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Home health care staffing, routing, and scheduling problem with multiple shifts and emergency considerations

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Abstract

Effective planning of human resources is critical in designing an efficient home healthcare system. In this study, we present a novel home healthcare staffing, routing, and scheduling problem inspired by a real-world application. The proposed problem addresses a set of patients, with varying daily visit requirements, being served by a set of caregivers with different qualification levels over a multi-day multi-shift planning horizon. The study aims to achieve three objectives: minimizing the number of additional shifts, maximizing the allocation of caregivers to emergencies, and minimizing the sum of route lengths. These objectives are optimized hierarchically while considering a set of restrictions, including time windows, skill matching, synchronicity, care continuity, and labor regulations. To tackle the problem, we introduce a mixed-integer linear programming model. The model is then extended and two sets of valid inequalities are incorporated to enhance its tightness. Computational experiments are conducted on a set of 20 instances. The results highlight the efficiency of the proposed extension in increasing both the number of instances that can be solved to optimality and the number of instances for which a feasible solution is found.
Keywords: Mixed-integer programming; Valid inequalities; Home healthcare; Human resource planning; Multiple shifts; Synchronization.

1 Introduction

Home healthcare (HHC) refers to nursing and care services received by patients and elderly people who do not reside in health institutions, such as hospitals or nursing homes, but in their own homes. In the past decade, the demand for home healthcare services has grown for several reasons, such as population growth and an aging population. According to a report by Norwegian Statistics Agency (2022), the number of HHC recipients increased by 11.2 % from 2017 to 2021.

The significance of human resource planning in the healthcare industry is increasing due to the shortage of caregivers faced by both municipalities and private companies. Efficient human resource planning involves optimizing staff work schedules and effectively routing, scheduling, and assigning caregivers to patients. In this study, we address the routing and scheduling problem of home health care providers in Norway, with a focus on organizational aspects of the workforce.

The proposed problem considers multiple shifts per day, varying daily patient visit requirements, the allocation of caregivers to emergencies, and the assignment of caregivers to additional shifts along the planning horizon. Furthermore, it addresses time windows, skill matching, synchronicity, and care continuity as hard constraints. To tackle the problem, we propose a mixed-integer linear programming model. The model is then extended and two sets of valid inequalities are added to enhance its tightness. A computational study is conducted on 20 instances to evaluate the performance of the extended model and identify its strengths and limitations.

The rest of this article is organized as follows. After reviewing previous works in Section 2, we details the proposed problem in Section 3. In Section 4, a mixed integer linear programming formulation is introduced. Next, the formulation is extended and valid inequalities are added in Section 5. Computational studies are provided in Section 6. Finally, our concluding remarks are found in Section 7.

2 Literature review

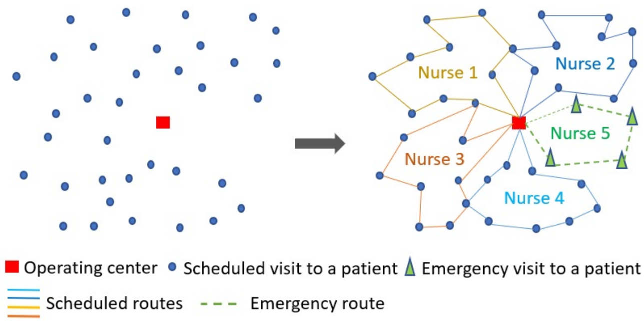
The home healthcare routing and scheduling problem (HHCRSP) extends the vehicle routing problem with time windows (VRPTW) by considering additional constraints specific to the home healthcare context. In this section, we provide a brief literature review on the vehicle routing problem and its variants, followed by a broader review of previous home health care routing and scheduling studies. Lastly, we discuss the contribution of our research problem to the literature.

2.1 Vehicle routing problems

The vehicle routing problem (VRP) was first explored by Clarke and Wright (1964). It involves the design of routes, starting and ending at a depot, for a fleet of vehicles that serve a set of customers to achieve specific objectives. In the HHC context, caregivers represent the fleet, patients are the customers, and the operating center serves as the depot. Figure 1 illustrates a potential solution to a VRP in the home healthcare context. The figure shows four routes, depicted by solid lines, assigned to four nurses for visiting 35 patients. Each patient is visited once, either by one nurse or concurrently by two nurses. Additionally, one nurse is exclusively dedicated to emergency visits and the route for this nurse is depicted as a dashed line. Given the extensive literature on this topic, we limit our discussion to VRP studies relevant to our research problem. For an in-depth review of the literature on VRP and its variants, we refer to the works of Sharma et al. (2018) and Liu et al. (2023).

The vehicle routing problem with time windows (VRPTW) extends the standard VRP by considering that each customer can only be visited within a given time interval. Kallehauge et al. (2005) presented the VRPTW in terms of its mathematical modeling, structure, and decomposition alternatives. Another VRP variant, known as the periodic vehicle routing problem (PVRP), was initially explored by Beltrami and Bodin (1974). The authors addressed the construction of efficient vehicle routes for

Figure 1: Example of a VRP solution in the HHC context



garbage collection over several days, aiming to minimize both the number of vehicles and total travel time. Later, Ren et al. (2010) studied a PVRP where each day in the planning horizon is divided into shifts and each route must start and end within the respective shift's time window.

Several considerations in our research problem have been addressed in previous VRP studies, including skill matching, precedence, and synchronization. Skill matching involves aligning personnel of different skill levels with customers possessing varying service requirements. For instance, Cappanera et al. (2011) tackled a network problem that aims to design routes for a set of technicians, each with a certain skill level, to service a set of customers, each with specific requirements. The authors assumed that the service at each customer can be operated by any technician that has at least a certain skill level. Precedence constraints in VRPs involve enforcing specific sequences in which certain visits must be carried out. Synchronization refers to the coordination of multiple vehicles to start the service at a specific customer simultaneously. Both precedence and synchronization were first investigated by Bredström and Rönnqvist (2008). Recently, Soares et al. (2023) conducted a literature review specifically on synchronization within VRPs.

The integration of human resource planning with the vehicle routing problem have been addressed in several fields. For example, De Bruecker et al. (2018) combined the problems of developing personnel shift schedules and vehicle routing in addressing a waste collection problem. Wang et al. (2023) studied the human resource allocation and vehicle routing in disaster response operations. For a more in-depth discussion on this topic, we refer to the review provided by Castillo-Salazar et al. (2016).

2.2 Home health care routing and scheduling problems

The home health care routing and scheduling problem (HHCRSP) was first explored by Begur et al. (1997), focusing on patient-to-patient travel costs and workforce allocation. Subsequently, there has been a growing interest among researchers in this field. Di Mascolo et al. (2021) reported that between 1997 and 2020, 165 studies were published on the topic. However, the problems studied in those articles vary significantly due to differences in national and organizational settings. For instance, Xiang et al. (2023a) investigated a HHC problem in China, where caregivers receive overtime wages if their working hours exceed the regular duration. In contrast, Shahnejat-Bushehri et al. (2021) explored a HHC problem in Canada, where caregivers can work entire additional periods instead of limited extra hours. In this section, we delve into several aspects of HHCRSPs that have received considerable attention in the literature. For a more comprehensive review, we refer to the work by Euchi et al. (2022).

2.2.1 Planning horizon

HHCRSPs are classified based on the planning horizon into two categories: single-period problems and multi-period problems. Single-period problems address caregivers' routing and scheduling throughout one working day. For example, Liu et al. (2017) investigated lunch break requirements within a single-day problem. Hahnejat-Bushehri et al. (2019) focused on the constraints associated with transferring collected biological samples to a laboratory within specific time frames. Li et al. (2021) explored the routing and scheduling of a group of doctors throughout a day. Each doctor is either assigned to a route or dedicated to providing services at a community care center. Ma et al. (2022) examined the cooperation between multiple home healthcare centers to meet the demand for a single day. In contrast, multi-period problems involve the routing and scheduling of caregivers over several days or weeks. For example, Demirbilek et al. (2021) investigated the dynamic routing and scheduling of multiple nurses. The study assumed that patient assignment decisions are made immediately upon the request arrival. Once accepted, patients are visited on the same days and times by the same nurse throughout the planning horizon. Xiang et al. (2023b) extended the work of Li et al. (2021) by considering fixed-frequency services for every patient throughout a multi-day planning horizon.

2.2.2 Optimization criteria

The optimization criteria considered in prior research can be categorized into three main categories: time, cost, and preferences. Time criteria focus on minimizing travel time, overtime, and waiting time. Cost criteria typically involve minimizing travel and workforce costs. Preferences criteria refer to maximizing satisfaction for both patients and caregivers.

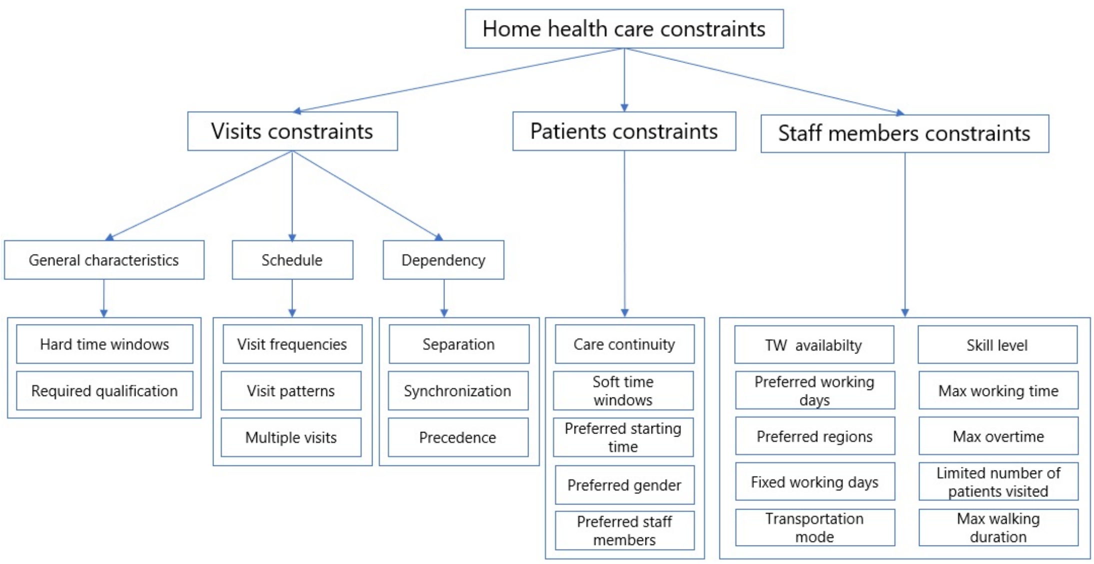
Most studies have incorporated multiple criteria into their objective functions, expressing them either as a weighted sum or by employing multi-objective optimization methods. For example, Liu et al. (2018) investigated the trade-off between minimizing operational costs and maximizing patient satisfaction. Di Mascolo et al. (2018) aimed to minimize a weighted sum of penalties linked to patient dissatisfaction, considering factors like gender and time windows, while also maximizing workload balance among staff members. Decerle et al. (2019) aimed to maximize service quality, minimize staff working time, and maximize workload balance among caregivers. In a single-day-multi-period problem addressed by Shahnejat-Bushehri et al. (2021), the objective is to minimize total traveling costs, service costs, and the number of employed nurses. Malagodi et al. (2021) explored the trade-off between minimizing mismatches between patients and caregivers and minimizing the overall working time. Shiri et al. (2021) aimed to minimize the cost of locating a healthcare center among several candidates, along with the total traveling cost, caregivers' overtime cost, and the number of visits conducted by overqualified caregivers. Yang et al. (2021) addressed three conflicting objectives: maximizing service quality, minimizing total traveling cost, and maximizing workload balance. Fu et al. (2024) studied a multi-objective problem where the cost of a visit depends on the qualifications of the caregiver performing the service. The primary objective is to minimize fixed staff cost, total traveling cost, and overall service cost. The secondary objective is to minimize penalty costs associated with violating time windows. Du and Li (2024) investigated a green HHCRSP with the objective of minimizing total operating cost and carbon emissions while maximizing patient and caregiver satisfaction.

