Decision Making in Manufacturing and Services
Vol. 16 • 2022 • pp. 67–90

Decision Support for Allocating Farmed Fish to Customer Orders Using a Bi-objective Optimization Model

Sunniva Haukvik Knudseth*, Even Molland*, Arild Hoff*, Lars Magnus Hvattum*, Johan Oppen*

Abstract. Aquaculture is an important industry in certain coastal areas. Focusing on the farming of salmon and trout, an operational planning problem arises with the goal of allocating a supply of fish to the demand that is expressed through customer orders. This paper provides a conceptual model of such a planning problem and defines a corresponding bi-objective mathematical programming model. The problem is novel with respect to the structure of fish transport and the rules for satisfying customer orders with respect to fish size, quality, certification, and health status. Computational experiments have been conducted to gain further insight into the use of the provided model to provide support for planners who are involved in operational decision-making. The results indicated that the bi-objective optimization model can be useful in situations where a supply is insufficient to cover all of the demand within a given planning horizon.

Keywords: augmented epsilon-constraint method, multi-objective model, fish supply chain, assignment problem, decision support

Mathematics Subject Classification: 90C29, 90B80

JEL Classification: Q22

Submitted: January 31, 2022

Revised: December 31, 2022

1. INTRODUCTION

The farming of salmon and trout in coastal areas has become an increasingly important food source, providing significant contributions to the economies of the producing countries (NOU, 2019). The aquaculture industry is subject to regulations regarding the maximum-allowed biomass and must address the health of the grown fish (Norwegian Food Safety Authority, 2020). In addition, exported fish may need certifications (Kiwa Norway, 2021).

*Molde University College, Faculty of Logistics, Norway, e-mails: sunniva.h.knudseth@himolde.no, logtf141@hotmail.com, arild.hoff@himolde.no, hvattum@himolde.no, oppenjohan@gmail.com

Figure 1 illustrates a typical supply chain for farmed fish. This paper addresses a planning problem that arises in the later parts of the chain. After the fish are slaughtered, they must be assigned to customer orders. These orders are either internal (as a company has its own processing facilities) or external (meaning that receiving customers are outside a company's facilities). The allocation to orders should maximize the amount of fish that is delivered while also fulfilling as many high-priority customer orders as possible.

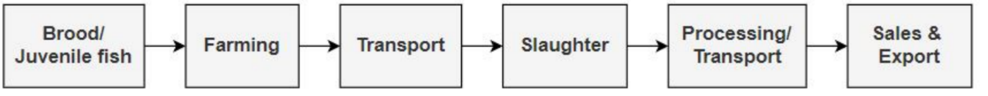

Fig. 1. *Supply chain in aquaculture*

Perishable products such as fish increase the complexity of supply chains because of their limited shelf-lives (Koldborg Jensen et al., 2010), by the fact that their values decrease while moving downstream (Musavi & Bozorgi-Amiri, 2017), and through the high variability of their price and demand (Ahumada & Villalobos, 2009). Koldborg Jensen et al. (2010) highlighted several characteristics in the supply chains of the fish industry. In general, different species of fish belong to different chains from the time of their catch to consumption, but some species may be interchangeable from the point of view of the end customer; this leads to an interdependence among the different chains. Aquaculture can occasionally be a parallel source to wild catch, but it can also result in independent supply chains from farm to fork.

At the upstream end of the supply chain, fish are caught along with the fish breeding in farms (Koldborg Jensen et al., 2010), while fresh and processed sales are at the downstream end. Between these, there are several agents who handle and process the fish and their products. Abedi and Zhu (2017) divided such supply chains that involve fish or other livestock into two parts; the warm chain, and the cold chain. The warm chain covers live fish, whereas the cold chain covers the products after harvest and processing. The latter is the focus of this paper.

Some related research has been conducted on the use of optimization models to support decisions in supply chains with perishable products. Ahumada and Villalobos (2011) presented a mixed-integer programming model to decide which agricultural products to harvest, how many times per week to harvest them, and on which days to harvest them. They also considered restrictions on time and labor and how harvest decisions affected the quality of the products. Amorim et al. (2012) developed a multi-objective mixed-integer programming model that covered the production and distribution of perishable goods. The focus was to minimize the total cost and maximize the mean remaining shelf life. Musavi and Bozorgi-Amiri (2017) considered a multi-objective optimization problem for a perishable food supply chain. They applied a heuristic solution method (NSGA-II) to generate an approximation of the Pareto front.

Abedi and Zhu (2017) provided a mixed-integer programming model to maximize the profit of a trout supply chain. The output of the model included the purchase quantity (of trout spawns), a harvest plan, and a distribution plan. The distribution plan also involved customer prioritization based on quantity in demand in order to efficiently find a way to deliver fish. The authors remarked that, up until that point in time, fish-farming companies had not taken much advantage of efficient distribution planning. Mosallanezhad et al. (2021) investigated a shrimp supply chain network, focusing on the locations of facilities such as distribution centers, wholesalers, factories, and markets as well as determining the flow of the products and waste within the network.

Outside of supply chains for perishable products, another stream of relevant research focuses on order allocation. This includes research on combined supplier selection and order allocation (Jia et al., 2020; Kaviani et al., 2020; Moheb-Alizadeh & Handfield, 2019; Sharma & Darbari, 2021). However, the existing literature considers allocating orders to suppliers from the point of view of the producer, whereas our research considers customer orders. Another direction of this type of research is the combination of location and allocation; for example, regarding nursing homes (Wang & Ma, 2018) or crisis situations (Ghasemi et al., 2019) (where one first locates facilities and then allocates flow).

Fan et al. (2019) tackled orders from customers at a brand manufacturer where the orders were aggregated before the brand manufacturer sent them to the equipment manufacturers. They had a multi product-period-equipment manufacturer problem and presented an integer nonlinear programming formulation. To solve their problem, a novel genetic algorithm was developed. Seitz et al. (2020) allocated supply to customer orders (as in our work) while taking into account the fact that the demand was forecast (which differed from our setting). In general, there is less research on order allocation under uncertainty; however, there are some examples (such as the allocation of uncertain customer orders to machines (Zhang et al., 2022)).

In our problem, we considered two objectives: maximizing deliveries, and the number of high-priority orders that are fulfilled. The ε-constraint method, the augmented ε-constraint method, or variants of these have been used to solve optimization models for many different types of supply chains, including the dairy industry (Gholizadeh et al., 2021), oil and gas (Ebrahimi & Bagheri, 2022), waste management (Abdollahi Saadatlu et al., 2022; Rabbani et al., 2020), nursing homes (Wang & Ma, 2018), and evacuation planning (Ghasemi et al., 2019). In a relevant article, Fasihi et al. (2021b) proposed a novel mathematical model for minimizing the cost of a closed-loop supply chain for fish.

The ε-constraint method tends to be time-consuming and not always able to solve all instances within a reasonable time limit; therefore, it is often compared to heuristic solution methods (Fasihi et al., 2021a). Ghasemi et al. (2019) found that the ε-constraint method provided good results (despite being slow), and Ebrahimi and Bagheri (2022) preferred the ε-constraint method over goal programming.

Our research is motivated by companies that farm, slaughter, and sell fish. The companies have planners who allocate slaughtered fish to customer orders; the combination of the relevant aspects of this planning problem has not been studied in the academic literature. This includes how fish transport is conducted and the rules for fulfilling orders with respect to fish quality, size, certification, and health status. This paper contributes to the literature by providing a conceptual model of the problem at hand and then producing a mixed-integer programming model for the problem. Finally, we report on computational experiments that were performed to assess the usefulness of the mathematical model. Based on the results, we aim to characterize situations where bi-objective optimization is beneficial and, on the other hand, situations where single-objective optimization is sufficient.

The remainder of this paper is structured as follows. Section 2 describes the problem that was modeled and solved. Next, Section 3 presents a mixed-integer programming model for the problem. A computational study to evaluate the model and the obtained solutions is contained in Section 4, followed by our conclusions in Section 5.

