An application of the multi-depot heterogeneous fixed fleet open vehicle routing problem

Abstract

This work describes an application of a multi-depot heterogeneous fixed fleet open vehicle routing problem. A contractor owns a fleet of vehicles with different capacities and running costs. The fleet is used to transport craftsmen from their homes to assigned project sites and back, with some of the craftsmen appointed as drivers while others are passengers. An optimization model is described that enables the contractor to minimize the transportation costs, and a computational study shows that the model can be solved to optimality for realistically sized instances using a standard mixed-integer programming solver.

Keywords: mixed integer programming; assignment; transportation.

1 Introduction

Vehicle routing problems appear in many real-world planning situations (Toth and Vigo, 2014). This paper is motivated by an application where a contractor is responsible for several long-term projects, each project being manned by a group of craftsmen. In practice, similar situations may appear for different types of companies, operating using workers such as bricklayers, decorators, electricians, or plumbers. The background for the problem is that the company owns a fleet of vehicles that the workers use both for transportation between project sites and their homes and as storage for some of their equipment.

The contribution of this paper is first to show that the planning problem of the transportation company leads to a novel variant of the open vehicle routing problem. Second, a mixed integer programming model is provided for the problem, and third, computational experiments are conducted showing that the model can be solved to optimality for range of realistically sized test instances. Finally, an extension of the problem is also considered, where one of the most important constraints is relaxed and the resulting extension is solved heuristically.

The remainder of this paper is structured as follows. In Section 2 the problem faced by the company is described. Relevant literature on open vehicle routing problems is discussed in Section 3. Section 4 contains a mathematical formulation of the specific open vehicle routing problem studied in this work. A computational study is presented in Section 5, and concluding remarks are given in Section 6.

2 Problem description

A company must transport workers to project sites and back using company-owned vehicles with different capacities and running costs. Workers drive the vehicles themselves. At the start of the day, each vehicle is initially located at the home address of a worker who is appointed as the driver. The company is free to decide which vehicle each driver should use, and also to decide which workers should act as drivers. On the way to the project site, each vehicle picks up workers at their respective home locations. The total number of workers to be picked up must not exceed the vehicle's capacity. On the way back from the project site, each vehicle delivers workers back to their homes with the last home in the path being the vehicle driver's home.

The company may have several projects running simultaneously. The assignment of workers to projects is predetermined. That is, for each worker, the home location is given together with the location of the project to which the worker is assigned. A vehicle can only pick up workers going to the same project. The reason is that the workers keep some of their equipment in the vehicle between working days.

The problem is to determine which vehicles and which paths should be used to transport the workers to their corresponding project sites at the minimum cost, while taking into consideration the following:

- The travelling costs in both directions are the same.
- The starting locations of the vehicles are not fixed (given).
- Each worker home must be visited only once.
- Vehicle capacity must not be exceeded.
- Vehicles use the same route to return to the original location at the end of the working day.
- Different types of vehicles (different in terms of capacity and cost) are used to transport the workers.
- The number of vehicles of each type is fixed and limited.

In Figure 1 a small example is provided, with two project sites, nine workers, and three vehicles. The problem as described here is based on a company where not all workers are equal in terms of their experience and qualifications. However, there could exist circumstances in which all workers can be treated as equal. In that case, it makes sense that the assignment of workers to projects is not fixed, but rather a decision taken together with driver assignments and routing decisions. This variant of the problem is briefly considered at the end of Section 5.