In addition, unique optimality criteria have been explored in the literature. Rest and Hirsch (2016) investigated caregivers' use of public transportation, aiming to minimize route lengths, staff overtime, and visits by overqualified caregivers. Xiao et al. (2018) considered subcontracting costs as part of the operating cost. Eching et al. (2019) addressed a multi-period problem that aims to minimize the number of unserved patients. Pereira et al. (2020) tackled scheduling patient visits, routing decisions, and determining visit days to minimize route makespan. Li et al. (2021) aimed to minimize door-to-door service travel costs, waiting penalties for out-patients, and maximize overall patient preference satisfaction. Ziya-Gorabi et al. (2022) presents a novel mathematical model aimed at minimizing environmental pollution resulting from caregivers' travel.

2.2.3 Constraints

The constraints examined in prior home healthcare routing and scheduling studies can be categorized into three main groups: those associated with visits, patients, and staff members. Figure 2 illustrates the most frequently cited restrictions in the literature, classified by category.

Figure 2: Considered constraints in the HHCRSPs



Previous studies have addressed various constraints related to visits. For example, Gaspero and Urli (2014) considered patient preferences for being visited before a given time. Malagodi et al. (2021) assumed that each patient has a preferred start time for visits. Liu et al. (2018) addressed a minimum required duration for each visit rather than adhering to a fixed duration. As a result, the extra time invested by a caregiver during a visit increases patient satisfaction. Moussavi et al. (2019) considered that each visit should be conducted at a given time rather than within a specific time window. Demirbilek et al. (2019) proposed scheduling each visit at a specific appointment time selected from several available options. Gomes and Ramos (2019) supposed that some patients prefer to receive a different caregiver at each visit (non-loyalty). Fathollahi-Fard et al. (2020) presented several restrictions related to the use of different means of transportation, such as a maximum distance traveled using each mode of transportation for each caregiver. In addition, Euchi (2020) explored imposing an upper limit on the number of visits that can be assigned to every caregiver.

Skill matching involves aligning caregiver qualifications with visit requirements. Shao et al. (2012) addressed a HHCRSP for therapists with varying qualifications, assuming that the duration of every visit depends on the therapist's qualification. Nasir and Dang (2020) considered that each caregiver is qualified to provide at least two types of services offered to patients. Demirbilek et al. (2021) categorized caregivers into various qualification levels based on their education and experience. The authors considered that caregivers with a high qualification level can also conduct visits that require caregivers with a low qualification level.

Service synchronization requires coordinating multiple caregivers for simultaneous visits. To address this challenge, Liu et al. (2019) generated multiple nodes for visits requiring multiple caregivers. Then, an identical start service time was enforced for nodes allocated to each visit. In a different approach, Yadav and Tanksale (2022) linked the start service time of each node requiring multiple caregivers with the arrival time of every caregiver to that node.

The care continuity restrictions ensure that every patient is served by a maximum number of caregivers throughout the planning horizon. Cappanera et al. (2018) imposed an upper limit on the number of different caregivers assigned to each patient over the planning horizon. In contrast, Carello et al. (2018) classified patients into three groups based on care continuity: hard (assigned to a single caregiver), partial (assigned to up to two caregivers), and none (open to any number of caregivers). Additionally, Lahrichi et al. (2022) mandated assigning one nurse to every patient along the planning horizon.

2.2.4 Solution methodology

Several methods have been employed for solving HHCRSPs. These methods, which are often developed for solving vehicle routing and staff scheduling problems, involves exact methods (e.g., branch-and-price, branch-and-bound), heuristics (e.g., local search metaheuristics, population-based metaheuristics), and hybrids (e.g., matheuristics, hybrid heuristics, hybrid exact methods).

The development of valid inequalities to enhance computational performance is a prevalent practice in previous HHC routing and scheduling studies. For instance, Cappanera and Scutellà (2015) introduced valid inequalities to tighten a mixed-integer programming model designed for a multi-period HHC problem. Another studies by Heching and Hooker (2016) and Naderi et al. (2023) explored the use of logic-based Benders decomposition to solve HHC problems, incorporating several sets of valid inequalities to reduce the number of Benders iterations.

For further details regarding the planning horizon, optimization criteria, constraints, and solution methods in the studies covered in our review, we refer to Table 1.

2.3 Contribution to the literature

In this article, we investigate a multi-day multi-shift home healthcare staffing, routing, and scheduling problem. Consequently, decisions made for each period impact decisions for subsequent periods. Each patient requires a number of visits per day, with specific requirements for each visit. Additionally, each caregiver is assigned a certain qualification level and scheduled to work specific shifts during the planning horizon. We address three objectives: minimizing the number of additional shifts, maximizing the allocation of caregivers to emergencies, and minimizing the sum of route lengths. These objectives are optimized hierarchically while considering a set of restrictions, including time windows, skill matching, synchronicity, care continuity, and labor regulations.

Considering variations in the number and requirements of patient visits across multiple shifts has received limited attention in existing literature. However, we observed two notable contributions in this domain. Yadav and Tanksale (2022) considered that each patient requires a certain number of procedures and every procedure should be provided a given number of times during multiple shifts. Unlike our research problem, the authors addressed a single-day planning horizon and assumed that all caregivers scheduled to work a shift are allocated to routes during that time. Moussavi et al. (2019) investigated a multi-day HHCRSP assuming that each patient requires a given number of visits per day, each day is divided into periods, and every visit should be performed within a given period. However, the study did not consider time windows for periods, skill matching, service synchronicity, care continuity, or staffing decisions.

In addition, we observed limited attempts to integrate staffing, routing, and scheduling decisions in the home healthcare literature. Notable contributions in this domain include the studies by Nikzad et al. (2021) and Naderi et al. (2023). However, both studies considered that each patient requires at most one visit per day. They also did not address the existence of shifts, time windows, and service synchronicity. Another study by Restrepo et al. (2020) addressed the integration of caregiver staffing and scheduling decisions but ignored the routing aspect.

Although most previous HHC studies considered labor regulations, but to the best of our knowledge, no previous study tackled similar regulations to those addressed in our research problem.

Table 1: The summery of the literature review

Reference	Planning horizon			Optimization criteria					Constraints					Solution
	SD	MD	MS	OC	SAT	NR	RL	Other	TW	SM	SY	CC	LR	methodology
Shao et al. (2012)	-	✓	-	✓	✓	-	-	-	✓	✓	-	-	-	GRASP - ALNS
Gaspero and Urli (2014)	-	✓	-	-	✓	-	-	-	✓	✓	-	-	✓	2OPT-Heuristic
Cappanera and Scutellà (2015)	-	✓	-	-	✓	-	-	-	✓	✓	-	✓	✓	MILP
Rest and Hirsch (2016)	✓	-	✓	-	-	-	✓	OT - OQ	✓	✓	-	-	✓	TS
Liu et al. (2017)	✓	-	-	✓	-	-	-	-	✓	-	-	-	✓	BP
Di Mascolo et al. (2018)	✓	-	-	-	✓	-	-	-	✓	✓	✓	-	-	MILP
Liu et al. (2018)	-	✓	-	✓	✓	-	-	-	-	✓	-	✓	✓	Heuristic
Xiao et al. (2018)	-	✓	-	✓	✓	-	-	-	✓	✓	-	-	✓	LRA
Demirbilek et al. (2019)	-	✓	✓	-	✓	-	-	-	-	✓	-	✓	-	SBA
Decerle et al. (2019)	✓	-	-	-	✓	-	✓	-	✓	✓	✓	-	✓	MA - ACO
Euchi (2020)	✓	-	-	-	✓	-	-	IT	✓	-	-	-	✓	K-means - ACO
Gomes and Ramos (2019)	-	✓	-	-	✓	-	✓	-	✓	-	-	-	✓	-
Grenouilleau et al. (2019)	-	✓	-	✓	✓	-	-	-	✓	-	-	-	✓	LNS
Eching et al. (2019)	-	✓	-	-	✓	-	-	-	✓	✓	-	-	✓	LBBD
Moussavi et al. (2019)	-	✓	✓	-	-	-	-	TD	-	-	-	-	✓	Matheuristic
Chaieb et al. (2020)	✓	-	-	-	✓	-	-	IT	✓	✓	-	-	✓	K-means, HA
Fathollahi-Fard et al. (2020)	-	✓	-	✓	✓	-	✓	-	✓	-	-	✓	✓	MILP-LRA
Frifita and Masmoudi (2020)	✓	-	-	-	-	-	✓	-	✓	-	✓	-	✓	VNS
Martin et al. (2020)	-	✓	-	-	-	-	✓	-	✓	-	-	-	-	ACO
Nasir and Dang (2020)	✓	-	-	✓	-	-	-	-	✓	✓	-	-	✓	MILP - VSN
Pereira et al. (2020)	-	✓	-	-	-	-	✓	-	✓	-	-	-	✓	ACO
Restrepo et al. (2020)	-	✓	✓	✓	-	✓	-	-	✓	✓	-	-	✓	MILP
Demirbilek et al. (2021)	✓	-	-	-	✓	-	-	-	✓	✓	-	-	-	SBA
Li et al. (2021)	✓	-	✓	✓	✓	✓	-	-	✓	✓	-	-	✓	GA
Liu et al. (2021)	✓	-	-	✓	-	-	-	-	✓	✓	✓	-	✓	MA - HSA
Malagodi et al. (2021)	✓	-	✓	-	✓	-	✓	OT	✓	✓	-	-	✓	CD
Nikzad et al. (2021)	-	✓	-	✓	-	-	-	IT	✓	✓	-	✓	✓	Matheuristic
Shahnejat-Bushehri et al. (2021)	-	✓	✓	✓	-	✓	-	-	✓	✓	✓	✓	✓	MA
Shiri et al. (2021)	-	✓	-	✓	-	-	-	OQ	✓	-	-	-	✓	Hybrid
Yang et al. (2021)	-	✓	-	✓	✓	-	-	-	✓	-	-	-	✓	LNS

