast

Brice Assimizele[a,d], Johannes O Royset[b], Robin T. Bye[c], and Johan Oppen[d]

[a]Faculty of Maritime Technology and Operations, Norwegian University of Science and Technology, Ålesund, Norway

[b]Operations Research Department, Naval Postgraduate School, Monterey, California, USA

[c]Faculty of Engineering and Natural Sciences, Norwegian University of Science and Technology, Ålesund, Norway

[d]Department of Logistics, Molde University College, Molde, Norway

Abstract

An important task of operators in Norwegian vessel traffic services (VTS) centers is to cleverly position tugboats before potential vessel distress calls. Here, we formulate a nonlinear binary-integer program, integrated in a receding horizon control algorithm, that minimizes the expected cost of grounding accidents by positioning tugboats optimally under uncertainty about vessel incidents and environmental conditions. Linearizations of the model lead to easy-to-compute bounds on the optimal value. Numerical experiments with real-world data demonstrate significant reduction in the expected cost, suggesting that the model can be used as a decision-support tool at VTS centers.

Keywords: OR in maritime industry; Mixed Integer Programming; Search and Rescue; Oil Spill

1 Introduction

During the last decades marine transportation of crude oil and petroleum products has increased considerably as well as its associated risk to the environment. Although accidental oil spills from tankers are relatively rare (Goerlandt and Montewka, 2014), oil transport remains one of the main concerns for various stakeholders in the protection of the marine environment (Dalton and Jin, 2010). Indeed, oil spills can result in severe consequences to the marine ecosystem (Lecklin et al., 2011). In addition, there are high socioeconomic costs, clean-up costs, and even possibility of loss of life. Most of the large oil spills are related to grounding and collision accidents of oil tankers (Vanem et al., 2008). About one third of commercial ship accidents are caused by ship grounding accidents (ITOPF, 2013).

In Norway, several hundred oil tankers travel each year along the northern coastline. To monitor this traffic, the Norwegian Coastal Administration (NCA) operates a center for vessel traffic services (VTS) in the town of Vardø in northeastern Norway. The VTS center is responsible for the coastline from the Russian border in the Barents Sea to Rørvik near Trondheim, a distance of more than 600 nautical miles. The region is environmentally sensitive due to important fisheries and increasing tourism. About 200 vessels are monitored daily by the VTS center of which five to six oil tankers receive special attention due to their size or risk of pollution. These tankers are required by law to sail along a predefined corridor about 27 nautical miles from the coast. The VTS center operates a fleet of two tugboats with the purpose to intercept any vessel that lose steering, propulsion, or power, and drift towards land. Dynamic information (e.g., position, heading, speed over ground, rate of turn) and static information (e.g., identity, dimensions, cargo, flag) of the ships entering the region are obtained every two seconds on average through the Automatic Identification System (AIS). In addition to AIS information, weather forecast, real-time measurements of ocean currents, wave height, and wind are available to predict drift trajectories. At any time, an oil tanker moving in the region may lose its maneuverability, e.g., through steering or propulsion failure. Thus, the tugboats have to be sufficiently close to hook-up with any drifting oil tanker before it runs ashore.

The increasing oil tanker traffic in the High North makes it difficult for the VTS operators to dynamically position the tugboats to locations where they can be the most effective. In an effort to improve the positioning of tugboats, the authors were invited by the NCA to visit the VTS center in Vardø and suggest improvements in the process. This paper reports models for the optimal tugboat positioning (OTP) problem as well as computational results obtained after the visit and subsequent meetings and exchanges of information with the VTS center and the NCA representatives. NCA is currently evaluating the possibility to implement the models in a decision support system operating at the VTS. This paper is the first to formulate the OTP problem and demonstrate the benefits from its solution using historical events.

The remainder of the paper is organized as follows. In Section 2, we introduce the related literature on emergency response and safety organization. In Section 3, formulate a nonlinear binary integer program (BIP) for the OTP problem with two linearizations. Section 4 gives methods for obtaining input data to the models, and Section 5 presents computational experiments. The paper ends with conclusions and a discussion of further research in Section 6.

2 Related Literature

We present a review on general resource location/allocation and patrol routing problems, where safety organization and emergency response systems are the primary concern, followed by specific literature on the OTP problem.

General models on resource allocation and patrol routing problems in the literature include p-median problems (p-MP) (Campbell, 1996; Church and ReVelle, 1976; Church et al., 2004; Ishfaq and Sox, 2010), p-center problems (p-CP) (Davidović et al., 2011; Drezner, 1984; Espejo et al., 2015; Suzuki and Drezner, 1996), covering problems that are categorized into maximal covering location problems (MCLP) (Balcik and Beamon, 2008; Church and ReVelle, 1974; Davari et al., 2011), set covering problems (SCP) (Badri et al., 1998; Beasley and Jørnsten, 1992; Caprara et al., 2000), maximum coverage patrol routing problems (MCPRP) (Capar et al., 2015; Dewil et al., 2015; Keskin et al., 2012; Li and Keskin, 2013) and police districting problems (PDP) (Camacho-Collados and Liberatore, 2015; D'Amico et al., 2002).

The OTP problem closely relates to maritime search and rescue (SAR) operations. SAR operations consist of search for missing or distressed vessels followed by their rescue. Basdemir (2004) proposes an MCLP that allocates SAR helicopters to candidate bases to satisfy predefined incidents regions. A combination of p-MP and p-CP models are used in Dawson et al. (2007) to determine the locations of security teams over a geographic area to maintain security for the United States air force intercontinental ballistic missile systems. The combined model minimizes both the distance traveled and the maximum distance from any missile site to required security forces. An optimization and simulation method is used in Afshartous et al. (2009) to determine the locations of the United States coast guard air stations to respond to emergency distress calls. They model the problem as a p-Uncapacitated Facility Location Problem (p-UFLP). The authors assume the demand for each client to be equal and served with a single resource. A similar problem is presented in Razi and Karatas (2016), but the demand for each incident varies and each demand can be covered from multiple resources. Radovilsky and Koermer (2007) develop an integer linear programming model to optimally allocate small boats to the United States coast guard (USCG) stations. Their objective function minimizes the shortage or excess capacity at the stations. An improved formulation called boat allocation tool (BAT) is developed by Wagner and Radovilsky (2012), but do not consider actual locations of incidents and the corresponding response time. Chircop et al. (2013) address the fleet sizing problem faced by the Royal Australian Navy (RAN) with a column generation algorithm incorporated into a branch-and-price framework. A fleet patrol boats should be able to provide complete coverage of a set of specified patrol regions. Moreover, Millar and Russell (2012) develop a binary integer programming model (BIP) for the fisheries surveillance patrol routing problem in the Canadian Atlantic offshore groundfish fishery. The primary goal of the fisheries patrol routing problem is to maximize the deterrent effect of a patrol vessel through routing over a network of fishing grounds. They are the first to formulate this problem, which relates to the selective traveling salesman problem, where the fishing grounds represent the cities, and all or a subset of grounds is visited on a given trip. Their model, however, focuses more on scheduling than boat positioning. Pelot et al. (2015) categorize SAR boats based on their capabilities and use historical incident data to solve the allocation problem for the Canadian coast guard. In their study, incidents are classified based on their severity and a response time requirement is established for each type. Similarly, Eide et al. (2007) develop a dynamic risk model that prioritize oil tankers based on their potential oil spill volume in case of grounding accidents and subdivide the northern Norwegian coastline in segments, where each segment has an associated risk level. The model estimates the environmental risk of a drift grounding accident occurring with a specific tanker, at a given location, and under current weather conditions. Drift trajectories with high risk can then be prioritized in the planning of tugboat positions. Abi-Zeid and Frost (2005) develop a geographic decision support tool (SRAPlan) based on search theory to assist the Canadian forces in the planning of search missions for missing aircrafts. A similar system is also developed for the Polish SAR teams (Wysokiński et al., 2014).

All these tools and models, despites their importance, do not suggest how and where the fleet of tugboats should move in order to minimize risk. Razi and Karatas (2016) on the other hand develop a tactical model for determining the optimal placement of SAR boats, however their model do not account for uncertainty related to vessel incidents and the dynamic nature of the SAR resources positioning, which are the primary concern of the OTP problem. To determine the optimal positions of tugboats in real time, researchers have developed methods both including genetic algorithms (Bye, 2012; Bye and Schaathun, 2014, 2015a,b) and an MIP model (Assimizele et al., 2013). Their algorithms and models assume oil tankers move along piecewise-linear corridors and approximately parallel to the coastline. Additionally, their proposed set of objective functions focuses mainly on the minimization of distances between future tugboat positions and locations where hook-up with a drifting vessel might be possible. However, in situations with multiple tugboats, sum of distances are not effective surrogates for the probability of successful hook-up between a tugboat and a vessel and the cost associated with failure to do so. In addition, a distance minimization will not capture the different consequences associated with each vessel type and grounding location. Moreover, a one-dimensional modeling approach also has less flexibility in geographical positioning. All these weaknesses are addressed in this paper with a two-dimensional nonlinear binary integer programming model.

In contrast to the optimization of SAR and related operations (see for example Alpern and Gal (2002); Pietz and Royset (2014); Royset and Sato (2010); Shechter et al. (2015); Stone et al. (2016) and references therein), where search for a vessel is a central aspect, operators in the present context know the location of vessels. In an OTP problem, a tanker in distress has a known current location due to the continuously transmitted AIS information. That is, the OTP problem is primarily a rescue mission. However, uncertainty about which vessel will need assistance and the subsequent drift trajectories and weather conditions add complexity to the process of planning current and future tugboat positions. Thus, our aim is to assist the VTS operators by developing a nonlinear BIP model integrated in a receding horizon control algorithm that minimizes the expected environmental cost associated with grounding vessels and utilizes a two-dimensional discretization of the coastal zone. Royset and Sato (2010) adopt a similar two-dimensional modeling approach by subdividing the region of interest into a finite set of cells in a discrete-time route-optimization problem, where searchers seek to detect randomly moving targets.

3 Model Formulation

We subdivide the High North region controlled by the VTS center in Vardø into a finite number of cells $\mathcal{C} = \{1,...,C\}$ and discretize the planning horizon into a finite set of time periods $\mathcal{T} = \{0,1,...,T\}$. Each vessel (oil tanker) in the set $\mathcal{V} = \{1,...,V\}$ occupies one cell at each time period and can move to any reachable cell in a time period depending on its speed, which is influenced by the weather conditions.

Every vessel $v \in \mathcal{V}$ is associated with a family of possible paths and times when it might become in distress. A path $p = (c_1, c_2, ..., c_T)$, $c_t \in \mathcal{C}$, is a sequence of cells representing the trajectory of the vessel over time. The pair $\omega_v = (t, p)$ gives a vessel scenario for vessel v, where p is the associated path and $t \in \mathcal{T} \cup \{T+1\}$ is the time the VTS center is alerted to the distress of vessel v. Typically, the vessel reports steering failure, loss of propulsion, and other issues through AIS. However, sometimes incidents go unreported and the VTS center might simply observe a change in heading and speed. We let Ω_v be the set of scenarios for vessel v. Typically, each vessel has a scenario (t, p) where the path p is the planned trajectory of the vessel in the absence of failure. In this case, there will be no distress call and the time t is set to $T+1$. In addition, we define $\bar{\omega} = (\omega_1, ..., \omega_V) \in \bar{\Omega}$, where $\bar{\Omega} = \Omega_1 \times .... \times \Omega_V$ is the collection of all scenarios.

Let $\mathcal{G} = \{1,...,G\}$ be the set of tugboats operated by the VTS center in Vardø. At the beginning of the planning horizon, each tugboat $g \in \mathcal{G}$ is positioned at cell $c_{0g} \in \mathcal{C}$. The tugboats can transit between reachable cells each time period. Specifically, let $\mathcal{F}_{tg}(c) \subset \mathcal{C}$ be the set of cells that are adjacent to $c \in \mathcal{C}$ in period t for tugboat g. That is, the set of cells reachable from cell c in one time period by tugboat g. The set $\mathcal{F}_{tg}(c)$ depends on the weather conditions in time period t and the maximum speed for tugboat g. Additionally, the fleet of tugboats are not allowed to move far away from the coastline because of a secondary escort mission; some of the ships in transit to ports located in the north of Norway need to be escorted by tugboats. This secondary task does not influence the model we develop as the available number of tugboats at each time period in the planning horizon is known well in advance.

A vessel might start drifting at any time period with a certain probability, which depends on internal factors of the vessel as well as the weather conditions (e.g., ocean current, wave height). Moreover, the path followed while drifting is also determined by environmental factors. In Section 4, we give details about how the specifics of a scenario can be computed. We let R_{ω_v} be the probability of scenario ω_v for vessel v. We assume that the probabilities of failure through steering or propulsion are independent between vessels. Although this assumption might not always be reasonable, here we justify it by the fact that vessels in distress are usually spatially separated with few common environmental factors. Hence, the probability for a scenario $\bar{\omega}$ is given by $R_{\bar{\omega}} = \prod_{v \in \mathcal{V}} R_{\omega_v}$. A critical component is a tugboat's ability to hook-up with a vessel that is drifting next to it. We let the probability of successful hook-up by tugboat g with vessel v, given vessel v follows scenario $\omega_v = (t, p)$ and tugboat g is in cell c at time of distress call t, be denoted by $Q_{gc\omega_v}$.

The aim is to move tugboats between cells in such a way that the expected cost of ship grounding accidents is minimized. Let K_{ω_v} be the grounding cost associated with vessel scenario $\omega_v = (t, p)$, i.e., the cost for a vessel following path p and no tugboat manages to hook-up with the drifting vessel. We note that K_{ω_v} is a deterministic quantity, but it is trivial to account for uncertainty in the cost by defining additional vessel scenarios. The cost mostly depends on the grounding location as well as the type and volume of the oil spill. This cost is equal to zero for vessel scenarios having $t = T + 1$, i.e., no failure occurs and the vessel follows a normal route in the corridor. We define x_{gct} as a binary variable that takes the value 1 if tugboat g is in cell c at time t, and 0 otherwise. For a tugboat g in cell c at time of distress t, the probability of not being able to hook-up with vessel v following scenario $\omega_v = (t, p)$ is $1 - Q_{gc\omega_v}$. Thus, the probability that no tugboat rescues vessel v if vessel scenario ω_v occurs equals

$$\prod_{g \in \mathcal{G}, c \in \mathcal{C}} (1 - Q_{gc\omega_v})^{x_{gct}}.$$

Note that subscript t in the variable x_{gct} is the time of distress in scenario $\omega_v = (t, p)$ and the probability of hook-up $Q_{gc\omega_v}$ is relative to that time. Then, the expected grounding cost for scenario $\bar{\omega}$ equal

$$\sum_{v \in \mathcal{V}} K_{\omega_v} \prod_{g \in \mathcal{G}, c \in \mathcal{C}} (1 - Q_{gc\omega_v})^{x_{gct}}. \tag{1}$$

The expected total cost across all scenarios follows as

$$\sum_{\bar{\omega} = (\omega_1, \ldots, \omega_V) \in \bar{\Omega}} R_{\bar{\omega}} \sum_{v \in \mathcal{V}} K_{\omega_v} \prod_{g \in \mathcal{G}, c \in \mathcal{C}} (1 - Q_{gc\omega_v})^{x_{gct}}, \tag{2}$$

which we denote by $f(\mathbf{x})$, where $\mathbf{x}$ is the vector with components x_{gct}. Let $\alpha_{gc\omega_v} = -\ln(1 - Q_{gc\omega_v})$ be the *hook-up rate*. The function f can be equivalently written as:

$$f(\mathbf{x}) = \sum_{\bar{\omega} = (\omega_1, \ldots, \omega_V) \in \bar{\Omega}} R_{\bar{\omega}} \sum_{v \in \mathcal{V}} K_{\omega_v} \exp\left(- \sum_{g \in \mathcal{G}} \sum_{c \in \mathcal{C}} \alpha_{gc\omega_v} x_{gct} \right). \tag{3}$$

Since costs are nonnegative, f is a convex function. In fact, the exponential function is convex and f is a sum of exponential functions. In the following subsections, we formulate the OTP problem as a nonlinear BIP and give linear approximations.

3.1 OTP Model

A nonlinear BIP model is developed next to minimize the objective function f in (3) subject to operational constraints.

OTP model:

Indices

t	time period
$c\ c'\ c_t$	cells
v	vessel
g	tugboat
p	path $p = (c_1, ..., c_T)$
ω_v	scenario for vessel v; $\omega_v = (t, p)$
$\bar{\omega}$	scenario for all vessels $\bar{\omega} = (\omega_1, ..., \omega_V)$

Sets

$\mathcal{C}$	set of cells $\mathcal{C} = \{1, ..., C\}$
$\mathcal{F}_{tg}(c) \subseteq \mathcal{C}$	set of cells reachable from cell c in period t for tugboat g
$\mathcal{V}$	set of vessels $\mathcal{V} = \{1, ..., V\}$
$\mathcal{G}$	set of tugboats $\mathcal{G} = \{1, ..., G\}$
$\mathcal{T}$	set of time periods $\mathcal{T} = \{0, 1, ..., T\}$
Ω_v	set of scenarios for vessel v
$\bar{\Omega}$	set of all possible scenarios $\bar{\Omega} = \Omega_1 \times .... \times \Omega_V$

Parameters

K_{ω_v}	grounding cost for vessel v in scenario $\omega_v = (t, p)$
R_{ω_v}	probability for vessel scenario $\omega_v = (t, p)$
$R_{\bar{\omega}}$	probability for scenario $\bar{\omega} = (\omega_1, ..., \omega_V)$, $R_{\bar{\omega}} = \prod_{v \in \mathcal{V}} R_{\omega_v}$
$Q_{gc\omega_v}$	probability of successful hook-up by tugboat g with vessel v, given tugboat g is in cell c at time of distress call t and vessel v follows scenario $\omega_v = (t, p)$
$\alpha_{gc\omega_v}$	hook-up rate with vessel v for tugboat g in cell c under scenario ω_v, $\alpha_{gc\omega_v} = -\ln(1 - Q_{gc\omega_v})$

Variables

x_{gct}	binary variable taking the value 1 if tugboat g is in cell c at time t, 0 otherwise

Formulation

$$\min f(\mathbf{x})$$

s.t.

$$\sum_{c\in\mathcal{F}_{tg}(c')} x_{gct-1} \geq x_{gc't} \qquad \forall g \in \mathcal{G}, \forall c' \in \mathcal{C}, \forall t \in \mathcal{T}\backslash\{0\} \tag{4}$$

$$\sum_{c\in\mathcal{C}} x_{gct} = 1 \qquad \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \tag{5}$$

$$x_{g,c_{0g},0} = 1 \qquad \forall g \in \mathcal{G} \tag{6}$$

$$x_{gct} \in \{0,1\} \qquad \forall g \in \mathcal{G}, \forall c \in \mathcal{C}, \forall t \in \mathcal{T} \tag{7}$$

Constraints (4) ensure tugboats move only between reachable cells. In addition, constraints (5) make sure tugboats are not located in more than one cell in any time period. Constraints (6) give initial positions of tugboats. That is, cell c_{0g} is the position of tugboat g at the beginning of the planning horizon.

3.2 Linearization of the Objective Function

Since a direct solution of the OTP model might be computationally costly, we develop two approaches to linearize the objective function f and obtain two resulting mixed-integer linear programming models MIP-L and MIP-U. A viable alternative could be the continuous relaxation (Branch-and-bound) to the convex problem using standard mixed-integer nonlinear program (MINLP) solvers. However, Royset and Sato, (2010) use a similar nonlinear convex exponential function to address the discrete-time route-optimization problem. They present two solutions approaches, one based on the cutting-plane (linearization) method and the other on continuous relaxation of the objective function. The cutting-plane approach, compared with the existing branch-and-bound algorithm (fail to find solutions), is able to solve many realistically sized problems instances in few minutes. Their specialized cut improves the solution time by about 50% and further reduce the solution time with about two orders of magnitude. In their study, standard MINLP solvers , Bonmin and DICOPT, have higher solution time compared to CPLEX with the linearized model. Moreover, the effective cut-building technology in mixed-integer linear program (MIP) are not available in the MINLP solvers.

3.2.1 Lower Linearization: MIP-L

New nonnegative variables z_{ω_v} are included to remove the nonlinearity in the objective function through the standard lower-bounding approximation (Ramos, 2007; Royset and Sato, 2010)

$$\exp(-y) \geq \max_{k\in\mathcal{K}}\{\exp(-y_k) - \exp(-y_k)(y - y_k)\} \ \forall y, y_k \in \mathbb{R}, k \in \mathcal{K}.$$

Accordingly, let $Y_{\omega_v k} \in \mathbb{R}$, $k \in \mathcal{K}$, where $\mathcal{K}$ represents the set of breakpoints. The resulting mixed-integer linear model (MIP-L) takes the following form.

Model MIP-L:

Additional Set

$\mathcal{K}$ — set of breakpoints $\mathcal{K} = \{1, ..., K\}$

Additional Parameters

$Y_{\omega_v k}$ — breakpoint number k, $k \in \mathcal{K}$ for vessel scenario ω_v

Additional Variables

z_{ω_v} — nonnegative variable used for linearization for vessel scenario ω_v

Formulation

$$\min \sum_{\bar{\omega}=(\omega_1,...,\omega_V)\in\bar{\Omega}} R_{\bar{\omega}} \sum_{v\in\mathcal{V}} K_{\omega_v} z_{\omega_v}$$

s.t.

(4)-(7) and

$$\exp(-Y_{\omega_v,k}) - \exp(-Y_{\omega_v,k})\left(\sum_{g\in\mathcal{G}}\sum_{c\in\mathcal{C}} \alpha_{gc\omega_v} x_{gct} - Y_{\omega_v,k}\right) \leq z_{\omega_v}\ \forall \omega_v = (t,p) \in \Omega_v, \forall v \in \mathcal{V},\ \forall k \in \mathcal{K}$$

$$z_{\omega_v} \geq 0 \quad \forall \omega_v = (t,p) \in \Omega_v,\ \forall v \in \mathcal{V}$$

Let θ_{OPT} and $\theta_{\text{MIP-L}}$ be the optimal value for the OTP and MIP-L models, respectively. Obviously, $\theta_{\text{MIP-L}} \leq \theta_{\text{OPT}}$.

3.2.2 Upper Linearization: MIP-U

It is well known (Bazaraa et al., 1995; Lin et al., 2013) that $\exp(-y)$ can be bounded from above on $[0, y_{max}]$ by a piecewise linear function that coincides with $\exp(-y_k)$ at points $y_k \in [0, y_{max}]$, $k \in \mathcal{K}$. We apply this approach to the term $\exp(-\sum_{g\in\mathcal{G}}\sum_{c\in\mathcal{C}} \alpha_{gc\omega_v} x_{gct})$ and set $y_{max} = \sum_{g\in\mathcal{G}} \max_{c,\omega_v} \alpha_{gc\omega_v}$. The piecewise linear function is most easily represented by nonnegative auxiliary variables that sums to one. Specifically, $\sum_{k\in\mathcal{K}} \lambda_k \exp(-y_k) \geq \exp(-\sum_{k\in\mathcal{K}} \lambda_k y_k)$ for $0 \leq \sum_{k\in\mathcal{K}} \lambda_k y_k \leq y_{max}$. Using these relations, we obtain the upper-bounding model MIP-U as follows:

Model MIP-U:

Additional Parameters

$Y_{\omega_v k}$, $F_{\omega_v k} = \exp(-Y_{\omega_v k})$ function values at $k \in \mathcal{K}$

Additional Variables

$\lambda_{\omega_v k}$ nonnegative variable used for linearization

Formulation

$$\min \sum_{\bar{\omega}=(\omega_1,\dots,\omega_V)\in\bar{\Omega}} R_{\bar{\omega}} \sum_{v\in\mathcal{V}} K_{\omega_v} \left(\sum_{k\in\mathcal{K}} \lambda_{\omega_v k} F_{\omega_v k} \right)$$

s.t.

$$(4)\text{-}(7) \text{ and}$$

$$\sum_{g\in\mathcal{G}} \sum_{c\in C} \alpha_{gc\omega_v} x_{gct} = \sum_{k\in\mathcal{K}} \lambda_{\omega_v k} Y_{\omega_v k} \quad \forall \omega_v = (t,p) \in \Omega_v,\ \forall v \in \mathcal{V}$$

$$\sum_{k=1}^{K} \lambda_{\omega_v k} = 1 \quad \forall \omega_v = (t,p) \in \Omega_v,\ \forall v \in \mathcal{V}$$

$$\lambda_{\omega_v k} \geq 0 \quad \forall \omega_v \in \Omega_v,\ \forall v \in \mathcal{V},\ \forall k \in \mathcal{K}$$

The optimal value of MIP-U provides an upper bound for θ_{OPT}. However, it is obviously better to use the true objective function value. Accordingly,

$$\theta_{\text{MIP-L}} \leq \theta_{\text{OPT}} \leq \min\{f(\bar{\mathbf{x}}), f(\underline{\mathbf{x}})\},$$

where $\bar{\mathbf{x}}$ and $\underline{\mathbf{x}}$ are the vectors of optimal x_{gct} for MIP-U and MIP-L, respectively. We observe that MIP-L suffices to generate both upper and lower bounds on θ_{OPT}, but we also include MIP-U as it solves much faster than MIP-L and sometimes yields better solutions.