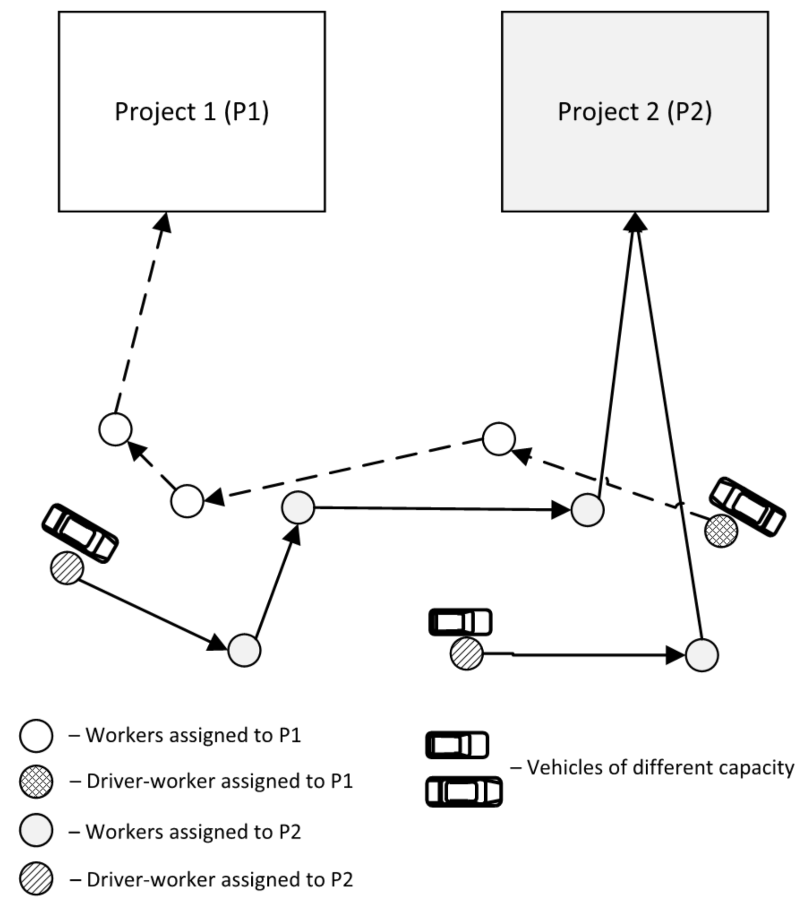

Figure 1: Conceptual representation of workers' routes to the project sites.

3 Literature Review

The *capacitated vehicle routing problem* (CVRP) is one of the most studied combinatorial optimization problems. The CVRP calls for the determination of the optimal set of routes to be performed by a fleet of vehicles to serve a given set of customers (Toth and Vigo, 2014) so that each customer is visited only once by only one vehicle and all the demand is satisfied. Each vehicle has a limited capacity. Typically, The CVRP can be used to effectively tackle problems related to delivery and collections of goods, solid waste collection, street cleaning, school bus routing, dial-a-ride systems, transportation of handicapped persons, routing of salespeople, and of maintenance units (Toth and Vigo, 2002).

In the CVRP, the transportation network is represented by a graph with arcs corresponding to road sections, and vertices corresponding to the depot and customer locations. The arcs can be either directed (traversable in only one direction) or undirected (traversable in both directions). Each arc has a cost, usually represented by the distance between its vertices, or the travel time (subject to the type of vehicle and period of the arc traversal).

In the *open vehicle routing problem* (OVRP), a vehicle is not required to return to the depot after the last customer is served (Li et al., 2007). The OVRP has many practical applications described in the literature, including delivery of goods by hired vehicles when the vehicles are not required to return to the depot, delivery of goods to customers and collection of goods from the same customers by the same vehicle on the way back, and planning a set of school bus routes, where pupils are picked up in the morning at various locations and then brought to school, and in the evening the pupils are taken back to homes in the reversed order. The latter case is similar to the problem described in Section 2, where workers are picked up at various locations to be delivered to the project sites and then delivered back to their homes after work using the reverse routes. The differences are that the OVRP considers a homogeneous fleet of vehicles (i.e., they have the same capacities and running costs), and that there is only one depot (school).

Recently the OVRP has been given increased attention from researchers and different algorithms have been developed. As the OVRP is the NP-hard problem, researchers mainly focus on meta-heuristic approaches. Li et al. (2007) reviewed OVRP algorithms and provided a variant of record-to-record travel.