Table 1 continued

Reference	Planning horizon			Optimization criteria					Constraints					Solution
	SD	MD	MS	OC	SAT	NR	RL	Other	TW	SM	SY	CC	LR	methodology
Bahadori-Chinibelagh et al. (2022)	✓	-	-	✓	-	-	-	TD	✓	-	-	-	-	Heuristics
Bazirha et al. (2022)	✓	-	-	✓	✓	-	-	-	✓	✓	-	-	✓	GA
Blais-Amyot (2022)	✓	-	✓	-	-	-	-	IT	✓	✓	-	-	✓	BP
Fathollahi-Fard et al. (2022)	✓	-	-	✓	✓	-	-	-	✓	-	-	-	✓	Hybrid
Krityakierne et al. (2022)	-	✓	-	-	✓	-	-	IT	✓	✓	-	-	✓	TS
Lahrichi et al. (2022)	-	✓	-	-	✓	-	-	-	✓	-	-	✓	✓	TS
Li et al. (2022)	-	✓	-	✓	✓	-	-	-	✓	✓	-	✓	✓	ALNAS
Ma et al. (2022)	✓	-	-	✓	-	-	-	-	✓	✓	-	-	✓	SSA
Qiu et al. (2022)	✓	-	-	✓	-	-	-	-	✓	✓	✓	-	-	BPC
Yadav and Tanksale (2022)	✓	-	✓	-	✓	-	-	REV	✓	✓	✓	✓	✓	Heuristic
Ziya-Gorabi et al. (2022)	✓	-	-	✓	✓	✓	-	CE	✓	-	-	-	✓	Hybrid
Clapper et al. (2023)	✓	-	✓	-	-	-	✓	-	✓	✓	-	-	✓	GA
Naderi et al. (2023)	-	✓	-	✓	-	-	-	-	✓	✓	-	-	✓	LBBD
Xiang et al. (2023a)	✓	-	-	✓	✓	-	-	-	✓	✓	-	-	✓	GA
Xiang et al. (2023b)	-	✓	-	✓	✓	-	-	-	✓	✓	-	-	✓	TS
Du and Li (2024)	✓	-	-	✓	✓	-	-	CE	✓	✓	-	-	✓	GA
Fu et al. (2024)	✓	-	-	✓	-	-	-	-	-	-	-	-	✓	MBO
Kummer et al. (2024)	✓	-	-	-	✓	-	-	TD	✓	✓	✓	-	✓	GA
Our problem	-	✓	✓	-	-	✓	✓	-	✓	✓	✓	✓	✓	MILP

Planning horizon: **SD**: Single day; **MD**: Multiple days; **MS**: Multiple shifts (periods) in a day.
Optimization criteria: **OC**: Operation cost; **SAT**: Patients and caregivers satisfaction; **NR**: Number of routes (required caregivers); **RL**: Sum of route lengths; **OT**: Overtime; **OQ**: Overqualified visits; **TD**: Travel distance; **IT**: Idle time (waiting time); **CE**: Carbon emissions; **REV**: Revenue.
Constraints: **TW**: time windows; **SM**: Skill matching; **SY**: Synchronicity; **CC**: Care continuity; **LR**: Labor regulations.
Solution methodologies: **ACO**: Ant Colony Algorithm; **ALNS**: Adaptive Large Neighborhood Search; **BP**: Branch and Price; **BPC**: Branch and Price and Cut; **CD**: Cluster-Based Decomposition; **CG**: Column Generation; **GA**: Genetic Algorithm; **GOA**: Gossip Algorithm; **GRASP**: Greedy Randomized Adaptive Search Procedure; **HA**: Hungarian Algorithm; **HSA**: Hybrid Simulated Annealing; **LBBD**: Logic-Based Benders Decomposition; **LRA**: Lagrangian Relaxation-Based Algorithm; **LNS**: Large Neighborhood Search; **MA**: Memetic Algorithm; **MBO**: Migrating Birds Optimization; **MILP**: Mixed Integer Linear Programming; **SBA**: Scenario-Based Approach; **SSA**: Stochastic Simulation Approach; **TS**: Tabu Search; **VNS**: Variable Neighborhood Search.

3 Problem description

In this section, we present a problem inspired by home health care operations in three Norwegian municipalities: Molde, Oslo, and Ås.

We consider a specified geographical area where a home healthcare center (referred to as the operating center) is responsible for providing medical services to a set of patients over a multi-day planning horizon. This is facilitated through a team of caregivers with varying qualification levels. Each day is divided into successive shifts where the end time of a shift corresponds to the beginning time of the next shift. Moreover, these shifts are equal in duration and remain fixed throughout the planning horizon. The travel times between patients and between the operating center and each patient are given.

Each caregiver is associated with a specific qualification level based on his/her education and is scheduled to work specific shifts during the planning horizon. According to labor regulations, caregivers are not allowed to work overtime as extra hours. Instead, they can be called upon to work entire additional shifts if needed. However, caregivers are not permitted to work two consecutive shifts, whether on the same day (e.g., morning and evening shifts during a two-shift day) or on consecutive days (e.g., Monday evening and Tuesday morning). Every caregiver working a regular shift must either be assigned to a route or solely dedicated to emergencies. In contrast, when a caregiver is assigned to work an additional shift, that caregiver should be assigned to a route during that shift. Overall, caregivers assigned to routes in a shift should start and end at the operating center within the shift's time window.

Each patient requires a given number of visits per day during the planning horizon. Every visit requires a certain number of caregivers, each with a specific qualification level. Additionally, every visit is characterized by a specified duration and a time window that may span at most two consecutive shifts. Patients may also require an interval between every two consecutive visits during a day. The planning process takes into account the continuity of care, meaning that each patient can at most be served by a given number of caregivers of each qualification level throughout the planning horizon. Synchronization is also considered, ensuring that when multiple caregivers are required for a visit, they must start and end the service at the same time.

Furthermore, patients may require emergency visits. The number and timing of these visits are unknown. Therefore, after minimizing the total number of additional shifts as the primary objective, decision-makers aim to maximize the allocation of caregivers to emergencies along the planning horizon. As a third objective, the sum of route lengths over the planning horizon should be minimized. Figure 3 represents the hierarchy of these three objectives.

Figure 4 shows an example of routes for two nurses, starting from the operating center, passing through several patients, before returning to the operating center. Three out of four patients should be visited

Figure 3: Hierarchy for the three considered objectives

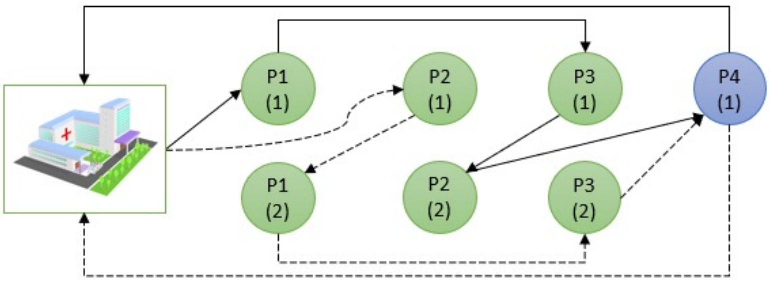
Priority 1: Minimize the number of additional required shifts

Priority 2: Maximize caregivers allocation to emergencies

Priority 3: Minimize the sum of route lengths

twice and one patient requires only one visit. Each node, represented by a circle, represents a visit to a patient, where the identifier of the patient comes first and the number of the visit is indicated in parentheses. The green nodes indicate visits that require one nurse and the blue node represents a visit that requires two nurses.

Figure 4: Example of a solution to a HHCRSP



As shown in Figure 4, the first nurse, following the solid arrows, starts from the operating center and goes to patient P1 for the first visit, then moves to patient P3 for the first visit, continues to patient P2 for the second visit, then travels to patient P4, and ends up at the operating center, as the route is completed. The second caregiver, following the dashed arrows, starts from the operating center and goes to patient P2 for the first visit, then moves to patient P1 for the second visit, continues to patient P3 for the second visit, then travels to patient P4, and ends up at the operating center, as the trip is completed.

4 Mathematical formulation

In this section, we present a mixed-integer linear programming model that addresses the network problem described in Section 3.

Let $\mathcal{C}$ represents a team of caregivers and $\mathcal{P}$ denotes a set of patients. Each caregiver $c \in \mathcal{C}$ is assigned to a route, dedicated to emergencies, or does not work at all in shift $s \in \mathcal{S}$ on day $d \in \mathcal{D}$. The pairs (i, v) and (j, u) on day d represent nodes in a complete directed graph, where (i, v) denotes visit v to patient i and (j, u) represents visit u to patient j on that day. The direct move from node (i, v) to node (j, u) on day d represents an arc in the network and is expressed by the tuple (i, v, j, u, d). The rest of our formulation is presented as follows.

Sets

$\mathcal{P}$	:	set of patients, indexed by i and j
$\mathcal{C}$	:	set of caregivers, indexed by c
$\mathcal{D}$	:	set of consecutive days, indexed by $d \in \{1, ..., \lvert\mathcal{D}\rvert\}$
$\mathcal{S}$	:	set of successive shifts per day, indexed by $s \in \{1, ..., \lvert\mathcal{S}\rvert\}$
$\mathcal{V}_{id}$	:	set of visits required by patient i on day d, indexed by $v \in \{1, \ldots, \lvert\mathcal{V}_{id}\rvert\}$
$\mathcal{Q}$	:	set of qualification levels, indexed by q

In addition, the following auxiliary sets are generated to facilitate formulating the model.

$\mathcal{C}''_q$	:	set of caregivers with qualification level q
$\mathcal{N}_d$	:	set of all nodes (i.e. patient visits) on day d
$\mathcal{N}''_{cd}$	:	set of all nodes on day d that can be visited by caregiver c
$\mathcal{M}_{cd}$	:	set of all possible moves (i, v, j, u) on day d that can be conducted by caregiver c

Node (i, v) on day d is included in set $\mathcal{N}''_{cd}$ if that node requires caregivers with the same qualification level as caregiver c. Thus, the sets $\mathcal{N}''_{cd}$ enable the model to express that certain caregivers are unable to visit specific nodes during the planning horizon. Similarly, the arc (i, v, j, u) on day d is included in set $\mathcal{M}_{cd}$ if nodes (i, v) and (j, u) can be visited consecutively by the same caregiver and both nodes require caregivers with the same qualification level as caregiver c. Hence, the sets $\mathcal{M}_{cd}$ empower the model to restrict specific caregivers from executing particular moves along the planning horizon.