2. PROBLEM DESCRIPTION

When fish is slaughtered, planners need to assign the fish to any pending customer orders; after this, the fish is transported to the customers. We focus on the decisions that planners must make when assigning fish to orders, the information that is available when making decisions, and the restrictions to which the decisions must adhere. The plans are made before the start of each week; then, the planning horizon is Monday through Friday (as illustrated in Figure 2).

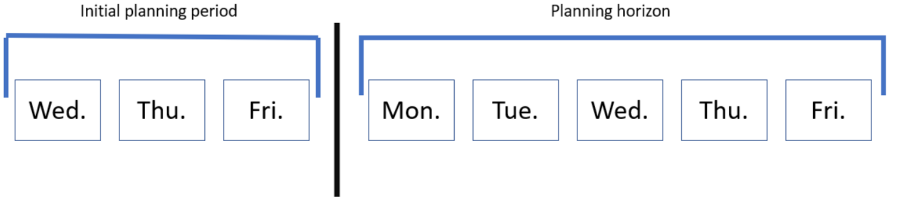

Fig. 2. *Initial planning period and planning horizon*

We next describe the physical flow of fish through the supply chain, followed by the flow of information with respect to the planning process and further details regarding the process of allocating the fish to the customers' orders.

2.1. Physical flow

The main entities that were considered in this research were fish farms, plants, and distribution centers. The physical flow of fish from farms is illustrated in Figure 3, which follows the flow from the start of an order to the end customer.

Fish farms are located along coasts and consist of facilities where fish are grown. At the time of harvest, the fish are loaded onto a well boat and transported from the farm to the plants. A plant consists of a slaughterhouse, a packing facility, and possibly a processing facility that is located in the same building. When arriving at the plant, the fish are unloaded and delivered to the slaughterhouse. When the slaughter process is completed, the fish are either forwarded to an internal-processing facility or transported out of the plant.

If the next step is the internal-processing facility, the fish are sent directly from the slaughterhouse to the facility and further processed into filets or other fish products. Otherwise, the fish must first be packed into boxes at the packing facility. A standard box weighs around 20 kg when fully packed. After being packed, the boxes are ready for storage or further transport. The storage capacity in a plant is limited, and the boxes are only stored there for a short period of time. When the boxes are shipped from the plant, the next destination can be a final customer or a distribution center.

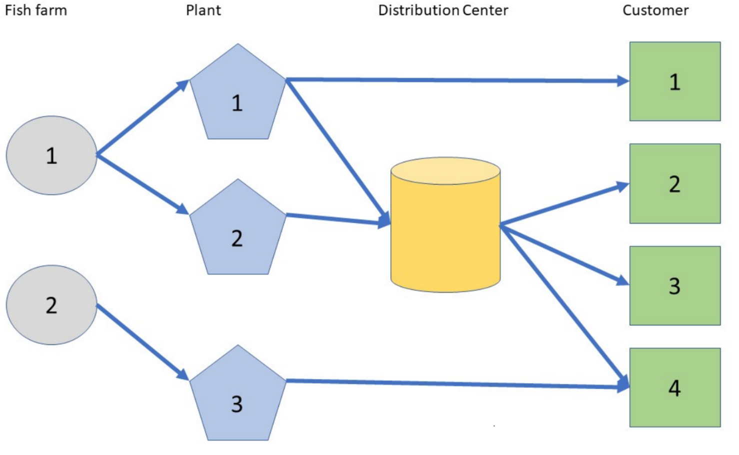

Fig. 3. *Simplified illustration of physical flow*

A distribution center is used as an intermediary hub and has the features of consolidation and storage. All of the incoming supply to the distribution center comes from the plants. There is a certain distance between a plant and a distribution center, and the lead time is one or two days. If necessary, it is possible to consolidate the supply from multiple plants prior to further transport. The distribution center has a significantly larger inventory capacity than the plants do, and long-term storage is allowed so that boxes can be stored up until the expiration dates of the fish.

2.2. Information flow

Fish are characterized by three attributes: species, quality, and size class. There are two species involved: salmon, and trout. Each is split into three different qualities and then subdivided into nine different size classes based on the weights of the fish.

A fish farm has a set of certifications and a given status for the health of the fish. The fish that comes from a given farm is associated with the same certificates and health status. When the fish have grown to an acceptable weight and size, they are ready for harvest. The fish are harvested on a specific day and sent directly to a plant for further processing. The planners know the day and time that the fish will arrive at the plant but cannot influence this time themselves.

The planners also have access to continuously updated forecasts of the amounts of fish that will be delivered to a plant. The species of the incoming fish is known, but the forecast has more-specific data on the total amount of fish, the amount of the fish from each size class, and the amount of the fish with a given quality. When the fish is slaughtered, it is also weighed and its quality checked. At this point, the planners will begin to receive more-accurate information about the available amounts of the supplies.

The demand for fish is expressed through orders. The main elements of an order are details about the requested species along with the size class, quality, number of boxes, certification, health status, delivery date, and delivery location. A sales department works closely with planners and handles all orders, customer relationships, contracts, and prices. In cooperation, the planners and sales department decide about the priorities of the individual orders.

An order contains details about the numbers of boxes that are requested for a given combination of species, size class, and quality. The orders can allow for flexibility in the numbers of boxes that are delivered for a given combination, which are given as upper and lower bounds on the numbers of boxes. If it is possible to deliver within the bounds, the order is considered to be fulfilled. While an order specifies a certain species and quality, it may allow for the delivery of multiple size classes.

Each order specifies the date on which the customer wants the fish to reach its destination. When allocating fish to orders, the planners use the day of delivery to decide when the supply should be sent from the plant. An order can allow for some flexibility with respect to the requested delivery date.

The orders can specify that the fish must have one or more certifications. The customers can also refrain from receiving fish with specific health statuses. The federal governments of certain countries have their own requirements for the health and certification of fish; this means that, if the fish is to be exported to such countries, then it needs to meet their specific requirements.

An important distinction is between external and internal orders. An internal order is an order that comes from the internal-processing facility at a plant. External orders have destinations that are outside the plants, which require the transportation of the fish from the plants to the final destinations (possibly via distribution centers).

2.3. Allocation process

The allocation process has the goal of allocating supply to demand in the best possible way. The planners go through five defined steps when creating a plan:

1) receiving orders,
2) checking orders,
3) prioritizing orders,
4) checking whether it is possible to fulfill orders,
5) delivering or canceling orders.

Planners receive order details from the sales department and check what is requested. The orders are then assigned their priorities based on contractual obligations, customer relationships, and prices. Sometimes, a customer must deal with deviations from their specified demand. The planners aim to deliver to customers with as little deviation as possible over a longer period of time. Some customers place regular orders, which opens up the possibility of leveling out the deviations over time. If a customer has a significant deviation from their demand in one week, their order for the following week can be set to a higher priority.

The available supply is known exactly or is estimated. Based on the above information, the planners can start the allocation of supply to demand while deciding how each order is to be fulfilled. If it is possible to fulfill an order, it can be accepted; otherwise, it must be cancelled. When all of the orders that can be fulfilled are processed, there might be unsold fish left. The sales department then tries to obtain additional orders (for example, in the spot market) so that all of the fish can be sold. If this is impossible, the fish is stored until the next planning horizon (usually at a distribution center).

In those weeks when there is an insufficient supply, the planners are forced to decide which orders to fulfill. In those weeks with surpluses of supply, however, challenges arise regarding where to store the boxes. The decision about finding appropriate locations to store the boxes is affected by the storage costs, the available inventory capacity, and the shelf life of the fish.

Two goals are studied in this paper. The first goal is to provide as much of the available supply as possible to orders. This will lower the total number of unsold boxes that are left at the end of a planning horizon. Lowering the numbers of remaining unsold boxes can potentially lower storage costs. The second goal is to fulfill as many highly prioritized orders as possible. As previously stated, the priority of an order is based on contracts, customer relationships, and prices. Satisfying this goal will please important customers and can increase profits. The interesting connection between the two goals is that they can be conflicting or nonconflicting depending on the balance among the supply, the demand, and the prioritization of the orders.

3. MATHEMATICAL MODEL

The following model intends to capture the fundamental essence of the fish-allocation process. It is designed for use just before weekly operations start and is not meant for dynamic replanning during a planning horizon.