Zachariadis and Kiranoudis (2010) proposed an innovative local search metaheuristic examining wide solution neighborhoods. Repoussis et al. (2010) presented a hybrid evolution strategy for solving the OVRP. Pichpibul and Kawtummachai (2013) proposed a heuristic approach based on the Clark-and-Wright algorithm. Their method includes several procedures: a modified Clark-and-Wright procedure, open-route construction, two-phase selection procedure, and route post-improvement.

The *multi-depot OVRP* (MDOVRP) is similar to the OVRP except that the vehicles can start in either of several depots. An integer linear programming model for the MDOVRP was proposed by Pichka et al. (2014). The objective function in the model aims to minimize total traveling costs while considering fixed activation costs of engaging vehicles. An ant colony optimization algorithm was been proposed by Yao et al. (2014) to solve a real-life problem of seafood delivery formulated as an MDOVRP. To deal with the multi-depot feature, they suggested to convert the problem into an OVRP by using a dummy central depot. Liu et al. (2014) presented a genetic algorithm and a mixed integer programming model for the MDOVRP, and Lalla-Ruiz et al. (2016) later provided an improved mixed integer programming model. Recently, Soto et al. (2017) proposed a metaheuristic algorithm to tackle the MDOVRP, namely a multiple variable neighborhood search hybridized with a tabu search. Shen et al. (2018) considered a version of MDOVRP with time windows, and developed a heuristic solution method based on particle swarm optimization and tabu search.

While most research on OVRPs consider a homogeneous fleet of vehicles, the *heterogeneous fixed fleet OVRP* (HFFOVRP) allows the vehicles in the fleet to vary in terms of capacities and costs. However, a very limited number of research studies cover this kind of problem. Among those few, Yousefikhoshbakht et al. (2016) proposed a mixed integer programming formulation as well as ant colony system algorithm. Moreover, they proposed a compound heuristic algorithm to solve the HFFOVRP. The algorithm includes the sweep algorithm; insert, swap, and 2-opt moves; a modified elite ant system; and column generation. Li et al. (2012) proposed a solution method for the HFFOVRP based on a multi-start adaptive memory programming metaheuristic.

4 Mathematical Formulation

The problem described in Section 2 can be interpreted as a multi-depot heterogeneous fleet open vehicle routing problem with a special structure. Below, a mathematical formulation for the problem is given, based on a model provided by Yousefikhoshbakht et al. (2015). However, some differences to this model arise for the following reasons:

- There are no fixed costs associated to using the vehicles, as the company uses its own fleet and all the fixed costs (insurance, registration, taxes) are supposed to be paid regardless whether a vehicle is used in the current transportation plan or not. It is also assumed that the workers do not get paid extra while performing the driver's function.
- There are multiple depots, which correspond to project sites.
- Each worker (customer) is assigned to a specific project site (depot). Hence, a sparse matrix of links between worker homes and project sites is used.
- Variable costs and distances are defined explicitly.

The model aims to find an optimal path for transporting workers from project sites to worker homes. The ending nodes generated as a result of solving the model will be used as starting points (initial location of vehicles) to travel to project sites. Hence, it is assumed that costs are symmetric.

Sets

V	Set of vehicle types
W	Set of worker homes
W_p	Set of worker homes assigned to project p
P	Set of project sites
A	Set of links within $W \cup P$
A_p	Set of links belonging to project p
N	Set of all nodes: $N = (1, 2, \ldots, \lvert P\rvert, \lvert P\rvert + 1, \ldots, \lvert P\rvert + \lvert W\rvert)$

Parameters

D_{ij}	Distance between node i and node j; $i, j \in N$
C_k	Driving cost per unit of distance of vehicles of type $k \in V$
Q_i^P	Number of workers to deliver at node $i \in N$
N_k	Number of vehicles of type $k \in V$
Q_k^V	Capacity of vehicles of type $k \in V$ (unit: number of workers)