4 Input Parameters

In this section, we present preliminary methods to obtain accurate input parameter to the OTP model

4.1 Hook-up Probabilities

Each drifting vessel is detected by the VTS center, which in turn informs the nearest tugboat. The time needed for a tugboat to reach the drifting vessel depends on the reaction and mobilization time, sailing time from the initial tugboat position to the vessel and the time required to hook-up the vessel to the tugboat. It takes on average 2 hours to hook-up the drifting vessel with the tugboat when they are next to each other, but this can increase in bad weather conditions (Eide et al., 2007). Once the vessel is reached by the tugboat, the time, t_l, left before it runs ashore will determine the probability of successful hook-up. Recall that $Q_{gc\omega_v}$ is the probability of successful hook-up by tugboat g to vessel v, given tugboat g is in cell c at time t and vessel v follows $\omega_v = (t, p)$. For every vessel scenario and tugboat position, we determine t_l using the maximum speed of the tugboat and the location of c relative to p, and set

$$Q_{gc\omega_v} = \frac{\beta_{\omega_v} \exp\left(\delta_{\omega_v}(t_l - t_{\min})\right)}{1 + \exp\left(\delta_{\omega_v}(t_l - t_{\min})\right)}.$$

The parameter $t_{\min}$ represents the minimal remaining drift time required to attempt hook-up. If t_l is less than $t_{\min}$, $Q_{gc\omega_v}$ is set to 0. In addition, $\beta_{\omega_v} \in [0,1]$ and $\delta_{\omega_v} \geq 0$ represent the influence of weather. This model is a preliminary attempt to estimate the hook-up probabilities. Further work is needed to fit the model using empirical data from field tests and actual accidents, which is nontrivial work and beyond the scope of the paper. Nevertheless, we have included the weather and current factors in the experiments with real-world data. In fact, the maximal operational speeds of the tugboats at each cells depend on the current and wind forces as well as wave height. Moreover, the formula simply transform the time left t_l , into a value between zero and one. In addition, for every time left t_{l_1} and t_{l_2} , we have the following condition: If $t_{l_1} \leq t_{l_2}$ then $Q_{gc\omega_v}(t_{l_1}) \leq Q_{gc\omega_v}(t_{l_2})$, which is a necessary and sufficient condition to optimally move tugboats in cells with higher response time.

4.2 Drift Trajectories

The motion of a drifting vessel is entirely determined by the sum of surface and body forces acting on it (Jankowski, 1992). The forces acting on the surface are caused by the buoyancy force, the sea surface current, surface wind and the waves. The gravitational force is the only body force acting on its center of mass.

The drift caused by the wind alone is termed the object's *leeway* (Hodgins and Mak, 1995). Because of the asymmetric shape of the vessel, the drag and lift component of the wind will cause the object to drift at an angle relative to the wind called "leeway angle". The Norwegian Meteorological Institute (NMI) developed a LEEWAY model as part of its oceanic trajectory models suite for Search and Rescue, Vessel Traffic Service and Environmental Protection Agency (Breivik and Allen, 2008).

Uncertainty parameters such as leeway divergence angle is obtained through Monte Carlo simulation (Breivik and Allen, 2008) and field investigation (Allen and Plourde, 1999). Ni et al. (2010) present a theoretical drift prediction based on the law of physics and non-probabilistic analysis of uncertainty. Consider a vessel in steady drift with velocity U_B subjected to a forcing field with constant wind velocity U_W and a constant current velocity U_C. The law of motion dictates that the relative wind force $U_W - U_B$ and the leeway force $U_B - U_C$ must be opposite to each other. In addition, the sum of the two forces are equal to zero for a steady drift. Thus, the drift velocity can be expressed as follow:

$$U_B = \frac{1}{1+\tau}U_C + \frac{\tau}{1+\tau}U_W = U_C + \mu(U_W - U_C),$$

where μ is the leeway rate and $\tau^2 = (C_D A\rho)_1/(C_D A\rho)_2$. The subscripts 1 and 2 refer to the in-air and in-water quantities respectively, where C_D is the drag coefficient, A is the cross-sectional area exposed, and ρ is the fluid density. More details can be found in Ni et al. (2010). This formula does not consider the wave drift force which can be expressed by $F = \frac{1}{2}\rho_2 f_g L C_{2W} a^2$. The wave amplitude is one-half of the wave height and is represented by a, C_{2W} denotes the wave drift coefficient and L is the vessel length and f_g is the gravitational force. Hence, the drift velocity with wave force included can be expressed as follow:

$$U_B = \tilde{U}_B - \frac{\tilde{U}_B - U_C}{1-\tau} + \sqrt{\left(\frac{\tilde{U}_B - U_C}{1-\tau}\right)^2 + \frac{\chi}{1-\tau^2}}, \tag{8}$$

where $\tilde{U}_B$ is the solution of the equation of motion in the absence of waves and the parameter

$$\chi = \frac{f_g L C_{2W} a^2}{(C_D A)_2}.$$

As the region is discretized into cells, it is possible to estimate, for every time period, the next position of a drifting vessel given information on the local wind, surface current of the initial position, and the shape and buoyancy of the vessel. For every vessel and time step, we can estimate a potential drift trajectory using (8), which is represented by a sequence of cells (see Figure 1). Specifically, we compute a path for a vessel as follows.

Step 0:
Set $i := 0$ and $p := (c_t)$, where c_t represents the position of the vessel at the time of distress call $t \leq T$.

Step 1:
Obtain the wind, current velocities and wave force as well as actual vessel velocity from the AIS for the current cell c_{t+i} and set $i := i+1$.

Step 2:
Determine the new actual vessel velocity, U_B, using the formula in (8) as illustrated in Figure 1.

Step 3:
Determine in which cell falls the new obtained vessel force U_B and denote it by c_{t+i}; and set $p := (c_t, ..., c_{t+i})$.

Step 4:
Go back to Step 1 or stop if the current cell c_{t+i} is ashore or outside the region of interest.

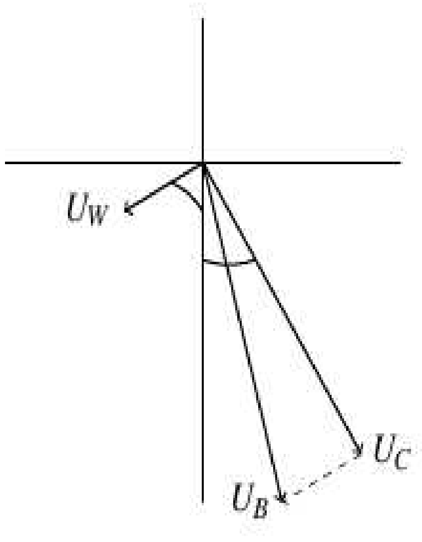


Figure 1: Drift velocity

The algorithm above does not generate the whole path followed by the vessel, but only from the time of distress call at cell c_t to shore. However, it is trivial to include the other parts of the path.

4.3 Environmental Costs and Drift Probabilities

Vessel grounding accidents can result in severe pollution from oil spills and damage to the environment. In addition, the oil spill highly depend on vessel type (Talley et al., 2012), capacity and grounding location. These consequences can be evaluated in terms of costs for a better classification of vessels and potential grounding locations.

About 34% of oil spills in European seas are caused by vessel grounding accidents (see Figure 2). One of the best known grounding-related oil spill accident is that of the Sea Empress in 1996, which ran aground in the entrance to Milford Haven, in the southwestern United Kingdom. The vessel released a total of 72,360 tonnes of oil into the sea (ITOPF, 2013).

The main factors influencing the cost of oil spills include the type of oil, amount of oil spilled and spillage rate, the physical, biological and economic characteristics of the spill location, and the weather and sea conditions at the time of the spill (Grey, 1999; Kontovas et al., 2010; Vanem et al., 2008; White and Molloy, 2003). The levels and types of cleanup capabilities to optimally respond to oil spills are outside of the scope of this paper (see Psaraftis et al. (1986) for related research in this area). Although a grounding vessel might not lose its entire cargo, the vessel size indicates the potential volume of oil spill. In addition, the amount of oil spill depends on the grounding location. A vessel running aground on hard rock will likely cause more oil spills than grounding on sand.

Vanem et al. (2008) develop a model that incorporates all costs of oil tanker spill accidents. They consider the spill amount as the major factor with a global average cleanup cost

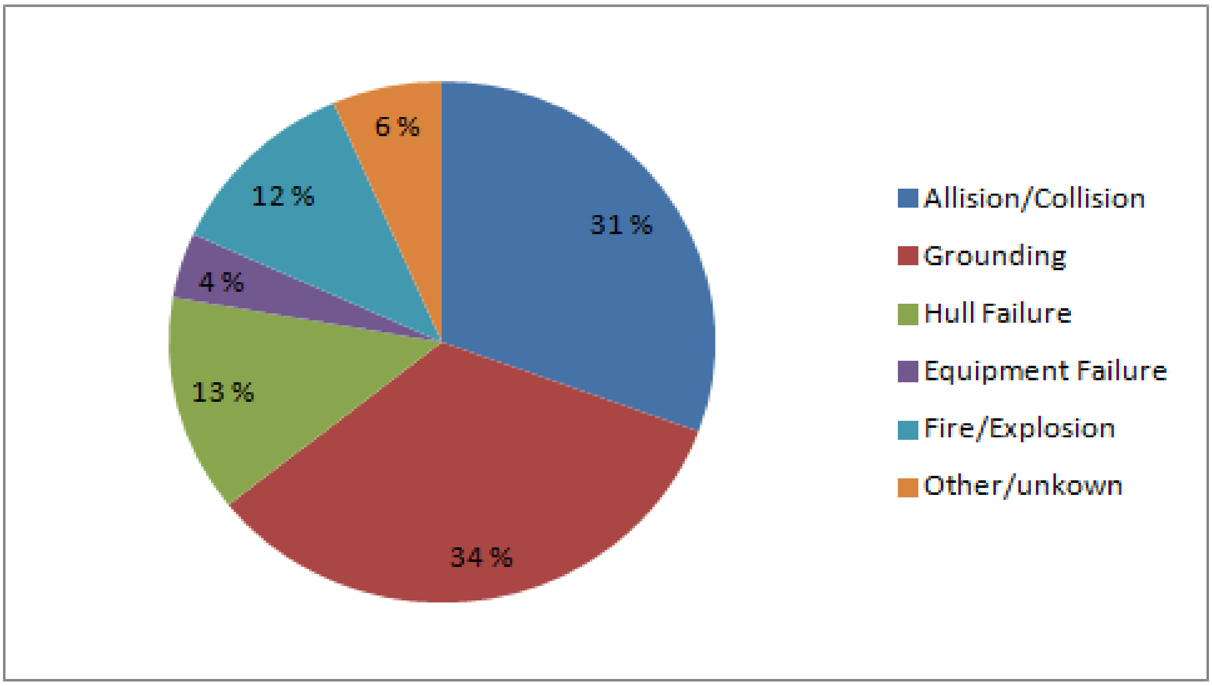


Figure 2: Volume of oil spilled per cause of accidents in European Seas for accidents above 7 tonnes per spill (Data Source: ITOPF 2013)

of USD $16,000/\text{tonne}$. In addition to the cleanup cost, average environmental damage and socioeconomic costs are estimated to be around USD 24000/tonne. Research conducted about Norwegian waters assesses the environmental damage to be almost twice the cost associated with cleanup and rescue operations (Viggo, 2003). Kontovas et al. (2010) use a regression analysis of oil spill cost with data from the IOPCF (International Oil Pollution Compensation Federation) and obtain a total cost $K = 51.432V^{0.728}$ in terms of V, the volume in tonne of oil spilled. The total cost, in USD, includes three cost categories: cleanup, socioeconomics losses and environmental costs. In this paper, the cost related to each vessel scenario in the OTP model are obtained from this total cost formula with V determined by the size of the vessel and the part of the coast it might hit as categorized by Eide et al. (2007).

In the model formulation, R_{ω_v}, with $\omega_v = (t, p)$, is the probability for vessel v to start drifting along the path p at time $t \in \mathcal{T}$. The probability mainly depends on human factors (experience of nautical officers, excessive fatigue, stress and usage of alcohol), type of vessel (size, wind exposure area of the ship, flag state, age of the vessel), weather conditions, and the characteristics of the route (length, depth and width of the waterway). Sophisticated methods, such as fault tree analysis (Kum and Sahin, 2015; Mokhtari et al., 2011; Senol et al., 2015), for determining R_{ω_v} are beyond the scope of this paper. We simply generate these probabilities randomly in the simulation experiments, based on historical information both about how often drifts actually occur on average, and also about how much the probabilities vary between vessels due to different characteristics, such as flag state, age, and previous incidents.

5 Case Studies

In this section, we discuss the effectiveness, efficiency and performance of the models through three different case studies. First, we present the general settings common to all cases. Second, an illustrative example of the model and output is presented in Case 1. Third, we compare the MIP-L and MIP-U solutions and discuss the computational costs, the number of breakpoints, the effect of time horizon on the solution quality and the sensitivity analysis with larger scale problems in Case 2. Fourth, three real-world examples with historical data from the NCA are presented in Case 3.

5.1 Computational Settings

We limit the set $\bar{\Omega}$ to those scenarios with exactly one distress call and also only consider one possible drift trajectory for each vessel in distress. The first assumption is reasonable as distress calls are quite rare. The second assumption is a simplification that places focus on the main source of uncertainty: time of distress call (Ni et al., 2010). A richer set of scenarios are easily included but its generation is beyond the scope of the paper. In Cases 1 and 2, the drift trajectories are randomly generated using a Markov chain. Specifically, we subdivide the region into 20 zones, where the sum of the wind and current force direction ($U_w + U_C$) for each zone is either north-east, north-west or north-south. Each zone is randomly associated with one of these three directions. Additionally, every cell in the region is directly connected from below with three cells, which are named Left, Straight and Right. The next cell in the path $p = (c_t, ..., c_T)$ of a drifting vessel scenario is randomly determined based on the zone where the current cell falls (see Table 1). For instance, if the current cell is located in the zone where the vector $U_w + U_C$ has a north-east direction, then the Left cell will be chosen as next cell in the path with a probability equal to 0.25. In addition, the wave magnitude at each cell is randomly given a value of 0 or 1 with equal probabilities, where the value of 0 represents low wave height and 1 that of large wave magnitude. The drifting vessel will spend two time periods at cells with large wave magnitude and only one time period at cells with small wave height. In Case 3, historical wave height, current and wind forces are used to determine each drift trajectory; see further details in Subsection 5.4.

Previous study in the Norwegian sea propose an average failure rate of 0.26 per ship-year (Hansson and Kiær, 1997). The research is not based on actual statistics but on a fault tree analysis, combining failure rates for components and expert judgment. It is not clear how these results can be implemented for different ships in a simulation experiment. Additionally, these data are old and may not be representative for today's accident scenarios. In the paper, these probabilities of failure are randomly generated with very low values at every time periods to reflect the actual scenarios. In the case studies with historical events, we have computed these probabilities based on the frequency of drift accidents but still include some randomness. Specifically, let p_s be the drifting rate for a specific period of time in the region of interest.

Then we generate the probabilities of failure, R_{ω_v}, for every vessel such that $p \in [0.01, 0.09]$ and $R_{\omega_v} = \frac{p}{p_s}, \sum R_{\omega_v} = p_s$, where $1 - p_s$ is the probability of no failure. Further challenging analytical and empirical researches are needed to effectively determine these probabilities. Nevertheless, we have done some sensitivity analysis with different distributions, which include different values of R_{ω_v}. Furthermore, the grounding cost for each vessel scenario is computed using the formula $K = 51.432V^{0.728}$, where the volume V is randomly generated according to a uniform random variable on [2187, 51704] plus a normal random variable with mean 15000 and standard deviation = 5000 (see Kontovas et al. (2010) for details on grounding costs and volume of oil spills). Moreover, the VTS center in Vardø currently operates with two tugboats. We use this number of tugboats for experiments in all the three cases. These tugboats have a secondary function of escorting vessels that are in transit to Norwegian ports. Thus, they are required to move relatively close to shore as reflected in our discretization of the area of interest and movement constraints. The problem of whether the resource capacity are optimal or not is out of the scope of this paper. We propose an optimal tugs policy based on the resource available. However, the fleet of tug was reduced from three to two a year ago by the NCA and could be justified by few main reasons. First, the accidents are very rare and most of the drift time are very long with 20 to 30 hours (slow drift) and fast drift count for only 10 hours Eide et al. (2007). Second, The NCA have acquired "ship arrestors", which considerably reduce the vessel's drifting speed and give more time to the tugs to hook-up with the vessel in time.

All computations are carried out on a personal computer with an Intel(R) Pentium(R) IV 3.0 CPU and 4.0 GB of RAM, running Windows 7. The optimization solver is Gurobi 5.5.0.

Table 1: Transition probabilities for path generation

$U_w + U_C$	North-east			North-west			North-south		
Cell	Left	Straight	Right	Left	Straight	Right	Left	Straight	Right
Prob	0.25	0.25	0.5	0.5	0.25	0.25	0.25	0.5	0.25

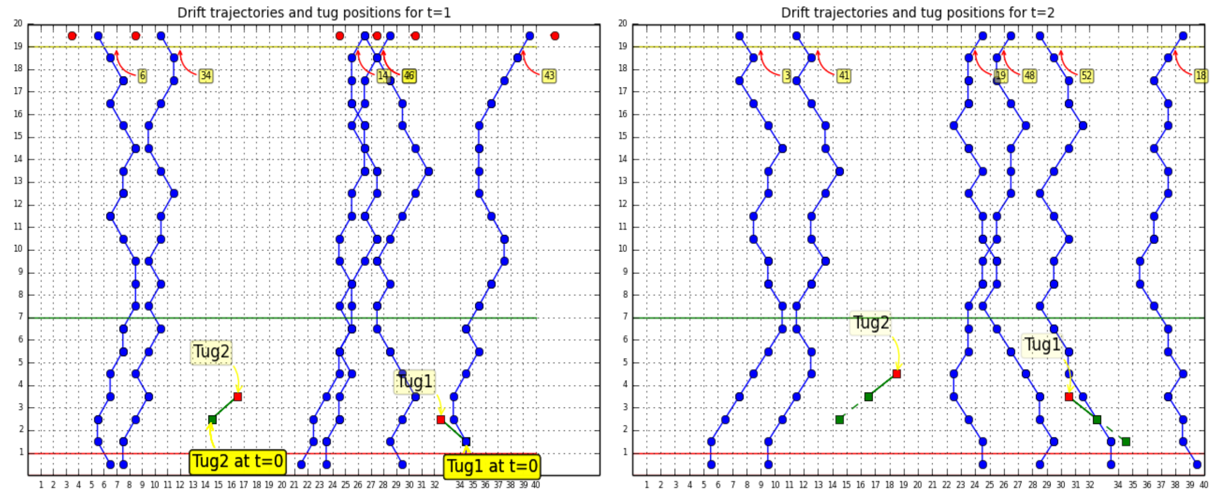
5.2 Case 1: Illustrative Example

We start by considering a corridor of length 40 kilometers and width 20 kilometers divided into 800 cells of 1-by-1 kilometer. Six vessels sail in the corridor 18 km off the coastline. These vessels are patrolled by two tugboats moving close to shore with a maximal speed of 10.7 knots (1 knot= 1.85 km/h). Each vessel moves in the corridor with an average speed between 6.5 and 19.5 knots. Moreover, a planning horizon of 6 hours is used with a time step of one hour. For this case, tugboats are constrained to move relatively slowly. Each cell c has a total of 15 reachable cells in $\mathcal{F}_{tg}(c)$, which represents the number of possible hops per time period. Figure 3 shows an optimal solution, where tugboats are represented by small squares and are allowed to move between shore and the green line. Tug2 is initially positioned 3 km off the coast and 15 km from the origin point with coordinate $(0,0)$ in the figure, while Tug1 is positioned 2 km away from the coast and 34 km from the origin point along the coast. The optimal positions of tugboats are also shown in Figure 3. In addition, the initial position of each vessel is represented by a red circle and the vessel scenarios for each time period as blue circles linked with blue sticks. Moreover, the corridor is delimited by a yellow line and the coast is shown by a red line. The cost, in million NOK, for each vessel scenario is labeled close to each drift trajectory. The number of scenarios, $|\bar{\Omega}|$, is equal to 36 for six vessels and six time periods. For every time period, Figure 3 illustrates the optimal decision of the model and the potential drift trajectories. In the figure for time period t, we only show the six drift trajectories that are associated with a distress call at t.

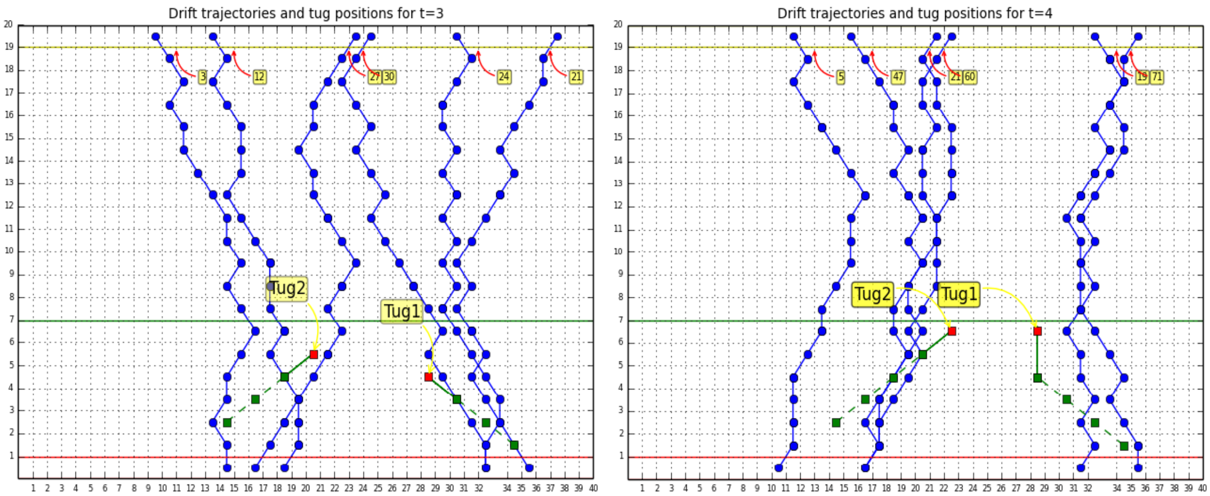
Both MIP-L and MIP-U give the same objective value of NOK 0.67 million as well as tugboat positions at each time period. This optimal solution is obtained in 15 seconds for MIP-U and 170 seconds for MIP-L. For this test instance, MIP-L model has a total number of $60,765$ constraints and 36 continuous variables, whereas MIP-U has a total number of 825 constraints and $36,000$ continuous variables. The two models have the same number of binary variables of $2,880$. At each time period in the planning horizon, the optimal decisions are influenced by the vessel scenarios of the next time periods. This is well illustrated in period 2, where Tug2 moves slightly away from the path with high cost of NOK 41 million. In fact, there are more scenarios with considerable cost on the east side of Tug2 both in periods 2, 3 and 4. Additionally, if we consider only vessel scenarios at the fourth time period, it might not be optimal to move Tug2 east, which is actually optimal when considering vessel scenarios in the next time periods. As expected, the tugboats move towards the corridor until they reach the green line limit (see Figure 3). The closer the tugboats move away from the coastline, the greater the probability of successful hook-up of potential drifting vessels.

5.3 Case 2: Computational Tests

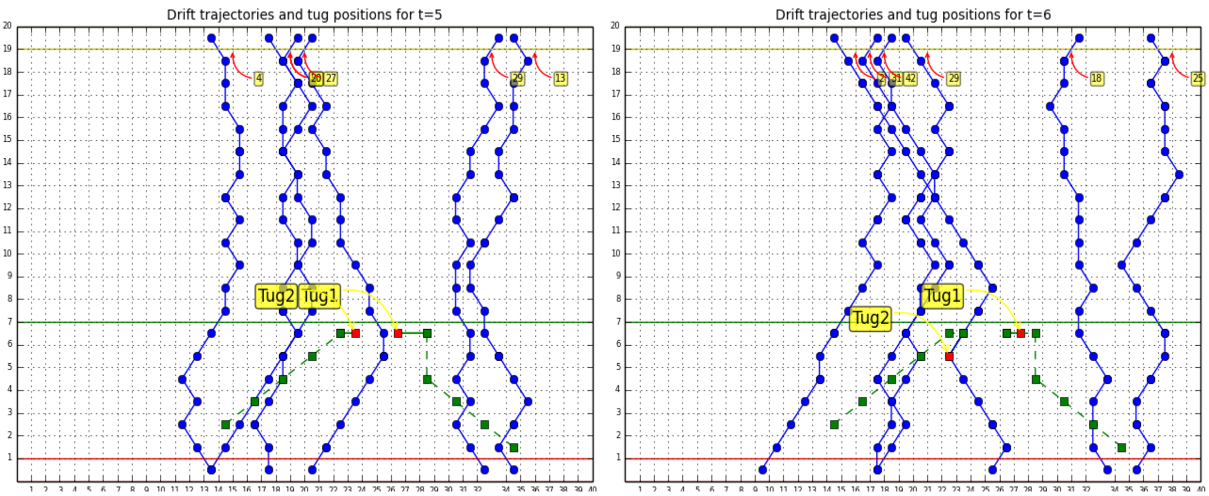
To evaluate the performance and quality of the models developed, we present results for realistically sized test instances. The cells for this case are built with geographical positions from the region of interest for the VTS. Clearly, we collect the geographical coordinates of the center position of each cell and transform them into Cartesian coordinates for calculations. Once the model is run, the optimal positions of tugboats as well as drift trajectories and vessels positions are transformed back to geographical coordinates. The region of interest covers about $1,100$ km of coastline and the corridor is on average 50 km off the coastline. We partition the area between the corridor and the coastline into cells of 5 by 5 km, which gives a total number of $2,200$ cells. In practice, the number of cells is slightly smaller than the number given above. This is explained by the fact that the corridor and the coastline are not straight lines and neither totally parallel to each other. Thus, we use few triangle cells with different sizes to better represent the region of interest. Vessels typically have an operating speed of 14 to 15 knots and



(a) Results for periods 1 and 2



(b) Results for periods 3 and 4



(c) Results for periods 5 and 6

Figure 3: Results for Case1: Illustrative Example. The green and blue solid lines represent the movement of tugboats and drift trajectories of vessels, respectively.

tugboats about 12 knots (Eide et al., 2007). In addition, the operators have subdivided the region of interest into two zones. The first tugboat is assigned to the first zone, Zone A, spanning from the border to Russia to Torsvåg, and Zone B from Torsvåg to Røst is patrolled by the second tugboat.