The fish are reared in individual farms; L denotes the set of all farms. Outbreaks of disease can occur on any farm; the set of possible diseases is written as E. The farms can be certified; the set of possible certifications is indicated by H.

After harvesting the fish from a farm, it is sent to a plant for slaughter and packing; A denotes the set of all plants. The fish are described using the following attributes: species, size class, and quality. To this end, we introduce the set S of the species, the set Z of the size classes, and the set Q of the possible qualities.

The model is designed for a seven-day schedule, but the notation is kept flexible. All of the plants are open from day 1 through day T. The lead time from plant a to a distribution center is R_a, which is at least one full day. Parameter F represents the longest lead time from a plant to a distribution center, with $F = max_{a \in A} \{R_a\}$. All of the supply that is sent from a plant on day T must arrive at a distribution center before a planning horizon ends. Therefore, the length of the planning horizon is $T + F$, and the distribution center is open from day 1 through day $T + F$.

There are two types of orders. External orders have destinations that are outside a plant and are elements of set O^{EX}. Internal orders are related to the processing facilities that are located in the same buildings as the plants and are elements of set O^{IN}. Parameter K_o is used to indicate the importance of an order o, with higher values indicating a higher priority when serving the orders.

External orders have two different methods of delivery, while internal orders have only one. Direct delivery is defined as sending boxes directly from one plant to a customer without the need to consolidate from other plants. Variable $x^A_{l,a,s,z,q,t,o}$ represents the number of boxes that contain fish from species s, size class z, and quality q that are harvested on farm l and sent directly from plant a on day t to cover order o. Binary variable $y^A_{a,t,o}$ takes a value of 1 if order o is delivered directly from plant a on day t; otherwise, this value is 0.

If the delivery of an order is made from a distribution center, there cannot also be a direct delivery from a plant to the same order. Variable $x^{DC}_{l,a,s,z,q,t,o}$ represents the number of boxes that contain fish from species s, size class z, and quality q that are harvested on farm l and sent from plant a through a distribution center to order o on day t. Binary variable $y^{DC}_{t,o}$ takes a value of 1 if order o is served from a distribution center on day t; otherwise, this value is 0.

Internal orders have only one method of delivery, which is in-house delivery. It is not allowed to serve an internal-processing facility in plant a from a distribution center or other plants. Variable $x^{IN}_{l,a,s,z,q,t,o}$ represents the number of boxes that contain fish from species s, size class z, and quality q that are harvested on farm l and delivered to the internal-processing facility of plant a to deliver order o on day t. Binary variable $y^{IN}_{t,o}$ takes a value of 1 if internal order o is served on day t; otherwise, the value is 0.

Parameters:

T – number of days that plant is open,
F – longest lead time from plant to distribution center,
$P_{l,a,s,z,q,t}$ – supply coming from farm l to plant a of species s, size class z, and quality q on day t,
$D^{TOTAL}_{s,q,o}$ – total demand of boxes of fish of species s and quality q in order o,
$D^{MAX}_{s,z,q,o}$ – maximum demand of boxes of fish of species s, size class z, and quality q in order o,
$D^{MIN}_{s,z,q,o}$ – minimum demand of boxes of fish of species s, size class z, and quality q in order o,
K_o – importance of order o,
G_o – relative lower bound on fish delivered to order o of given species and quality,
$\sigma^A_{t,o}$ – 1 if order o does not allow delivery on day t, 0 otherwise,
$\sigma^{DC}_{t,o}$ – 1 if order o does not allow delivery on day t, 0 otherwise,
$\sigma^{IN}_{t,o}$ – 1 if order o does not allow delivery on day t, 0 otherwise,
$I^A_{l,a,s,z,q}$ – initial number of boxes of fish of species s, size class z, and quality q from farm l in inventory at plant a,
$I^{DC}_{l,a,s,z,q}$ – initial number of boxes of fish of species s, size class z, and quality q coming from farm l through plant a in inventory at distribution center,

C_a^A – inventory capacity at plant a,
R_a – lead time from plant a to distribution center,
$\delta_{a,o}^A$ – 1 if plant a can deliver to order o, 0 otherwise,
$U_{o,e}^O$ – 1 if order o does not accept fish from farm with disease e, 0 otherwise,
$U_{l,e}^L$ – 1 if fish comes from farm l without disease e, 0 otherwise,
$\pi_{o,h}^O$ – 1 if order o requires fish with certificate h, 0 otherwise,
$\pi_{l,h}^L$ – 1 if fish from farm l have certificate h, 0 otherwise,
J^A – weight of making direct delivery,
J^{DC} – weight of making delivery through distribution center,
M – large constant used to ensure linear constraints.

Variables:

$x_{l,a,s,z,q,t,o}^A$ – number of boxes of fish of species s, size class z, and quality q from farm l sent directly from plant a to order o on day t,
$x_{l,a,s,z,q,t,o}^{DC}$ – number of boxes of fish of species s, size class z, and quality q from farm l sent from plant a through distribution center to order o on day t,
$x_{l,a,s,z,q,t,o}^{IN}$ – number of boxes of fish of species s, size class z, and quality q from farm l sent from plant a to internal order o on day t,
$y_{a,t,o}^A$ – 1 if delivery to order o on day t directly from plant a is done, 0 otherwise,
$y_{t,o}^{DC}$ – 1 if delivery to order o on day t directly from distribution center is done, 0 otherwise,
$y_{t,o}^{IN}$ – 1 if delivery to internal order o on day t from plant is done, 0 otherwise,
$\mu_{l,a,s,z,q,t}^A$ – number of boxes of fish of species s, size class z, and quality q from farm l stored at plant a on day t,
$\mu_{l,a,s,z,q,t}^{DC}$ – number of boxes of fish of species s, size class z, and quality q from farm l sent from plant a stored at distribution center on day t,
$b_{l,a,s,z,q,t}$ – number of boxes of fish of species s, size class z, and quality q from farm l sent from plant a to distribution center on day t.

3.1. Objective functions

We define two objective functions ($W1$ and $W2$) as representing the total number of boxes that have been delivered to all orders and the total value of the prioritized orders that have been fulfilled, respectively. Both objective functions are to be maximized.

$$\max W1 = \sum_{l\in L}\sum_{l\in L}\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q}\left(\sum_{t=1}^{T}\sum_{o\in O}^{\varepsilon x} J^A x_{l,a,s,z,q,t,o}^A + \right.$$
$$\left. + \textstyle\sum_{t=1}^{T+F} J^{DC} x_{l,a,s,z,q,t,o}^{DC} + \sum_{t=1}^{T}\sum_{o\in O^{IN}} x_{l,a,s,z,q,t,o}^{IN}\right). \tag{1}$$

The first goal is to deliver as much as possible of the available supply to the orders, which corresponds to maximizing the total number of boxes that are delivered to all orders. It is possible for planners to predefine which delivery method is preferred for any external orders. Weights for direct delivery J^A and delivery through distribution center J^{DC} are multiplied with their respective variables. A relatively higher weight on one of the two parameters indicates which of the delivery methods is preferred; each weight is a positive number that is less than or equal to 1.

$$\max W_2 = \sum_{o\in O^{\varepsilon x}} K_o \left(\sum_{a\in A} \sum_{t=1}^{T} y^A_{a,t,o} + \sum_{t=1}^{T+F} y^{DC}_{t,o} \right) + \sum_{o\in O^{IN}} K_o \sum_{t=1}^{T} y^{IN}_{t,o} . \tag{2}$$

The second goal is to fulfill as many highly prioritized orders as possible; that is, maximizing the total value of the prioritized orders that have been fulfilled.

3.2. Constraints

Initial inventory

$$\mu^A_{l,a,s,z,q,t} = I^A_{l,a,s,z,q}, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q, t = 0\,, \tag{3}$$

$$\mu^{DC}_{l,a,s,z,q,t} = I^{DC}_{l,a,s,z,q}, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q, t = 0\,. \tag{4}$$

The first set of constraints governs the initial inventories; Constraints (3) make sure that the initial inventory is set in a plant, while Constraints (4) handle the initial inventory in a similar fashion in a distribution center.