Variables

x_{ijk}	Equals 1 if a vehicle of type k goes directly from node i to node j $(i \neq j)$, and 0 otherwise
y_{ijk}	The quantity of workers that a vehicle of type k is carrying after leaving node i to service node j

Model

$$\min \sum_{k \in V} \sum_{i \in N} \sum_{j \in N} D_{ij} C_k x_{ijk} \tag{1}$$

$$\sum_{k \in V} \sum_{(i,j) \in A_p} x_{ijk} = 1 \qquad p \in P, j \in W_p \tag{2}$$

$$\sum_{k \in V} \sum_{j \in W} x_{ijk} \leq 1 \qquad i \in W \tag{3}$$

$$0 \leq \sum_{(i,j) \in A} x_{ijk} - \sum_{(j,i) \in A} x_{jik} \leq 1 \qquad j \in W, k \in V \tag{4}$$

$$\sum_{p \in P} \sum_{(p,j) \in A_p} x_{pjk} \leq N_k \qquad k \in V \tag{5}$$

$$\sum_{k \in V} \sum_{i \in N} y_{ijk} - \sum_{k \in V} \sum_{i \in N} y_{jik} = Q_j^P \qquad j \in W \tag{6}$$

$$Q_j^P x_{ijk} \leq y_{ijk} \leq (Q_k^V - Q_i^P) x_{ijk} \qquad i, j \in N, i \neq j, k \in V \tag{7}$$

$$\sum_{p \in P} \sum_{i \in W} x_{ipk} = 0 \qquad k \in V \tag{8}$$

$$x_{ijk} \in \{0, 1\} \qquad i, j \in N, i \neq j, k \in V \tag{9}$$

$$y_{ijk} \geq 0 \qquad i, j \in N, k \in V \tag{10}$$

The objective (1) is to minimize the total transportation cost. Constraints (2) ensure that each worker home must be visited once. Constraints (3) make sure that at most one vehicle departs from any worker home. Constraints (4) enforces that if a vehicle visits a customer, the vehicle can either remain there (the ending point) or depart from it. Constraints (5) ensure that the number of vehicles of each type does not exceed the predefined amount. Constraints (6) guarantee that all customers' demands (number of workers to be delivered at each customer) are fully satisfied. Constraints (7) ensure that

Table 1: Data on the composition of the vehicle fleet

k	Q_k^V	C_k	N_k	$Q_k^V N_k$
s2	2	100	14	28
s3	3	120	18	54
s5	5	130	5	25
s6	6	160	7	42
s9	9	179	1	9
Total:			45	158

vehicle capacity is not exceeded. Constraints (8) exclude any arc from any of the customers to the depot. Constraints (9) state that an arc has the value 1 if it belongs to the optimal solution, and has the value 0 otherwise. Constraints (10) ensure that the flow is non-negative.

5 Computational study

This section describes computational experiments using the model presented in Section 4. All computations were performed on a virtual server with 32GB RAM and 4-core Xeon E5-2698 2.3Ghz CPU. Using real-world data from the company that inspired this work, Section 5.1 contains results from tests where the assignment of workers to projects is fixed. As an extension of this, Section 5.2 presents some heuristics to improve the assignment of workers, in the case that workers can replace each other. Computational results from these heuristics are then presented and discussed.

5.1 Fixed worker assignments

The instances used to test the mathematical model presented in Section 4 are based on data from a Norwegian company. A typical working day is used as the basis for the project sites, the number of available workers and their home locations, as well as the assignment of workers to projects. The example week used to generate the instances consisted of 43 workers being assigned to 11 different projects. Table 1 shows the total vehicle fleet of the company. The fleet consists of five types of vehicles, with different capacities and driving costs.

A real problem instance with 11 projects and 43 workers was used in computational experiments with different combinations of vehicle types. The obtained results are shown in Table 2. The notation used for vehicle types contains the capacity information (number of seats in the vehicle, or the number of workers the vehicles of this type can transport), for example, vehicles of type s2 can transport two workers, vehicles of type s3 can transport three workers, and so on.