Parameters

R_{qivd}	:	required number of caregivers with qualification level q for visit v to patient i on day d, $q \in \mathcal{Q}$, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}_d$
T_{ij}	:	time needed to travel from patient i to patient j, $i, j \in \mathcal{P}$
T''_i	:	time required to travel between the operating center and patient $i \in \mathcal{P}$
E_{ivd}	:	service duration required for node (i, v) on day d, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}_d$
G_{ivd}	:	minimum time required between the end time of visit v to patient i and the start time of the next visit to the same patient on day d, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}_d : \lvert\mathcal{V}_{id}\rvert \geq 2$
A_{ivd}	:	earliest time for starting visit v to patient i on day d, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}_d$
B_{ivd}	:	latest time for starting visit v to patient i on day d, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}_d$
$\overrightarrow{I}_s$	:	begin time of shift $s \in \mathcal{S}$
$\overleftarrow{I}_s$	:	end time of shift $s \in \mathcal{S}$
L	:	shift length
J_{csd}	:	1 if caregiver c is scheduled to work a regular shift in shift s on day d, 0 otherwise,

$c \in \mathcal{C}$, $s \in \mathcal{S}$, $d \in \mathcal{D}$

W_{csd} : 1 if caregiver c is scheduled to work a regular shift in shift s on day d, 2 otherwise, $c \in \mathcal{C}$, $s \in \mathcal{S}$, $d \in \mathcal{D}$

K_{cq} : 1 if caregiver c has qualification level q, 0 otherwise, $c \in \mathcal{C}$, $q \in \mathcal{Q}$

O_c : small positive number representing the time priority for caregiver $c \in \mathcal{C}$. The domain of these parameters is specified as follows: $O_c \in \left(0, \frac{1}{L|\mathcal{C}||\mathcal{S}||\mathcal{D}|}\right)$ for all $c \in \mathcal{C}$.

H_{qi} : maximum number of different caregivers with qualification level q allowed to visit patient i during the planning horizon, $q \in \mathcal{Q}$, $i \in \mathcal{P}$,

Decision variables

y_{csd} : 1 if caregiver c is assigned to a route in shift s on day d, 0 otherwise, $c \in \mathcal{C}$, $s \in \mathcal{S}$, $d \in \mathcal{D}$

$\overrightarrow{x}_{csivd}$: 1 if node (i, v) on day d is the first visited node in the route of caregiver c in shift s (i.e. the caregiver moves directly from the operating center to this node), 0 otherwise, $c \in \mathcal{C}$, $s \in \mathcal{S}$, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}''_{cd}$

$x_{csivjud}$: 1 if nodes (i, v) and (j, u) on day d are visited consecutively by caregiver c in shift s, 0 otherwise, $c \in \mathcal{C}$, $s \in \mathcal{S}$, $d \in \mathcal{D}$, $(i, v, j, u) \in \mathcal{M}_{cd}$

$\overleftarrow{x}_{csivd}$: 1 if node (i, v) on day d is the last visited node in the route of caregiver c in shift s before the return to the operating center, 0 otherwise, $c \in \mathcal{C}$, $s \in \mathcal{S}$, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}''_{cd}$

z_{csivd} : 1 if node (i, v) on day d is visited by caregiver c during shift s, 0 otherwise, $c \in \mathcal{C}$, $s \in \mathcal{S}$, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}''_{cd}$

t_{ivd} : begin time of visit v to patient i on day d, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}_d$

$\overrightarrow{t}_{csd}$: start time of the route performed by caregiver c in shift s on day d, $c \in \mathcal{C}$, $s \in \mathcal{S}$, $d \in \mathcal{D}$

$\overleftarrow{t}_{csd}$: end time of the route performed by caregiver c in shift s on day d, $c \in \mathcal{C}$, $s \in \mathcal{S}$, $d \in \mathcal{D}$

In addition, the following auxiliary variables are needed to tackle the care continuity restrictions.

f_{cid} : 1 if caregiver c visits patient i during day d, 0 otherwise, $c \in \mathcal{C}$, $i \in \mathcal{P}$, $d \in \mathcal{D}$

h_{ci} : 1 if caregiver c visits patient i during the planning horizon, 0 otherwise, $c \in \mathcal{C}$, $i \in \mathcal{P}$

To clarify the possible work status for each caregiver in every shift throughout the planning horizon, we illustrate the relationship between parameters J_{csd} and variables y_{csd} in Figure 5.

Figure 5: Possible work status for each caregiver in every shift throughout the planning horizon.

if $J_{csd} = 1$ and $y_{csd} = 1$ →	Caregiver c works a regular shift in shift s on day d and is allocated to a route during that shift.
if $J_{csd} = 1$ and $y_{csd} = 0$ →	Caregiver c works a regular shift in shift s on day d and is devoted to emergencies during that shift.
if $J_{csd} = 0$ and $y_{csd} = 1$ →	Caregiver c works an additional shift in shift s on day d and is allocated to a route during that shift.
if $J_{csd} = 0$ and $y_{csd} = 0$ →	Caregiver c is off during shift s on day d.

Objective function

$$\min \sum_{c\in\mathcal{C}} \sum_{s\in\mathcal{S}} \sum_{d\in\mathcal{D}} \Big(W_{csd} y_{csd} + O_c(\overleftarrow{t}_{csd} - \overrightarrow{t}_{csd}) \Big) \tag{1}$$

The objective function (1) expresses the minimization of two parts. The first part represents the number of scheduled routes during the planning horizon. To minimize the number of additional shifts and maximize the allocation of caregivers to emergencies, a fixed cost is associated with assigning each caregivers to a route. Specifically, variables y_{csd} are multiplied by the corresponding parameters W_{csd} to ensure that the cost of assigning a route to a caregiver during a regular shift is 1, during an additional shift is 2, and the cost of devoting a caregiver to emergencies during a regular shift is 0. The second part denotes the total route lengths over the planning horizon. To ensure optimal allocation of resources, the sum of route lengths for each caregiver is multiplied by the corresponding time priority coefficient O_c. This last parameter may be used to prioritize the assignment of caregivers to routes. Caregivers with lower O_c will be more often used in routes while caregivers with higher O_c will be more often reserved for emergencies. The optimization of this function is subject to a set of constraints, which are categorized into groups as follows.

Routing constraints

$$y_{csd} - \sum_{(i,v)\in\mathcal{N}''_{cd}} \overrightarrow{x}_{csivd} = 0 \qquad c\in\mathcal{C}, s\in\mathcal{S}, d\in\mathcal{D} \tag{2}$$

$$z_{csivd} - \overrightarrow{x}_{csivd} - \sum_{(j,u,i,v)\in\mathcal{M}_{cd}} x_{csjuivd} = 0 \qquad c\in\mathcal{C}, s\in\mathcal{S}, d\in\mathcal{D}, (i,v)\in\mathcal{N}''_{cd} \tag{3}$$

$$z_{csivd} - \overleftarrow{x}_{csivd} - \sum_{(i,v,j,u)\in\mathcal{M}_{cd}} x_{csivjud} = 0 \qquad c\in\mathcal{C}, s\in\mathcal{S}, d\in\mathcal{D}, (i,v)\in\mathcal{N}''_{cd} \tag{4}$$

$$\sum_{c\in\mathcal{C}''_q} \sum_{s\in\mathcal{S}} z_{csivd} = R_{qivd} \qquad q\in\mathcal{Q}, d\in\mathcal{D}, (i,v)\in\mathcal{N}''_{cd} \tag{5}$$

Constraints (2)–(4) represent the flow of caregivers within the network. These constraints resemble a multi-commodity flow problem, where each operator is linked to a specific commodity and is restricted to traveling within the sub-graph defined by the nodes they are authorized to serve. Specifically, constraints (2) ensure that when a caregiver is assigned to a route during a shift, the caregiver must begin the route from the operating center. Constraint (3) enforces that a caregiver visiting a node must travel either from the operating center or from another node. Constraint (4) ensures that after a caregiver visits a node, that caregiver must either proceed to another node or return to the operating center. Constraints (5) confirm that each scheduled node is visited by the required number of caregivers of each qualification level.

Time constraints

$$
\begin{aligned}
& t_{jud} \geq (t_{ivd} + E_{ivd} + T_{ij}) - (B_{ivd} + E_{ivd} + T_{ij})(1 - x_{csivjud}) && c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D}, (i,v,j,u) \in \mathcal{M}_{cd} && (6)\\
& t_{ivd} \geq (\overrightarrow{t}_{csd} + T''_i) - (\overleftarrow{I}_s + T''_i)(1 - \overrightarrow{x}_{csivd}) && c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D}, (i,v) \in \mathcal{N}''_{cd} && (7)\\
& \overleftarrow{t}_{csd} \geq (t_{ivd} + E_{ivd} + T''_i) - (B_{ivd} + E_{ivd} + T''_i)(1 - \overleftarrow{x}_{csivd}) && c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D}, (i,v) \in \mathcal{N}''_{cd} && (8)\\
& t_{ivd} \geq A_{ivd} && d \in \mathcal{D}, (i,v) \in \mathcal{N}_d && (9)\\
& t_{ivd} \leq B_{ivd} && d \in \mathcal{D}, (i,v) \in \mathcal{N}_d && (10)\\
& t_{i,v+1,d} \geq t_{ivd} + E_{ivd} + G_{ivd} && i \in \mathcal{P}, d \in \mathcal{D}, v \in \mathcal{V}_{id} \setminus |\mathcal{V}_{id}| : |\mathcal{V}_{id}| \geq 2 && (11)\\
& \overrightarrow{t}_{csd} \geq \overrightarrow{I}_s y_{csd} && c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D} && (12)\\
& \overleftarrow{t}_{csd} \leq \overleftarrow{I}_s y_{csd} && c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D} && (13)
\end{aligned}
$$

Constraints (6)–(9) define the start and end time of each route and the start service time at each node. They also enforce service synchronization for visits that require multiple caregivers. Specifically, constraints (6) link the start time of visit u to patient j on day d with the end time of visit v to patient i on that day when a caregiver moves directly from node (i, v) to node (j, u). Constraints (7) connect the start time of visit v to patient i on day d with the departure time of caregiver c when the caregiver travels directly from the operating center to that node. Thus, the start service time at every node is determined by the arrival time of each caregiver to that node. Constraints (8) ensure that the end time of the route conducted by caregiver c in shift s on day d is at least equal to the end time of the service conducted at the last node plus the travel time required to return to the operating center. Constraints (9) and (10) define the time window for each visit to every patient on each day. Constraints (11) enforce the time interval required by a patient between two consecutive visits within a day. Constraints (12)–(13) ensure that each route assigned to a caregiver during a shift starts and ends within the shift's time window. These constraints also guarantee zero route length when a caregiver is devoted to emergencies or is off during a shift.