Inventory capacity constraints at plant

$$\sum_{l\in L} \sum_{s\in S} \sum_{z\in Z} \sum_{q\in Q} \mu^A_{l,a,s,z,q,t} \le C^A_a, \quad a \in A,\ t \in \{0,\dots,T\}\,. \tag{5}$$

The value of variables $\mu^A_{l,a,s,z,q,t}$ must be less than or equal to the inventory capacity at plant a, as is enforced by Constraints (5). The distribution center has no inventory-capacity constraint due to the assumption that its capacity is infinite.

Balance constraints at plants

$$\begin{gathered} P_{l,a,s,z,q,t} + \mu^A_{l,a,s,z,q,t-1} - \sum_{o\in O^{\varepsilon x}} x^A_{l,a,s,z,q,t,o} - \\ - \textstyle\sum_{o\in O^{IN}} x^{IN}_{l,a,s,z,q,t,o} - b_{l,a,s,z,q,t} = \mu^A_{l,a,s,z,q,t}, \\ l \in L, a \in A, s \in S, z \in Z, q \in Q, t = 0\,. \end{gathered} \tag{6}$$

Constraints (6) balance the inventory and the incoming and outgoing flows for plant a. The available supply in period t consists of incoming supply $P_{l,a,s,z,q,t}$ and the inventory from previous period $\mu^A_{l,a,s,z,q,t-1}$. If there is no available supply for a given combination of farm, plan, species, size class, and quality, all of the variables with this combination are set to 0. Variables $x^A_{l,a,s,z,q,t,o}$ and $x^{IN}_{l,a,s,z,q,t,o}$ state the number of boxes delivered directly or in-house, respectively. Variables $b_{l,a,s,z,q,t}$ represent the number of boxes that are sent from plant a to a distribution center. Sending boxes from a plant to a distribution center is done in order to ensure that there are enough boxes that are available for further delivery or to store the boxes at the distribution center instead of at the plant. All of the boxes that are left on day t at plant a are stored for the next period.

Balance constraints at distribution center

$$\begin{gathered}\mu^{DC}_{l,a,s,z,q,t-1} - \sum_{o\in O^{\varepsilon x}} x^{DC}_{l,a,s,z,q,t,o} = \mu^{DC}_{l,a,s,z,q,t},\\ l\in L, a\in A, s\in S, z\in Z, q\in Q, t\in\{1,\dots,R_a\},\end{gathered} \tag{7}$$

$$\begin{gathered}b_{l,a,s,z,q,t-R_a} + \mu^{DC}_{l,a,s,z,q,t-1} - \sum_{o\in O^{\varepsilon x}} x^{DC}_{l,a,s,z,q,t} = \mu^{DC}_{l,a,s,z,q,t},\\ l\in L, a\in A, s\in S, z\in Z, q\in Q, t\in\{1+R_a,\dots,T+R_a\},\end{gathered} \tag{8}$$

$$\begin{gathered}\mu^{DC}_{l,a,s,z,q,t-1} - \sum_{o\in O^{\varepsilon x}} x^{DC}_{l,a,s,z,q,t,o} = \mu^{DC}_{l,a,s,z,q,t},\\ l\in L, a\in A, s\in S, z\in Z, q\in Q, t\in\{T+R_a+1,\dots,T+F\}.\end{gathered} \tag{9}$$

Three sets of constraints are used to force the correct balance of inventory, incoming flow, and outgoing flow, respectively, at a distribution center. All of the incoming flow to the distribution center comes from the plants. All of the outgoing flow is the number of boxes that are delivered from the distribution center to the orders.

The lead time from plant a to a distribution center is R_a, which means that it takes R_a days to send the supply from plant a to the distribution center. The distribution center is open from Day 1, but the incoming supply can only arrive R_a days after leaving plant a. This affects the incoming flow at the distribution center. Constraints (7) ensure that there is balance in inventory and out-going flow in the periods before new supply can arrive from plant a.

The first day that the supply that is sent from plant a can arrive at a distribution center is on day $1 + R_a$. Constraints (8) ensure the balance of inventory, incoming flow, and outgoing flow at the distribution center from period $1 + R_a$ through $T + R_a$. Variables $b_{l,a,s,z,q,t-R_a}$ represent the number of boxes that are sent on day $t - R_a$ from plant a and arrive on day t at the distribution center.

The distribution center is open in Period 1 through $T + F$, but the constraint set (8) is only valid from $1 + R_a$ to $T + R_a$. If R_a is less than F for plant a, then this causes a problem. Variables $\mu^{DC}_{l,a,s,z,q,t-1}$, $x^{DC}_{l,a,s,z,q,o}$, and $\mu^{DC}_{l,a,s,z,q,t}$ that are related to plant a are unbounded in the periods after $T + R_a$. Constraints (9) address this problem and ensure that, from $T + R_a + 1$ through $T + F$, all of the variables that are related to those plants with lead times that are shorter than F are bounded.

Variables $b_{l,a,s,z,q,t-R_a}$ are omitted from Constraints (9). An example is used to explain why the variables are omitted. If the lead time of plant a is one day and it is now day $T + 2$, then variables $b_{l,a,s,z,q,t-R_a}$ are related to the number of boxes that left plant a on day $T + 1$. The plant is only open from day 1 through day T, so no boxes leave from plant a after day T. It is therefore redundant to have variables that are related to the number of boxes that are sent on day $T + 1$. This is the reason for not including $b_{l,a,s,z,q,t-R_a}$ in the constraint set.

Delivery

$$\sum_{a\in A}\sum_{t=1}^{T} y^{A}_{a,t,o} + \sum_{t=1}^{T+F} y^{DC}_{t,o} \le 1, \quad o\in O^{\varepsilon x}, \tag{10}$$

$$\sum_{t=1}^{T} y^{IN}_{t,o} \le 1, \quad o\in O^{\varepsilon x}. \tag{11}$$

Constraints (10) make sure that, if an external order is accepted, only one of the two available delivery methods for external orders is used. If the direct delivery from a plant is used, then only one plant can deliver to the order. It is only allowed to deliver one time to an order throughout the entire planning horizon regardless of the chosen delivery method. If an internal order is accepted, it is only served once throughout the planning horizon, as is indicated by Constraints (11).

Day of delivery

$$\sum_{l\in L}\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q} x^{A}_{l,a,s,z,q,t,o} \le M(1-\sigma^{A}_{t,o}), \quad t\in\{1,\dots,T\}, o\in O^{\varepsilon x}, \tag{12}$$

$$\sum_{l\in L}\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q} x^{DC}_{l,a,s,z,q,t,o} \le M(1-\sigma^{DC}_{t,o}), \quad t\in\{1,\dots,T+F\}, o\in O^{\varepsilon x}, \tag{13}$$

$$\sum_{l\in L}\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q} x^{IN}_{l,a,s,z,q,t,o} \le M(1-\sigma^{IN}_{t,o}), \quad t\in\{1,\dots,T\}, o\in O^{IN}. \tag{14}$$

The day of delivery is the day that the supply must be sent so it can arrive on the day that is specified in the order. Constraints (12) ensure that it is only possible to deliver directly from a plant to an external order on the allowed day of delivery. Similarly, Constraints (13) state that it is only possible to deliver from the distribution center to an external order on the allowed day of delivery. Finally, Constraints (14) ensure that it is only possible to deliver to an internal order on the allowed day of delivery.

Demand – upper bound

$$\begin{gathered}\sum_{l\in L}\sum_{z\in Z} x^{A}_{l,a,s,z,q,t,o} \le D^{TOTAL}_{s,q,o} y^{A}_{a,t,o},\\ a\in A, s\in S, q\in Q,\ t\in\{1,\dots,T\}, o\in O^{\varepsilon x},\end{gathered} \tag{15}$$

$$\begin{gathered}\sum_{l\in L}\sum_{a\in A}\sum_{z\in Z} x^{DC}_{l,a,s,z,q,t,o} \le D^{TOTAL}_{s,q,o} y^{DC}_{t,o},\\ s\in S, q\in Q,\ t\in\{1,\dots,T+F\}, o\in O^{\varepsilon x},\end{gathered} \tag{16}$$

$$\begin{gathered}\sum_{l\in L}\sum_{a\in A}\sum_{z\in Z} x^{IN}_{l,a,s,z,q,t,o} \le D^{TOTAL}_{s,q,o} y^{IN}_{t,o},\\ s\in S, q\in Q,\ t\in\{1,\dots,T\}, o\in O^{IN}.\end{gathered} \tag{17}$$

The model allows an order to have flexibility in the number of boxes that are delivered and the possibility of substituting the size classes that are used in the delivery. Flexibility is allowed in the number of boxes that are specified in an order. Parameter $D^{TOTAL}_{s,q,o}$ represents the demand for a specific combination of species s and quality q for order o. Three sets of constraints are related to separate methods of delivery and the upper bound on this demand, effectively stating that the deliveries to an order cannot exceed the maximum amounts of fish of a given species and quality levels that are requested.