5.3.1 Case 2A: Many Breakpoints

In this subsection, we mainly compare the MIP-U and MIP-L models with regards to solution quality and run time, and present the gap between the optimal OTP value and optimal MIP-U or MIP-L values. Accordingly, a test set of 6 vessels and 2 tugboats over a period of 20 hours with one hour time steps are randomly generated. In this test set, the tugboats cannot move more than 25 km away from the shore. That is, the patrol zone accounts for about $1,100$ cells along the coastline. The test case constitutes 30 instances and the results for the two MIPs are presented in Table 2. The run time is in minutes and the objective function in million NOK.

Preliminary calculations indicate that $K = 1,000$ is the minimum number of breakpoints in MIP-L and MIP-U, that gives an optimal solution of OTP. Consequently, we use $2,000$ breakpoints in order to be highly confident that MIP-L and MIP-U give the optimal solutions of the OTP model. For each instance, the total number of binary variables for both MIPs are $22,974$ with 120 and $240,000$ continuous variables respectively for MIP-L and MIP-U. The total number of constraints for MIP-L is $261,924$ and $22,045$ for MIP-U.

As presented in Table 2, the MIP-U model is about 93 times faster than MIP-L. Additionally, the variability in run times for MIP-L is larger than that of MIP-U. The average solution value is NOK 0.5797 million for MIP-U and NOK 0.5792 for MIP-L. Although the objective values of the two MIPs are slightly different, the optimal decisions for the tugboat positions are the same for every test result. In addition, the relative optimality gap defined by $(\min\{f(\bar{\mathbf{x}}), f(\underline{\mathbf{x}})\} - \text{MIP-L})/\text{MIP-L}$ is the negligible 0.03%, with a maximum of 0.05%.

Table 2: Case 2A. Test results for 30 instances

GAP=$(\min\{f(\bar{\mathbf{x}}), f(\underline{\mathbf{x}})\} - \text{MIP-L})/\text{MIP-L}$

	MIP-U		MIP-L		$\min\{f(\bar{\mathbf{x}}), f(\underline{\mathbf{x}})\}$	%GAP
	Obj.Val	Time (min)	Obj.Val	Time (min)		
Avrg	0.5797	7.57	0.5792	707.07	0.5796	0.029
Std.dev	0.3955	10.43	0.3954	95.42	0.3955	0.019
Min	0.1251	1.49	0.1250	573.72	0.1251	0.006
Max	1.5497	52.18	1.5485	888.63	1.5493	0.052

5.3.2 Case 2B: Few Breakpoints

The choice of 2000 breakpoints leads to optimal tugboat positions in all instances examined at the expense of high run times. As presented in Table 2, it takes between 1.5 and 52.2 minutes to obtain a solution of MIP-U, and even longer for MIP-L. In order to assess the solution quality with relatively few breakpoints, the two MIPs models are run with another set of 50 instances in the same manner as in Case 2A, with 500 breakpoints for MIP-U and 200 for MIP-L. This gives a total number of $82,974$ variables and $22,045$ constraints for MIP-U, and $23,094$ variables with $45,924$ constraints for MIP-L.

The average relative optimality gap is 0.51%, with a maximum of 5.4% (see Table3). The solution are obtained in 1.99 minute and 13.17 minutes on average for MIP-U and MIP-L, respectively. Moreover, the runtime for MIP-U is less than 9 minutes for every test instance. The MIPs are able to obtain optimal decisions on tugboat positions for 15 instances out of 50. Although the number of vessels are the same for each instances, the initial vessel positions along the corridor are different for each test case. Consequently, the number of scenarios varies for every test instance. Clearly, some vessels might leave the region before the end of the planning horizon, and thus reduce the number of vessel scenarios. This explains the high standard deviation both on the objective values and runtime presented in Table 3. However, these results remain practically reasonable as one might run the model every hour within a receding horizon framework discussed in the next subsection.

Table 3: Case 2B. Test results for 50 instances with small number of breakpoints GAP=$(\min\{f(\bar{\mathbf{x}}), f(\underline{\mathbf{x}})\} -$ MIP-L$)/$MIP-L

	MIP-U		MIP-L		$\min\{f(\bar{\mathbf{x}}), f(\underline{\mathbf{x}})\}$	%GAP
	Obj.Val	Time (min)	Obj.Val	Time (min)		
Avrg	0.6901	1.9958	0.6855	13.1669	0.6886	0.5152
Std.dev	0.7139	1.9709	0.7092	7.0475	0.7120	0.9421
Min	0.1762	0.1517	0.1746	2.4903	0.1757	0.0003
Max	2.0669	8.4818	2.0560	26.9707	2.0616	5.4455

5.3.3 Case 2C: Effect of Time Horizon

The number of scenarios increases with the length of the planning horizon. We use one large test instance from Case 2A and run the MIP models for different planning horizons, ranging from two to 22 hours. The result in Figure 4 shows how the computational time increases with the length of the time horizon and highlights the run times performance of MIP-U compared to that of MIP-L. Furthermore, it is clear from Figure 4 that MIP-U better copes with larger instance size than MIP-L. However, MIP-L is of course essential in computing a relative optimality gap.

The weather forecast is available in real time at the VTS center and dynamic information from vessels are transmitted on average every 2 seconds (Eide et al., 2007). This information can be used for repeated updates of the model resulting in better predictions of future vessel scenarios; see for example Park et al. (2009) and Wang et al. (2007) for background on such receding horizon control. Accordingly, MIP-U could be run in real time, with parameters updated every time period. For instance, the model could be run for 20 hours, while only the first hour is being implemented. Then we update the parameters and run the model again for the next 20 hours and so forth.

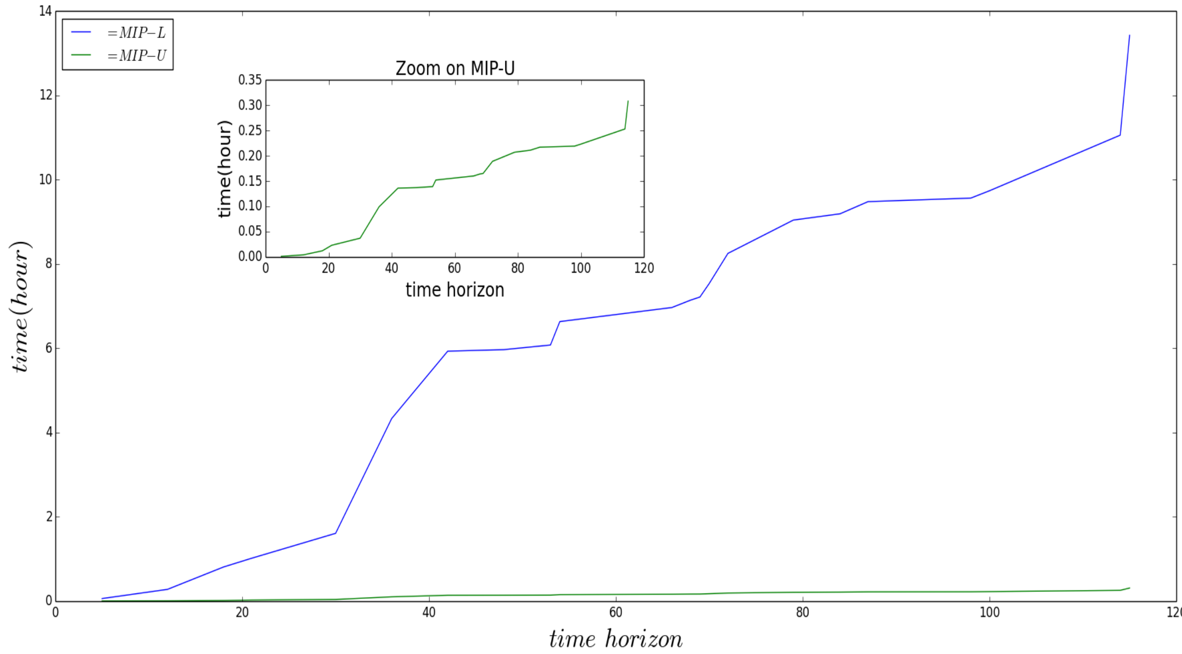


Figure 4: Case 2C. Influence of the number of scenarios on the computational time. The blue solid line represents the performance of the MIP-L model and the green solid line is that of the MIP-U model.

5.3.4 Case 2D: Sensitivity Analysis with Larger Scale Problems

To analyze the sensitivity of the MIP-U model as well as its scalability, we run different test instances with a total of 10 vessels for a number of tugboats ranging from one to six. For each number of tugboats and distribution of drift trajectories, we run the MIP-U model and compare its solution value with two different distributions for the same tugboat positions. These settings allow us to analyze the sensitivity of the solution value to different probabilities of failure R_{ω_v}, grounding costs K_{ω_v} for each vessel scenario and fleet size.

In Figure 5, the green solid line represents the optimal cost distribution while the red and blue solid lines represent the variation of the expected cost for two different distribution of drift trajectories. Unsurprisingly, the solution values are very sensitive with changes in the failure probabilities and grounding costs. This is mainly due to the high uncertainty about weather conditions and ocean currents. Although the integration of the MIP-U model with the receding horizon control algorithm, described in Section 4.2, could considerably address this issue, more accurate parameters estimation are required. Additionally, the expected environmental cost obviously decreases with higher number of tugboats as shown in Figure 5. Increasing the fleet size will bring additional acquisition costs, which are strategic decisions not discussed in this paper. Our model, however, focuses on operational decision level by proposing optimal real-time allocation and positioning of tugboats based on the available fleet size. Nevertheless, the NCA has decided to reduce the speeds of drifting vessels by acquiring new ”ship arrestors" and reduce its fleet size from three to two tugboats. Moreover, the maximum runtime for these larger scale instances is less than 45 minutes, which is sufficient to run the model every one or two hours with the receding horizon control algorithm. Sophisticated heuristics algorithms might reduce this computing time further. For cases where larger fleet sizes are needed, this scalability issue can also be easily addressed by subdividing the region of interest into smaller zones and optimally assign tugboats to each zone. The new problem will then be very similar to that of the location/allocation and deployment of ambulances in the EMS systems.

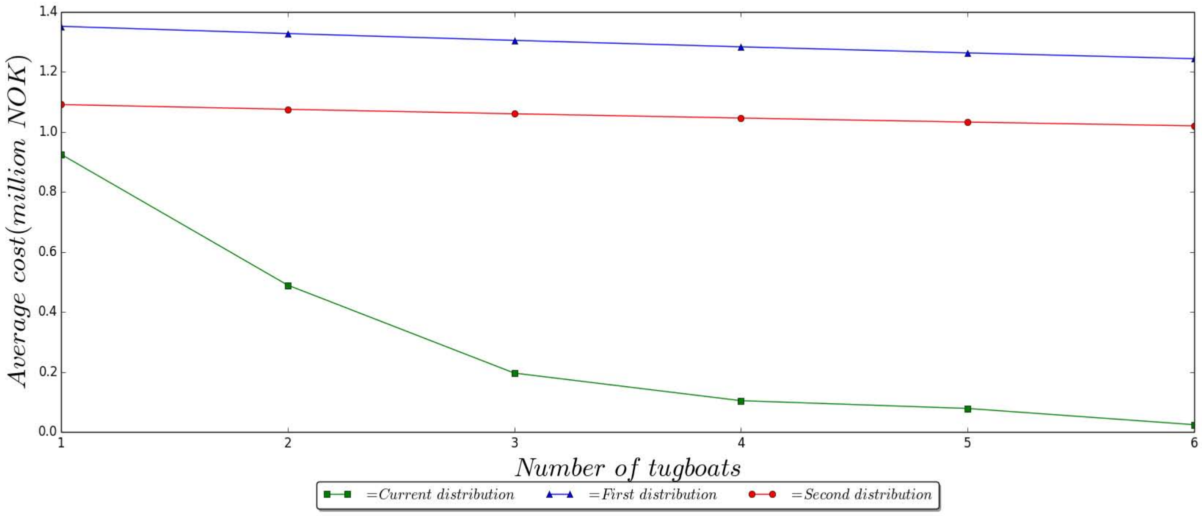


Figure 5: Case 2D. Influence of the number of tugboats and distributions of the drift trajectories on the expected environmental cost. The green solid line represents the optimal costs for the current distribution while the red and blue solid lines represent the variation in the costs with different distributions.

5.4 Case 3: Historical Events

In this subsection, we discuss three real-world cases with historical data from the NCA. Case 3A involves no grounding incident, but highlights the potential cost saving opportunities that could be gained by having solutions of the OTP problem guide decisions. In Cases 3B and 3C, we present two different instances where an accident actually occurred and run the model for 15 hours prior to the time of distress.

A path $p = (c_1, ..., c_T)$ for each vessel scenario ω_v is generated using AIS and NMI information with the algorithm presented in Section 4.2. Specifically, we collect the wind and current velocities, and wave force of the center point of each cell at each time period of the planning horizon and use the algorithm described in Section 4.2. to generate a path for each vessel scenario ω_v. In addition, the number of vessels in the region and their geographical positions for every time period, the time of distress and grounding locations are collected from AIS. Moreover, we use the basemap library in python to plot and draw the map with vessel scenarios and tugboat positions.

5.4.1 Case 3A: May 7, 2014

On the 7th of May 2014, six vessels sailed along the coastline of the High North. Their initial positions, speed over the ground (SOG) and direction at 1:30am are given in Table 4. At each time period of one hour, a potential vessel scenario is randomly generated, based on the historical wind and current directions and the model presented in Section 4.2, for every vessel. The problem size for this case is the same as Case 2A.

Table 4: Case 3A. Initial vessel positions and speeds

	Vessel 1	Vessel 2	Vessel 3	Vessel 4	Vessel 5	Vessel 6
Direction	North-west	West-north	West-north	North-west	West-north	North-west
Latitude	$N68^o02$	$N68^o15$	$N68^o21$	$N71^o07$	$N71^o22$	$N70^o38$
Longitude	$E009^o44$	$E010^o34$	$E010^o50$	$E019^o24$	$E021^o49$	$E032^o10$
SOG	12.4	10.5	13.7	13.2	11.5	13.3

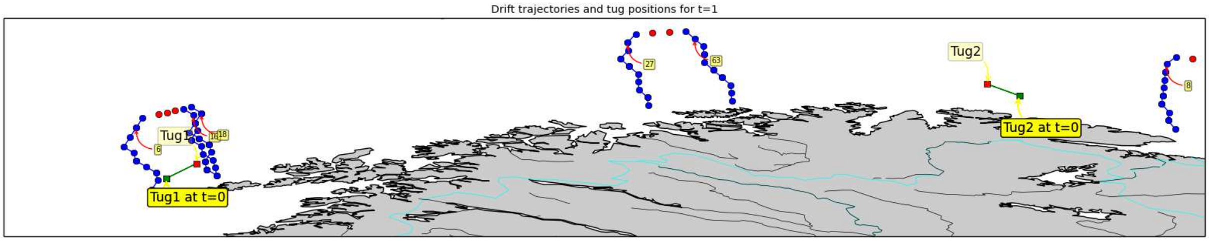
The optimal solution for MIP-U was found in 5.6 minutes with an objective value of NOK 0.34 million. The actual events had the first tug boat located at $70^o58'N-025^o51'E$ and the second one at $69^o40'N-018^o59'E$, and they were stationary during the whole planning horizon of 20 hours in accordance with current VTS policy.

The optimal locations for the first six time periods are presented in Figure 6. The priority is given to vessel scenarios with high cost. In the first time period, Tug2 moves north because of the high costs located in that direction. Tug1 moves west toward a vessel with small cost of NOK 15 millions in period 4, leaving a vessel with higher cost of NOK 41 millions, but this is due to the high cost of NOK 82 and NOK 51 million that appear in periods 5 and 6, respectively.

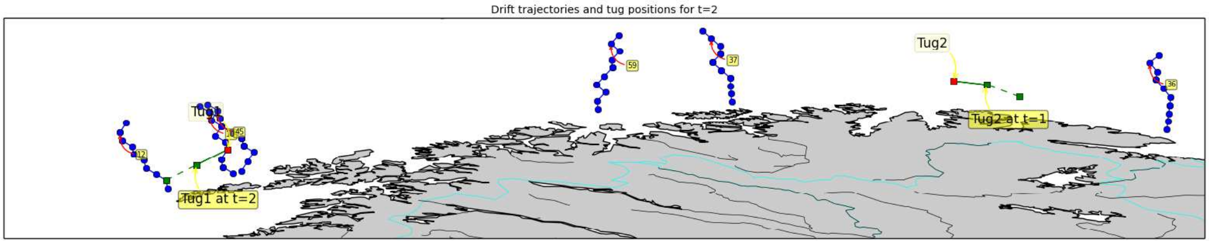
In this instance where no accident happened, the real cost was of course equal to zero. However, the expected cost under this policy is actually NOK 0.75 million, significantly higher than the optimized of NOK 0.34 million.

5.4.2 Case 3B: March 21, 2014

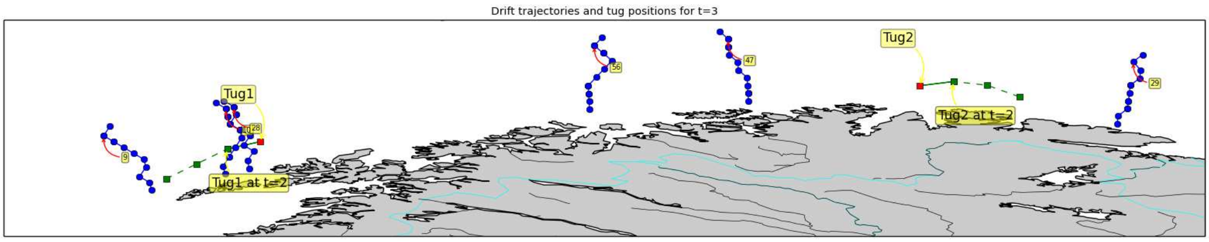
On the 21st of Mars 2014 at 11:10pm, a vessel ran aground at $71^o01.06'N-028^o27.46'E$ after about 15 hours of drifting time. At the time of distress, 07:55am, the nearest tugboat was located at $70^o40'N-023^o40'E$, and was not able to reach the drifting vessel on time. The tugboat moved toward the vessel but was 142.8 km away at the time of grounding. We run the MIP-U model with this case for 15 hours prior to the time of distress and present the results for the first and last time periods in Figure 7. The blue lines in Figure 7 represent the predicted drift trajectories for all the vessels that moved into the region in that time horizon, including the one that ran ashore, while the actual path followed by the drifting vessel is presented in green solid line. The two directed paths in red solid lines are the actual positions of tugboats from



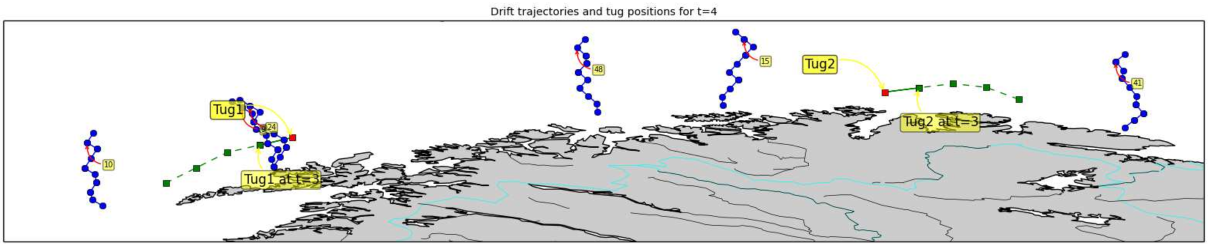
(a) t=1



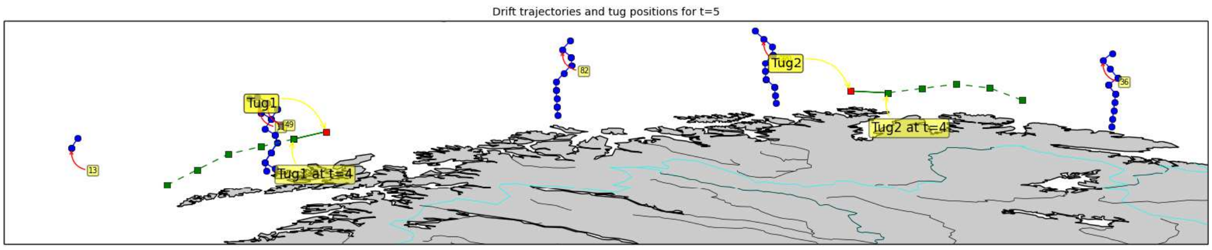
(b) t=2



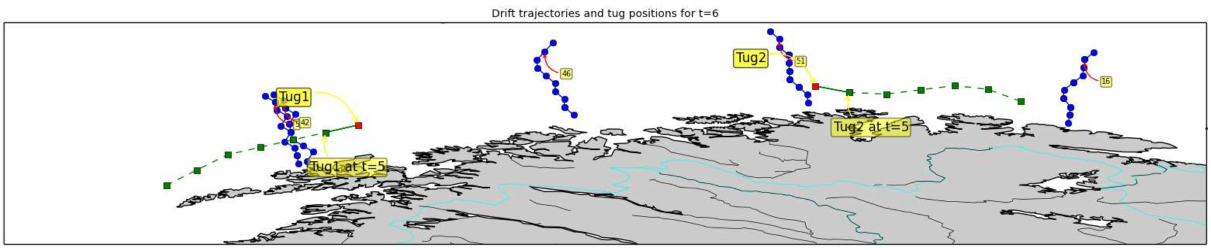
(c) t=3



(d) t=4



(e) t=5

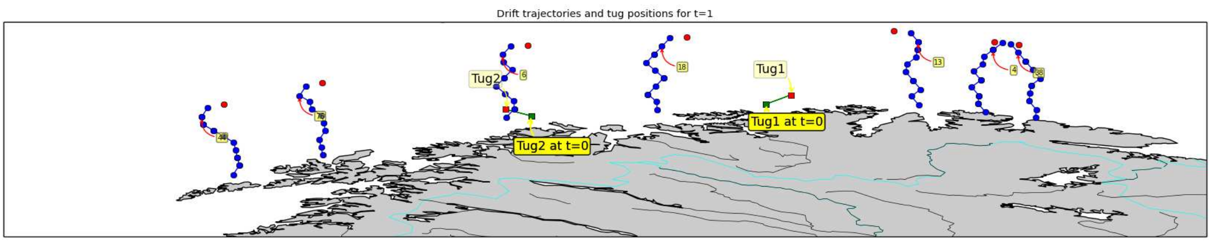


(f) t=6

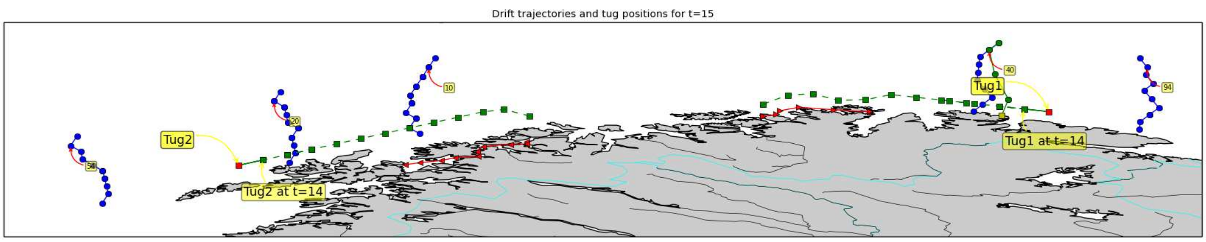
Figure 6: Case 3A. Illustration of the first six time periods. The dashed green lines represent tugboat movements at each time period and the blue solid lines represent vessel trajectories.

the time of distress to the time of grounding and the paths in dashed green line are suggested positions of tugboats by the model prior to the distress call. It is important to note that, in these green paths, the last positions represent the location of tugboats at time of distress of the vessel that ran ashore, not at the grounding time. A zoomed-in view of the grounding location of the drifting vessel as well as the nearest tugboat (Tug1) are presented in Figure 8, where the distance between the predicted and actual grounding location is about 17 km. Although this value is not large, further research as highlighted in the last section, needs to be done for accurate drift trajectories.

The probability of successful hook-up of the grounded vessel by Tug1 with the predicted drift trajectory and the MIP-U model is 0.79 and that of the actual drift trajectory is equal to 0.86. That is, the grounded vessel had 86% chance to be rescued if the MIP-U model was implemented at that time. Based on the actual position of Tug1 at time of distress from the current policy (see the first position of the red path in Figure 8) the vessel had only 0.22 probability to be hooked-up. For this real-world instance, the expected cost is equal to NOK 0.19 million if using the OTP model and NOK 0.28 million for the actual movement of tugboats.