Labor regulation constraints

$$J_{csd} + y_{csd}(1 - J_{csd}) + J_{c,s+1,d} + y_{c,s+1,d}(1 - J_{c,s+1,d}) \leq 1 \qquad c \in \mathcal{C}, s \in \mathcal{S} \setminus \{|\mathcal{S}|\}, d \in \mathcal{D} : |\mathcal{S}| \geq 2 \quad (14)$$

$$J_{c,|\mathcal{S}|,d} + y_{c,|\mathcal{S}|,d}(1 - J_{c,|\mathcal{S}|,d}) + J_{c,1,d+1} + y_{c,1,d+1}(1 - J_{c,1,d+1}) \leq 1 \quad c \in \mathcal{C}, d \in \mathcal{D} \setminus \{|\mathcal{D}|\} : |\mathcal{S}|, |\mathcal{D}| \geq 2 \quad (15)$$

Constraints (14) ensure that caregivers do not work two consecutive shifts within a single day. Constraints (15) are equivalent to constraints (14) but address situations where the two consecutive shifts fall on two consecutive days (e.g., Monday evening and Tuesday morning).

Care continuity constraints

$$\sum_{v \in \mathcal{V}_{id}} \sum_{s \in \mathcal{S}} z_{csivd} \leq |\mathcal{V}_{id}| f_{cid} \qquad c \in \mathcal{C}, i \in \mathcal{P}, d \in \mathcal{D} \quad (16)$$

$$h_{ci} \geq f_{cid} \qquad c \in \mathcal{C}, i \in \mathcal{P}, d \in \mathcal{D} \quad (17)$$

$$\sum_{c \in \mathcal{C}''_q} h_{ci} \leq H_{qi} \qquad q \in \mathcal{Q}, i \in \mathcal{P} \quad (18)$$

Constraints (16) enforce the variable f_{cid} to be 1 if caregiver c visits patient i on day d. Constraints (17) ensure that the variable h_{ci} equals 1 if the variable f_{cid} is equal to 1 for any day during the planning horizon. Constraints (18) guarantee that the total number of different caregivers of qualification level q visiting patient i during the planning horizon does not exceed the maximum allowed.

Variable domains

$$y_{csd} \in \{0, 1\} \qquad c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D} \quad (19)$$

$$\overrightarrow{x}_{csivd} \in \{0, 1\} \qquad c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D}, (i, v) \in \mathcal{N}''_{cd} \quad (20)$$

$$x_{csivjud} \in \{0, 1\} \qquad c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D}, (i, v, j, u) \in \mathcal{M}_{cd} \quad (21)$$

$$\overleftarrow{x}_{csivd} \in \{0, 1\} \qquad c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D}, (i, v) \in \mathcal{N}''_{cd} \quad (22)$$

$$z_{csivd} \in \{0, 1\} \qquad c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D}, (i, v) \in \mathcal{N}''_{cd} \quad (23)$$

$$t_{ivd} \geq 0 \qquad d \in \mathcal{D}, (i, v) \in \mathcal{N}_d \quad (24)$$

$$\overrightarrow{t}_{csd} \geq 0 \qquad c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D} \quad (25)$$

$$\overleftarrow{t}_{csd} \geq 0 \qquad c \in \mathcal{C}, s \in \mathcal{S}, d \in \mathcal{D} \quad (26)$$

$$f_{cid} \in \{0, 1\} \qquad c \in \mathcal{C}, i \in \mathcal{P}, d \in \mathcal{D} \quad (27)$$

$$h_{ci} \in \{0, 1\} \qquad c \in \mathcal{C}, i \in \mathcal{P} \quad (28)$$

5 Extended formulation

The model formulated in Section 4 describes the problem at hand. However, as will be demonstrated in Section 6, it exhibits some shortcomings when solved using a commercial solver employing the branch-and-bound method. To enhance the model's tightness and reduce the search space, we propose an extension to the model supplemented by two sets of valid inequalities.

5.1 Caregiver restrictions based on visit requirements

In this section, we expand the model by introducing additional constraints that serve two purposes. First, they determine the minimum number of caregivers assigned to routes in every shift throughout the planning horizon. Furthermore, they ensure that each patient is assigned the necessary number of caregivers along the planning horizon. The constraints are formulated based on the required number of caregivers and their qualification levels for each scheduled visit. The generation of these constraints requires introducing the following auxiliary notation.

α_{sivd} : 1 if the time window of node (i, v) on day d allows it to be visited during shift s on that day, 0 otherwise, $s \in \mathcal{S}$, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}_d$

β_{sivd} : 1 if node (i, v) on day d is decided to be visited during shift s, 0 otherwise, $s \in \mathcal{S}$, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}_d$

Formally, parameters α_{sivd} are calculated as follows:

$$\alpha_{sivd} = \begin{cases} 1 & \text{if } (A_{ivd} + E_{ivd} + T''_i \leq \overleftarrow{I}_s) \text{ and } (B_{ivd} \geq \overrightarrow{I}_s + T''_i) \\ 0 & \text{otherwise}, \quad s \in \mathcal{S},\ d \in \mathcal{D},\ (i, v) \in \mathcal{N}_d \end{cases} \tag{29}$$

Expressions (29) guarantee that node (i, v) on day d can be visited during shift s if two conditions are met. First, the earliest possible starting time of that node allows caregivers to conduct the service and return to the operating center before the shift's end time. Second, caregivers coming directly from the operating center can arrive at the patient location before the latest possible starting time of that node.

Now, the additional constraints are formulated as follows:

$$\sum_{c \in \mathcal{C}''_q} y_{csd} f_{cid} \geq R_{qivd} \beta_{sivd} \qquad q \in \mathcal{Q}, s \in \mathcal{S}, d \in \mathcal{D}, (i, v) \in \mathcal{N}_d \tag{30}$$

$$\sum_{s \in \mathcal{S}} \beta_{sivd} = 1 \qquad d \in \mathcal{D}, (i, v) \in \mathcal{N}_d \tag{31}$$

$$\beta_{sivd} \leq \alpha_{sivd} \qquad s \in \mathcal{S}, d \in \mathcal{D}, (i, v) \in \mathcal{N}_d \tag{32}$$

$$\beta_{sivd} \in \{0, 1\} \qquad s \in \mathcal{S}, d \in \mathcal{D}, (i, v) \in \mathcal{N}_d \tag{33}$$

Constraints (30) ensure that if node (i, v) on day d is scheduled during shift s, there must be a minimum number of caregivers with qualification level q working that shift and assigned to the patient. This minimum number corresponds to the required number of caregivers with that qualification level for that node. Constraints (31) guarantee that every node is scheduled during a single shift. Constraints (32) state that node (i, v) on day d can be visited during shift s if the time window of that node allows. Constraints (33) define the binary nature of variables β_{sivd}.

Constraints (30) are non-linear due to a product of two binary variables. To linearize these constraints, we replace the product $(y_{csd} f_{cid})$ by a new binary variable denoted as μ_{csid}. The new variable is then defined by generating the following constraints.

$$\mu_{csid} \leq f_{cid} \qquad c \in \mathcal{C}, s \in \mathcal{S}, i \in \mathcal{P}, d \in \mathcal{D} \tag{34}$$

$$\mu_{csid} \leq y_{csd} \qquad c \in \mathcal{C}, s \in \mathcal{S}, i \in \mathcal{P}, d \in \mathcal{D} \tag{35}$$

$$\mu_{csid} \in \{0, 1\} \qquad c \in \mathcal{C}, s \in \mathcal{S}, i \in \mathcal{P}, d \in \mathcal{D} \tag{36}$$

Constraints (34)–(36) dictate that variable μ_{csid} can equal 1 if both h_{cid} and y_{csd} are also equal to 1.

5.2 Caregiver restrictions based on travel and service times

In continuation of the constraints outlined in Section 5.1, we present a set of valid inequalities designed to determine the minimum number of caregivers for each qualification level assigned to routes throughout the planning horizon. These inequalities are introduced based on shift length, travel times, and visit duration. The development of these inequalities requires the following auxiliary notation.

- δ_{sivd} : 1 if node (i, v) on day d must be visited during shift s, 0 otherwise, $s \in \mathcal{S}$, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}_d$
- θ_{sivd} : 1 if node (i, v) on day d can be visited during shift s or $s + 1$, 0 otherwise, $s \in \mathcal{S} \setminus \{|\mathcal{S}|\}$, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}_d : |\mathcal{S}| \geq 2$
- $\mathcal{F}_{qsd}$: set of patients that must be visited by caregivers of qualification level q during shift s on day d, $q \in \mathcal{Q}$, $s \in \mathcal{S}$, $d \in \mathcal{D}$
- χ_{qivd} : set of patients requiring visits from caregivers with qualification level q on day d with the condition that these visits can occur prior to visit v to patient i on that day, $q \in \mathcal{Q}$, $d \in \mathcal{D}$, $(i, v) \in N_d : R_{qivd} \geq 1$
- $\vec{\omega}_{qsd}$: time required to travel from the operating center to the nearest patient that should be visited by caregivers of qualification level $q \in \mathcal{Q}$ in shift $s \in \mathcal{S}$ on day $d \in \mathcal{D}$
- ϕ_{qivd} : minimum travel time required for a caregiver of qualification level q to travel toward node (i, v) on day d, $q \in \mathcal{Q}$, $d \in \mathcal{D}$, $(i, v) \in \mathcal{N}_d : R_{qivd} \geq 1$

For clarity, expressions (37) illustrate the relationship between three auxiliary parameters: α_{sivd}, δ_{sivd}, and θ_{sivd}.

$$\text{if } \sum_{s\in\mathcal{S}} \alpha_{sivd} = \begin{cases} 1 & \rightarrow \theta_{sivd} = 0 \text{ and } \delta_{sivd} = \begin{cases} 1 & \text{if } \alpha_{sivd} = 1 \\ 0 & \text{otherwise}, \quad s \in \mathcal{S},\ d \in \mathcal{D},\ (i,v) \in \mathcal{N}_d \end{cases} \\ 2 & \rightarrow \delta_{sivd} = 0 \text{ and } \theta_{sivd} = \begin{cases} 1 & \text{if } \alpha_{sivd} + \alpha_{s+1,i,v,d} = 2 \\ 0 & \text{otherwise}, \quad s \in \mathcal{S} \setminus \{|\mathcal{S}|\},\ d \in \mathcal{D},\ (i,v) \in \mathcal{N}_d : |\mathcal{S}| \geq 2 \end{cases} \end{cases} \tag{37}$$