The number of boxes that are delivered directly from plant a are fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s and quality q; this controlled by Constraints (15). Constraints (16) ensure that the number of boxes that are delivered from the distribution center are fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s and quality q, while Constraints (17) ensure that the number of boxes that are delivered in-house are fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s and quality q.

Demand – lower bound

$$\sum_{l \in L} \sum_{z \in Z} x^{A}_{l,a,s,z,q,t,o} \geq D^{TOTAL}_{s,q,o} G_o y^{A}_{a,t,o}, \quad a \in A, s \in S, q \in Q,\ t \in \{1, \dots, T\}, o \in O^{\varepsilon x}, \tag{18}$$

$$\sum_{l \in L} \sum_{a \in A} \sum_{z \in Z} x^{DC}_{l,a,s,z,q,t,o} \geq D^{TOTAL}_{s,q,o} G_o y^{DC}_{t,o}, \quad s \in S, q \in Q,\ t \in \{1, \dots, T+F\}, o \in O^{\varepsilon x}, \tag{19}$$

$$\sum_{l \in L} \sum_{a \in A} \sum_{z \in Z} x^{IN}_{l,a,s,z,q,t,o} \geq D^{TOTAL}_{s,q,o} G_o y^{IN}_{t,o}, \quad s \in S, q \in Q,\ t \in \{1, \dots, T\}, o \in O^{IN}. \tag{20}$$

Parameter G_o is a value that is within a range of 0 to 1. If parameter $D^{TOTAL}_{s,q,o}$ is multiplied by G_o, the lower bound on the demand for a specific combination of species s and quality q is found for order o. Constraints (18) force the number of boxes that are delivered directly from plant a to be more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s and quality q. Constraints (19) ensure that the number of boxes that are delivered from the distribution center are more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s and quality q. The number of boxes that are delivered in-house must be more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s and quality q; this is expressed by Constraints (20).

Demand – upper bound (size class)

$$\sum_{l \in L} x^{A}_{l,a,s,z,q,t,o} \leq D^{MAX}_{s,z,q,o}\, y^{A}_{a,t,o}, \quad a \in A, s \in S, z \in Z, q \in Q,\ t \in \{1, \dots, T\}, o \in O^{\varepsilon x}, \tag{21}$$

$$\sum_{l \in L} \sum_{a \in A} x^{DC}_{l,a,s,z,q,t,o} \leq D^{MAX}_{s,z,q,o}\, y^{DC}_{t,o}, \quad s \in S, z \in Z, q \in Q,\ t \in \{1, \dots, T+F\}, o \in O^{\varepsilon x}, \tag{22}$$

$$\sum_{l \in L} \sum_{a \in A} x^{IN}_{l,a,s,z,q,t,o} \leq D^{MAX}_{s,z,q,o}\, y^{IN}_{t,o}, \quad s \in S, z \in Z, q \in Q,\ t \in \{1, \dots, T\}, o \in O^{IN}. \tag{23}$$

Parameter $D^{MAX}_{s,z,q,o}$ represents the upper bound on a given size class z that can be delivered with a combination of species s and quality q for order o. The parameter must be set to less than or equal to $D^{TOTAL}_{s,q,o}$. If $D^{MAX}_{s,z,q,o}$ is set to zero, it is not allowed to deliver anything of this size class z for this combination of species s and quality q for order o. If an order is accepted, Constraints (18)–(20) control the total amount that is delivered for the given species and quality of the fish. Combined with Constraints (21)–(26), a company has some flexibility in terms of which size classes are used to meet the demand.

Constraints (21) state that the number of boxes of size class z that are delivered directly from plant a must be fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q. Next, Constraints (22) ensure that the number of boxes of size class z that are delivered from the distribution center are fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q. Then, Constraints (23) ensure that the number of boxes of size class z that are delivered internally are fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q.

Demand – lower bound (size class)

$$\begin{gathered}\sum_{l \in L} x^{A}_{l,a,s,z,q,t,o} \geq D^{MIN}_{s,z,q,o}\, y^{A}_{a,t,o},\\ a \in A, s \in S, z \in Z, q \in Q,\ t \in \{1,\dots,T\}, o \in O^{\varepsilon x},\end{gathered} \tag{24}$$

$$\begin{gathered}\sum_{l \in L}\sum_{a \in A} x^{DC}_{l,a,s,z,q,t,o} \geq D^{MIN}_{s,z,q,o}\, y^{DC}_{t,o},\\ s \in S, z \in Z, q \in Q,\ t \in \{1,\dots,T+F\}, o \in O^{\varepsilon x},\end{gathered} \tag{25}$$

$$\begin{gathered}\sum_{l \in L}\sum_{a \in A} x^{IN}_{l,a,s,z,q,t,o} \geq D^{MIN}_{s,z,q,o}\, y^{IN}_{t,o},\\ s \in S, z \in Z, q \in Q,\ t \in \{1,\dots,T\}, o \in O^{IN}.\end{gathered} \tag{26}$$

Parameter $D^{MIN}_{s,z,q,o}$ represents the lower bound of the boxes of a given size class s that must be delivered with a combination of species s and quality q for order o. The value of parameter $D^{MIN}_{s,z,q,o}$ must be less than or equal to $D^{MAX}_{s,z,q,o}$. Then, Constraints (24) enforce that the number of boxes of size class z that are delivered directly from plant a are more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q.

Constraints (25) ensure that the number of boxes of size class z that are delivered from a distribution center are more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q. For internal orders, Constraints (26) state that the number of boxes of size class z that are delivered in-house must be more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q.

Disease

$$\begin{gathered}\sum_{a \in A}\sum_{s \in S}\sum_{z \in Z}\sum_{q \in Q}\left(\sum_{t=1}^{T} x^{A}_{l,a,s,z,q,t,o} + \sum_{t=1}^{T+F} x^{DC}_{l,a,s,z,q,t,o}\right) \leq M\left((1 - U^{O}_{o,e}) + U^{L}_{l,e}\right),\\ L \in L, o \in O^{\varepsilon x}, e \in E,\end{gathered} \tag{27}$$

$$\begin{gathered}\sum_{a \in A}\sum_{s \in S}\sum_{z \in Z}\sum_{q \in Q}\left(\sum_{t=1}^{T} x^{IN}_{l,a,s,z,q,t,o}\right) \leq M\left((1 - U^{O}_{o,e}) + U^{L}_{l,e}\right),\\ L \in L, o \in O^{IN}, e \in E.\end{gathered} \tag{28}$$

If external order o is to receive fish that was grown in farm l, then Constraints (27) ensure that the health status of disease e is what is specified in the order. Similarly, if internal order o is to receive fish that was bred in farm l, Constraints (28) ensure that the health status of disease e is what is specified in the order.

Certifications

$$\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q}\left(\sum_{t=1}^{T} x^{A}_{l,a,s,z,q,t,o} + \sum_{t=1}^{T+F} x^{DC}_{l,a,s,z,q,t,o}\right) \le M\left((1-\pi^{O}_{o,h}) + \pi^{L}_{l,h}\right), \quad L\in L, o\in O^{\varepsilon x}, h\in H, \tag{29}$$

$$\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q}\left(\sum_{t=1}^{T} x^{IN}_{l,a,s,z,q,t,o}\right) \le M\left((1-\pi^{O}_{o,h}) + \pi^{L}_{l,h}\right), \quad L\in L, o\in O^{IN}, h\in H. \tag{30}$$

If external order o is to receive fish that was raised in farm l, then Constraints (29) force this fish to meet the certification requirements h that are specified in the order. Constraints (30) deal with internal orders in a similar way: if internal order o is to receive fish that was bred in farm l, then this fish must meet the corresponding requirements.