(a) t=1



(b) t=15

Figure 7: Case 3B. Results for the first instance with grounded vessel. The dashed green lines represent the suggested movements of tugboats by the MIP-U model and the predicted drift trajectories in blue solid lines. In addition, the actual drift trajectory of the vessel that ran aground is represented by green solid lines. The paths in red are the actual positions of tugboats from the time of distress to the time of grounding.

5.4.3 Case 3C: September 12, 2014

In this case, a vessel ran ashore at $71^o02.97'N - 023^o53.89'E$ on September 12 2014 at 2:25pm. The nearest tugboat, located at $70^o41.58'N - 023^o19.21'E$, stayed static for the whole drifting time of about 9 hours. As explained by the operators at the VTS center, vessels are

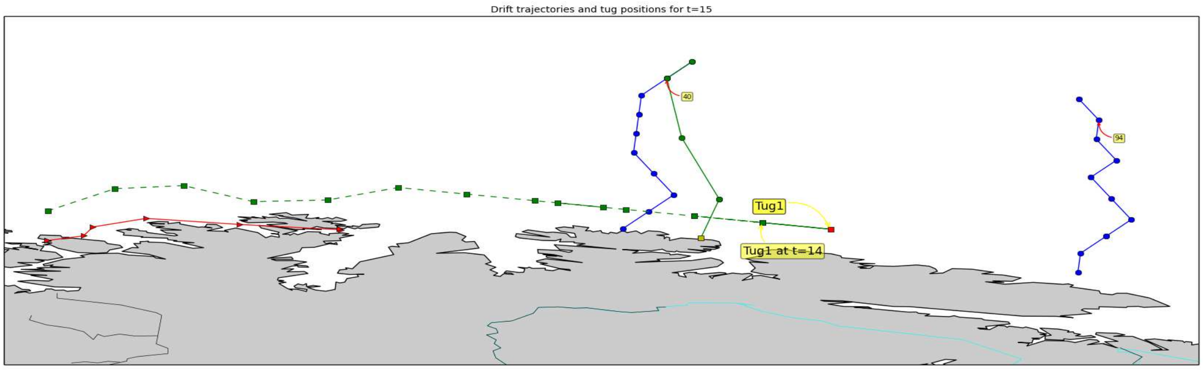
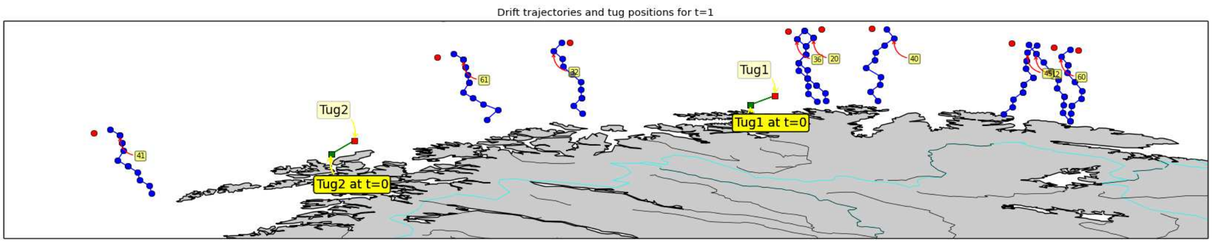
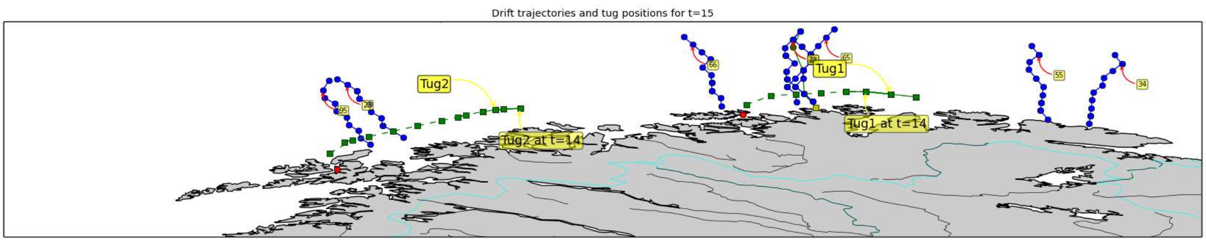


Figure 8: Zoom on the grounded vessel for case 3B. The dashed green lines represent the suggested movements of the nearest tugboat by the MIP-U model and the predicted drift trajectories are represented in blue solid lines. In addition, the actual drift trajectory of the vessel that ran aground is represented in green solid lines. The path in red is the actual positions of the nearest tugboat from the time of distress to the time of grounding and the estimated grounding cost in million NOK for each drift trajectory is labeled in yellow.

not given the same priority and sometimes vessels moving out of the corridor will "misinform" the VTS center that they are not in trouble but merely fishing (or some other false message) while they are in fact drifting with the risk of grounding. This peculiar situation often stems from misunderstandings about the cost of rescue to the ship owner. The rescue is in fact free for the ship owner, but this is not widely known. The MIP-U model is also run for 15 hours prior to the time of distress of the grounding vessel. The results for the first and last time periods



(a) t=1



(b) t=15

Figure 9: Results for Case 3C. The dashed green lines represent the suggested movements of the tugboats by the MIP-U model and the predicted drift trajectories are represented in blue solid lines. In addition, the actual drift trajectory of the vessel that ran aground is represented in green solid lines. The two cycles in red are the actual static position of the tugboats from the time of distress to the time of grounding and the estimated grounding cost in million NOK for each drift trajectory is labeled in yellow.

are presented in Figure 9 and the zoom on the grounding location is presented in Figure 10,

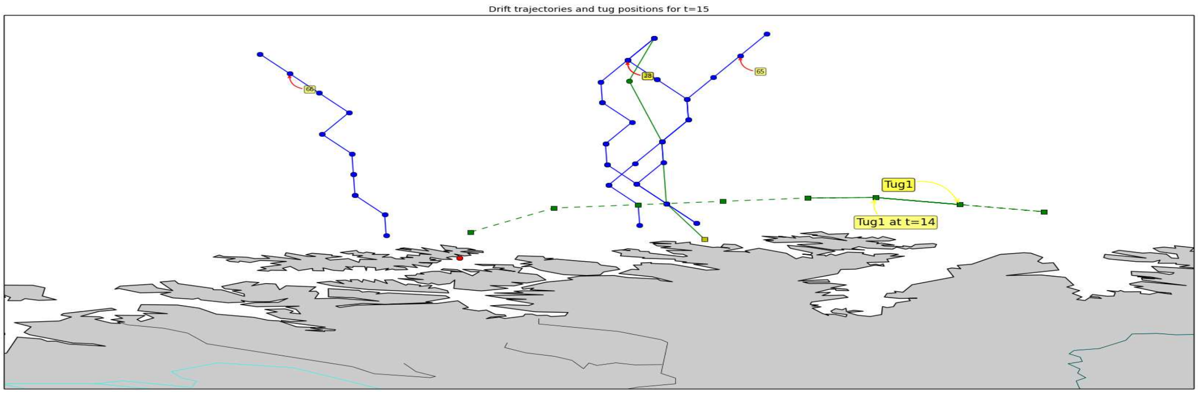


Figure 10: Zoom on the grounded vessel for case 3C. The dashed green lines represent the suggested movements of the nearest tugboat by the MIP-U model and the predicted drift trajectories are represented in blue solid lines. In addition, the actual drift trajectory of the vessel that ran aground is represented in green solid lines. The cycle in red is the actual static position of the nearest tugboat from the time of distress to the time of grounding and the estimated grounding cost in million NOK for each drift trajectory is labeled in yellow.

where the red circle represents the position of the nearest tugboat. The predicted drift trajectory has a grounding position 43 km away from the location where the vessel actually run aground. For this particular case, the model and the current policy have a close probability of successful hook-up of 0.79 and 0.69, respectively. This is mainly due to the actual static position of the tugboat and the grounding location, which in this case is rather fortunate. In addition, the MIP-U model does not know which drift will occur in advance and thus, tries to minimize the overall expected cost. Indeed, there are two drift trajectories with high costs on the right side of the nearest tugboat. This is why the tugboat does not move closer to the drift trajectory that actually occurred. The expected cost for this real-world instance is equal to NOK 0.07 million if using the OTP model and NOK 0.24 million for the current policy.

6 Conclusions

In this article, we developed a nonlinear binary integer programming model to minimize the clean-up costs, socioeconomic losses and environmental costs associated with oil spill from grounding accidents. Two linearizations of the model lead to mixed integer models that bound the optimal value of the original problem with practically near zero optimality gaps. The paper also presents methods for obtaining input data to the model. Preliminary results for small and realistically-sized instances indicate noteworthy features of our approach. Optimal tugboat positions are obtained in less than two minutes for realistic instances with a small number of breakpoints. A test with a real-world instance in Case 3A indicates a total clean-up and socioeconomic costs saving opportunity of 45%. Moreover, tests with three representative historical data sets highlight the importance and benefits of implementing the MIP-U model at the NCA. Specifically, we demonstrate that on a single day in 2014, decision support by the proposed model might have reduced the expected cost from grounding accidents that day from NOK 0.75 million to NOK 0.34 million. In two other studies of actual grounding incidents in 2014, we predict that adoption of the decision support tool in the hours prior of the grounding events might have increased the probability of avoiding those accidents from 22% to 86% in the first incident and from 69% to 79% in the second. The NCA finds these estimates exceptionally interesting and the proposed decision support tool highly promising as the basis for a real-time automated system that can assist the NCA's VTS operators. Although the model is not yet implemented in practice, the examples use real data, which provide an indicate of what we can expect.

It is recommended that further studies are initiated in order to obtain more accurate input for the model. First, the region should be partitioned into an "optimal" number of cells. A large cell size will reduce the precision on tugboat positions as well as vessel scenarios. Conversely, small cells size gives good precision at the expense of high model complexity. Second, more than one vessel scenario at each time period and scenario could be considered, which might lead to a need for Monte Carlo sampling techniques. Third, and most importantly, there is a need for better assessments of grounding costs at each coastline segment as well as estimates of probabilities of distress calls. Furthermore, the model can be extended and incorporate a receding horizon control algorithm to effectively address the uncertainty and dynamic environment of the problem. In fact, there might be new vessels entering and leaving the region during the planning horizon. In addition, one of the tugboats could be out of patrol because of several reasons such as escort of vessels and crew shift. All this information needs to be updated dynamically. Finally, different objective functions could be used for the OTP problem. For instance, minimizing the superquantile instead of the expected cost could be of great interest to the NCA.

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Paper 3

A Risk-Averse Decision-Making Tool for Dynamic Positioning of Patrol Tugs along the Northern Norwegian Coastline

A risk-averse decision-making tool for dynamic positioning of patrol tugs along the northern Norwegian coastline

Brice Assimizele[a,c], Johannes O Royset[b], Johan Oppen[c], and Robin T. Bye[d]

[a]Faculty of Maritime Technology and Operations, Norwegian University of Science and Technology, Norway

[b]Operations Research Department, Naval Postgraduate School, Monterey, California, USA

[c]Department of Logistics, Molde University College, Molde, Norway

[d]Faculty of Engineering and Natural Sciences, Norwegian University of Science and Technology, Norway

Abstract

The Norwegian coastal administration (NCA) operates centers for vessel traffic services (VTS) to prevent grounding accidents along the highly environmentally sensitive northern coast of Norway. At any time, vessels moving in the region might lose steering, propulsion or power and need to be rescued, by a fleet of tugs controlled by the VTS center, before they run aground. The constantly changing traffic and weather conditions add to the challenge for the VTS operators. To prevent rare, but very high environmental costs, a risk-based methodology is adopted and the corresponding problem integrated into a non-linear program. The linearized model, run with historical events of about $50,000$ potential vessel drifting scenarios, illustrates a considerable reduction of the expected worst-case environmental costs.

Keywords: OR in maritime industry; Mixed Integer Programming; Search and Rescue; Oil Spill; α-superquantile ; Risk-Averse; Sustainable operations; Risk management; Decision support systems

1 Introduction

The significant global reliance on oil consumption leads to an increased likelihood of spillages. Oil spills cause serious damage to a wide range of economic sectors and the environment. They affect fishing, tourism and other related sectors of the economy. As described in Assimizele et al. (2016), grounding and collision are the main cause of oil spills. The severity of oil tanker incidents are highlighted in various impact assessment studies (Alló and Loureiro, 2013; Chapman and Hanemann, 2001; Loureiro et al., 2006; Moore et al., 1998). In order to mitigate these risks and hazards within maritime operations, a variety of rules and regulations are being adopted by maritime stakeholders. Despite these considerable efforts to enhance safety in crude oil transports, the statistics reveal that there are still a numbers of significant incidents every year in the marine industry (EMSA, 2013). Indeed, providing cost effective and sustainable marine transportation is challenging and complex. Sustainability is surely the greatest challenge of our current and next generation. Addressing this environmental challenge requires commitment from both private and public sector. Due to this emerging concern, companies are under serious pressure to evaluate their impact on the environment, and to engage in evaluating the triple bottom line (people, profit, and planet) (Ansaripoor et al., 2014).

In order to address this complex environmental problem, the Norwegian Coastal Administration (NCA) operates a center for vessel traffic services (VTS) in the town of Vardø in northeastern Norway. Each year, several hundred oil tankers travel along the northern coastline. These oil tankers are subject to possible drift grounding accidents along an environmentally sensitive region of about 600 nautical miles. A fleet of tugboats, controlled by the VTS center, patrol the coastline in order to rescue any potentially drifting oil tanker before it runs aground. Approximately 200 vessels are monitored every day by the VTS center of which five to six oil tankers receive special attention due to their size and risk of pollution. These tankers sail, by law, along a predefined corridor of about 27 nautical miles away from the coast. In addition, the VTS center obtains dynamic information (e.g., position, heading) and static information (e.g., identity, dimensions, cargo, flags) of the ships entering the region every two seconds on average, through the automatic identification system (AIS). Moreover, weather forecast, real-time measurements of ocean currents, wave height, and wind are available to predict potential drift trajectories. An oil tanker moving in the region of interest might lose its maneuverability (e.g., from steering or propulsion failure) and the tugboats have to be close enough to effectively rescue the oil tanker before it runs ashore (Assimizele et al., 2016). Dynamic tugboat locations at the VTS center is currently difficult because of the increased oil traffic in the High North. In addition, preventing the worst-case incidents is of great concern but challenging to risk-averse NCA managers. To assist the VTS operators in addressing this optimal tugboat positioning (OTP) problem, we develop models with the s-risk measure of risk concept discussed next and analyze computational results with real-world cases.

Assimizele et al. (2016) first described the OTP problem using a two-dimensional modeling approach and focused on the minimization of the expected grounding costs over a planning horizon. The choice of the expectation as a measure of risk by Assimizele et al. (2016) ignores the variability of the environmental costs and places the focus on the average outcome. In addition, only one drift trajectory is used in their model to predict the path followed by a drifting vessel. Consequently, the enhancement we propose from previous work is twofold. First, we consider more than one trajectory for each drifting vessel for practical implementation, which considerably increases the number of scenarios and provides better tugboat policy. Obviously, accurate predictions of drifting vessel trajectories lead to a higher probability of successful rescue operations. Second, we provide flexibility to the operators at the VTS center by modeling different levels of risk-averseness. Specifically, these operators are faced with an environmental grounding cost represented by a random variable Y. As we change tugboat policies, we can modify the random variable and in particular its distribution. Our goal is to select a tugboat policy that minimizes Y. However, since Y is a random variable, it is unclear how to compare two candidate random variables Y and $Y^{'}$, or to make an assessment whether a given cost Y resulting in a chosen policy is sufficiently low, say below a threshold c.

Although not the only possibility, s-risks enable a comparison of random variables for these purposes. The question of how to address and rank uncertain quantities represented by random variables is highly important in many areas of operations research, engineering, and economics (Rockafellar and Royset, 2015). S-risks, and more generally risk measures, enable comparison across random variables. A *measure of risk* is a functional $\mathcal{R}$ that assigns to a random variable Y a value $\mathcal{R}(Y)$ in $(-\infty,\infty]$ as a quantification of the risk in it. The comparison of two random variables Y and $Y^{'}$ then reduced to that of comparing the numbers $\mathcal{R}(Y)$ and $\mathcal{R}(Y^{'})$.

Due to the abundance of possible risk measures, coherency (Artzner et al., 1999) is a concept that stands out as guidance on what would constitute a good and useful measure of risk (see many examples in Rockafellar and Uryasev (2013)). Superquantile risk (s-risk) is a coherent measure of risk in the sense of Artzner et al. (1999) and has improved properties over those of quantile risk (value-at-risk) (Rockafellar and Royset, 2015). It was initially proposed for financial applications under the name conditional value-at-risk (Rockafellar and Uryasev, 2000, 2002a). For risk averse decisions, s-risk is more appealing because it takes into account the contribution from the very rare but very large losses. Thus, s-risk allows the managers at the VTS center to safeguard against worst case scenarios. Specifically, we say that a random variable Y is safely $\leq c$, if the α-superquantile of Y is $\leq c$, where $\alpha \in [0,1]$. Thus, if we select a tugboat policy with cost Y and Y is safely $\leq c$, then we have safeguarded against high cost. On average over the worst $(1-\alpha)\%$ outcomes, the realized environmental cost will be $\leq c$. Indeed, we assist the decision maker faced with random cost by shaping the upper tail of the distribution of Y for various decisions. For $\alpha = 0$, the problem corresponds to the minimization of the expectation of Y addressed in Assimizele et al. (2016), while $\alpha = 1$ represents a tugboat policy with all the realization of $Y \leq c$, which might not be feasible to achieve.

The value-at-risk (VaR) is the minimum potential loss that a company can tolerate with a certain probability during a finite period, whereas s-risk is the conditional expectation of the losses beyond the VaR (Ansaripoor et al., 2014). For cases where the loss exceeds the VaR value, s-risk provides a better indication of the potential losses exceeding the predefined confidence level (Ran et al., 2015). For the same confidence level, VaR is a lower bound for s-risk. In optimization applications, s-risk is superior to VaR as it captures tail behavior and preserves convexity in optimization models. Because of its coherency as opposed to VaR, s-risk is well-suited as scalar representations of a random variable in risk-averse decision making. Moreover, it can be expressed by a minimization formula suggested by Rockafellar and Uryasev (2000), which can be incorporated in optimization problems. It helps to achieve significant shortcuts through linear programming techniques by preserving the crucial problem features such as convexity and makes many large-scale calculations possible (Rockafellar and Uryasev, 2002b; Sarykalin et al., 2014). Indeed, a s-risk of a random variable is more stable than the corresponding quantiles under parametric perturbation of that variable.

Superquantile, introduced by Rockafellar and Uryasev (2000), is presently one of the widely used risk measures suggested by theoreticians and market practitioners (Huang et al., 2008; Sarykalin et al., 2014). It is recognized as an important approach in risk analysis and stochastic optimization (Rockafellar and Royset, 2013). The concept is used in various portfolio optimization problems (Tong et al., 2010) and is known under a variety of names such as "conditional value-at-risk," "average value-at-risk," "tail value-at-risk," and "expected shortfall," with some minor variations in definitions. In many applications, distributional information about a random variable are most of the time incomplete, which makes superquantiles especially important for stability (Rockafellar and Royset, 2013). This is why superquantile is well suited for risk-averse decision making and optimization. It has been applied in financial engineering (Alexander et al., 2006; Balbás et al., 2010; Uryasev et al., 2010; Wang and Uryasev, 2007), structural engineering (Haukaas and Mahsuli, 2013; Minguez et al., 2013; Rockafellar and Royset, 2010), military operations (Commander et al., 2007; Kalinchenko et al., 2011; Molyboha and Zabarankin, 2012), supply chains (Ansaripoor et al., 2014; Tomlin, 2006; Verderame and Floudas, 2011), and energy systems (Carrion et al., 2007; Conejo et al., 2008). We refer to Krokhmal et al. (2011); Rockafellar (2014); Sarykalin et al. (2014) for reviews of risk measures and superquantiles.

The rest of the paper is organized as follows. Section 2 presents the s-risk measure with focus on its application to optimization. We formulate a risk-averse OTP problem in Section 3. The numerical results with real-world data are presented in Section 4. Conclusions and recommendations for further research follow in Section 5.

2 Superquantile Risk Measure in Optimization

2.1 Definition of Superquantile

Let Y be a real-valued random variable representing loss or cost and characterized by its cumulative distribution function F_Y An equivalent characterization is in terms of the quantile function

$$q_\alpha(Y) = \min\{y|F_Y(y) \geq \alpha\}\forall\alpha \in (0,1).$$

That is, the α-quantile $q_\alpha(Y)$ or value-at-risk ($VaR_\alpha(Y)$) is the lowest y such that $prob\{Y \geq y\} \leq 1-\alpha$. It is a lower α-percentile of the random variable Y. The s-risk concept introduced by Rockafellar and Uryasev (2000) under the term conditional value-at-risk (CVaR) is defined as another description of a random variable Y derived from the function $\bar{q}_\alpha(Y) : (0,1) \to (-\infty, \infty]$, which is the expectation in the upper α-tail distribution of Y, for $\alpha \in [0,1]$. Thus, the s-risk function is equivalently given by

$$\bar{q}_\alpha(Y) = \frac{1}{1-\alpha}\int_\alpha^1 q_{\alpha'}(Y)d\alpha' \;\forall \alpha \in (0,1).$$

From this expression, s-risk equals the conditional expectation of Y subject to $Y \geq VaR_\alpha(Y)$. Note that for $\alpha = 0$ and 1, $\bar{q}_0(Y) = E[Y]$ and $\bar{q}_1(Y) = Sup[Y]$, respectively. Although the definition of $\bar{q}_\alpha(Y)$ holds for any integrable random variable Y and $\alpha \in [0,1]$, it relates to applications where high realizations of Y is of special concern. This type of situations might arise when Y describes "cost," "loss," or "damage" associated with some future actions and the risk-averse decision maker's aim is to reduce the upper tail of the distribution.

The definition of s-risks indicates difficulties for evaluation and practical implementation. Fortunately, a trade-off formula developed by Rockafellar and Uryasev (2000, 2002a) reduces the task to that of evaluating expectations and minimizing over a scalar thereby eliminating that concern. Specifically,

$$\bar{q}_\alpha(Y) = \min_y\{y + \mathcal{V}_\alpha(Y-y)\}, \text{ where } \mathcal{V}_\alpha(Y) = \frac{1}{1-\alpha}E[\max\{0,Y\}].$$

S-risks provide an adequate picture of risks reflected in extreme tails (Sarykalin et al., 2014). It is a useful tool in modeling and optimization.

2.2 S-risks Optimization

Let us consider a random loss function $f(x,\omega)$ that depends on the decision vector x and a random vector of risk factors ω. The main idea in Rockafellar and Uryasev (2000) is to define a function

$$F_\alpha(x,\zeta) = \zeta + \frac{1}{1-\alpha}E\{\max\{0, f(x,\omega) - \zeta\}\},$$

that can be used instead of superquantile. They prove that:

1. $F_\alpha(x,\zeta)$ is convex with respect to α;
2. $VaR_\alpha(x)$ is a minimum point of function $F_\alpha(x,\zeta)$ with respect to ζ;
3. minimizing $F_\alpha(x,\zeta)$ with respect to ζ gives $\bar{q}_\alpha(x)$. That is $\bar{q}_\alpha(x) = \min_\zeta F_\alpha(x,\zeta)$.

S-risks can be used to shape the risk in an optimization model with specified risk level (Sarykalin et al., 2014). It approximately equals the average of some percentage of the worst-case loss scenarios. For practical calculations, an ordinary approximation of $F_\alpha(x,\zeta)$ is done by the scenario method. Rockafellar and Uryasev (2002a) suggest a mathematical model that simultaneously minimizes the s-risk and calculates the corresponding VaR as follow:

$$\min_{x\in X, z_\omega\in\mathbb{R}, \beta_\alpha\in\mathbb{R}} \bar{q}_\alpha(x) = \min_{x\in X, z_\omega\in\mathbb{R}, \beta_\alpha\in\mathbb{R}} \beta_\alpha + \frac{1}{1-\alpha}\sum_{\omega\in\Omega} z_\omega$$

s.t.

$$f(x,\gamma_\omega) - \beta_\alpha \leq z_\omega \quad \forall \omega \in \Omega$$

$$z_\omega \geq 0 \quad \forall \omega \in \Omega, x \in X$$

In this model, β_α denotes the *VaR* for the confidence level of α, and Ω represents the set of scenarios, γ_ω is the vector of stochastic variables in scenario ω and x is the vector of decision variables. Moreover, z_ω are positive dummy variables, and f denotes the loss function. The solution of the above linear programming model simultaneously gives the optimal value $\bar{q}_\alpha(x^*)$, the value-at-risk β^*_α and x^*.

3 Risk-Averse Formulation for the OTP

We discretize the planning horizon into a finite set of time periods $\mathcal{T} = \{0, 1, ..., T\}$ and subdivide the region of interest controlled by the VTS center in Vardø into a finite number of cells $\mathcal{C} = \{1, ..., C\}$. Every vessel in the set $\mathcal{V} = \{1, ..., V\}$ occupies one cell at each time period and can move to any reachable cell within a time period depending on the vessel's speed, which is influenced by the weather conditions.