Parameters $\overrightarrow{\psi}_{qsd}$ and ϕ_{qivd} are calculated as follows:

$$\text{if } |\mathcal{F}_{qsd}| \geq 1 \text{ then } \overrightarrow{\omega}_{qsd} = \min_{i\in\mathcal{F}_{qsd}} (T''_i) \text{ else } \overrightarrow{\omega}_{qsd} = 0 \qquad q \in \mathcal{Q}, s \in \mathcal{S}, d \in \mathcal{D} \tag{38}$$

$$\text{if } R_{qivd} \geq 1 \text{ then } \phi_{qivd} = \min_{j\in\chi_{qivd}} (T_{ji}, T''_i) \text{ else } \phi_{qivd} = 0 \qquad q \in \mathcal{Q}, d \in \mathcal{D}, (i,v) \in \mathcal{N}_d \tag{39}$$

Now, the valid inequalities can be formulated as follows:

$$\sum_{c\in\mathcal{C}''_q} y_{csd} \geq \left\lceil \frac{\overrightarrow{\omega}_{qsd} + \sum_{(i,v)\in\mathcal{N}''_{cd}} (\delta_{sivd} \cdot R_{qivd} \cdot (\phi_{qivd} + E_{ivd}))}{L} \right\rceil \qquad q \in \mathcal{Q}, s \in \mathcal{S}, d \in \mathcal{D} \tag{40}$$

Constraints (40) specify a lower bound on the number of caregivers with qualification level q that must be assigned to routes during shift s on day d. This is based on the minimum travel and service time required to conduct the required visits during that shift.

$$\sum_{c\in\mathcal{C}''_q} (y_{csd} + y_{c,s+1,d}) \geq \left\lceil \frac{\overrightarrow{\omega}_{qsd} + \overrightarrow{\omega}_{q,s+1,d} + \sum_{(i,v)\in\mathcal{N}''_{cd}} ((\delta_{sivd} + \delta_{s+1,i,v,d} + \theta_{sivd}) \cdot R_{qivd} \cdot (\phi_{qivd} + E_{ivd}))}{L} \right\rceil$$

$$q \in \mathcal{Q}, s \in \mathcal{S} \setminus \{|\mathcal{S}|\}, d \in \mathcal{D} : |\mathcal{S}| \geq 2 \tag{41}$$

Constraints (41) are equivalent to constraints (40) but they consider determining the minimum number of caregivers required for each pair of consecutive shifts.

5.3 Routing restrictions

In this section, we generate a set of valid inequalities to determine the minimum sum of route lengths for each shift within the planning horizon. Additionally, logical constraints are formulated to constrain the search space for the routing part in the model. These inequalities are formulated as follows.

$$\sum_{c\in\mathcal{C}''_q}(\overleftarrow{t}_{csd}-\overrightarrow{t}_{csd})\geq\sum_{(i,v)\in\mathcal{N}''_{cd}}(\delta_{sivd}\cdot R_{qivd}\cdot(\phi_{qivd}+E_{ivd}))\qquad q\in\mathcal{Q},s\in\mathcal{S},d\in\mathcal{D}\tag{42}$$

$$\sum_{c\in\mathcal{C}''_q}(\overleftarrow{t}_{csd}-\overrightarrow{t}_{csd}+\overleftarrow{t}_{c,s+1,d}-\overrightarrow{t}_{c,s+1,d})\geq$$
$$\sum_{(i,v)\in\mathcal{N}''_{cd}}((\delta_{sivd}+\delta_{s+1,i,v,d}+\theta_{sivd})\cdot R_{qivd}\cdot(\phi_{qivd}+E_{ivd}))\qquad q\in\mathcal{Q},s\in\mathcal{S}\setminus\{|\mathcal{S}|\},d\in\mathcal{D}:|\mathcal{S}|\geq 2\tag{43}$$

Constraints (42) ensure that the sum of route lengths for caregivers with qualification level q during shift s on day d is at least equal to the minimum time required for nodes that must be visited during that shift and necessitate caregivers of that skill level. Constraints (43) are equivalent to constraints (42) but they consider the minimum sum of route lengths for each pair of consecutive shifts.

$$\sum_{(i,v,j,u)\in\mathcal{M}_{cd}}\sum_{s\in\mathcal{S}}\left(x_{csivjud}+\overleftarrow{x}_{csivd}\right)\leq 1\qquad c\in\mathcal{C},d\in\mathcal{D},(i,v)\in\mathcal{N}''_{cd}\tag{44}$$

$$\sum_{(j,u,i,v)\in\mathcal{M}_{cd}}\sum_{s\in\mathcal{S}}\left(x_{csjuivd}+\overrightarrow{x}_{csivd}\right)\leq 1\qquad c\in\mathcal{C},d\in\mathcal{D},(i,v)\in\mathcal{N}''_{cd}\tag{45}$$

$$\sum_{(i,v)\in\mathcal{N}''_{cd}}z_{csivd}\leq\sum_{(i,v)\in\mathcal{N}''_{cd}}R_{qivd}y_{csd}\qquad q\in\mathcal{Q},c\in\mathcal{C}''_q,s\in\mathcal{S},d\in\mathcal{D}\tag{46}$$

Constraints (44) ensure that caregiver c can move from node (i,v) on day d to another node or to the operating center at most once throughout all shifts. Similarly, constraints (45) guarantee that caregiver c on day d can travel toward node (i,v) at most once along all shifts. Constraints (46) confirm that caregiver c can be assigned to nodes during shift s on day d only if that caregiver is assigned to a route during that shift.

6 Computational study

This section provides an overview of the test instances utilized in the experiments, along with their corresponding computational results and interpretations. Two models are considered in the study: the original model (1–28) and the extended model (1–46). All experiments have been performed on a computer with a 1.90 GHz Intel i7-8350U CPU, 64 GB of RAM. The models were coded in Python and solved using Gurobi 10.0.0. A time limit of 3600 seconds was imposed for each experiment.

6.1 Test instances description

Due to the novelty of the proposed problem, no benchmark instances were available. Therefore, we generated twenty instances based on inputs obtained from three healthcare centers in Norway. These instances were designed to closely resemble real-life problems in municipalities of different sizes.

In general, the instances vary in terms of the planning horizon (1 to 7 days), the number of shifts (2 or 3), the number of caregivers (4 to 15), the number of qualification levels (2 or 3), the number of patients (15 to 75), and the number of visits (20 to 300). Each instance is associated with a unique name, starting with the letter A if the number of daily shifts is two and the letter B if the number of shifts is three, followed by the number of days, caregivers, qualification levels, patients, and visits required during the planning horizon. Table 2 provides more details for the twenty instances.

Table 2: Test instances

Instance name	$\|\mathcal{S}\|$	$\|\mathcal{D}\|$	$\|\mathcal{C}\|$	$\|\mathcal{Q}\|$	$\|\mathcal{P}\|$	No. of visits	Avg. T_{ij} (min)
A-1-4-2-15-20	2	1	4	2	15	20	14.36
B-1-6-2-20-30	3	1	6	2	20	30	17.12
A-2-6-2-15-30	2	2	6	2	15	30	14.89
A-2-6-3-20-50	2	2	6	3	20	50	13.93
B-2-8-2-25-75	3	2	8	2	25	75	19.45
A-3-6-2-20-50	2	3	6	2	20	50	18.87
A-3-6-2-25-75	2	3	6	2	25	75	15.96
A-3-8-3-30-100	2	3	8	3	30	100	16.40
A-3-8-3-35-150	2	3	8	3	35	150	20.13
B-3-10-2-40-150	3	3	10	2	40	150	18.68
A-5-8-2-30-75	2	5	8	2	30	75	17.46
A-5-8-2-35-100	2	5	8	2	35	100	15.56
A-5-10-3-40-150	2	5	10	3	40	150	14.58
A-5-10-3-50-200	2	5	10	3	50	200	17.60
B-5-12-2-60-200	3	5	12	2	60	200	19.55
A-7-10-2-40-150	2	7	10	2	40	150	14.81
A-7-12-3-50-200	2	7	12	2	50	200	18.70
A-7-12-3-60-250	2	7	12	3	60	250	18.48
A-7-15-3-75-300	2	7	15	3	75	300	17.64
B-7-15-2-75-300	3	7	15	2	75	300	15.73

In all instances and corresponding parameters, minute-based time units are used to accurately represent temporal aspects. For instances with two shifts per day, each shift lasts eight hours. The first shift spans from 480 minutes (8:00) to 960 minutes (16:00), followed by the second shift from 960 minutes (16:00) to 1440 minutes (24:00). Similarly, instances with three shifts per day have each shift lasting six hours. The first shift starts at 360 minutes (06:00) and ends at 720 minutes (12:00), followed by the second shift from 720 minutes (12:00) to 1080 minutes (18:00), and finally, the last shift spans from 1080 minutes (18:00) to 1440 minutes (24:00).

Three qualification levels are considered in the instances: nurses, assistants, and health workers. Each instance includes at least two of these qualifications, evenly distributed among the caregivers. Moreover, caregivers are assigned to shifts under the assumption that each caregiver is scheduled to work at least one shift during the planning horizon and at most one shift per day. In every instance, a complete network is considered where all patients are connected with each other and with the operating center.

Travel times between patients and between the operating center and each patient, measured in minutes, are calculated based on the Euclidean distance between randomly selected locations. However, the observed travel time between two locations ranges from a minimum of two minutes to a maximum of 30 minutes. The average travel time between two locations in each instance is presented in the last column of Table 2. The remaining parameters for each instance were randomly generated based on the obtained inputs, as detailed in Table 3.

Table 3: Inputs for generating parameter values

Parameter	Value	Parameter	Value
$\lvert\mathcal{V}_{id}\rvert$	$rand\{0, 1, 2, 3\}$	J_{csd}	$rand\{0, 1\}$
R_{qivd}	$rand\{1, 2, 3\}$	K_{cq}	$rand\{0, 1\}$
E_{ivd}	$rand\{5, 15, 30, 45, 60\}$	H_{qi}	$rand\{2, 3, 4\}$
A_{ivd}	$rand\{360 \text{ or } 480, +30, ..., 1380\}$	B_{ivd}	$A_{ivd} + rand\{30, 60, 90, 120\}$

Based on the obtained inputs, we assumed that the percentage of daily visits requiring more than one caregiver varies between 15% and 20% in each instance. Moreover, an equivalent percentage of daily visits have time windows spanning two shifts. The objective function coefficients are defined to establish a hierarchy among the multiple objectives. Specifically, parameter W_{csd} is set to 1 if caregiver c is scheduled to work during shift s on day d, 2 otherwise. Thus, minimizing the number of additional shifts is prioritized over maximizing the allocation of caregivers to emergencies. Additionally, coefficient O_c is assigned a value of 0.00001 for all caregivers to ensure the sum of route lengths remains below 1. Consequently, the objective function value for each instance comprises two parts: an integer value denoting the number of routes and a non-integer value expressing the sum of route lengths.