Internal-processing facility

$$\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q}\sum_{t=1}^{T} x^{IN}_{l,a,s,z,q,t,o} \le M\delta^{A}_{a,o}, \quad a\in A, o\in O^{IN}. \tag{31}$$

An internal order is related to only one plant. Constraints (31) are used to enforce that only the plant that is related to order o is allowed to deliver fish. If $\delta^{A}_{a,o}$ is 1, then order o is related to plant a; otherwise, this value is 0.

Domain of variables

$$x^{A}_{l,a,s,z,q,t,o} \ge 0, \quad l\in L, a\in A, s\in S, z\in Z, q\in Q,\ t\in\{1,\dots,T\}, o\in O^{\varepsilon x}, \tag{32}$$

$$x^{DC}_{l,a,s,z,q,t,o} \ge 0, \quad l\in L, a\in A, s\in S, z\in Z, q\in Q,\ t\in\{1,\dots,T+F\}, o\in O^{\varepsilon x}, \tag{33}$$

$$x^{IN}_{l,a,s,z,q,t,o} \ge 0, \quad l\in L, a\in A, s\in S, z\in Z, q\in Q,\ t\in\{1,\dots,T\}, o\in O^{IN}, \tag{34}$$

$$y^{A}_{a,t,o} \in \{0,1\}, \quad a\in A, t\in\{1,\dots,T\}\, o\in O^{\varepsilon x}, \tag{35}$$

$$y^{DC}_{t,o} \in \{0,1\}, \quad t\in\{1,\dots,T+F\}\, o\in O^{\varepsilon x}, \tag{36}$$

$$y_{t,o}^{IN} \in \{0,1\}, \quad t \in \{1,\ldots,T\} \, o \in O^{IN}, \tag{37}$$

$$\mu_{l,a,s,z,q,t}^{A} \geq 0, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q, \; t \in \{1,\ldots,T\}, \tag{38}$$

$$\mu_{l,a,s,z,q,t}^{DC} \geq 0, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q, \; t \in \{1,\ldots,T+F\}, \tag{39}$$

$$b_{l,a,s,z,q,t} \geq 0, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q, \; t \in \{1,\ldots,TF\}. \tag{40}$$

All of the variables are non-negative.

3.3. Augmented ε-constraint method

The augmented ε-constraint method (Mavrotas, 2009) is used to solve a model with both objectives. First, a pay-off table is calculated in which each objective function is optimized individually, providing the best possible value for each objective function. The table also provides the worst value for each objective function, since the other objective functions are prioritized first according to the lexicographic method (Hwang & Masud, 1979). Let parameter r_k be the range of objective function k in the pay-off table.

In our implementation of the augmented ε-constraint method, the objective function that maximizes the total number of boxes that are delivered is made the primary objective:

$$\max \sum_{l \in L} \sum_{a \in A} \sum_{s \in S} \sum_{z \in Z} \sum_{q \in Q} \left(\sum_{t=1}^{T} \sum_{o \in O^{\varepsilon x}} J^{A} x_{l,a,s,z,q,t,o}^{A} + \sum_{t=1}^{T+F} \sum_{o \in O^{\varepsilon x}} J^{DC} x_{l,a,s,z,q,t,o}^{DC} + \sum_{t=1}^{T} \sum_{o \in O^{IN}} x_{l,a,s,z,q,t,o}^{IN} \right) + \lambda \left(\frac{S_2}{r_2} \right), \tag{41}$$

where S_k are the slack or surplus values, and the parameter λ is given in the interval $\left[10^{-6}, 10^{-3}\right]$.

The objective function that maximizes the total value of the prioritized orders is transformed into a constraint:

$$\sum_{o \in O^{\varepsilon x}} K_o \left(\sum_{a \in A} \sum_{t=1}^{T} y_{a,t,o}^{A} + \sum_{t=1}^{T+F} y_{t,o}^{DC} \right) + \sum_{o \in O^{IN}} K_o \sum_{t=1}^{T} y_{t,o}^{IN} - S_2 = \varepsilon_2, \tag{42}$$

where ε_k is calculated as follows:

$$\varepsilon_k = f_k^{MIN} + t \left(\frac{r_k}{q_k} \right),$$

where f_k^{MIN} is the minimum that is obtained from the payoff table, and t is the counter for the specific objective function. The number of intervals for objective function k^{th}, q_k, influences the potential number of different Pareto-optimal solutions that can be found.

4. COMPUTATIONAL STUDY

This section reports on our computational experiments using the mathematical model that was provided in Section 3. The model was implemented using AMPL and solved using CPLEX (Version 20.1.0.0 – 64-bit). The tests were run on a computer with 16 GB RAM and a 3.7 GHz Intel Core i5-9600K CPU with six cores.

4.1. Test instances

Based on the operations of our focal company, we created ten different test instances. The instances varied in terms of the numbers of farms, plants, species, size classes, qualities, certifications, diseases, and orders; the values for each of these elements are presented in Table 1. While the number of plants, species, size classes, qualities, certifications, and diseases are based on real-world data, the number of orders, the demand and supply, and the order priorities were generated to span a range of potential realistic situations.

Table 1. *Test instances*

Entity	**Instance**				
	1A, 1B	2A, 2B	3A, 3B	4A, 4B	5A, 5B
Farm	2	3	4	5	6
Plant	2	3	4	5	6
Species	2	2	2	2	3
Size class	3	4	4	6	6
Quality	2	3	4	4	4
Diseases	2	3	4	4	5
Certifications	2	3	4	4	5
Orders	50	100	150	200	250

In the instances, the orders with low demand had a high priority, and the orders with high demand had a low priority. The prioritization coefficients had values from a discrete interval of integer numbers from 1 to 10 (where 10 was the highest). The orders were divided into groups of ten orders, each requesting the same combination of fish but a different number of boxes. The demand for each combination in an order was set to be a random number between 100 and 1000 boxes.

The required days of delivery were set so it was possible to deliver all of the orders on any day throughout the planning horizon. The requirements about diseases and certifications were set so that all of the orders accepted all fish (independently from its health status and certifications). The only source of incoming supply was set to be farms. The initial inventories were set to zero for all of the plants and the distribution center. The inventory capacity was set to zero for all of the plants, so any supply that needed to be stored was required to be sent to the distribution center.

All of the orders allowed for a deviation of 10% from the number of boxes that were requested for a given combination; this allowed for flexibility in demand. All of the orders were set to be external orders. The weights for the delivery methods for the external orders were set so that direct delivery was weighted higher than delivery through the distribution center. Parameter T was set to 5, the lead time from a plant to the distribution center was set to 1 or 2 days, and the longest lead time F was 2.

The ten instances were divided into two sets on the basis of the balance between the supply and demand for a given combination of fish. In the first set (A), the five instances had more supply than demand, and in the second set (B), the five instances had less supply than demand. The instances in the two sets were identical except for their available supplies. The size of each instance is presented in Table 2.

Table 2. *Problem size*

Instance	Variables	Constraints
1A, 1B	30,562	28,934
2A, 2B	265,504	138,142
3A, 3B	935,378	337,942
4A, 4B	2,909,200	744,230
5A, 5B	7,834,500	1,608,570

4.2. Results

This section presents an analysis of the results for each problem instance as was found by using the proposed solution method. We started by providing pay-off tables for the instances in Tables 3 and 4. The payoff tables were calculated as described by Chowdhury and Tan (2004), meaning that we first optimized each individual objective separately as a primary objective and then optimized for each remaining objective subject to fixing the value of the current primary objective to its optimal value using a hard constraint.