Table 2 presents results for instances with 11 projects and 43 workers, using variations of the

Table 2: Computational experiments for instances with 11 projects and 43 workers.

Exp. no.	Tot. cap.	Vehicle quantity s2	s3	s5	s6	s9	#V	#C	Total cost	Sec.	#R
1	158	14	18	5	7	1	2680	3084	3603.70	7.9	14
2	149	14	18	5	7	0	2144	2493	3628.56	11.7	15
3	107	14	18	5	0	0	1608	1902	4876.27	15.6	16
4	82	14	18	0	0	0	1072	1311	5191.50	1.2	19
5	48	24	0	0	0	0	536	720	6366.20	1.2	24
6	99	0	0	0	0	11	536	720	4248.77	0.5	11
7	45	11	1	1	1	1	2680	3084	3690.12	27.8	15
8	45	8	3	1	1	1	2680	3084	3656.29	13.9	14
9	48	3	14	0	0	0	1072	1311	5226.95	1.0	17
10	43	10	1	1	1	1		infeasible			
11	45	0	15	0	0	0		infeasible			

available vehicle fleet. That is, the fleet has a much larger capacity than what is required, and it is therefore interesting to see how the results change when modifying the fleet. The table shows an identifier for the instance solved ("Exp. no."), the total capacity of the fleet ("Tot. cap."), the number of vehicles of each type, the number of variables ("#V") and constraints ("#C") in the corresponding mixed integer programming formulation, and regarding the optimal solution: the total cost, the time used to solve the instance to optimality ("Sec."), and the number of routes in the solution ("#R").

As seen in Table 2, the solution time for the instances is quite low, due to a relatively low amount of workers per project (about four on average). To assess the effect on solution times of having larger projects, Table 3 shows corresponding results where one additional project has been added to the instance. The project added is relatively big, requiring 19 workers, yielding a total of 12 projects and 62 workers.

The results provided in Tables 2 and 3 show that a limited variety of vehicles and restricted total capacity, that is total capacity equal to the number of workers or only slightly larger than the number of workers, tend to negatively affect the total cost or lead to instances with no feasible solutions. The instances can be infeasible even if the capacity is sufficient (e.g. 43 workers vs 43/45 seats). This can be explained by the fact that vehicles are not allowed to travel between project sites. Each vehicle chosen by the model is only used by workers assigned to the same project. Therefore, there might be cases of unused capacity. For example, if a project has only three workers assigned to it and a five-seat vehicle has been chosen to transport the three workers, there will be unused capacity corresponding to two workers, and this vehicle cannot be used for any other route. The unused capacity depends

Table 3: Computational Experiments 12 projects, 62 workers

Exp. no.	Tot. cap.	Vehicle quantity s2	s3	s5	s6	s9	#V	#C	Total cost	Sec.	#R
12	158	14	18	5	7	1	6480	7041	4095.83	353.4	19
13	149	14	18	5	7	0	5184	5670	4133.18	348.0	20
14	107	14	18	5	0	0	3888	4299	5454.80	1319.6	23
15	82	14	18	0	0	0	2592	2928	5799.30	10.3	26
16	68	34	0	0	0	0	1296	1557	7065.51	6.2	34
17	126	0	0	0	0	14	1296	1557	4842.92	9.0	14
18	65	21	1	1	1	1	6480	7041	4304.23	137.4	25
19	65	12	7	1	1	1	6480	7041	4235.77	616.5	22
20	65	7	17	0	0	0	2592	2928	5818.35	496.4	24

largely on the capacities of the available vehicles.

Even when the instance has feasible solutions, an insufficient quantity of a particular vehicle type can also lead to a higher total cost, simply because the replacement chosen from the remaining vehicle types might have a higher fuel cost per km or a lower capacity. Moreover, an insufficient quantity of higher capacity vehicles appears to make more negative impact on the total cost than insufficient quantity of lower capacity vehicles. The lowest cost is obtained using the full fleet of vehicles in the input (the very first row in the table) and therefore it appears that the variety of capacities and a sufficient quantity is critical for finding a good solution.