6.2 Computational time analysis

This section evaluates the computational time required to solve the test instances. Table 4 presents the lower bound (LB), upper bound (UB), optimality gap, and computational time for each instance solved using both the original and extended models.

As shown in Table 4, the extended model exhibits better computational efficiency compared to the original model. Specifically, the extended model solved instances A-1-4-2-15-20, B-1-6-2-20-30, A-2-6-2-15-30, and A-2-6-3-20-50 in 1, 10, 2, and 10 seconds, respectively, while these instances required 2, 44, 14, and 122 seconds, respectively, using the original model. Furthermore, the extended model achieved optimality when solving instances A-3-6-2-20-50, A-3-8-3-30-100, B-3-10-2-40-150, A-5-8-2-30-75, and A-7-10-2-40-150 in 12, 1465, 3531, 12616, and 1375 seconds, respectively. In contrast, the original model failed to prove optimality for these instances within the proposed time limit. For instances A-3-8-3-35-150, A-5-10-3-40-150, A-5-10-3-50-200, B-5-12-2-60-200, A-7-12-3-60-250, A-7-15-3-75-300, and B-7-15-2-75-300, the extended model successfully found feasible solutions, whereas the original model failed within the proposed time limit. For instances that could be solved using both models, the upper

Table 4: Results comparison of the original and extended model

	Original model				Extended model			
Instance	LB	UB	GAP	Time (s)	LB	UB	GAP	Time (s)
A-1-4-2-15-20	4.01297	4.01297	0.00%	2	4.01297	4.01297	0.00%	1
B-1-6-2-20-30	12.01757	12.01757	0.00%	44	12.01757	12.01757	0.00%	10
A-2-6-2-15-30	8.02480	8.02480	0.00%	14	8.02480	8.02480	0.00%	2
A-2-6-3-20-50	11.02411	11.02411	0.00%	122	11.02411	11.02411	0.00%	10
B-2-8-2-25-75	17.01201	20.03596	15.10%	3600	19.02611	19.03581	0.05%	3600
A-3-6-2-20-50	14.01535	16.02848	12.56%	3600	16.02848	16.02848	0.00%	12
A-3-6-2-25-75	12.02616	15.04299	20.06%	3600	15.03776	15.04298	0.03%	3600
A-3-8-3-30-100	20.04068	20.05080	0.05%	3600	20.05080	20.05080	0.00%	1465
A-3-8-3-35-150	17.02841	-	-	3600	26.05965	26.07409	0.06%	3600
B-3-10-2-40-150	38.05157	38.07662	0.07%	3600	38.07662	38.07662	0.00%	3531
A-5-8-2-30-75	19.36220	21.04529	7.99%	3600	21.04529	21.04529	0.00%	1216
A-5-8-2-35-100	24.03771	28.05091	14.31%	3600	28.04020	28.04887	0.03%	3600
A-5-10-3-40-150	23.03202	-	-	3600	36.06648	36.07343	0.02%	3600
A-5-10-3-50-200	19.03139	-	-	3600	39.07378	39.08226	0.02%	3600
B-5-12-2-60-200	46.34824	-	-	3600	55.06800	61.10129	9.87%	3600
A-7-10-2-40-150	28.02662	33.07966	11.14%	3600	32.07928	32.07928	0.00%	1375
A-7-12-3-50-200	45.06266	49.13078	8.28%	3600	49.11251	49.13059	0.04%	3600
A-7-12-3-60-250	31.77639	-	-	3600	54.10598	54.12271	0.03%	3600
A-7-15-3-75-300	32.53326	-	-	3600	56.12348	56.15371	0.05%	3600
B-7-15-2-75-300	66.47156	-	-	3600	66.14055	74.17168	10.83%	3600

bound was found faster when the extended model was used. For example, the best upper bound for instance A-3-8-3-30-100 was achieved in 25 seconds using the extended model, while the same upper bound was obtained in 150 seconds using the original model.

6.3 Feasibility and optimality gap analysis

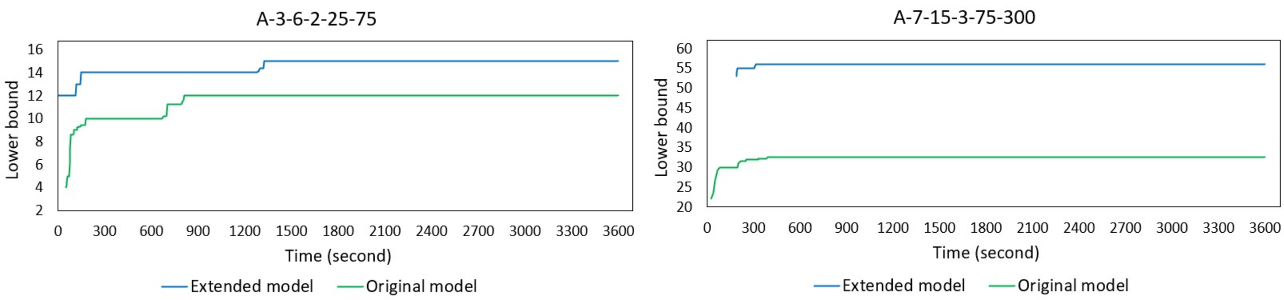
In this section, we compare the performance of the original model and the extended model in terms of feasibility and optimality gap.

Table 4 illustrates that the original model yielded proven optimal solutions for four instances, feasible solutions without achieving optimality for nine instances, and failed to provide any feasible solutions for seven instances within the imposed time limit. In contrast, the extended model provided proven optimal solutions for nine instances and feasible solutions without achieving optimality for the remaining instances. Furthermore, the extended model outperformed the original model in solving five instances, either by reducing the number of required routes (as observed in instance B-2-8-2-25-75) or by decreasing the total route lengths (as evident in instance A-5-8-2-35-100).

For seven out of nine instances where the original model could not achieve optimality, the absolute optimality gaps exceeded 1, with relative optimality gaps ranging from 7.99% to 20.06%. Hence, the original model failed to achieve optimality for both the number of caregivers assigned to routes and the total route lengths for these instances. In contrast, for nine out of eleven instances where the extended model failed to provide proven optimal solutions, the absolute optimality gaps were less than one, with relative optimality gaps ranging from 0.02% to 0.05%. This implies that the extended model succeeded in proving the optimal number of caregivers assigned to routes but failed to prove the optimality of the sum of route lengths for these instances. For the remaining two instances (B-5-12-2-60-200 and B-7-15-2-75-300), the optimality gaps exceeded 1. This indicates that the extended model encountered more difficulties when tackling instances involving three shifts per day during the planning horizon.

Thus, the model's extension led to notable improvements in solving instances, as evidenced by the increased number of instances solved to optimality and the ability to solve additional instances within the imposed time limit. These enhancements can be primarily attributed to the impact of the additional constraints on the problem bounds and search space. Specifically, the introduction of the first and second sets of constraints (denoted as S1 and S2, respectively) raised the initial lower bound for the number of caregivers assigned to routes. Additionally, the third set of additional constraints (denoted as S3) increased the initial lower bound for the sum of route lengths and reduced the solution space. Figure 6 presents two diagrams illustrating the variations in lower bounds when solving two instances using both the original and extended models. In each diagram, the green line represents the lower bound obtained with the original model and the blue line depicts the lower bound obtained when the three sets of additional constraints were incorporated.

Figure 6: Comparison of lower bounds for two instances solved using the original and extended models



To further analyze the impact of the additional constraints, we conducted experiments by removing each set of constraints individually from the extended model and then running the model for each instance. Table 5 presents the upper bound and optimality gap for each instance in each experiment.

Table 5 shows that removing the first additional constraint set, presented in Section 5.1, led to a worse upper bound or an increase in the optimality gap for eight instances. Similarly, removing the second set of additional constraints, presented in Section 5.2, resulted in a worse solution in ten instances. In contrast, the exclusion of the third set of additional constraints, presented in Section 5.3, had a more significant impact, affecting solving fifteen out of the twenty instances.

Table 5: Results obtained when testing the effect of removing each additional set of constraints

Instance	Extended model without S1		Extended model without S2		Extended model without S3	
	UB	GAP	UB	GAP	UB	GAP
A-1-4-2-15-20	4.01297	0.00%	4.01297	0.00%	4.01297	0.00%
B-1-6-2-20-30	12.01757	0.00%	12.01757	0.00%	12.01757	0.00%
A-2-6-2-15-30	8.02480	0.00%	8.02480	0.00%	8.02480	0.00%
A-2-6-3-20-50	11.02411	0.00%	11.02411	0.00%	11.02411	0.00%
B-2-8-2-25-75	20.03628	15.10%	19.03581	0.05%	19.03581	0.07%
A-3-6-2-20-50	16.02848	0.00%	16.02848	0.00%	16.02848	0.04%
A-3-6-2-25-75	15.04298	6.67%	15.04298	0.03%	15.04298	20.1%
A-3-8-3-30-100	20.05080	0.00%	20.05080	0.00%	20.05080	0.05%
A-3-8-3-35-150	26.07421	0.06%	26.07486	0.06%	-	-
B-3-10-2-40-150	40.07455	4.99%	38.07662	0.07%	38.07667	0.09 %
A-5-8-2-30-75	21.04529	0.00%	21.04529	0.01%	21.04529	0.03%
A-5-8-2-35-100	28.04887	3.59%	28.04887	0.03%	28.04887	0.06%
A-5-10-3-40-150	36.07343	0.02%	36.07343	0.02%	-	-
A-5-10-3-50-200	39.08228	0.02%	39.08232	0.02%	-	-
B-5-12-2-60-200	-	-	-	-	-	-
A-7-10-2-40-150	32.07928	0.00%	32.07928	0.02%	32.07928	0.11%
A-7-12-3-50-200	49.13059	0.04%	49.13059	0.04%	49.13085	0.13%
A-7-12-3-60-250	54.12277	1.86%	54.12301	0.03%	-	-
A-7-15-3-75-300	56.15428	3.63%	59.15429	5.13%	-	-
B-7-15-2-75-300	74.17264	10.91%	-	-	-	-

6.4 Solution structure analysis

In this section, we present further details about the solution structure for each instance, along with an examination of the cost of care continuity service. Table 6 presents the solution structure for each instance under care continuity restrictions, including the number of additional required shifts, the number of times caregivers are devoted to emergencies, the number of generated routes, the sum of route lengths, and the total traveling and waiting time as a percentage of the sum of route lengths.