Table 3. *More supply than demand*

Instance	Max		Value	Seconds
1A	W_1	W_1	27,706	0.1
		W_2	275	0.1
	W_2	W_1	27,706	0.1
		W_2	275	0.1
2A	W_1	W_1	56,665	0.1
		W_2	550	0.3
	W_2	W_1	56,665	0.1
		W_2	550	0.0
3A	W_1	W_1	82,728	0.1
		W_2	825	0.4
	W_2	W_1	82,728	0.1
		W_2	825	0.0
4A	W_1	W_1	110,210	0.2
		W_2	1100	0.6
	W_2	W_1	110,210	0.3
		W_2	1100	0.1
5A	W_1	W_1	141,377	0.2
		W_2	1375	0.9
	W_2	W_1	141,377	0.3
		W_2	1375	0.1

Table 4. *Less supply than demand*

Instance	Max		Value	Seconds
1B	W_1	W_1	19,398	0.1
		W_2	252	2.9
	W_2	W_1	18,756	267.7
		W_2	260	0.2
2B	W_1	W_1	39,673	1.2
		W_2	506	3.4
	W_2	W_1	38,301	31.9
		W_2	520	0.1
3B	W_1	W_1	57,919	0.4
		W_2	763	6.1
	W_2	W_1	56,186	1,551.3
		W_2	782	0.2
4B	W_1	W_1	77,160	0.7
		W_2	1021	11.3
	W_2	W_1	74,823	8,344.4
		W_2	1044	0.8
5B	W_1	W_1	98,978	0.6
		W_2	1275	7.0
	W_2	W_1	96 385	1,727.9
		W_2	1304	0.4

If a solution that was found in an instance was the same regardless of the order in which the objectives were solved, then only one Pareto-optimal solution existed for that given instance. In those instances with more supply than demand, the solutions within each instance were the same for both objectives regardless of the order in which they were solved. Both objectives had different values to maximize, but their overall intention was to serve all orders. Having more supply than demand enabled the possibility of fulfilling all orders and caused no conflict between the objectives.

The opposite could be observed for those instances with less supply than demand: the solutions were affected by the order in which the objectives were solved. The improvement of one objective led to a worsening of the other objective, which was indicative of the existence of more than one Pareto-optimal solution. Having less available supply than demand prompted the need to decide which order was most important to fulfill. The two objectives differed in how they treated orders; one objective considered the number of boxes that were specified in an order to be an indicator of importance, while the other objective focused on the value of the priority coefficient.

The time that was required to solve each instance was relatively low in those cases with more supply than demand. When solving those instances with less supply than demand, however, the solution time increased. This was especially evident when maximizing the total number of boxes that were delivered to all orders (W_1), while the optimal total value of the fulfilled prioritized orders (W_2) was used as a constraint.

Having observed that each of the instances with less supply than demand had at least two Pareto-optimal solutions, a further analysis was conducted for this case. We found that the same number of orders was fulfilled regardless of the order in which the objectives were handled; the only difference was in the individual orders that were fulfilled. In general, when maximizing the number of boxes that were delivered first, more low-priority orders were fulfilled as compared to when maximizing the total values of the prioritized orders first. This was due to the fact that those orders with low prioritization coefficients requested the greatest number of boxes for a given combination of fish attributes. Also observed was the pattern that the highly prioritized orders for a given combination were always fulfilled (independently from the order in which the objectives were solved). This was due to the fact that higher-priority orders requested relatively low numbers of boxes as compared to the available supply, making it possible to fulfill many orders with high priorities.

To further investigate the conflict between the two objective functions for those instances with less supply than demand, we next used the augmented ε-constraint method to generate the Pareto fronts for the five instances with this characteristic. This meant that the model was solved multiple times for each instance in order to generate different Pareto-optimal solutions. In each run, we used a maximum time limit of 700 seconds and specified an acceptable optimality gap. The acceptable optimality gap was set slightly higher (for instance, 5B, as this instance is larger and more difficult to solve). Table 5 summarizes the settings that were used to solve each instance.

Table 5. *Optimality gap and grid points*

Instance	Optimality gap [%]	Grid points
1B	0.01	9
2B	0.01	15
3B	0.01	20
4B	0.01	24
5B	0.05	30

Having instances of increasing problem sizes opened up observations into whether any iteration was unable to meet the required optimality gap before its time limit was reached. Table 6 gives a summary of the number of unique solutions for each instance, the maximum and minimum optimality gaps that were reached, and the number of times that the time limit was reached. Since the number of found unique solutions was identical to the number of grid points that were used, the resulting approximate Pareto fronts indicated that there may have been many more Pareto-optimal points that were not found.

Table 6. *Summary of instance solutions*

Instance	Unique solutions	Max gap [%]	Min gap [%]	Number of times time limit reached
1B	9	0.01	< 0.001	0
2B	15	0.01	0	0
3B	20	0.017	0	1
4B	24	1.265	< 0.001	18
5B	30	1.161	0.033	18

When creating the Pareto fronts for Instances 1B and 2B, all of the iterations found Pareto-optimal solutions with optimality gaps of 0.01% or lower within the time limit. As the size of the problem increased, fewer iterations could find a solution that satisfied the optimality gap that was required within the time limit. This was especially apparent for Instances 4B and 5B – both of which having 18 runs in which the required optimality gap was not met within the time limit. At most, two of the solutions had optimality gaps of more than 1%. This caused an issue regarding the certainty of having actually found a Pareto-optimal solution since the gap to optimality was so large. The estimated Pareto fronts for Instances 2B and 5B are illustrated in Figure 4. For the other instances, the Pareto fronts had similar shapes but with varying numbers of different points (as indicated in Table 6).

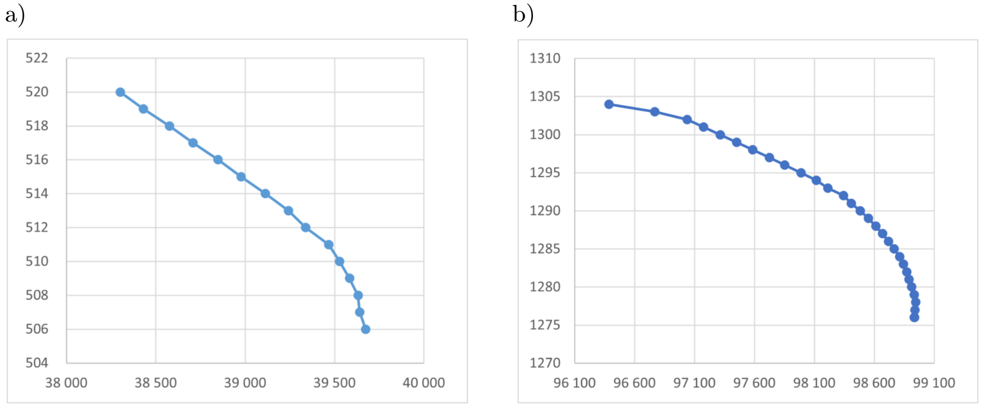

Fig. 4. *Pareto fronts for two individual instances, with W_1 on first axis and W_2 on second axis: a) Instance 2B; b) Instance 5B*

5. CONCLUDING REMARKS

This paper has presented a conceptual model of the allocation process that takes place in a company in the fish-farming industry. The company allocates freshly harvested fish to customer orders in order to satisfy the demand of its customers. A bi-objective mathematical programming model was formulated to capture the essence of the planning problem, aiming to provide decision support for the planners at the company.

The two objectives that were modeled were as follows:

1) maximize total number of boxes of fish delivered to all orders,
2) maximize total value of fulfilled prioritized orders.

These two objective functions represent a realistic allocation dilemma. When reviewing the existing literature, no previous multi-objective assignment model was found that considered any similar allocation problem. To solve the instances of the mathematical model, the augmented ε-constraint method (AUGMECON) was used, as it allowed us to find Pareto-optimal solutions in an efficient manner.

A computational study was performed using ten different instances. The study demonstrated an example where those orders that requested high numbers of boxes were set to have low priorities, while those orders that requested low numbers of boxes had high priorities. The results showed that, in a scenario with more supply than demand, no conflict could be observed between the two objectives of the model. This showed that, in cases where there is more supply than demand, a single-objective formulation may be sufficient.

When there was less supply than demand, conflicts occurred; these resulted in different Pareto-optimal solutions depending on which objective was solved first. In these cases, using multi-objective optimization was appropriate. The computational effort that was required to solve the instances also increased significantly when there was less supply than demand. Approximate Pareto fronts were created for all instances with lower supply than demand. The results showed that, as the size of the problem grew, fewer solutions with optimality gaps that satisfyed the requirement were found before the time limits were reached.