In contrast with Assimizele et al. (2016), we consider more than one trajectory for each potential drift. Thus, we define $\mathcal{P}_{vt}$ as the set of possible paths followed by vessel $v \in \mathcal{V}$ in period $t \in \mathcal{T}$. In addition, $\omega_v = (t, p) \in \Omega_v$ represents the scenario for vessel v following the path $p \in \mathcal{P}_{vt}$, where Ω_v denotes the set of possible scenarios for vessel v and $t \in \mathcal{T} \cup \{T+1\}$ is the time the VTS center notices or is alerted to the distress of vessel v. In the absence of failure, there will be no distress call and the time t is set to $T+1$. For a given planning horizon, we define $\bar{\omega} = (\omega_1, ..., \omega_V) \in \bar{\Omega}$, where $\bar{\Omega} = \Omega_1 \times .... \times \Omega_V$ is the collection of all vessel scenarios.

Let $\mathcal{G} = \{1, ..., G\}$ denote the set of tugboats operated by the VTS center in Vardø. Each tugboat $g \in \mathcal{G}$ is initially positioned at cell $c_{0g} \in \mathcal{C}$ and can transit between reachable cells at each time period. Accordingly, let $\mathcal{F}_{tg}(c) \subset \mathcal{C}$ be the set of cells reachable from cell c in one time period by tugboat g. The set $\mathcal{F}_{tg}(c)$ depends on the weather conditions at a current time period t as well as the maximum speed for tugboat g. In addition, the fleet of tugboats are not allowed to move very far away from the coastline because of the secondary escort mission. In fact, some of the ships in transit to ports located in the north of Norway need to be escorted by tugboats. Nevertheless, this does not affect our model because the available number of tugboats at each time period in the planning horizon is known well in advance.

At any time period, a vessel might start drifting with a certain probability which depends on internal factors of the vessel itself as well as the weather conditions (e.g., ocean current, wave height). Let R_{ω_v} be the probability of scenario ω_v for vessel v. Assuming that the probability of vessel scenarios are independent of each other, the probability for a scenario $\bar{\omega}$ is defined by $R_{\bar{\omega}} = \prod_{v\in\mathcal{V}} R_{\omega_v}$. This assumption is justified by the fact that vessels in distress are usually spatially separated with few common environmental factors. In addition, a drifting vessel has to be rescued by tugboats before it run aground. Thus, we let the probability of successful hook-up by tugboat g with vessel v, given vessel v follows scenario $\omega_v = (t, p)$ and tugboat g is in cell c at time of distress call t, be denoted by $Q_{gc\omega_v}$.

The main objective is to determine tugboat positions at any time period in the planning horizon such that the expected cost of the worst-case scenarios is minimized. Therefore, let K_{ω_v} be the grounding cost associated with vessel scenario $\omega_v = (t, p)$ if vessel v follows path p and no tugboat manages to hook it up. The uncertainty in this cost can be easily handled by defining additional vessel scenarios. This cost mostly depends on the grounding location as well as the type and volume of the oil spill and is equal to zero for vessel scenarios with $t = T+1$ (i.e., no failure occurs and the vessel follows a normal route in the corridor). Additionally, we define x_{gct} as a binary variable that takes the value 1 if tugboat g is in cell c at time t, 0 otherwise. The probability that no tugboat rescues vessel v if vessel scenario ω_v occurs equals

$$\prod_{g\in\mathcal{G},c\in\mathcal{C}} (1 - Q_{gc\omega_v})^{x_{gct}}.$$

Moreover, let $\rho(\mathbf{x}, \bar{\omega})$ denotes the grounding cost function for scenario $\bar{\omega}$, where $\mathbf{x}$ represents a vector with components x_{gct}. Then,

$$\rho(\mathbf{x}, \bar{\omega}) = \sum_{\omega_v\in\bar{\omega}} K_{\omega_v} \prod_{g\in\mathcal{G},c\in\mathcal{C}} (1 - Q_{gc\omega_v})^{x_{gct}}, \tag{1}$$

where subscript t in the variable x_{gct} denotes the time of distress in scenario $\omega_v = (t, p)$ and the probability of hook-up $Q_{gc\omega_v}$ is relative to that time. This function can be equivalently written as:

$$\rho(\mathbf{x}, \bar{\omega}) = \sum_{\omega_v\in\bar{\omega}} K_{\omega_v} \exp\left(-\sum_{g\in\mathcal{G}}\sum_{c\in\mathcal{C}} \alpha_{gc\omega_v} x_{gct} \right) \tag{2}$$

where $\alpha_{gc\omega_v} = -\ln(1 - Q_{gc\omega_v})$ represents the *hook-up rate*. For a probability $\alpha \in (0, 1)$, the s-risk of the grounding cost, $\bar{q}_\alpha(\rho(\mathbf{x}, \bar{\omega}))$, can be written as follows.

$$\bar{q}_\alpha(\rho(\mathbf{x}, \bar{\omega})) = \min_{\zeta} \zeta + \frac{1}{1-\alpha} E\{\max\{\rho(\mathbf{x}, \bar{\omega}) - \zeta\}\} \tag{3}$$

This expression is equivalent to:

$$\min_{\mathbf{x},\beta_\alpha,z_{\bar{\omega}}} \beta_\alpha + \frac{1}{1-\alpha} \sum_{\bar{\omega}\in\bar{\Omega}} R_{\bar{\omega}} z_{\bar{\omega}}$$

s.t.

$$\rho(\mathbf{x},\bar{\omega}) - \beta_\alpha \leq z_{\bar{\omega}} \quad \forall \bar{\omega} \in \bar{\Omega}, \tag{4}$$

where $z_{\bar{\omega}}$ is a dummy variable for each scenario $\bar{\omega}$.

The objective function $\rho(\mathbf{x},\bar{\omega})$ is linearized from below and above by two different linearization methods, which lead to two mixed integer linear programs denoted by MIP-1 and MIP-2. These two linear models allow to compute the optimality gap and, give optimal decision policies and close to optimal environmental costs in a reasonable time. The details of the nonlinear model with its two linearizations are presented in the appendix.

4 Numerical Results and Case Studies

In this section, we discuss the input settings and present simulation results with realistic test instances and cases studies with historical events.

4.1 Input Data and Settings

Most of the input parameters used in this paper are the same as those of Assimizele et al. (2016). Therefore, we only present a summary with few enhancements and refer the reader to Assimizele et al. (2016); Kontovas et al. (2010); Ni et al. (2010) and the references therein for details on the input settings.

All computations are carried out on a personal computer with an Intel(R) Pentium(R) IV 3.0 CPU and 4.0 GB of RAM, running Windows 7. In addition, Gurobi 6.0.5 is used as optimization solver on Python 2.7.3 with Pyomo 4.2.

4.1.1 Failure and Hook-up Probabilities

As opposed to Assimizele et al. (2016), the probability $R_{\bar{\omega}}$ is randomly generated such that $\sum_{\bar{\omega}\in\bar{\Omega}} R_{\bar{\omega}} = 1$, where the scenario with no drifting vessel is considerably larger than those with potential incidents. This gives a more accurate representation of the real movement of vessels with, consequently, different average costs form previous work.

The probability of a successful rescue operation of a drifting vessel depends on the time left, t_l, once the vessel is reached by the tugboat. For every vessel scenario, we determine the hook-up probabilities using the following formula.

$$Q_{gc\omega_v} = \frac{\beta_{\omega_v} \exp\left(\delta_{\omega_v}(t_l - t_{\min})\right)}{1 + \exp\left(\delta_{\omega_v}(t_l - t_{\min})\right)}.$$

The influence of the weather is represented by $\beta_{\omega_v} \in [0,1]$ and that of "ship arrestors" by $\delta_{\omega_v} \geq 0$. In fact, the NCA recently acquired new " ship arrestors" used by helicopters to reduce the vessel's drifting speed, which considerably increases the probability of successful rescue operations. Additionally, the parameter $t_{\min}$ represents the minimal remaining drift time required to attempt a hook-up and $Q_{gc\omega_v} = 0$ if $t_l \leq 0$.

4.1.2 Grounding Costs and Drift Trajectories

The main factor factor influencing the environmental costs are related to the volume of oil spilled as well as the biological and economical characteristics of the spill location. In addition, the grounding cost for each vessel scenario is computed with the formula $K = 51.432V^{0.728}$, where the volume, V, is randomly generated according to a uniform random variable on $[2187, 51704]$ plus the absolute value of a normal random variable with mean 15000 and standard deviation 5000 (see Assimizele et al. (2016)).

The drift trajectories are generated as in Assimizele et al. (2016), using weather information from the Norwegian Meteorological Institute (NMI) and the Leeway formula. Moreover, the model is run dynamically using a rolling horizon algorithm. The information about the vessel entering and leaving the region as well as local wind, surface current and wave height are determined at each time period and updated to the model (see Assimizele et al. (2016)).

4.2 Computational Results

The two variants of the risk averse OTP model, MIP-1 and MIP-2, are run with 50 realistic randomly chosen instances for 10 hours and $\alpha = 0.95$. These instances are run for two tugboats presently operated by the VTS center and 6 vessels. Additionally, we use a total of 500 and 200 breakpoints (see Appendix A) for MIP-1 and MIP-2 respectively, which give a relative optimality gap of 0.4 as presented in Table 1. The main objective of these numerical results is to present the value-at-risk and s-risk values, which could be of interest to the managers at the VTS center in Vardø.

For a time horizon of 10 hours, the probability that the environmental costs, in case of drift grounding accidents, exceeds 6.646 millions Norwegian kroner (NOK) is less than 0.05. In addition, the expected worst case costs will not exceed NOK 8.670 millions. Moreover, we have a maximum worst environmental cost of NOK 19.605 millions, which is indeed attractive for the risk averse managers at the NCA . The MIP-1 is by far faster than the MIP-2 model, with an average computational speed of only 19.93 seconds (see Table 1), and is used for the rest of the numerical results.

4.3 Case Studies with Historical Events

In this subsection, we discuss the results of the risk averse OTP model run with real-world data from the NCA. We present two cases in the first subsection, where an accident actually occurred and discuss the tugboats policy for different values of α. These first cases consider

Table 1: Numerical results for 50 realistic instances with $\alpha = 0.95$, where the costs are in million NOK.

	MIP-1			MIP-2			%GAP
	S-risk	Time (sec)	VaR	S-risk	Time (sec)	VaR	
Avg	8.670	19.932	6.646	8.640	105.769	6.614	0.420
Std	5.182	32.512	4.831	5.177	23.657	4.844	0.612
Min	1.782	5.755	0.000	1.779	76.582	0.006	0.612
Max	19.605	105.602	19.140	19.605	159.313	19.130	0.612

one drift trajectory per vessel scenario. Then, in the next subsection, we explore the solution quality with more scenarios with the same two cases. Specifically, we use three drift trajectories for each vessel scenario.

For these cases, the area between the corridor and the coastline is partitioned into cells of 5 by 5 km and covers about $1,100$ km of coastline. Moreover, the corridor is on average 50 km off the coastline and the total number of $2,200$ cells are built with geographical positions . Additionally, historical current, wind forces and wave height are collected from the AIS and the NMI to generate drift trajectories as in Assimizele et al. (2016). The operators at the VTS center have subdivided the region into two zones patrolled by two operational tugboats, where the first zone spanning from the border to Russia to Torsvåg is assigned to the first tugboat and the second zone from Torsvåg to Røst is patrolled by the second tugboat. Moreover, the maps with vessel scenarios and tugboat positions are plotted using the basemap library in python.

4.3.1 Cases with few number of Scenarios

We consider two historical events in this subsection, where only one drift trajectory is generated for each vessel scenario and discuss the tugboat policy for different risk averse decisions. The total number of scenarios considered for each historical event in these cases is more than 5000.

4.3.2 Event 1A: March 21, 2014

In this first case, a vessel ran aground at $71^o01.06'N - 028^o27.46'E$ on the 21st of Mars 2014 at 11:10pm, after about 15 hours of drifting time. The nearest tugboat, located at $70^o40'N - 023^o40'E$, was not able to reach the drifting vessel on time. The tugboat unsuccessfully moved toward the vessel and was 142.8 km away at the time of grounding. To evaluate the decisions on tugboat policy with this historical event, we run the MIP-1 model for six different values of α ranging from 0.00 to 0.95. The model is run for 15 hours prior to the time of the distress call and include all the seven vessels that sailed in the region during the planning horizon.

The numerical results in Table 2 present a great reduction in worst case (maximum) environmental cost. In addition, the initial cost in Table 2 represents the environmental cost for the

Table 2: Numerical results for Event 1A with costs presented in million NOK.

α-s-risk	0.00	0.25	0.50	0.75	0.90	0.95	Initial cost
Avg	2.724	2.730	2.735	2.754	2.755	2.841	13.733
Std	1.643	1.526	1.464	1.357	1.262	1.231	5.077
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max	12.165	10.335	10.335	8.317	6.349	6.349	27.361

cases with no tugboats positioned along the coast. For $\alpha = 0.00$, which corresponds to the minimization of the expectation, the maximum cost is equal to NOK 12.165 millions, whereas that of $\alpha = 0.95$ in only NOK 6.349 millions. We notice, however, that the average cost increases with α. That is, the managers at the VTS center will have to make a trade-off between avoiding the high cost of a worst case scenario at the expense of higher expected cost. In addition, the standard deviation for this test case decreases with higher values of α (see Table 2). The risk averse OTP model gives, indeed, more options with regards to tugboats policy.

The histograms in Figure 1 present the distribution of the potential environmental cost for different decisions that are computed using different values of α as well as that of the initial cost in case of no action from tugboats. As shown in Figure 1(a), there is a considerable average and maximum cost reduction for $\alpha = 0.00$ as opposed to the initial cost, which represent the cost of having no operational tugboats in the region. In Figure 1(b-c), we see a great reduction of the upper tail for the risk averse distribution. This might not be representative as we only have very few worst case scenarios. Nevertheless, we clearly see, in Figure 1(c), how the worst case cost distribution is almost contained in that of the risk neutral. That is, the policy for $\alpha = 0.95$ shape the cost distribution by reducing the maximum, but also increasing the minimum value.

In Figure 2, we present a zoomed-in view of the grounding location with the nearest tugboat positions for $\alpha = 0.00$ and $\alpha = 0.95$. The blue solid lines in Figure 2 represent the predicted drift trajectories and the actual trajectory followed by the drifting vessel is represented in green solid line. In addition, the directed path in red solid lines are the actual positions of the nearest tugboat from the time of distress to the time of grounding. The paths in dashed green line with squares and triangles are suggested positions of the nearest tugboat by the model prior to the distress call for $\alpha = 0.00$ and $\alpha = 0.95$ respectively. Note that, in these green paths, the last positions in red square and triangle represent the tugboat positions at the time of distress call, not at the grounding time. For the risk neutral policy ($\alpha = 0.00$), the probability of successful hook-up of the grounded vessel if the risk-averse model was implemented is equal to 0.86, whereas that of $\alpha = 0.95$ is equal to 0.38. In fact, the risk averse policy for $\alpha = 0.95$ focuses on the minimization of the very high costs. This is why the tugboat moves toward the potential drifting vessel with high cost of about NOK 23 millions and leaves the vessel with potential cost of about NOK 10 millions. On the contrary, the risk neutral that minimizes the expectation tries to balanced the overall average cost. The nearest tugboat would have had a high successful

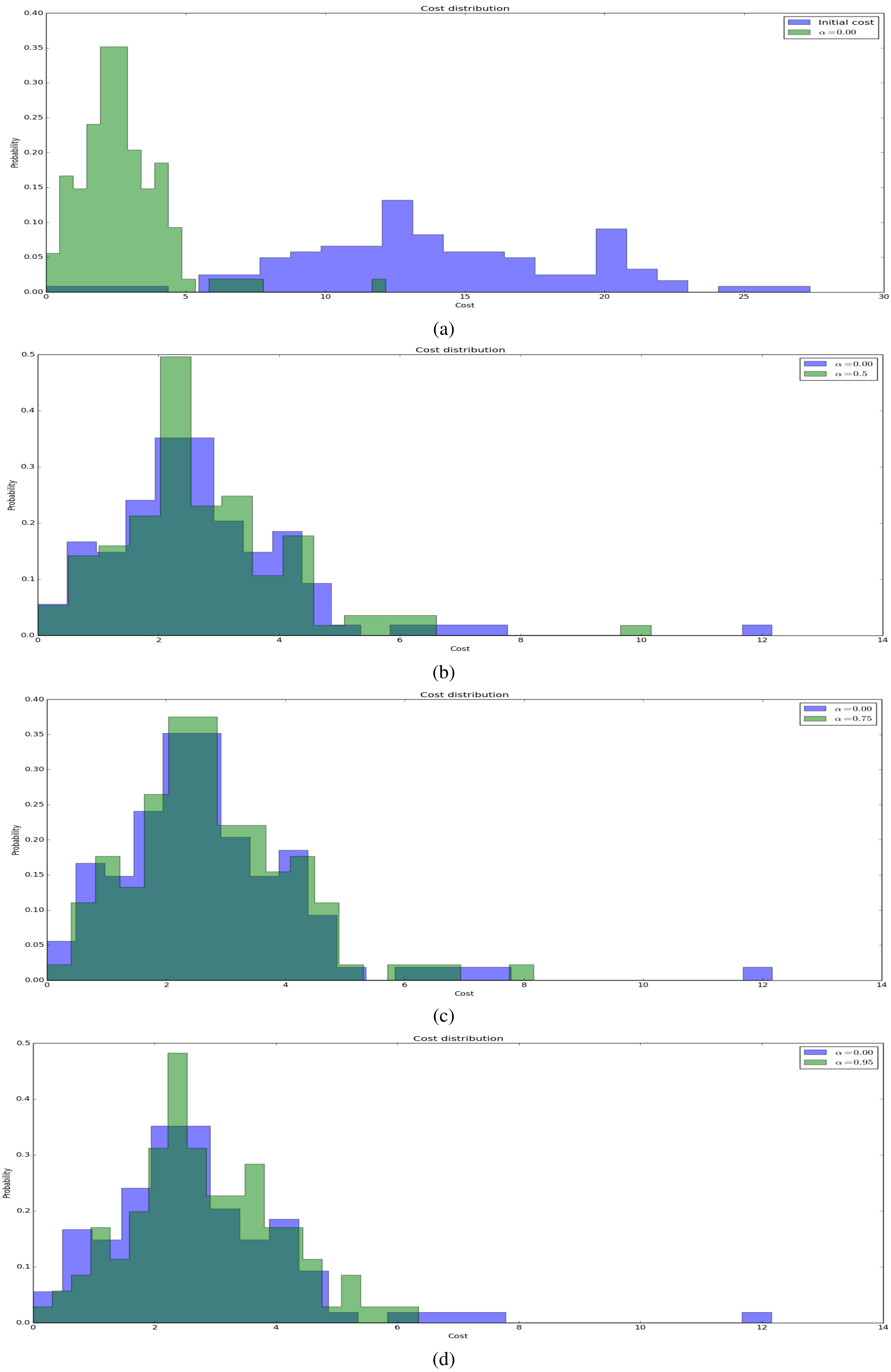


Figure 1: Event 1A. Distribution of the cost for different value of α. The histograms show the reduction of the tail of the distribution as α increases.

rescue probability if the worst case scenario with a cost of NOK 23 millions happened instead. By implementing the risk averse OTP model, the mangers at the VTS center have the flexibility and opportunity to influence these probabilities of successful hook-up for different values of α, which represents the level of risk averseness.

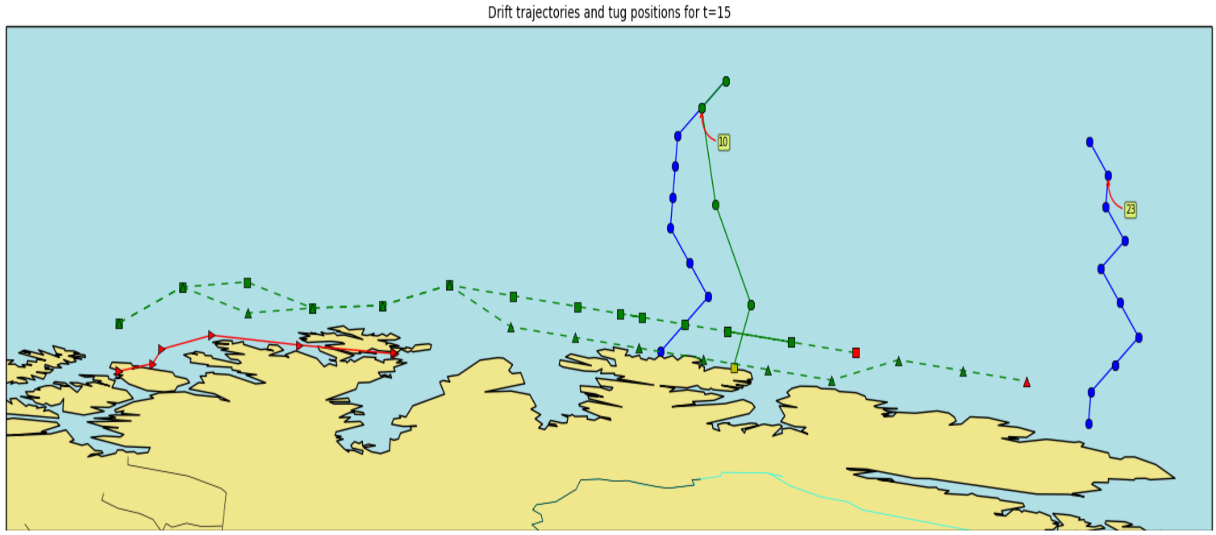


Figure 2: Zoom on the grounded vessel for Event 1A. The predicted drift trajectories are represented in blue solid lines and the actual drift trajectory of the vessel that ran aground is represented in green solid lines. In addition, the estimated grounding cost in million NOK for each drift trajectory is labeled in yellow and the path in red is the actual positions of the nearest tugboat from the time of distress to the time of grounding. The dashed green lines represent the suggested movements of the nearest tugboat by the MIP-1 for $\alpha = 0.00$ and $\alpha = 0.95$, where the small squares and triangle represent the positions of the nearest tugboat for $\alpha = 0.00$ and $\alpha = 0.95$ respectively.

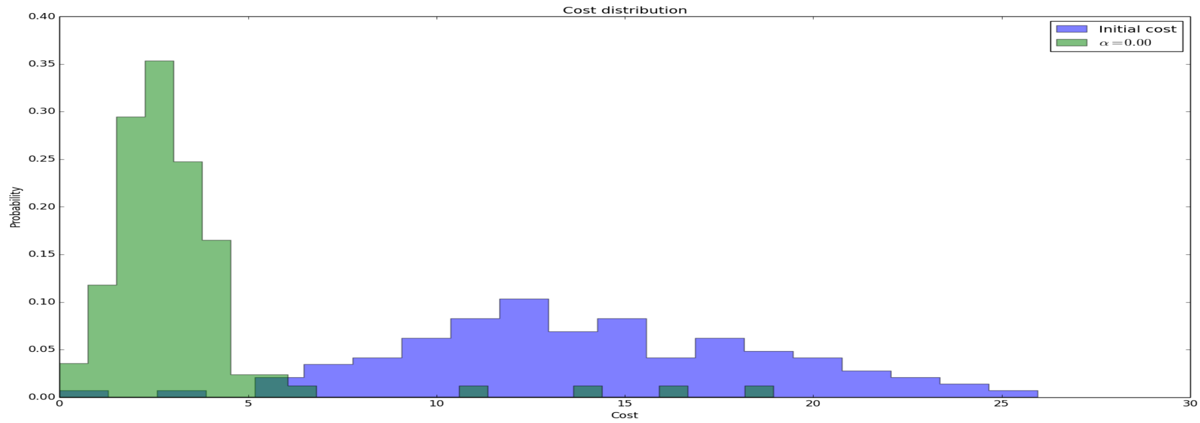
4.3.3 Event 1B: September 12, 2014

On September 12 2014 at 2:25pm, a vessel ran ashore at $71^o 02.97' N - 023^o 53.89' E$ after about 9 hours of drifting time. The nearest tugboat stayed static for the whole drifting (see Assimizele et al. (2016) for detailed explanation) time. We run the MIP-1 model with this case for 15 hours prior to the time of distress of the grounded vessel. A total of nine vessels moved in the region within the 15 hours considered and the corresponding predicted drift trajectories are included in the MIP-1 model, which is run for different levels of risk averseness.

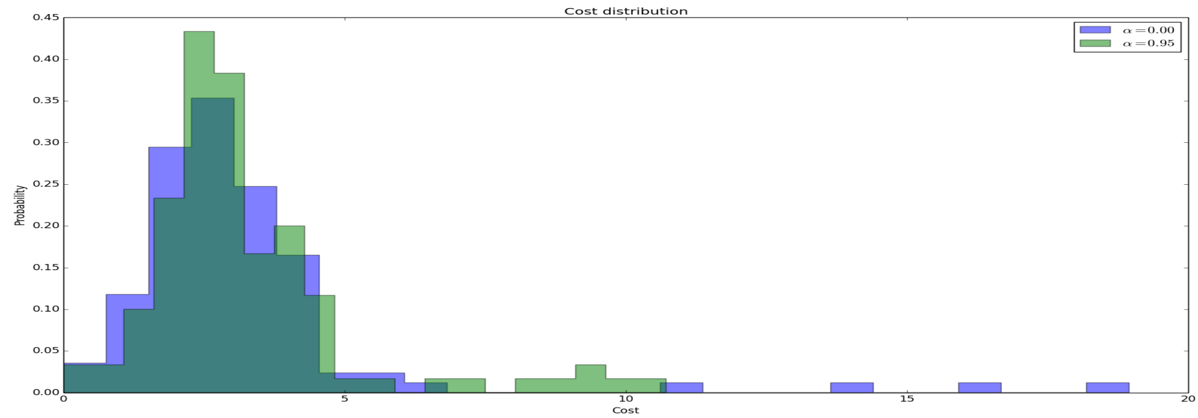
Table 3: Numerical results for Event 1B with costs presented in million NOK.