On the other hand, variety greatly affects computational time, as it is directly linked to the total number of variables and constraints that need to be handled by the solver. The number of variables in a given problem instance depends on the following:

- The total number of links, which is affected by the total number of nodes and the number of projects. Due to absence of links between nodes belonging to different projects, for the same total amount of workers, a higher number of projects results in a lower number of links. For each project $p \in P$, the number of links is equal to $(|W_p| + Q_p^P)(|W_p| + Q_p^P - 1)$. The total number of links for a given instance is equal to the sum of all project cluster links.
- The number of different variables defined for each link, in this case two, corresponding to x_{ijk} and y_{ijk}.
- The number of vehicle types, equal to the size of set V in the model.

Hence, the total number of variables for the model can be calculated as $2|V| \sum_{i \in P}(|W_p| + Q_p^P)(|W_p| + Q_p^P - 1)$. As the number of variables is higher when workers are assigned to fewer different projects,

Table 4: Computational Experiment with 1 projects and 77 workers

Exp.	Tot.	Vehicle quantity							Total		
no.	cap.	s2	s3	s5	s6	s9	#V	#C	cost	Sec.	#R
21	123	39	0	0	0	5	24024	24413	3589.14	11441	19

an instance with a single project and 77 workers was generated to test the capabilities of the model. This particular instances contained just 2 vehicle types, resulting in 24024 variables. The results are displayed in Table 4.

As can be seen, it took 11,441 seconds or 3.17 hours for the solver to find the solution. This can still can be considered as an acceptable duration. However, adding just one more vehicle type, the calculations might take an unacceptably long time, and for such cases a heuristic approach may be relevant.

5.2 Free worker assignments

The assignment of workers to projects is given and is not part of the model from Section 4. That is, the company assigns workers to projects according to some criteria before deciding on a transportation plan. Sometimes, there are good reasons why certain workers are assigned to specific projects. For example, a worker may already be familiar with the project, the worker may have specific skills that are required by the project, or there are personal preferences among the workers that are important to follow.

However, if for a particular planning period, the only concern is the minimization of transportation cost, it is of interest to check if the costs can be reduced by modifying the assignment of workers to projects. To this end, three heuristics are defined that can be used to improve the assignments. The heuristics are structured as follows:

1. Start by using the given assignment of workers to projects, and obtain the corresponding optimal solution of the model for fixed assignments.

2. Use a heuristic rule to create a new assignment of workers to projects, and solve the corresponding problem.

3. Repeat Step 2 until all possible variants of worker assignments have been generated, or until another stopping criterion has been met.

4. After having considered many different assignments, return with the best solution found.

Below is the description of three different heuristic rules used to generate alternative assignments of workers to projects. The rules are applied for a given sequence of projects, and Step 2 above is

Table 5: Results of applying heuristics to decide assignments of workers to projects.

Exp. no.	Tot. cap.	Vehicle quantity s2	s3	s5	s6	s9	Initial solution	Sh. dist. to proj.	Greedy chain	Combined nearest
22	158	14	18	5	7	1	3603.7	3483.95	3458.2	3458.2

repeated for all permutations of projects, or until a time limit has been reached:

Shortest distance to project. The workers for each project are selected according to the distance from each worker's home location to the project site. For each project in turn, the worker homes nearest to the project are selected from a list of unassigned workers. This is repeated until the demand of the project has been met. For the project first in turn, the whole set of workers is available to choose from, whereas for the subsequent projects the list of unassigned workers grows shorter. Hence, the order in which projects take turns in selecting homes is important and each possible sequence of projects is considered in turn, until all alternatives have been exhausted.