Solving a problem instance that is described with the model provides a plan for the allocation of fish to customer orders for a given horizon. This plan can be used to help planners determine how their supplies should be allocated to their orders; this may help their companies make better trade-offs in situations where planning is difficult (such as when the supply is insufficient to cover the demands from all of the orders).

Additional objectives could have been included in our study. As an example, a focal company may wish to maximize its profits in addition to the amount of fish that is delivered or the number of prioritized orders that are fulfilled; this includes selling their fish to the spot market for a better price. A limitation of our study was the lack of economic data on the transactions. For future research, we also propose to investigate the effect of uncertainty on supply forecasts. The quality and size of harvested fish are only known probabilistically until the fish is slaughtered this uncertainty may influence the quality of the planned order allocations.

REFERENCES

Abdollahi Saadatlu E., Barzinpour F. & Yaghoubi S. (2022). A sustainable model for municipal solid waste system considering global warming potential impact: A case study. *Computers & Industrial Engineering*, **169**, art. no. 108127. DOI: 10.1016/j.cie.2022.108127.

Abedi A. & Zhu W. (2017). An optimisation model for purchase, production and distribution in fish supply chain – A case study. *International Journal of Production Research*, **55**(12), pp. 3451–3464. DOI: 10.1080/00207543.2016.1242800.

Ahumada O. & Villalobos J.R. (2009). Application of planning models in the agri-food supply chain: A review. *European Journal of Operational Research*, **196**(1), pp. 1–20. DOI: 10.1016/j.ejor.2008.02.014.

Ahumada O. & Villalobos J.R. (2011). Operational model for planning the harvest and distribution of perishable agricultural products. *International Journal of Production Economics*, **133**(2), pp. 677–687. DOI: 10.1016/j.ijpe.2011.05.015.

Amorim P., Günther H.-O. & Almada-Lobo B. (2012). Multi-objective integrated production and distribution planning of perishable products. *International Journal of Production Economics*, **138**(1), pp. 89–101. DOI: 10.1016/j.ijpe.2012.03.005.

Chowdhury M. & Tan P. (2004). A multi-objective decision-making framework for transportation investments. *Journal of the Transportation Research Forum*, **43**(1), pp. 91–104. DOI: 10.22004/ag.econ.206723.

Ebrahimi S. & Bagheri E. (2022). Optimizing profit and reliability using a bi-objective mathematical model for oil and gas supply chain under disruption risks. *Computers & Industrial Engineering*, **163**(100), art. no. 107849. DOI: 10.1016/j.cie.2021.107849.

Fan Z., Li S. & Gao Z. (2019). Multiobjective sustainable order allocation problem optimization with improved genetic algorithm using priority encoding. *Mathematical Problems in Engineering*, **2019**, art. no. 8218709. DOI: 10.1155/2019/8218709.

Fasihi M., Tavakkoli-Moghaddam R., Najafi S. & Hajiaghaei M. (2021a). Optimizing a bi-objective multi-period fish closed-loop supply chain network design by three multi-objective meta-heuristic algorithms. *Scientia Iranica*. DOI: 10.24200/sci.2021.57930.5477.

Fasihi M., Tavakkoli-Moghaddam R., Najafi S. & Hajiaghaei-Keshteli M. (2021b). Developing a bi-objective mathematical model to design the fish closed-loop supply chain. *International Journal of Engineering*, **34**(5), pp. 1257–1268. DOI: 10.5829/ije.2021.34.05b.19.

Ghasemi P., Khalili-Damghani K., Hafezalkotob A. & Raissi S. (2019). Uncertain multi-objective multi-commodity multi-period multi-vehicle location-allocation model for earthquake evacuation planning. *Applied Mathematics and Computation*, **350**(100), pp. 105–132. DOI: 10.1016/j.amc.2018.12.061.

Gholizadeh H., Jahani H., Abareshi A. & Goh M. (2021). Sustainable closed-loop supply chain for dairy industry with robust and heuristic optimization. *Computers & Industrial Engineering*, **157**, art. no. 107324. DOI: 10.1016/j.cie.2021.107324.

Hwang C.-L. & Masud A.S.M. (1979). *Multiple Objective Decision Making. A State-of-the-Art Survey*. Springer Verlag. DOI: 10.1007/978-3-642-45511-7.

Jia R., Liu Y. & Bai X. (2020). Sustainable supplier selection and order allocation: Distributionally robust goal programming model and tractable approximation. *Computers & Industrial Engineering*, **140**, art. no. 106267. DOI: 10.1016/j.cie.2020.106267.

Kaviani M.A., Peykam A., Khan S.A., Brahimi N. & Niknam R. (2020). A new weighted fuzzy programming model for supplier selection and order allocation in the food industry. *Journal of Modelling in Management*, **15**(2), pp. 381–406. DOI: 10.1108/JM2-11-2018-0191.

Kiwa Norway (2021). *Mattrygghet og akvakultur* [in Norwegian]. https://www.kiwa.com/no/no/vaare-tjenester/sertifisering/mattrygghet-og-akvakultur/ [10.05.2021].

Koldborg Jensen T., Nielsen J., Larsen E.P. & Clausen J. (2010). The fish industry – toward supply chain modeling. *Journal of Aquatic Food Product Technology*, **19**(3–4), pp. 214–226. DOI: 10.1080/10498850.2010.508964.

Mavrotas G. (2009). Effective implementation of the ε-constraint method in Multi-Objective Mathematical Programming problems. *Applied Mathematics and Computation*, **213**(2), pp. 455–465. DOI: 10.1016/j.amc.2009.03.037.

Moheb-Alizadeh H. & Handfield R. (2019). Sustainable supplier selection and order allocation: A novel multi-objective programming model with a hybrid solution approach. *Computers & Industrial Engineering*, **129**, pp. 192–209. DOI: 10.1016/j.cie.2019.01.011.

Mosallanezhad B., Hajiaghaei-Keshteli M. & Triki C. (2021). Shrimp closed-loop supply chain network design. *Soft Computing*, **25**(11), pp. 7399–7422. DOI: 10.1007/s00500-021-05698-1.

Musavi M. & Bozorgi-Amiri A. (2017). A Multi-objective sustainable hub location-scheduling problem for perishable food supply chain. *Computers & Industrial Engineering*, **113**(100), pp. 766–778. DOI: 10.1016/j.cie.2017.07.039.

Norwegian Food Safety Authority (2020). *Fish health.* https://www.mattilsynet.no/fisk_og_akvakultur/fiskehelse/ [10.05.2021].

NOU (2019). *Skattlegging av havbruksvirksomhet. Taxation of aquaculture activities,* Finansdepartement.

Rabbani M., Amirhossein Sadati S. & Farrokhi-Asl H. (2020). Incorporating location routing model and decision making techniques in industrial waste management: Application in the automotive industry. *Computers & Industrial Engineering*, **148**, art. no. 106692. DOI: 10.1016/j.cie.2020.106692.

Seitz A., Grunow M. & Akkerman R. (2020). Data driven supply allocation to individual customers considering forecast bias. *International Journal of Production Economics*, **227**, art. no. 107683. DOI: 10.1016/j.ijpe.2020.107683.

Sharma R. & Darbari J.D. (2021). Integrated optimization model for sustainable supplier selection and order allocation in food supply chain. In: A. Tiwari, K. Ahuja, A. Yadav, J.C. Bansal, K. Deep & A.K. Nagar (Eds.). *Soft Computing for Problem Solving*, Springer, pp. 557–572.

Wang S. & Ma S. (2018). Efficient methods for a bi-objective nursing home location and allocation problem: A case study. *Applied Soft Computing*, **65**, pp. 280–291. DOI: 10.1016/j.asoc.2018.01.014.

Zhang Z., Guo C., Ruan W., Wang W. & Zhou P. (2022). An intelligent stochastic optimization approach for stochastic order allocation problems with high-dimensional order uncertainties. *Computers & Industrial Engineering*, **167**, art. no. 108008. DOI: 10.1016/j.cie.2022.108008.