α-s-risk	0.00	0.25	0.50	0.75	0.90	0.95	Initial cost
Avg	3.194	3.236	3.231	3.228	3.240	3.233	13.943
Std	2.627	1.875	1.857	1.830	1.849	1.850	4.802
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max	18.948	10.735	10.735	10.735	10.735	10.714	25.959

For this particular case, there is no considerable difference on the average cost for each value of α as presented in Table 3. Additionally, the maximum value is constant for $\alpha \geq 0.25$. This means that for some particular instances, the managers at the VTS center will not have much options in terms of risk averseness level. This may rise the need for an increase on the fleet of tugboat, which was reduced from three to two in about a year ago. The optimal fleet problem is out of the scope of this paper. Despite the constant maximum value of the cost for a risk averse policy, the difference between that of the risk neutral is about NOK 8.213 millions. In addition, the standard deviation for the risk averse policy is NOK $752,000$ lower that the risk neutral policy. As presented in Figure 3, there is a considerable environmental expected and maximum cost reduction with the use of the risk averse OTP model for $\alpha = 0.00$ and $\alpha = 0.95$.



(a)



(b)

Figure 3: Event 1B. Distribution of the cost for different value of α. The histograms show the reduction of the tail of the distribution as α increases.

The zoom on the grounding location is presented in Figure 4, where the red circle represents the position of the nearest tugboat. For this case, the predicted drift trajectory has a grounding position about 43 km from the location where the vessel actually run aground. The nearest tugboat position for the policy with $\alpha = 0.0$ and $\alpha = 0.95$ are almost the same. In fact, there are more potential drift trajectories on the left side of the tugboat, and while the risk neutral policy tries to balance the overall expected cost, the risk averse policy attempts to move closer to the drift trajectories with slightly high cost. The probabilities of successful hook-up with the grounded vessel if the MIP-1 model was implemented are equal to 0.69 and 0.72 for $\alpha = 0.00$ and $\alpha = 0.95$ respectively. Additionally, the risk averse policy has more tugboat movement and consequently larger total travel distance. Obviously, a significant reduction in the worst case scenarios require more active smart-movement of tugboats.

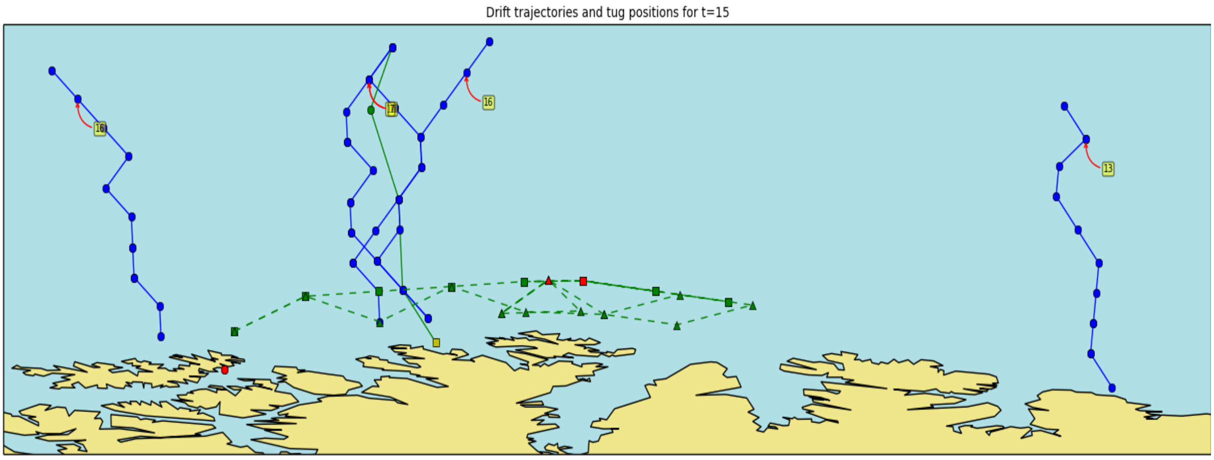


Figure 4: Zoom on the grounded vessel for case 1B. The predicted drift trajectories are represented in blue solid lines and the actual drift trajectory of the vessel that ran aground is represented in green solid lines. In addition, the estimated grounding cost in million NOK for each drift trajectory is labeled in yellow and the red cycle is the actual static position of the nearest tugboat at the time of distress call. The dashed green lines represent the suggested movements of the nearest tugboat by the MIP-1 for $\alpha = 0.00$ and $\alpha = 0.95$, where the small squares and triangle represent the positions of the nearest tugboat for $\alpha = 0.00$ and $\alpha = 0.95$ respectively.

4.3.4 Cases with large number of Scenarios

In this subsection, we use the same historical events from the previous subsection, but with larger number of scenarios of more than $50,000$ for each case. We consider exactly three potential predicted drift trajectories for each vessel scenario ω_v $v \in \mathcal{V}$. Indeed, having more than one predicted drift trajectory allows to better address the uncertainly in the weather condition and improves accuracy on the potential environmental cost estimate. There is a significant difference between the amount of oil spills from the vessel that runs aground on sands and the one that runs rocks. Thus, inaccurate predicted drift trajectory could be misleading on the environmental costs evaluation.

4.3.5 Event 2A: March 21, 2014

The maximum worst case scenarios presented in Table 4, decreases as α increases and is equal to NOK 13.931 millions for $\alpha = 0.95$. Additionally, the expected cost for this event with large number of scenarios in Table 4 range from NOK 3.480 millions to NOK 3.525 millions for $\alpha \in [0.00, 0.95]$, which is about NOK $700,000$ higher than that of Event 1A. Moreover, the standard deviations for this case are about a million NOK higher than those of smaller number of scenarios in Event 1A.

Table 4: Numerical results for Event 2A with costs presented in million NOK.

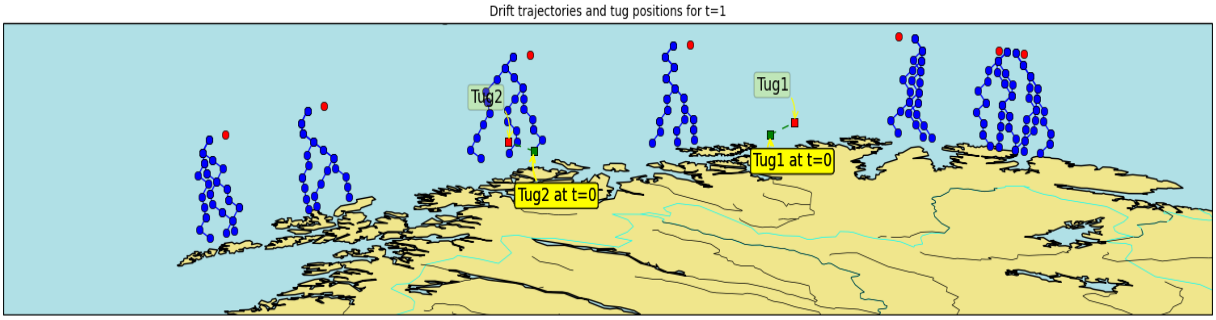
α-s-risk	0.00	0.25	0.50	0.75	0.90	0.95	Initial cost
Avg	3.480	3.494	3.494	3.497	3.503	3.525	13.749
Std	2.800	2.784	2.676	2.649	2.526	2.534	4.664
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max	16.991	16.991	15.991	14.517	14.480	13.931	29.892

To evaluate the actual solutions difference between Event 1A and Event 2A, we compute the costs with large scenarios number with tugboat policy from Event 1A and present the results, for $\alpha = 0.0$ and $\alpha = 0.95$, in Table 5. The expected maximum environmental cost for the policy with few scenarios is equal to NOK 21.335 millions and NOK 20.632 millions for $\alpha = 0.00$ and $\alpha = 0.95$ respectively. This maximum cost is smaller for the policy with large scenarios numbers and is equal to only NOK 16.991 millions and NOK 13.931 millions for $\alpha = 0.00$ and $\alpha = 0.95$ respectively. Clearly, the use of single drift trajectory to predict the path followed by a drifting vessel ignores the uncertainty associated the ocean currents and wind forces, and wave magnitude. The policy for few number of scenarios significantly underestimates the potential environmental cost (see Table 2 and Table 5).

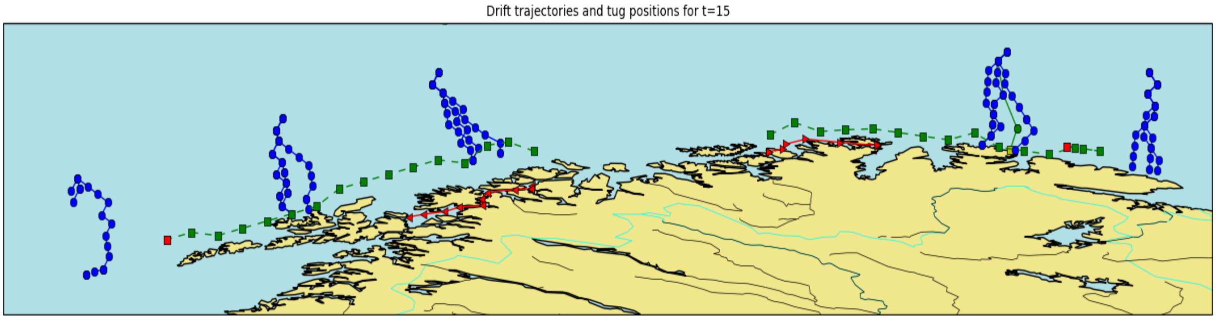
Table 5: Comparison between Event 1A and Event 2A
with costs presented in million NOK.

	Few Scenarios		More Scenarios	
α-s-risk	$\alpha = 0.00$	$\alpha = 0.95$	$\alpha = 0.0$	$\alpha = 0.95$
Avg	4.662	4.924	3.480	3.525
Std	3.118	3.356	2.800	2.534
Min	0.000	0.000	0.000	0.000
Max	21.335	20.632	16.991	13.931

In Figure 5, we present the results of the first and the last time period for $\alpha = 0.95$. The solution is obtained in 18.55 minutes and the 0.95-superquantile value for this case is equal to NOK 8.37 millions, which correspond to the expectation of the worst case environmental cost. The red cycles in Figure 5(a) represent the initial positions of the vessels and the blue solid line are the predicted drift trajectories of the potential vessels in distress. Moreover, the suggested tugboat positions by the risk averse model are represented in dashed green line, while the their actual positions are in red directed paths (see Figure 5). For this particular case, the probability of a successful hook-up with the vessel that run ashore is equal to 0.81, if the MIP-1 model was implemented. This is surely greater than 0.38 probability for the Event 1A of few scenarios number.



(a) t=1



(b) t=15

Figure 5: Results of the first and last time period for Event 2A with $\alpha = 0.95$. The dashed green lines represent the suggested movements of tugboats by the s-risk policy and the predicted drift trajectories are in blue solid lines. Additionally, the actual drift trajectory of the vessel that ran aground is represented by green solid lines. The paths in red are the actual positions of tugboats from the time of distress to the time of grounding.

4.3.6 Event 2B: September 12, 2014

This historical event is the same as Event 1B, but with larger number of scenarios. The results for the MIP-1 model run with six different values of α are presented in Table 6. As in the previous cases, the average environmental cost increases with higher values of α. In fact, the main concern of the risk averse model is not to minimize the expected cost but the rare and very expensive worst case scenarios. This is done at the slight expense of the expected environmental cost. In Table 6, the maximum cost for $\alpha = 0.95$ is about NOK 3.694 millions smaller than that of the risk neutral policy.

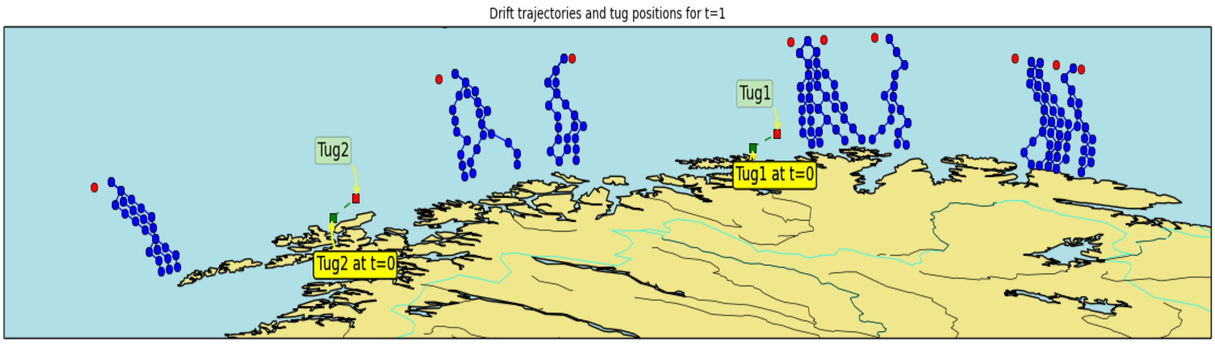
Table 6: Numerical results for Event 2B with costs presented in million NOK.

α-s-risk	0.00	0.25	0.50	0.75	0.90	0.95	Initial cost
Avg	3.694	3.694	3.712	3.714	3.898	4.040	12.938
Std	4.015	4.013	3.968	3.936	3.722	3.647	6.310
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max	22.302	22.302	20.170	20.170	18.687	18.608	28.411

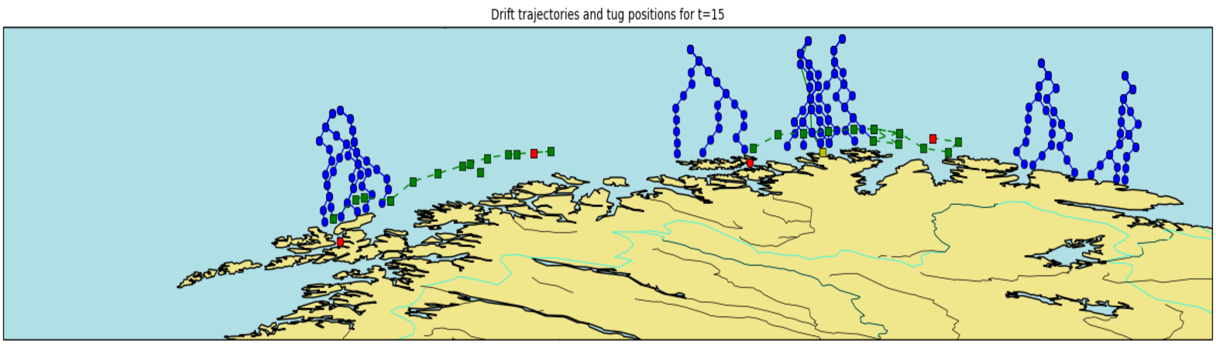
To evaluate the difference between the tugboat policy in Event 1B and Event 2B, we compute the environmental costs with tugboat policy in Event 1B for large number of scenarios from Event 2B. The numerical results in Table 7 present a higher average cost in Event 1B compare to Event 2B for the two different values of α. Moreover, the maximum cost for the case with more scenarios number is about NOK 8 millions smaller that the one with few number of scenarios for $\alpha = 0.95$. This maximum environmental cost difference is about NOK 4 millions for $\alpha = 0.00$. These results highlight the benefit of addressing the uncertainty associated with the weather conditions by predicting more than one single drift trajectory for the potential vessels in distress.

Table 7: Comparison between Event 1B and Event 2B with costs presented in million NOK.

	Few Scenarios		More Scenarios	
α-s-risk	$\alpha = 0.00$	$\alpha = 0.95$	$\alpha = 0.0$	$\alpha = 0.95$
Avg	4.270	4.409	3.694	4.040
Std	4.266	4.465	4.015	3.647
Min	0.000	0.000	0.000	0.000
Max	26.187	26.381	22.302	18.608



(a) t=1



(b) t=15

Figure 6: Results of the first and last time period for Event 2B with $\alpha = 0.95$. The dashed green lines represent the suggested movements of tugboats by the s-risk policy and the predicted drift trajectories are in blue solid lines. Additionally, the actual drift trajectory of the vessel that ran aground is represented by green solid lines. The two red cycles in Figure 6(b) are the actual static positions of tugboats.

We present the first and last time period of the results of the MIP-1 model run with this case for $\alpha = 0.95$ in Figure 6, where the decision on tugboat positions are represented by solid green lines. The expected worst case environmental cost is equal to NOK 14.078 millions and the optimal solution is obtained in 25.70 minutes. In addition, the probability of a successful rescue of the grounded vessel, if the MIP-1 was implemented, is equal to 0.67. This probability could be increased by reducing the degree of risk averseness.

5 Conclusions

This article analyzes the importance and application of s-risk measure in maritime operations. We develop a new nonlinear mixed integer programming model, build on previous work, that minimizes the expectation of the worst case scenarios. The linearized models, run with 50 realistic and randomly chosen instances, indicate a NOK 8.670 millions expected maximal cost. In addition, we are 95% confident that in case of grounding accident, the average environmental cost is less than NOK 6.646 millions. We present the flexibility that managers at the VTS center could have by modeling difference level of risk averseness with historical events for few scenarios. Moreover, we account for uncertainty on weather conditions by considering more than one predicted drift trajectory of the vessels in distress. The results with historical events run with the MIP-1 for more than $50,000$ scenarios present a considerable expected cost improvement of about one million NOK compared to previous work that uses few scenarios number. For every case, with different risk averse levels, of these historical events, the average cost is less than NOK 4 millions.

We recommend in-depth study on failure probabilities of vessels moving in the region of interest. A possible approach among others, in that respect, could be the use of Fault tree analysis (FTA). Moreover, there is a need of an empirical research on the environmental cost from oil spill assessment. In that way, one could come out with a model that is specific and consequently more accurate to the high environmental sensitive region of interest. Additionally, it will be important to evaluate the hook-up probability formula with more historical events to obtain significantly accurate model parameters. A discretization of the region is done with a 5 by 5 km cells size. Further research could explore smaller cells size, which will definitely improve the tugboat policy, but at the expense of higher complexity and solution run times. The high complexity, however, leaves room to meta heuristics algorithms such as Unified Tabu Search. Finally, one could also develop alternatives models with different objectives functions that focus of the environmental risk as opposed to the environmental costs addressed in this paper.

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A S-risk OTP Model

OTP model:

Indices

t	time period
$c\ c'\ c_t$	cells
v	vessel
g	tugboat
p	path; $p = (c_1, ..., c_T)$
ω_v	scenario for vessel v; $\omega_v = (t, p)$
$\bar{\omega}$	scenario for all vessels $\bar{\omega} = (\omega_1, ..., \omega_V)$

Sets

$\mathcal{C}$	set of cells $\mathcal{C} = \{1, ..., C\}$
$\mathcal{F}_{tg}(c) \subseteq \mathcal{C}$	set of cells reachable from cell c in period t for tugboat g
$\mathcal{V}$	set of vessels $\mathcal{V} = \{1, ..., V\}$
$\mathcal{G}$	set of tugboats $\mathcal{G} = \{1, ..., G\}$
$\mathcal{T}$	set of time periods $\mathcal{T} = \{0, 1, ..., T\}$
Ω_v	set of scenarios for vessel v
$\bar{\Omega}$	set of all possible scenarios $\bar{\Omega} = \Omega_1 \times .... \times \Omega_V$

Parameters

K_{ω_v}	grounding cost for vessel v in scenario $\omega_v = (t, p)$
R_{ω_v}	probability for vessel scenario $\omega_v = (t, p)$
$R_{\bar{\omega}}$	probability for scenario $\bar{\omega} = (\omega_1, ..., \omega_V)$, $R_{\bar{\omega}} = \prod_{v \in \mathcal{V}} R_{\omega_v}$
$Q_{gc\omega_v}$	probability of successful hook-up by tugboat g with vessel v, given tugboat g is in cell c at time of distress call t and vessel v follows scenario $\omega_v = (t, p)$
$\alpha_{gc\omega_v}$	hook-up rate with vessel v for tugboat g in cell c under scenario ω_v, $\alpha_{gc\omega_v} = -\ln(1 - Q_{gc\omega_v})$
α	confidence level, $\alpha \in (0, 1)$
β_α	VaR for a confidence level α
c_{0g}	initial position of tugboat g

Variables

x_{gct}	binary variable taking the value 1 if tugboat g is in cell c at time t, 0 otherwise
$z_{\bar{\omega}}$	dummy variables

Formulation

$$\min_{\mathbf{x}, \beta_\alpha, z_{\bar{\omega}}} \beta_\alpha + \frac{1}{1-\alpha} \sum_{\bar{\omega} \in \bar{\Omega}} R_{\bar{\omega}} z_{\bar{\omega}}$$

s.t.

$$\sum_{c\in\mathcal{F}_{tg}(c')} x_{gct-1} \geq x_{gc't} \qquad \forall g\in\mathcal{G}, \forall c'\in\mathcal{C}, \forall t\in\mathcal{T}\setminus\{0\} \tag{5}$$

$$\sum_{c\in\mathcal{C}} x_{gct} = 1 \qquad \forall g\in\mathcal{G}, \forall t\in\mathcal{T} \tag{6}$$

$$x_{g,c_{0g},0} = 1 \qquad \forall g\in\mathcal{G} \tag{7}$$

$$x_{gct} \in \{0,1\} \quad \forall g\in\mathcal{G}, \forall c\in\mathcal{C}, \forall t\in\mathcal{T} \tag{8}$$

$$z_{\bar\omega} \geq 0 \quad \forall\bar\omega\in\bar\Omega \tag{9}$$

$$\rho(\mathbf{x},\bar\omega) - \beta_\alpha \leq z_{\bar\omega} \quad \forall\omega\in\Omega \tag{10}$$

Constraints (5) ensure tugboats move only between reachable cells and constraints (6) make sure tugboats are not located in more than one cell in a time period. Constraints (7) give initial positions of tugboats at the beginning of the planning horizon.

A.1 Linear Formulations of the Risk-Averse OTP Model

The risk-averse OTP model is nonlinear because it contains a nonlinear, but fortunately convex function $\rho(\mathbf{x},\bar\omega)$, which is the sum of exponential functions. We use two linearization techniques to obtain two mixed-integer linear programing models MIP-1 and MP-2.

A.1.1 Lambda-Formulation: MIP-1

The function $\exp(-y)$ can be bounded from above on $[0, y_{max}]$ by a piecewise linear function coinciding with $\exp(-y_k)$ at points $y_k \in [0, y_{max}]$, $k\in\mathcal{K}$, where $\mathcal{K}$ denotes the set of breakpoints. We apply this method to the term $\exp(-\sum_{g\in\mathcal{G}}\sum_{c\in\mathcal{C}}\alpha_{gc\omega_v}x_{gct})$ contained in $\rho(\mathbf{x},\bar\omega)$. In addition, we set $y_{max} = \sum_{g\in\mathcal{G}}\max_{c,\omega_v}\alpha_{gc\omega_v}$ and use the inequality $\sum_{k\in\mathcal{K}}\lambda_k\exp(-y_k) \geq \exp(-\sum_{k\in\mathcal{K}}\lambda_k y_k)$ for $0 \leq \sum_{k\in\mathcal{K}}\lambda_k y_k \leq y_{max}$ to obtain the upper-bonding model MIP-1 presented below.

Risk-Averse MIP-1:

Additional Set

$\mathcal{K}$	set of breakpoints $\mathcal{K} = \{1,...,K\}$

Additional Parameters

$Y_{\omega_v k}$, $F_{\omega_v k} = \exp(-Y_{\omega_v k})$	function values at $k\in\mathcal{K}$

Additional Variables

$\lambda_{\omega_v k}$	nonnegative variable used for linearization

Formulation

$$\min_{\mathbf{x},\beta_\alpha,z_{\bar\omega}} \beta_\alpha + \frac{1}{1-\alpha}\sum_{\bar\omega\in\bar\Omega} R_{\bar\omega} z_{\bar\omega}$$

s.t.

$$(4)\text{-}(7) \text{ and}$$

$$\sum_{k\in\mathcal{K}} \lambda_{\omega_v k} F_{\omega_v k} - \beta_\alpha \le z_{\bar{\omega}} \quad \forall \omega \in \Omega$$

$$\sum_{g\in\mathcal{G}} \sum_{c\in\mathcal{C}} \alpha_{gc\omega_v} x_{gct} \ge \sum_{k\in\mathcal{K}} \lambda_{\omega_v k} Y_{\omega_v k} \quad \forall \omega_v = (t,p) \in \Omega_v,\ \forall v \in \mathcal{V}$$

$$\sum_{k=1}^{K} \lambda_{\omega_v k} = 1 \ \forall \omega_v = (t,p) \in \Omega_v,\ \forall v \in \mathcal{V}$$

$$\lambda_{\omega_v k} \ge 0 \ \forall \omega_v \in \Omega_v,\ \forall v \in \mathcal{V},\ \forall k \in \mathcal{K}$$

A.1.2 Taylor Expansion: MIP-2

In order to remove the nonlinearity in $\rho(\mathbf{x}, \bar{\omega})$, we use the standard lower-bounding approximation

$$\exp(-y) \ge \max_{k\in\mathcal{K}} \exp(-y_k) - \exp(-y_k)(y - y_k) \ \forall y \in \mathbb{R}.$$

Thus, let $Y_{\omega_v k} \in \mathbb{R}$, $k \in \mathcal{K}$. The corresponding mixed-integer linear model, MIP-1 can then be written as follow.