Greedy chain. This is a simple, greedy rule. The workers for each project are selected according to the distance from the previous node to their home location. The first worker added is the one with a home location nearest to the project site. The second worker is the worker with a home location nearest to home location of the first worker added, and so on. As for the first heuristic rule, the order of projects for which workers are added is important and all possible sequences of projects are considered in turn.

Combined nearest. This heuristic combines the two methods described above: The first worker added is the one whose home location is nearest to the project site, and each subsequent worker is the nearest to either of the project site and the previous worker home added. If the distances are the same, for two workers, priority is given to the worker nearest to the previous home location.

To test the effect of freely assigning workers to projects, the original instances with 11 projects and 43 workers is used, with the complete vehicle fleet being available. In our experiments, to change the initial assignment of workers and projects, the three heuristics "Shortest distance to projects", "Greedy chain", and "Combined nearest" were used. The number of possible sequences of projects is very large, and the improvement process was stopped after one hour. The obtained results are shown in Table 5.

The solution obtained for the problem instance with fixed worker assignments, with a cost of 3603.7, is used as a basis for comparisons. After one hour of running time the heuristics were able to improve the initial solution by around 4 %. The "Greedy chain" and "Combined nearest" methods performed best for this particular instance. After solving various other instances, with different home locations, it was observed that the "Combined nearest" method performed at least as well as the "Greedy chain" method, and depending on the location of worker homes relative to the project sites may give better

Table 6: Results of applying heuristics to decide assignments of workers to projects on a reduced instance (5 projects, 18 workers)

Exp.	Tot.	Vehicle quantity					Initial	Sh. dist.	Greedy	Combined
no.	cap.	s2	s3	s5	s6	s9	solution	to proj.	chain	nearest
23	158	14	18	5	7	1	385.93	317.18	353.21	296.84

results. In some cases, the method "Shortest distance to project" gave better results than the "Greedy chain" method. Table 6 shows an example with five projects and 18 workers (a subset of the instance in Table 5), and applying the same three heuristics.

6 Concluding remarks

In this work, a mixed integer mathematical model for solving a multi-depot open vehicle routing problem with heterogeneous vehicles was developed. This model provided a way to solve a real-life problem of a company that needs to find a least-cost solution for transporting workers between their home locations and their assigned project sites. Apart from the mathematical model that provides an exact solution to the posed problem, several heuristics were developed for a variant of the problem where workers are not preassigned to specific projects.

Computational experiments reveal that realistically sized instances can be solved to optimality within reasonable running times. The running times are, however, sensitive to the ratio of workers to projects, with higher ratios providing a larger computational challenge. As an example, an instance with only one project and 77 workers, the running time exceeded 3 hours on a standard desktop computer. For even larger instances, work on heuristic solution methods could be warranted.

In the main problem, workers are assigned to projects by the management before planning the transportation. A variant of the problem was considered where the selection of workers for each project is free, and can be adjusted to minimize the total transportation costs. This problem was tackled by three different heuristics, called "Shortest distance to projects", "Greedy chain", and "Combined nearest", with the latter heuristic showing the best performance.

There are further variants of the problem that may be relevant for decision makers in companies that operate under similar constraints. For example, when considering a longer time horizon, the project portfolio is in constant change. This means that the workers may change from one project to another, that workers may be split into different teams, and that it may become beneficial to change which workers are appointed as drivers. A transition from one set of routes to another set of routes must then take place, with some workers handing their vehicles over to different workers. If such transitions take place frequently, it may influence the optimal routes, given that extra driving must take place to change from one set of routes to the next.

It may also be relevant to explore alternative objective functions in the current setting: minimizing transportation costs will in general lead to routes where each vehicle is transporting as many workers as possible. However, this may mean that some workers (such as the driver) spend much time in the vehicle. By using more vehicles with fewer workers per route, the costs may increase, but it may be possible to reduce the total time spent on the road for the workers, thereby increasing the amount of time that can be spent working on the projects. This can be achieved by minimizing the total time spent on the road for all the workers. Balancing these two criteria naturally leads to a multi-objective optimization problem.

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