The findings presented in Table 6 indicate a considerable variation in the solutions obtained across different instances. This variation can be attributed to the differences in the sizes of the instances. The number of additional required shifts varies from 0 to 12 and the number of times caregivers are devoted to emergencies ranges from 0 to 20. Similarly, the range of generated routes extends from 4 to 66, with total route lengths varying between 1297 to 17168 minutes. Furthermore, the cumulative travel and waiting times constitute between 23.44% and 40.77% of the sum of route lengths. These percentages are high compared to those in previous studies and may be explained by the number of different constraints that each route must satisfy in the problem being tackled.

The proposed resource allocation approach demonstrates its effectiveness in generating feasible solu-

Table 6: Instance solution structures when the care continuity restrictions are enforced

Instance	N.Ext	N.Emerg	N.Routes	L.Routes	T.W.Time
A-1-4-2-15-20	0	0	4	1297	32.15%
B-1-6-2-20-30	3	0	9	1757	34.78%
A-2-6-2-15-30	0	1	8	2480	31.60%
A-2-6-3-20-50	1	2	10	2411	23.44%
B-2-8-2-25-75	3	4	16	3581	39.48%
A-3-6-2-20-50	3	0	14	2848	38.91%
A-3-6-2-25-75	1	4	14	4298	30.38%
A-3-8-3-30-100	0	4	20	5080	32.45%
A-3-8-3-35-150	4	1	22	7409	40.77%
B-3-10-2-40-150	4	5	34	7662	39.65%
A-5-8-2-30-75	0	8	21	4529	35.80%
A-5-8-2-35-100	3	9	25	4887	31.35%
A-5-10-3-40-150	3	2	33	7343	28.53%
A-5-10-3-50-200	6	5	33	8226	35.55%
B-5-12-2-60-200	12	13	49	10129	38.08%
A-7-10-2-40-150	0	16	32	7928	27.22%
A-7-12-3-50-200	1	20	48	13059	38.78%
A-7-12-3-60-250	7	19	47	12271	38.55%
A-7-15-3-75-300	2	18	54	15371	37.39%
B-7-15-2-75-300	8	17	66	17168	29.48%

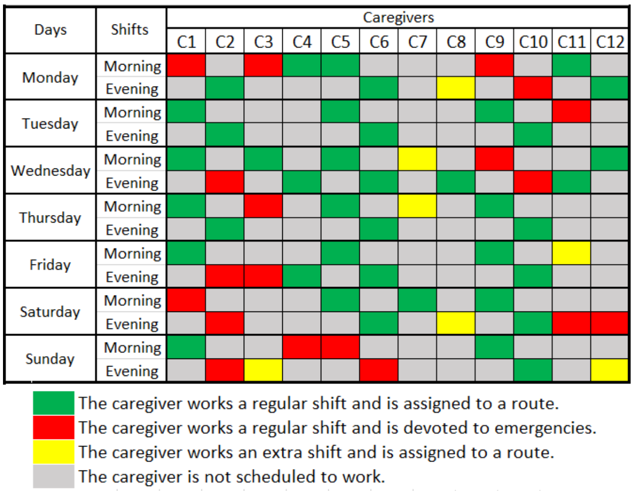
N.Ext: Number of extra shifts. N.Emerg: Number of times caregivers are devoted to emergencies. N.Routes: Number of routes. L.Routes: Sum of route lengths, in minutes, along the planning horizon. T.W.Time: Total traveling and waiting time as a percentage of the sum of route lengths.

tions. For example, during the evening shift of the seventh day in instance A-7-12-3-60-250, a nurse and an assistant are devoted to emergencies, while an additional nurse and a health worker are scheduled to work an extra shift during that period. This allocation is necessary because there are insufficient available health workers on that shift to meet the demand. Additionally, despite the availability of another nurse, a nurse is scheduled for an extra shift to adhere to continuity of care restrictions. Figure 7 presents the caregivers' work schedule for instance A-7-12-3-60-250, illustrating which caregivers are assigned to routes, devoted to emergencies, and scheduled for extra shifts along the planning horizon.

To investigate the impact of care continuity restrictions on solutions, we conducted additional experiments by excluding Equations (16)–(18) from the extended model. Table 7 presents the solution structure for each instance without enforcing the care continuity restrictions. Additionally, the changes in solution structure compared to Table 6 are indicated in parentheses.

Table 7 shows that excluding the care continuity constraints leads to modifications in the solution structure for 15 out of 20 instances. These alterations appear as a reduction in the number of additional shifts, an increase in the allocation of caregivers to emergencies, a decrease in the sum of route lengths, or a combination thereof. For example, ignoring care continuity constraints in instance B-7-15-2-75-300 leads to a decrease in the number of additional shifts by 2, an increase in the number of times caregivers

Figure 7: Caregivers' work schedule for instance A-7-12-3-60-250



are devoted to emergencies by 3, a decrease in the total number of routes by 5, and an increase in the sum of route lengths by 550 minutes. It was also observed that some patients are scheduled to be visited by four different nurses during the planning horizon, whereas only two different nurses are assigned when the care continuity restrictions are imposed. However, the deterioration in care continuity varies among patients within each instance and also differs from one instance to another.

7 Conclusions

Effective planning of human resources is critical in designing an efficient home healthcare system. In this work, we studied a staffing, routing, and scheduling problem inspired by home health care operations in Norway. The proposed problem considers a multi-day planning horizon where each day is divided into shifts and a set of caregivers are scheduled to work in every shift. In addition, patients require varying numbers of daily visits with different requirements. The study aimed to achieve three objectives: minimizing the number of additional shifts, maximizing the allocation of caregivers to emergencies, and minimizing the sum of route lengths over the planning horizon. These objectives were optimized hierarchically while considering a set of restrictions, including time windows, skill matching, synchronicity, care continuity, and labor regulations. To tackle the problem, we introduced a mixed-integer linear programming model. The model was then extended and two sets of valid inequalities were integrated to enhance its tightness. Both the original and extended models were tested on a set of 20 instances. The computational experiments demonstrate the efficiency of the extended model in achieving optimality and finding feasible solutions for a greater number of instances compared to the original model. Specifically, incorporating the extension increased the number of instances solved to optimality from 4 to 9 and produced feasible solutions for 7 instances that for which no feasible solution was found with the original model within the proposed time limit.

Table 7: Instance solution structures when the care continuity restrictions are disregarded

Instance	N.Ext	N.Emerg	N.Routes	L.Routes	T.W.Time
A-1-4-2-15-20	0 (0)	0 (0)	4 (0)	1297 (0)	32.15% (0.00%)
B-1-6-2-20-30	3 (0)	0 (0)	9 (0)	1658 (-99)	30.68% (-4.10%)
A-2-6-2-15-30	0 (0)	1 (0)	8 (0)	2480 (0)	31.60% (0.00%)
A-2-6-3-20-50	1 (0)	2 (0)	10 (0)	2265 (-146)	18.12% (-5.32%)
B-2-8-2-25-75	1 (-2)	3 (-1)	15 (-1)	3484 (-97)	37.35% (-2.13%)
A-3-6-2-20-50	3 (0)	0 (0)	14 (0)	2570 (-278)	31.49% (-7.42%)
A-3-6-2-25-75	1 (0)	4 (0)	14 (0)	4298 (0)	30.38% (0.00%)
A-3-8-3-30-100	0 (0)	6 (+2)	18 (-2)	4991 (-89)	30.42% (-2.03%)
A-3-8-3-35-150	3 (-1)	1 (0)	21 (-1)	7282 (-127)	38.09% (-2.68%)
B-3-10-2-40-150	1 (-3)	4 (-1)	32 (-2)	7026 (-636)	33.82% (-5.83%)
A-5-8-2-30-75	0 (0)	8 (0)	21 (0)	4529 (0)	35.80% (0.00%)
A-5-8-2-35-100	3 (0)	9 (0)	25 (0)	4887 (0)	31.35% (0.00%)
A-5-10-3-40-150	3 (0)	2 (0)	33 (0)	7015 (-328)	24.49% (-4.04%)
A-5-10-3-50-200	3 (-3)	4(-1)	31 (-2)	7938 (-288)	32.15% (-3.40%)
B-5-12-2-60-200	8 (-4)	15 (+2)	43 (-6)	10949 (+820)	42.23% (+4.15%)
A-7-10-2-40-150	0 (0)	17 (+1)	31 (-1)	7966 (+38)	28.10% (+0.88%)
A-7-12-3-50-200	0 (-1)	19 (-1)	48 (0)	13059 (0)	38.78% (0.00%)
A-7-12-3-60-250	6 (-1)	18 (-1)	47 (0)	12271 (0)	38.55% (0.00%)
A-7-15-3-75-300	1 (-1)	20 (+2)	51 (-3)	15121 (-250)	35.04% (-2.35%)
B-7-15-2-75-300	6 (-2)	20 (+3)	61 (-5)	17718 (+550)	31.71% (+2.23%)

N.Ext: Number of extra shifts. N.Emerg: Number of times caregivers are devoted to emergencies. N.Routes: Number of routes. L.Routes: Sum of route lengths, in minutes, along the planning horizon. T.W.Time: Total traveling and waiting time as a percentage of the sum of route lengths.

However, there are still some limitations to this work. The primary challenge lies in the computational time required to solve larger and more complex instances. In further experiments, we noticed that the extended model failed to provide any feasible solutions within the proposed time limit for instances A-3-8-3-35-150, B-5-12-2-60-200, A-7-12-3-60-250, A-7-15-3-75-300, and B-7-15-2-75-300 when tighter care continuity restrictions were imposed. Additionally, it failed to generate feasible solutions for larger instances that had a greater number of visits unless the time limit was increased. Other limitations that could impede the practical application of the proposed expanded model pertain to overtime work and workload balance. Within our instance solutions, we observed cases where a caregiver is assigned to work an additional shift solely to conduct one visit. While overtime for home care providers in Norway is typically limited to entire additional shifts, it might be feasible to assign limited overtime hours instead, as seen in much of the previous literature on home health care in other countries. Furthermore, it was observed in most instances that certain caregivers are devoted to emergencies or assigned to additional shifts more than others. This can be attributed to the impact of care continuity restrictions on staffing decisions. Therefore, while considering workload balance among caregivers can enhance the model's applicability, tackling it as a hard constraint may increase the likelihood of infeasibility.

For future work, we propose exploring the potential of utilizing column generation and logic-based Benders decomposition for solving the proposed problem. These two-stage methods have demonstrated effectiveness in similar optimization problems. Additionally, investigating the continuity of care from a rolling horizon perspective can provide a valuable extension to our study. Lastly, considering caregivers' preferences, such as workload balancing and willingness to work additional shifts, can increase the applicability of our work.

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