Model MIP-2:

Additional Parameters

$Y_{\omega_v k}$ breakpoint number k, $k \in \mathcal{K}$ for vessel scenario ω_v

Formulation

$$\min_{\mathbf{x},\beta_\alpha,z_{\bar{\omega}}} \beta_\alpha + \frac{1}{1-\alpha} \sum_{\bar{\omega}\in\bar{\Omega}} R_{\bar{\omega}} z_{\bar{\omega}}$$

s.t.

$$(4)\text{-}(7) \text{ and}$$

$$\exp(-Y_{\omega_v,k}) - \exp(-Y_{\omega_v,k}) \left(\sum_{g\in\mathcal{G}} \sum_{c\in\mathcal{C}} \alpha_{gc\omega_v} x_{gct} - Y_{\omega_v,k} \right) - \beta_\alpha \le z_{\omega_v} \forall \omega_v = (t,p) \in \Omega_v, \forall v \in \mathcal{V},\ \forall k \in \mathcal{K}$$

Paper 4

Minimizing the Environmental Risk from Oil Tanker Grounding Accidents in the High North

Minimizing the Environmental Risk from Oil Tanker Grounding Accidents in the High North

Brice Assimizele[a,b]

[a]Faculty of Maritime Technology and Operations, Norwegian University of Science and Technology, Aalesund Campus, Norway

[b]Department of Logistics, Molde University College, Molde, Norway

Abstract

To address the high environmental risk related to the increased oil tanker traffic in the High North, the Norwegian Coastal Administration (NCA) manages one of its vessel traffic service (VTS) centers in the town of Vardø, Norway. The fleet of tugboats, controlled by the VTS center operators, patrols the coastline to hook-up with any potential drifting oil tanker in the region of interest, before it runs ashore. Presently, the tugboats are controlled manually, which is not only challenging but less effective. In this paper, we develop two alternative binary integer programming models that give better tugboat policies in less computational time compared to previous work. Promising results with historical data illustrate great potential for optimal environmental risk reduction along the northern coast of Norway.

Keywords: OR in Maritime Industry; Mixed Integer Programming; Search and Rescue; Oil Spill; Risk management; Decision Support Systems ; Dynamic Resource Allocation

1 Introduction

Maritime transportation plays an essential role to the international trade as it provides a cost-effective means to transport large cargo volumes. It is, however, characterized by a high level of uncertainty, which creates various risks in terms of fatalities, environmental pollution, and loss of property. In particular, oil spills from oil tankers grounding accidents have a devastating effect on the marine ecosystem (Lecklin et al., 2011), they involve prohibitive clean-up operations costs (Montewka et al., 2013) and have a significant impact on the economic activities of the local communities (Crotts and Mazanec, 2013). Ship grounding accidents are generally caused by technical and mechanical failures, environmental factors and human errors.

Presently, there is no consensus on the statistical distribution of the causes of shipping accidents (Goerlandt and Montewka, 2014), due to the different viewpoints of accident analysis. Thus, prevention remains the primary way of addressing the environmental issues related to maritime oil transport.

In pursuit of a sustainable sea transportation in the High North, the Norwegian coastal administration (NCA) administers one of its vessel traffic service (VTS) centers in the town of Vardø, Norway. About 200 vessels are monitored daily by the VTS center of which five to six oil tankers receive special attention due to their size or risk of pollution. Through the automatic identification system (AIS), the VTS center obtains static information (cargo, identity, dimensions) and dynamic information (heading, position) from oil tankers moving in the region. Additionally, dynamic models of wind, ocean currents, wave heights and weather forecasts are used to predict potential drift trajectories and grounding locations of vessels. Moreover, the oil tankers are required by law to move along a predefined corridor approximately 50 km away from the coastline. Any oil tanker that loses s its maneuverability through steering or propulsion failure is immediately assigned to the closest patrol tugboat for rescue operation before it runs ashore. The size of the zone of interest is about 1100 km of coastline, and the number of tankers entering the region makes it difficult to effectively move the tugboats at the right place in time.

Previous work conducted by Assimizele et al. (2013); Bye (2012); Bye and Schaathun (2015) consider a one-dimension modeling approach and focus on the minimization of the distances between potentially drifting vessels and the nearest tugboat by means of genetic algorithms and a mixed integer programming (MIP). Their model and algorithms allocate tugboats to oil tankers, but do not give information on the probability of successful hook-up. In addition, the implementation of a one-dimension modeling approach is problematic, as it would give inaccurate geographical positions. Moreover, the dynamic risk model developed by Eide et al. (2007) prioritizes oil tankers based on their potential oil spill volume in case of accidents, but does not suggest tugboat positions. Assimizele et al. (2016) develop a two-dimension mathematical model, with hook-up probabilities, that minimizes the expected cost of grounding accidents. Despite these improvements, they do not account for uncertainty on weather conditions and use only one drift trajectory to predict the path followed by the potential drifting vessel. Moreover, the discretization of the region into cells of 5 by 5 km in their models is very large for optimal tugboat policies. In this paper, we address these issues by developing two alternative mathematical models that use more than one drift trajectory and smaller cells size for optimal decisions on tugboat positions in less computational time.

The remainder of this paper is organized as follows. Section 2 formulates the tugboat positioning problem and presents the two linear integer models that minimize the environmental risk from oil tanker grounding accidents. We discuss the integration of a receding horizon control (RHC) into the mathematical models in Section 3. In addition, we present the numerical results with realistic test instances as well as case studies with historical events in Section 4. Finally, conclusions and further research are provided in Section 5.

2 Model formulation

Following the formulation approach from Assimizele et al. (2016), we discretize the time horizon into a finite set of time periods $\mathcal{T} = \{1,2,...,T\}$ and subdivide the region of interest controlled by the VTS center into a finite number of cells $\mathcal{C} = \{1,...,C\}$. Each tugboat or oil tanker occupies one cell at each time period and can move to neighborhood cells depending on the speed, which is influenced by the weather conditions. The non-drifting oil tankers move on cells defined in the corridor and tugboats in the zone close to shore and approximately parallel to the corridor. Furthermore, the cells are constructed in a way that any tugboat will not need more than a time period to move from a given cell, except cells with very bad weather conditions at specified time periods.

For every oil tankers (vessels) v, in the set $\mathcal{V}$, entering the region of interest, we consider independent potential drift trajectories at each time period in the defined time horizon. Let $\bar{\Omega}$ represent the set of possible scenarios in the planning horizon. Obviously, $\bar{\omega} \in \bar{\Omega}$ is a combination of drift trajectories (vessel scenarios), or normal routes in absence of an incident, followed by each vessel. That is, $\bar{\omega} = (\omega_1,...,\omega_v)$, where $\omega_v \in \Omega_v$ denotes the vessel scenario for vessel v in a given time period and Ω_v is the set of all possible scenarios for vessel v. In case of drift, a vessel will follow a path denoted by $p = (c_1, c_2, ..., c_T)$, $c_t \in \mathcal{C}$, which is a succession of cells followed by the drifting vessel. Although the model inputs are updated every time period as discussed in Section 3, uncertainty on drift trajectories is addressed by predicting more than one single potential path. That is, $\omega_v = (t, p_i) \forall p_i \in \mathcal{P}_{\omega_v t}$, where $\mathcal{P}_{\omega_v t}$ represents the set of all predicted path for vessel scenario ω_v at time of distress call t and we denote by N the cardinality of $\mathcal{P}_{\omega_v t}$. Thus, $\omega_v = (t, p_i)$ represents the potential scenario for vessel v, where $t \in \mathcal{T} \cup \{T+1\}$ is the time the VTS center notices or is alerted to the distress of vessel v and p_i are the predicted path followed by the drifting vessel. In the absence of incident, t is set to $T+1$.

Let $\mathcal{G}$ be the set of tugboats run by the VTS center in the town of Vardø. At the beginning of the planning, each tugboat $g \in \mathcal{G}$ is positioned at initial cell $c_{0g} \in \mathcal{C}$. The tugboats can only transit between neighborhood cells at each time period, which is determined by their maximal speeds in the planning horizon. Accordingly, let $\mathcal{F}(c) \subset \mathcal{C}$ be the set of cells that are adjacent to $c \in \mathcal{C}$. Thus, $\mathcal{F}(c)$ represents cells that are reachable from cell c within one time period.

The main objective is to determine the position of tugboats at each time period such that the expected environmental consequence of oil tanker grounding accidents is minimized. Thus, let K_{ω_v} denote the environmental consequence associated with oil tanker v if vessel scenario ω_v occurs and no tugboat manage to rescue it before it runs ashore. In the next subsection, we present the risk model $Risk_{\omega_v}$ for any vessel scenario ω_v that helps to derive the risk for all scenarios $\bar{\omega}$, which represents the main function to be minimized in the two binary integer programming (BIP) models presented in Subsection 2.2.

2.1 Environmental Risk Modeling for Drift Grounding Vessels

A risk is a combination of the probability of an event and its consequence. In drift grounding accidents, the risk model for each potential vessel scenario ω_v is the product of the probability of failure R_{ω_v}, the probability of grounding given that it is adrift, $1-Q_{\omega_v}$ and the environmental consequence, K_{ω_v}:

$$Risk_{\omega_v} = R_{\omega_v}(1-Q_{\omega_v})K_{\omega_v} \tag{1}$$

An oil tanker might start drifting at any time period with a certain probability, R_{ω_v}, that depends on the internal factors from the oil tanker itself as well as wind and current forces, and wave heights.

In equation (1), Q_{ω_v} represents the probability of successful hook-up of the drifting vessel with the nearest tugboat. The VTS center detects every drifting oil tanker and informs the nearest tugboat. Practically, the tugboat response time is determined by three main factors: (1) preparation time (reaction time and mobilization time), (2) sailing time and (3) connection or towing time. Eide et al. (2007) illustrate the need for further analysis on the weather dependent towing time which is about 2 hours. Once the drifting vessel is reached by the tugboat, the time t_l left before it runs ashore will determine the probability of successful hook-up. Thus, the probability of successful hook-up with the nearest tugboat given that the vessel is adrift, denoted by Q_{ω_v}, mainly depends on t_l. Accordingly, let $Q_{gc\omega_v}$ denote the probability of successful hook-up by tugboat g with drifting vessel v, given tugboat g is in cell c at time of distress call t and vessel v follows scenario $\omega_v = (t, p_i)$. This probability depends on the position of the nearest tugboat at time of distress call, currents, wind, waves, distance of the vessel to shore and property of the drifting vessel such as type, draft, size and loading condition. All these dependencies are captured in $\lambda_{gc\omega_v}$, which is the predicted time left once tugboat g, in cell c at time of distress call t, reaches the drifting vessel in scenario $\omega_v = (t, p_i)$. As in Assimizele et al. (2016), we determine $\lambda_{gc\omega_v}$ using the maximal operational speed of the nearest tugboat and its location relative to the drifting vessel's trajectory, and set

$$Q_{gc\omega_v} = \frac{\beta_{\omega_v}\exp\left(\delta_{\omega_v}(\lambda_{gc\omega_v} - t_{\min})\right)}{1+\exp\left(\delta_{\omega_v}(\lambda_{gc\omega_v} - t_{\min})\right)}.$$

The parameter $t_{\min}$ represents the minimal remaining drift time required to attempt a hook-up. If $\lambda_{gc\omega_v}$ is less than $t_{\min}$, $Q_{gc\omega_v}$ is set to 0. In addition, $\beta_{\omega_v} \in [0,1]$ and $\delta_{\omega_v} \geq 0$ represent the influence of weather conditions (Assimizele et al., 2016).

The environmental consequence of a drift grounding accident depends on the expected oil spill size (S) and the impact (I) of one tonne of oil on the environment (Eide et al., 2007), such that $K_{\omega_v} = S_{\omega_v} I_{\omega_v}$. It is important to note that spill size and spill impact include both bunker and cargo spill. The spill size depends on the vessel type, size, loading condition and on whether the ship is single or double hulled. It is found by combining the probability of an oil spill τ_{ω_v}, given that the vessel run aground with the expected oil outflow in the event of oil spill, O_{ω_v}, in scenario ω_v: $S_{\omega_v} = \tau_{\omega_v} O_{\omega_v}$. Moreover, $O_{\omega_v} = \alpha_{\omega_v}\gamma_{\omega_v}\mathrm{Dwt}$, where α_{ω_v} is the expected outflow rate given as a percentage of the tank content volume and γ_{ω_v} is the volume of cargo and bunker oil as a percentage of vessel dead-weight tonnage Dwt.

The oil spill impact per tonne depends on the type of oil spilled and the vulnerability of the affected area. This is modeled as environmental sensitivity index, E_{ω_v} and oil type significance index, L_{ω_v} ($I_{\omega_v} = E_{\omega_v} L_{\omega_v}$). The value E_{ω_v} depends on oil type and incorporates the vulnerability and ecological significance of the geographical area. In addition, L_{ω_v} describes the significance of the oil type spilled. In case of drift for a given vessel scenario $\omega_v = (t, p_i)$, the impact of an oil spill will depend on the distance to shore, the weathering processes, the chemical composition of the oil, and the drift trajectory, which depends on the local wind and current condition.

2.2 Binary Integer Programming Models

In this subsection, we present two different binary integer models that minimize the expected environmental consequences from oil tanker grounding accidents. The first model, BIP-A1, allocates the potential drifting vessels to the nearest tugboat, while the second model, BIP-A2, focuses on the number of vessels that could not be rescued within a predefined threshold.

2.3 BIP-A1 Model

We denote by $z_{gc\omega_v}$ a binary variable taking the value 1 if tugboat g is in cell c and is the nearest tugboat at time of distress call t of vessel scenario $\omega_v = (t, p_i)$, and 0 otherwise. In addition, we assume that the probability of vessel scenarios ω_v are mutually independent. This assumption may not always be reasonable, however, we justify it by the fact that vessel in distress are usually spatially separated with few common environmental factors (Assimizele et al., 2016). Thus, the probability for a scenario $\bar{\omega}$ is given by $R_{\bar{\omega}} = \prod_{\omega_v \in \bar{\omega}} R_{\omega_v}$. In addition, we define x_{gct} as a binary variable taking the value 1 if tugboat g is in cell c at time t, and 0 otherwise. The environmental risk function to be minimized is then written as followed:

$$f(\bar{z}) = \sum_{\bar{\omega}=(\omega_1,\ldots,\omega_v)\in\bar{\Omega}} R_{\bar{\omega}} \sum_{\omega_v\in\bar{\omega}} \sum_{g\in\mathcal{G}} \sum_{c\in C} (1 - Q_{gc\omega_v}) K_{\omega_v} z_{gc\omega_v},$$

where $\bar{z}$ denotes the vector with components $z_{gc\omega_v}$. The binary integer programming model below is developed to optimally minimize the objective function $f(\bar{z})$ subject to some constrains.

Indices

t	time period
$c\ c'\ c_t$	cells
v	vessel
g	tugboat
p_i	path; $p_i = (c_1, ..., c_T)$
ω_v	scenario for vessel v; $\omega_v = (t, p_i)$
$\bar{\omega}$	scenario for all vessels $\bar{\omega} = (\omega_1, ..., \omega_v)$

Sets

$\mathcal{C}$	set of cells
$\mathcal{V}$	set of vessels
$\mathcal{G}$	set of tugboats
$\mathcal{T}$	set of time period
$\mathcal{F}(c) \subseteq \mathcal{C}$	set of cells adjacent to cell c
$\mathcal{P}_{\omega_v t}$	set of paths for vessel scenario $\omega_v = (t, p_i)$
Ω_v	set of scenarios for vessel v
$\bar{\Omega}$	set of all possible scenarios $\bar{\Omega} = \Omega_1 \times ... \times \Omega_v$

Parameters

K_{ω_v}	environmental consequence associated with vessel v in scenario ω_v
R_{ω_v}	failure probability for vessel scenario ω_v
$R_{\bar{\omega}}$	probability for scenario $\bar{\omega} = (\omega_1, ..., \omega_v)$, $R_{\bar{\omega}} = \prod_{v \in \mathcal{V}} R_{\omega_v}$
$Q_{gc\omega_v}$	probability of successful hook-up by tugboat g with vessel v, given tugboat g is in cell c at time of distress call t and vessel v follows scenario $\omega_v = (t, p_i),\ p_i \in \mathcal{P}_{\omega_v t}$
c_{0g}	initial position of tugboat g

Variables

x_{gct}	binary variable taking the value 1 if tugboat g is in cell c at time t, 0 otherwise
$z_{gc\omega_v}$	binary variable takes the value 1 if tugboat g is in cell c at time of distress call t and is allocated (nearest) to vessel v doing scenario ω_v

Formulation

$$\min f(\bar{z})$$

s.t.

$$\sum_{c \in \mathcal{F}_{tg}(c')} x_{gct-1} \geq x_{gc't} \qquad \forall g \in \mathcal{G}, \forall c' \in \mathcal{C}, \forall t \in \mathcal{T} \backslash \{0\} \tag{2}$$

$$\sum_{c \in \mathcal{C}} x_{gct} = 1 \qquad \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \tag{3}$$

$$x_{g,c_{0g},0} = 1 \qquad \forall g \in \mathcal{G} \tag{4}$$

$$z_{gc\omega_v} \leq x_{gct} \qquad \forall g \in \mathcal{G}, \forall c \in \mathcal{C}, \omega_v = (p,t) \in \Omega_v, v \in \mathcal{V} \tag{5}$$

$$\sum_{g \in \mathcal{G}, c \in \mathcal{C}} z_{gc\omega_v} = 1 \qquad \forall \omega_v \in \bar{\omega}, v \in \mathcal{V} \tag{6}$$

$$x_{gct}, z_{gc\omega_v} \in \{0,1\} \qquad \forall g \in \mathcal{G}, \forall c \in \mathcal{C}, \forall t \in \mathcal{T}, \forall \omega_v \in \Omega_v, v \in \mathcal{V} \tag{7}$$

Constrains (2) ensure tugboats move only between neighborhood cells. In addition, constrains (3) make sure tugboats are located in only one cell at each time period. Constraints (5) and (6) allocate nearest tugboats to vessel scenarios and ensure that each vessel scenario ω_v is allocated to only one tugboat. The initial positions of tugboats are given in constrains (4), such that cell c_{0g} is the position of tugboat g at the beginning of the time horizon.

2.4 BIP-A2 Model

The main objective of this model is to minimize the expected environmental consequence associated with the potential drifting vessel scenarios that could not be rescued within a predefined threshold ρ. From the previous approach,

$$Q_{gc\omega_v} = \frac{\beta_{\omega_v} \exp\left(\delta_{\omega_v}(\lambda_{gc\omega_v} - t_{\min})\right)}{1 + \exp\left(\delta_{\omega_v}(\lambda_{gc\omega_v} - t_{\min})\right)},$$

where $\lambda_{gc\omega_v}$ represents the estimated drift time left once the vessel is reached by the tugboat. Thus, we define $H_{gc\omega_v}$ as a binary parameter taking the value 1 if tugboat g is at cell c at time of distress call t and is not able to hook-up with vessel v under scenario $\omega_v = (t, p_i)$, within a predefined threshold time ρ and 0 otherwise. Additionally, let y_{ω_v} be a variable that takes the value 1 if no tugboat is able to hook-up with vessel v, doing scenario ω_v, within a predefined threshold ρ and 0 otherwise. The expected environmental consequence to be minimized in then written as follow.

$$g(\bar{x}) = \sum_{\bar{\omega}=(\omega_1,\ldots,\omega_V) \in \bar{\Omega}} R_{\bar{\omega}} \sum_{\omega_v \in \Omega_v, v \in \mathcal{V}} K_{\omega_v} y_{\omega_v}$$

where $\bar{x}$ denotes a vector with components x_{gct}.

In addition to $g(\bar{x})$, we define $u(\bar{x})$ as a function that give incentive to tugboats to optimally position themselves once the threshold is reached.

$$u(\bar{x}) = \sum_{\bar{\omega}=(\omega_1,\ldots,\omega_V)\in\bar{\Omega}} \sum_{\omega_v\in\Omega_v, v\in\mathcal{V}} \sum_{c\in\mathcal{C}} \sum_{g\in\mathcal{G}} \frac{K_{\omega_v} x_{gct}}{\lambda_{gc\omega_v}}.$$

The BIP-A2 model that minimizes the objective functions $g(\bar{x})$ and $u(\bar{x})$ is presented below.

Additional Parameters

$\lambda_{gc\omega_v}$	Drift time left once tugboat g reaches vessel v, given tugboat g is in cell c at time of distress call t and vessel v follows scenario $\omega_v = (t, p_i), p_i \in \mathcal{P}_{\omega_v t}$
$H_{gc\omega_v}$	Takes the value 1 if tugboat g is at cell c and is not able to hook-up with vessel v under scenario ω_v, within a predefined threshold ρ, and 0 otherwise.

Additional Variables

y_{ω_v}	binary variable taking the value 1 if no tugboat is able to rescue vessel v in scenario ω_v, 0 otherwise

Formulation

$$\min g(\bar{x}) + u(\bar{x})$$

s.t.

(2)-(4) and

$$\sum_{c\in\mathcal{C}} \sum_{g\in\mathcal{G}} H_{gc\omega_v} x_{gct} \le y_{\omega_v} + \mathbf{card}(\mathcal{G}) - 1 \quad \forall \omega_v \in \bar{\Omega} \tag{8}$$

$$x_{gct}, y_{\omega_v} \in \{0,1\} \qquad \forall g \in \mathcal{G}, \forall c \in \mathcal{C}, \forall t \in \mathcal{T}, \forall \omega_v \in \Omega_v, v \in \mathcal{V} \tag{9}$$

Constraints (8) capture the vessels that could not be reached within the predefined threshold and the other constraints are the same as in BIP-A1.

3 Integrating the BIP Models with the RHC

In this section, we integrate the RHC algorithm with the BIP models to account for uncertainty in weather conditions and dynamic changes of the input parameters. A RHC is a class of algorithms that make use of explicit process models to predict future response of a system, with optimizations as intermediate step. The main idea is to dynamically run the BIP model in real time, while implementing only the first time period over the whole planing horizon (see (Assimizele et al., 2016) and references therein). Indeed, updating the parameters with new accurate values improve the output quality of the BIP model. For instance, information about vessels entering and leaving the region of interest as well as available operational tugboats need to be updated at every time period. Moreover, it is certainly efficient to pro-actively include next time periods in the BIP model. We then implement the RHC algorithm as present in table 1.

Table 1: MPBIP algorithm.

Step 1:
a- Let $t := 1$; $x_{g,c_{0g},0} :=$ initial value $\forall g \in \mathcal{G}$
b- Obtain the predicted drift trajectories and velocity of vessels and tugboats.
c- Run the BIP model to obtain the optimal positions of patrol tugs.
d- Implement only the first period of the BIP solution.
Step 2:
a- Let $t := t+1$; $x_{g,c_{0g},0} := x_{gct-1} \forall g \in \mathcal{G}$
b- Update the predicted drift trajectories and velocity of vessels and tugboats. Additionally, update the current number of oil tankers moving along the zone of interest as well as the available number of tugboats. Update the probability of successful hook-up matrix .
c- Run the BIP model to obtain the optimal tugboats policy.
d- Implement the first period of the new BIP solution.
Step 3: Go back to Step 2 or stop if $t = T+1$

4 Test Cases

We present the numerical settings in this section and discuss the quality and performance of the BIP models, compared with previous work, run with realistic test cases. In addition, the promising results with a historical event highlight the important features derived from the BIP models as a decision support tool to the NCA managers.

4.1 Computational Settings

The region of interest covers about $1,100$ km of coastline and the corridor is on average 50 km away from the coast. We discretize the region by collecting the center position of each cell and transform them into Cartesian coordinates for input to the model. Once the optimal solution of the BIP model is obtained, the drift trajectories, oil tanker and tugboat positions are transformed back to geographical coordinates. The drift trajectories are obtained using the AIS and Norwegian Meteorological Institute (NMI) information with the algorithm presented in Section 3 (see (Assimizele et al., 2016) for details). Presently, the VTS center operate a fleet of two tugboats with an average operating speed of 12 knots and vessels typically have an operating speed of $14-15$ knots. The VTS center subdivided the region into two zones, where each zone is assigned to one tugboat. The first zone spans from the border to Russia to Torsvåg and the second zone from Torsvåg to Røst. Moreover, the hook-up probabilities are computed using the formula presented in Section 2.1. We set the threshold $\rho = 5$ hours, which is lager than the 2 hours average towing time. This threshold value for BIP-A2 can be changed according to the VTS center operators needs as discussed in Section 4.2.4.

Previous research, on the same region of interest, conducted by Eide et al. (2007) presents a resource specific environmental sensitivity index ranging from 1 to 9. The Coastal segments used for summarizing index values, are presented in Figure 1. Because of data accessibility, we randomly generate the environmental consequence according to a uniform random variable on [1, 9] times an absolute normal random variable with mean 5 and standard deviation 2. This is reasonable assumption as the main goal is to dynamically assess tugboat positions according to different consequence levels. Moreover, the only unavailable input data required to compute the environmental consequence is the Deadweight Tonnage (Dwt) of each vessel, which is accessible to the VTS center operators for practical implementation of the BIP model. The failure probability are randomly chosen between [0.01, 0.09] as in Assimizele et al. (2016). Specific settings to each case are presented in the corresponding subsections.

All computations are carried out on a personal computer with an Intel(R) Pentium(R)IV 3.0 CPU and 4.0 GB of RAM. The optimization software Gurobi 6.0.5 is used as a solver, with Python 2.7.3 and Pyomo 4.2., on Microsoft Windows 7.

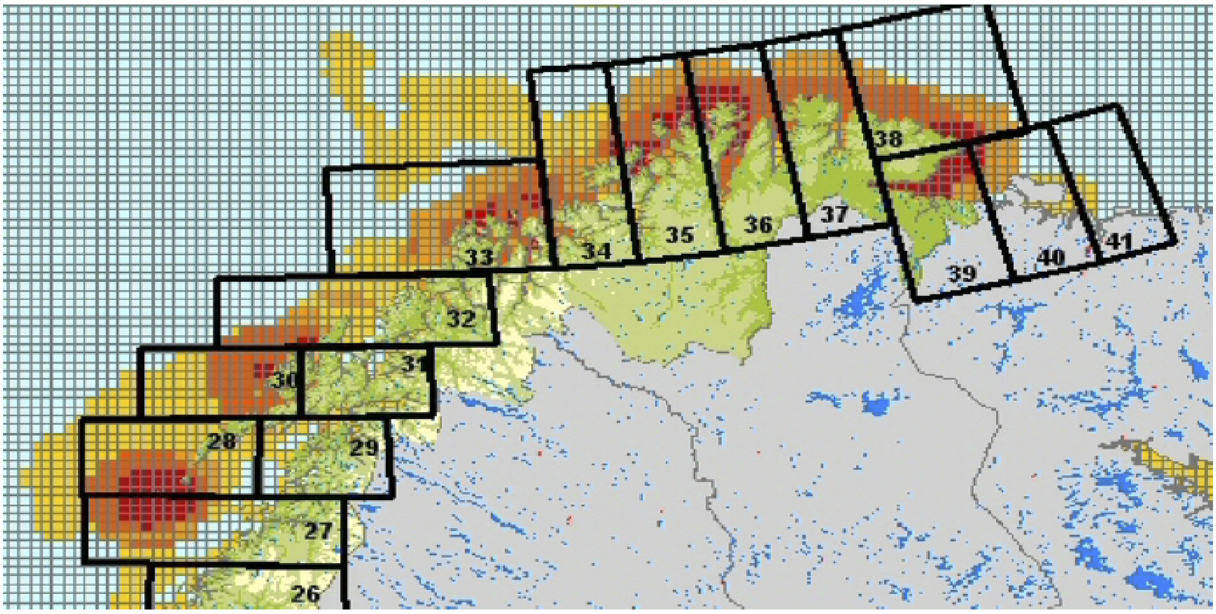


Figure 1: Coastal segments used for summarizing index values, shown in color codes on 10 by 10 km squares (Eide et al., 2007).

4.2 Test Cases with Realistic Data

This subsection discuss the numerical results for three different cases. For each case, the models are run for a total of 24 hours with 100 different instances and the environmental risk associated with each scenario is computed according to the tugboat positions from each model policy. A total number of 6 oil tankers, which correspond to the current average daily number, is used with random geographical positions, directions and speeds.

4.2.1 Case 1: Large Cell Size

In this case, we use a large cell size of 5 by 5 km as in Assimizele et al. (2016) and compare the quality and performance of their MIP-U model with those of BIP-A1 and BIP-A2 models. In order to allow comparison with the MIP-U model from previous work, we only use one predicted drift trajectory for vessel scenario ω_v. The numerical results in Table 2 present the statistical values of the computational time and environmental risk related to each scenario from the test instances for each model.

Table 2: Numerical results for 5 by 5 km cells size.

	MIP-U		BIP-A1		BIP-A2	
	Risk	Time (min)	Risk	Time (min)	Risk	Time (min)
Avrg	11.758	1.91	11.838	0.565	11.818	0.077
Std.dev	7.992	1.91	8.055	0.565	8.119	0.077
Min	1.835	5.589	1.835	4.535	1.835	1.378
Max	41.244	11.786	41.244	6.633	41.648	1.744

As presented in Table 2, the average environmental risk is almost the same for the three models with a value of 11.8. In addition, the standard deviations from the BIP-A1 and BIP-A2 models are slightly higher than that of MIP-U. The computational time is, however, very high for MIP-U model. In fact, the BIP-A1 is about three times faster than MIP-U model. Moreover, the BIP-A2 performance is by far better than those of MIP-U and BIP-A1. Nevertheless, the performances of these three models are acceptable for this case with small scenarios number and large cells size. Essentially, the models are run dynamically, where only the first step of one hour is implemented, as described in Table 1, and each of these model can be run every hour. The very small standard deviation of the computational time in BIP-A2 model is a good indication of its great performance when considering lager scenarios number discussed in the next subsection.

4.2.2 Case 2: Smaller Cell Size

In this subsection, we consider smaller cells size, of 2 by 2 km, compared to that of Case 1. We also use a single path as in Case 1 to predict the drift trajectory of each potential drifting vessel. The computational results for MIP-U and BIP-A2 models are presented in Table 3. Actually, the BIP-A1 model could not give optimal solution after two hours of run time with each test instance of this case.

Table 3: Numerical results for 2 by 2 km cells size.

	MIP-U		BIP-A2	
	Risk	Time (min)	Risk	Time (min)
Avrg	2.209	39.246	2.253	5.495
Std.dev	4.784	42.937	5.070	3.717
Min	4.034	5.589	3.577	1.378
Max	43.983	91.458	43.983	11.069

Noticeably, the computational time for both MIP-U and BIP-A2 models have considerably increased compared to the values in Case 1. Indeed, smaller cells size increases the overall number of cells, which consequently expand the problem size. The average run-time for the BIP-A2 model is equal to 5.5 minutes with a standard deviation of only 3.7 minutes. These values are significantly smaller than those of MIP-U model. The average performance of almost 40 minutes, with a maximum of 91.5 minutes, in MIP-U model makes it impossible to be run dynamically and account for uncertainty with the algorithm in Table 1. Thus, the BIP-A2 model is well suited for large number of cells as well as possible extension of the current region of interest. Additionally, the environmental risk for this case with small cells size has considerably decreased compared to that of Case 1. In fact, large number of cells increases the flexibility of tugboats and allow for better positions that minimize the environmental risk.

4.2.3 Case 3: Large Scenarios Number

This case study uses the same cells size of 2 by 2 km as in Case 2. The main difference, however, is on the total number of scenarios. In this case, we use three drift trajectories to predict the path followed by each vessel scenario ω_v. In addition, we consider up to two possible vessel scenarios ω_v for each scenario $\bar{\omega}$. This gives more than $50,000$ total number of scenarios.

Table 4: Results for 2 by 2 km cells size with BIP-A2 model.

	Initial Risk	Optimal Risk	Time (min)
Avrg	31.172	3.769	12.116
Std.dev	10.518	5.762	2.808
Min	8.637	0.349	5.589
Max	57.159	46.940	14.498

For this case, none of the MIP-U and BIP-A1 models are able to provide solutions in less than three hours. Thus, the numerical results presented in Table 4 are those of the BIP-A2 model only. The initial risk column in Table 4 represent statistical values of the potential environmental risk if no action is taken from tugboats. Fortunately, the computational time is less than 15 minutes for all the instances considered in this case, which makes it possible to combine the

BIP-A2 model with the receding horizon control algorithm described in Table 1. Obviously, the increase in the computational time compared to Case 2 is due to the lager number of scenarios and cells, which also increase the complexity of the model. Additionally, the average and standard deviation of the risk have increased from 2.253 to 3.769 and 5.070 to 5.762, respectively (see Table 3 and Table 4). This could be misleading, but it is important to note that in Case 2, only one drift trajectory is used to predict the path followed by a drifting vessel, which do not account for uncertainty such as wave heights, ocean currents and wind forces, and underestimates the real potential risk.

4.2.4 Case 4: Different values of the threshold

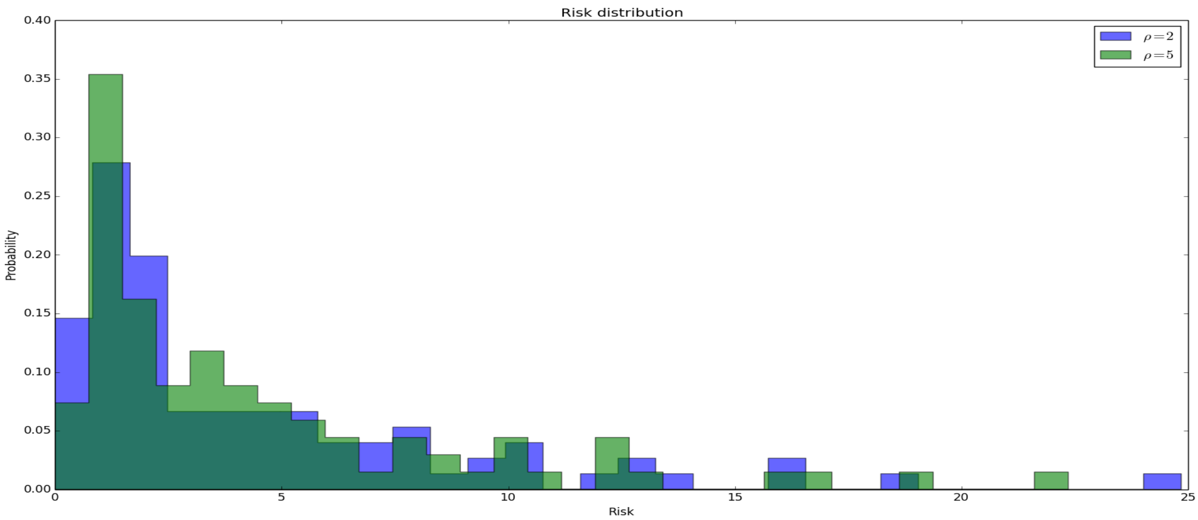
In order to assess the effect of the threshold parameter on the solution values, we run the BIP-A2 model with a real world instance. The test case consists of 9 vessels that moved along the coast over a time period of 15 hours. In addition, we discretize the region of interest into small cells with large scenarios number as in Case 3. The expected potential risk for each value of the threshold $\rho \in \{2,5,8,11\}$ are presented in Table 5.

Table 5: Results for different values of the threshold with BIP-A2 model.

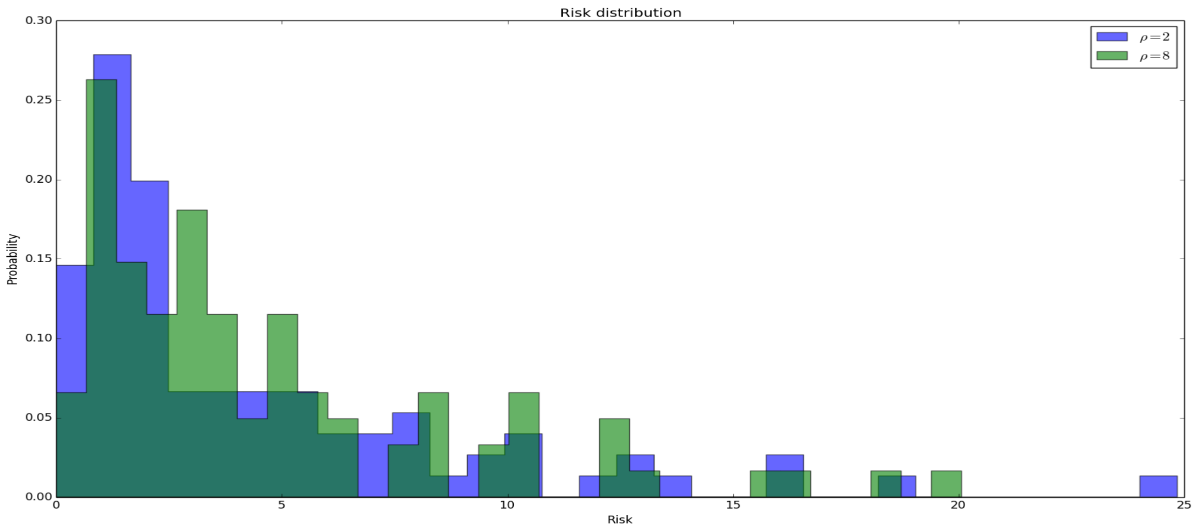
Threshold	2	5	8	11
Avrg	4.396	4.440	4.726	4.919
Std.dev	4.622	4.448	4.294	4.122
Min	1.036	1.245	1.928	3.105
Max	24.848	22.355	20.065	18.711

As presented in Table 5, the average potential environmental risk increases with higher values of the threshold. For $\rho = 2$ the expected risk is equal to 4.39, whereas that of $\rho = 11$ is equal to 4.92. We notice, however, that the standard deviation of the risk decreases with higher values of the threshold. For $\rho = 2$ the standard deviation of the risk is equal to 4.62, whereas that of $\rho = 11$ is equal to 4.12. Moreover, the maximum value of the risk decreases from 24.85 to 18.71 while the minimum risk value increase from 1.04 to 3.11 for a threshold value of 2 and 11, respectively. The threshold parameter gives more options with regards to tugboats policy and level of risk. That is, the managers at the VTS center will have to make a trade-off between having smaller standard deviation of the risk and avoiding the high risk of worst case scenarios at the expense of higher expected potential risk.

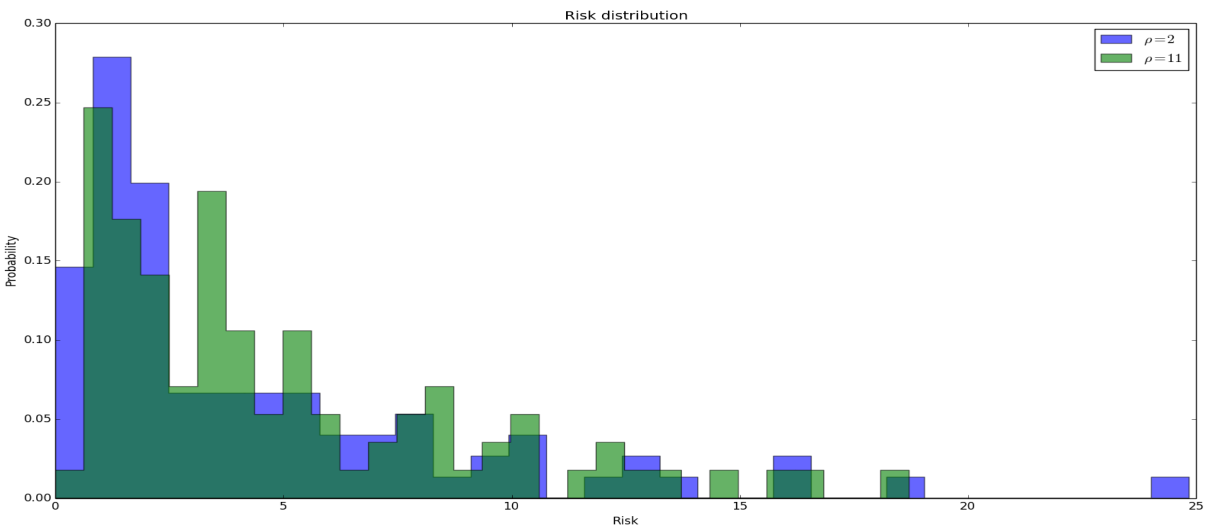
The histograms in Figure 2 present the distribution of the potential environmental risk for different decisions that are computed using different values of ρ. As shown in Figure 2(a-c), the tails of the distribution reduces with higher values of ρ. This is mostly seen on the upper tails, which represent the very rare but high risk scenarios. The policy for $\rho = 11$ in Figure 2(c) considerably shapes the distribution of the risk by reducing the maximum, but also decreases the probability of smaller risk values.



(a)



(b)



(c)

Figure 2: Case 4. Distribution of the risk for different value of ρ. The histograms show the reduction of the tail of the distribution as ρ increases.

4.3 Test Case with Historical Data

This case is based on real-world data collected from the AIS and the NMI. In addition, we use the basemap library in python to plot and draw the maps with drift trajectories, oil tanker and tugboat positions.

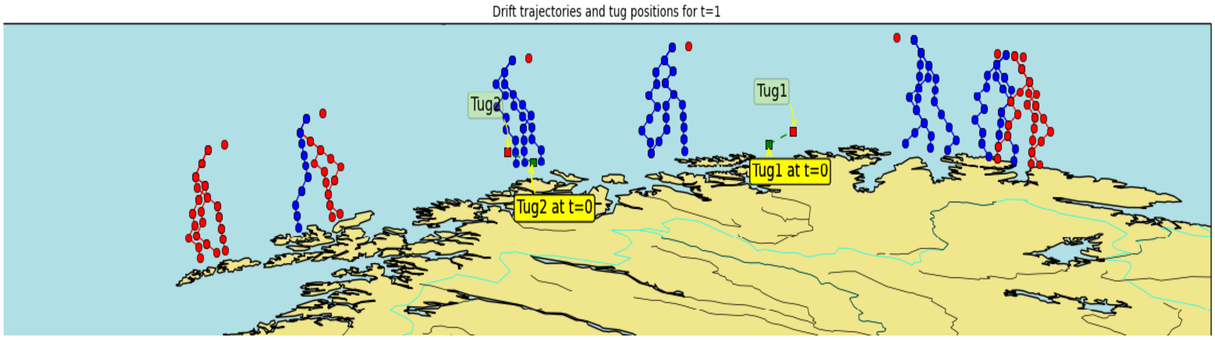
On the 21st of Mars 2014 at 11:10pm, a vessel ran aground at $N71^o01.06'N - 028^o27.46'E$ after 15 hours of drifting time. At the time of distress call, the nearest tugboat was located at $N70^o40'N - 023^o40'E$ and unsuccessfully tried to reach the drifting vessel. The tugboat was located about 142.8 km away from the vessel at the time of grounding. We run the BIP-A2 model for 15 hours prior to the time of distress call for more than $50,000$ scenarios. A total number of 7 vessels, including the one that ran ashore, sailed along the region during the considered planning horizon. Their corresponding directions, latitudes, longitudes and speeds over ground (SOG) at the beginning of the planning horizon are presented in Table 6.

Table 6: Case 3A. Initial vessel directions, positions and SOG.

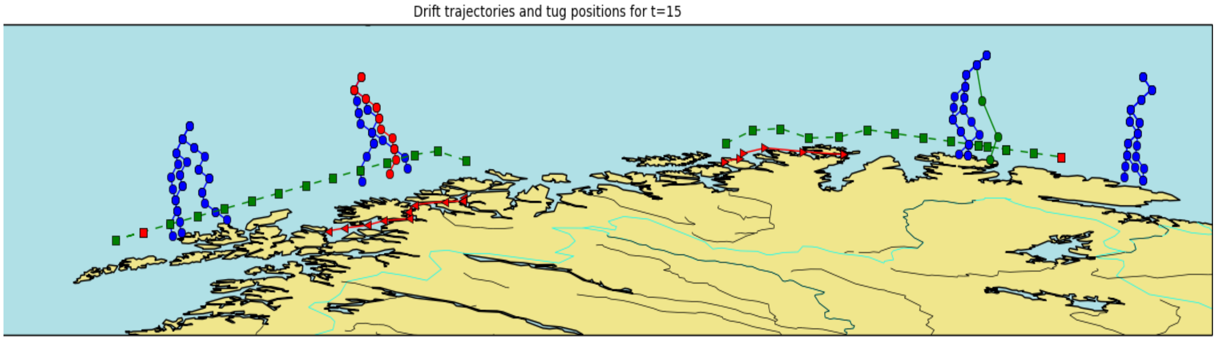
Vessel 1	Vessel 2	Vessel 3	Vessel 4	Vessel 5	Vessel 6	Vessel 7
North-west	North-west	West-north	West-north	North-west	West-north	North-west
$N71^o1$	$N71^o51$	$N69^o28$	$N71^o27$	$N71^o25$	$N71^o42$	$N68^o55$
$E028^o02$	$E022^o56$	$E013^o51$	$E028^o29$	$E026^o27$	$E019^o48$	$E012^o10$
12.4	13.4	12.5	12.6	13.0	13.1	9.1

The results for the first and last time period are presented in Figure 3, where the initial positions of vessels are presented in red cycles. In addition, the drift trajectories for vessel scenarios that could be rescued within the threshold of $\rho = 5$ hours are in blue solid lines while those of the vessels that could not be hook-up with tugboats within the threshold are in red solid line. Moreover, the actual drift trajectory followed by the grounded vessel is represented by green solid lines. Furthermore, the two red directed lines in Figure 3(b) are the actual paths followed by the tugboats from the time of distress call to the time of grounding while the green solid lines linked with small squares represent the suggested movement of the tugboats by the BIP-A2 model. The risk values associated with each vessel scenario is not presented in the figure because of the small visibility.

A zoomed-in view of the grounded location in Figure 4 shows actual and predicted drift trajectories of the grounded vessel. Additionally, the small square in red represent the suggested position of the nearest tugboat by the BIP-A2 model at the time of distress call. The probability of successful rescue of the grounded vessel by the nearest tugboat with the BIP-A2 model is about 0.70 while that of the actual tugboat policy is equal to 0.2. That is, the grounded vessel had 70% chance to be rescued if the BIP-A2 model was implemented. For this particular case, the hook-up probability is slightly smaller than that of the MIP-U model of 0.86 in Assimizele et al. (2016). In fact, their model do not account for the uncertainty on drift trajectory. Thus, in



(a) t=1



(b) t=15

Figure 3: Results of the first and last time period. The dashed green lines represent the suggested movements of tugboats by the BIP-A2 model and the predicted drift trajectories that can be rescued within the threshold $\rho = 5$ hours are in blue solid lines while those in red solid line represent the drift trajectories of the vessels that could not be hook-up within the threshold. Additionally, the actual drift trajectory of the vessel that ran aground is represented by green solid lines. The two directed red lines close to shore in Figure 3(b) are the actual positions of tugboats from the time of distress call to the time of grounding.

the long run more vessels in distress will have very low probability of successful hook-up with tugboats as very few number of scenarios are considered.

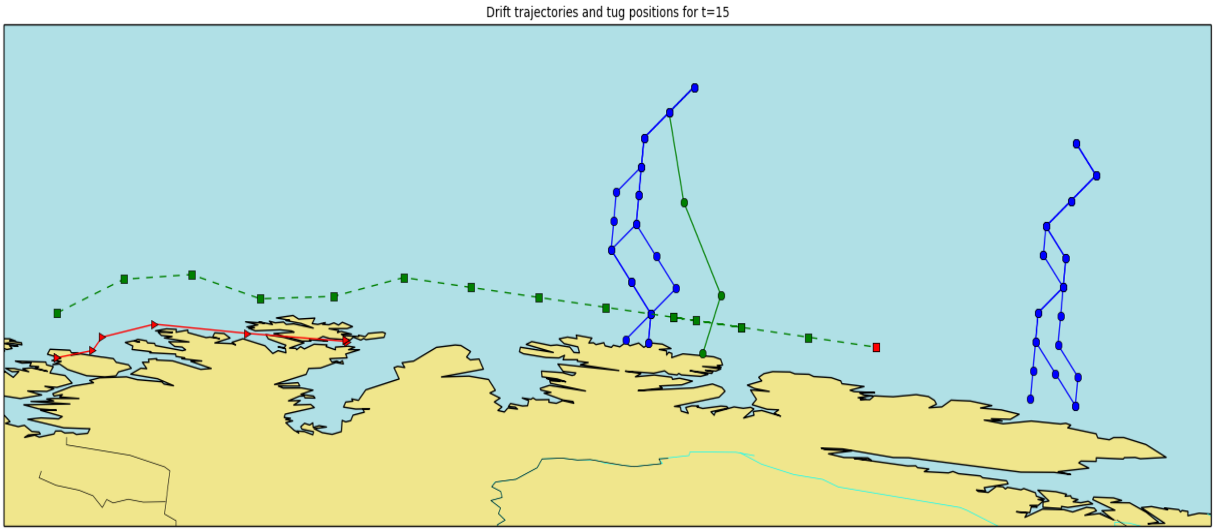


Figure 4: Zoom on the grounded vessel. The predicted drift trajectories are represented in blue solid lines and the actual drift trajectory of the vessel that ran aground is represented in green solid lines. In addition, the red directed path represent the positions of the nearest tugboat from the time of distress call to the time of grounding. The dashed green lines are the suggested movements of the nearest tugboat by the BIP-A2 model.

5 Conclusions

In this paper we address the environmental risk related to oil tankers traffic along the northern Norwegian coast. We propose two alternative models that could be used as a decision support tool at the vessel traffic service center in the town of Vardø, for a better rescue operation of vessels in distress. These models are combined with a receding horizon control algorithm to account for uncertainty in weather conditions and to dynamically update the constantly changing input parameters. For a large cells size of 5 by 5 km and smaller scenarios number, the BIP-A1 and BIP-A2 models outperform the MIP-U model from previous work. In addition, the BIP-A model is by far faster than the other models for large scenarios number and small cells size, which considerably add complexity to the models. Moreover, the BIP-A2 model gives flexibility to the operators at the VTS center by allowing different threshold levels. The results with a historical event indicate better decisions on tugboat patrol operations.

It is recommended that further research is done to determine the optimal fleet of tugboat required as well as extension of the BIP models to consider other search and rescue operations. Additionally, more research is needed to better assess the failure probabilities of vessels, oil spill rates, probability of oil spill given that an accident has occurred and environmental consequence of the region of interest. Furthermore, the hook-up probability formula could be better estimated with empirical data including new features such as ship arrestor newly acquired by the NCA to reduce the speed of the drifting vessels.

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Molde University College
Specialized University in Logistics
P.O. Box 2110
NO-6402 Molde
Norway
www.himolde.no

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